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CHOOSING TELESCOPE SYSTEM MIX

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FINAL REPORT

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INSTRUMENTATION DIRECTORATE
US ARMY WHITE SANDS MISSILE RANGE
WHITE SANDS MISSILE RANGE, NEW MEXICO 88002

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SUMMARY

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OBJECTIVE

The present task involved the evaluation of the White Sands Missile Range (WSMR) requirements for an integrated photo-optical instrumentation system for attitude data, vector miss distance and event readings. This included the available resources consisting of Versatile Tracking Mounts (VTMs), the alternatives available for their modernization, the possibility of acquiring new VTMs used in fixed or mobile capacity and, finally, the possibility of acquiring the Distant Object Attitude Measurement System (DOAMS), a new proposed class of optical instrumentation. Following an economic analysis in which the initial investment costs and the 10 year life cycle discounted recurring costs were incorporated, a linear programming model was formulated to arrive at an optimum mix of each kind of instruments so that a minimum total cost over the 10 year life cycle is achieved while simultaneously maintaining acceptable levels of aggregate versatility and quality.

It is assumed that the reader of the present report has familiarity with the overall prevailing situation, as well as the DOAMS Study Committee report entitled "System Validation Report" dated 15 April 1976.

AVAILABLE ALTERNATIVES AND NONRECURRING COSTS

The problem studied consisted in the identification of alternative courses of action available in the integrated telescope system and in the determination of the optimum mix of telescopes to achieve an economic objective. Specific alternatives considered were the following:

1. Partial or total retention of existing VTM units.
2. Partial or total modification of existing VTM units.
3. Acquisition of new VTMs used as mobile units.
4. Acquisition of new VTMs used as fixed units.
5. Acquisition of DOAMS.

A brief elaboration on each available alternative will be given in the sequel with some emphasis on critical factors affecting the present problem.

Assuming that the existing Intercept Ground Optical Recorders (IGORs) and Modified IGORs (MIGORs) have reached an obsolescence level which warrants their retirement, the present telescope resources consist of 25 Photo-Sonics

Cine-Sextant telescopes, often shortly referred to as 25 VTMs. These telescopes utilize a manual focusing device and have capability to accommodate up to 180 inch focal length (f1) optics. It is important, however, to note that the present inventory of high powered optics consists of

1. five 180 inch f1 athermalized optics,
2. twenty-three 100 inch f1 athermalized optics, and
3. twenty-seven 50 inch f1 athermalized optics.

One also should note that a hitch and suspension system modification program for the VTM was implemented and field evaluated. Despite the uncertainty for an overall hitch and suspension modification, funds for such a program were considered to be sunk cost.

In a program accepting any VTM units without other changes or modifications in the next decade, no capital investment will be incurred, and the only costs experienced will be associated with the operation, repair, and maintenance of the units.

Modification of existing VTM units would include, in addition to an improved hitch and suspension system, such features as follows:

1. Automatic focusing devices to capture and produce high quality images at all trajectory levels.
2. Exposure control.
3. Microprocessors to replace the Instrument Data Converter (IDC) and Film Data Recording System (FDRS) and also control the focusing and exposure functions.
4. 180 inch f1 athermalized optics.

The aggregate modernization program can be achieved at a price of \$144,000 per unit.

New mobile VTMs with 17-bit encoders, tracking scope, 180 and 100 inch athermalized optics, 2 cameras, automatic focusing devices and peripheral equipment (generator, shop van, prime mover, modified trailer) which can be acquired at a price of \$553,500 per unit.

These new mobile VTMs would essentially possess the same characteristics as the modified VTMs except that they will be less subject to failure.

As an added alternative, an option in which new VTMs can be acquired and used on fixed site was considered. Each unit would include the basic mount, 19-bit encoders, a tracking scope, 180 and 100 inch f1 athermalized optics, 2 cameras, and automatic focusing devices and could be acquired at a price of \$455,500 per unit. There would be an associated construction cost of \$153,400 per site.

Finally, the new proposed class of photo-optical telescope instrumentation system known as DOAMS was considered. This is a transportable type telescope used however more frequently in the capacity of a fixed unit. It incorporates an automatic variable focusing device as well as the capability to accommodate 100 and 200 inch f1 athermalized optics. The DOAMS telescope possesses, in addition, several other qualitative and optical features which are not available on the VTM telescope. In particular, the pointing accuracy is of the order of 1 arc minute, the elevation axis will dump 180 degrees, veiling glare is minimized, higher image resolution and contrast can be achieved, the transmittance is $T/11.87$ for the 200 inch system, the wavefront requirement is $1/5\lambda$ rms, etc. Inflationary prices have substantially increased the unit procurement price. At a reduced performance equal to or better than the prototype, the unit price is \$695,168 for 4 units or less and \$624,049 for 5 units or more. The DOAMS telescope requires special sites and, on the average, it was assumed that the number of DOAMS sites is about 1.5 times the number of DOAMS telescopes. Two existing IGOR sites can be modified at a cost of \$20,000 each. The purchase of any additional DOAMS sites will cost about \$153,400 per site. The DOAMS telescope will also require the acquisition of special transporters at a cost of \$50,000 per transporter.

RECURRING COSTS

The development of an objective function was based on an economic criterion consisting of the total discounted (present value) cost over a life cycle of 10 years (1976 to 1986). In addition to the initial investment costs, all recurring costs associated with the retention of each telescope, site, etc., over these 10 years, were incorporated. These recurring costs included the following:

1. Maintenance, repair, and operation of telescopes.
2. Purchase, processing, analysis and reading of films which provide insufficient information to be of any value (estimated at \$.70/ft of film).
3. Maintenance cost of sites.
4. Others.

Important parameters which played a significant role in the identification and definition of those costs were as follows:

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1. Intrinsic instrument reliability (p). This quantity is solely determined by types and frequency of failure of the instrument. Conservative estimates of reliability were taken as .950 for DOAMS, .950 for fixed VTM units, .900 for new mobile VTM units, .800 for any modified or existing VTM units.

2. Average time elapsed from the moment certain types of failures in the instrument occur until action is taken to correct such failure (c). Presently, this delay is estimated to be 10 working days, although parametric values of 2, 5, and 10 days were considered.

3. Average delay associated in repairing a failure (b). This was taken to be 2 days.

4. Average number of missions a telescope is used in a given day (a). This was parametrized and given values of .2 and .5.

At this stage, it is important to differentiate between the intrinsic instrument reliability and the proportion of quality data acquired. These two quantities are different. For example, if the intrinsic instrument reliability is .900, then, for $a = .5$ missions/day, $c = 10$ days, the corresponding proportion of quality data acquired is 66.6 percent.

Among others, it was assumed that 2 men are necessary to operate any of the instruments. Despite the controversy surrounding this particular topic, it was felt that factors related to occupational safety and environmental hazard will ultimately dictate in the near future the level of manpower utilized.

OTHER FACTORS

Due consideration was given to retaining an acceptable balance between fixed and mobile units in order to maintain the level of versatility and mobility dictated by operational requirements. In addition, the availability of existing sites for mobile units was related to the present and potential level of mobile VTMs. The existence of the prototype DOAMS was also an accepted condition.

Finally, the effect on geometric dilution of precision (GDOP) of attitude estimates was considered. The complex GDOP expression involved the number of each type of telescope available in the mixed instrumentation system, as well as their f_l and a relative qualitative factor. This factor was selected to be unity for all VTMs. For the DOAMS telescope, the factor called the DOAMS Qualitative Enhancement Factor (DQEF) reflects the relative improvement of the DOAMS over the VTM. Thus, a DQEF of 1.00 means essentially that VTMs and DOAMS have the same qualitative and optical features (except for f_l). This factor was introduced as a parameter since a value of relative improvement could not be established analytically.

PROBLEM SOLUTION

A linear objective function was formulated which consisted of the sum of the total life cycle costs associated with each of the alternatives available. The unknown variables were the quantity of each species in the instrumentation mix considered. A total of 18 linear inequality constraints was incorporated. The resultant linear programming problem was run for various values of the parameters a , c , and DQEF. After accounting for the various fixed costs and making the appropriate adjustments, the results obtained for the optimal solution are exhibited in Tables 3 and 4.

For example, if on the average each telescope is used in one mission a week ($a = .20$) and the average delay between failure occurrence and identification is 10 days, then, for a DQEF = 1.2, the optimal program mix is to acquire 7 new fixed VTMs, 4 mobile new VTMs and modify 17 of the existing VTMs at a total life cycle cost of about \$12,500,000.

The impact of the delay factor c on the life cycle cost can be noted. A reduction of this factor from the present level of 10 days to 2 days can generate a significant reduction in the amount of wasted film, hence, a corresponding substantial reduction in the total life cycle cost.

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OBJECTIVE

The present task involved the evaluation of the WSMR requirements for an integrated photo-optical instrumentation system for attitude data, vector miss-distance and event recordings. This included the available resources consisting of VTMs, the alternatives available for their modernization, the possibility of acquiring new VTMs used in fixed or mobile capacity and, finally, the possibility of acquiring DOAMS, a proposed new class of optical instrumentation.

The initial extensive analysis of the DOAMS is not repeated for the other classes of photo-optical systems since, for the most part, the steps involved in identifying variables, deriving equations, and formulating cost components remain essentially the same. For these cases, the final expressions and main results are given.

Following an economic analysis in which the initial investment costs and a 10 year life cycle discounted recurring costs are incorporated, a linear programming model is formulated to arrive at an optimum mix of each kind of instruments so that a minimum total cost over the 10 year life cycle is achieved while simultaneously maintaining acceptable levels of aggregate versatility and quality.

Specific alternatives considered were the following:

1. Partial or total retention of existing VTM units.
2. Partial or total modification of existing VTM units.
3. Acquisition of new VTMs used as mobile units.
4. Acquisition of new VTMs used as fixed units.
5. Acquisition of DOAMS.

GENERAL REMARKS

The following general remarks pertain to the analysis and are made at the outset in order to explain the reasons underlying some of the assumptions made, as well as to provide a common denominator in making a comparative economic analysis between the various proposed systems.

It is assumed that the existing IGORs and MIGORs have reached an obsolescence level warranting their immediate retirement. Thus, at the time of the present analysis, the telescope resources consist of 25 Photo-Sonics

Cine-Sextant telescopes, often referred to as VTMs. These telescopes utilize a manual focusing device and have the capability to accommodate up to 180 inch fl optics. It is important, however, to note that the present inventory of high powered optics consists of

1. five 180 inch fl athermalized optics (approximately T/11),
2. twenty-three 100 inch fl athermalized optics (approximately T/11), and
3. twenty-seven 50 inch fl athermalized optics.

Although a hitch and suspension system modification program for the VTM was implemented and field evaluated, uncertainty surrounds an overall implementation for such a program. However, it was agreed to consider the funds for this program as sunk cost and, hence, such costs were not incorporated in the analysis.

To establish a common basis for a trade-off analysis, particularly in relation to the nonrecurring cost factors, it was assumed that the equipment has a life or usage term extending to the end of the 10th calendar year of operation (to coincide with the anticipated end of future fiscal years) since acquisition. If, as it is assumed, acquisition is made in a fiscal year starting July 1 (and that is accepted as the first project year), then the actual life cycle costing will be over 9.5 years.

In general, one should not expect to obtain identical qualitative or quantitative final results when selecting different horizon periods. Evidently, the selection of a longer horizon period, say of the order of 15 or 20 years, will obviously result in lower averaged total discounted costs for all programs. However, the primary disadvantage of such a selection is the well known cost carryover effect if a future modernization program is introduced at any future point in time. The likelihood of a technologically improved system taking over the present proposed system is much higher when selecting, say, a 20 year horizon period than a 10 year horizon period. The impact of this accepted accounting practice cannot be neglected and could significantly affect the economic justification of any proposed future modernization program.

It was assumed that 2 men are necessary to operate any of the telescope instruments, DOAMS or VTM. From a strictly design standpoint, a single trained person can operate the DOAMS telescope. Although, as a general practice, the support of a VTM requires 2 operators, it was possible to identify several past instances when, the need arising, a single operator was used. Despite the present controversy surrounding this particular topic, it is felt that in the forthcoming years factors related to occupational safety and environmental hazard will ultimately dictate the minimum level of manpower utilized. Such factors as driving and transporting

equipment in a desert environment, extended period of isolation, size of equipment to operate, servo-drives of components, unexpected situations arising under certain operating conditions, and the like are adequate reasons to justify an operating team of not less than two persons.

