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A STUDY OF SEARCH AND SCREENING WITH HYDROFOIL VEHICLES.(U)
JAN 62 W T KINSELLA, E J FOOTE, J D YOUNG
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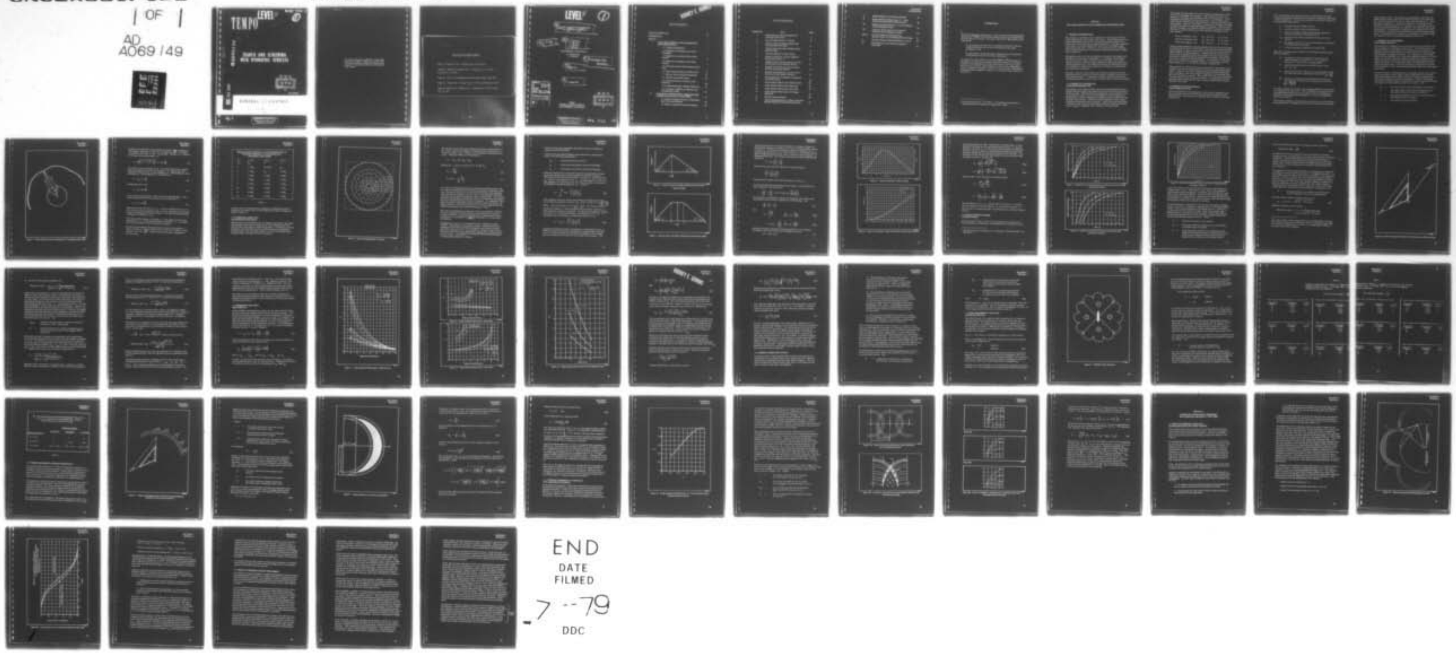

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SEARCH AND SCREENING WITH HYDROFOIL VEHICLES

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GENERAL ELECTRIC

SANTA BARBARA, CALIFORNIA

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Errata Sheet to RM 62 TMP-5

Page 17. Equation (14). Change "xin" to read "sin".

Page 24. Figure 14, lowest curve. Change "T_d = 6.0 min." to read "T_d = 60 min".

Page 32. Table 3. At beginning of second line of title, add "d".

Page 41. Figure 20a. Change "$R_c/d = 0.45$" to read "$R_c/d = 0.9$".

Page 41. Figure 20 c, Middle curve. Change "$P_s = 0.66$" to read "$P_s = 0.67$".

LEVEL II

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A STUDY OF SEARCH AND SCREENING WITH HYDROFOIL VEHICLES.

⑫ 57p.

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W. T. Kinsella,
E. J. Foote,
J. D. Young
G. A. Bosanko

Acknowledgment of Contribution:

P. J. McDonough
V. Nishihara

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INTRODUCTION

This study has been conducted by a team from the Professional Staff Operation of TEMPO for the Heavy Military Electronics Department, General Electric Company, Syracuse, New York. It consists of two basic parts:

1) an analysis of task force screening by hydrofoil vehicles, using intermittent search with convergence zone sonar propagation; and

(2) an analysis of the interaction between enemy submarines and hydrofoil vehicles escorting a task force.

The objective of this study is to provide HMED with parametric information to permit the evaluation of the effectiveness of potential hydrofoil craft operating as an anti-submarine screen for a naval task force or a convoy, with emphasis on the former. This document is intended to parallel a report of a similar study conducted by Boeing Airplane Company* relative to conventional sonar search by hydrofoils.

