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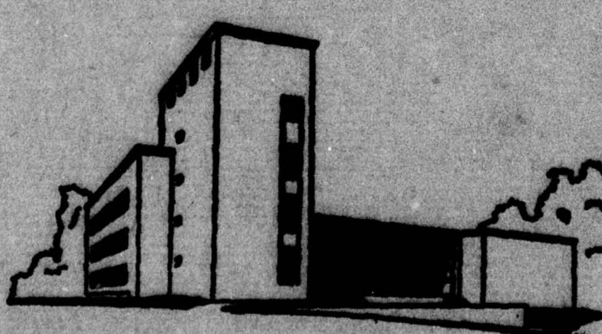

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A GOAL FOCUSING APPROACH TO ANALYSIS OF TRADE-OFFS AMONG HOUSEHOLD PRODUCTION OUTPUTS*

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ABSTRACT

The ideas of goal programming are combined with efficient point considerations to yield a computational approach for use in tradeoff analysis and evaluations for goals accounting analysis. The orientation is toward an efficient point which comes closest to a set of interrelated goals. Attention is thereby restricted to a subset of the possible efficient points. Shadow prices to guide further analyses are also automatically supplied for guiding subsequent tradeoff analyses. Relations to the utility theoretic and household production considerations of Gary Becker and Kelvin Lancaster are indicated and an illustrative example involving econometric estimates for pertinent quality-of-life dimensions is supplied.

KEY WORDS

Household Production

Efficient Points

Goal Programming

Complementarity

Shadow Prices

Tradeoffs

Goals Accounting

Quality-of-Life

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1. Introduction

N.E. Terleckyj in [8] proposes an approach to an economic analysis of the quality of life. That approach rested on the theory of household production as formulated by Becker [1] (See also Michael and Becker [5] as well as Stigler and Becker [7].) We here join these ideas to those of Lancaster [4] as a start toward developing an operational and wholly computational approach for use in goals accounting analysis. Our purpose is twofold: First, to provide a suitable way of identifying and evaluating phenomena such as substitution ratios, shadow prices and the efficiency frontiers which enter into the multidimensional context of a quality-of-life analysis. Second, to provide a common frame of reference for a variety of related research endeavors. In conclusion we supply numerical examples to provide concrete guidance and illustration of the theories and methodological developments which occupy the earlier part of the paper. Hopefully, this will invite discussion and critical attention.

The organization of the paper is as follows. In sections 2 through 5 we provide an expository development of the household production possibility set following similar developments in Becker [1] and Lancaster [4]. In sections 6 through 8 these ideas are extended and given operational form via a goal programming formulation that we shall refer to as "goal focusing". Ways of locating efficiency frontiers and computing the related substitution ratios and shadow prices were analyzed and illustrated in the context of such a goal focusing model and its related "goal artifacts." These we rated to the approaches of Nestor Terleckyj in [8] and [9] which we shall note passing en route to our conclusions. We conclude with illustrative applications in section 9 followed by a summary and suggestions for further research.

2. Definitions and Notations

We initiate the analysis by introducing definitions and notations which are consistent with those used in [1] and [3]. Let

z_j = vector of household products, which result from the production of household j.

A typical element z_{ij} in z_j is defined as follows:

$$z_{ij} = f_{ij}(x,g,t), \tag{1}$$

where

x = a vector of market goods,

g = a vector of public goods and services,

t = a vector of time inputs.[1/]

Thus (1) defines a production (function) relation in which the outputs z_{ij} enter as arguments for the utility of the jth household defined by

$$U_j = U_j(z_j,E), \tag{2}$$

where

E = a vector of external goods produced by other organizations outside the household,

and z_j is as already defined in the development leading to (1).

We now let

V_j = the sum of non-wage income of household j,

w = the "wage rate", common to all households,[2/]

T_j = time available,

1/ In the sense of Becker [1]. See Terleckyj [3] page 2.

2/ This assumption of a uniform wage can, of course, be relaxed if desired.

so that

$$S_j = wT_j + V_j \tag{3}$$

is a characterization of the full income concept in the sense of Becker [1].

We next define

$t_{\ell j}$ = the time spent in the ℓth activity (either consuming x's or g's) by the jth household,

p_k = price of kth market good,

so that

$$S_j = \sum_{k,\ell} (wt_{\ell j} + p_k x_{kj}) = \sum_{\ell} wt_{\ell j} + \sum_{k} p_k x_{kj} \tag{4}$$

Thus, from (3) and (4)

$$wT_j + V_j = \sum_{k,\ell}(wt_{\ell j} + p_k x_{kj}). \tag{5}$$

In other words, the value of total household time plus its nonwage income, on the left, equals the value of time spent in household production plus the expenditures for purchased goods and services. The former, i.e., the expression on the left, can be interpreted as the total budget, composed of wage and non-wage income. The expression on the right can be interpreted as the total cost of producing the z_j resulting from the jth household's production function (1).

3. A Simple Household Production Process

We now briefly consider the household production functions in (1). Such functions have been derived and tested in a variety of forms. In this paper, however, we shall follow Lancaster [4] and consider only functions of the form

$$z = By \tag{9}$$

where

B = a household production technology matrix,

and

$$y = \begin{bmatrix} t \\ x \end{bmatrix}.$$

t, x and z are as defined in section 2. We can, of course, include public goods, g, in this analysis explicitly. Here, however, we treat it as a constant vector so that the input variables in the household production function are x and t as in Becker [1].

