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THE DESIGN OF ADAPTIVE RECEIVERS FOR OPTIMUM DETECTION OF TEMPO--ETC(U)  
JAN 63 R E WILLIAMS

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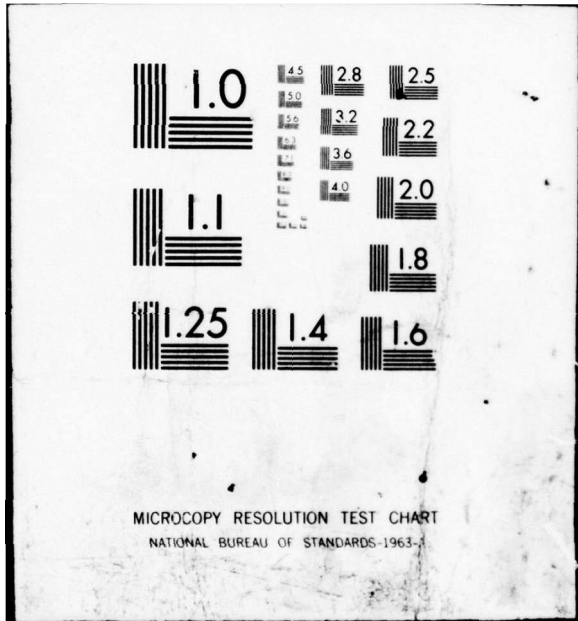
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The Design of Adaptive Receivers  
for Optimum Detection  
of  
Temporally Distorted Signals

by  
Ross E. Williams

Hudson Laboratories, Columbia  
University Technical  
Memorandum No. 65

Contract Nonr-266(84)

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✓ CONTRACT Nonr-266(84)

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Dobbs Ferry, New York

Robert A. Frosch  
Director

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9 Technical Memorandum No. 65

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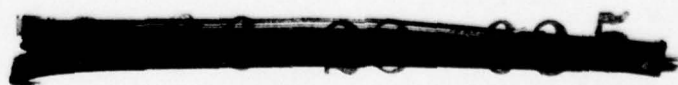
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It was shown in the preceding paper<sup>1</sup> that an optimum receiver for a signal subject to temporal distortions is an adaptive correlator which maximizes the quantity

$$\frac{r}{\sigma_n} \sum_{i=1}^{2WT} f_i \left( \frac{i}{2W} \right) s_i \left[ k_o \left( \frac{i}{2W} \right) - \tau_o - \tau_i \left( \frac{i}{2W} \right) \right] - \frac{1}{2\sigma_r} \sum_{i,j=1}^{2WT} \frac{\tau_i^c \rho_{i,j}^c \tau_j}{|\rho|} = \text{Max.} \quad (1)$$

where  $f_i \left( \frac{i}{2W} \right)$  = sample at time  $t = \frac{i}{2W}$  of the received waveform  $f(t)$ ,

where  $f(t) = n(t) + r s(t)$  and

$n(t)$  = additive noise

$r$  = attenuation coefficient

$s(t)$  = signal transmitted.

$s_i \left( \frac{i}{2W} \right)$  = sample of signal  $s(t)$  at time  $t = \frac{i}{2W}$

$W$  = bandwidth of receiver and  $s(t)$

$\tau_o$  = time delay due to round-trip transit time of signal, averaged over signal duration.

$k_o$  = average time base scale factor imposed by motion of target.

$\tau_i \left( \frac{i}{2W} \right)$  = sample at time  $t = \frac{i}{2W}$  of the time base distortion factor  $\tau(t)$  imposed by motion of the medium and target accelerations.

$\rho_{i,j}^c$  = cofactor of the  $(i,j)$  component of the normalized correlation matrix for  $\tau(t)$ .

$|\rho|$  = determinant of the normalized correlation matrix for  $\tau(t)$ .

<sup>1</sup> Ross E. Williams, "Criteria for an Optimum Receiver for Use with a Temporally Unstable Medium" (Hudson Laboratories, Columbia University Technical Memorandum No. 64, July 24, 1962; revised January 14, 1963).

$\sigma_n^2$  = variance of the additive noise probability distribution.  
 $\sigma_\tau^2$  = variance of the  $\tau(t)$  probability distribution.

This memorandum will be concerned with electronic and optical methods to determine the value of parameters in Eq. (1) for which the right side of the equation has maximum value. In other words, it is concerned with the design of an optimum receiver which Technical Memorandum No. 64<sup>1</sup> attempted to show was an adaptive receiver in the following sense: The right side of Eq. (1) will have maximum value when the timebase of the received waveform, which contains additive noise as well as signal, is adjusted to that of the transmitted signal\* under the constraints of the known  $\tau(t)$  correlation matrix ( $\rho$ ) and the variances  $\sigma_n^2$  and  $\sigma_\tau^2$ .

Three somewhat dissimilar embodiments of adaptive receivers are described in this memorandum. They are by no means inclusive of all the possibilities for adaptive timebase compensation, but they do illustrate the wide range of design and hardware approaches to this problem. It should be noted that the function of these receivers is to establish the presence or absence of a target in a fluctuating medium which provides an adequate signal power, but an inadequate coherence time, for conventional detection techniques.

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\* Obviously it is immaterial as to whether the received waveform is adjusted to the transmitted signal, as stated here and instrumented in Fig. 3, or the transmitted signal adjusted to the received waveform, as specified in Eq. (1).

## I Adaptive Deltic Correlator

### A. Conventional Deltic

The usual Deltic cross correlator<sup>2</sup> uses two or more recirculating delay lines for dynamic storage and multiplication of a reference function and a signal function in a time-compressed format. Figure 1 shows the essential components of a Deltic in block diagram form. The upper delay line of Fig. 1 receives hard clipped samples of the signal function at sample intervals  $T = (N + 1) \Delta$ , where  $N$  is the total number of samples which can be stored on the line at one time and  $\Delta$  is the time interval per sample on the line. The output of the line is connected back to the input so that at a time  $N\Delta$  seconds later the line will contain exactly the same information in the same order. At a time  $(N + 1) \Delta$  seconds later all samples have moved to the right one sample position and a new sample is received from the left, replacing the oldest sample in the line. Thus the signal function in this line precesses by an amount  $\Delta$  to the right on each complete circulation in the sample interval  $T = (N + 1) \Delta$ , and is updated by receiving one new sample on each complete circulation. The lower delay line in Fig. 1 holds the reference function, but this function is neither precessed nor updated as it makes each circulation. This line is exactly  $(N + 1) \Delta$  seconds long, so that the information merely circulates without precessing in the signal sample interval  $T = (N + 1) \Delta$ . The 0 and 1 outputs of the two lines are connected to two AND gates, which in turn are connected through an OR gate to a counter. A pulse input to the counter signifies an identical polarity (0 or 1)

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<sup>2</sup> V. C. Anderson, Technical Memorandum No. 37 (NR-014-903), Harvard University Acoustics Research Laboratory, January 5, 1956.

at the instantaneous output of the two lines. This is a digital multiplication of the form  $r(n\Delta)s(n\Delta+\tau_0)$ , where  $n$  can have any value from 1 to  $(N+1)$  and  $\tau_0$  represents the state of relative precession between the two delay lines. The counter output provides:

$$\phi(\tau_0) = \sum_{n=1}^{N+1} r(n\Delta)s(n\Delta+\tau_0) \quad (2)$$

for each circulation of  $(N+1)\Delta$  seconds. Thus a correlation coefficient  $\phi(\tau_0)$  for hard-clipped signals is formed in each sample interval  $T = (N+1)\Delta$ , where  $T$  is adequately short for proper sampling (less than one half the period of the highest frequency component of the incoming signal function).

#### B. Adaptive Deltic

The Deltic correlator can be modified to force it to act as an adaptive correlator, as shown in Fig. 2. Here the signal function delay line has a number of closely spaced outputs, one for each bit position near the end of the line. Adjustment of the signal function timebase is accomplished by varying the switch position with the proper rate, amplitude, and polarity (with respect to the center switch tap) to remove the timebase distortions imposed by the medium as the signal function is reinserted at the beginning of the delay line. Because the sampling interval  $T = (N+1)\Delta$  is short compared to the characteristic periods of the signal function and the signal samples are therefore redundant, the effect of the switch adding or removing samples from the signal will not be detrimental to the correlation function. Improvement in the correlation output at point A of Fig. 2 serves as a monitor on the progress of the correcting action of the switch. The signal

and reference functions are recirculated until the output at point A is large enough to stop the iterative correction and provide a true adaptive correlator output.