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* Boeing Document No. D 2-10868, "A Technique for Analysis of Intermittent Search Operations Applicable to ASW."

SECTION I

TASK FORCE SCREENING WITH CONVERGENCE ZONE PROPAGATION

1.1 GENERAL CONSIDERATIONS

This study is an analysis of anti-submarine screening by hydrofoil craft equipped with convergence zone sonar. The problems of localization and attack were for the most part beyond the scope of the study; they are touched upon in part in Section II. Primary emphasis in the study is directed towards the mission of screening a high speed naval task force. However, the analytical model is flexible, and is equally applicable to screening of merchant convoys or other ship formations.

Recognition is given to the uncertainties in tactical and weapon capabilities of future enemy submarines. Accordingly, the distance out from the force center to which sonar coverage is to be maintained (a function of enemy weapon range) and submarine speed are treated parametrically, and different submarine penetration tactics are considered. Submarine speeds up to 35 knots are examined in the numerical illustrations.

Similarly, hydrofoil acceleration and maximum speed are treated parametrically in order to provide a range of potential vehicle operating characteristics. A hydrofoil maximum speed of 50 knots is considered feasible, and this value is used in the numerical examples.

1.2 PROPERTIES OF CONVERGENCE ZONE SOUND PROPAGATION

Convergence zone sonars detect submarines by transmitting sound signals through certain long range acoustic paths that are present in the deep ocean. With this type of propagation, the variations in the sound velocity profile of the ocean medium cause the sound beam periodically to refract downward to a depth of about 12,000 feet and back up to the surface. Where the beam reappears at the surface, a submarine detection zone is formed. These zones take the shape of annular rings concentric about the sound source and at distances from it equal to multiples of the distance to the first zone.

The ranges to these zones vary in the different geographical areas where such propagation is possible. The width of the zone is a function of the range, and increases with increasing depth. In the illustrations that will be used in this study, a near-surface zone width has been selected as representative of normal submarine operating depths. However, the width is a parameter and may be varied to study the effects of deep submergence if desired. Where used in specific examples, the following representative numerical values of the width w and the radius R_c of the various convergence zones were chosen:

First convergence zone:	$R_c = 30$ n. mi.	$w = 3$ n. mi.
Second convergence zone:	$R_c = 60$ n. mi.	$w = 6$ n. mi.
Third convergence zone:	$R_c = 90$ n. mi.	$w = 9$ n. mi.

Under normal conditions, the transmission time for the sound energy to travel from the source to the first zone above and return would be approximately 77 secs.; travel time to and from the second and third zones would be multiples thereof. Considering this travel time, a sonar mode of operation where three pings were sent out in rapid succession would increase the probability of detection and not interfere with receiving the returning signal. Accordingly, for the purpose of this study, the minimum dip time to search each zone has been chosen as 1.5, 3.0, and 4.5 minutes, respectively.

Advancements in vehicle self-noise control may in time permit convergence zone sonar search while the vehicle is making appreciable speed. However, this study recognizes the difficulties associated with convergence zone sonar signal recognition, and presumes that the hydrofoil stops to search. This condition is the most constrained from the point of view of hydrofoil tactics. Any speed that escorts can make while searching will lead to greater effectiveness than indicated in this study.

1.3 APPROACH TO CONVENTIONAL SONAR SCREEN ANALYSIS

As was mentioned in the introduction, this study is intended to parallel a similar study of hydrofoil search with conventional sonar. The approach to the study of conventional sonar search is reviewed briefly here.

The conventional sonar study develops a methodology to predict average probability of detection of a submarine randomly penetrating a line of hydrofoils. The tactical situation is described in terms of six independent parameters:

V: average hydrofoil transit velocity

S: speed of advance of the screened force

U: speed of submarine target (in direction opposite to direction of advance of screened force)

T_d: dip time (time taken by hydrofoil during one sonar search)

R: the range at which a 50 per cent probability of detection is obtained with three pings

C: screen unit spacing (expressed in units of R)

With these six parameters defined, the following dependent quantities are obtained:

T_t: transit time; time for hydrofoil to transit to next search position; a function of V, S, T_d; also a function of C in the case of zigzag tactics

T: cycle time; total time per hydrofoil search cycle $(T = T_t + T_d)$

A: distance of advance of screen or screened body during one hydrofoil cycle, expressed in units of R: $\left(A = \frac{ST}{R}\right)$

β: distance of relative movement between search line and target per search cycle, expressed in units of R: $\left(\beta = \frac{T(S+U)}{R}\right)$.

The basic output of the study is the average probability of detection of a submarine by the screen. This probability is a function of the six independent parameters plus a known curve of probability of detection versus range (the parameter R is the 0.5 probability point of this curve).