Equation (9) might also be regarded as the reduced form of a system of structural equations of an econometric model[3/]

$$\Gamma z + Ax = u \tag{9.a}$$

where z denotes a vector of endogenous variables, x denotes a vector of exogenous variables, u is a vector of random errors, Γ is a nonsingular square matrix of coefficients for the relationship between the endogenous variables and A, also a matrix, provides a mapping from the exogenous vector, x, to the vector of endogenous variables z. Hence,

$$z = Bx + \Gamma^{-1}u \tag{9.b}$$

3/ See e.g., Tinbergen [11], Theil [10], Johnston [3].

where

$$B = -\Gamma^{-1}A. \qquad (9.c)$$

With suitable assumptions for $\Gamma^{-1}u$, (9.a)-(9.c) provide access to econometric methods of estimation and identification of equation (9). See Section 9 below.

4. The Household Production Possibility Set

The relationships specified by (9) characterize the household production process as a transformation or mapping from a resource space, R, for x and t into a household space, H, with points z. This transformation will be submitted to a budget constraint, as in (5), via a development which proceeds as follows. Formally, we have a budget set in R-space (resource space) defined as

$$R \equiv \{x,t: \ [we^T, p^T]y = T_j w + V_j, \qquad x,t \geq 0,\} \tag{10}$$

where the superscript T indicates transposition, e^T is the row vector with unity for each of its elements, and p^T is a row vector of prices.[4/]

The image in H-space (household space) for the By transformation is the set

$$C \equiv \{z: \ z = By, \ [we^T, p^T]y = T_j w + V_j, \ y \geq 0\} \tag{11}$$

Our analysis in this section will be concerned with the general properties of C and its relationship to the production technology matrix B, the price and wage vector $[we^T, p^T]$ and the full (household) income $T_j w + V_j$.

Following the analysis given in Lancaster [4] we list some of the properties of C as follows:

(a) The budget set in R is the set of all convex combinations of the following extreme points

---

4/ To simplify the exposition we exclude here additional constraints from the budget set (10) such as $\sum_\ell t_{\ell j} = T_j$ and possible inequality constraints for x.

An illustrative application of related considerations is provided in section 9.

$$\begin{bmatrix} \frac{T_j w + V_j}{w} \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ \vdots \\ \frac{T_j w + V_j}{w} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ \frac{T_j w + V_j}{P_1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ \frac{T_j w + V_j}{P_N} \end{bmatrix} .$$

(b) C is the image set of R and consists of all convex combinations of the images of the extreme points of the budget set.

(c) Every extreme point of C is the image of an extreme point in the budget set. An extreme point of the budget set is not necessarily an extreme point of C, however.

(d) Let z^s denote the image of an extreme point y^s in (5), and let B^s denote the sth column of B. Then the expression

$$z^s = \frac{T_j w + V_j}{P_s} B^s \qquad (12)$$

gives the extreme points in H, the household space, where P_s is either p_k or w as required.

We turn next to a simple illustrative example. Let,

$$B = \begin{bmatrix} .8 & .7 & .08 & .2 \\ .4 & .6 & .125 & 0 \end{bmatrix}$$

$$w = 10$$

$$p_1 = 2$$

$$p_2 = 4$$

and

$$T_j w + V_j = 100.$$

The extreme points in R are

$$\begin{bmatrix} 10 \\ \\ \\ \\ \end{bmatrix}, \begin{bmatrix} \\ 10 \\ \\ \\ \end{bmatrix}, \begin{bmatrix} \\ \\ 50 \\ \\ \end{bmatrix}, \begin{bmatrix} \\ \\ \\ 25 \end{bmatrix}$$

The image of these in H is

$$\begin{bmatrix} 8 \\ 4 \end{bmatrix}, \begin{bmatrix} 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \end{bmatrix},$$

which gives rise to the convex polyhedral set potrayed in Figure 1. The polytope $\{z_{x_1}, z_{x_2}, z_{t_1}, z_{t_2}\}$ in H defines the feasible region of household production. In summary, then, the mapping obtained from B is from points x,t in R to points z in H with an image set C consisting of all feasible points permitted by the budget constraint.[5]

---

[5] We follow here the common form of household production analysis with a household budget (10) in the form of an equality (as in Becker [1]) rather than an inequality (as in Lancaster [4]) so that the origin point in Figure 1 is not part of the polyhedral set of production possibilities.

Additional constraints in R, such as $t_1 > L$, will of course alter the set of extreme points of the full income set in R and hence also produce a different image set C in H.

Of course the concept of "full income" extends beyond money (wage) income only, and increases or decreases in V_j will also produce a shift of the feasible production region in H. Thus, for example, an alteration from $T_jw + V_j = 100$ to $T_jw + V_j = 150$ due to a 50% increase in income from varying only V_j will produce a shift in the household production region. See Figure 2.