In computing the present worth of recurring expenditures experienced over the life cycle of the equipment, an annual interest rate of 10 percent was used which generates a discount factor of $1/(1 + .10) = .90909$ per year. This figure is consistent with DOD guidelines. Inflationary factors were not incorporated to retain consistency with the accounting procedure used in the "System Validation Report." Under these assumptions, the present worth of the aggregate contribution of a recurring cost of \$1.00/annum incurred over the life cycle of the equipment (terminating at the end of its 10th year of operation and assuming that the first fiscal year starts July 1) is

$$(.954 + .867 + .788 + .717 + .652 + .592 + .538 + .489 + .445 + .405) = 6.447 .$$

This figure will be used throughout the report for computing the present worth of various recurring costs. By so doing, one retains consistency with the anticipated gradual changeover in the fiscal year guidelines to coincide eventually with the calendar year.

The following additional assumptions were made:

1. Average number of days missions are run per year = 250 days.
2. Average number of days operation per day of routine maintenance performed on a given telescope = 5 days.
3. Number of labor hours per day = 8 hours.
4. Hourly wage for labor associated with repair and maintenance, including fringe benefits = \$9.41.
5. Hourly wage for labor associated with the telescope operation, including fringe benefits = \$12.74.

THE DOAMS

The DOAMS is a proposed new class of photo-optical telescope instrumentation system. This is a transportable type telescope to be used more often in the capacity of a fixed unit, rather than a mobile unit. It possesses, however, several distinct characteristics which make it amenable to be dismounted from a given site, moved, and relocated to another especially designed site. Its mobility feature, on the other hand, is not comparable to the inherent mobility of the VTM.

The DOAMS unit incorporates an automatic variable focusing device, as well as the capability to accommodate 200 inch f1 athermalized optics. The DOAMS telescope possesses, in addition, several other qualitative and optical features which are not available on the VTM telescope. In particular, the pointing accuracy is of the order of 1 arc minute, the elevation axis will dump 180 degrees, veiling glare is minimized, higher image resolution and contrast can be achieved, the transmittance is $T/11.87$ for the 200 inch system, the wavefront requirement is $1/5\lambda$ rms, etc. Inflationary trends have substantially increased the unit procurement price. At a reduced performance equal to or better than the prototype, the unit price is \$695,168 for 4 units or less and \$624,049 for 5 units or more.

The DOAMS telescopes require special sites. Although two existing IGOR sites can be modified at a cost of \$20,000 each to accommodate the DOAMS telescopes, nevertheless, no other present sites are being contemplated to be modified and the full construction of any additional DOAMS site will cost about \$153,400 per site. Moreover, in order to fully utilize the transportable capability of the DOAMS telescope and ensure a broader and more diversified deployment of these telescopes when necessary, it becomes important to build more sites than available telescopes. The establishment of an average number of sites equalling 1.5 times the number of available DOAMS telescopes is an accepted figure which has been used in the analysis.

DOAMS telescopes will also require the acquisition of 2 special transporters at a cost of \$50,000 each.

The acquisition of the new DOAMS prototype and its testing has enabled the acquisition of valuable information in assessing the technical, optical, and operational performance of the telescope in general. This added knowledge cannot be ignored in an overall validation study. It should supplement the present analysis which incorporates basic economic parameters, as well as such factors as manpower requirements, reliability, downtime, etc.

The field performance of the present prototype has called for a diminution in the requirements of the transmittance ($T/11.87$ for the 200 inch system and $T/5.5$ for the 100 inch system) and the wavefront requirement ($1/15\lambda$ rms). The retention of the more rigid standards called by the original specifications does not seem to be warranted particularly in view of a corresponding significant increase in the unit procurement price of the telescope.

TELESCOPES

The telescopes (basic DOAMS excluding cameras) are characterized by their number and their unit cost.

Number ($X_1 - 1$)

It will be assumed that the additional total number of required DOAMS telescopes is an unknown variable ($X_1 - 1$) to be determined. Presently, one prototype has been acquired and contractual option calls for 9 additional units although the encumbered funding level will not allow more than 4 extra added units. The option, however, is not of a binding nature as a proviso exists for initiating a revised contract to purchase any number less than 9, or not purchasing any further units at all. In addition, any acquired units may be disposed of through a mutually agreeable working arrangement between WSMR and other agencies and/or concerns.

The cost of the DOAMS prototype has already been absorbed as a research and development (R&D) expenditure. This cost will be considered a sunk cost and will not be accounted for in any of the cost functions or equations.

Present production capability will only allow delivery of DOAMS telescopes at intervals over a number of years, rather than a delivery on a batch basis. This limitation is dictated by the fact that at Contraves-Goerz Corporation the DOAMS telescope is still a job shop custom ordered product rather than a mass produced one. For example, an order level of 9 extra units will be so filled that the first unit will be delivered on 1 February 1978 with succeeding units following at two months intervals and finally the 9th unit will be delivered on 1 July 1979. Any additional acquisition will have to be delayed beyond 1980. Since the planning horizon is taken to be the time frame 1976 through 1986, it will be assumed that adequate notice can be given to the manufacturer to fill in orders of any practical magnitude within this time frame. Equivalently, without loss in generality, it will be assumed that there is no production limitation.

Additional considerations, such as contractual obligations, availability of range real estate, command policies, and similar restrictions, do not appear to be factors in generating an upper bound level on the total number of DOAMS telescopes that can potentially be acquired.

We shall define the following variable

X_1 = total number of additional new telescopes to be procured.

Note that since a prototype telescope is already in the field, the total number of existing DOAMS telescopes on the range would be X_1 . This last quantity would be the one that would normally be used in the inequality constraints that will appear in the mathematical formulation of the problem.

Based on the previous analysis, a lower bound for X_1 is 1 while the upper bound is infinity. For parametric analysis, it may be convenient to set

the lower bound to a given positive quantity a_1 , and to set the upper bound to another positive given quantity B_1 , ($B_1 > a_1$). Thus,

$$a_1 \leq X_1 \leq B_1$$

Unit Procurement Cost of DOAMS Telescope P_1

The unit acquisition cost for a telescope, excluding cameras, is a known quantity and hence a given input parameter. It will be assumed at the outset that the total initial acquisition cost (nonrecurring) is proportional to the total number of units acquired. This assumption is, of course, an initial approximation since in practice two important factors should be accounted for.

1. It is a well accepted fact that inflationary forces governing the national economy will tend to escalate the cost of unitary production of the DOAMS telescopes in the years to come. For example, an estimated price increase of about 42 percent occurred between the August 1973 price quotation (\$380,000) and the May 1975 price quotation (\$540,000). The final firm price quotation (on a 9 unit basis) at a reduced performance equal to or better than the prototype is \$624,049. The unit price (based on a 9 unit basis) in full compliance with the specifications is quoted at \$740,918. This substantial increase in the unit price has been the main factor in calling for reduced performance specifications.

2. In general, the unit procurement cost depends on the total number of units acquired. For example, the acquisition of only 4 units at the reduced performance level is quoted at \$695,168 as of July 1976. The acquisition of a number of units larger than a total of 10 need not necessarily be accompanied by a reduction in the unit price. On the one hand, as more units are produced, the manufacturer will tend to improve technologically its production and run it more efficiently through a learning process, thus reducing the manufacturing cost. On the other hand, the delivery of the units subsequent to the 10th will occur on or about 1980, and the inflationary forces will likely wipe out any anticipated cost reduction. This becomes particularly critical if one considers the fact that the optical glass industry is a higher user of energy*. It is evident that the ultimate final unit price will have to be mutually agreeable to both WSMR and Contraves-Goerz. In summary, it is very likely that unit price for quantities greater than 10 will be about \$624,000, assuming reduced performance level.

For the present analysis, the cost term function is

$$P_1(X_1 - 1) = \begin{cases} 695,168 X_1 - 695,168 & 1 \leq X_1 \leq 4 \\ 624,049 X_1 - 624,049 & 5 \leq X_1 < \infty \end{cases}$$

*Conversation with Mr. G. A. Economou, Contraves-Goerz Corporation.

OPERATIONS AND MAINTENANCE COSTS

We refer here to the entries of Appendices A and B. The following symbols are defined:

p_1 = telescope reliability in a given mission.

a_1 = average number of missions a telescope is used in a given day.

c_1 = average number of days elapsed between failure occurrence and failure detection in a telescope.

b_1 = average number of days for troubleshooting and repair.

We assume that the mechanical maintenance aspect for DOAMS telescopes is negligible and that the dominating cost in maintenance is of electronic nature. The three basic cost components for operations and maintenance are

1. maintenance cost,
2. repair cost, and
3. operations and maintenance cost associated with missions.

Maintenance Cost

Material cost per telescope per annum-----\$167.95

Labor cost per telescope per annum-----\$775.80

Total material and labor cost per telescope per annum---\$943.75

Ten year discounted cost per telescope is

$$(6.447)(943.75) = \$6,084.35 \quad (1)$$

Repair Cost

We assume that repair involves only labor cost (minor repair). The proportion of time the telescope is in a repair state is

$$\frac{a_1 b_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)}$$

The average number of days the telescope is in a repair state per year, excluding routine maintenance of 5 days a year, is

$$(250 - 5) \frac{a_1 b_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)}$$

Assuming that 1.5 men are assigned to attend the repair, the total number of manhours per year necessary to repair a single DOAMS telescope is (assuming that one labor day consists of 8 hours)

$$(250 - 5) \frac{a_1 b_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)} (1.5)(8)$$

$$= 2940 \frac{a_1 b_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)}$$

At \$9.41 per labor day, the corresponding annual cost is

$$(9.41)(2940) \frac{a_1 b_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)}$$

$$= \$27,665.4 \frac{a_1 b_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)}$$

Thus, the 10 year discounted cost for repair of a single DOAMS telescope is

$$(6.447)(27,665.4) \frac{a_1 b_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)}$$

$$= 178,358.83 \frac{a_1 b_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)} \quad (2)$$

Operations and Maintenance Cost Associated with Missions

Although DOAMS is designed so that from an operational standpoint one skilled laborer can perform the necessary tasks for target tracking, nevertheless, as mentioned earlier in this report, it was assumed that this will more likely involve two laborers. The cost of operations and maintenance associated with a mission (System Validation Report) is at a laborer rate of \$12.75 per manhour:

Pre-mission:	(\$12.75)(2 men)(1 hr)	= \$25.48
Post mission:	(\$12.74)(2 men)(.5 hr)	= \$12.74
Operation:	(\$12.74)(2 men)(.75 hr)	= \$18.68
	TOTAL	\$56.90

On an average, 3.4 gallons of gasoline are consumed in transporting personnel to a telescope site. Using a cost of gasoline of \$0.39 per gallon, the fuel cost per mission is

$$(3.4 \text{ gallons})(\$0.39) = \$1.326$$

Thus, total cost of utilizing one DOAMS telescope per mission is

$$(\$56.90) + (\$1.326) = \$58.226$$

Assuming that missions are run on the average of 250 days a year, and that the average downtime per telescope per year for routine maintenance is 5 days, then the average number of missions a telescope is used in a given year is

$$(250 - 5)(\text{availability})(\text{average number of missions a telescope is used in a given day})$$

Since the availability is (see Appendix B)

$$(\text{availability}) = \frac{1 + a_1 c_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)}$$

it follows that the total average cost of utilizing one DOAMS telescope per year is

$$(\$58.226)(245) \frac{1 + a_1 c_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)} (a_1)$$

The corresponding 10 year discounted cost for operations and maintenance associated with missions per DOAMS telescope is

$$\begin{aligned} (6.447)(\$58.226)(245) \frac{1 + a_1 c_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)} (a_1) \\ = 91,968.84 \frac{[1 + a_1 c_1 (1 - p_1)] a_1}{1 + a_1 (b_1 + c_1) (1 - p_1)} \end{aligned} \quad (3)$$

Total Operations and Maintenance Costs

The total 10 year discounted cost of operations and maintenance for each DOAMS telescope is the sum total of (1), (2), and (3). We shall express this sum by the symbol C_1 and write

$$C_1 = \left\{ 6,084.35 + 178,358.83 \frac{a_1 b_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)} \right. \\ \left. + 91,968.84 \frac{[1 + a_1 c_1 (1 - p_1)] a_1}{1 + a_1 (b_1 + c_1) (1 - p_1)} \right\}$$

Since there are X_1 DOAMS, the total 10 year discounted cost of operations and maintenance is

$$C_1 X_1 = \left\{ 6,084.35 + 178,358.83 \frac{a_1 b_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)} \right. \\ \left. + 91,968.84 \frac{[1 + a_1 c_1 (1 - p_1)] a_1}{1 + a_1 (b_1 + c_1) (1 - p_1)} \right\} X_1 \quad (4)$$

COST ASSOCIATED WITH UNACCEPTABLE FILM QUALITY

Let

L_1 = average length of film used per mission (in feet),

e = cost of acquiring, processing, editing, storing one foot of film (in dollars).