The exact operation of the Adaptive Deltic is better described in terms of the more detailed diagram of Fig. 3. Each output tap on the signal function delay line is connected to a digital multiplier of the type shown in Fig. 1, described above, whose other input comes from the reference delay line output. Each multiplier feeds an integrator and envelope detector circuit of adjustable time constant  $T'$ . The outputs of all the integrators and envelope detectors provide multiple inputs to a "greatest of" selector of the type shown in Fig. 4. The most positive signal at the control grids of the tubes in this circuit will assume control by cutting off all other tubes, and a coded output from this circuit indicates which of the various signal delay line output taps provided, in conjunction with the reference function output, the largest short term correlation coefficient

$$\phi(\tau_0) = \sum_{n=1}^{N'} r(n\Delta) s(n\Delta + \tau_0) \quad (3)$$

where  $N'\Delta$  represents the adjustable time constant of the integrators. This coded output, which measures the location of the proper output tap on  $S_2$  and therefore the timebase distortion factor  $r(t)$ , is sent to a third recirculating delay line with total delay equal to  $(N + 1)\Delta$ . This delay line is called the  $\tau(t)$  memory, and serves to program the electronic switch  $S_2$  on subsequent recirculations so as to control the effective length of the signal function delay line in accordance with the just measured values of  $\tau(t)$ . Since the information in the  $\tau(t)$  memory circulates synchronously

with the signal function, its coded output can operate switch  $S_2$  in such a way that the local timebase distortions of the signal function are corrected instantaneously (through the electronic switching operation) as they are fed back to the signal function delay line input. Thus it is possible to extend or compress the effective length of the signal function delay line under control of the  $\tau(t)$  memory. The correction is not fully accomplished in one circulation of the lines; rather, it is an iterative operation in which improvements are made on the signal function on each circulation until a maximum value is attained, and no further improvement occurs, in the long term correlation coefficient at the output point A, Fig. 3.

The steps in the iterative process proceed as follows:

1. Scanning operation. The tap at  $N\Delta$  on the signal function delay line is connected to one pole of switch  $S_1$  for normal Deltic operation until sufficient precession of the distorted signal function has occurred to bring it into approximate alignment with the reference function.

2. Mixed integration operation. When the sum  $\Phi(\tau_0)$  of any sequential combination of the short term ( $T' = N'\Delta$ ) integrator outputs  $\phi_i(\tau_{0,i})$ , Eq. 3, performed over the full signal span by the  $T = (N + 1)\Delta$  integrator,

$$\Phi(\tau_0) = \sum_{i=1}^{(N+1)/N'} \phi_i(\tau_{0,i}) \quad (4)$$

where  $\tau_0 \equiv \langle \tau_{0,i} \rangle_i$  = average over the full signal time span of the short term, coherent correlation time displacements

becomes large enough to exceed a predetermined threshold, even though widely separated  $S_2$  taps are used for the short term integrations  $\phi_i(\tau_{0,i})$

making up the sum,  $S_1$  is switched to its other position in which it will accept a feedback from the tap of  $S_2$  that is selected by the  $\tau(t)$  memory. This summation is performed by the summing amplifier (Fig. 4) whose inputs are obtained one at a time from the gates of the "greatest of" circuit. Only that gate connected to a plate circuit below ground potential will allow the corresponding grid signal to pass to the summing amplifier. This incoherent summing of many short integration time, coherent correlations is sometimes referred to as a mixed integration. It is used here as a partial improvement on the usual long term coherent correlation as a means of deciding that a signal may be present and a change to the iterative procedure of timebase adjustment is warranted. The T integrator, Fig. 3, is followed by a discriminator which operates  $S_1$  when the mixed integration sum  $\Phi(\tau_0)$ , Eq. 4, exceeds the predetermined threshold. By virtue of the taps on  $S_2$ , this mixed integration has the advantage of achieving slightly different  $\tau_{0,i}$  time displacements in the sum of Eq. 4, and therefore one characteristic of a distorted time base. The mixed integration also decreases the frequency resolution of the signal by using much shorter integration times  $N'\Delta$  in each of the  $\phi_i(\tau_{c,i})$ , Eq. 3, and therefore lessens the attenuation of the correlation peak which would otherwise result from small distortions of the time base. However, in itself the mixed integrator is not an optimum receiver, as defined by Eq. 1, because it includes only displacement effects and not a "rubberizing," or  $\tau(t)$ , distortion of the timebase. Furthermore, it requires no continuity of the  $\tau_{0,i}$  from one short integration time  $N'\Delta$  to the next, an obvious requirement for signal distortions imposed by the medium or target motion. In spite of these shortcomings, it is an excellent indicator that the signal is so aligned with respect to the reference that a significant correlation

coefficient might be achieved by subsequent adjustment of the signal timebase.

3. Timebase adjustment through iteration. With switch  $S_1$  in its lower position, the timebase of the received signal function is adjusted continuously by  $S_2$  under control of the recirculating  $\tau(t)$  memory. The  $\tau(t)$  memory is constantly revised as the received signal function, reference function, and  $\tau(t)$  delay lines recirculate synchronously and the signal timebase distortions are progressively removed. On each recirculation only the incremental change  $[\tau_k(t) - \tau_{k-1}(t)]$ , in  $\tau(t)$ , with respect to its form on the preceding  $(k-1)$  circulation, is detected by the "greatest of" selector, Fig. 4. As shown in Fig. 3, this change is converted to a voltage level in the encoder, to digital form in the A/D converter, and then added to  $\tau_{k-1}(t)$ , the  $\tau(t)$  function on the preceding,  $(k-1)$ , circulation. The digitized output of the adder,  $\tau_k(t)$ , which is sent to the  $\tau(t)$  memory, is not restricted by the frequency restraints of the medium or the target motion. It is merely a cumulative digital record of the time varying outputs of the "greatest of" selector. However, the speed with which electronic switch  $S_2$  changes from one tap to another is set by the  $\tau$  amplifier (Fig. 3) frequency response, which in turn is matched to  $\rho$  of Eq. (1). The amplitude of the switching motion, or the frequency with which  $S_2$  connects to extreme tap positions, corresponding to large  $[\tau_k(t) - \tau_{k-1}(t)]$ , is controlled by allowing the known  $\frac{\sigma_r^2}{\sigma_n^2}$  ratio to establish the shape of the non-linear, compressed (e.g., logarithmic) gain characteristic of the  $\tau$  amplifier. The exact operating point on this gain characteristic is set by the  $\tau_k(t)$  output from the adder after passing through a D/A converter. The larger the instantaneous value of  $\tau_k$ .

the smaller is the differential gain of the amplifier. The non-linear, compressed shape of the gain characteristic and the location of the operating point on this characteristic, as controlled by  $\tau_k$ , reflect the fact that relatively large  $\tau(t)$  values are improbable, and are more likely to be present at the "greatest of" selector output because of adaptation to the additive noise than to an actual signal timebase distortion. In more precise terms, making the left side of Eq. (1) a maximum requires time-base adjustment of the received signal function (to maximize the first term, Eq. [1]) tempered by rejection of relatively large and improbable  $\tau(t)$  values (to minimize the second term, Eq. [1]). The importance of the second term in Eq. (1), and therefore the degree of rejection, is set by the ratio  $\frac{\sigma_\tau^2}{\sigma_n^2}$ . The role of this ratio in Eq. (1) is simulated in the Adaptive Deltic by its control of the gain characteristic of the  $\tau$  amplifier. A large  $\sigma_\tau^2$  characteristic of abrupt target motions or large boundary motions of the medium, and/or small  $\sigma_n^2$  characteristic of low noise content in the received waveform, permit relatively frequent excursions of  $S_2$  to extreme tap positions. On the other hand, a small  $\frac{\sigma_\tau^2}{\sigma_n^2}$  ratio leads to a much higher probability for  $S_2$  to contact a centrally located tap. Thus the tap positions actually assumed by  $S_2$  can differ significantly from those called for by the  $\tau(t)$  memory because of the frequency and amplitude response of the  $\tau(t)$  amplifier.

The actual correlation output is presented continuously at the output point A, Fig. 3. As a signal timebase adjustment proceeds from one iteration (recirculation around the delay lines) to the next, the output at point A should increase in magnitude. This progressive increase can be used to lengthen the time constant  $T'$  of all the short term integrators feeding the "greatest of"

selector. This is equivalent to lengthening the coherent integration times in each of the  $\phi_i$ , Eq. 3, which is justified as the timebase adjustment provides progressively longer coherence times of the received waveform with respect to the reference function. Eventually the short term time constant  $T'$  should become equal to the long term signal duration (in compressed Deltic form)  $T$ , and switch  $S_2$  should stay connected to the central,  $(N + 1)\Delta$ , feedback tap as the signal timebase becomes fully corrected. The average relative velocity of the target, and therefore the value of  $k_0$  in Eq. (1), can be found from the average slope, over the signal duration  $T$ , of the  $\tau(t)$  function stored in the  $\tau(t)$  memory.