The same kinds of change will be observed if T_j is varied with w fixed. However, changes in prices or wages will also effect the household production region but in a different manner. We illustrate these effects by comparing two households. Let B, p, and $S_j = T_jw + V_j$, be as previously defined. But let Household No. 1 have a wage rate $W_1 = 10$ and Household No. 2 have $W_2 = 12.5$.[6] Then, respectively, the extreme points of each household are:

<u>Household No. 1</u> <u>Household No. 2</u>

$$\begin{bmatrix}3\\4\end{bmatrix}, \begin{bmatrix}7\\6\end{bmatrix}, \begin{bmatrix}4\\6\end{bmatrix}, \begin{bmatrix}5\\0\end{bmatrix}; \quad \begin{bmatrix}6.4\\3.2\end{bmatrix}, \begin{bmatrix}5.6\\4.8\end{bmatrix}, \begin{bmatrix}4\\6\end{bmatrix}, \begin{bmatrix}5\\0\end{bmatrix}$$

As in Figure 3, this variation in wages causes a tilt away from the original position of Household 1 and 2 so that the latter moves to the shaded region while the former remains, as at the start, in the unshaded set C. Thus we observe that an increase in wage rates has shrunk the feasible household production region for Household No. 2. If the above

[6] Although we assume that both households have the same total budget $S_j = T_jw+V_j$, since the wage, w, is different the households necessarily differ also in either T_j or V_j or both.

25% increase in the wage rate of household 2 is also accompanied by related adjustments in T_j and for V_j then the total income would increase as well. Figure 4, below depicts the shift in the household production possibility frontier resulting from 50% increase in S_j and 25% increase in W_j of household 2.

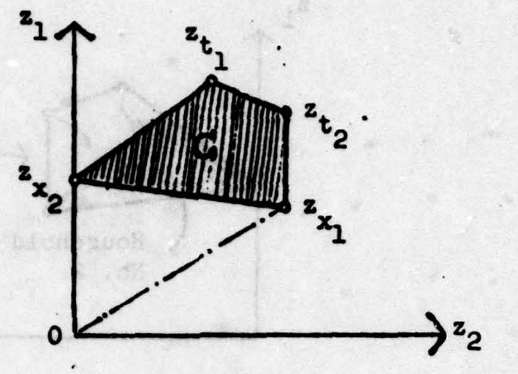
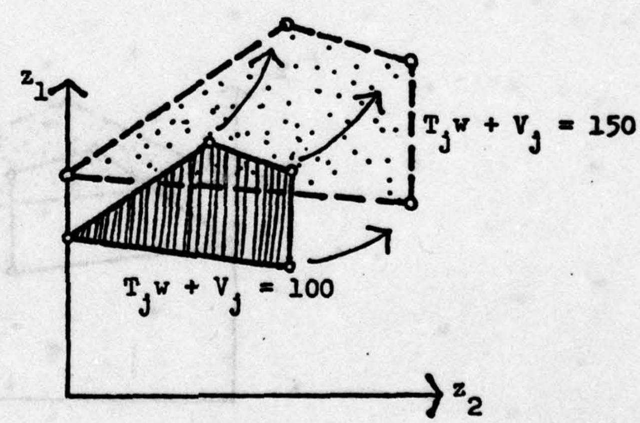


Figure 1



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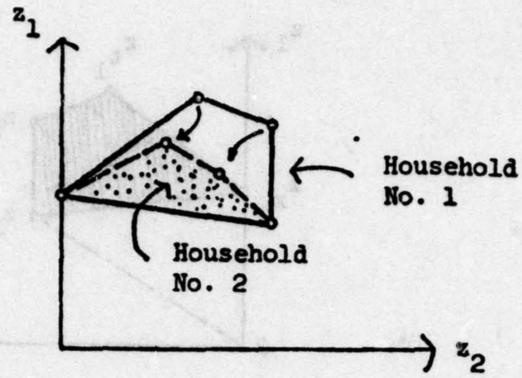

Figure 3

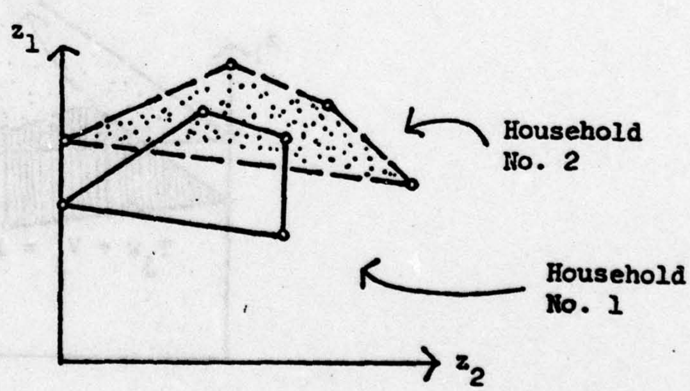

Figure 4

5. Efficient Household Production

Having discussed the properties of the feasible set in H, we proceed next to discuss the efficient frontier of C. Since C is the feasible set in H the producers problem is

$$\begin{aligned} &\text{maximize} \quad U(z,E) \\ &\text{subject to} \quad z\varepsilon C \end{aligned} \tag{13}$$

where "ε" refers to "element of" so that "zεC" refers to the z which are elements of C. Assuming that U is differentiable with $\frac{\partial U(z,E)}{\partial z} > 0$ for all i and any E, an optimum point z* will necessarily be a boundary point of C.

We illustrate this via Figure 4 for which

$$\frac{\partial U(z,E)}{\partial z} \equiv \left[\frac{\partial U(z,E)}{\partial z_1}, \quad \frac{\partial U(z,E)}{\partial z_2} \right]^T > 0, \tag{14}$$

so that utility increases in each of the indicated dimensions, separately, for each E. Evidently U(z,E) is on the efficiency frontier of Household 1 but not of Household 2. Relating this to Figure 5 (see page 21) we may say that the increase in wage rate for the latter caused it to move away from its previous efficiency frontier. As a result of its increased wage from w = 10 to w = 12.5 the cost of producing z_{ij} increases and the satisfaction level is correspondingly lowered in these dimensions due to the constancy of the income constraint $T_j w + V_j$.