The proportion of unacceptable quality data is (see Appendix B)

$$\frac{a_1 c_1 (1 - p_1)}{1 + a_1 c_1 (1 - p_1)}$$

Now, the average length of film used per telescope per year is

(250 - 5) (proportion of time a telescope is in use)
 (average number of missions per day per telescope)
 (average film length per mission)

$$= (245) \frac{1 + a_1 c_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)} (a_1) (L)$$

Thus, the average length of wasted film per telescope per year is

$$(245) \frac{1 + a_1 c_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)} (a_1) (L) \frac{a_1 c_1 (1 - p_1)}{1 + a_1 c_1 (1 - p_1)}$$

$$= 245 \frac{a_1^2 L c_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)}$$

The annual cost of wasted film per DOAMS telescope is

$$245 \frac{a_1^2 L c_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)} e$$

The 10 year discounted cost of wasted film per telescope is

$$(6.447)(245)(e) \frac{a_1^2 L c_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)}$$

$$= 1580e \frac{a_1^2 L c_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)}$$

The total 10 year discounted cost for the X_1 telescopes is

$$1580e \frac{a_1^2 L c_1 (1 - p_1)}{1 + a_1 (b_1 + c_1) (1 - p_1)} X_1 \quad (5)$$

Remark: Calculation of e (Total cost per foot of film)

70mm price per foot of color film-----	\$.4548
70mm price per foot for processing the film-----	<u>\$.2000</u>
	\$.6548 per foot

Others

Loss assessment per roll-----	\$.370
Loss reading time per roll-----	\$15.340
Reread 1.5 times per roll-----	\$23.010
Lost analyst time per roll-----	<u>\$ 7.620</u>
	\$46.34 per roll
CPU times for 3 reruns (using 6 stations) per roll-----	\$13.125 per roll

At 1,200 feet per roll, the extra additional cost per foot of film is

$$\frac{46.34 + 13.125}{1,200} = \$.0500 \text{ per foot}$$

The total cost per foot of wasted film is \$.7048 per foot.

DOAMS SITES

Initial Investment Cost

Investment capital is required to purchase DOAMS telescope sites. This will imply the following:

1. For the first two sites, modification of two existing telescope sites at a cost of \$20,000 per site.
2. Additional DOAMS sites to be purchased at a cost of \$153,400 per site.

We shall let Y_D denote the total number of DOAMS sites required. This number will depend not only on the total number of DOAMS units procured which is X_1 , but also on a future modernization program. This modernization program may involve the acquisition of new telescope (transportable or mobile). It is also possible that future missions may require flight paths and ground instrumentation geometry support different from the present ones. In any case, the construction of the DOAMS sites to meet future requirements is uncertain. Further the construction task is more likely to be evolving over the next 10 years.

Another added difficulty stems from the fact that the number of sites is also dictated in part by operational requirements. For example, 16 DOAMS telescopes would likely require less frequent relocation than 6 DOAMS telescopes, assuming that the system incorporates other supportive telescopes.

Conversation with individuals knowledgeable in operational support led to the conclusion that the number of sites should be of the order of one and a half times the number of DOAMS telescopes. Throughout the study, it was assumed that

$$Y_D = 1.5X_1$$

It was also assumed that the capital required for construction cost is presently encumbered. This leads to the following cost function for DOAMS site construction costs:

$$\begin{aligned}
 (2)(20,000) + (Y_D - 2)(153,000) &= (2)(20,000) + (1.5X_1 - 2)(153,000) \\
 &= 229,500X_1 - 266,000 \quad , \quad X_1 \geq 2 \quad . \quad (6)
 \end{aligned}$$

Operating Costs

A separate budget is set aside every year by the National Range Operations Directorate, Data Collection Division (NR-D), for the maintenance (including repair, routine maintenance, etc.) of all range sites. This allocation is not categorized by sites and does not make allowance for site location or complexity. The Facilities Engineering Directorate at WSMR is responsible for all aspects of this maintenance program except for the electronic control system of the astrodomes which is the responsibility of the optics shop. A nominal figure of \$300.00 per year would be considered an average maintenance cost for each site.

The total 10 year discounted cost for maintaining $Y_D = 1.5X_1$ DOAMS sites is then

$$(6.447)(300)(1.5X_1) = 2,901.15X_1 \quad . \quad (7)$$

Total Costs for DOAMS Sites

The total 10 year discounted cost for purchasing and operating DOAMS sites is the sum of (6) and (7) yielding

$$(229,500X_1 - 266,000) + (2,901X_1) = 232,401X_1 - 266,000 \quad , \quad X_1 \geq 2 \quad .$$

Other Fixed Costs Including Transporters

The operation of the DOAMS telescopes will require the acquisition of special transporters to resite the DOAMS when the need arises, as well as to carry the telescopes to the maintenance shops, if required. The cost of these transporters is estimated at \$50,000 each.

It is estimated that two transporters can adequately handle the logistics requirements of the DOAMS telescopes at an estimated cost of

$$(2)(50,000) = \$100,000.00 \quad .$$

In addition, it was assumed that \$1,000.00 a year is necessitated for operations and maintenance of each transporter resulting in a 10 year discounted cost of

$$(6.447)(2)(1,000.00) = \$12,894.00 \quad .$$

Finally, it was assumed that \$250.00 per year is necessary as vehicle maintenance for transporting personnel to sites, resulting in a 10 year discounted cost of

$$(6.447)(250) = \$1,612.00 \quad .$$

Total other fixed costs is \$114,473.00.

SUMMARY OF LIFE CYCLE COSTS FOR DOAMS TELESCOPES

Telescope Procurement

$$695,168X_1 - 695,168 \quad , \quad 1 \leq X_1 \leq 4$$

$$624,049X_1 - 624,049 \quad , \quad 5 \leq X_1 < \infty \quad .$$

Operations and Maintenance Costs

$$\{6,084 + \frac{178,359[a_1b_1(1-p_1)] + 91,969[1 + a_1c_1(1-p_1)]a_1}{1 + a_1(b_1 + c_1)(1-p_1)}\} X_1 \quad .$$

Cost Associated with Unacceptable Film Quality

$$1580eL \frac{a_1^2 c_1 (1-p_1)}{1 + a_1(b_1 + c_1)(1-p_1)} X_1 \quad .$$

DOAMS Sites Investment and Operating Costs

$$232,401X_1 - 266,000 \quad , \quad X_1 \geq 2 \quad .$$

Other Fixed Costs Including Transporters

$$114,473.00 \quad .$$

THE FIXED NEW VTM SYSTEM

ANALYSIS OF THE SYSTEM

We let X_2 denote the total number of fixed new VTMs. The procurement price for each X_2 unit, with optics, is estimated at \$455,000 and would include the following:

Basic mount, less trailer-----	\$179,000
19-bit encoder-----	55,000
Tracking scope-----	6,500
100 inch f1 lens (athermalized optics) (approximately T/11)-----	25,000
180 inch f1 lens (athermalized optics) (approximately T/11)-----	70,000
10B camera-----	25,000
10R camera-----	35,000
Two automatic focus tables (@ \$30,000 each)-----	60,000
	<u>\$455,000</u>

The other costs are the same as for DOAMS with the exception that each VTM will require exactly one site at a cost of \$153,400 per site and no transporters would be needed.

SUMMARY OF LIFE CYCLE COSTS FOR FIXED NEW VTMs

Telescope Procurement

$$455,500X_2$$

Operations and Maintenance Costs

$$\{6,084 + \frac{178,359[a_2b_2(1 - p_2)] + 91,969[1 + a_2c_2(1 - p_2)]a_2}{1 + a_2(b_2 + c_2)(1 - p_2)}\}X_2$$

Cost Associated with Unacceptable Film Quality

$$1580eL \frac{a_2^2c_2(1 - p_2)}{1 + a_2(b_2 + c_2)(1 - p_2)} X_2$$

VTM Sites Investment and Operating Costs

$$153,400X_2 + 2,901X_2 = 156,301X_2$$

Other Fixed Costs

Vehicle maintenance cost for transporting personnel to sites: \$1,612.

THE MOBILE NEW VTM SYSTEM

ANALYSIS OF THE SYSTEM

We let X_3 denote the total number of mobile new VTMs. The procurement price for each unit, with optics, is estimated at \$553,000 and would include the following:

Basic mount, less trailer-----	\$179,000
17-bit encoder-----	45,000
Tracking scope-----	6,500
100 inch f1 lens (athermalized optics)-----	25,000
180 inch f1 lens (athermalized optics)-----	70,000
10B camera-----	25,000
10R camera-----	35,000
Two automatic focus tables (@ \$30,000 each)-----	60,000
Generator-----	19,000
Support shop van-----	41,000
Prime mover-----	8,000
Modified trailer-----	<u>40,000</u>
TOTAL	\$553,500

In computing the operations and maintenance costs, the mechanical aspect of the maintenance cost cannot be neglected for mobile units. Again, we compute three basic components for operations and maintenance, namely,

1. maintenance cost,
2. repair cost, and
3. operations and maintenance cost associated with missions.

Maintenance Cost

Both mechanical and electronic aspects of maintenance are accounted for (see Appendix A).

Material cost per telescope per annum: $83.98 + 167.96 = \$ 251.94$

Labor cost per telescope per annum: $387.90 + 775.80 = \underline{\$1,163.70}$

Total material and labor cost per telescope per annum = $\$1,415.64$

Ten year discounted cost per telescope is

$$(6.447)(1,415.64) = \$9,126.63 \quad (8)$$

Repair Cost

The underlying assumptions and the general derivation are the same as the previous classes of telescopes. The 10 year discounted cost for repair of a single mobile new VTM unit is

$$178,358.83 \frac{a_3 b_3 (1 - p_3)}{1 + a_3 (b_3 + c_3) (1 - p_3)} \quad (9)$$

Operations and Maintenance Cost Associated with Missions

The analysis proceeds along the same line as in the "Systems Validation Report" by Russell, et al. We have the following:

1. Fuel Cost Analysis:

a. Average number of gallons of gasoline used to transport personnel to a VTM site is 3.4 gallons.

b. Average number of gallons of diesel fuel required to move VTM to a site is 6.7 gallons.

c. Average number of gallons of diesel fuel required to run a generator for a standard mission is 5.1 gallons.

d. Average number of gallons of diesel fuel required to run generator for real-time operation is 6.9 gallons.

e. Assume real-time data system used 80 percent of time.

f. Cost of fuel to transport VTM and personnel:

$$(6.7 \text{ gallons per mission})(111.3 \text{ mission per year})(\$.31 \text{ per gallon}) \\ = \$231.17 \text{ per year per site.}$$

g. Cost of fuel to run a generator for a real-time data mission:

$$(6.9 \text{ gallons per mission})(111.3 \text{ missions per year})(.80)(\$.31 \text{ per gallon}) \\ = \$190.00 \text{ per year per site.}$$

h. Cost of fuel to run a generator for a standard mission:

$$(5.1 \text{ gallons per mission})(111.3 \text{ missions per year})(.20)(\$.31 \text{ per gallon}) \\ = \$35.00 \text{ per year per site.}$$

i. Cost of diesel fuel for transport and generator per sites is

$$(231.00 + 190.00 + 35.00) = \$456.00.$$

Cost of fuel for transport per mission = $456.00/111.3 = \$4.10$ per mission.