4. End of iteration. The iteration process is brought to a stop when:

- a)  $S_2$  ceases to change tap positions, or
- b) the output at point A stops increasing, or
- c) the output at point A exceeds a predetermined threshold.

All three of these conditions satisfied simultaneously indicate a successful adaptive match of received and reference functions, and therefore the presence of a target. The following significant outputs, in addition to the correlation coefficient at point A, are also available:

- a') Range to target, as indicated by the time at which  $S_1$  switched to lower position to start the iteration and stop the regular precession of the signal function.
- b') Average relative target velocity, as indicated by the average slope of the stored  $\tau(t)$  function.

However, it is also possible that one of the conditions (a) or (b) may occur without a completely satisfactory correlation of received and reference waveforms. This could occur if distortions other than those of the timebase, such

as signal fading, are present. In such an event, the signal function may adjust satisfactorily over isolated portions of its timebase but these isolated portions may not connect smoothly to provide a continuous function  $\tau(t)$ . In the "bad" intervals the "greatest of" selector either fails to pass any signal exceeding the threshold of the discriminator, or those  $\tau(t)$  values passed are random and so widely separated that they correspond to impossible temporal changes in the medium. Then it is desirable to construct a continuous  $\tau(t)$  function by interpolating between those portions of the timebase which provide a strong and unambiguous indication of  $\tau(t)$  in the "greatest of" selector. To do this, it is necessary to know the value of  $\tau(t)$  at the start of the next "good" portion of the timebase when finishing the last "good" portion, the two being separated by a "bad" interval. This can be done by storing the  $\tau(t)$  value at the beginning of the oldest "good" portion of the timebase (that "good" portion which is about to appear next at the output of the  $\tau(t)$  memory by virtue of the recirculation), and sending it to the  $\tau(t)$  amplifier as a bias level to be approached during the "bad" interval. The rate of approach, or of switching  $S_2$ , will be determined by the difference in  $\tau(t)$  at the beginning and end of the "bad" interval and by the length of the "bad" interval. This linear interpolation will allow  $S_2$  to be in a position to accept the  $\tau(t)$  values for the next "good" interval, the first part of which would have been lost to it if it had to approach these well established  $\tau(t)$  values ( $S_2$  tap positions) suddenly through the low frequency response of the  $\tau(t)$  amplifier. Obviously the output at point A will be reduced in this case, and a decision must be made with regard to (c) above, the threshold level, as to whether a target is present or not. However, the adaptive receiver is still an optimum one for correcting timebase distortions whether or not multiplicative noise (fading, etc.) is present on the signal.

Since the timebase adjustment involves an iterative operation, in general it cannot be carried out in real time. However, in many detection situations sufficient time is available between transmissions to operate in off-line fashion when examining probable target returns. The off-line operation is begun, and the relative precession of the received signal and reference delay lines stopped, only when the mixed integration output of the "greatest of" selector indicates a target may be present. The operation returns to a real time search, and switch  $S_1$  assumes its upward (original) position, whenever any of the conditions (a), (b) or (c) occur. If all three have occurred, a target is declared present.

Although the  $\tau(t)$  memory delay line and the other delay lines have approximately the same length, the binary coding of the  $\tau(t)$  data requires more than one sample position for each sample, whereas the hard-clipped signal and reference samples are fully described by one bit each. However, this does not require a higher packing density on the  $\tau(t)$  memory line because the highest frequency component in  $\tau(t)$  is much lower than the received signal and reference upper frequencies. Therefore several bit positions can be assigned to a single  $\tau(t)$  sample while still maintaining an adequate sampling rate.

It is not intended that the way in which  $\rho$ ,  $\sigma_{\tau}^2$  and  $\sigma_n^2$  control the  $\tau$  amplifier, Fig. 3, should exactly simulate the role of these parameters in Eq. (1). In general terms,  $\rho$  controls the time constant, or high frequency response, while  $\frac{\sigma_{\tau}^2}{\sigma_n^2}$  controls the non-linear shape of the gain characteristic of the  $\tau$  amplifier, but neither does so exactly in the manner prescribed by Eq. (1). The exact form of Eq. (1) need not be simulated since an iterative

procedure is being followed to approach a maximum output level at point A, Fig. 3, and this maximum will occur for a particular final adjustment of the signal timebase regardless of the detailed nature of the iterative steps which approach it. It is only necessary that no secondary maxima occur at point A during the iterative approach to the true maximum, for if such were the case the process would stop once a secondary maximum was achieved. However, when a target is actually present and the signal-to-noise ratio is adequate for detection this is an unlikely event, for adjustment of the timebase of a target return should lead smoothly to a true maximum. It is more likely to occur when the iterative process is triggered by a fortuitous match between receiver channel noise and the reference function, in which case the iteration will soon stop by virtue of cause (a) or (b) of step 4 above, and a target will be declared not present by failure to exceed the threshold, cause (c).

## II Digital Computer Program

All of the logical operations ascribed to the Adaptive Deltic correlator in the previous section can be programmed on a digital computer. Moreover, a general purpose computer allows a flexibility of programming which would permit desirable deviations from this logic in specific cases. The computer has the further advantage over the Adaptive Deltic of treating multi-level quantizing of the received analog waveform  $f(t)$  and the reference function  $s(t)$  with greater ease. Obviously the disadvantage of the computer is its high initial and operating costs, and for special purpose operations such as correlation this is seldom justified when the operations are to be repeated many times. Nevertheless, a digital computer with an analog-to-digital converter at the input is a useful system for exploring the capability of adaptive correlation techniques.

Operations which constitute major portions of such a computer program for adaptive correlation are the same as those performed by the Adaptive Deltic:

- 1) Multiplication and integration of samples from a fixed and a pre-processing information channel, as in a normal Deltic program.
- 2) A mixed integration consisting of the sum of several outputs from a "greatest of" selector whose inputs are the envelopes of a series of coherent correlations involving integration times limited to the stability time of the medium (presumably shorter than the signal duration).
- 3) An adaptive change in the timebase of  $f(t)$  in accordance with an iteration of 2) above, with the constraints of the following quantities known a priori:

- a)  $\rho$  , the normalized autocorrelation of  $\tau(t)$  , which determines the permissible frequency range of  $\tau(t)$  .
  - b)  $\sigma_{\tau}^2$  , the variance of  $\tau(t)$  , which determines the amplitude range of  $\tau(t)$  .
  - c)  $\sigma_n^2$  , the average noise power, which determines the extent to which a) and b) above can be allowed to constrain the timebase adjustment of  $f(t)$  .
- 4) An end of the iteration process when either the output correlation function or the  $\tau(t)$  memory ceases to change significantly.

Computer programs have already been written for conventional Deltic correlation,<sup>3</sup> and their extension to an adaptive operation is not difficult.

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<sup>3</sup> C. N. Pryor, "A High-Speed Cross-Correlation and Fourier Transform Subroutine Package for IBM 7090," U. S. Naval Ordnance Laboratory Memorandum-to-File, 21 June 1962.

### III Adaptive Optical Correlator

The same general philosophy but quite different physical principles can be used to arrive at the conceptual design of the correlator which performs an adaptive timebase correlation by optical methods. Such an approach retains the advantages of multi-channel capability and large time-bandwidth capacity characteristically associated with optical correlators while adding the adaptive timebase feature. The problem is somewhat more complicated with optical correlators because no two channels will in general have exactly the same  $\tau(t)$  timebase distortion factor. In fact, if the information on the various channels arrives at the receiver by grossly different propagation paths, or if differently accelerating targets appear on different channels, the various  $\tau_j(t)$ , where the subscript  $j$  refers to channel number, will be completely independent. In such a case it is necessary to adjust the timebases of all channels independently and simultaneously. Normally this would seem to be a far more difficult task than adjustment of a single timebase, as in the Adaptive Deltic. However, the partially coherent panchromatic optical correlator is particularly suited to this task because the time derivatives of the  $\tau_j(t)$  appear naturally at its output, and these can be sensed and fed back to control the writing of the various signal functions  $s_j(t)$ . The partially coherent panchromatic correlator is essentially a combination of two optical correlator types described previously: 1) the panchromatic optical correlator<sup>4</sup> in which time distortions in the received signal are compensated by the light source spectrum, and 2) the partially coherent optical correlator<sup>5</sup> in which only partial light coherence is maintained along the time axis of the

<sup>4</sup> Ross E. Williams, "The Panchromatic Principle in Optical Filtering" (Hudson Laboratories, Columbia University Technical Memorandum No. 68, January 30, 1963).

<sup>5</sup> Ross E. Williams, "Partially Coherent Data Processing in Optical Systems" (Hudson Laboratories, Columbia University Technical Memorandum No. 69, February 28, 1963).

received signal function so that a mixed integration analogous to Eq. 3 is performed in the output plane.