The tactical situation is described as a search by a screen of hydrofoils stationed in a straight line normal to the direction of advance of the surface force. Two tactics are investigated. The first is a zigzag search pattern where the hydrofoils are spaced at intervals 2C. The line of hydrofoils zigs or zags a lateral distance C, while advancing a distance A, during each transit. The second tactic involves a leapfrog operation by a double line of hydrofoils, where each line alternatively advances a distance 2A while the other stops and searches. Probabilities of detection are considered to be identical when the two tactics are described in terms of the parameters above. The probability of detection is expressed as a function of β in a family of curves representing different values of C.

1.4 APPROACH TO CONVERGENCE ZONE SCREEN ANALYSIS

The concept of a straight line screen front may not be suitable in the case of search by convergence zone sonar. In this latter case, the spacing between screen units (and the number of units required) depends upon the radial distance from the task force center to which antisubmarine coverage is desired. For example, if a small task group is to be defended from a submarine launched missile of 20-mile range, the 30-mile ring of coverage provided from a single escort station with first convergence zone sonar will more than provide the desired coverage. If, on the other hand, the submarine can attack with a 40-mile weapon, it will be necessary to search from several screen stations to cover the necessary distance with first convergence zone sonar.

For each radius of desired task force coverage, and for each convergence zone radius, there exists a unique disposition of screening units that minimizes the number of screen stations required to provide sonar coverage at the desired distance. Figure 1 illustrates how to determine the number and position of escort stations necessary to achieve a certain minimum radius of coverage. Let:

R_c = the inner radius of the convergence zone annulus

r = the radius of the circle (from formation center) upon which the screen units are stationed

L = the minimum distance from the formation center protected by sonar coverage

n = the number of screen stations.