2. Operation and Maintenance Cost:

$$\text{Relocation and premission: } (\$12.74 \text{ per hour})(2 \text{ men})(2 \text{ hours}) = \$50.96$$

$$\text{Post mission: } (\$12.74 \text{ per hour})(2 \text{ men})(.5 \text{ hour}) = \$12.74$$

$$\text{Operation: } (\$12.74 \text{ per hour})(2 \text{ men})(\text{support time of } .75 \text{ hour}) = \underline{\$19.11}$$

\$82.81/mission

$$\text{Total cost of operation per mission: } (\$4.10) + (\$82.81) = \$86.91$$

Assuming that missions are run on the average 250 days a year and that the average downtime per telescope per year for routine maintenance is 5 days, then the average number of missions a telescope is used in a given year is

$$(250 - 5)(\text{availability})(\text{average number of missions a telescope is used in a given day})$$

Since the availability is (see Appendix B)

$$(\text{availability}) = \frac{1 + a_3 c_3 (1 - p_3)}{1 + a_3 (b_3 + c_3) (1 - p_3)}$$

it follows that the total average cost of utilizing one new mobile VTM telescope per year is

$$(\$86.91)(245)(\text{availability})(a_3)$$

The corresponding 10 year discounted cost for operations and maintenance associated with missions per telescope is

$$\begin{aligned}
& (6.447)(\$86.91)(245) \frac{1 + a_3 c_3 (1 - p_3)}{1 + a_3 (b_3 + c_3) (1 - p_3)} (a_3) \\
& = 137,275 \frac{[1 + a_3 c_3 (1 - p_3)] a_3}{1 + a_3 (b_3 + c_3) (1 - p_3)} \quad . \quad (10)
\end{aligned}$$

Total Operations and Maintenance Costs

The total 10 year discounted cost of operations and maintenance for each new mobile VTM telescope is the sum of (8), (9), and (10). We shall express this sum by C_3 and write

$$\begin{aligned}
C_3 X_3 = & (9,127 + 178,359 \frac{a_3 b_3 (1 - p_3)}{1 + a_3 (b_3 + c_3) (1 - p_3)} \\
& + 137,275 \frac{[1 + a_3 c_3 (1 - p_3)] a_3}{1 + a_3 (b_3 + c_3) (1 - p_3)}) \quad .
\end{aligned}$$

Since there are X_3 new mobile VTM units, the total 10 year discounted cost of operations and maintenance is

$$\begin{aligned}
C_3 X_3 = & (9,127 + 178,359 \frac{a_3 b_3 (1 - p_3)}{1 + a_3 (b_3 + c_3) (1 - p_3)} \\
& + 137,275 \frac{[1 + a_3 c_3 (1 - p_3)] a_3}{1 + a_3 (b_3 + c_3) (1 - p_3)}) X_3 \quad .
\end{aligned}$$

Other Costs

The other costs are the same as before. Each VTM requires on the average 4 sites (see Systems Validation Report) at a cost of \$62,000 per site. Let Y_3 be the number of sites, then the total site cost is $\$248,000 Y_3$.

SUMMARY OF LIFE CYCLE COSTS FOR NEW MOBILE VTM UNITS

Telescope Procurement

$$553,500 X_3 \quad .$$

Operations and Maintenance Costs

$$(9,127 + \frac{178,359 [a_3 b_3 (1 - p_3)] + 137,275 [1 + a_3 c_3 (1 - p_3)] a_3}{1 + a_3 (b_3 + c_3) (1 - p_3)}) X_3 \quad .$$

Cost Associated with Unacceptable Film Quality

$$1580eL \frac{a_3^2 c_3 (1 - p_3)}{1 + a_3 (b_3 + c_3) (1 - p_3)} X_3 .$$

VTM Sites Investment and Operating Costs

$$248,000Y_3 + 1290Y_3 = 249,290Y_3 .$$

Other Fixed Costs

None.

THE MODIFIED (MOBILE) EXISTING VTM SYSTEM

ANALYSIS OF THE SYSTEM

We let X_4 denote the total number of existing VTMs to be modified and used as mobile units. The modification price for each unit, including a 180 inch f1 lens optics is estimated at \$144,000 and would include the following:

180 inch f1 lens (athermalized optics)-----	\$ 70,000
Two auto-focusing (@ \$21,000 each)-----	\$ 42,000
Two controlled shutters (@ \$8,000 each)-----	\$ 16,000
Replacement of existing IDC and FDRS with microprocessors-----	<u>\$ 16,000</u>
TOTAL	\$144,000

Other costs are the same as the mobile new VTM system.

SUMMARY OF LIFE CYCLE COSTS FOR MODIFIED EXISTING VTM SYSTEM

Telescope Procurement

$$144,000X_4 .$$

Operations and Maintenance Costs

$$(9,127 + \frac{178,359[a_4 b_4 (1 - p_4)] + 137,275[1 + a_4 c_4 (1 - p_4)] a_4}{1 + a_4 (b_4 + c_4) (1 - p_4)}) X_4 .$$

Cost Associated with Unacceptable Film Quality

$$1580eL \frac{a_4^2 c_4 (1 - p_4)}{1 + a_4 (b_4 + c_4) (1 - p_4)} X_4 .$$

VTM Sites Investment and Operating Costs

Let Y_4 be the number of VTM sites required; then

$$248,000Y_4 + 1290Y_4 = 249,290Y_4 .$$

Other Fixed Costs

None.

THE EXISTING VTM SYSTEM

ANALYSIS OF THE SYSTEM

We let X_5 denote the total number of existing VTMs to retain their status quo. It is evident that the sum of the total number X_4 of VTMs to be modified and the total number X_5 of VTMs to retain the status quo should not exceed 25, which is the total number of existing VTMs. Thus,

$$X_4 + X_5 \leq 25 .$$

There are no initial costs associated in this class of VTM systems. The other costs would essentially remain the same. Further, if Y_5 is the number of sites associated with this class of VTMs, then the corresponding VTM sites investment and operating costs are

$$248,000Y_5 + 1290Y_5 = 249,290Y_5 .$$

SUMMARY OF LIFE CYCLE COSTS FOR MODIFIED EXISTING VTM SYSTEM

Telescope Procurement

None.

Operations and Maintenance Costs

$$(9,127 + \frac{178,359[a_5 b_5 (1 - p_5)] + 137,275[1 + a_5 c_5 (1 - p_5)]a_5}{1 + a_5 (b_5 + c_5) (1 - p_5)}) X_5 .$$

Cost Associated with Unacceptable Film Quality

$$1580eL \frac{a_5^2 c_5 (1 - p_5)}{\gamma + a_5 (b_5 + c_5) (1 - p_5)} X_5 .$$

VTM Sites Investment and Operating Costs

$$248,000Y_5 + 1290Y_5 = 249,290Y_5 .$$

Other Fixed Costs

None.

INEQUALITY CONSTRAINTS AND COSTS FOR SITES FOR MOBILE VTMs

The following variables were identified for sites for mobile VTMs:

Y_3 = number of sites for mobile VTMs newly acquired,

Y_4 = number of sites for modified existing VTMs, and

Y_5 = number of sites for existing VTMs.

These 3 variables could be reduced to 1 if we let Y_1 = number of new sites additions for VTMs = $Y_3 + Y_4 + Y_5$. The existing levels of VTMs is denoted by Y . Clearly,

$$Y \leq 150 .$$

Assuming that on the average each mobile VTM unit would require 4 sites, then

$$Y + Y_1 = 4(X_3 + X_4 + X_5) ,$$

with the proviso that $Y_1 \geq 0$. Hence, the inequality

$$4X_3 + 4X_4 + 4X_5 - Y \geq 0 .$$

The three previous expressions will appear as the second, third, and fourth constraints in the mathematical model for linear programming.

Note also that since the variables Y_3 , Y_4 , and Y_5 were grouped into a single variable Y_1 , the associated cost for mobile VTM sites can be integrated into a single cost. Noting that the 150 existing sites require a 10 year discounted cost of maintenance of \$1,290 per site, the total cost for VTM sites can be expressed as

$$62,000Y_3 + 1,290Y_3 + 62,000Y_4 + 1,290Y_4 + 62,000Y_5 + 1,290Y_5 \\ + (1,290)(150) = 63,290(Y_3 + Y_4 + Y_5) + 199,947 .$$

THE OPTICS SYSTEM: ITS AVAILABILITY AND COST

It is necessary to identify the available number of optics which can be utilized on VTM systems. This presently consist of the following:

1. Five 180 inch f1 athermalized optics.
2. Twenty-three 100 inch f1 athermalized optics.
3. Twenty-seven 50 inch f1 athermalized optics.

Adequate supply of the 50 inch optics does not necessitate consideration of any further procurement for the existing VTM system. This is likely to hold true also for the 100 inch optics. The 180 inch optics has a substantially low inventory and a modernization program should consider new acquisitions. Present prices are as follows:

1. For the 180 inch f1 optics: \$70,000.
2. For the 100 inch f1 optics: \$30,000.

THE CAMERA SYSTEM

Two basic camera units can be mounted on any of the DOAMS or VTM telescopes. These are

1. the 70mm, Model 10-B, rotary prism recording camera, and
2. the 70mm, Model 10-R, recording camera.

Both of these are manufactured by Photo-Sonics. Although any of these cameras can be adapted to any of the telescope units, be it VTM or DOAMS, it is most suitable to adapt the 10-R to the optical unit with the larger f1 (say the 200 inch on the DOAMS telescope) and the 10-B to the optical unit with the smaller f1 (say the 100 inch on the DOAMS telescope). We briefly digress in the sequel on each of these recording camera units.

THE 70MM, MODEL 10-B, ROTARY PRISM RECORDING CAMERA

The camera is designed for high speed motion pictures at frame rates of 90, 180, and 360 frames per second. As an optional feature, it is possible to obtain half-frame rates of 180, 360, and 720 frames per second. Images are captured on 70mm films. These cameras can also be positioned on fixed camera installations in the range.

The present level of inventory of the 70mm, Model 10-B, recording cameras is as follows:

1. 10 old units due to retire.
2. 20 additional existing units in operable condition.
3. 11 recently procured existing units.

Thus, for all practical purposes, approximately 31 units of the Model 10-B cameras should be considered as available in inventory. The procurement of the 11 new units was made for an anticipated acquisition of 10 new DOAMS telescope systems, with one backup (float) camera unit. Present purchase price per unit of the Model 10-B cameras is estimated at \$28,000.00.

THE 70MM, MODEL 10-R, RECORDING CAMERA

This camera utilizes intermittent pin-registered movement operation to record motion pictures on 70mm frames. The frame rate is infinitely variable ranging from 6 to 125 frames per second. These cameras can also be utilized on fixed camera installations on the range.

The range modernization program called for the acquisition of 20 of these cameras to be used on 20 of the existing VTMs. This was done in 1975. An additional twelve 70mm, Model 10-R cameras were purchased for the anticipated 10 new DOAMS telescope systems, 2 of these being backup (float) camera units. Thus, a total of thirty-two 70mm, Model 10-R, recording cameras are presently available in inventory. Present purchase price per unit of these cameras with FDRS capability is estimated to be \$34,000.00.

To summarize, the following inventory of cameras is available:

Model 10-B cameras: 30 + 1 backup = 31.

Model 10-R cameras: 30 + 2 backups = 32.

ACCOUNTING FOR CAMERA COSTS

The present inventory of Model 10-B and 10-R cameras is adequate to take care of the existing 20 VTMs, as well as an additional 10 telescopes. Acquisition of any further telescopes would require a corresponding

additional acquisition of both Model 10-B and 10-R cameras. From the standpoint of life cycle costing, expenditures incurred so far to build up the present inventory of cameras are considered as sunk cost.

Since the price of some of the alternative telescope systems, such as new fixed and new mobile VTM units already include the cost of the 10-B and the 10-R cameras, special provision was made to account for this situation in the problem minimum considered without unduly increasing the formulation of the problem. In order to simplify the structure of the mathematical model, the costing of any additional cameras required on and above the present inventory level, has not been explicitly incorporated as a cost variable in the objective function. However, the final cost tabulation is adjusted to account for any additional camera acquisition necessitated and as dictated by the optimal solution to the life cycle cost problem. For example, let the optimal solution yield as answer 1 DOAMS, 9 new fixed VTMs, 2 new mobile VTMs, the modification of 16 existing VTMs, and the retention of 9 of the existing VTMs. Then, recalling the present level of cameras and the fact that the new fixed and mobile VTMs already incorporate camera units, the following is obtained:

Excess 10-B cameras: $(31) + (11) - (37) = 5$ at \$28,000 per unit .

Excess 10-R cameras: $(32) + (11) - (37) = 6$ at \$34,000 per unit .

Thus, 5 of the new VTMs (fixed or mobile) will be purchased without 10-B cameras and 6 of them will be purchased without 10-R cameras. The total minimum cost dictated by the optimal solution would be reduced by

$$(5)(28,000) + (6)(34,000) = \$344,000 .$$

INEQUALITIES GOVERNING THE EQUIVALENCE BETWEEN FIXED (TRANSPORTABLE) AND MOBILE TELESCOPES

From a strictly operational standpoint, based on daily scheduling requirements and in order to provide proper instrumentation support to implement the various mission testing, it is evident that some type of an operational equivalence exists between fixed (transportable) and mobile telescopes. This equivalence could be established heuristically in the form of a relationship which would reflect the versatility of the mobile telescopes in its usage over different missions.