We shall return to this analysis at a later point but here we can now formally define the efficient set for household productions as

$$\{y : By\varepsilon C^*, \ (we^T, p^T)y = V_j + T_j w, \ y > 0\} \tag{15}$$

where C* denotes the efficiency frontier in C.

6. Shadow Prices of Household Products

Now we denote these efficient z^* for the household production level as $z^*\epsilon C^*$ and then seek to locate the necessary set of inputs y^* (i.e., x^* and t^*) which satisfy the income set and produce efficient outputs. This problem can then be formulated in an ordinary linear programming form

$$\begin{aligned} &\text{minimize} && [we^T, p^T]y \\ &\text{subject to} && \\ & && By = z^* \\ & && y \geq 0,\ z^*\epsilon C^* \end{aligned} \tag{16}$$

with optimality clearly obtained when $T_j w + V_j = [we^T, p^T]y^*$.

From (16) we have the dual problem

$$\begin{aligned} &\text{maximize} && \pi z^* \\ &\text{subject to} && \\ & && \pi B \leq [we^T, p^T] \end{aligned} \tag{17}$$

where the components of π are otherwise unrestricted. Note that the usual algorithms supply solutions to both problems simultaneously, i.e., if the solutions for both problems exist then we can obtain the optimal π^* values as shadow prices for the household's production output and at the same time we obtain the y^* values as the optimal input.

Via our preceding development, it also follows from the linear programming duality theory, at an optimum we have for each household j:

$$V_j + T_j w = \sum_{\ell,k} (wt^*_{\ell j} + p_k x^*_{kj}) = \sum_i \pi^*_{ij} z^*_{ij}, \tag{18}$$

where π^*_{ij} are the shadow prices of the jth household products.

It is of particular interest to develop the close correspondence between (18) and the aggregate "weighted average shadow prices" developed by Terleckyj in [3]. To do this we proceed to aggregate (18) over all households in the economy,

$$\sum_j (V_j + T_j w) = \sum_j \sum_{\ell k} (wt^*_{\ell j} + p_k x^*_{kj}) = \sum_j \sum_i \pi^*_{ij} z^*_{ij} \quad (19)$$

Now letting

$$\tilde{V} = \sum_j V_j \quad (20)$$

$$\tilde{T} = \sum_j T_j \quad (21)$$

$$\tilde{T}_\ell = \sum_j t^*_{\ell j} \quad (22)$$

$$\tilde{x}_k = \sum_j x^*_{kj} \quad (23)$$

$$\tilde{z}_i = \sum_j z^*_{ij} \quad (24)$$

we finally obtain

$$\tilde{V} + \tilde{T}w = w(\sum_\ell \tilde{T}_\ell) + \sum_k p_k \tilde{x}_k = \sum_i \tilde{\pi}_i \tilde{z}_i, \quad (25)$$

where

$$\tilde{\pi}_i = \frac{\sum_i \pi^*_{ij} z^*_{ij}}{\sum_j z^*_{ij}} = \sum_j \pi^*_{ij} \frac{z^*_{ij}}{\sum_j z^*_{ij}}$$

$$= \sum_j \mu_j \pi^*_{ij}, \quad (26)$$

and $\mu_j \geq 0$, $\sum_j \mu_j = 1$. Hence, $\tilde{\pi}_i$ are (weighted) average shadow prices as interpreted by Terleckyj [8];[7/] and thus might be used to construct price indexes for the quality of life products produced within households.

7/ Terleckyj in [3] eq. (11) stipulates a special case of (25). The latter, i.e., (25), provides a generalization to the aggregate average shadow price concept advanced by Terleckyj.

7. Substitution and Complementarity of Household Products

The unrestricted signs of the shadow prices obtained in program (17) need further interpretation.[8/] In the customary interpretation of shadow price vectors a typical component π_i^* reflects the sensitivity of the cost function in (16) to marginal variation in the ith product z_i^* of a household. Now, however, we want to arrange matters so that adjustments in z^* are always made so that we consider only $z^* \epsilon C^*$ for which $[we^T, p^T]y = T_j w + V_j$. Thus to stay on the surface of the production possibility set we apply the following condition

$$\sum_i \Delta z_i^* \pi_i^* = 0 \tag{27}$$

where Δz_i^* denotes the marginal change in z_i^*.

We now proceed less formally to consider an application of the orthogonality condition (27) to simultaneous changes only in two household products, say z_i^* and z_k^*. Then

$$\Delta z_i^* \pi_i^* + \Delta z_k^* \pi_k^* = 0 \tag{27.a}$$

or

$$-\frac{\Delta z_i^*}{\Delta z_k^*} = \frac{\pi_k^*}{\pi_i^*}$$

There are now the following possible cases:

(i) If $\pi_i^* = 0$ then at least one of π_k^* or $\Delta z_k^* = 0$. See (27a). If $\pi_k^* \neq 0$ then $\Delta z_k^* = 0$ and substitution does not occur.

(ii) If $\pi_i^* \neq 0$ and $\pi_k^* \neq 0$ have the <u>same</u> signs

$$\Delta z_i^* > 0 \text{ implies } \Delta z_k^* < 0$$

$$(\Delta z_i^* < 0 \text{ implies } \Delta z_k^* > 0)$$

8/ If desired we can replace $By = z^*$ with $By \geq z^*$ in (16) in order to restrict the π variables in (17) to non-negative values.

In other words, we have the case in which the ith and kth household products are substitutes.

