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RESPONSE OF STRUCTURES
TO DYNAMIC LOADING

by

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Norman/Jones

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Abstract

This survey opens with some introductory comments on the response of structures subjected to large dynamic loads which cause plastic behaviour. The rigid plastic method of analysis is then introduced, together with a few observations on the limitations and advantages of this simplified procedure. This is followed by a discussion of recent studies into the influence of transverse shear and rotatory inertia on the dynamic plastic stable response of beams and circular plates. The dynamic plastic buckling behaviour of various structural members and an idealised model is then examined in section 4, and the review closes with a discussion and some concluding remarks.

1. Introduction

This article focuses on the response of structural members subjected to dynamic loads, or sudden displacements, which are sufficiently severe to cause plastic flow of the material and permanent displacements or structural damage. This topic has received considerable attention, particularly in recent years, and is discussed in textbooks, (Goldsmith 1960, Johnson 1972) and review articles (Rawlings (1963, 1971), Symonds 1967, Krajcinovic 1973, Lee 1974, Baker 1975, Jones (1975, 1978), Ross et al.1977), including one by Rawlings (1974) at the last Oxford Conference. Nevertheless, a greater understanding of this subject is required in order to provide engineers and designers with more information so that they can properly design structures to withstand effectively the severe dynamic loads which arise in a variety of practical situations.

Standard static methods of analysis with dynamic magnification factors, for example, are not adequate in many dynamic plastic structural problems, as demonstrated by the static and dynamic tests on idealised buses conducted by Lowe et al. (1972). The roof of the bus shown in Figure 7 of Lowe et al. (1972) (see also plate 25 in Johnson 1972) essentially collapsed as an Euler column when subjected to a static longitudinal load, while the front end was crushed severely under a dynamic longitudinal load with little damage to the roof elsewhere. An analysis of a bus subjected to static longitudinal loads might therefore be of little

relevance to the design of a crashworthy vehicle. On the other hand, there are practical structures which respond in a quasi-static manner when subjected to dynamic loads and might, therefore, be studied using the methods of static plastic analysis. This circumstance may occur when an external dynamic transverse load acts on a beam or plate, for example, with a duration which is long compared to the corresponding natural period of elastic vibration. It is straightforward to demonstrate this observation for a simple one-degree-of-freedom elastic spring mass system (Biggs 1964) when subjected to pressure pulses and it has been shown by Jones (1973, 1976a) that static methods of plastic analysis do provide surprisingly good agreement with the results of slamming damage experiments conducted on ships and catamarans. It has also been suggested that quasi-static methods of plastic analysis should suffice for predicting the structural damage sustained during many types of ship collisions (Jones 1976b). Notwithstanding these comments, there are numerous practical situations in which a dynamic plastic analysis is, unfortunately, unavoidable.

A transverse dynamic load on a structural member causes stress waves to propagate through the thickness as well as producing an overall structural response. The propagation of stress waves through the structural thickness can cause failure by spalling, or delamination of composite structures, when the shock or impact loads are sufficiently severe. This phenomenon occurs in the same order of time that it takes a stress wave to propagate through the thick-

ness (H), which is approximately $H(\rho/E)^{1/2}$ and $H(\rho/E_t)^{1/2}$ for elastic and elastic-linear work-hardening materials, respectively. Thus, this type of failure usually occurs within microseconds of initial impact and is sometimes referred to as "early time response" to distinguish it from the gross structural behaviour which occurs at later times. It is customary practice to uncouple the early time wave propagation behaviour from the long time or gross structural response because the time durations of these two phenomena usually differ by a few orders of magnitude. The early time response of structural members and stress wave propagation is not considered further but has been discussed by Kolsky (1953), Goldsmith (1960), Johnson (1972), Hopkins (1974) and others, including a review by Kolsky (1974) at the last Oxford Conference and various articles on the topic at this Conference. Thus, this article focuses on the long-term behaviour (typically of the order of milliseconds) of structures for which the external dynamic load is assumed to impart momentum instantaneously to the middle surface of a structure (i.e., transverse wave propagation is disregarded).

The gross inelastic structural response of fully clamped beams which are subjected to transverse impulsive velocities has been examined by Menkes and Opat (1973) and Jones (1976c). Menkes and Opat observed that within a certain range of impulsive velocities the beams responded with large inelastic deformations which they called a mode I ductile response. A mode II behaviour which involves tearing (tensile fracture) of the beam material at the sup-

ports eventually occurs when the impulsive velocity reaches a sufficiently large value. At still higher velocities a mode III response occurs due to transverse shear failure of the beam material at the supports. It appears likely that similar types of failure modes exist for other kinds of structures. However, due to the paucity of theoretical and experimental information on mode II and mode III responses, most of this article out of necessity is restricted to a discussion of mode I behaviour which involves large inelastic deformations of structures. Incidentally, the perforation of structures has been discussed by Goldsmith (1960), Cristescu (1967), Johnson (1972), Backman and Goldsmith (1978) and others.

This review concentrates largely on theoretical methods of analysis and experimental results for the mode I stable plastic response and the dynamic plastic buckling of structures. It might be remarked at this juncture that simple theoretical methods are extremely useful for the preliminary stages of design and might even be adequate for the final design in many cases. In fact, rigid-plastic methods of analysis have proved remarkably accurate in those situations in which the underlying assumptions of the theory are recognised and satisfied, as remarked in section 2. The overall response features and major conclusions can be reached for many problems using quite simple calculations: a situation familiar for the static plastic behaviour of structures. This provides insight into the structural problem

which might permit a designer to make a quick and sensible choice between various competing designs. If more accurate results are required for the final design, or additional information which is not predicted by an analytical method, then the dimensions of the preliminary design might be used as input in a numerical scheme (e.g., Witmer et al., 1963, Leech et al., 1968, Morino et al., 1971, Hashmi et al., 1972, Wu 1974, Ni and Lee 1974, etc.). In this sense, the analytical and numerical approaches are both necessary, complementary, and together provide a powerful design tool.

One difficulty associated with all theoretical and numerical studies on the dynamic inelastic behaviour of structures lies with establishing the characteristics of the dynamic loads. Another difficulty is related to the paucity of information on the constitutive equations for materials, especially in the dynamic regime. It has been remarked by Jones (1974) that the multi-dimensional constitutive equations are usually constructed according to the properties observed during uniaxial tests. Moreover, the form of the multi-dimensional constitutive equations for elastic-plastic materials is not yet clear, even for static problems as observed recently by Hunsaker et al. (1976). This shortcoming is compounded for dynamic problems because little experimental information exists for strain rate history effects or combined loadings (Campbell (1970, 1973), Schapery et al. 1978, Senseny et al. 1978, etc.).

It is not possible to provide a conspectus of the entire field of dynamic plasticity in this article. Fortunately, however, many aspects have been adequately examined in previous review articles, as remarked earlier, so that only a few topics of more recent interest will therefore be discussed. A general discussion of the rigid-plastic idealisation and concomitant accuracy is given in the next section, which is followed in section 3 by a review of recent investigations into the influence of transverse shear and rotatory inertia effects on the dynamic plastic response of structures. The important subject of dynamic plastic buckling is then taken up in section 4 and the article closes with a discussion and some concluding remarks.

2. Rigid-Plastic Idealisation

2.1 Introduction

The rigid-plastic idealisation for the static behaviour of structures is well known and the techniques of analysis have been incorporated successfully into many design procedures and design codes (Massonnet and Save 1965, Horne 1971, Plastic Design in Steel 1971, etc.). The dynamic plastic response of structures might also be examined using rigid-plastic methods of analysis which lead to significant simplifications for many problems and yet provide surprisingly accurate predictions when the limitations of the procedure are respected. Several features of the rigid-plastic method for dynamic problems are discussed in the remainder of this section.

2.2 Material Elasticity

In its simplest form, a rigid-plastic method employs a rigid perfectly plastic constitutive equation which disregards both material elasticity and strain hardening. The neglect of material elasticity appears reasonable provided the energy ratio is larger than about 3 (Bodner and Symonds, 1962), when the energy ratio is defined as the external dynamic energy divided by the maximum amount of strain energy which can be absorbed by a structure in a wholly elastic manner. An estimate of the elastic strain energy when the entire structure just reaches the yield stress is

usually made, rather than an elastic calculation, to obtain the maximum amount of elastic strain energy absorbed by a structure. This is a very simple calculation (e.g., appendix in Jones et al., 1972), which generally leads to conservative energy ratios as observed by Jones and Guedes Soares (1978) for beams. Nevertheless, material elasticity might be important for external pressure pulses with durations which are not short compared to the natural period of elastic vibration (Symonds 1967) and when the energy ratio is so small that the dynamic loads do not cause extensive plastic behavior of the material. Indeed, there exists a threshold dynamic load for a given problem below which the response is entirely elastic.

Recently, Forrestal et al., (1976, 1977, 1978) have developed a simple, surprisingly accurate and therefore attractive design procedure for incorporating the approximate influence of material elasticity on the dynamic plastic response of beams. An exact elastic analysis is first undertaken for a dynamic beam problem which remains valid until the maximum stress reaches yield. If the beam material is strain rate sensitive, then this yield stress is calculated from the Cowper-Symonds constitutive law (equation (1) here), using the corresponding strain rate predicted by the elastic analysis. The subsequent plastic behaviour is controlled by a constant yield stress, a simplification introduced by Perrone (1965). Various other assumptions were introduced in this work, particularly when catering for material strain hardening, but Forrestal et al. appear

to have captured the principal features of behaviour because of the excellent agreement with the peak displacements recorded during experiments on simply supported 1018 steel, 304 stainless steel, and aluminum 6061 T6 beams.

2.3 Material Strain Hardening

Material strain hardening is not important for moderate strains or permanent transverse deflections of beams unless a material hardens significantly. Symonds and Jones (1972) estimated the correction required in a rigid perfectly plastic analysis to account for strain hardening and found that it was relatively unimportant compared to other effects in an impulsively loaded mild steel beam with a transverse displacement equal to seven times the beam thickness. This conclusion is also supported by the theoretical analysis of Jones (1968) on an annular plate and by Wu (1974), who obtained excellent agreement between his finite-element predictions without strain hardening and the experimental results of Jones et al. (1970) on aluminum 6061 T6 rectangular plates. It might be expected intuitively that the presence of material strain hardening would always reduce the final displacements of a structure. However, this might not always occur, because the increase of strength could cause a transfer of energy from one mode of deformation to another mode of deformation which is a less efficient absorber of energy.

2.4 Strain Rate Sensitivity

If the constitutive equations of a material are dependent on the rate of straining, then a rigid perfectly plastic approximation might require further refinement. Hot rolled mild steel, for example, is notoriously strain rate sensitive since the yield stress is double the corresponding "static" yield stress at a strain rate of 40 sec^{-1} , approximately (e.g., Marsh and Campbell 1963, Symonds 1965, Campbell and Cooper 1966, etc.). This phenomenon should, therefore, be catered for in the basic equations which govern the behaviour of structural members made from mild steel. On the other hand, aluminum 6061 T6 is essentially strain rate insensitive at the usual strain rates encountered in practice.

Various features of the phenomenon of strain rate sensitivity are being explored actively by many research groups around the world, as reported at the last and current Oxford Conferences. However, Cowper and Symonds (Symonds 1965, 1967) found that the simple empirical expression

$$\frac{\sigma_y'}{\sigma_y} = 1 + \left(\frac{\dot{\epsilon}}{D}\right)^{1/p} \quad (1)$$

with $D = 40.4 \text{ sec}^{-1}$, and $p = 5$, which is a special case of the more general relations of Perzyna (1966), provided a reasonable estimate of the dynamic flow stress (σ_y') recorded during many dynamic uniaxial tensile and compressive constant strain rate ($\dot{\epsilon}$) tests on mild steel, where σ_y is the associated static yield stress. The values

$D = 6500 \text{ sec}^{-1}$ and $p = 4$ are commonly used for aluminum alloy, but there is some uncertainty about these particular values because of conflicting test data, as remarked by Jones (1974). Comparisons with test data reveal that equation (1), with $D = 120 \text{ sec}^{-1}$ and $p = 9$, describes the constant strain rate behaviour of alpha-titanium (Ti-50A) (Symonds and Chon 1974), while $D = 100 \text{ sec}^{-1}$ and $p = 10$ is suggested by the data recorded on 304 stainless steel (Forrestal and Sagartz 1978).

Aspden and Campbell (1966) integrated equation (1) through the thickness (H) of a beam using the usual assumptions of beam theory and found that the dynamic bending moment (M') is related to the associated curvature rate ($\dot{\kappa}$) according to the expression

$$\frac{M'}{M_0} = 1 + \frac{2p}{2p+1} \left(\frac{\dot{\kappa}H}{2D} \right)^{1/p}, \quad (2)$$

where $M_0 = \sigma_y H^2/4$ is the static collapse moment. The dynamic bending moments for plastic flow predicted by equation (2) provide a reasonable average measure of the much more complicated behaviour observed in the corresponding test results on mild steel beams. The test results exhibited more pronounced upper yield strengths than the corresponding uniaxial tests and are ignored by equations (1) and (2).

Equation (1) and its generalised counterpart

$$\frac{\sigma_e'}{\sigma_y} = 1 + \left(\frac{\dot{\epsilon}_e}{D} \right)^{1/p}, \quad (3)$$

where $\sigma_e' = (3S'_{ij}S'_{ij}/2)^{1/2}$ and $\dot{\epsilon}_e = (2\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}/3)^{1/2}$, and equation (2) do not, of course, reflect the complicated strain rate sensitive behaviour of materials such as strain rate history effects discussed by Campbell (1973) and others. Nevertheless, the Cowper-Symonds constitutive relation and its various derivative forms, including the linear case, when $p = 1$ (see also equations (19) - (25) in Symonds and Jones 1972 for the combined influence of bending moment and axial force), are used almost exclusively in theoretical and numerical studies on the dynamic plastic behaviour of structures made from strain rate sensitive materials. The almost universal acceptance of this equation stems from the observation that analytical and numerical predictions agree remarkably well with experimental tests on beams (Bodner and Symonds 1962, Symonds and Jones 1972, Forrestal and Sagartz 1978, etc.).