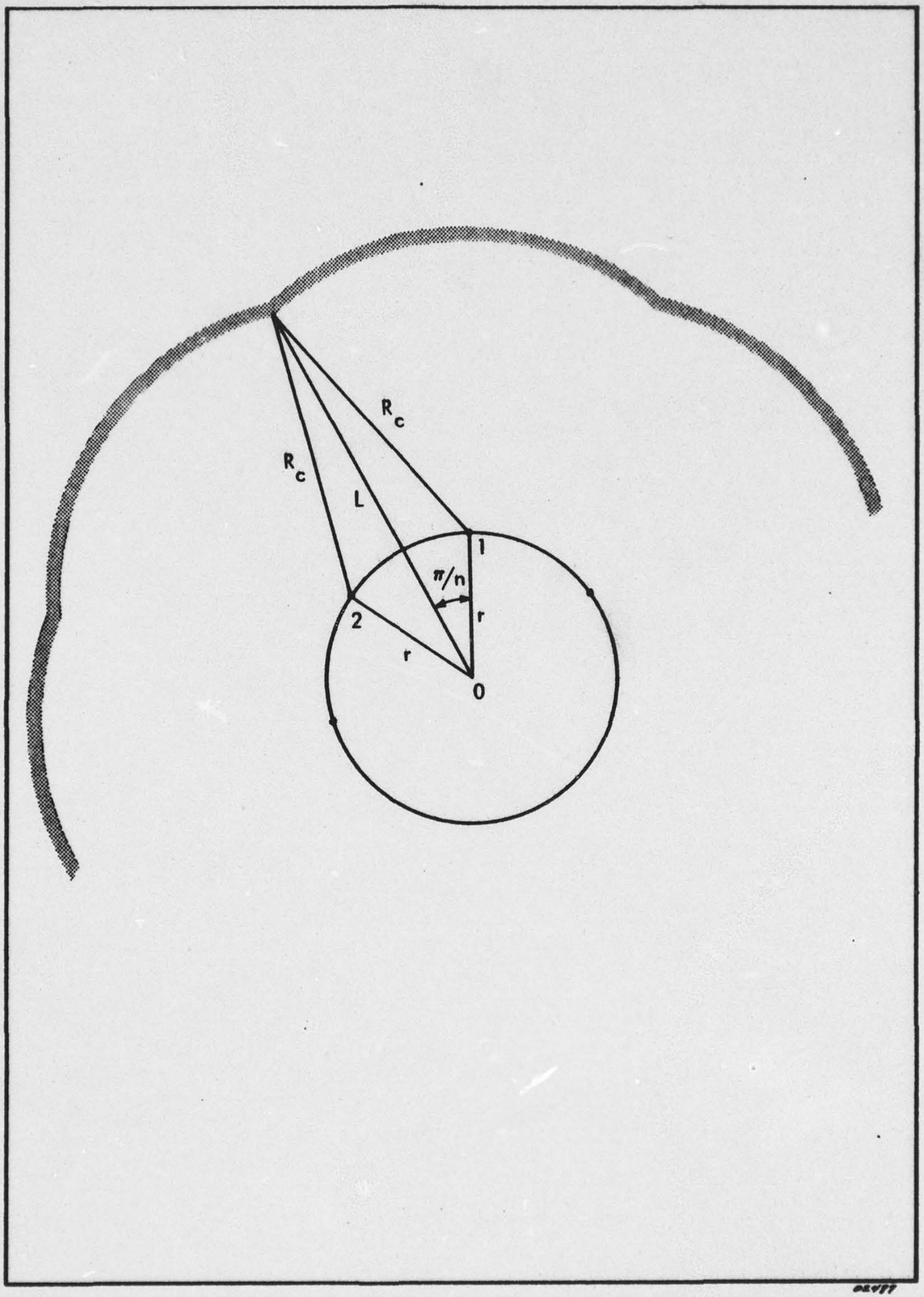

Figure 1. Screen Stations for Escorts Equipped with Convergence Zone Sonar

Consider two adjacent screen stations at points 1 and 2 of Figure 1. The angular separation of these two units will be $\frac{2\pi}{n}$. The angle subtended by the lines L and r is then $\frac{\pi}{n}$. The line L, in terms of the other three parameters, is:

$$L = \sqrt{R_c^2 - r^2 \sin^2 \frac{\pi}{n}} + r \cos \frac{\pi}{n} \tag{1}$$

The radius of the screening circle r is to be varied so as to maximize the minimum distance of coverage L for a given R_c and n. Taking the derivative of L with respect to r, and equating to zero, the value of r that maximizes L is found to be:

$$r = R_c \cot \frac{\pi}{n} \tag{2}$$

substituting (2) in (1),

$$L = R_c \csc \frac{\pi}{n} \quad . \tag{3}$$

It may be seen from Figure 1 that the screen unit spacing d, (the linear distance between two adjacent screen stations) is:

$$d = 2r \sin \frac{\pi}{n} \tag{4}$$

Equations (2) and (3) hold for $n \geq 2$. For the excluded case of $n = 1$, it is obvious from the geometry that a screen consisting of one escort would be stationed at formation center ($r = 0$) and will provide sonar coverage to a distance $L = R_c$.

The values of the radius of coverage, L, the radius of the screen station circle, r, and the distance between stations, d, are shown for various numbers of screen stations in Table I.

It may be noted that for large numbers of escorts, the radius of sonar coverage L and the radius of the screen circle station r both approach the value $\frac{nR_c}{\pi}$ while the escort spacing d approaches the value $2R_c$.

Radius of sonar coverage L ; screen station circle r ; and escort spacing d , for n escorts with convergence zone sonar

n	L	r	d
1, 2	R_c	0	0
3	$1.15R_c$	$0.577R_c$	R_c
4	$1.41R_c$	R_c	$1.41R_c$
5	$1.70R_c$	$1.38R_c$	$1.62R_c$
6	$2.00R_c$	$1.73R_c$	$1.73R_c$
7	$2.30R_c$	$2.07R_c$	$1.79R_c$
10	$3.24R_c$	$3.08R_c$	$1.91R_c$
15	$4.80R_c$	$4.70R_c$	$1.96R_c$
20	$6.39R_c$	$6.31R_c$	$1.98R_c$

Table 1.

Typical sonar coverage patterns attained by stationing escorts to maximize the radius of the circle of task force sonar coverage are shown in Figure 2.

1.5 INTERMITTENT SEARCH WITH CONVERGENCE ZONE SONAR

During each cycle of search with convergence zone sonar, the hydrofoil stops and searches an annular area, then transits to its next search station and stops to repeat the process. To protect a moving formation, the hydrofoils must successively search out adjacent annular areas as the formation moves, in order to maintain intact a ring of sonar coverage about the formation.