(iii) If $\pi_i^* \neq 0$ and $\pi_k^* \neq 0$ have <u>different</u> signs

$$\Delta z_i^* > 0 \text{ implies } \Delta z_k^* > 0$$

$$(\Delta z_i^* < 0 \text{ implies } \Delta z_k^* < 0) \quad .$$

In this case, complementarity holds between the ith and kth household products and again substitution does not occur.

Excluding those household products with zero for shadow price indices we may partition the remaining household products into two sets

$$Z^+ = \{z_i : \pi_i^* > 0\}$$

$$Z^- = \{z_j : \pi_j^* < 0\}.$$

Only products which belong to the same set are substitutable. Given our assumption that both (16) and (17) have solutions, then Z^+ will not be empty. We can, therefore, replace (27) by the following two orthogonality conditions,

$$\sum_i \Delta z_i^* \pi_i^* = 0 \qquad z_i^* \varepsilon Z^+ \qquad (27.b)$$

$$\sum_j \Delta z_j^* \pi_j^* = 0 \qquad z_j^* \varepsilon Z^- \qquad (27.c)$$

The rate of substitution given by π_i^*/π_k^* reflects the marginal rates of trade-off between household products i and k, which pairs are constrained to be considered only for products in Z^+ or in Z^- only.

8. A Goal Focusing Approach

A major difficulty with operationalizing our formulation is the lack of knowledge on the household utility function U(z,E). One approach would perhaps be via revealed preferences.[9] Here we consider instead a "goal focusing" approach. Suppose, for instance, that we have some desired preference relations, e.g., 2 weeks vacation for every 25 weeks of employment, 250 square feet of floor area per every additional child, etc. Then we could consider desired proportions

$$z_i = \rho_{ir} z_r , \tag{28}$$

for specified pairs, i and r, where ρ_{ir} is a scalar constant reflecting the desired relationship between z_i and z_r. The idea is to replace the non-operational U(z,E) by (23) relaxed to a goal statement which we may refer to as a "goal artifact."

Bearing in mind, however, that generally there are many efficient points and a utility function focuses our attention on only a few of them, we have performed a similar focusing by the use of these goal artifacts. In addition, we may employ other constraints to aid in the focusing process. For example, we may utilize upper and lower bounds on the inputs via

$$\underline{h} \leq Gy \leq \overline{h}, \tag{29}$$

where G is a matrix and $\underline{h}$ and $\overline{h}$ are corresponding vectors. We shall also refer to these as part of our goal focusing approach.

Thus we shall be replacing the utility function by a functional of goal deviations. We also wish to ensure that we are at the efficiency frontier. We can do this by inserting a further term involving a scalar, $-\eta$, in the functional and adjoining the relation

$$\eta = e^T z$$

9/ See, e.g., Samuelson [6], chapter VI.

where e and z are column vectors with e^T indicating the transposition of a vector with unity for all its elements. Because $z \geq 0$, it follows that η is also non-negative. Incorporating the scalar, η, into the functional we shall be minimizing on $-\eta$ and hence maximizing η. Multiplying this by a suitably large positive constant will ensure attainment of the efficiency frontier for the z^{*}.

In this approach we want to be "as close as possible" to all goals ("Goal focusing") but with all z_j <u>also</u> at the highest possible level.10/ An illustrative case is given in Figure 6 where with $\rho_{12} = 1$ we seek to be as close as possible to $z_1 = z_2$. Note that this condition is satisfied for all points where the ray for which $z_1 = z_2$ intersects the shaded interior which is the feasible region. Here we (a) want to move as far as possible to the northeast along this ray and (b) be on an efficiency frontier. We do this via the following goal focusing model,

$$\text{minimize} \quad -\alpha\eta + \sum_{ir} (\sigma^{+}_{ir}\, \delta^{+}_{ir} + \sigma^{-}_{ir}\, \delta^{-}_{ir}) \tag{30}$$

subject to

$$z_i - \rho_{ir} z_r - \delta^{+}_{ir} + \delta^{-}_{ir} = 0,$$

$$e^T z = \eta$$

$$By = z$$

$$[we^T, p^T]y = V + Tw$$

$$\underline{h} \leq Gy \leq \overline{h}$$

$$z, y, \delta^{+}_{ir}, \delta^{-}_{ir} \geq 0$$

10/ This is done for <u>each</u> household to obtain contact with the analysis in [3] and not because efficiency is to be distinguished from optimality at the individual household level. For considerations relating efficient points and goal programming, see Appendix B and Chapter X in [2]. See also A. Charnes and W.W. Cooper "Goal Programming and Multiple Objective Optimizations." Management Sciences Research Report 381, Pittsburgh, Pa., 15213: Graduate School of Industrial Administration, Carnegie-Mellon University, November 1975.

where $\alpha = \sum_{ir} (\sigma^+_{ir} \delta^+_{ir})$, and σ^+_{ir}, σ^-_{ir} are non-negative weights associated with the positive and negative deviations $\delta^+_{ir}, \delta^-_{ir} \geq 0$, from the goal in the goal focus constraint associated with z_i.

Application of (30) is not limited to a single household. It could be applied to a collective set of households with each household j submitting its own $\{\rho_{irj}\}$. In that case, each household would have its own set of constraints, such as those displayed in (30). The socioeconomic system in which these households might also be subject to additional sets of constraints which emanate from scarce resources or national (system-wide) goals and priorities. Two such examples are

$$\sum_j x_{kj} \leq X_k, \tag{31}$$

designating a bound on the kth activity level in the economy, and a set of policy constraints

$$\sum_j z_{ij} \leq z_i, \tag{32}$$

denoting, for example, a desired upper bound on energy consumption or other explicit policy goals.