2.5 Finite-Deflections

If the dynamic loads produce a stable response in a structural member with transverse deflections larger than the corresponding thickness, approximately, then the influence of finite-deflections, or geometry changes, can introduce in-plane or membrane forces which might exercise an important influence on the structural behaviour. This phenomenon is particularly important for axially restrained beams and cylindrical shells and for circular and rectangular plates

subjected to transverse dynamic loads.

Simple theoretical rigid-plastic methods which retain the influence of geometry changes have been developed for beams and plates (Jones 1971) and shells (Walters and Jones 1972) and have provided reasonable estimates of the available experimental results. In order to illustrate the simplicity of this theoretical method and its potential value for design purposes, the particular case of a fully clamped strain rate insensitive rectangular plate subjected to an impulsive velocity (V_0) is now briefly discussed. Jones (1971) used a rigid perfectly plastic analysis, which included the influence of geometry changes, to predict the maximum transverse displacement (W_m) at the plate centre

$$\frac{W_m}{H} = \frac{(3 - \xi_0) \{ (1 + \Gamma)^{1/2} - 1 \}}{2 \{ 1 + (\xi_0 - 1)(\xi_0 - 2) \}}, \quad (4)$$

where

$$\Gamma = \frac{\lambda \beta^2}{6} (3 - 2\xi_0) \left(1 - \xi_0 + \frac{1}{2 - \xi_0} \right), \quad \beta = \frac{B}{L},$$

$$\xi_0 = \beta \{ (3 + \beta^2)^{1/2} - \beta \}, \quad \lambda = \frac{4\rho V_0^2 L^2}{\sigma_y H^2}$$

and $2L$, $2B$, and H are the plate length, width and thickness, respectively, and σ_y is the static uniaxial tensile yield stress. Equation (4) with $\beta \rightarrow 0$ predicts the behaviour of a beam with a span $2B$. Thus,

$$\frac{W_m}{H} = \frac{1}{2} \left\{ \left(1 + \frac{3\lambda}{4} \right)^{1/2} - 1 \right\} \quad (5)$$

when relabelling the beam span as $2L$ rather than $2B$ so that λ is still defined as above.

It is evident from Figure 1 that equation (5) provides reasonable agreement with the maximum permanent transverse displacements recorded on strain rate insensitive beams (aluminum 6061 T6), while equation (4) compares favourably with test results obtained from aluminum 6061 T6 rectangular plates with various aspect ratios in the range $0.25 \leq \beta \leq 1.00$, as indicated in Figure 2 for $\beta \approx 3/5$ (Jones and Baeder 1972, Jones 1975). The same theoretical rigid-plastic procedure has been used to generate simple formulae for fully clamped rectangular plates subjected to rectangular and triangular shaped pressure-time histories, which arise in a number of practical situations (Jones 1971, 1973). In addition, theoretical results have been reported for the simply supported case, but the method of Jones (1971) is sufficiently simple that many more beam and arbitrarily shaped plate problems are amenable to analysis.

Symonds and Jones (1972) examined the simultaneous influence of geometry changes and material strain rate sensitivity on the dynamic plastic response of fully clamped beams loaded impulsively. It transpired that surprisingly accurate predictions for the influence of strain rate sensitivity in this particular finite-deflection problem were obtained by using a modified yield stress $n\sigma_y$, where

$$n = 1 + \left\{ \frac{H^2 \lambda^{3/2}}{48 DL^3} \sqrt{\frac{\sigma_y}{\rho}} \right\}^{1/p} \quad (6)$$

Thus, equation (5) with λ/n instead of λ is used to estimate the maximum permanent transverse displacement of a fully clamped beam made from a strain rate sensitive material as indicated in Figure 1.

The permanent transverse displacements of the impulsively loaded fully clamped beams presented in Figure 1 indicate the relative importance of material strain rate sensitivity and geometry changes. It is clear that a classical infinitesimal rigid-plastic theory is totally inadequate for this particular problem, when the deflections are larger than the beam thickness. The theoretical predictions of equation (5) and equation (5) with σ_y replaced by $0.618\sigma_y$ for an inscribing yield criterion, provide a simple way of bounding a more exact theory developed by Jones (1971) and give acceptable bounds for the experimental results recorded on the strain rate insensitive beams (aluminum 6061 T6). The theoretical results of equations (5) and (6), which retain the simultaneous influence of geometry changes and material strain rate sensitivity, agree quite well with the corresponding predictions according to the numerical scheme of Witmer et al. (1963) and the experimental results on the strongly strain rate sensitive mild steel beams. It is evident from Figures 1 and 2 that geometry changes exercise a dominant effect on the structural response, while material strain rate sensitivity causes a further significant reduction of the permanent transverse displacements of the mild steel beams.

2.6 Pseudo-Shakedown

It was remarked by Jones (1973) that a type of "pseudo-shakedown" is possible in a rigid perfectly plastic structure which undergoes geometry changes. If a dynamic pressure pulse with a peak value (p_{eq}) and a short duration acts on a structure, then the maximum permanent transverse displacement (W_1) might, under certain circumstances, be smaller than the maximum permanent transverse displacement (W_s) predicted by a finite-deflection rigid-plastic analysis for the same peak load (p_{eq}) supported statically. Now, the static load carrying capacity of a deformed structure with an initial transverse deflection (W_1) is p_1 (say), where $\bar{p}_c < p_1 < p_{eq}$, and \bar{p}_c is the corresponding static rigid-plastic collapse load for infinitesimal deflections. Thus, if the same dynamic load, with a peak value p_{eq} , were to be repeated, then the initially deformed rigid perfectly plastic structure would not commence to deform further until the external load reached the new static load carrying capacity p_1 . The maximum permanent transverse displacement W_2 (say) might still be less than W_s when the loading duration was sufficiently short. It is evident that this process could continue until the permanent set or damage from repeated dynamic impacts with the same peak value (p_{eq}) and duration eventually equalled the quasi-static value (W_s). The structure would then have reached a "pseudo-shakedown" state and, according to a rigid perfectly plastic analysis, would not then deform for further

repetitions of the same pressure pulse.

Yuhara (1975) conducted some impact tests on rectangular plates in order to simulate the bow damage sustained by large ships having full form as described by Jones (1977). Some of these experimental results are presented in Figure 3. The saturated values in Figure 3 were recorded by Yuhara after repeated impacts on the rectangular plates and are similar to the theoretical predictions according to a static rigid plastic theory, which retains the influence of geometry changes (Jones 1977). Thus, the experimental results of Yuhara (1975) provide a practical demonstration of the phenomenon of "pseudo-shakedown" suggested by Jones (1973) and discussed above.

2.7 Concluding Remarks

This section provides a brief overview of the rigid-plastic method for estimating the dynamic plastic response of structures. Many more experimental investigations, theoretical studies and successful applications, which are far too numerous to discuss here, have been obtained by various groups. However, various aspects of the dynamic plastic response of structures are explored in the reviews by Rawlings (1963, 1971, 1974), Symonds (1967), Johnson (1972), Krajcinovic (1973), Baker (1975), Jones (1975, 1978) and Ross et al. (1977), while the various theorems in dynamic plasticity are discussed by Wierzbicki (1970), Martin (1972), Symonds and Chon (1974), Maier and Corradi (1974), Wierzbicki (1975), and others.

3. Influence of Transverse Shear Force and Rotatory Inertia

It is well known that sufficiently large transverse shear forces might influence the static plastic behaviour of beams, and, to account for this effect, several theoretical and empirical formulae have been developed and incorporated in various design procedures and codes (Horne 1971, Plastic Design in Steel 1971, Neal 1977). The transverse shear force according to a classical (bending only) theory is initially infinite at the supports of a simply supported beam immediately after loading with an impulsive velocity distributed uniformly over the entire span. However, by way of contrast, the transverse shear force at the supports of a statically loaded beam is finite because it must be in equilibrium with the total external transverse load. This situation is responsible for the observation that transverse shear forces exercise a more important influence on the response of dynamically loaded rigid-plastic beams than on the static plastic behaviour (Symonds 1968). Indeed, it has been demonstrated experimentally by Menkes and Opat (1973), and shown theoretically by Jones (1976c), that shear failures can develop at the supports of uniform isotropic beams loaded impulsively. The numerical results of Jones and Guedes Soares (Table 1, 1978) also indicate that transverse shear forces can be significant in beams that undergo higher modal dynamic responses. Although the excitation of pure modal responses

is unlikely for most practical problems, unless deliberately activated as in a specially designed energy absorbing system, it is apparent that the behaviour of complex structures loaded dynamically might involve complicated deformation fields having some of the features of higher modal responses. Transverse shear forces also dominate the dynamic plastic response of strongly anisotropic beams (Spencer 1974, Jones 1976d).

It appears that there is considerable uncertainty in the literature on many aspects of the precise role of transverse shear forces, even for the yielding of rigid perfectly plastic beams loaded statically. Indeed it has been demonstrated that interaction curves relating bending moment (M) and transverse shear force (Q) are not proper yield curves and, as further support to this view, interaction curves for I-beams have been constructed which are not convex (Heyman 1970). Nevertheless, the finite transverse shear strength of beams loaded statically is catered for in practical design, as remarked earlier, and several formulae are available which, in the case of an I-beam, assume that the maximum transverse shear force equals the carrying capacity of the web alone.

The role of transverse shear forces on the plastic yielding of beams was examined in a recent note by Gomes de Oliveira and Jones (1978a), in which some justification was given for using convex yield curves for I-beams within the setting of engineering or classical beam theory. A

suitable compromise from an engineering viewpoint between the simple local (stress resultant) and more rigorous non-local (plane stress, plane strain) theories might be achieved for I-beams when using a local theory (e.g. Hodge 1957) with a maximum transverse shear force based only on the web area. It is evident from Figure 4 of Gomes de Oliveira and Jones (1978a) that Hodge's (1957) revised theoretical results now provide an inscribing lower bound curve in the $M/M_0 - Q/Q_0$ plane which, because of its simplicity, might be acceptable for many theoretical studies on beams. Furthermore, the theoretical predictions of Heyman and Dutton (1954) and of Ranshi, Chitkara and Johnson (1976) and others are reasonably well approximated by a square yield curve, which has been used for solving various problems in dynamic plasticity. In fact, Hodge's (1957) revised results and a square yield curve provide two simple methods for essentially bounding the actual yield curve for an I-beam.

Fortunately, it is shown in Figure 5 of Gomes de Oliveira and Jones (1978a) that a number of local and nonlocal theories give similar curves in the $M/M_0 - Q/Q_0$ plane for beams with rectangular cross-sections so, in this case, one may select whichever theory is the most convenient.

Symonds (1968) has examined the influence of transverse shear forces on the dynamic plastic response of an infinitely long beam struck by a mass travelling with an initial velocity V_0 . Symonds simplified his theoretical

work with the aid of a square yield criterion which relates the values of bending moment (M) and transverse shear force (Q) required for plastic yielding. More recently, Nonaka (1977) used a similar theoretical procedure for a beam simply supported across a span of finite length. The beam was subjected to a blast type loading (with a peak value at $t = 0$ and decreasing monotonically with time t) distributed uniformly across the entire span, and detailed theoretical results were presented for the particular cases of impulsive loading, rectangular pulse loading and exponentially decaying loading. Generally speaking, Nonaka's observations lend further support to those of Symonds in that transverse shear effects can be important for beams with non-compact cross-sections, regardless of the type of dynamic loading, while they are important for compact beams when subjected to dynamic pressures which are much larger than the corresponding static plastic collapse pressure (e.g. impulsive loading).

No restrictions were placed on the amount of shear sliding at the stationary plastic "hinges" which developed in the theoretical analyses presented by Symonds (1968) and Nonaka (1977). However, it is clear that complete severance has occurred when the amount of shear sliding equals the beam thickness, as remarked by Jones (1976c). Thus, it is necessary to ensure that this mode III type of failure does not intervene and control the response of a particular beam.

It might be shown that the transverse shear force at

the simple supports of the impulsively loaded rigid perfectly plastic circular plate examined by Wang (1955) is infinitely large when motion commences. Thus, the influence of transverse shear on the dynamic plastic response of circular plates is of potential importance and has been studied recently by Jones and Gomes de Oliveira (1978). It was assumed in this work that the transverse shear force and circumferential and radial bending moments required for plastic flow were controlled by the simplified yield criterion which was first used by Sawczuk and Duszek (1963) to examine the static plastic behaviour of circular plates. A simple theoretical procedure, which is exact within the setting of classical plasticity for the selected yield criterion, predicted the results presented in Figure 4 for a simply supported circular plate subjected to an impulsive velocity V_0 . The parameter

$$\nu = \frac{Q_0 R}{2M_0} \quad (7)$$

is a measure of the transverse shear strength of the plate cross-section relative to that in bending, where Q_0 and M_0 are the values of the transverse shear force and bending moment (per unit length) individually required for plastic collapse, and R is the plate radius. \bar{W}_1 is the dimensionless permanent transverse displacement at the plate supports, while \bar{W}_f is the dimensionless maximum permanent transverse displacement at the plate centre ($\bar{W} = 12M_0 W / \rho H V_0^2 R^2$, ρ is density, H is plate thickness). These theoretical re-

sults demonstrate that transverse shear effects are more important for dynamic loads than for static loads since Sawczuk and Duszek (1963) showed that, when $\nu \geq 3/2$, the static collapse behaviour of a simply supported circular plate loaded uniformly is independent of transverse shear effects which were retained according to the same simplified yield criterion. However, it is important to ensure that the dimensionless shear sliding at the supports (\bar{w}_1) does not cause a mode III shear failure in an actual circular plate as discussed earlier for beams (Jones 1976c, Jones and Gomes de Oliveira 1978).