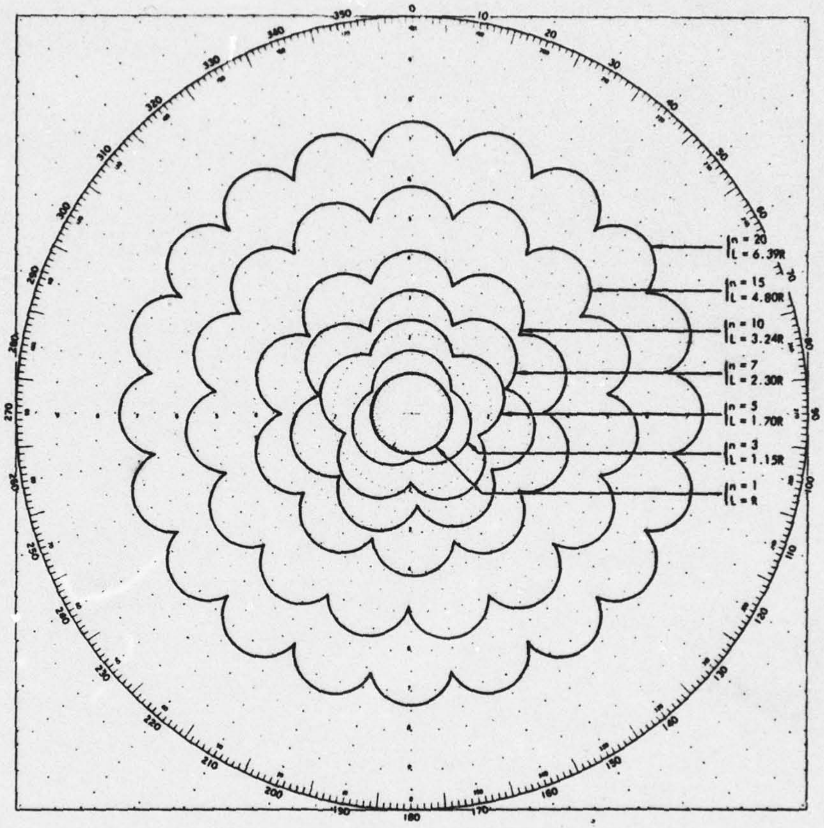

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Figure 2. Sonar Coverage Patterns of n Escorts

The distance travelled by the hydrofoil during the transit portion of the cycle is equal to the distance travelled by the protected force during the entire cycle. Define D as the distance travelled in one cycle. Using the notation of Paragraph 1.3:

$$D = VT_t = ST = ST_t + ST_d \tag{5}$$

Solving for T_t and D in terms of V, S, and T_d;

$$T_t = \frac{ST_d}{V-S} \tag{6}$$

$$D = VT_t = \frac{T_d}{\frac{1}{S} - \frac{1}{V}} \quad . \tag{7}$$

It is seen from Equation (7) that the distance travelled in one cycle, D, may be varied within limits by the operational commander. However, the lower limit on D is fixed by three factors—the minimum time required to complete a sonar "look," the maximum hydrofoil velocity, and the speed of advance of the screened body. All of these factors will probably be beyond the control of the tactical commander who must decide what search tactics to employ. It is apparent, then, that the tactical commander may be unable to select the value of D that he desires, since the lower limit on D may be greater than the width of the annular area searched during one "look."

The ratio of the distance covered per cycle to the annular width can serve as a measure of effectiveness of one hydrofoil in a moving screen. However, before proceeding with the development of such a measure, it is desirable to ~~refine~~ redefine the meaning of V, the hydrofoil velocity.

In Equations (5) to (7), V was implied to be a constant average velocity, independent of the length of transit. Since it may be advantageous for hydrofoils with convergence zone sonar to transit only a few miles per cycle, it is prudent to recognize that acceleration and deceleration may significantly change the average velocity in short cycles. Consultation with Boeing* indicates that hydrofoil

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* Conversation with R. E. Nichols.

velocity can be quite adequately described in terms of linear acceleration and deceleration.

Transit time and transit distance may therefore be expressed in terms of the following parameters:

V_m = maximum hydrofoil velocity

a_1 = acceleration (a_1 positive when accelerating)

a_2 = deceleration (a_2 positive when decelerating).

When the transit time is too short to permit the hydrofoil to attain maximum velocity, the velocity profile is as shown in Figure 3. The hydrofoil accelerates from speed 0 at rate a_1 until some time t_0, whence it decelerates at rate a_2, reaching speed 0 at time T_t. The integral of V with respect to t (or the area under the curve) is the distance travelled during time T_t. That is:

$$D = \int_0^{T_t} V dt = \frac{T_t^2}{2}\left(\frac{a_1 a_2}{a_1 + a_2}\right) \qquad (8a)$$

This condition exists as long as the maximum ordinate of the curve is less than the maximum velocity V_m; that is, when $T_t < V_m\left(\frac{1}{a_1} + \frac{1}{a_2}\right)$. When, on the other hand, $T_t > V_m\left(\frac{1}{a_1} + \frac{1}{a_2}\right)$, the velocity profile is as shown in Figure 4. Here the hydrofoil accelerates until time t_1 when maximum velocity is attained. It continues at maximum velocity until time t_2, and then decelerates, reaching speed 0 at T_t. In this latter case, the distance travelled during time T_t is

$$D = V_m T_t - \frac{V_m^2}{2}\left(\frac{a_1 + a_2}{a_1 a_2}\right) \qquad (8b)$$

A particular hydrofoil under consideration is anticipated to have a 50-knot maximum speed with acceleration characteristics that permit it to increase speed from standstill to 50 knots in 40 seconds, and to decrease from 50 knots to zero speed in 60 seconds. The