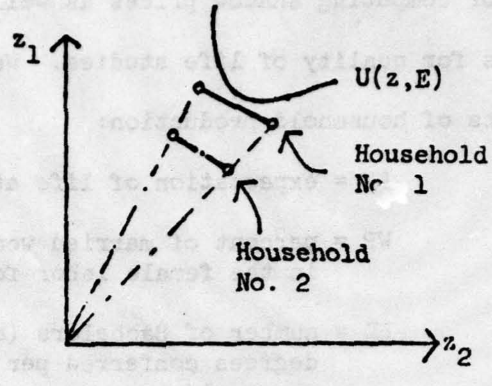

Figure 5

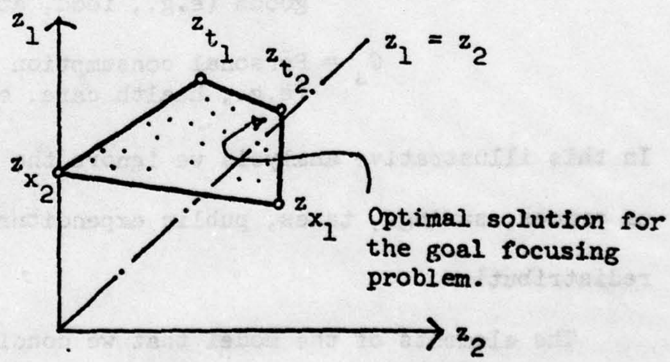

Figure 6

9. Illustrative Example

We now want to develop a numerical example in order to illustrate the procedure for computing shadow prices as well as their interpretation and implications for quality of life studies. We consider in this section three outputs of household production:

LE = expectation of life at birth in years

WP = percent of married women (with husband present) in the female labor force

BD = number of Bachelors (or first professional) degrees conferred per 1000 persons age 23 years old.

These outputs are here assumed to depend on household consumption of durable goods, nondurable goods and services. The distribution of personal consumption expenditures give rise to the budget constraint

$$C_d + C_n + C_s = \left\{ \begin{array}{c} \text{personal consumption} \\ \text{expenditures budget} \end{array} \right\} \tag{33}$$

where

C_d = Personal consumption expenditures on durable goods (e.g., housing, transportation)

C_n = Personal consumption expenditures on non-durable goods (e.g., food, attire)

C_s = Personal consumption expenditures on services (e.g., health care, education)

In this illustrative analysis we ignore the household's time input as well as prices, savings, taxes, public expenditures and other aspects of income redistribution.

The elements of the model that we consider consist of the following relations

$$\begin{bmatrix} LE \\ WP \\ BD \end{bmatrix} = \begin{bmatrix} 65.39 \\ 22.69 \\ 0 \end{bmatrix} + \begin{bmatrix} .00547 & .00278 & 0 \\ .01535 & 8 & .04229 \\ 0 & 8 & .263 \end{bmatrix} \begin{bmatrix} C_d \\ C_n \\ C_s \end{bmatrix} \tag{34}$$

which are based on econometric estimates obtained from USA data in [12] for the years 1947 through 1970, incorporating only statistically significant coefficients. These are intended, of course, only for illustrative purpose. The first equation shows life expectancy at birth as a function of expenditures on durable and non-durable goods. The second equation shows the participation of married females in the labor force as a function of consumption of durable goods and services. The third equation shows education to be directly proportional to expenditures on services.

Table 1 summarizes data reflecting inputs (consumption of goods and services) and outputs of household production for the U.S. in 1950, 1960 and 1970. The output data of the table are then used to construct proportional constraints of the form given by (28) in order to describe the relationship between various household products during each year. Thus, we have

$$\begin{aligned} (BD_{1960}) &= 3.34 \quad (WP_{1960}) \\ (BD_{1960}) &= 2.61 \quad (LE_{1960}) \\ (LE_{1960}) &= 1.28 \quad (WP_{1960}) \end{aligned} \tag{35}$$

which here describe a consistent[11/] set of proportions between the various household products in 1960. See (28).

The figures for consumption expenditures in Table 1 are presented in 1958 constant dollars. These can be used, in turn, to define minimum levels of each expenditure category, e.g.,

$$C_d \geq 200,\ C_n \geq 700,\ C_s \geq 500 \tag{36}$$

as additional constraints for use in program (30). See also (29).

11/ Of course this set need not be consistent. See discussion in previous section.

Our purpose in this example is to compute the shadow prices and trade-offs among the above household products. In the absence of an explicit utility function we proceed to locate a possible point of tangency on the household production possibility set. See Figure 4. Recalling that we want to use the goal focusing model (30) in lieu of (13) we show via Table 1 how the relevant constraints can be constructed with 1960 data. Under each of the variables at the top of the table we list the coefficients for use in formulating the relations for (30) as follows. First we list the coefficients which enter into the objective function to be minimized. Then we list the coefficients for the constraints. The first set of constraints reflects the output proportion goal artifacts of equations (35) in goal programming form. The second set of constraints describes the weights used to attain the efficiency frontier.[12/] The third set of constraints describes the transformation of consumer expenditures into household products using the econometric model (34). The next constraint is the $1,723 consumer budget limit and the final three constraints reflect the lower bounds on various expenditures provided in (36).