It turns out that $\nu = R/H$ for the particular case of a circular plate having a solid homogeneous cross-section with $Q_0 = \sigma_y H/2$ and $M_0 = \sigma_y H^2/4$. On the other hand, if a circular plate is constructed with a sandwich cross-section, then an inner core of thickness h and a shear yield stress τ_y supports a maximum transverse shear force $Q_0 = \tau_y h$ (per unit length), while thin exterior sheets of thickness t can independently carry a maximum bending moment $M_0 = \sigma_y t(h + t)$, where σ_y is the corresponding tensile yield stress. In this circumstance equation (7) gives

$$\nu = \frac{R}{H} \frac{\tau_y}{\sigma_y/2} \left\{ \frac{h/H}{1 - (h/H)^2} \right\} \quad (8)$$

when $H = h + 2t$. Thus, a sandwich plate with $2R/H = 15$, $\sigma_y/2\tau_y = 8$ and $h/H = 0.735$ (e.g., a 0.5 in (1.27 cm) thick core with 0.1 in. (0.254 cm) sheets gives $h/H = 0.714$) gives $\nu = 1.5$, for which transverse shear effects are very impor-

tant according to the results in Figure 4. In fact, the maximum permanent transverse displacement is only two-thirds of the value predicted by a classical bending only theory. Furthermore, there is a transverse displacement of equal magnitude around the boundary, which is zero in the classical theory of Wang (1955) without transverse shear effects.

The role of rotatory inertia on the dynamic elastic response of beams and plates has been studied extensively by many authors but the corresponding dynamic plastic behaviour has only received attention recently. Jones and Gomes de Oliveira (1979) developed a theoretical procedure to explore the simultaneous influence of rotatory inertia and transverse shear forces on the dynamic plastic behaviour of beams. The behaviour of a long beam struck by a mass and an impulsively loaded simply-supported beam were examined using a square yield criterion.

The theoretical procedure of Jones and Gomes de Oliveira (1979) was used to obtain the results in Figure 5, which show the temporal variation of the energy partition for an impulsively loaded, simply supported beam of length $2L$ and mass m per unit length. The variable

$$I^2 = \frac{I_r}{mL^2} \quad (9)$$

is the dimensionless rotatory inertia, where I_r is the actual rotatory inertia for the cross-section and

$$v = \frac{Q_o L}{2M_o} \quad (10)$$

It is evident that a considerable portion of the initial kinetic energy is absorbed through shear deformations when $I = 0$ and is even greater when $I \neq 0$. The value $\nu = 1.5$ was used to obtain the results in Figure 5 and corresponds to span-to-depth ratios of 3 and 22.01 for beams with rectangular and wide-flanged I (W14 x 87) cross-sections, respectively. However, the particular results in Figure 5, which include rotatory inertia ($I = 0.1924$), are associated with a beam having a rectangular cross-section. The theoretical results in Figure 5 indicate that the incorporation of transverse shear force, as well as bending moment, in the yield criterion could exercise a significant effect on the dynamic plastic response of beams, while the influence of rotatory inertia is less important but not negligible.

It turns out that the angular rotation at the supports is particularly sensitive to the presence of the transverse shear force in the yield criterion, while the maximum permanent transverse displacements of simply supported impulsively loaded beams with $\nu \geq 3/2$ are identical to the bending only solution if $I = 0$ and up to eleven per cent. smaller when rotatory inertia is considered. The influence of rotatory inertia is less important for a long beam struck by a mass, which suggests that the effect of rotatory inertia on the dynamic plastic response of beams is sensitive to the kind of boundary conditions and the type of loading.

The theoretical analysis derived by Jones and Gomes de Oliveira (1979) was based on a simplified yield criterion relating the transverse shear force and bending moment. Gomes de Oliveira and Jones (1978b) developed a simple numerical procedure for the dynamic plastic response of beams to explore the sensitivity of the yield criterion on the influence of transverse shear and rotatory inertia. Any yield condition could be examined with this method, but the influence of the particular one due to Ilyushin-Shapiro (Shapiro 1961), which is considered exact, was explored in detail.

Typical results for a long beam struck by a mass G travelling with an initial velocity V_0 are shown in Figure 6, which reveals that consideration of the more exact Ilyushin-Shapiro yield criterion leads to a significant change in the dimensionless slope ($\bar{\theta}$) underneath the striker predicted by the much simpler analysis of Jones and Gomes de Oliveira (1979). However, a relatively marginal improvement is observed for the dimensionless maximum transverse displacements (\bar{W}). It turns out that the numerical results for an impulsively loaded simply supported beam are virtually indistinguishable from the theoretical predictions of Jones and Gomes de Oliveira (1979). Thus, it appears, from the two beam problems examined by Gomes de Oliveira and Jones (1978b), that a simple theoretical solution based on a square yield criterion is adequate for design purposes when only the maximum transverse displace-

ments are required. The retention of rotatory inertia effects and the influence of transverse shear forces in the numerical calculations using the Ilyushin-Shapiro yield condition might lead to a reduction of 10 per cent. in the slope of the impact problem, and reductions of up to 17 per cent. and 10 per cent. for the slopes and transverse displacements of the impulsive case, respectively, when compared to the numerical results for $I = 0$.

Jones and Gomes de Oliveira (1978) also explored the role of rotatory inertia on the dynamic plastic response of impulsively loaded simply supported circular plates. In this case the dimensionless rotatory inertia is

$$I = \frac{6I_r}{\mu R^2}, \quad (11)$$

where μ is the mass per unit surface area. It is evident from Figure 4 that the inclusion of rotatory inertia in the governing equations and the retention of transverse shear as well as bending effects in the yield criterion leads to an increase in the permanent transverse shear sliding at the plate supports and a decrease in the maximum final transverse displacement which occurs at the plate center. However, the inclusion of I gives rise to respective changes in these quantities of approximately 11.5% and 14.2% at most. Thus, the simpler theoretical analysis of Jones and Gomes de Oliveira (1978), with $I = 0$, would probably suffice for most practical purposes.

It was observed earlier from equation (8) that trans-

verse shear effects might be important for a circular plate with a sandwich cross-section because ν is small for practical values of R/H . On the other hand, transverse shear forces do not play an important role in the dynamic plastic response of circular plates with solid homogeneous cross-sections because equation (7) then predicts $\nu = R/H$.

The value of rotatory inertia per unit area for a solid homogeneous cross-section of thickness H and material density ρ is

$$I_r = \frac{\rho H^3}{12} \quad \text{and} \quad I = \frac{H^2}{2R^2}, \quad (12a,b)$$

when $\mu = \rho H$, while

$$I_r^s = \frac{\rho_c h^3}{12} + \frac{\rho_s t h^2}{2}, \quad \text{and} \quad I^s = \frac{1}{2R^2} \left\{ \frac{\rho_c h^3 + 6\rho_s t h^2}{\rho_c h + 2\rho_s t} \right\} \quad (13a,b)$$

with $\mu = \rho_c h + 2\rho_s t$ for the same sandwich cross-section which was used to derive equation (8), where ρ_c and ρ_s are the densities of the material in the core and outer sheets, respectively. Thus, a sandwich cross-section with $\rho_s = k\rho_c$ gives

$$\frac{I^s}{I} = \frac{(h/H)^3 + 6k(t/H)(h/H)^2}{h/H + 2k(t/H)}. \quad (14)$$

For example, a sandwich cross-section with $h = 0.5$ in. (1.27 cm), $t = 0.1$ in (0.254 cm) (i.e., $H = 0.7$ in (1.778 cm) and $k = 2$, gives $I^s/I = 0.9637$. Thus, the influence of rotatory inertia is only slightly less important for the sandwich cross-section with the parameters considered above

than for a solid homogeneous cross-section.

Generally speaking, it is evident that transverse shear effects would be more important for the dynamic plastic response of beams and plates with sandwich cross-sections, while the influence of rotatory inertia in circular plates depends on the values of the parameters in equation (14).

4. Dynamic Plastic Buckling

4.1 Introductory Comments

The work described in the previous sections focuses on the inelastic behaviour of structures which have a "stable" response when subjected to dynamic loads. However, dynamic plastic buckling or unstable behaviour, which is characterised by wrinkling as in static buckling, can occur when certain structures are subjected to large external loads. Johnson and Mamalis (1978) have discussed many examples of dynamic plastic buckling which develop during motor vehicle, train, aircraft and ship collisions. This aspect of motor vehicle crashworthiness has received a great deal of attention and has been examined by Postlethwaite and Mills (1970), Newman and Rawlings (1973), Miles (1976), McIvor et al. (1977a,b,c), Wierzbicki (1977), Wierzbicki et al. (1978) and many others. Dynamic plastic buckling might also occur in various aerospace, nuclear and petroleum engineering problems (Al-Hassani 1974, Flo-

rence and Abrahamson 1977) and in other branches of engineering. The suddenly applied dynamic loading, which occurs in most of the above practical situations, is known as pulse loading to distinguish it from oscillatory or repetitive loadings. Large oscillatory loads might also develop in certain practical situations for which information concerning a steady state response would be of primary interest. However, this section examines the transient response of simple structural members subjected to pulse loads.

Although the dynamic buckling of elastic structures has received a great deal of attention, the topic is not clearly understood as remarked by Svalbonas and Kalnins (1977), who compared various predictions for the dynamic buckling of elastic spherical caps. Indeed, it appears that the influence of initial geometrical imperfections in an elastic spherical cap has been examined only recently (Kao and Perrone 1978), even though, in 1964, Budiansky and Hutchinson demonstrated the importance of initial imperfections on the dynamic elastic response of an idealised column.

The influence of material plasticity further complicates the dynamic behaviour of structures, and Bathe et al. (1975) observed that it exercised a considerable influence on the dynamic axisymmetric response of a perfect spherical cap. Hartzman (1974) found that the dynamic

buckling load of a particular elastic-plastic perfect spherical dome subjected to a step pressure pulse of unlimited duration was larger than the corresponding static collapse pressure in contradistinction to the observations of other authors for the elastic case. However, Hartzman's finite-element numerical scheme is restricted to axisymmetric behaviour so that any possible asymmetric buckling modes are suppressed, which occurred, for example, in the experimental tests of Witmer et al. (1962) on hemispherical shells impacted by projectiles. Huffington and Wortman (1975) conducted a parametric study using a finite-difference scheme to provide some physical insight into the relative importance of various parameters on the dynamic buckling of elastic perfectly plastic fixed-ended geometrically perfect cylindrical shells subjected to frontal cosine impulsive loadings. The procedure in this particular article is suitable for generating engineering design data over a wider range of parameters than is found usually in articles based on wholly numerical schemes.

4.2 Simple Perturbation Method and Some Experimental Results

In order to simplify investigations on dynamic plastic buckling, the rigid-plastic idealisation discussed in section 2 forms the basis of a theoretical procedure which

has been used to examine the dynamic plastic buckling of flat plates subjected to axial loads (Goodier 1968, Ramsey and Vaughan 1971), cylindrical shells subjected to external dynamic pressures (Vaughan and Florence 1970) and axial loads (Florence and Goodier 1968), and the external impulsive loading of spherical shells (Jones and Ahn 1974a). However, the influence of material strain hardening was retained in all these analyses. Elasticity effects in cylindrical and spherical shells were examined by Stuiver (1965) and Jones and Ahn (1974b), respectively, while Florence (1968, 1970) and Wojewodzki (1972, 1973) investigated viscoplastic effects in cylindrical shells. The theoretical analyses in these articles and others were simplified by first obtaining the unperturbed or dominant behaviour and then considering dynamic buckling as a perturbation of this otherwise uniform motion. It was also assumed that the strains associated with the unperturbed motion dominated the strains introduced by the perturbation so that no strain reversal occurred until buckling was well developed. Unlike classical static buckling analyses, a distinct value of the dynamic load which causes structural instability is not predicted by these theoretical analyses. Rather, an expression is obtained which indicates how the displacement profile of a structure grows with time for different levels of dynamic load. Buckling is said to occur when the dynamic load reaches a threshold or critical value which is associated with the minimum unacceptable or

or maximum acceptable deformation, the magnitude of which is defined arbitrarily.

Jones and Okawa (1976) investigated the dynamic plastic buckling of a cylindrical shell made from a rigid linear strain-hardening material when subjected to a uniformly distributed, almost axisymmetric external impulsive velocity field. A particularly simple solution was found for an infinitely long cylindrical shell which offered the advantage that various characteristics of the response could be examined analytically and the approximations in the theoretical procedure studied critically.

The theoretical predictions for the dominant behaviour, critical mode numbers, and threshold impulses from all known previous studies on the dynamic plastic buckling of cylindrical shells and rings subjected to external impulsive velocities were examined by Jones and Okawa (1976). In addition, experimental results were presented from a test program on hot-rolled mild steel and aluminum 6061 T6 rings subjected to axisymmetric external impulsive velocity fields. The experimental values were compared with all known experimental results and theoretical predictions for the dynamic plastic buckling of rings and cylindrical shells. It is evident from these comparisons that the various theoretical predictions are widely divergent, some giving good agreement with the corresponding experimental values, while others are unsuitable.

Generally speaking, the simple theoretical predictions

for the permanent radial displacements of rigid perfectly plastic rings subjected to an axisymmetric velocity field (equations (28) and (33) of Jones and Okawa 1976) agree reasonably well with the permanent average radial displacements recorded in the experimental tests, provided any material strain rate sensitivity is catered for, as suggested by Perrone (1965) and demonstrated for shells by Jones (1974).