In performing the analysis, we assume that the set of fixed telescopes have, except for the mobility factor, the same characteristics as the set of mobile telescopes. In addition, we assume that adequate manpower level exists to cope with all the scheduling requirements and that all telescopes

are in perfect operating condition without being subject to failure. Two extreme scenarios could be conceived which would adequately define the mobility factor. An intermediate third scenario is also discussed.

SCENARIO 1

We assume here that all available telescopes are of the fixed type. And one asks the question: "How many of the fixed telescopes are necessary in order to fulfill all mission requirements over the next decade?" The idea here is to establish over the entire range complex adequate fixed sites to accommodate fixed telescopes. Thus, with the provision that adequate workforce exists to man these telescopes, an adequate inventory of fixed telescopes exists to cope with any of the daily schedules over the next decade. It is evident that under such circumstances, the scheduling function would still persist but at a reduced level. The adoption of such an extreme situation would still involve, under workforce limitation, the allocation of manpower to the various fixed telescope sites.

The ensuing qualitative analysis provides an insight on the problem structure.

At the present approximately 700 telescope sites are scattered throughout the range to accommodate the mobile VTM telescopes. These sites are utilized at one time or another as a telescope site. Thus, from a strictly theoretical standpoint, if one is attempting to cover the entire range with permanently fixed telescopes, 700 of them would be necessary with all existing 700 telescope sites reconstructed to accommodate all of them. This extreme situation does not, however, accept the transportable feature of the DOAMS telescopes and is of no practical value. Alternatively, one could conceive of 150 transportable telescopes scattered over the range with a very low aggregate utilization factor of the order of 10 to 15 percent, such that the telescopes would be relocated with a frequency of the order of once or twice a year. This alternative would imply the construction of more than 150 sites to accommodate these telescopes. This again is not within the domain of feasibility, and certainly does not capitalize more fully on the transportability of the telescopes.

More realistically, one could think of a total of 60 transportable telescopes with a utilization factor of the order of 50 to 60 percent and a frequency of relocation of the order of once a week. Approximately half of the telescopes would be permanently sited in the Stallion area. The remaining half would be used and shared as the necessity arises among the remaining sites.

Thus, a plausible alternative is to have at least 60 transportable telescope units with no mobile units.

SCENARIO 2

In this situation, we assume that all telescopes are of the mobile type and again one poses the question on how many of them are necessary to fulfill all mission requirements over the next decade without relying on fixed (transportable) telescopes. The underlying assumptions remain the same as for Scenario 1 and the methodology to perform the analysis is again of the same kind. Two sources of information were used.

The first source assesses an approximate total of 50 mobile telescope units with an aggregate utilization factor of 70 to 80 percent. Obviously, realizing that some of the sites are more frequently used than others, many of the telescopes, even though mobile in their characteristics, will rarely be resited, and would thus be defacto permanent. On the other hand, the remaining units would likely be resited at an average frequency of once a day.

The second source of information is the System Validation report:

"From an analysis of the total October 1975 monthly mission forecast (21 work days), the range was presented, on four different days, with user requirements for telescope instrumentation support necessitating the use of from 23 to 32 VTMs. Some days contained such a difficult mix of mission types forecast that as many as 17 VTMs would require relocation to different sites at least once daily to provide adequate geometry to satisfy the different project data collection requirements. These relocation moves for VTMs require from three to five and one-half hours depending on the distance and immediate availability of personnel after completion of a mission to transport the equipment to another site to make ready for a later mission."

To account for an extreme situation of the aforementioned type and in order to minimize the frequency of daily relocation under this extreme situation, a conservative total of what is presently available (25 mobile units) plus 17 extra ones was believed to substantially take care of the requirements. This yielded a total of 42 mobile units.

As an arbitrary compromised level between these two sources, of mobile telescope units necessary with no transportable units, the number 45 was chosen.

SCENARIO 3

This intermediate scenario is the one recommended by the System Validation Report prepared by the DOAMS Study Committee which favors the purchase of not less than 12 DOAMS type instruments, in addition to the existing 25 mobile telescope units.

AGGREGATION AND INTERPOLATION OF SCENARIOS

The three previously discussed scenarios form the basis for identifying and quantitatively defining the mobility factor. More explicitly, we assume that all three scenarios are equivalent in the mobility sense, and that the mix level of instruments in each scenario constitutes the minimum acceptable level. We further assume that the aggregate of three sets of data points provided by the respective scenarios can be aggregated and used to obtain a mix level between fixed and mobile telescopes which is equivalent to any of the mobility levels dictated by the scenarios.

To obtain an acceptable mix level, we hypothesize that the three data points are generated by a quadratic function. Let

F = number of fixed telescopes in the mix, and

M = number of mobile telescopes in the mix.

Then we assume a relation of the form

$$F = aM^2 + bM + c \quad ,$$

where a , b , and c are constants to be determined given the (F, M) data points $(60, 0)$, $(12, 25)$, and $(0, 45)$. Three equations in the three unknowns a , b , and c can be obtained yielding

$$F = .029333M^2 - 2.65333M + 60 \quad .$$

For computational convenience, this quadratic relation has been piecewise linearized to yield the following system of linear equations:

1. $25 \leq F \leq 60$, $0 \leq M \leq 16$
 $16F + 35M - 960 = 0$
2. $20 \leq F \leq 25$, $16 \leq M \leq 19.20$
 $3.2F + 5M - 160 = 0$
3. $18 \leq F \leq 20$, $19.20 \leq M \leq 20.50$
 $1.3F + 2M - 64.4 = 0$
4. $15 \leq F \leq 18$, $20.50 \leq M \leq 22.60$
 $2.1F + 3M - 99.3 = 0$
5. $12 \leq F \leq 15$, $22.60 \leq M \leq 25.00$
 $2.4F + 3M - 103.8 = 0$

6. $10 \leq F \leq 12$, $25.00 \leq M \leq 26.80$
 $1.8F + 2M - 71.6 = 0$
7. $8 \leq F \leq 10$, $26.80 \leq M \leq 28.70$
 $1.9F + 2M - 72.6 = 0$
8. $7 \leq F \leq 8$, $28.70 \leq M \leq 29.75$
 $1.05F + M - 37.1 = 0$
9. $5 \leq F \leq 7$, $29.75 \leq M \leq 32$
 $2.25F + 2M - 75.25 = 0$
10. $3 \leq F \leq 5$, $32 \leq M \leq 35$
 $3F + 2M - 79.0 = 0$
11. $1 \leq F \leq 3$, $35 \leq M \leq 39$
 $2F + M - 41.0 = 0$
12. $0 \leq F \leq 1$, $39 \leq M \leq 45$
 $6F + M - 45.0 = 0$

To summarize, in order to sustain the proper level of mobility and versatility, a certain minimum mix must exist between the total number of fixed telescopes (F) and the total number of mobile telescopes (M). Although the minimum level of this mix is dictated for three situations heuristically by operational requirements based on judgmental factors, nevertheless, it is possible to quantify the minimum level of the mix using a reasonable interpolation scheme.

Since a mix which exceeds the minimum requirements level is still acceptable, the implementation of any of the above 12 equations will necessitate their conversion to reflect this situation. For example, the first expression would be written as

$$16F + 35M - 960 \geq 0$$

Thus, a grand total of twelve inequalities will have to be satisfied.

As a final remark, one should note that the twelve relations are a piecewise linear approximation to a quadratic function. Other such approximations can be constructed using finer subdivisions. These, however, will not add any further information to the problem.

THE FINAL SET OF INEQUALITIES

In the actual problem, we have

X_1 = number of DOAMS telescopes,

X_2 = number of fixed new VTM telescopes,

X_3 = number of mobile new VTM telescopes,

X_4 = number of modified VTM mobile telescopes, and

X_5 = number of existing VTM mobile telescopes.

Thus,

$$F = X_1 + X_2 \quad ,$$

$$M = X_3 + X_4 + X_5 \quad .$$

The simplex which will reflect quantitatively the stated mobility and versatility requirements can be represented by a set of twelve inequalities in five variables. Since the set of points upon which the sequence of line segments is built is convex, no additional new variables need to be introduced, and we have the following:

$$\begin{aligned} 16X_1 + 16X_2 + 35X_3 + 35X_4 + 35X_5 &\geq 960 \\ 3.2X_1 + 3.2X_2 + 5X_3 + 5X_4 + 5X_5 &\geq 160 \\ 1.3X_1 + 1.3X_2 + 2X_3 + 2X_4 + 2X_5 &\geq 64.4 \\ 2.1X_1 + 2.1X_2 + 3X_3 + 3X_4 + 3X_5 &\geq 99.3 \\ 2.4X_1 + 2.4X_2 + 3X_3 + 3X_4 + 3X_5 &\geq 103.8 \\ 1.8X_1 + 1.8X_2 + 2X_3 + 2X_4 + 2X_5 &\geq 71.6 \\ 1.9X_1 + 1.9X_2 + 2X_3 + 2X_4 + 2X_5 &\geq 72.6 \\ 1.05X_1 + 1.05X_2 + X_3 + X_4 + X_5 &\geq 37.1 \\ 2.25X_1 + 2.25X_2 + 2X_3 + 2X_4 + 2X_5 &\geq 75.25 \\ 3X_1 + 3X_2 + 2X_3 + 2X_4 + 2X_5 &\geq 79.0 \\ 2X_1 + 2X_2 + X_3 + X_4 + X_5 &\geq 41.0 \\ 6X_1 + 6X_2 + X_3 + X_4 + X_5 &\geq 45.0 \end{aligned}$$

V-ANGLE BIAS AND ERROR IN V-ANGLE MEASUREMENTS

V-ANGLE BIAS

A reasonably exhaustive discussion of V-angle bias is included in Lockheed Electronics Co. (LEC) Report¹. Due to certain natural bounding conditions, the observed V-angles on the photographic frames will appear to have rotated from their true position. This bias is "a result of the illumination of an object changing its apparent shape and orientation due to the interactions of the contrast and resolution as seen by the observing telescopes."¹ The observed V-angle depends in general on

1. the geometry of the target object,
2. the sum illumination of the object relative to the observer, and
3. the target aspect angle relative to the observer's optical system.

Simulation results in the LEC Report have shown that the first two are the dominant factors. There are several theoretical and practical techniques and procedures to considerably reduce these bias errors in any given reading. We shall assume that one or more of these correction techniques will be implemented to eliminate this bias source of "error" in the final attitude estimate of the target object.

Another possibility to minimize the V-angle bias error is to exploit the geometric siting of the instruments in a multistation configuration so that through some symmetrical pairing the biases are oppositely equalized and thus minimized.

ERROR IN ATTITUDE ESTIMATE DUE TO ERRORS IN V-ANGLE MEASUREMENTS

Error in V-angle measurements, excluding the bias factor, can be attributed to five basic factors, namely,

1. quality of the image (object contrast, signal-to-noise ratio of recorded image, etc.),
2. size of the image,
3. accuracy of the V-angle reader (Telereadex film reader),

1. C. Richey, R. Willoughby, G. Abravanel, and M. Beeman, "Distant Optical Attitude Measurement Telescope System," LEC Theoretical Report 70-1, WAO 2003, July 1970.

4. individual performing the actual measurement, and
5. resolution relative to the image size.

It is evident that in a multistation system involving simultaneous measurements, errors in the attitude estimation is generated from errors in the V-angle measurements. Moreover, the relative positions of the instruments with respect to each other, as well as their position relative to the target, will have an impact on the estimated attitude. Theoretically, all these factors can be incorporated in a GDOP expression for attitude. In this respect, the contribution by D. W. Comstock in calculating error in two station attitude is to be noted (see Appendix C).

However, similar work involving more than two stations does not exist. As a result, the ensuing discussion will simply attempt to highlight some of the problems encountered and discuss them in a heuristic fashion. No derivations will be attempted; the formulas derived will be somewhat representative of what one may ultimately expect.

Part of the analysis will rely on some previous work². In this work, GDOP expressions are obtained on position estimates for cinetheodolites to reflect the interrelationships between station configuration, station mix, slant range of target, number of instruments in each mix, and image quality.

The analysis for attitude GDOP proceeds as follows. We consider a mix of m classes of telescopes (e.g., DOAMS, VTM, modified VTM, etc.) tracking a stationary target. The instruments in a particular class have the same quality and are of the same type. Each class of instruments is symmetrically deployed relative to the approximate target position. Hence, for each class of instruments an equi-elevation target-site position is retained. Direct recordings from telescopes yield images where the V-angle measurements have errors which are independently and normally distributed.