Figure 5 shows a schematic diagram of the partially coherent panchromatic optical correlator. Each point of the area light source illuminates only a restricted region of the signal film, and the coherence of this illumination falls off radially from the center of the region according to a coherence factor  $\gamma(t-t')$  which has a roughly Gaussian shape (actually a first order Bessel function divided by its argument). The totality of all such overlapping, coherently illuminated regions produces a mixed integration at the output of the form

$$I(\tau_0) = \iint \gamma(t-t') f(t-\tau_0) s(t) f(t'-\tau_0) s(t') dt dt' \quad (5)$$

This result is equivalent to a superposition of many coherent correlators, each with a restricted aperture of tapered transmittivity  $\gamma(t-t')$  overlapping one another, whose outputs are summed incoherently. That part of the optical correlator to the left of P1 in Fig. 5 shows the partially coherent illumination, the extent of whose coherence (the width of the  $\gamma(t-t')$  factor) can be varied by axial motion of the lenses L1 which causes each light source point in plane P0 to be imaged closer to or further from plane P1, thereby changing the coherently illuminated area formed by the intersection of plane P1 with the cone of light rays from the source point.

Thus the partial coherence principle produces a true mixed integration of adjustable coherence time which may serve, as in the Adaptive Deltic correlator, as the starting point for an adaptive technique. That part of the optical correlator to the right of plane P1 in Fig. 5 is a conventional panchromatic correlator.<sup>4</sup>

A brief statement of some of the principles of the panchromatic optical correlator is in order to facilitate an understanding of its use as an adaptive device. As its name suggests, the panchromatic correlator uses a light source with a finite spectral width rather than a monochromatic source. With the received signal function  $f_j(t)$  inserted in a spatial plane and the reference function in a frequency plane, the diffraction grating  $G_1$  bends the optic axis to simulate a frequency shift when the time-distorted received signals are heterodyned to a lower frequency band before recording on film in order to use the low pass spectrum of the correlator more efficiently. The panchromatic correlator produces a correlation function at its output for any  $\tau_j(t)$  distortion of the signals  $f_j[t-\tau_j(t)]$  provided each  $\frac{d\tau_j(t)}{dt}$  is constant over the optical aperture represented by  $\gamma(t-t')$  and regardless of whether the  $\tau_j(t)$  are identical from channel to channel. In fact, each frequency in the light source corresponds to a particular  $\frac{d\tau(t)}{dt}$ , and thus the various  $\tau_j(t)$  distortion factors will be compensated by different predominant light frequencies in the various channels at the output. These are separated by the color separation prism to the left of plane P4 in Fig. 5. The output format in plane P4 is an area display as shown in Fig. 6, with the distortion  $\frac{d\tau_j(t)}{dt}$  plotted vertically for each input channel, and range, or real time, horizontally. As the signal film moves, the display changes in real time, thereby providing the  $\tau_0$  dependence (see Eq. 1) in the correlation function  $\phi(\tau_0)$  on P4. The record in plane P4 of a received signal with variable timebase distortion would appear as a wiggly trace of greater intensity than the background, wandering among various neighboring  $\frac{d\tau_j(t)}{dt}$  channels as shown in Fig. 6, provided the coherence interval represented by  $\gamma(t-t')$ , Eq. 5, is short enough that the received signal essen-

tially has a single timebase distortion in each time span. Thus, with the coherence time suitably small, the wiggly trace constitutes a record of instantaneous timebase distortions of the received signal over its time duration. A scanning device (e. g., a flying spot scanner or vidicon camera tube) which senses the shape of this trace (see Fig. 7) converts the vertical or  $\frac{d\tau_j(t)}{dt}$  deflection of the trace in Fig. 6 to a timebase correction (speeding up or slowing down) of the original received signal function as it is recorded on film. A feedback correction for this purpose to a flying spot scanner which records the received signal function on film is shown in Fig. 7. Without a feedback signal, the flying spot scanner provides an intensity modulated line scan which is imaged across the width of the film, i. e., across the many independent channels which are to be processed simultaneously. By multiplexing samples from each of the independent receiver channels onto the control grid of the flying spot scanner, a single scanner can expose all the channels on the film and synchronously demultiplex them with the cross-channel line scan shown in Fig. 7. The film is advanced at a rate which causes each successive line scan to be imaged on the film parallel to the last scan image and displaced one resolvable distance along the time axis from it.

The camera tube (e. g., vidicon) scans the correlator output plane in the vertical or  $\frac{d\tau_j(t)}{dt}$  direction synchronously with the vertical line scan of the flying spot scanner exposing the signal film. (Alternatively, a flying spot scanner-photomultiplier combination can be used to "read" the correlator output plane information if it is already recorded on film.) When the output plane intensity is below a threshold level in this vertical scan, or when the intensity exceeds the threshold indicating the presence of a correlation peak but the peak occurs at a  $\frac{d\tau_j(t)}{dt} = 0$  location, no signal is sent via the feedback path to the

flying spot scanner exposing the signal film. However, when a correlation peak occurs for a non-zero  $\frac{dr_j(t)}{dt}$ , the feedback signal to the deflection plates of the flying spot scanner causes the spot image to deflect along the time dimension (lengthwise dimension) of the film, thereby adjusting the timebase of that particular channel to compensate for the local timebase distortion. Thus the line scan is no longer a straight line scan across the width of the film, but departs in the time direction whenever necessary to correct for recognized time distortions. The direction and magnitude of the timewise deflection of the spot are, of course, set by the sign and magnitude of the local  $\frac{dr_j(t)}{dt}$  encountered in the correlator output plane, and because of the low inertia of the CRT electron beam and high frequency response of the deflection amplifier all signal channels can be adjusted simultaneously and independently. As in the Adaptive Deltic correlator, care must be taken to avoid adapting the received signal channel timebases to noise in the channels, and the proper restraints incorporated in the low pass filter and non-linear amplifier in Fig. 7 serve the same purpose as the equivalent ones in the Adaptive Deltic schematic, Fig. 3.

When the feedback technique is used it is obvious that real-time operation is no longer possible, for the feedback operates upon a pre-existing signal to correct it for a more coherent correlation: As the signal timebase becomes better adapted to remove unwanted distortions by successive iterations through the feedback path, as described above, the coherence interval  $(t-t')$  can be made longer and longer until eventually, when all distortions have been removed, it occupies the full aperture of the correlator. This variation of the coherence factor  $\gamma(t-t')$  is produced by axial adjustment of lenses L1 in Fig. 7. Starting with L1 so positioned that the coherence area on P1 is

small (plane P0 nearly imaged upon P1) the lenses L1 are gradually moved toward P0 until they are a focal length  $f$  from it, when parallel light rays from any given point in P0 illuminate the full aperture of P1 coherently and a completely coherent correlation is performed. At the same time, the wiggly trace in the output plane P4, Fig. 6, should straighten, becoming completely straight when completely coherent correlation is achieved.

The process actually does proceed in real time until a partially coherent correlation exceeds the threshold level for signals in plane P4, since no feedback is present until that time. When the threshold level is exceeded and feedback occurs, the signal inputs to the flying spot scanner control grid must be switched from the signal channels, where new signals continue to arrive, to a recorded version of the previous signals which first gave a partially coherent correlation indication. This is done most readily by recording all received signals on a loop of magnetic tape whose length is matched to the maximum time-aperture of the correlator, and, when the threshold is exceeded, replaying the loop into the flying spot scanner rather than continuously erasing and recording new signals, as in the case with no correlation present above the threshold.