The critical mode numbers observed during the current tests are compared in Figures 9 and 10 of Jones and Okawa (1976) with the results of all previous relevant experimental investigations known. The results are reasonably consistent, notwithstanding the differences in yield stresses of the materials, experimental techniques, and despite the fact that the buckled profiles of cylindrical shells and rings are irregular. The experimental critical mode numbers tend to increase with increase in the length to radius ratio and with increase in the radius to thickness ratio.

The experimental results in Figure 11 of Jones and Okawa (1976) indicate that respect of the threshold impulses estimated by Florence and Vaughan (1968) (equation (49) of Jones and Okawa 1976) ensures that the permanent wrinkles in the deformed profiles of the rings remain small. However, the experimental results in Figure 12 reveal that the ratio of wrinkle (buckle) amplitude to average permanent radial displacement decreases with in-

crease in the impulse magnitude, while Figure 14 shows that the ratio of the wrinkle amplitude to the magnitude of the initial imperfection increases with impulse.

As mentioned earlier, the compressive membrane strain in the simple theory is assumed to dominate the tensile strains which develop as a result of bending associated with the growth of imperfections in the initial geometry and impulsive velocity distribution. Thus, the total strain is decreasing throughout the wall thickness everywhere in a structural member so that a tangent modulus formulation is valid and is largely responsible for the simplicity of the method. In order to examine the influence of strain-rate reversal, which is neglected in the simple theory, Lindberg and Kennedy (1975) used a numerical procedure to obtain the dynamic behaviour of an elastic-plastic cylindrical shell subjected to an external impulsive velocity. It transpires that the simple theory predicts reasonable estimates for the threshold impulses but overestimates the critical mode number.

Abrahamson (1974) and Florence and Abrahamson (1977) have explored the behaviour of a cylindrical shell subjected to a large constant impulsive velocity and allowed for an increase in the wall thickness as the mean radius decreases. Stability of a cylindrical shell is improved as the wall thickness increases during deformation so that a critical velocity might be found, above which the shell is stable in the sense that the amplification of imperfec-

tions in the initial geometric shape and initial velocity fields lies within acceptable limits. A shell is unstable for velocities less than the critical velocity because the stabilising influence of the increase in wall thickness during deformation is insufficient to prevent growth of the imperfections beyond acceptable values. It is interesting to note again that the general trend of the experimental results in Figure 12 of Jones and Okawa (1976) shows that the ratio of the wrinkle (buckle) amplitude to the average permanent radial displacement of an impulsively loaded ring decreases with increase in impulse.

4.3 Behaviour of Simple Models

Budiansky and Hutchinson (1964), and also Hutchinson and Budiansky (1966), and Budiansky (1966) have examined the dynamic elastic buckling characteristics of an idealised imperfection-sensitive column. It has been shown that the dynamic buckling load associated with a step load of infinite duration equals the static buckling load for a perfect model and is a decreasing fraction of the corresponding static collapse load for models with increasingly larger imperfections, but is never less than three-quarters the static collapse load for a quadratic structure. Later, Danielson (1969) refined this idealised column and retained the inertia of the prebuckling response in order to obtain theoretical predictions for dynamic elastic buckling of greater generality.

Huang and Tsai (1969) used a phase-plane procedure to investigate the dynamic snap-through behaviour of a simple shallow elastic perfectly plastic truss. Huang and Tsai neglected bending and axial inertia effects and found that the dynamic buckling load increases with increase in yield stress for materials having a small yield stress. Dynamic buckling is controlled by elastic effects when the material yield stress is large, but backward snap-through of the truss might not occur when the applied load is sufficiently large.

Hutchinson (1972) examined the static post-buckling behaviour of a simple imperfection-sensitive model which is made from an elastic linear work hardening material. This idealised model combines the essential features of Shanley's (1947) model for the static plastic buckling of columns and other models which have been used to study the static buckling of imperfection-sensitive elastic structures. It turns out that, for small imperfections, this model, when loaded statically, is more imperfection-sensitive in the plastic range than in the elastic range. It has also been observed that an initial imperfection might cause the simple model to buckle in a wholly elastic manner even though a perfect model would buckle plastically.

The conclusions of the studies described above relate only to the idealised models used in their development and cannot be employed without modification to predict the behaviour of other structures. Nevertheless, the behaviour of these models do reveal some valuable insight

into the various phenomena and provide a basis for understanding the characteristics of more realistic static and dynamic buckling problems. It is in this spirit that Jones and Reis (1979) have combined the essential features of Danielson's (1969) dynamic elastic and Hutchinson's (1972) static plastic models in order to examine the dynamic plastic buckling of the idealised column illustrated in Figure 7.

The various members in Figure 7(a) are rigid and weightless and the only masses, m_0 and m_1 , are concentrated at H and A, respectively. The unloaded model has a stress-free initial imperfection $\bar{\xi}$, while the member FHG is constrained to remain horizontal. Member FHG and pin B are constrained to move vertically in frictionless guides. Frictionless pins are located at A, B, and I and the behaviour of the softening non-linear spring at A is governed by the relation $F = \alpha \xi^2$, where $\xi + \bar{\xi}$ is the total horizontal displacement of A, as indicated in Figure 7(b). The material behaviour is simulated by springs 1 and 2 with the load-displacement characteristics shown in Figure 8 and, for convenience, it is assumed that the springs have identical characteristics.

Jones and Reis (1979) have shown that the static plastic behaviour of the model in Figure 7 confirms the characteristics reported by Hutchinson (1972) for another idealised column.

Jones and Reis (1979) used a phase plane procedure to

examine the dynamic buckling of the model for the particular case when $m_0 = 0$ and the external load P is a step loading of unlimited duration. A step loading was selected for this study because it is the most severe form of dynamic load as noted by Kao and Perrone (1978) and others. It transpires that the results for dynamic elastic buckling are identical to those reported by Budiansky and Hutchinson (1964). This wholly elastic analysis remains valid provided the force in spring number 2 is less than the yield force ($F_2 = K\Delta_Y$), or

$$\frac{P_D}{P_C} \leq \frac{4a\Delta_Y/3L_1 - 1}{2ar/3 - 1}, \quad (15)$$

where P_D is the dynamic buckling load, and

$$P_C = KrL_1, \quad r = L_1/L_2 \quad \text{and} \quad a = L_2\alpha/2Kr^2. \quad (16a-c)$$

Dynamic loads which violate inequality (15) cause plastic flow to commence in spring number 2 at some intermediate time (t_1) during the response. Thus, a wholly elastic analysis is valid until t_1 , after which spring number 1 continues to behave elastically, while spring number 2 responds plastically. Jones and Reis (1979) used a phase plane method to obtain the dynamic elastic-plastic buckling behaviour under these circumstances. This analysis is valid provided the force in spring number 1 remains elastic which, together with the equality in (15), requires

$$\frac{4a \Delta_y / 3L_1 - 1}{2ar/3 - 1} \leq \frac{P_D}{P_C} \leq \frac{2\Delta_y}{rL_1} \quad (17)$$

If the external dynamic load P is sufficiently large, then both springs 1 and 2 may yield plastically before the lateral motion (ξ) of the mass m_1 commences. However, immediately upon a movement ξ , spring 1 unloads elastically while spring 2 continues to load plastically in order to support the time-independent external load P . A phase plane analysis predicts the dynamic plastic-elastic buckling load (P_D)

$$\left(2\Omega - \frac{P_D}{P_C} \right)^2 = \frac{16a\bar{z}}{3} \frac{P_D}{P_C} \quad (18)$$

where

$$\Omega = \gamma/(1+\gamma), \quad \gamma = K_t/K, \quad \text{and} \quad \bar{z} = \bar{\xi}/L_2 \quad (19a-c)$$

and remains valid provided

$$\frac{2\Delta_y}{rL_1} \leq \frac{P_D}{P_C} \leq 2\Omega \quad (20)$$

Equation (18) with $\gamma = 1$ (i.e., $2\Omega = 1$) gives the dynamic buckling predictions for the wholly elastic case which are the same as those found by Budiansky and Hutchinson (1964) for a different model. Thus, the dynamic plastic-elastic buckling load is smaller than the dynamic elastic buckling load for a given dimensionless imperfection ($a\bar{z}$) and the difference increases as the strain hardening parameter γ decreases.

Typical phase plane diagrams for dynamic elastic,

elastic-plastic and plastic-elastic buckling are presented in Figures 9 and 10 ($z = \xi/L_2$, $z' = (m_1/2Kr^2)^{1/2} (dz/dt)$). A composite curve for the dimensionless dynamic buckling loads P_D/P_C associated with the dimensionless initial imperfections $a\bar{z}$ is given in Figure 11 for a particular set of parameters.

Inequality (15) is an equality at point c in Figure 11 so that dynamic buckling of the model in Figure 7 is controlled by wholly elastic effects for larger initial imperfections (i.e., curve cd), while inequality (17) is satisfied on the portion bc for which dynamic elastic-plastic buckling prevails. The point b is associated with the left hand side of equality (20). Thus, dynamic plastic-elastic buckling governs the behaviour of the model for small imperfections which lie on the curve ab.

It is evident from Figure 11 that the dynamic buckling of an imperfect model may be elastic even though a perfect model having the same parameters would buckle plastically. Hutchinson (1972) observed a similar phenomenon for the static buckling of an idealised elastic-plastic column. Moreover, equation (18) indicates that initial imperfections ($a\bar{z}$) are as important for dynamic plastic-elastic buckling as they are for dynamic elastic buckling, which is given by equation (18) with $\gamma = 1$ (i.e., $2\Omega = 1$).

A static buckling curve might be constructed for this model and is similar in form to the corresponding dynamic curve as shown in Figure 11. However, despite the

similarities between the two curves, the sequence of plastic loading and elastic unloading in springs 1 and 2 are different for both cases. If the initial imperfection is sufficiently small in the static case, then both springs respond elastically until spring 2 yield plastically. Loading continues with spring 1 elastic and spring 2 plastic until spring 1 yield plastically. Further static loading continues with both springs responding plastically until spring 1 commences to unload elastically. Finally, spring 2 continues to load plastically and spring 1 unloads elastically from its earlier plastic state until the maximum load carrying capacity is reached.

The dynamic buckling load according to equation (18) for the perfect model equals the reduced modulus load ($2\Omega P_c$) because the inertia of the prebuckling response is neglected (i.e., $m_0 = 0$). Thus, the external step loading P can be accommodated by springs 1 and 2 prior to any lateral movement (ξ) of mass m_1 . In fact, the immediate response of both springs 1 and 2 is identical for a given value of P , regardless of whether the model is initially perfect ($a\bar{z} = 0$) or has initial stress-free imperfections ($a\bar{z} \neq 0$). The dynamic buckling loads in Figure 11 are therefore larger than the corresponding static ones for small imperfections. This is due largely to the different elastic-plastic deformation histories in the springs during the static and dynamic responses. It is interesting to note that Hartzman (1974) found that the dynamic buck-

ling pressure of a geometrically perfect elastic-plastic spherical dome was larger than the corresponding static buckling pressure. However, the dynamic buckling loads for the model in Figure 7(a) are smaller than the associated static ones when the initial imperfections are larger than those corresponding to point f in Figure 11.

It is interesting to remark that the simple perturbation scheme which has been used to study successfully the dynamic plastic buckling of various structural members as described earlier cannot be used to examine the dynamic plastic-elastic behaviour of the model in Figure 7, with $m_0 = 0$, because spring 1 unloads elastically immediately after the motion of m_1 commences. Elastic unloading is precluded from a simple perturbation procedure which is based on a tangent modulus from of the constitutive equations. This situation also prevails for the dynamic elastic-plastic case, with $m_0 = 0$, since one spring remains elastic throughout the entire motion, while the other spring is plastic for the final part of the response. In fact, vertical equilibrium of the model in Figure 7 demands that the sum of the two spring forces equal P when $m_0 = 0$ and P is time-independent. Thus, it is impossible for two identical elastic work-hardening springs to respond plastically when $\xi > 0$ and P remains constant.

Jones and Reis (1979) employed a numerical procedure to examine the behaviour of the model in Figure 7 when prebuckling inertia is retained in the basic equations

(i.e., $m_0 \neq 0$). Numerical results are presented by Jones and Reis (1979) which show the variation of the dynamic buckling load with m_0/m_1 . It turns out that the character of dynamic buckling is similar to that found for the model without prebuckling inertia (i.e., $m_0 = 0$) within the range $0 < \omega_1/\omega_0 < 0.5$, where $\omega_1/\omega_0 = r\sqrt{m_0/m_1}$. However, parametric resonance occurs at $\omega_1/\omega_0 = 0.5$, for which the associated buckling load is zero. The dynamic buckling load then increases with increase in ω_1/ω_0 until $\omega_1/\omega_0 = 1$, when parametric resonance again intervenes. The numerical results reveal that, when material plasticity occurs during the dynamic response for loads less than the dynamic buckling load, its importance is confined largely to the first half-cycle of the response of m_1 and the model responds essentially elastically thereafter.

It is interesting to observe from the particular curves in Figure 11 for models with initial imperfections lying within the range $0.1230 \leq a\bar{z} \leq 0.1372$ that static loads cause plastic buckling while dynamic loads are responsible for elastic buckling.