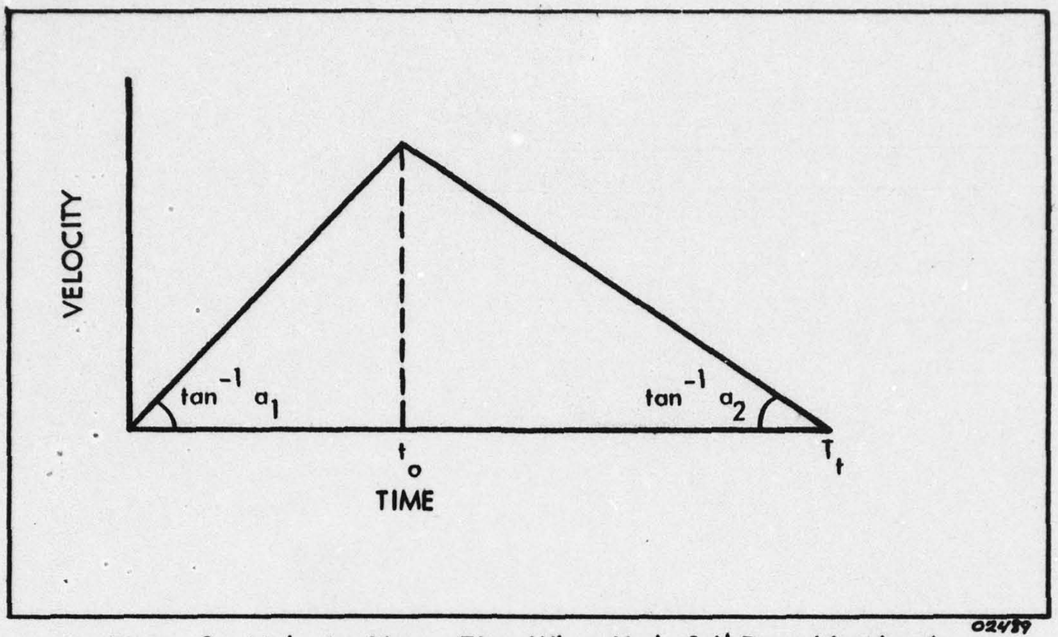

Figure 3. Velocity Versus Time When Hydrofoil Does Not Attain Maximum Speed

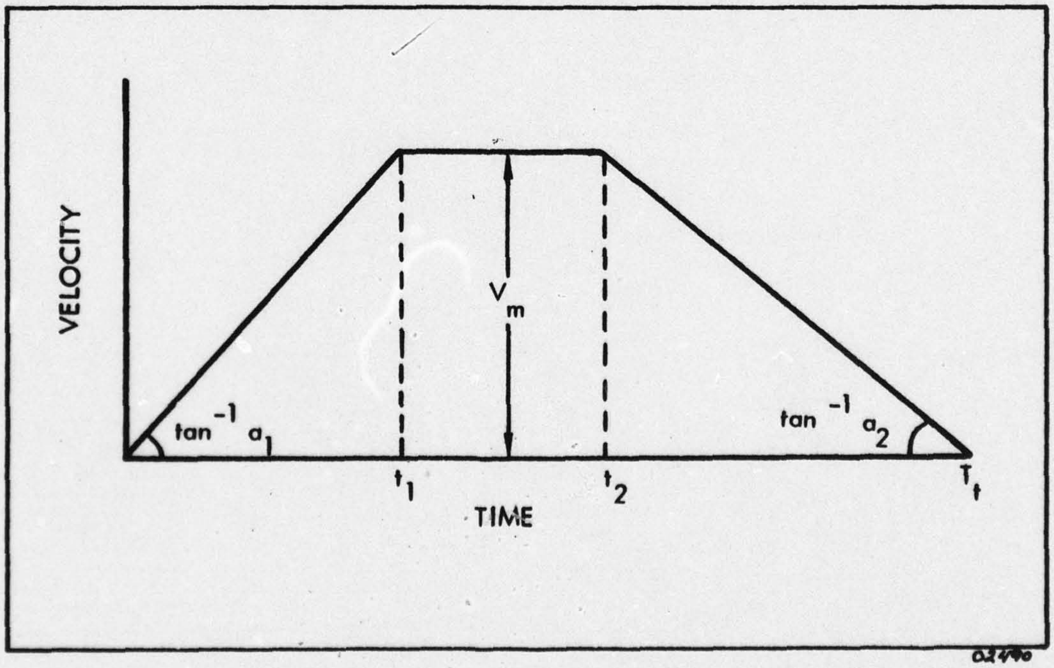

Figure 4. Velocity Versus Time When Hydrofoil Attains Maximum Speed

velocity profile of this "nominal hydrofoil" is shown in Figure 5, and the total distance travelled as a function of transit time is shown in Figure 6. The distance travelled is seen from Figure 6 to increase parabolically from 0 to 0.695 miles in 1 2/3 minutes, thereafter increasing linearly at the rate of 5/6th of a mile a minute. In the generalized case, the distance travelled increases parabolically until time

$$T_t = V_m \left(\frac{1}{a_1} + \frac{1}{a_2} \right)$$

at which time the hydrofoil will have covered a distance

$$\frac{V_m^2}{2} \left(\frac{a_1 + a_2}{a_1 a_2} \right) .$$

Thereafter the distance increases linearly at a rate V_m .