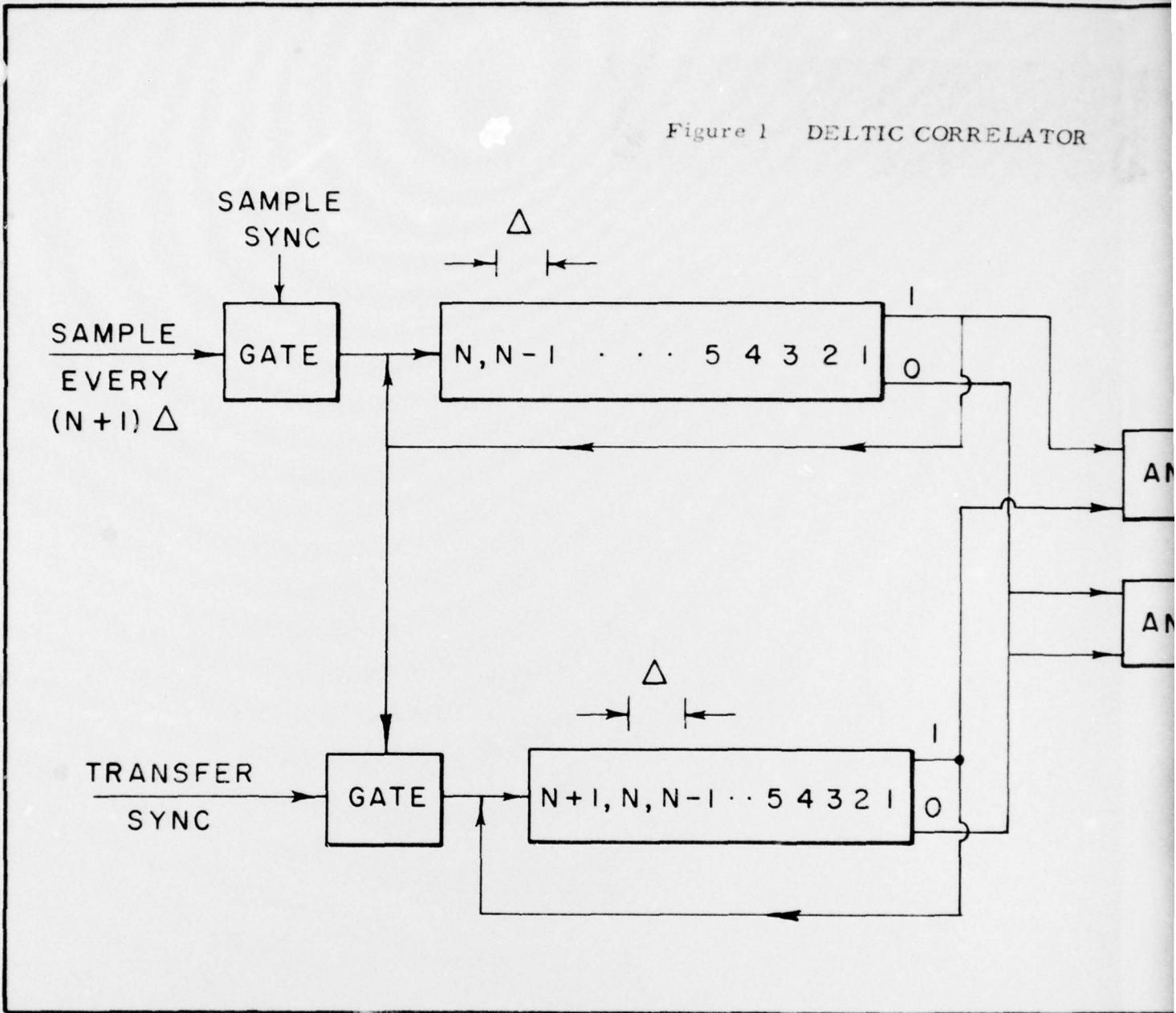
The purpose of each of the adaptive correlators described above is to determine whether the relatively low level correlation resulting from a partially coherent integration of the signal timebase actually is associated with a target or not. If it is, the correlation peak should rise as the timebase is adjusted for local distortions, reaching a maximum when the correlation becomes completely coherent. On the other hand, when the partially coherent result exceeds the first threshold only because of a fortuitous matching of the received signal channel noise with the reference function (a not unlikely event) no amount of timebase adjustment through successive iterations will enhance the correlation peak when the feedback is made with proper frequency and amplitude restraints. In this case the iterations can be cut-off, and normal real-time operations restored, when the correlation peak fails to exceed a second, higher threshold after a fixed number of iterations (step 4b above).

It is important to note that the problem in adaptive timebase correlation is not primarily one of enhancing the signal-to-noise ratio, but rather one of increasing the time coherence of the signal so that an already adequate signal-to-noise ratio will provide a suitably strong correlation peak after timebase compensations have been made. Thus the technique is not primarily designed for weak signal detections, but for greater probability of detection when the signal-to-noise ratio is adequate but either target or boundary motions limit the coherence of the signal.

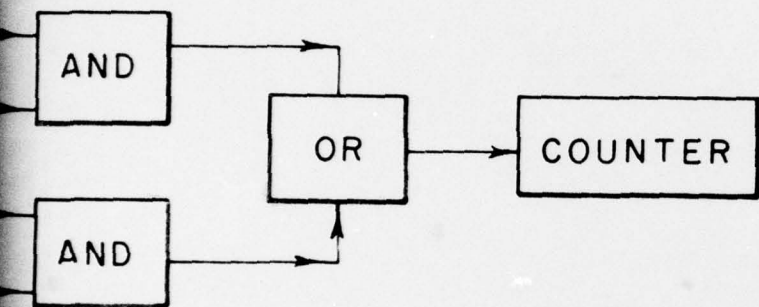
### Acknowledgment

The author would like to thank Mr. Donald Brown for several helpful discussions on the design of "greatest of" circuits.

Figure 1 DELTIC CORRELATOR



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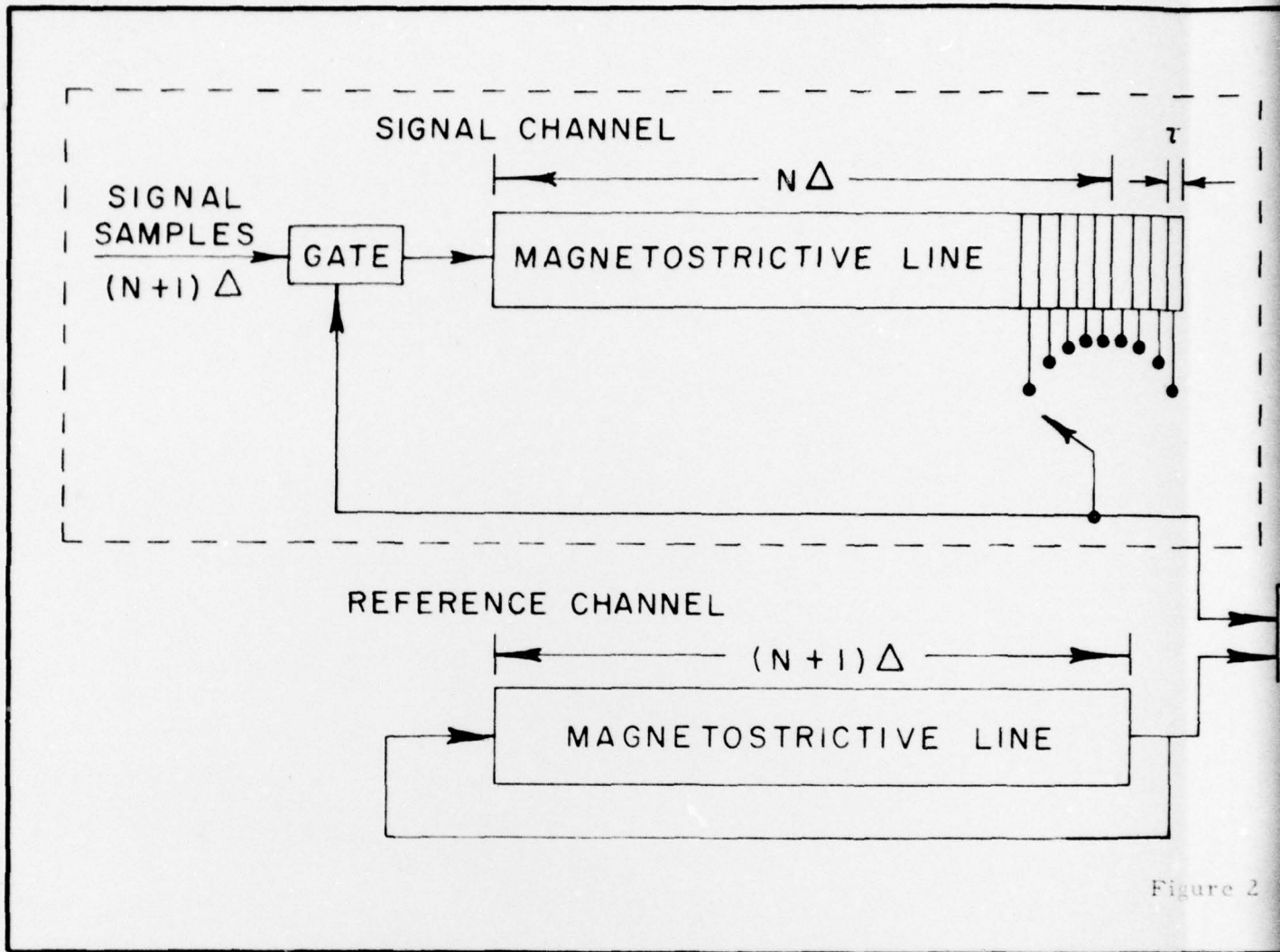