4.4 Quasi-bifurcation Criterion

Lee (1975) explored the bifurcation and uniqueness of elastic-plastic continua loaded dynamically from a general viewpoint, because random initial imperfections, or perturbed motion, might be insufficient to describe the dynamic plastic buckling of some structures. Lee (1977)

further examined this subject and developed a quasi-bifurcation criterion for the stability of elastic-plastic continua loaded dynamically. A quasi-bifurcation of motion develops at a certain time t_{cr} when a nontrivial perturbed motion exists, which makes the functional defined by equation(51) of Lee (1977) an extremum. Lee showed that his dynamic quasi-bifurcation criterion reduces to Hill's bifurcation theory for quasi-static loads. It was shown by Lee (1978) that his quasi-bifurcation criterion predicted modes at the onset of instability which are in qualitative agreement with the final buckling modes observed by Abrahamson and Goodier (1966) in aluminum alloy rods impacted axially against heavy steel slabs. However, Lee plans to examine the post-bifurcation behaviour in order to improve the agreement with experimental results.

5. Discussion

It was remarked in section 1 that it was not the aim of this article to provide a conspectus of the entire field of dynamic plasticity. After some general comments on the rigid-plastic method for solving dynamic plasticity problems in section 2, attention was focussed in section 3 on some recent investigations and applications involving a stable dynamic plastic response. The dynamic plastic unstable response of various structures was discussed in section 4. Many other important areas of dynamic structural plasticity are actively being developed which are not discussed here, but have been reviewed elsewhere as remarked in section 1. However, a few brief and necessarily incomplete comments on some of this work are now given.

The dynamic plastic behaviour of ideal fibre-reinforced, or strongly anisotropic, beams has been studied, using a rigid-plastic procedure, by Spencer (1974), Jones (1976d), Shaw and Spencer (1977), Laudiero and Jones (1977) and reviewed by Jones (1978). The simplifications contained in the material idealisation for an ideal fibre-reinforced beam furnish a yield criterion for plastic flow in terms of the transverse shear force with the bending moment as a reaction. Recently, Shaw and Spencer (1978) and Spencer (1979) have extended these general ideas in order to study the transverse impact of ideal fibre-reinforced rigid-plastic plates. It appears from these theo-

retical investigations that fibre-reinforcement is responsible for significant reductions in the permanent deflection or weight of structures. However, this benefit is achieved at the expense of higher decelerations which might be a disadvantage for certain problems as remarked by Jones (1976d).

Parkes (1974, 1978) has examined the behaviour of rigid-plastic beams when subjected to sudden temperature gradients such as those which large neutron pulses cause in fissile materials. A temperature gradient may give rise to curvature changes of beam, plate, and shell structural members in addition to various other effects. The structural response is governed by the equations of dynamics when these curvature changes develop in a sufficiently short time. Thus, the dynamic structural response may be estimated with the aid of rigid-plastic methods of analysis when the temperature accelerations are sufficiently large. This work is discussed further by Jones (1978).

Lepik and Mróz (1977) used a mode approximation procedure to obtain the optimal design of rigid-plastic structures subjected to dynamic loads. The objective was to seek the design which gave the minimum permanent displacements of a structure with a given constant volume of material. The particular case of a stepwise constant thickness beam subjected to a uniformly distributed pressure-pulse with a rectangular pressure-time history was examined in some detail. It was found that the maximum permanent deflec-

tion of an optimal two-step (per half span) beam was one-half the corresponding permanent deflection of a uniform beam having the same volume. The impulsive loading of beams and circular plates was also examined by Lepik and Mróz (1977). Lepik and Mróz (1978) have also explored the role of asymmetric mode motions on the optimal design of an impulsively loaded stepped beam. Jones and Wierzbicki (1976) conducted experiments on the higher modal response of uniform beams, while Jones and Guedes Soares (1978) developed theoretical rigid plastic solutions for uniform beams undergoing modal responses of any symmetric or antisymmetric order.

Although pure modal responses are not likely to be excited in most practical problems unless deliberately activated (as in a specially designed energy absorbing system), it is nevertheless apparent that the behaviour of complex structures loaded dynamically can involve complicated deformation fields having some of the features of higher modal responses. Furthermore, Lepik and Mróz (1978) remark that simplified design calculations using mode forms are sometimes entirely adequate, particularly when only limited information is available on the dynamic loading characteristics.

Various energy absorbing devices and systems, which utilise a wide variety of concepts, have been studied in the literature and examined largely for potential applications in land and space vehicles and aircraft. Ezra and

Fay (1972) investigated the characteristics of a number of material deformation, friction and extrusion energy absorbing devices from the viewpoint of improvement in aircraft crashworthiness. Ezra and Fay (1972) also identified the essential characteristics of energy absorption which are indispensable for revealing and comparing the merits of different energy absorbing devices. Recently, Johnson and Reid (1978) have reviewed some of the available literature on the dynamic plastic impact of metallic structures to assess the energy absorption capability of various simple structural elements which may form part of energy absorption systems. Silva-Gomes et al. (1978) have examined the dynamic plastic behaviour of a chain of rings subjected to a dynamic axial load in order to explore the potential for arresting a moving mass in a controlled fashion.

A recent brief review on approximate theoretical rigid-plastic methods of analysis, fluid-structure interaction and numerical methods for dynamic plasticity was presented by Jones (1978).

6. Concluding Remarks

This review does not survey the entire field of dynamic structural plasticity but is rather an attempt to complement the many available reviews which are cited in section 1. In particular, emphasis is given in section 3 to recent studies into the influence of transverse shear and rotatory inertia on the dynamic plastic stable response of structures, while dynamic plastic instability is examined in section 4. The limitations of dynamic plastic methods of analysis are discussed in section 2, while various applications in many fields are mentioned throughout the text. The field of dynamic plasticity is growing rapidly and no doubt many more applications will be found as engineers strive to design new structures which must be as light and safe as possible yet withstand large dynamic loads arising in many practical situations.

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Titles of Figures

- Figure 1. Maximum permanent transverse displacements (W_m) of a fully clamped beam loaded impulsively with a velocity V_0 .
- $\square, \Delta, \circ, \diamond$ experimental results on aluminum 6061 T6 beams after Jones et al. (1971).
- $\blacksquare, \blacktriangle, \blacktriangledown, \blacklozenge$ experimental results on mild steel beams after Symonds and Jones (1972).
- \bullet a - e, numerical finite-difference results of Witmer et al. (1963) from Symonds and Jones (1972).
- ① Infinitesimal analysis (bending only).
- ② Equation (5) (circumscribing yield criterion) after Jones (1971).
- ③ Equation (5) with σ_y replaced by $0.618\sigma_y$ (inscribing yield criterion).
- ④ Theoretical solution of Jones (1971) for "exact" yield criterion.
- ⑤ Equation (5) with strain rate correction factor according to equation (6) and $H = 0.1$ in (0.254 cm) (circumscribing yield criterion)
- ⑥ as ⑤ but with σ_y replaced by $0.618\sigma_y$ (inscribing yield criterion).

- Figure 2. Maximum permanent transverse displacements (W_m) of a fully clamped rectangular plate loaded impulsively with a velocity V_0 and $\beta = 0.593$.
- \circ, \square, Δ experimental results on aluminum 6061 T6 rectangular plates after Jones et al. (1970).

●, ■, ▲ experimental results on hot-rolled mild steel rectangular plates after Jones et al. (1970).

- ① Approximate infinitesimal analysis (equation (12) of Jones et al. 1970).
- ② Equation (4) (circumscribing yield criterion) after Jones (1971) for a strain rate insensitive material.
- ③ Equation (4) with σ_y replaced by $0.618\sigma_y$ (inscribing yield criterion).
- ④ Theoretical solution of Jones (1971) for "exact" yield criterion and a strain rate insensitive material.

Figure 3. Pseudo-Shakedown of bow models.

○, □ experimental impact tests of Yuhara (1975)
 — rigid, perfectly plastic analysis for a fully clamped rectangular plate with geometry changes and p_{eq} applied statically (Jones 1977).

Figure 4. Dimensionless permanent transverse displacements at centre (\bar{w}_f) and supports (\bar{w}_1) of a simply supported circular plate loaded impulsively with an initial velocity V_0 .

Figure 5. Variation of dimensionless energy ratios with time for a simply supported beam with $\nu = 1.5$ subjected to an impulsive velocity V_0 .
 — · — · — simple bending theory

----- $I = 0$ after Nonaka (1977) ($I^2 = I_R/mL^2$).

————— $I = 0.1924$ after Jones and Gomes de Oliveira (1979).

R_B , R_K , and R_S are bending, kinetic, and shear energies, respectively, divided by mLV_0^2 , and $T = M_0 t/mL^2 V_0$.

Figure 6. Dimensionless slope ($\bar{\theta}$) and displacement (\bar{w}) of an infinitely long beam hit by a mass G travelling with an initial velocity V_0 .

(a) $\lambda_0 = 3.319$

1. Ilyushin-Shapiro yield curve with $I = 0.3194$ after Gomes de Oliveira and Jones (1978b).
2. as 1. but $I = 0$.
3. simple bending solution
4. square yield curve with $I = 0.3194$ after Jones and Gomes de Oliveira (1979).
5. as 4 but $I = 0$ (Symonds 1968).

(b) $\lambda_0 = 24.357$

- 1 - 5. as above but with $I = 0.4861$.

$$\lambda_0 = 6mM_0/GQ_0, \quad \bar{\theta} = 6M_0\theta/GV_0^2, \quad \bar{w} = 12mM_0w/G^2V_0^2,$$

$$T = 12mM_0t/G^2V_0, \quad I^2 = mI_R/G^2.$$

Figure 7. Simple model of Jones and Reis (1979).

(a) initial position, (b) deformed position.

Figure 8. Elastic-plastic characteristics of springs 1 and 2.

Figure 9. Phase plane trajectories for the dynamic elastic case with $m_0 = 0$, $a = 10$, and $a\bar{z} = 0.25$.

Figure 10. (a) Phase plane trajectories for dynamic elastic-plastic case with $m_0 = 0$, $a = 10$, $\gamma = 0.75$, $a\bar{z} = 0.05$, $r = 1$ and $\Delta_Y L_2/L_1^2 = 0.268$.

----- elastic, ——— elastic-plastic

(b) Phase plane trajectories for dynamic plastic-elastic case with $m_0 = 0$, $a = 10$, $\gamma = 0.75$ and $a\bar{z} = 0.025$.

Figure 11. Comparison of dynamic (P_D) and static (P^*) buckling loads when $m_0 = 0$, $a = 10$, $r = 1$, $\gamma = 0.75$ and $\Delta_y L_2 / L_1^2 = 0.268$.

————— dynamic, ----- static.