The results of applying program (30) to 1960 U.S.data[13/] are summarized in the third column of Table 3. The first and second columns of the table compare the actual levels of various household products with those predicted by the model via equations (34). Note that while the model slightly overpredicts the participation rate of married women (WP) it underpredicts the life expectancey (LE) and education (BD). The efficient solution, obtained via program (30), is much closer to the actual indicators of 1960 than the

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12/ We have used an altered form of (30) with $\alpha=1$ and the constraint $e^T z=\eta$ replaced by $\varepsilon e^T z=\eta$ in which "ε" is here given the value of $\varepsilon=.005$. This is equivalent to a choice of $\alpha=.005$ in program (30).

13/ In this analysis we use unity for all nonnegative discretionary weights associated with positive and negative deviations from the goal artifacts in (35). Variations in these weight may, of course, produce different results.

predicted values, though it shows a slight decrease in both LE and WP and an increase in BD. This result also requires some reallocation of expenditures: a reduction in the expenditure level on durable goods from 243 to 200 coupled with a $33.24 and $15.76 increase in consumption of nondurable goods and services, respectively.

The shadow prices are displayed in the fourth column of Table 3. These were obtained from an application of program (17) by using the efficient solution for (30), as displayed in Table 3, for its objective function and the coefficients of (34) and (36) to form the constraint set. We next note that the shadow prices of LE and BD are negative while the shadow price assoicated with WP is positive. Hence, from our development in section 7, we can conclude that only substitutions between LE and BD are feasible, whereas, WP is complementary with respect to LE and BD. The rate of substitution or the trade-off between years of life expectance and number of Bachelors degrees is 359.71/13.94 or 25.8 Bachelors degrees (per 1000 persons age 23 years old) for every single yearoof life expectancy at birth.

The complementarity between the participation of married women (with husbands present) in the female labor force (WP) and the level of education of the 23-year-old cohort suggests that generally the more educated a society is, at the college level, the greater the participation of married women in the labor market. This might be considered as an indicator of a social trend toward "economic equality" of the sexes and the relation studied accordingly. The shadow prices suggest that each additional Bachelors degrees may be complemented by an increase of .22 percent (= - 13.94/63.04) in the participation rates of married women.

The complementarity between WP and the expectation of life at brith might be explained as follows. Higher life expectancy implies a longer life for married man and therefore a higher proportion of married women, with children who have left the household, thus available for employment. Alternatively, it might be argued that working couples enjoy a higher level of income and may therefore afford better care for their children, which, in turn, may be associated with a higher expectation of life at birth. The shadow prices suggest that each percent change in women participation may be complemented with a .175 change in years of life expectancy.

We should like to caution the reader, at this point, that the above interpretations are only plausible. The figures reflect what might be called the latent socioeconomic structure in households and are not the manipulative possibilities that terms like "substitution ratios" and "shadow prices" suggest. Actual variation in household products of a complementary nature are not feasible within a given household production possibility set. See equations (27a)-(27b).

The feasibility of marginal rates of substitution between years of life expectancy and the number of Bachelors degrees which are obtained from our model applied to (34) might be assoicated with the fact that no common resources are utilized in the household production of LE and BD. Reallocation of resources between the products of each household would then imply a substitution between LE and BD.

Finally, in Tables 4 and 5 we summarize results for the data of 1950 and 1970. In both cases the shadow prices are the same as those which were obtained with the 1960 data.[14] This bears on the issue of the sensitivity of our model results to data variations. We therefore used parametric programming to check the stability of the shadow prices when

[14] Even when the tight constraint $C_d \geq 200$ was relaxed the same shadow prices were obtained.

WP was held constant and LE and BD were substituted at a rate 1/25.8 = 13.94/359.71. The values of the shadow prices remain constant over the following range of life expectancy

$$69.15 > LE_{1950} \geq 69.00$$

$$70.08 > LE_{1960} \geq 69.30$$

$$73.98 > LE_{1970} \geq 70.56$$

When LE was made equal to the value on the left the 1/25.8 trade-off rate between LE and BD was replaced by a new 1/48.1 = 3.80/182.81. Concomitantly, the tight constraint $C_d = 200$ was replaced by $C_n = 700$.

10. Conclusions

In this set of preliminary notes we have attempted to sketch a framework for use in studying goals accounting systems. This has involved a new development which might be called a "goal focusing approach" to enable us to secure the values of selected "efficient substitution ratios" without recourse to an explicitly formulated "utility function." The particular device used in this paper relied on specified output proportions which we here regard as "goal artifacts." Other parts of these artifacts used upper and lower bounding constraints as part of our goal focusing approach. Still other devices could be employed. In any case, we have now outlined a way to explicitly examine particular portions of the efficiency frontier with full access to readily available computational algorithms and programs.

Other parts of this paper address issues such as aggregation of shadow prices, partitioning of household outputs into relevant substitution classes and weighting of "goals" and "goal focusing artifacts." These are all problems requiring further attention. There are also further opportunities such as extensions involving e.g., growth and/or other kinds of dynamic considerations along with the possibility of including cohort analyses and more explicit attention to household time budgets, etc. As noted in our introduction, the purpose of this paper is to provide a framework for ongoing endeavors of the research consortium such as analysis of trade-offs regarding future retirements in the U.S. or the development of behavioral data through survey research.

References

[1] Becker, G.S., "A Theory of the Allocation of Time," The Economic Journal, Vol. 75, No. 299, September 1965.

[2] Charnes, A. and W.W. Cooper, Management Models and Industrial Applications of Linear Programming, John Wiley and Sons, 1961.