Figure 2

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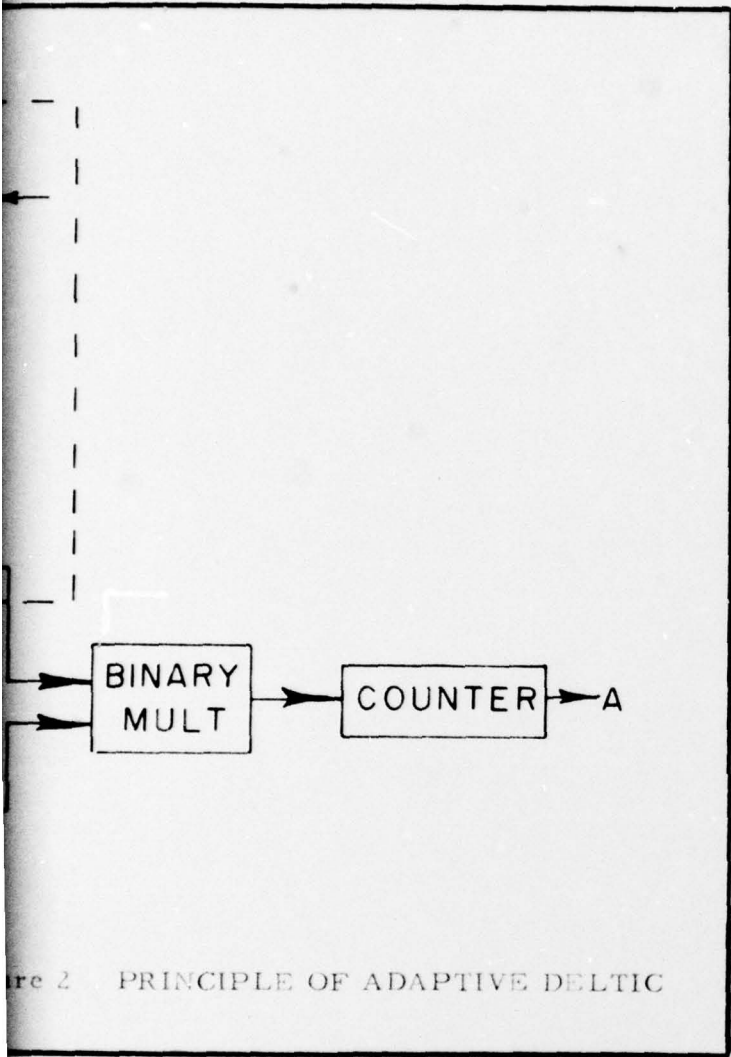
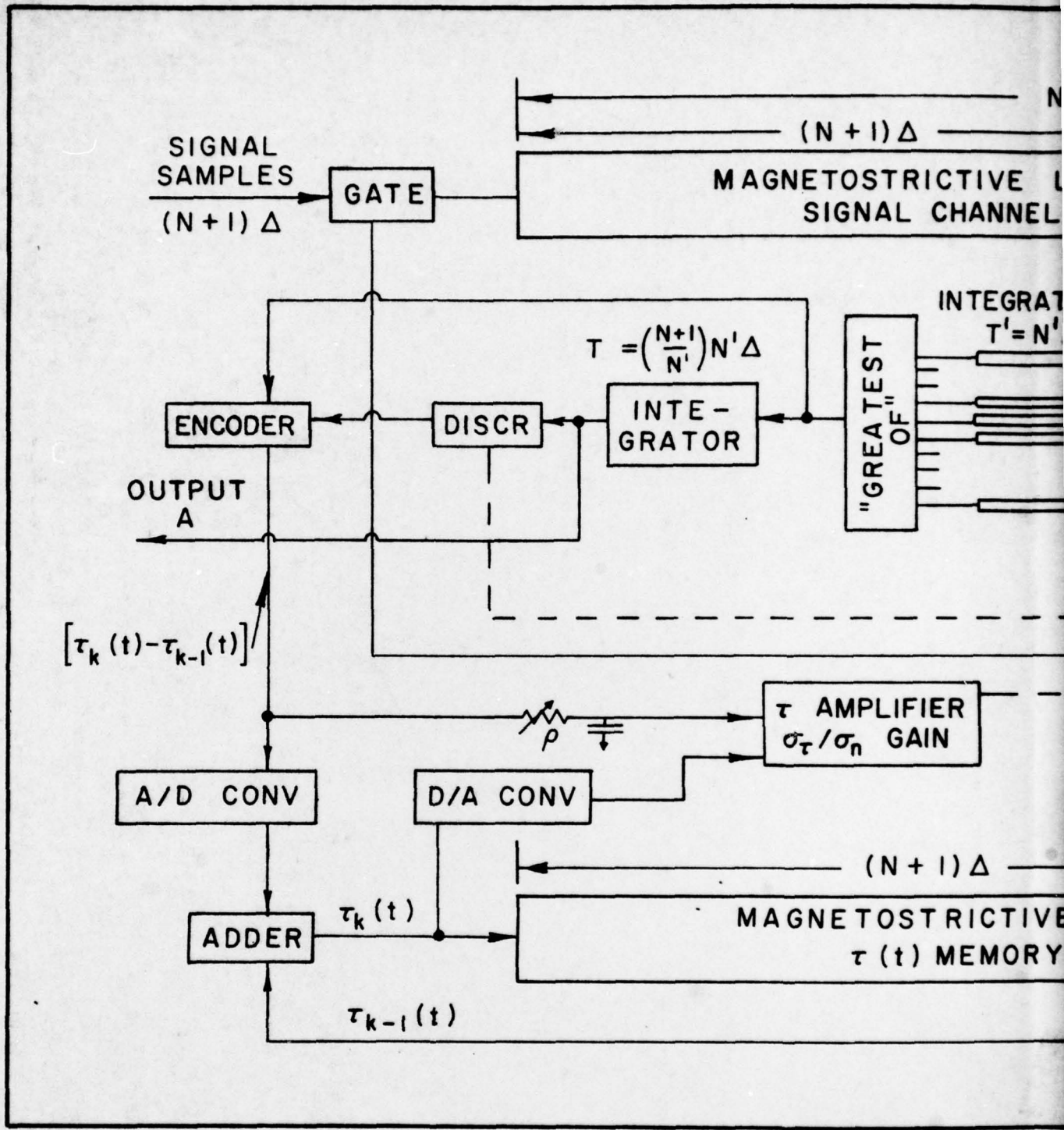