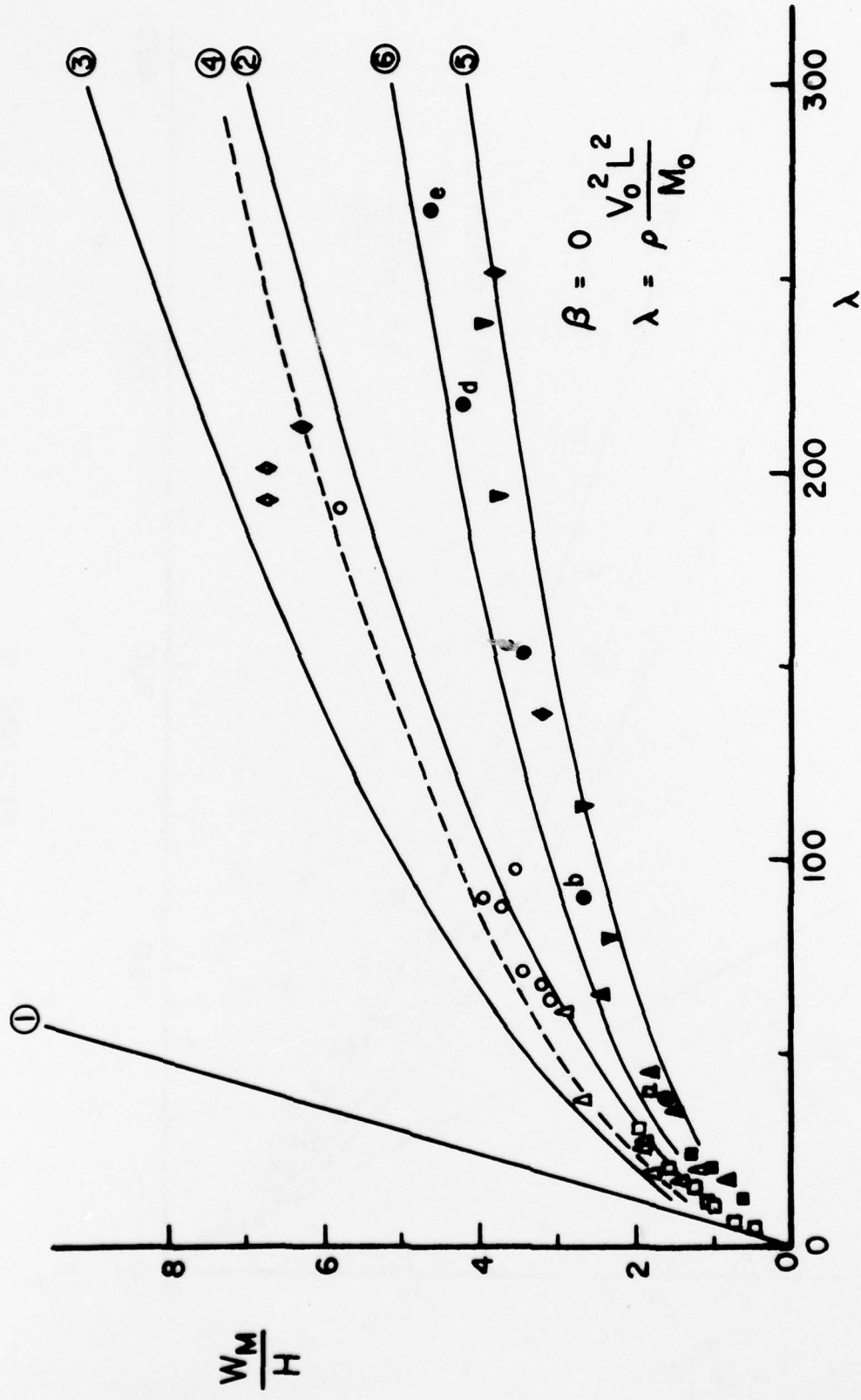


FIGURE I

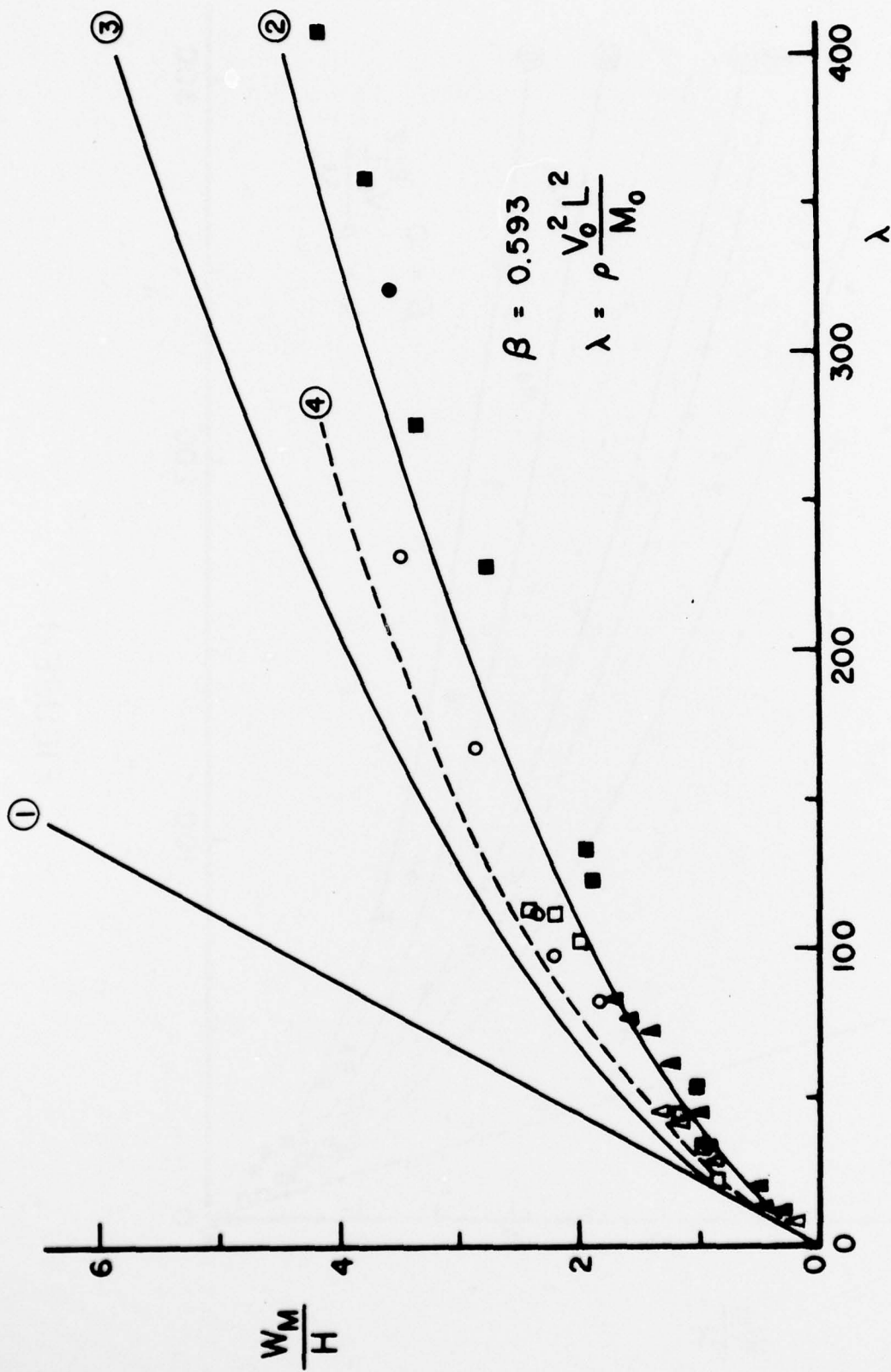


FIGURE 2

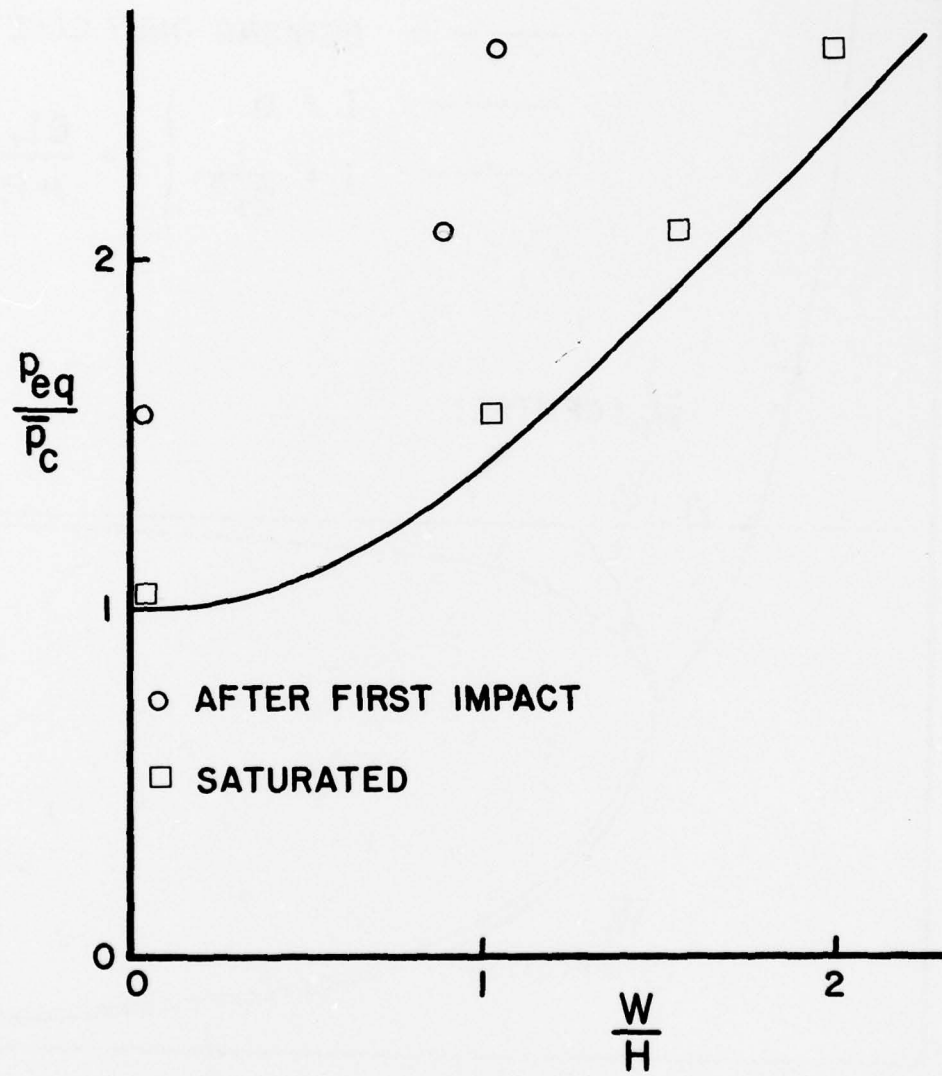


FIGURE 3