The acceleration and deceleration terms appear in the general distance equations only in the form

$$\left(\frac{1}{a_1} + \frac{1}{a_2} \right) \text{ or its reciprocal } \left(\frac{a_1 a_2}{a_1 + a_2} \right) .$$

It is therefore convenient to replace the separate acceleration and deceleration terms with an equivalent acceleration α. Let

$$\frac{1}{\alpha} = \frac{1}{a_1} + \frac{1}{a_2} \tag{9}$$

then

$$D = \frac{T_t^2 \alpha}{2} \qquad \left(\text{for } T_t \leq \frac{V_m}{\alpha} \right) \tag{10a}$$

$$= V_m T_t - \frac{V_m^2}{2\alpha} \qquad \left(\text{for } T_t > \frac{V_m}{\alpha} \right) \tag{10b}$$

In terms of speed of advance of the protected force, the distance travelled in one cycle was shown in Equation (5) to be:

$$D = ST_t + ST_d . \tag{5}$$

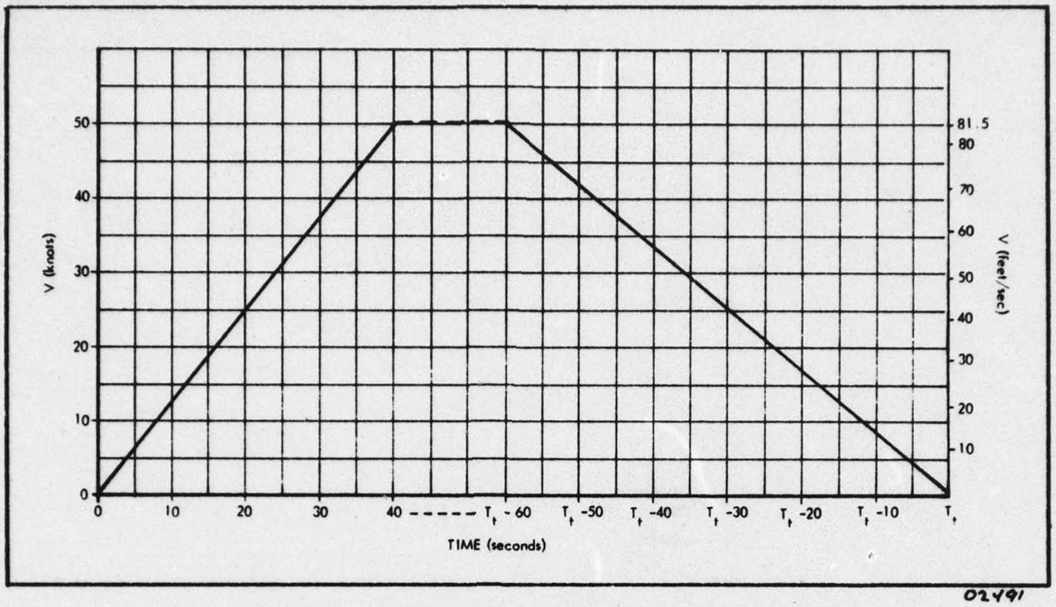

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Figure 5. "Nominal Hydrofoil" Velocity Profile