We assume that the azimuth and elevation angles of the instruments are so selected that the attitude GDOP is minimized. The GDOP is a measure of the error in attitude estimate using the actual V-angle measurements. Let then

m = number of classes of telescopes,

n_k = number of instruments in class k ($k = 1, 2, \dots, m$),

s = slant range, and

σ_k = standard error in V-angle measurement for instrument of class k .

2. B. D. Sivazlian, "Effect of Mixed Instrumentation in Tracking Targets with Minimal Error," Research Report No. 76-10, Department of Industrial and Systems Engineering, The University of Florida.

Then, we conjecture that the minimum achievable GDOP quantity, say GDOP*, is given by the following expression:

$$\text{GDOP}^* \propto \frac{s}{\frac{n_1}{\sigma_1^2} + \frac{n_2}{\sigma_2^2} + \dots + \frac{n_m}{\sigma_m^2}} \quad (11)$$

The quantities s and n_k , $k = 1, 2, \dots, m$, are known. To determine σ_k , we shall perform an analysis of the image size in relation to slant range and the fl of the telescope involved in the measurement.

IMAGE SIZE ANALYSIS

Image size I is inversely proportional to slant range s and proportional to fl F of the telescope. We can thus write

$$I \propto \frac{F}{s} \quad (12)$$

We now attempt to formulate a relationship between the standard error σ in V-angle measurements and the image size I . It is intuitively clear that the curve of σ as a function I should be as shown in Figure 1.

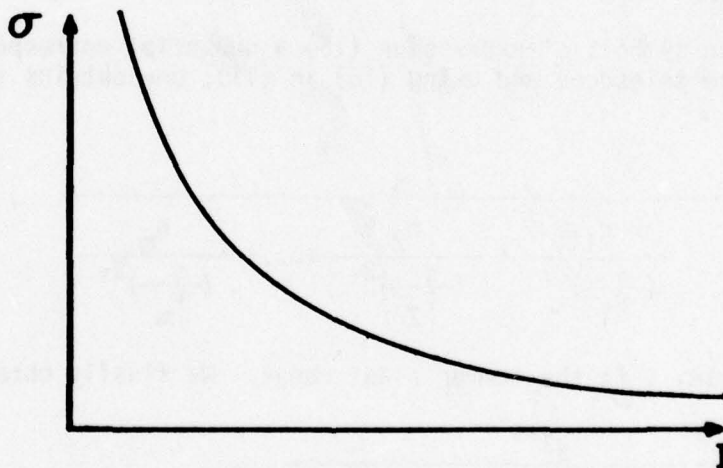


FIGURE 1. STANDARD ERROR VERSUS IMAGE SIZE

When the image size I is zero (a point), the error in measurement is infinity. Theoretically, as the image size increases, the value of σ tends to zero. However, in practice, the film frame size, as well as the image blur size, imposes some natural constraints on the maximum achievable image size. As an empirical relation, one may conjecture that

$$\sigma \propto \frac{1}{I^\gamma} \quad \gamma > 0 \quad . \quad (13)$$

The value of the constant γ is determined empirically.

Results of experimental investigations performed to determine the relationship between the standard deviation of V-angle reading and image size for various missile length to width ratios are tabulated in Table 1. These results were compiled from observations gathered through two types of Telereadexes, namely, the Grid Lines and the Bruning Arm.

In estimating the value of γ , we have selected the most common type of missile length to width ratio, 8:1 and assumed Grid Lines Telereadex. By taking logarithms on both sides of expression (13), one obtains a linear relationship in γ . The plot of $\ln \sigma$ $\ln I$ is as shown in Figure 2. The slope of the line gives an estimate of $\gamma = .5625$. Thus, Equation (13) becomes

$$\sigma \propto \frac{1}{I^{.5625}} \quad . \quad (14)$$

In general, if one combines expressions (12) and (13), one obtains

$$\sigma \propto \left(\frac{s}{F}\right)^\gamma \quad . \quad (15)$$

A HEURISTIC RELATION

Apposing on the symbols of expression (15) a subscript corresponding to the appropriate telescope and using (15) in (11), one obtains the following for (GDOP)*:

$$(GDOP)^* \propto \frac{s}{\frac{n_1}{\left(\frac{s}{F_1}\right)^{2\gamma}} + \frac{n_2}{\left(\frac{s}{F_2}\right)^{2\gamma}} + \dots + \frac{n_m}{\left(\frac{s}{F_m}\right)^{2\gamma}}}$$

where, of course, s is the common slant range. We finally obtain

$$(GDOP)^* \propto \frac{s^{\gamma+1}}{n_1 F_1^{2\gamma} + n_2 F_2^{2\gamma} + \dots + n_m F_m^{2\gamma}} \quad . \quad (16)$$

For a given slant range, a minimum prespecified level of least GDOP, say A , will be achieved if the following condition holds:

TABLE 1. IMAGE SIZE VERSUS V-ANGLE READING STANDARD DEVIATION

MISSILE LENGTH TO WIDTH RATIO	IMAGE LENGTH (mm)				
	1/2	1	2	4	8
	σ_v (Degrees)				
σ_{v_1} Telereadex (Grid Lines)	0.61	0.37	0.54	0.38	0.17
σ_{v_2} Telereadex (Bruning Arm)	0.26	0.08	0.25	0.32	0.22
<hr/>					
4:1 Ratio	σ_v (Degrees)				
σ_{v_1}	0.41	0.27	0.25	0.18	0.11
σ_{v_2}	0.21	0.08	0.12		0.16
<hr/>					
8:1 Ratio	σ_v (Degrees)				
σ_{v_1}	0.37	0.27	0.14	0.12	0.08
σ_{v_2}	0.25	0.38	0.13	0.05	0.11
<hr/>					
16:1 Ratio	σ_v (Degrees)				
σ_{v_1}	0.16	0.19	0.09	0.11	0.07
σ_{v_2}		0.08	0.03	0.09	0.08

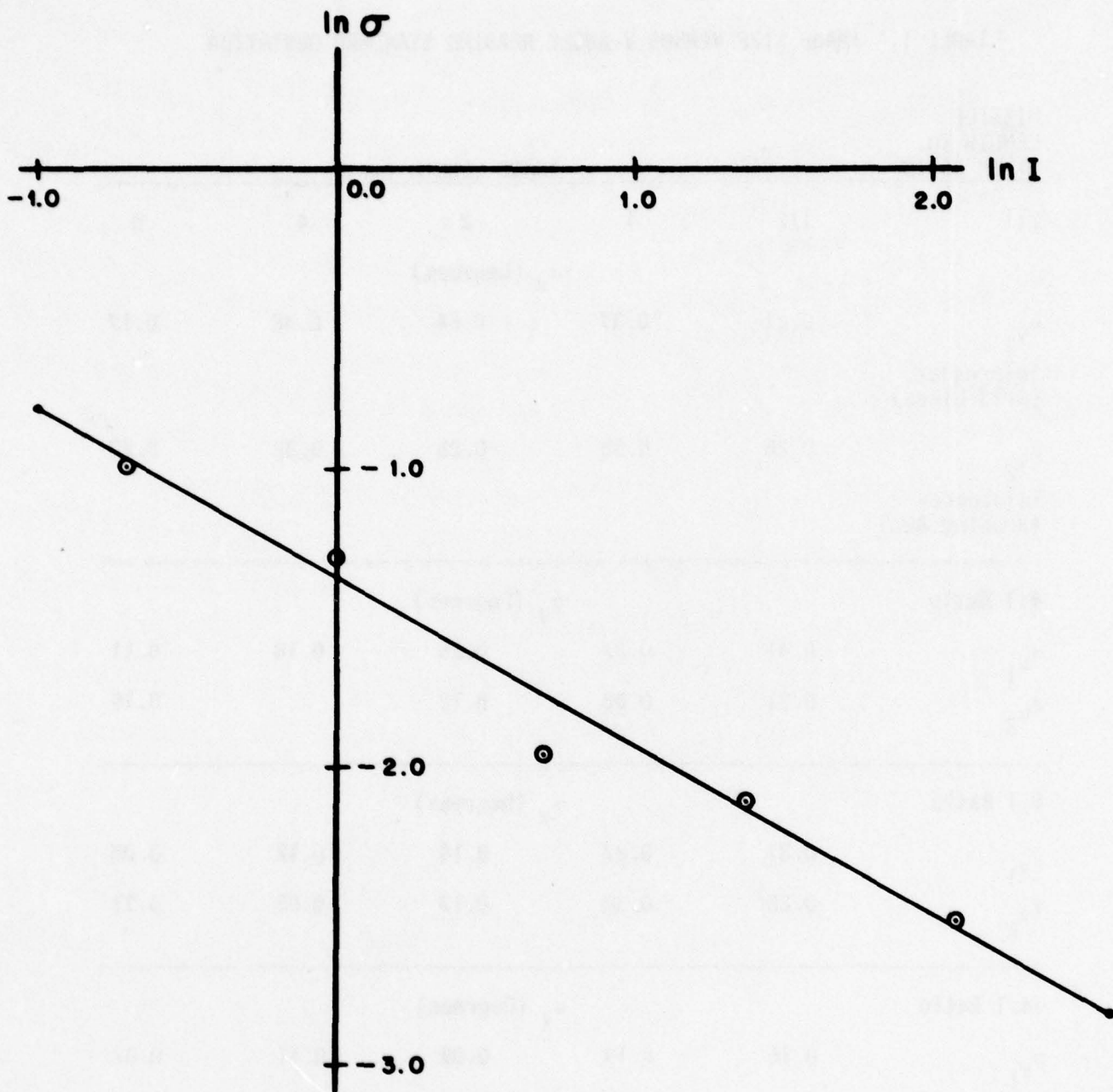


FIGURE 2. PLOT OF THE LOGARITHM OF THE STANDARD DEVIATION OF THE V-ANGLE READING VERSUS THE LOGARITHM OF IMAGE SIZE.

$$\frac{s^{\gamma+1}}{n_1 F_1^{2\gamma} + n_2 F_2^{2\gamma} + \dots + n_m F_m^{2\gamma}} \leq A$$

or equivalently,

$$n_1 F_1^{2\gamma} + n_2 F_2^{2\gamma} + \dots + n_m F_m^{2\gamma} \geq k \quad (17)$$

where k is a constant to be determined.

Thus, given that there are m telescope systems participating in a given mission and tracking a given target, such that n_i and F_i ($i = 1, 2, \dots, m$) are, respectively, the number of units in system i and F_i the fl, the required mix to achieve a prescribed least GDOP level should satisfy an inequality of the form (17). Note that the only nominal telescope characteristics affecting inequality (17) is the fl. It is conceivable, of course, that each telescope system has some additional features, such as optical, mechanical, qualitative or otherwise. Such extra features are not accounted for explicitly in expression (17).

In order to be able to reflect these other factors, relation (17) could be modified by attaching to each term in the left-hand side a weighting factor, say d_i ($i = 1, 2, \dots, m$), to be determined qualitatively. In this modified form, relation (17) becomes

$$d_1 n_1 F_1^{2\gamma} + d_2 n_2 F_2^{2\gamma} + \dots + d_m n_m F_m^{2\gamma} \geq k \quad (18)$$

Suppose now that the telescopes are scattered on the range in groups such that each group consists of the same mix of instruments. Let for $k = 1, 2, \dots, m$

Y_k = total number of instruments of class k , and

B = number of groupings.

If now we assume that the instruments in each group are symmetrically deployed with an equi-elevation target site position, then

$$Y_k = B n_k \quad k = 1, 2, \dots, m$$

Substitution in relation (18) yields

$$d_1 Y_1 F_1^{2\gamma} + d_2 Y_2 F_2^{2\gamma} + \dots + d_m Y_m F_m^{2\gamma} \geq \frac{k}{B} \quad (19)$$

THE DOAMS QUALITATIVE ENHANCEMENT FACTOR

The application of inequality (19) in practice needs some further considerations. Relation (19) assumes that each telescope is in perfect operating condition and not subject to failure and associated downtimes. This

implies that each telescope is available at any point in time and is capable of always providing quality data. In reality, this is not the case: telescopes are subject to malfunction and hence quality data is acquired only a fraction of the time by each individual telescope.