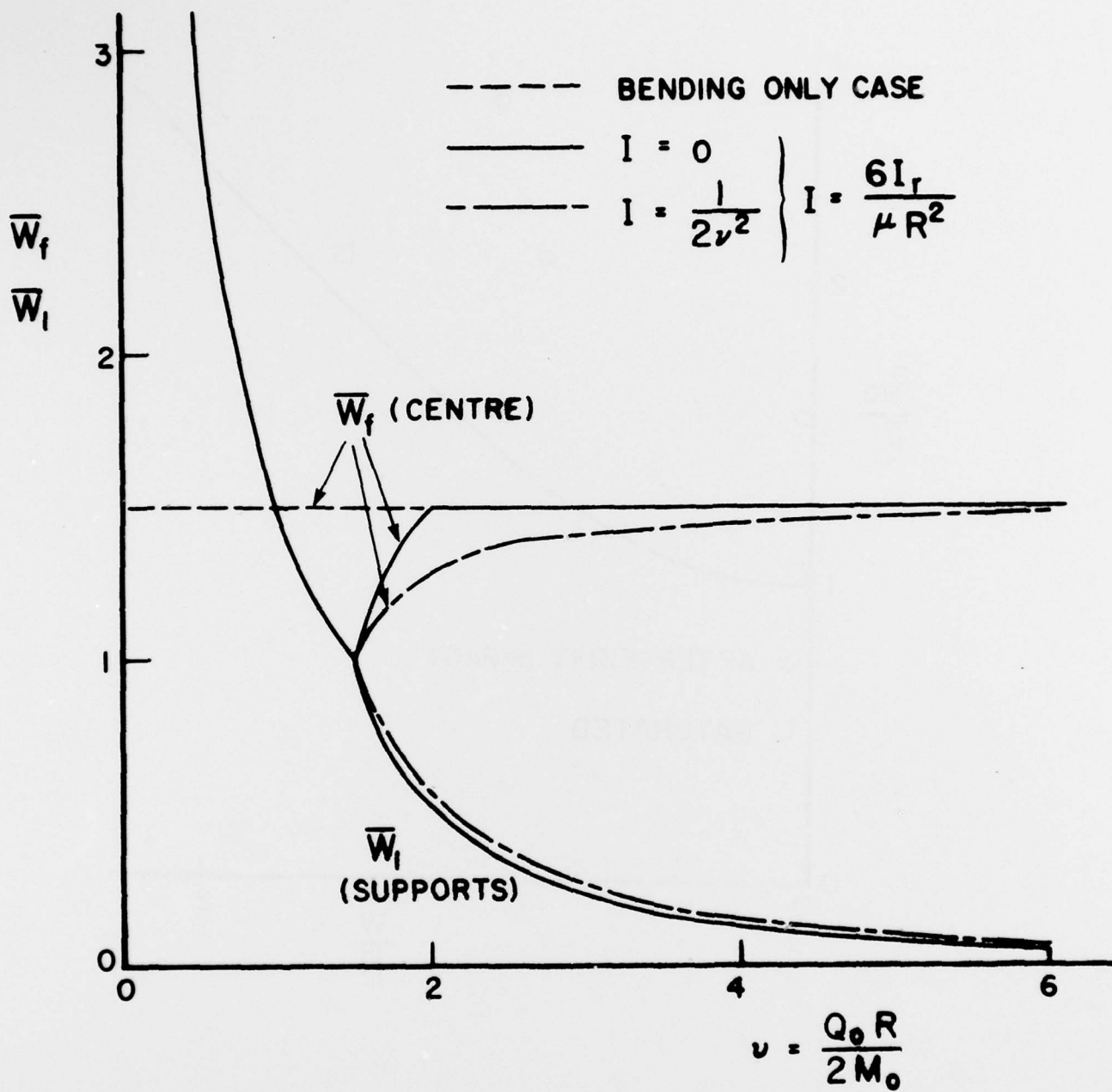


FIGURE 4

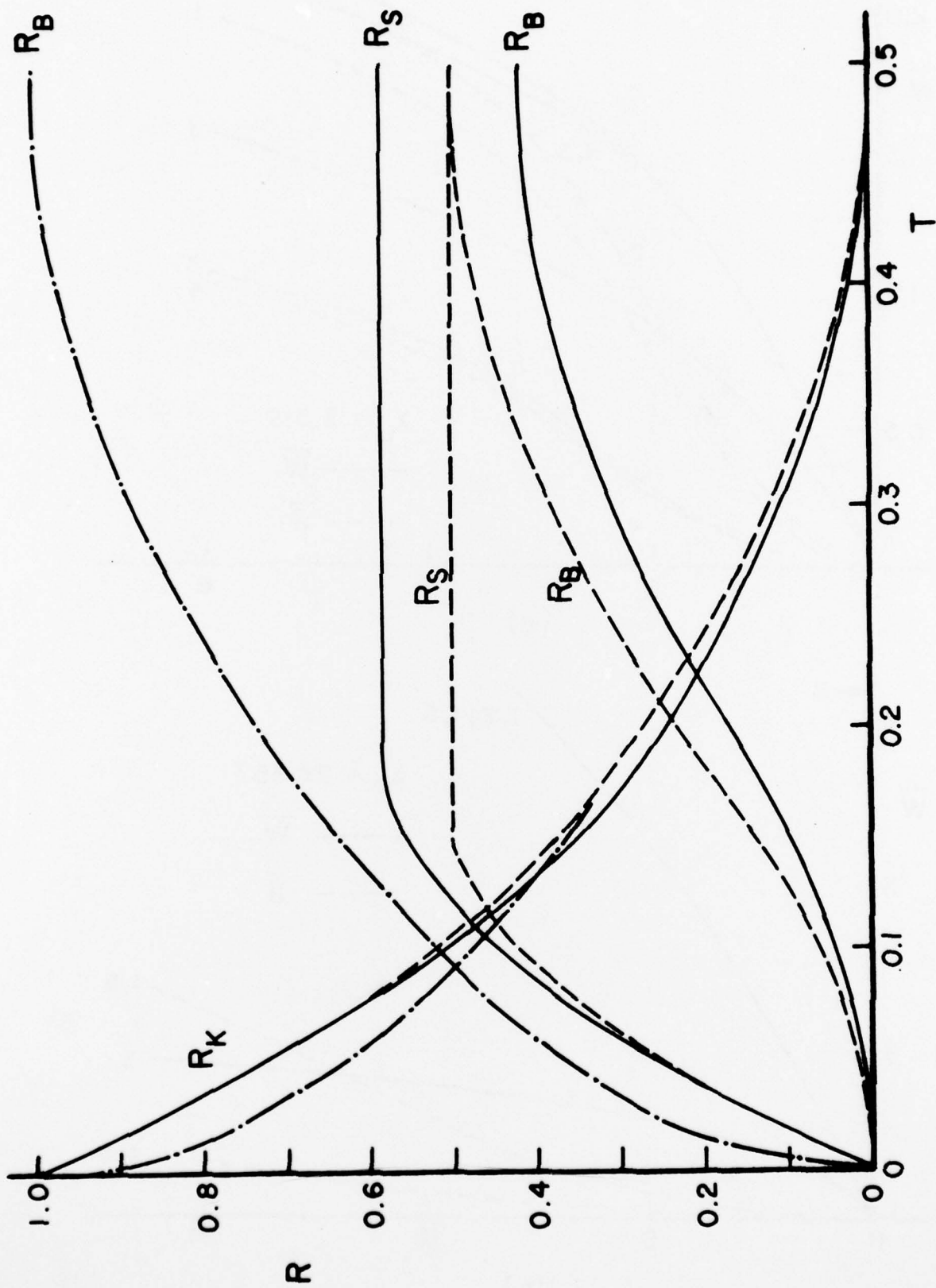


FIGURE 5

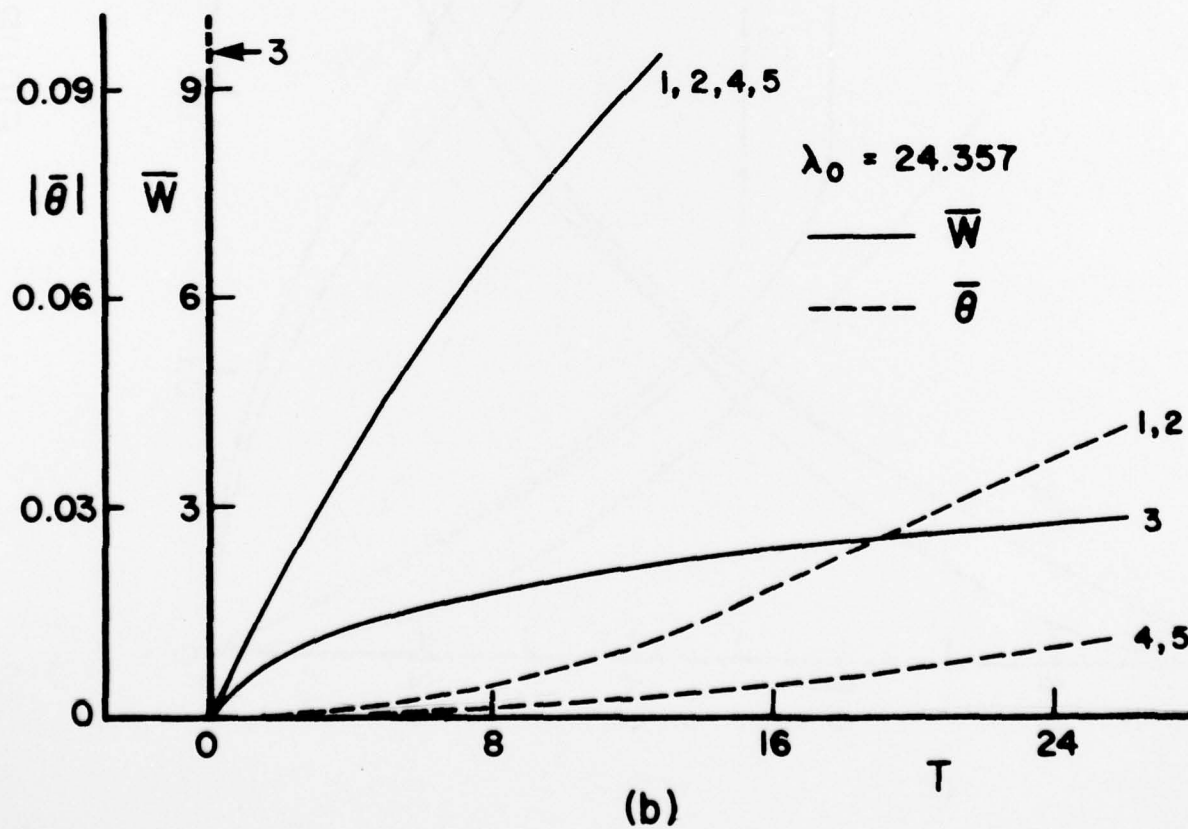
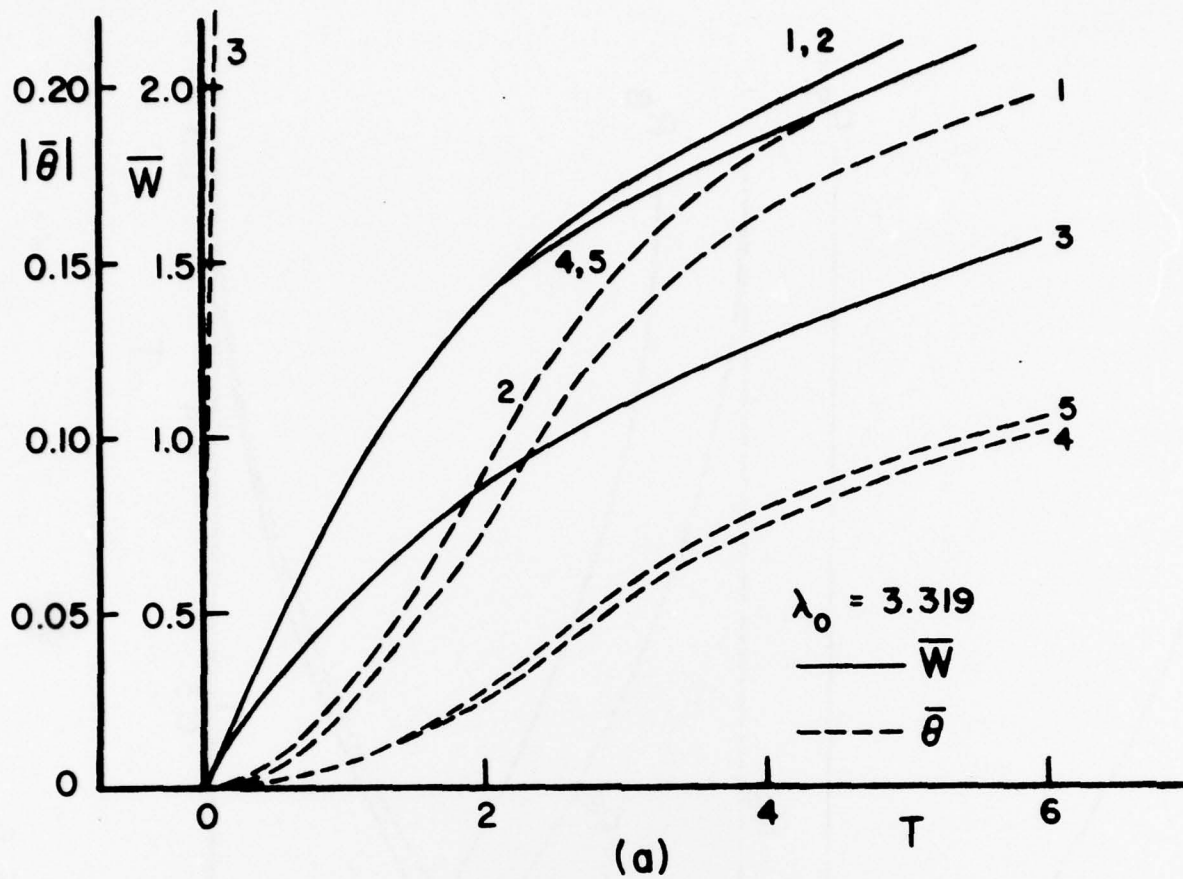