Figure 2 PRINCIPLE OF ADAPTIVE DELTIC



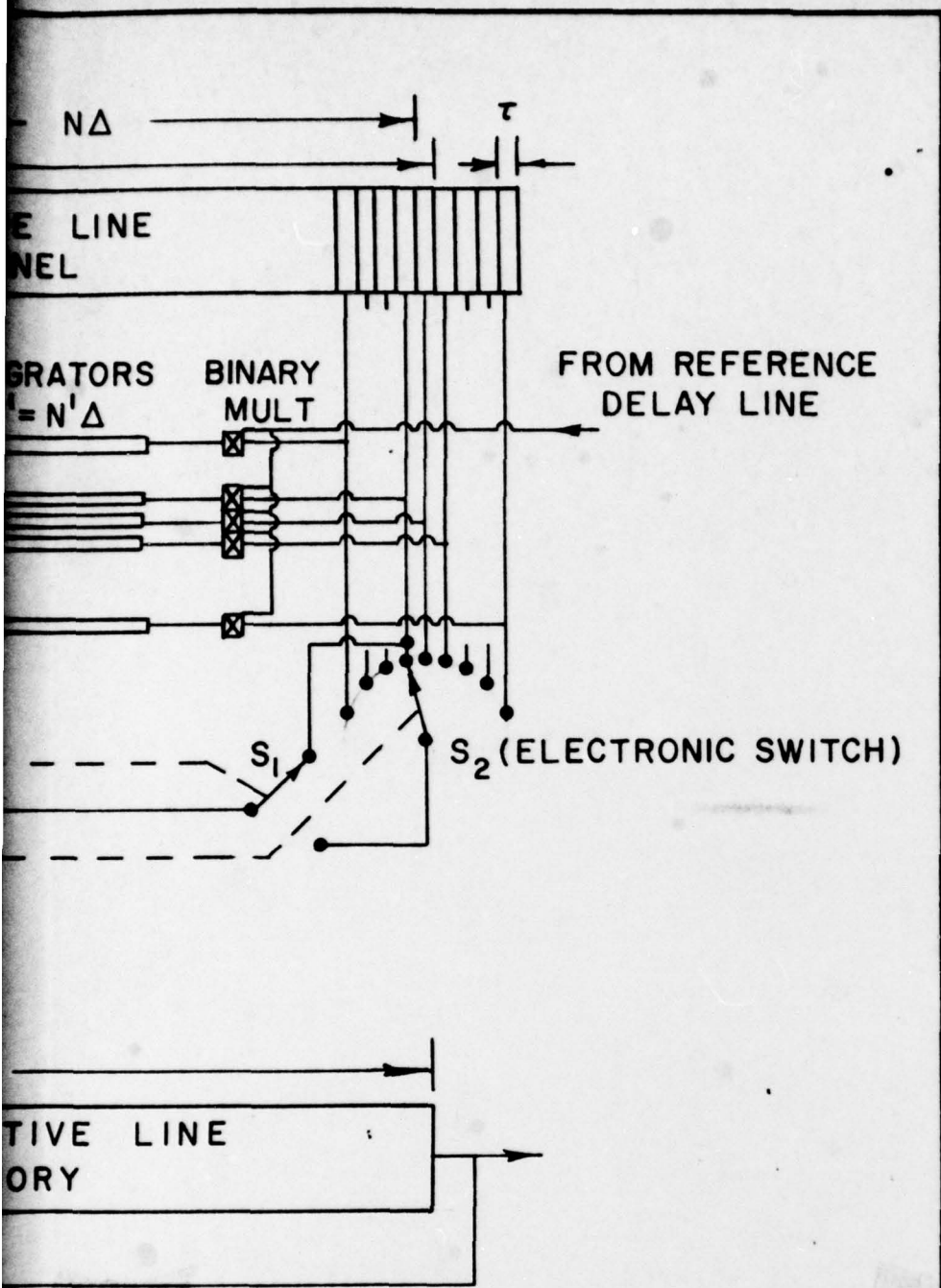


Figure 3 ADAPTIVE DELTIC CORRELATOR

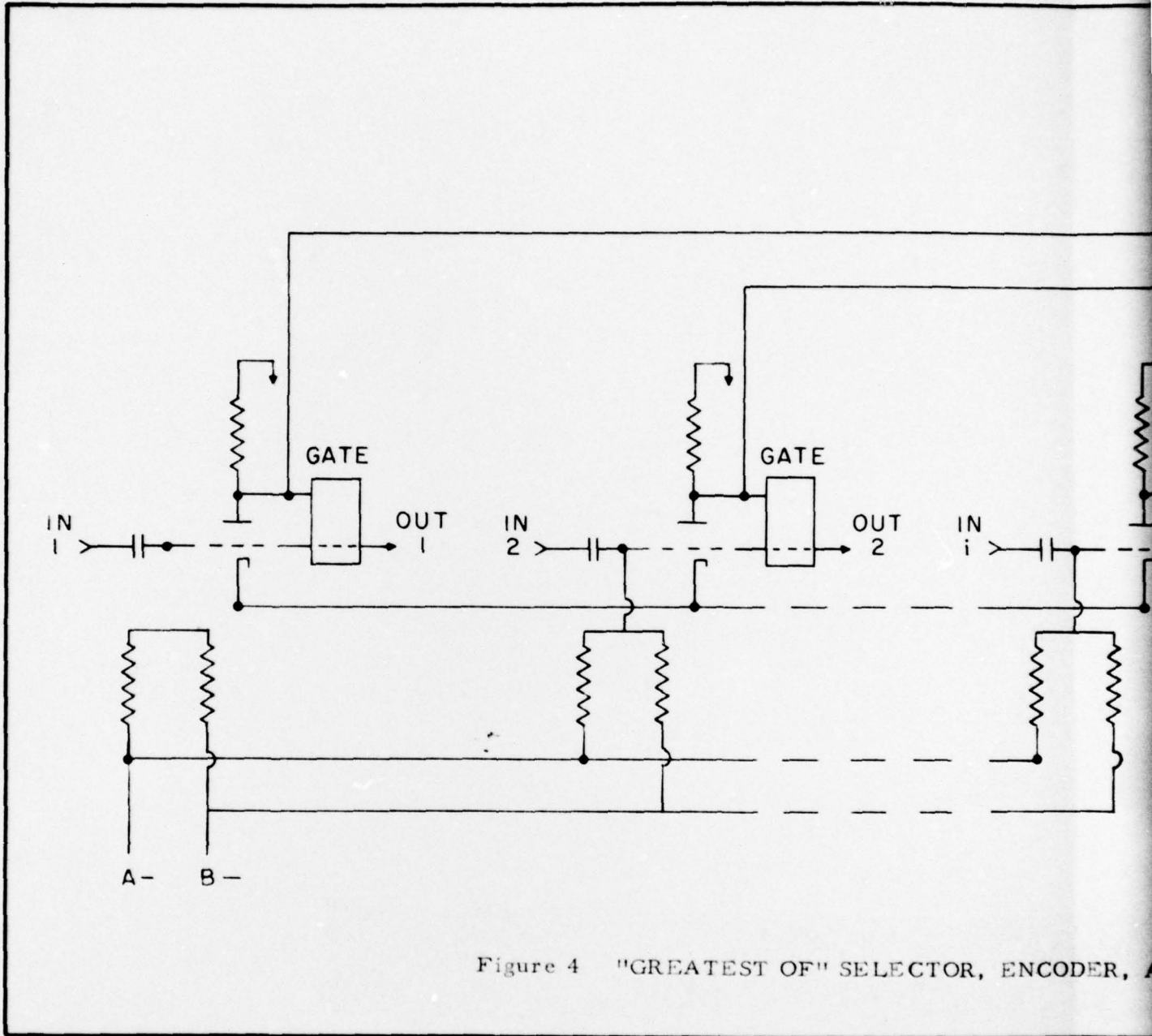
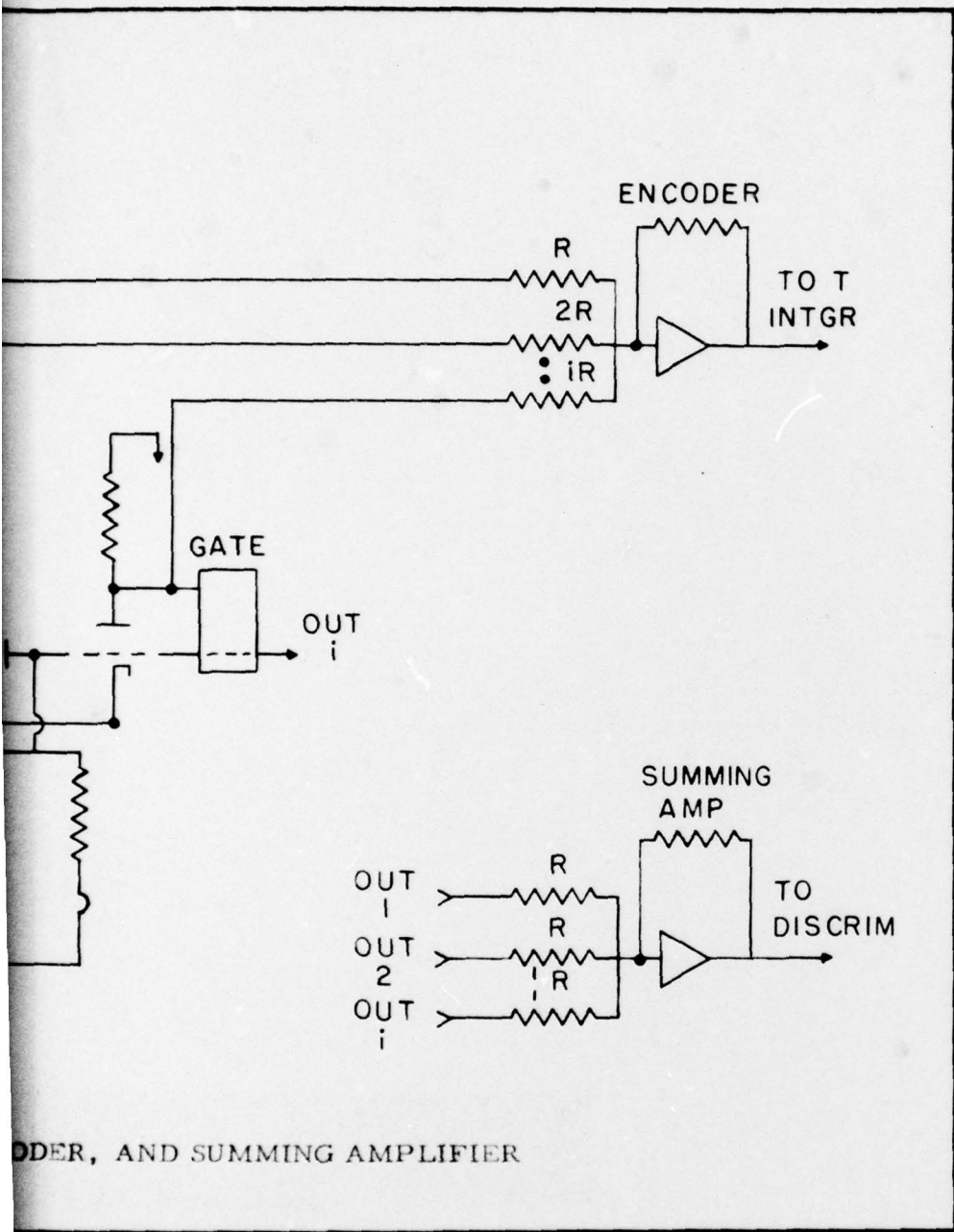