[3] Johnston, J. Econometric Methods, McGraw-Hill, New York, 1972.

[4] Lancaster, K., Consumer Demand, Columbia University Press, 1971.

[5] Michael, R.T., and G.S. Becker "On the New Theory of Consumer Behavior" Swedish Journal of Economics, Dec. 1973, Vol. 75, pp. 378-396.

[6] Samuelson, P.A., Foundations of Economic Analysis, Harvard University Press, Cambridge, Mass., 1948, Chapter VI.

[7] Stigler, G.J., and G.S. Becker "De Gustibus Non Est Disputandum," The American Economic Review, March 1977, Vol 67, pp. 76-90.

[8] Terleckyj, N.E., An Economic Analysis of the Quality of Life, (extended outline), National Planning Association, November 26, 1975.

[9] ______, Improvements in the Quality of Life, National Planning Association, Washington, D.C., 1975.

[10] Theil, H., Principles of Econometrics, John Wiley and Sons, New York, 1971. Chapters 9 and 10.

[11] Tinbergen, J. Statistical Testing of Business - Cycle Theories, Vols. I and II, League of Nations Geneva, 1939.

[12] U.S. Bureau of the Census, Historical Statistics of the United States, Colonial Times to 1970, Bicentennial Edition, Parts 1 and 2, Washington, D.C., 1976.

Table 1

Quality of life indicators and Personal Consumption expenditures patterns of the U.S.A. for 1950, 1960 and 1970

Quality of Life Indicators:	1950	1960	1970
Years of Life expectancy (LE)	68.2	69.7	70.9
Percent of Married Women Participation (WP)	48.0	54.4	58.8
Number of Bachelor (BD) Degrees	154*	182	223

Personal Consumption expenditures:
(in 1958 constant dollars)

	1950	1960	1970
Durable goods (C_d)	229	248	409
Nondurable goods (C_n)	752	802	1008
Services (C_s)	539	673	914
Total	1520	1723**	2331

* The actual 1950 value for BD is 182 which is extremely high compared to the graduation rates in the 50's. We have chosen instead the value for 1949.

** Note that the personnel consumption expenditures budget ($1723) is less than the per-capita disposable personal income for that year ($1883) and substantially below the total personal income figure ($2157), all in 1958 dollars.

Table 2

Matrix of Constraints for the Goal Programming Problem (30)

LE	WP	BD	C_d	C_n	C_s	η	δ_1^+	δ_2^+	δ_3^+	δ_1^-	δ_2^-	δ_3^-		
Objective Function:														
0	0	0	0	0	0	-1	1	1	1	1	1	1		
Constraints:														
0	-3.34	1					-1	0	0	1	0	0	=	0
-2.61	0	1					0	-1	0	0	1	0	=	0
1	-1.28	0					0	0	-1	0	0	1	=	0
.005	.005	.005				-1							=	0
-1	0	0	.00547	.00278	0								=	-65.89
0	-1	0	.01535	0	.04229								=	--22.69
0	0	-1	0	0	.263								=	0
			1	1	1								=	1723
			1	0	0								≥	200
			0	1	0								≥	700
			0	0	1								≥	500

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Table 3

Household Products and Shadow Prices for 1960

	Actual Household Products for 1960	Predicted Household Products by Econometrics Model (34)	Efficient Solution z^* from Goal Focusing Model (30)	Shadow Prices π^* from Program (17)
LE	69.70	69.47	69.30	-359.71
WP	54.40	54.96	54.84	63.04
BD	182.00	177.00	180.88	-13.94

	Actual Expenditures	Efficient Expenditures y^*	Lower Bounds
C_d	248	200	≥ 200
C_n	800	835.24	≥ 700
C_s	673	688.76	≥ 500
	1723	1723	

Table 4

Household Products and Shadow Prices for 1950

	Actual Household Products for 1950	Predicted Household Products by Econometrics Model (34)	Efficient Solution z* from Goal Focusing Model (30)	Shadow Prices π* from Program (17)
LE	68.2	69.22	69.00	-359.71
WP	48.0	49.00	50.83	63.04
BD	154	141.76	155.94	-13.94

	Actual Expenditures	Efficient Expenditures y*	Lower Bounds
C_d	229	200	≥ 200
C_n	752	727.05	≥ 700
C_s	539	592.95	≥ 500
	1520	1520	

Table 5

Household Products and Shadow Prices for 1970

	Actual Household Products for 1960	Predicted Household Products by Econometrics Model (34)	Efficient Solution z* from Goal Focusing Model (30)	Shadow Prices π* from Program (17)
LE	70.9	70.92	70.56	-359.71
WP	58.8	67.62	61.39	63.04
BD	223	240.38	221.57	-13.94

	Actual Expenditures	Efficient Expenditures y*	Lower Bounds
C_d	409	200	≥ 200
C_n	1008	1288.54	≥ 700
C_s	914	842.46	≥ 500
	2331	2331	

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The ideas of goal programming are combined with efficient point considerations to yield a computational approach for use in tradeoff analysis and evaluations for goals accounting analysis. The orientation is toward an efficient point which comes closest to a set of interrelated goals. Attention is thereby restricted to a subset of the possible efficient points. Shadow prices to guide further analyses are also automatically supplied for guiding subsequent tradeoff analyses. Relations to the utility theoretic and household production

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considerations of Gary Becker and Kelvin Lancaster are indicated and an illustrative example involving econometric estimates for pertinent quality-of-life dimensions is supplied.