Thus, if X_k , $k = 1, 2, \dots, m$, represents in actuality, the total number of telescopes of class k , and if $[1 + a_k c_k (1 - p_k)]^{-1}$ represents the proportion of quality data acquired by a telescope of class k (see Appendix B), then interpreting in (19) Y_k to be the long time average number of telescopes of class k which at any instant of time provides quality data, we should have

$$\frac{Y_k}{X_k} = \frac{1}{1 + a_k c_k (1 - p_k)} \quad , \quad k = 1, 2, \dots, m \quad . \quad (20)$$

Substituting (20) in (19) yields the inequality

$$\sum_{k=1}^m \frac{d_k F_k^2 Y_k X_k}{1 + a_k (b_k + c_k) (1 - p_k)} \geq \frac{k}{\beta} \quad . \quad (21)$$

In applying inequality (21), the following assumptions were made:

1. There are 5 basic classes of telescopes ($m = 5$) corresponding to the following:

- a. Index 1 for DOAMS telescopes.
- b. Index 2 for fixed new VTM telescopes.
- c. Index 3 for mobile new VTM telescopes.
- d. Index 4 for modified VTM mobile telescopes.
- e. Index 5 for existing VTM mobile telescopes.

For example, X_2 refers to the total number of fixed new VTM telescopes and F_1 corresponds to the fl of the DOAMS telescope.

2. The quantity a_k corresponding to the average number of missions a telescope is used in a given day was considered to be the same for all telescopes, i.e.,

$$a_1 = a_2 = a_3 = a_4 = a_5 = a \quad .$$

The quantity a was parametrized.

3. The quantity b_k corresponding to the average number of days necessary for troubleshooting and repair in a telescope was assumed to be 2 days for all telescopes:

$$b_1 = b_2 = b_3 = b_4 = b_5 = b = 2 \text{ days} .$$

4. The quantity c_k corresponding to the average number of days elapsed between failure occurrence and failure detection in a telescope was assumed to be the same for all telescopes and was parametrized

$$c_1 = c_2 = c_3 = c_4 = c_5 = c .$$

5. The quantity p_k corresponding to the telescope reliability in a given mission was taken as follows:

$$p_1 = .95 , p_2 = .95 , p_3 = .90 , p_4 = .80 , p_5 = .80 .$$

6. The quantity F_k corresponding to the fl of telescope k was taken as follows:

$$F_1 = 200 , F_2 = F_3 = F_4 = 180 \text{ and } F_5 = 100 .$$

7. The quantity γ was taken to be $\gamma = .5625$.

8. The quantity d_k was selected to be the same for all VTM telescopes, i.e., $d = d_k$, $k = 2, 3, 4, 5$. Dividing both sides of expression (21) by d and substituting for the parameter values, one obtains

$$\begin{aligned} & \frac{(d_1/d)F_1^{1.125}}{1 + a(2+c)(1-p_1)} X_1 + \frac{F_2^{1.125}}{1 + a(2+c)(1-p_2)} X_2 \\ & + \frac{F_3^{1.125}}{1 + a(2+c)(1-p_3)} X_3 + \frac{F_4^{1.125}}{1 + a(2+c)(1-p_4)} X_4 \\ & + \frac{F_5^{1.125}}{1 + a(2+c)(1-p_5)} X_5 \geq \tilde{k} , \end{aligned} \quad (22)$$

where $\tilde{k} = k/bd$. The quantity d_1/d , denoted by d_f , was called the DQEF. The factor d_f reflects the relative merits of the quality and optics of the DOAMS telescope. Thus, a DQEF = 1 = d_f means essentially that VTMs and DOAMS have the same qualitative and optical features (except for fl). The DQEF factor was parametrized in the final computation to assess on a cost basis the qualitative importance one should attach to the DOAMS telescope.

DETERMINATION OF \bar{k}

In order to be able to use relation (22) as one of the inequalities in the set of constraints for the linear programming, it is necessary to determine the value of \bar{k} . Note in particular that expression (22) tells us that the aggregate combination of factors characteristic of each telescope as appearing in the left-hand side of inequality (22) do weight the number of telescopes in each class (X_k) in such a way that the left-hand side should be at least equal to \bar{k} . We have assumed that the mix of telescopes which would constitute a norm dictating the value of \bar{k} would be the mix dictated by the "Systems Validation Report" prepared by the DOAMS Study Committee, 15 April 1976. The recommendation is to have at least 12 DOAMS telescopes and retain 5 of the existing VTM telescopes and modify 20 of the VTMs. Thus, \bar{k} is determined on the basis that

$$X_1 = 12, X_2 = 0, X_3 = 0, X_4 = 5, \text{ and } X_5 = 20.$$

THE MATHEMATICAL MODEL

INTRODUCTION

The mathematical model was expressed as a linear programming problem involving the following:

1. Seven variables: $X_1, X_2, X_3, X_4, X_5, Y_1$, and Y .
2. A linear objective function expressing the 10 year total discounted costs in the acquisition and operation of the mix of the five classes of specified telescopes, as well as the cost of supporting systems.
3. A set of 18 linear inequalities expressing site constraints, equivalence between fixed and mobile telescopes, minimum number of DOAMS telescopes, and qualitative requirements.
4. The objective is to determine the optimum mix of the five classes of telescopes so as to minimize the total 10 year discounted cost.

THE MODEL

The configuration of the general model is given in Table 2. Unfilled blank spaces referring to coefficients of variables are computed on the basis of the parametric analysis as previously discussed. The following definitions for inequalities were used:

TABLE 2. GENERAL SYSTEM CONFIGURATION INEQUALITIES

	X_1	X_2	X_3	X_4	X_5	Y_1	Y	
COST				1.0	+1.0			\leq 25.0
VIM OLD				X_4	X_5		Y	\leq 150.0
SIT OLD				X_4	X_5		+1.0 Y	
SIT, TE			+4.0 X_3	X_4	X_5	-1.0 Y_1	Y	= 0.0
SIT POS			+4.0 X_3	X_4	X_5		-1.0 Y	> 0.0
TE, EQ1	16	+16 X_2	+35.0 X_3	+35.0 X_4	+35.0 X_5			> 960.00
TE, EQ2	3.2	+3.2 X_2	+5.0 X_3	+5.0 X_4	+5.0 X_5			> 160.00
TE, EQ3	1.3	+1.3 X_2	+2.0 X_3	+2.0 X_4	+2.0 X_5			> 64.4
TE, EQ4	2.1	+2.1 X_2	+3.0 X_3	+3.0 X_4	+3.0 X_5			> 99.3
TE, EQ5	2.4	+2.4 X_2	+3.0 X_3	+3.0 X_4	+3.0 X_5			> 103.8
TE, EQ6	1.8	+1.8 X_2	+2.0 X_3	+2.0 X_4	+2.0 X_5			> 71.6
TE, EQ7	1.9	+1.9 X_2	+2.0 X_3	+2.0 X_4	+2.0 X_5			> 72.6
TE, EQ8	1.05	+1.05 X_2	+1.0 X_3	+1.0 X_4	+1.0 X_5			> 37.1
TE, EQ9	2.25	+2.25 X_2	+2.0 X_3	+2.0 X_4	+2.0 X_5			> 75.25
TE, EQ10	3.0	+3.0 X_2	+2.0 X_3	+2.0 X_4	+2.0 X_5			> 79.0
TE, EQ11	2.0	+2.0 X_2	+1.0 X_3	+1.0 X_4	+1.0 X_5			> 41.0
TE, EQ12	6.0	+6.0 X_2	+1.0 X_3	+1.0 X_4	+1.0 X_5			> 45.0
DOAMS	1.00							> 1.0
QUAL	X_2	X_2	X_3	X_4	X_5			>

VTM OLD: number of existing VTMs should not exceed 25.

SIT OLD: number of existing VTM sites should not exceed 150.

SIT, TE: relation between total number of mobile VTMs and required sites.

SIT POS: inequality stating that new VTM sites can only be added to 150, not subtracted.

TE, EQ1: inequalities governing the relation between mobile and
TE, EQ12: fixed telescopes.

DOAMS: the prototype DOAMS is for keeps.

QUAL: inequality governing the qualitative requirements.

To retain model simplicity and manageability, no additional constraints were added to the problem. However, the level of manpower was checked after obtaining the optimal solution to assure that such level was not violated.

In addition, since integer solution is not guaranteed, a fractional solution was rounded off to the nearest integer. Further, the level of 5 existing 180 inch optics was to be ascertained, and the inventory of cameras was to be properly balanced.

Thus, following the computation of the "optimal" solution and the rounding off, the total cost was computed based on the rounded off solution and the aforementioned adjustments. The sample computation format is as shown in Figure 3.

THE OPTIMAL SOLUTION

The obtained solutions, as well as the corresponding costs, were displayed for various values of a, c, and DQEF in Tables 3 and 4.

Values of a were taken as .20 and .50 missions per day. Values of c were taken to be 2, 5, and 10 days. Finally, a DQEF value of 1.2, 1.4, 1.6, and 1.8 were used. It is evident that the higher the DQEF, the more likely the DOAMS telescopes would be favored as solution to the problem. This, of course, is amply illustrated in the solution to the problem as appearing in Tables 3 and 4.

Nonrecurring Cost Plus 10 Year Discounted Recurring Cost

1. DOAMS telescope ()	-----	\$
2. New fixed VTM ()(includes cameras)	-----	\$
3. New mobile VTM ()(includes cameras)	-----	\$
4. Modify existing VTM ()	-----	\$
5. Existing VTM ()	-----	\$
	SUBTOTAL	\$

Other Costs

1. 5 existing 180 inch optics (5)(70,000)	-----	\$350,000
2. 10 year discounted cost for 150 mobile unit sites	-----	\$193,410
3. Vehicle maintenance for transporting personnel to sites	----	\$ 8,059
4. Camera adjustment cost	-----	\$
	SUBTOTAL	\$
	TOTAL	\$

DOAMS Cost ($X_1 =$)

Procurement ()()	-----	\$
Maintenance, repair, operations ()()	-----	\$
Wasted film ()()	-----	\$
Site procurement	-----	\$
Site maintenance	-----	\$
Transporter cost	-----	\$
Operations and maintenance for transporter	-----	\$
	TOTAL	\$

New Fixed VTM Cost ($X_2 =$)

Procurement ()()	-----	\$
Maintenance, repair, operations ()()	-----	\$
Wasted film ()()	-----	\$
Site procurement ()()	-----	\$
Site maintenance ()()	-----	\$
	TOTAL	\$

FIGURE 3. FORMS USED TO ESTIMATE SYSTEM COSTS

New Mobile VTM ($X_3 =$)

Procurement ()()	-----	\$
Maintenance, repair, operations ()()	-----	\$
Wasted film ()()	-----	\$
		<hr/>
	TOTAL	\$

Modify Existing VTM ($X_4 =$)

Modification Cost ()()	-----	\$
Maintenance, repair, operations ()()	-----	\$
Wasted film ()()	-----	\$
		<hr/>
	TOTAL	\$

Existing VTM ($X_5 =$)

Maintenance, repair, operations ()()	-----	\$
Wasted film ()()	-----	\$
		<hr/>
	TOTAL	\$

FIGURE 3. (Cont'd)

TABLE 3. OPTIMUM MIX OF INSTRUMENTS WITH ASSOCIATED COSTS

a = .20 missions per day; c = 2 days

	DQEF			
TELESCOPES	1.2	1.4	1.6	1.8
DOAMS	1	1	12	12
New fixed VTM	6	6	0	0
New mobile VTM	5	5	0	0
Modify existing VTM	15	20	0	0
Existing VTM	10	5	25	25
COST (\$)	10,576,252	11,296,252	12,311,973	12,311,973

a = .20 missions per day; c = 5 days

	DQEF			
TELESCOPES	1.2	1.4	1.6	1.8
DOAMS	1	12	12	12
New fixed VTM	2	0	0	0
New mobile VTM	10	0	0	0
Modify existing VTM	13	0	0	0
Existing VTM	12	25	25	25
COST (\$)	11,449,938	12,957,942	12,957,942	12,957,942

a = .20 missions per day; c = 10 days

	DQEF			
TELESCOPES	1.2	1.4	1.6	1.8
DOAMS	1	1	12	12
New fixed VTM	7	7	0	0
New mobile VTM	4	4	0	0
Modify existing VTM	17	25	0	0
Existing VTM	8	0	25	25
COST (\$)	12,467,100	13,619,100	13,805,353	13,805,353

Optimum mix of telescopes and corresponding minimum total 10 year life cycle discounted cost for various values of a and c.

a = average number of daily missions per telescope.

c = average delay in days between failure occurrence and detection in a telescope.

DQEF = DOAMS Qualitative Enhancement Factor.