Figure 4 "GREATEST OF" SELECTOR, ENCODER, A

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2-19-68 J. Fiore



ENCODER, AND SUMMING AMPLIFIER

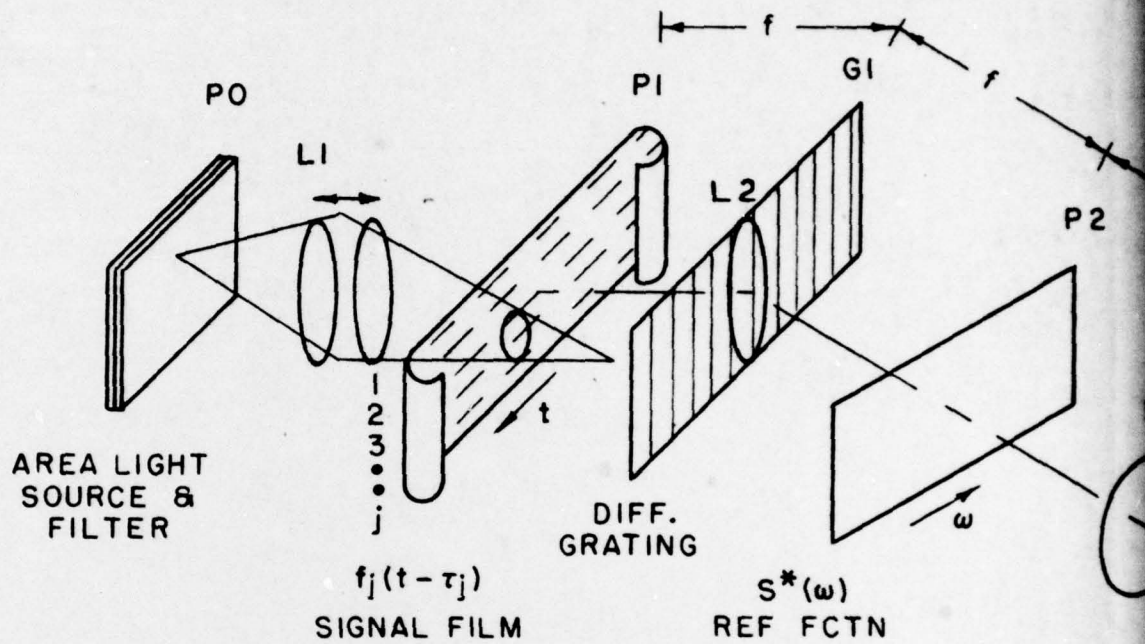


Figure 5 PARTIALLY COHERENT PANCHROMATIC



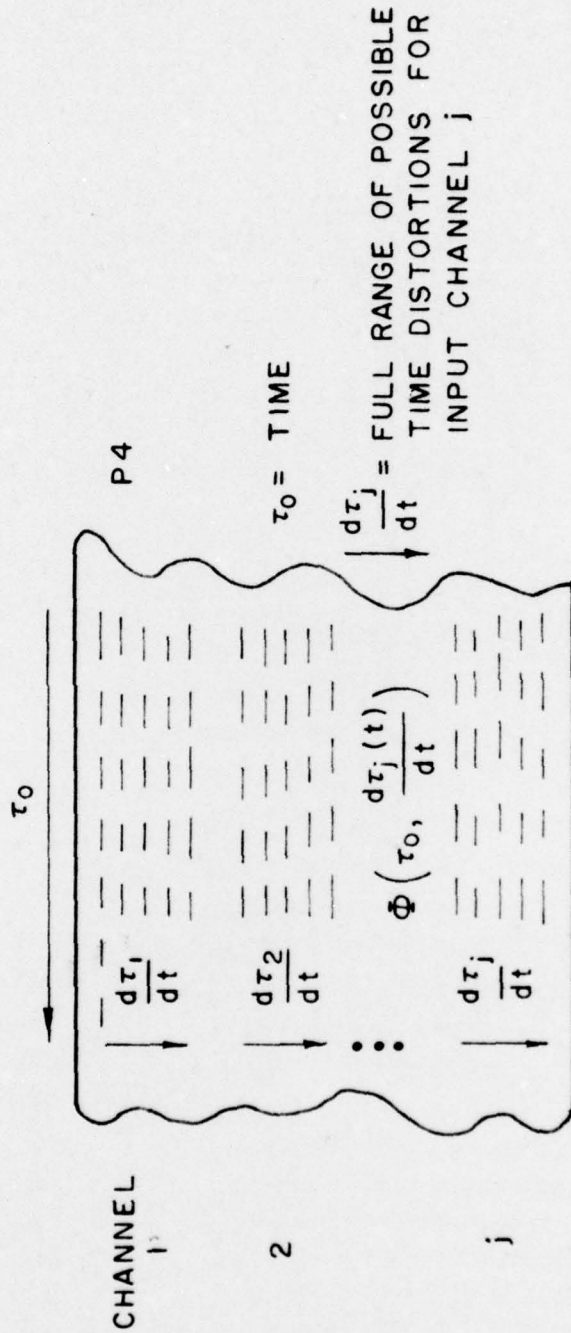


Figure 6 FORMAT OF PLANE P4

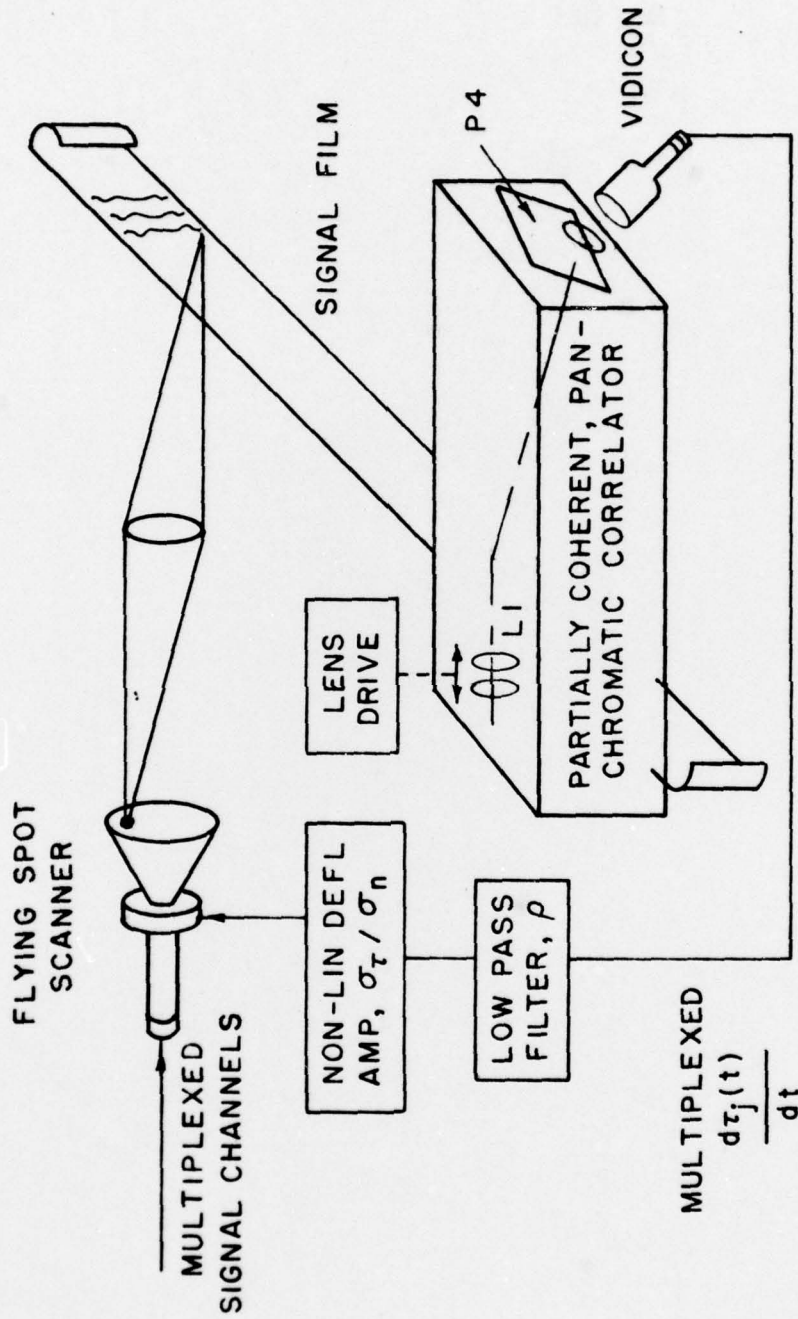


Figure 7 ADAPTIVE OPTICAL PROCESSOR