FIGURE 6

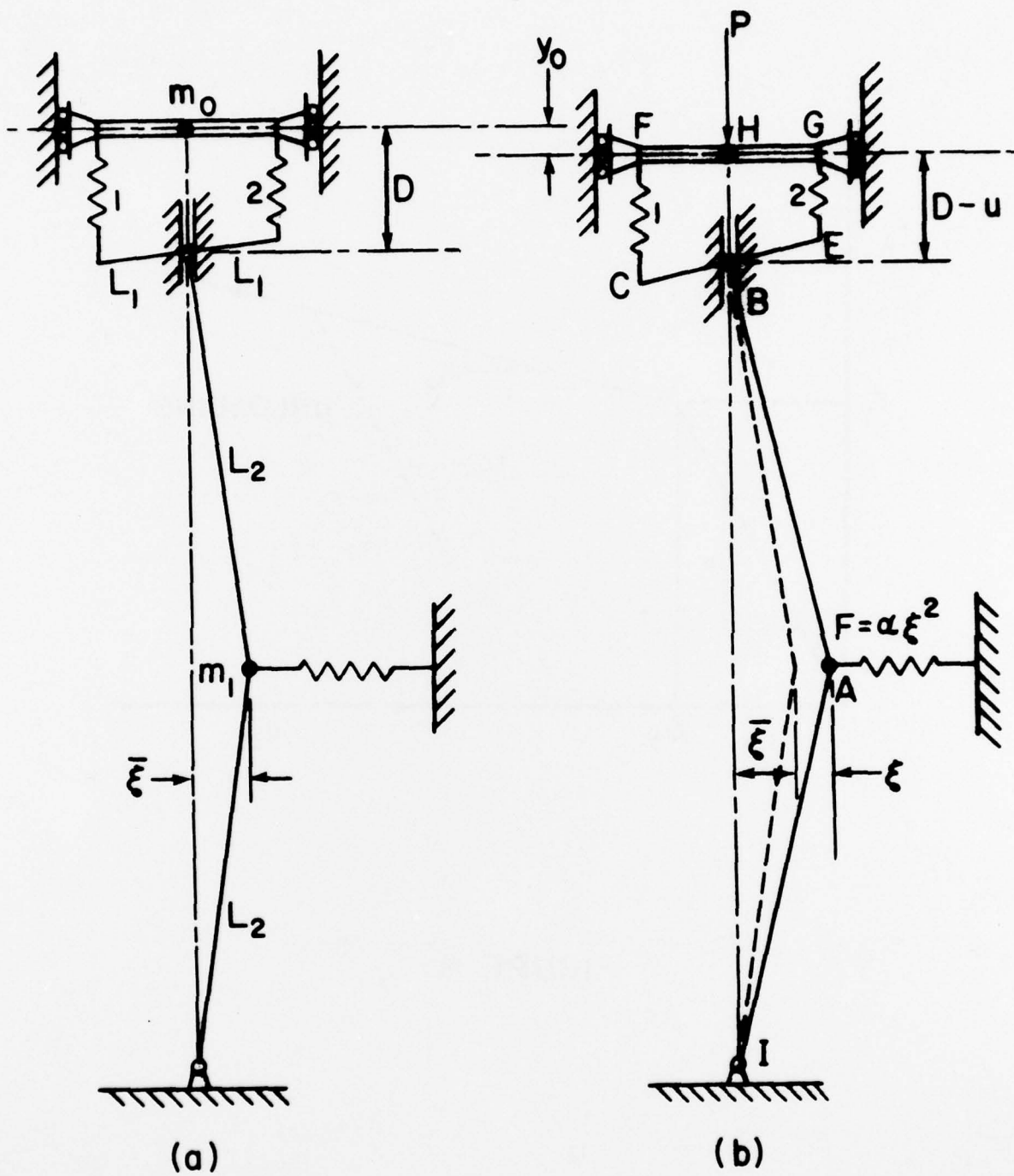


FIGURE 7

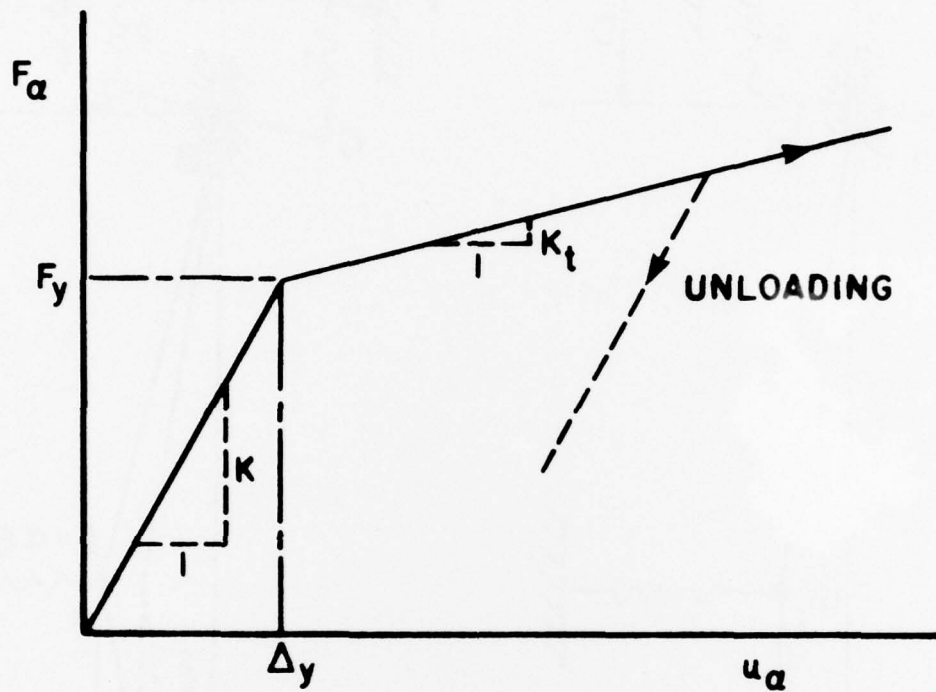


FIGURE 8

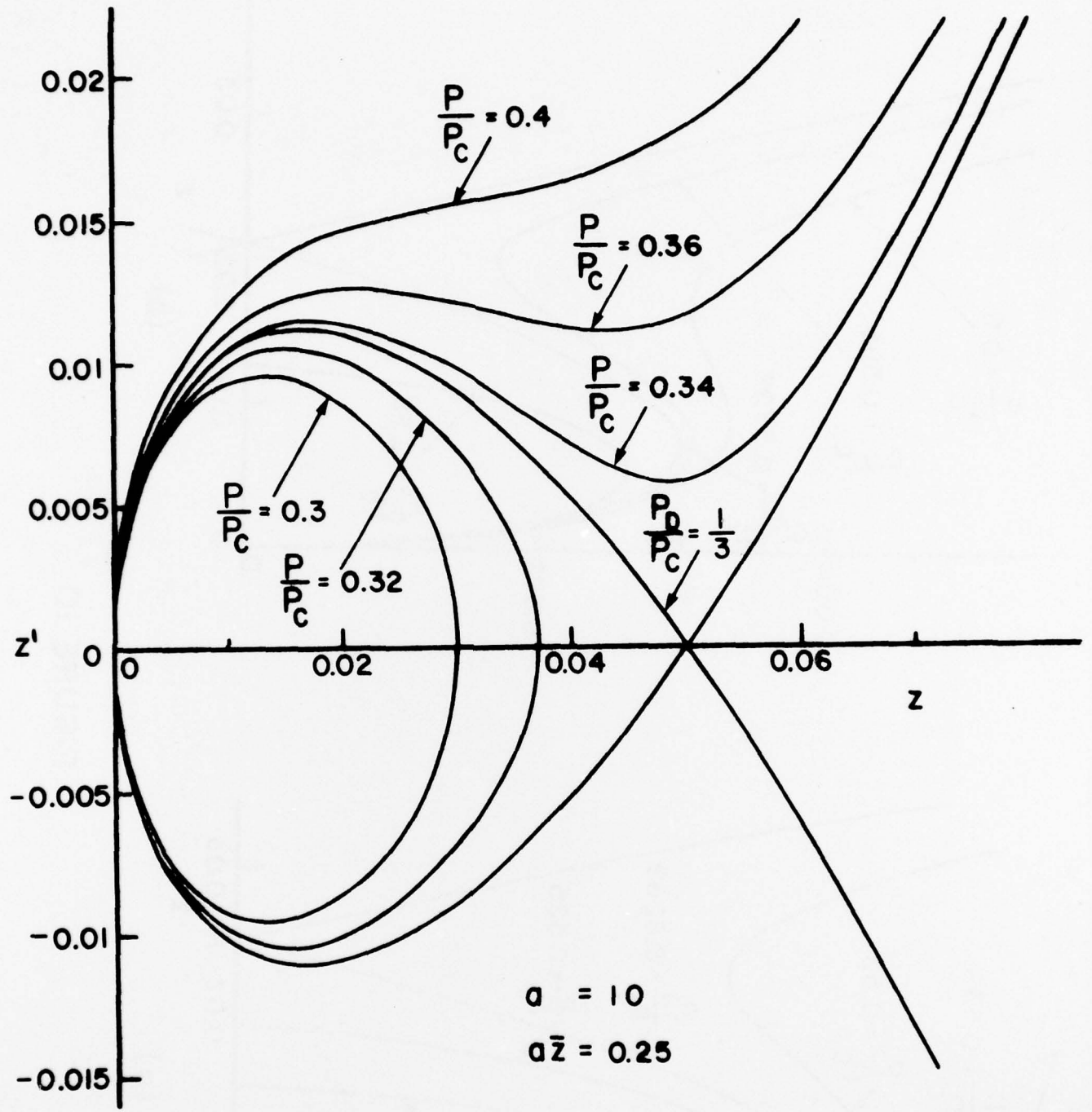
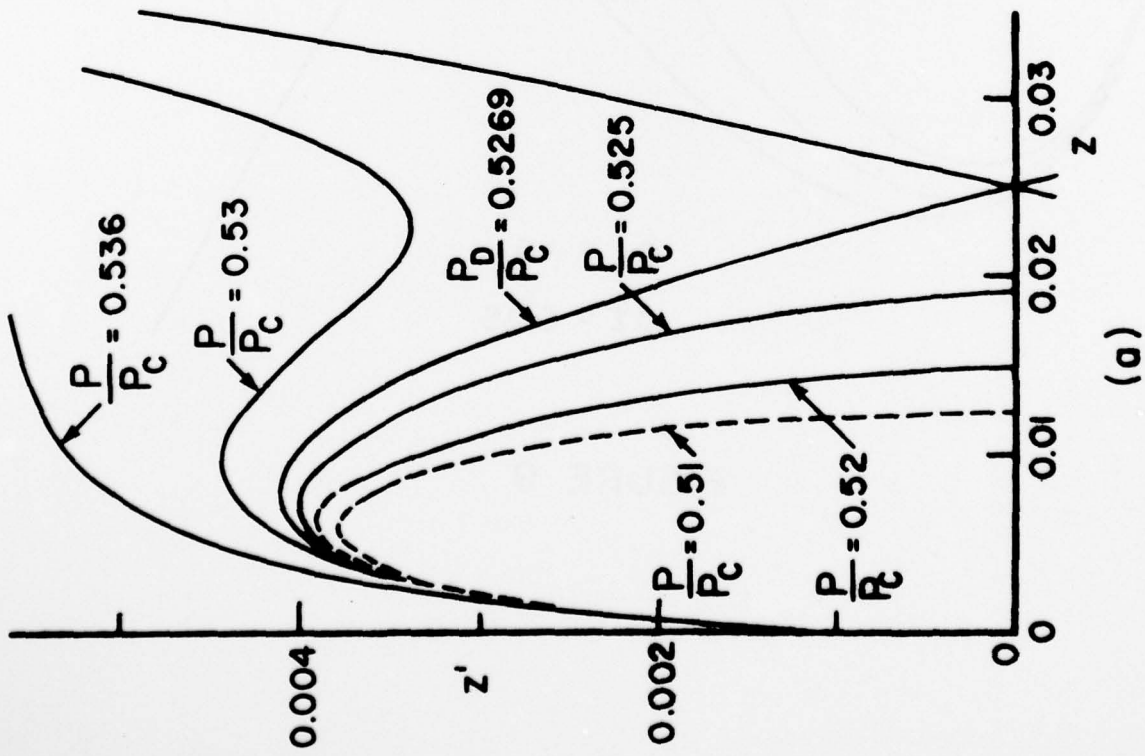
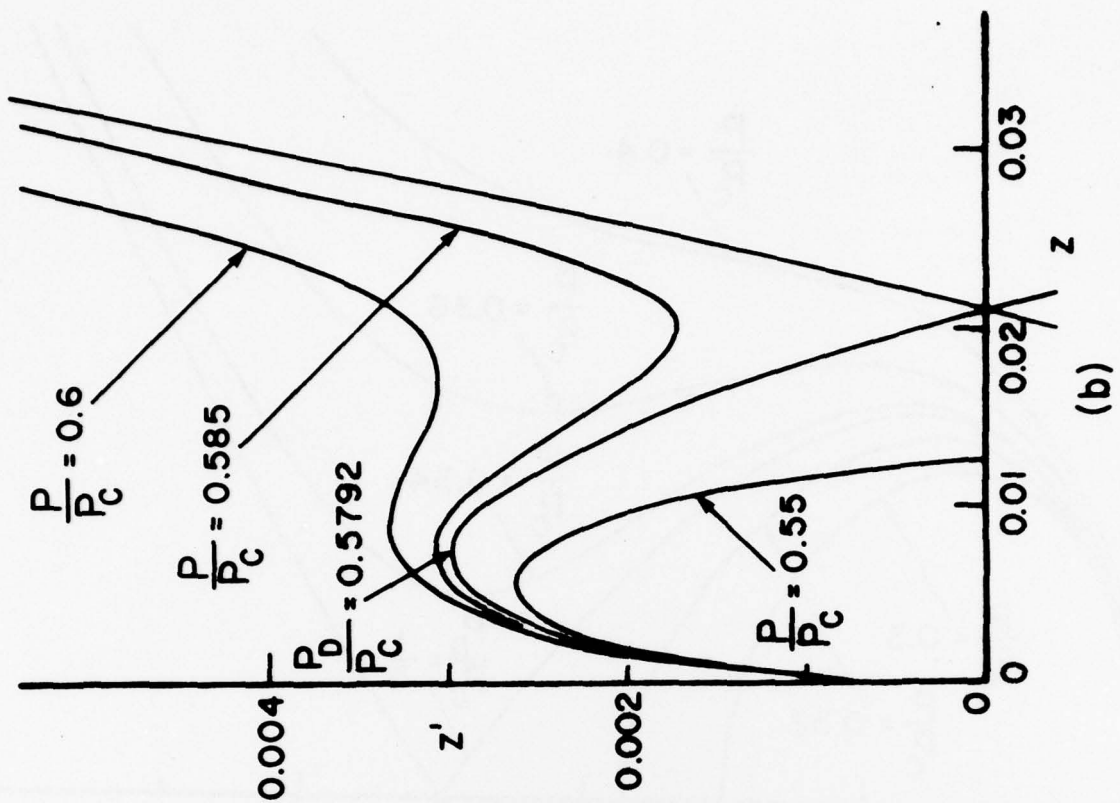


FIGURE 9



(a)



(b)

FIGURE 10

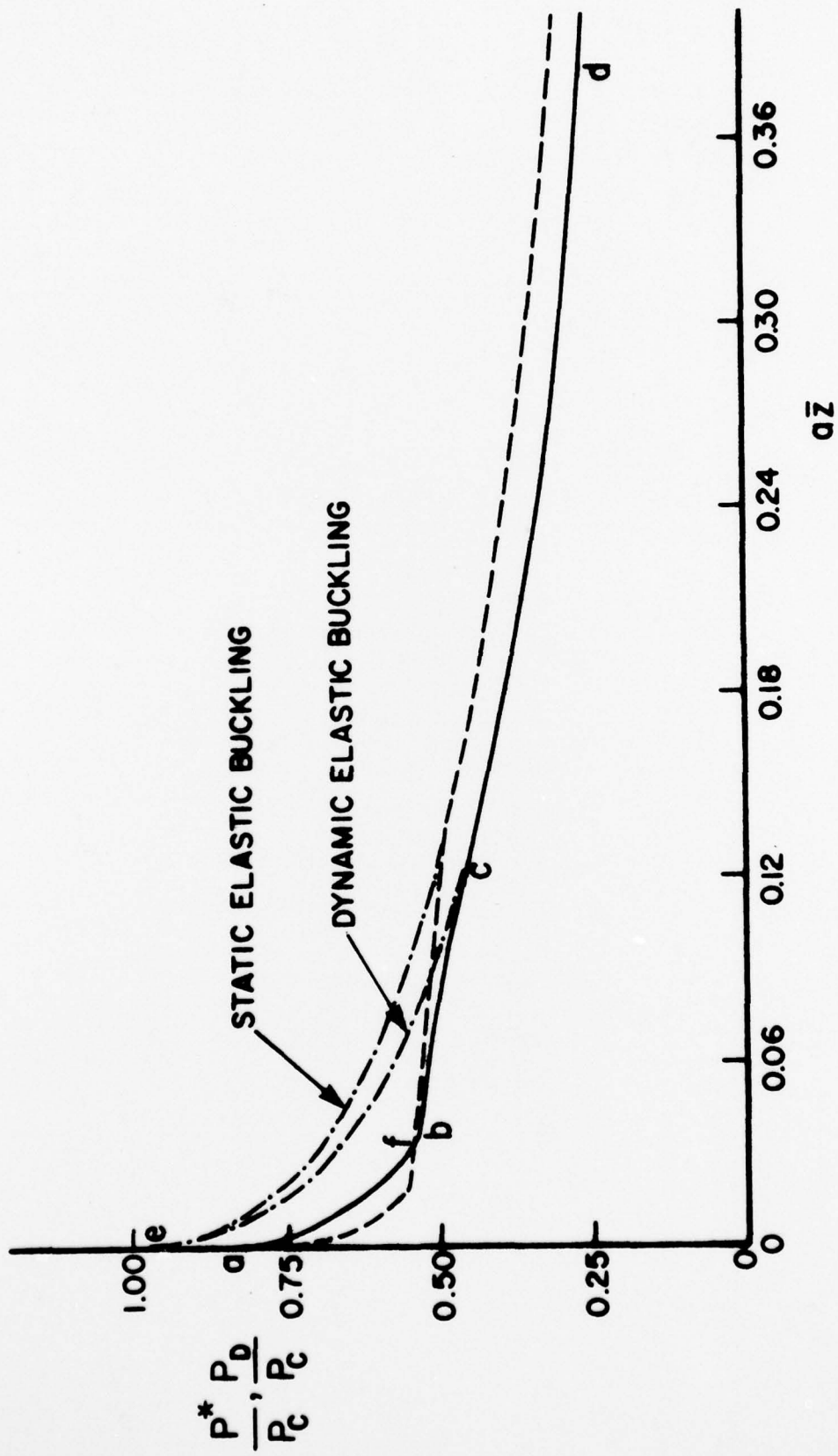


FIGURE II

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