TABLE 4. OPTIMUM MIX OF INSTRUMENTS WITH ASSOCIATED COSTS

a = .50 missions per day; c = 2 days

TELESCOPES	DQEF			
	1.2	1.4	1.6	1.8
DOAMS	1	1	12	12
New fixed VTM	9	9	0	0
New mobile VTM	2	2	0	0
Modify existing VTM	16	23	0	0
Existing VTM	9	2	25	25
COST (\$)	14,658,786	15,666,786	16,050,411	16,050,411

a = .50 missions per day; c = 5 days

TELESCOPES	DQEF			
	1.2	1.4	1.6	1.8
DOAMS	1	1	12	12
New fixed VTM	11	11	0	0
New mobile VTM	0	0	0	0
Modify existing VTM	17	25	0	0
Existing VTM	8	0	25	25
COST (\$)	17,756,101	18,908,101	19,015,592	19,015,592

a = .50 missions per day; c = 10 days

TELESCOPES	DQEF			
	1.2	1.4	1.6	1.8
DOAMS	1	12	12	12
New fixed VTM	11	0	0	0
New mobile VTM	0	0	0	0
Modify existing VTM	19	0	0	0
Existing VTM	6	25	25	25
COST (\$)	21,332,929	22,304,420	22,304,420	22,304,420

Optimum mix of telescopes and corresponding minimum total 10 year life cycle discounted cost for various values of a and c.

a = average number of daily missions per telescope.

c = average delay in days between failure occurrence and detection in a telescope.

DQEF = DOAMS Qualitative Enhancement Factor.

APPENDIX A

MATERIAL AND LABOR COSTS, AND AVAILABLE MANPOWER FOR TELESCOPES

ESTIMATING MATERIAL AND LABOR REQUIREMENTS

Data output for the South Range for the fiscal year 1975 provided the basis for obtaining estimates on material and labor costs on telescope maintenance and repair. These data yielded the following for 15 mobile Photo-Sonics telescopes:

1. Total annual manhours used: 1,855.
2. Total annual material cost: \$3,779.

Thus, on the average, the total manhours per year used on each telescope are

$$\frac{1,855}{15} = 124 .$$

Approximately 1/3 of the manhours are detailed for mechanical maintenance and 2/3 for electronic maintenance. Thus, for mechanical maintenance,

$$\text{manhours per telescope per year} = \frac{1}{3} \times 124 = 41.33 ,$$

and for electronic maintenance,

$$\text{manhours per telescope per year} = \frac{2}{3} \times 124 = 82.66 .$$

Assuming 2 workers are detailed for each aspect of this maintenance and that they are taking place simultaneously, the time necessary to complete a single maintenance on a given telescope is that necessary for the electronic maintenance that is

$$\frac{82.66}{8 \times 2} \approx 5 \text{ working days} .$$

Thus, out of the possible total number of 250 working days in a year, the average number of days a telescope is either operationally available or in a state of repair (excluding maintenance) is

$$(250 - \text{maintenance days per year}) = 245 \text{ days} .$$

Equivalently, we can say that the average number of days a telescope is operationally available is

$$(245)(\text{availability factor}) ,$$

and that the average number of days a telescope is in a state of repair is

$$(245)(1 - \text{availability factor})(\text{excluding maintenance})$$

ESTIMATING MATERIAL AND LABOR COSTS

Material cost per telescope per year for mechanical maintenance is

$$\left(\frac{1}{15}\right)(3,779)\left(\frac{1}{3}\right) = \$83.98$$

Material cost per telescope per year for electronic maintenance is

$$\left(\frac{1}{15}\right)(3,779)\left(\frac{2}{3}\right) = \$167.96$$

The standard labor cost is computed on the basis of \$9.41 per hour. Thus, labor cost per telescope per year for mechanical maintenance is

$$\left(\frac{1}{15}\right)(9.41)(1,855)\left(\frac{1}{3}\right) = \$387.50$$

The labor cost per telescope per year for electronic maintenance is

$$\left(\frac{1}{15}\right)(9.41)(1,855)\left(\frac{2}{3}\right) = \$775.80$$

ESTIMATING AVAILABLE MANHOURS PER YEAR FOR MAINTENANCE AND REPAIR IN THE SOUTH AND NORTH RANGES

South Range

Seven maintenance workers are presently available in the South Range for combined maintenance and repair work on telescope units. The following assumptions will be made:

1. Number of weeks per year = 50.
2. Number of hours per week = 40.
3. Allowance for leave = 15 percent.

The total number of manhour contribution per worker per year is

$$(50)(40)(.85) = 1,700 \text{ manhours}$$

The total number of manhours available per year in the South Range is

$$(7)(1,700) = 11,900 \text{ manhours}$$

This will be assumed to be apportioned on the basis of 1/3 on mechanical work and 2/3 on electronic work. Thus, the following estimates will be utilized. For mechanical maintenance and repair work, the total number of available manhours per year is

$$\left(\frac{1}{3}\right)(11,900) = 3,967 \text{ manhours per year .}$$

For electronic maintenance and repair work

$$\left(\frac{2}{3}\right)(11,900) = 7,933 \text{ manhours per year .}$$

North Range

Same number of maintenance workers are available in the North Range as in the South Range. Using the same data, we obtain the following.

For mechanical maintenance and repair work, the total number of available manhours per year is

$$\left(\frac{1}{3}\right)(11,900) = 3,967 \text{ manhours per year .}$$

For electronic maintenance and repair work,

$$\left(\frac{2}{3}\right)(11,900) = 7,933 \text{ manhours per year .}$$

Although the North Range is operated by contractors, approximately the same number of instruments are allocated to both North and South Ranges and the basic manpower available, as well as cost and wages are assumed to be the same.

The total combined manhours available per year in the North and South Ranges for maintenance and repair work are as follows:

<u>RANGE</u>	<u>MECHANICAL WORK</u>	<u>ELECTRONIC WORK</u>
South	3,967	7,933
North	<u>3,967</u>	<u>7,933</u>
TOTAL	7,934 manhours	15,866 manhours

APPENDIX B

AVAILABILITY ANALYSIS AND RELIABILITY ESTIMATION FOR TELESCOPES

In this appendix, we develop mathematical expressions for the availability of a telescope unit, as well as other expressions for the proportion of time the telescope is in a particular state, such as repair. In addition, a procedure is suggested whereby the true reliability of a telescope can be estimated based on available data. This is used to estimate the reliability of existing VTMs.

In general when a telescope is used on a particular mission, there exists a possibility that it fails to perform its function. We shall let

p = telescope reliability in a given mission. This, of course, is the probability of successful operation of the telescope during a mission. The quantity $q = 1 - p$ is the probability that the telescope will fail during a mission.

Clearly, the average number of missions the telescope will operate successfully before failure occurs is

$$\sum_{r=1}^{\infty} rp^{r-1}q = \frac{1}{q} = \frac{1}{1-p}$$

Let now

a = average number of missions a telescope is used in a given day.

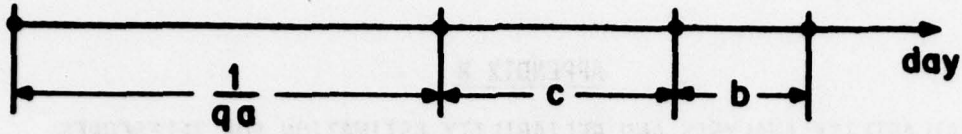
Then the average number of days a telescope is operative (up time) is

$$\frac{1}{a(1-p)}$$

One of the main features of telescope failures is that very often they cannot be immediately detected and identified as the film records proceed through image analysis. In other words, the quality and characteristics of images obtained from post mission developed films are the sources of information to detect failure occurrence of telescopes. Following such image analysis, the telescope may be retrieved from operation for troubleshooting and repair. Let

c = average number of days elapsed between failure occurrence and failure detection in a telescope, and

b = average number of days for troubleshooting and repair.



During the c days elapsed from failure until detection occurs, the telescope is utilized in the field, but the quality of information acquired on the film has no operational value.

AVAILABILITY ANALYSIS

availability = proportion of time the telescope is in use

$$= \frac{\frac{1}{qa} + c}{\frac{1}{qa} + c + b} = \frac{1 + ac(1 - p)}{1 + a(b + c)(1 - p)}$$

Proportion of quality data acquired is

$$\frac{\frac{1}{qa}}{\frac{1}{qa} + c} = \frac{1}{1 + ac(1 - p)}$$

Proportion of time the telescope is in a repair state is

$$\frac{b}{\frac{1}{qa} + c + b} = \frac{ab(1 - p)}{1 + a(b + c)(1 - p)}$$

RELIABILITY ESTIMATION

Reliability estimates for a telescope based on records taken at times when failure is detected are spurious since they do not account for the time for failure detection. For a given telescope, let

\hat{R} = spurious reliability estimate, and

\hat{p} = true reliability estimate.

Then, based on the number of missions, we have the following equality:

Average number of missions a telescope is used following downtime = average number of missions the telescope is used following downtime, until failure occurs + average number of missions the telescope is used between failure occurrence and failure detection.

Hence,

$$\frac{1}{1 - \hat{R}} = \frac{1}{1 - \hat{p}} + ac$$

Thus, the true reliability estimate for the telescope is

$$\hat{p} = 1 - \frac{1}{\frac{1}{1 - \hat{R}} - ac}$$

EXAMPLE

Let $\hat{R} = .912$, $c = 10$ days, $a = .14$ missions per day. Then

$$\hat{p} = 1 - \frac{1}{\frac{1}{1 - .912} - (.14)(10)} = .90$$

RELIABILITY ESTIMATION FOR EXISTING VTM

From 1 March to 25 August 1976, out of 3013 missions involving VTM telescopes, 251 failures occurred due to either mechanical or electronic malfunctions in the telescopes. Thus, the spurious reliability estimate of VTMs is

$$\hat{R} = 1 - \frac{251}{3013} = .9167$$

The true reliability estimate is

$$\hat{p} = 1 - \frac{1}{\frac{1}{1 - \hat{R}} - ac}$$

If the average number of missions was for example $a = .14$ missions per day and $c = 10$ days, then $\hat{p} \approx .90$.

APPENDIX C

ERROR IN TWO STATION ATTITUDE SOLUTIONS

The standard deviation of the V-angles may be propagated through the two station solution to yield the standard deviation of the missile's attitude angles (σ_α , σ_ϵ). This is not done for two station solutions since we cannot obtain an estimate of the V-angle error for two stations. However, the results can be expressed as a propagation factor times the standard deviation of the V-angle. This was done for two cases.

CASE I

The azimuth angles from station to missile is held constant while the elevation angle varies from 5 to 75 degrees. Stations are located on opposite sides of the missile.

$\alpha_1 = 90^\circ,$	$\alpha_2 = 270^\circ,$	$\alpha_A = 0^\circ,$	$\epsilon_A = 0^\circ$
$\epsilon_1 = \epsilon_2$	σ_α		σ_ϵ
5°	$8.1\sigma_V$		$.7\sigma_V$
15°	$2.7\sigma_V$		$.7\sigma_V$
30°	$1.4\sigma_V$		$.8\sigma_V$
45°	σ_V		σ_V
60°	$.8\sigma_V$		$1.4\sigma_V$
75°	$.7\sigma_V$		$2.7\sigma_V$

CASE II

The elevation angle is held constant at 45 degrees while the azimuth angle is allowed to vary. Again stations are on opposite sides of the missile.

$\epsilon_1 = 45^\circ,$	$\epsilon_2 = 45^\circ,$	$\alpha_A = 0^\circ,$	$\epsilon_A = 0^\circ$
α_1	α_2	σ_A	σ_ϵ
90°	270°	σ_V	σ_V
60°	300°	$.9\sigma_V$	σ_V
30°	330°	$.6\sigma_V$	$1.3\sigma_V$
15°	345°	$.5\sigma_V$	$2.1\sigma_V$
5°	355°	$.5\sigma_V$	$5.8\sigma_V$

CASE III

The elevation angle is held constant at 45 degrees. The azimuth of station one is held constant at 90 degrees while the azimuth of station two varies.

$\epsilon_1 = 45^\circ,$	$\epsilon_2 = 45^\circ,$	$\alpha_A = 0^\circ,$	$\epsilon_A = 0^\circ$
α_1	α_2	σ_α	α_ϵ
90°	0°	$.7\sigma_V$	$1.6\sigma_V$
90°	30°	$2.3\sigma_V$	$3.3\sigma_V$
90°	45°	$5.0\sigma_V$	$6.0\sigma_V$
90°	60°	$13.0\sigma_V$	$14.0\sigma_V$

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