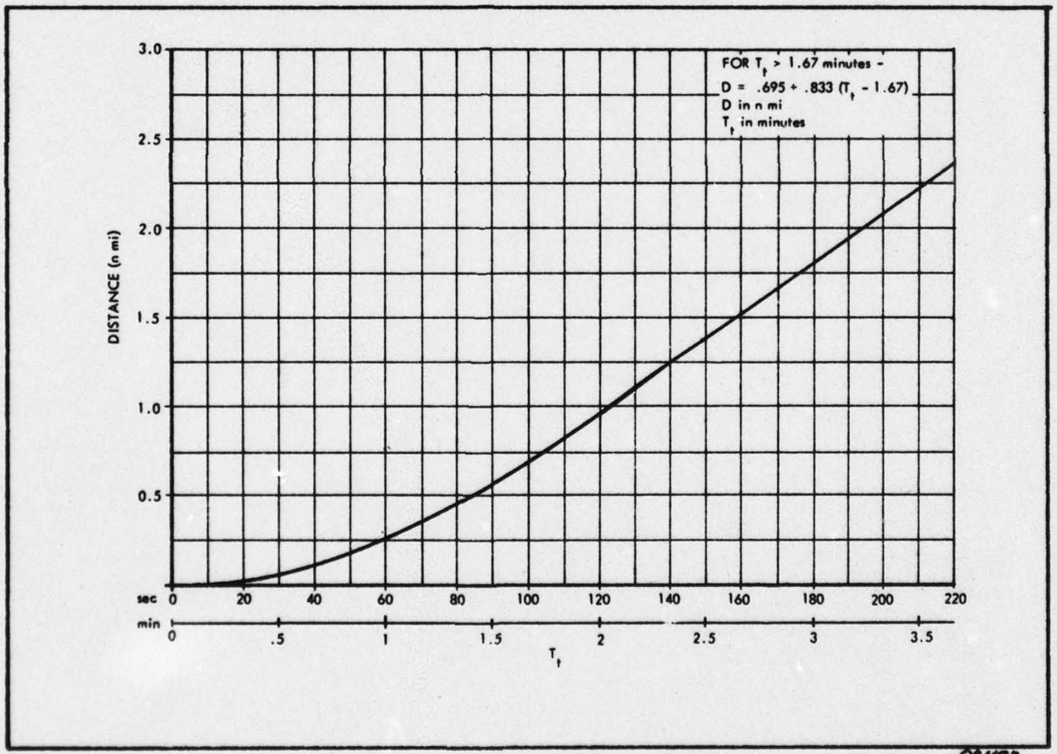

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Figure 6. Distance Travelled by "Nominal Hydrofoil" during Transit Period T_t

By solving Equations (10a), (10b) and (5) simultaneously, we may establish the duration of the hydrofoil transit period, T_t and the distance travelled, D , in terms of the four parameters (minimum dip time, speed of advance, hydrofoil acceleration and maximum hydrofoil speed) that are beyond the control of the tactician. The solutions for the transit time and distance when the hydrofoil does not attain maximum velocity are:

$$T_t = \frac{S}{\alpha}\left(1 + \sqrt{\frac{2\,T_d\,\alpha}{S} + 1}\right) \tag{11a}$$

$$D = \frac{S^2}{\alpha}\left(1 + \frac{\alpha T_d}{S} + \sqrt{1 + \frac{2\alpha T_d}{S}}\right) \tag{12a}$$

and when part of the transit is at maximum speed:

$$T_t = \frac{2ST_d + \frac{V_m^2}{\alpha}}{2(V_m - S)} \tag{11b}$$

$$D = \frac{V_m}{V_m - S}\left(ST_d + \frac{V_m^2}{2\alpha}\right) - \frac{V_m^2}{2\alpha} \tag{12b}$$

The dependence of T_t , D , and the total cycle time T , upon the speed of advance S and the dip time T_d is shown in Figures 7, 8, and 9 for the "nominal hydrofoil" described by the curves of Figures 5 and 6.

1.6 SINGLE HYDROFOIL MEASURE OF EFFECTIVENESS

It may be shown* that in order for the submarine to minimize its time of exposure to detection, it will penetrate the detection zone in

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* See Search and Screening by B. O. Koopman, OEG Report No. 56; Section 9.3.

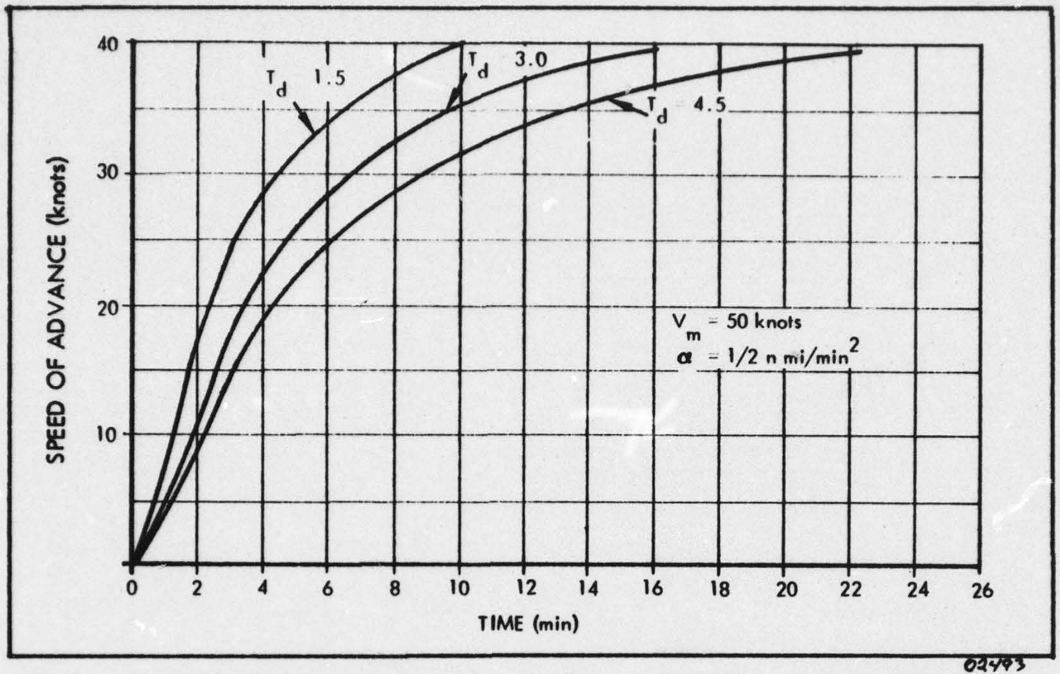

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Figure 7. Hydrofoil Transit Time Per Cycle as a Function of Sonar Dip Time and Speed of Advance

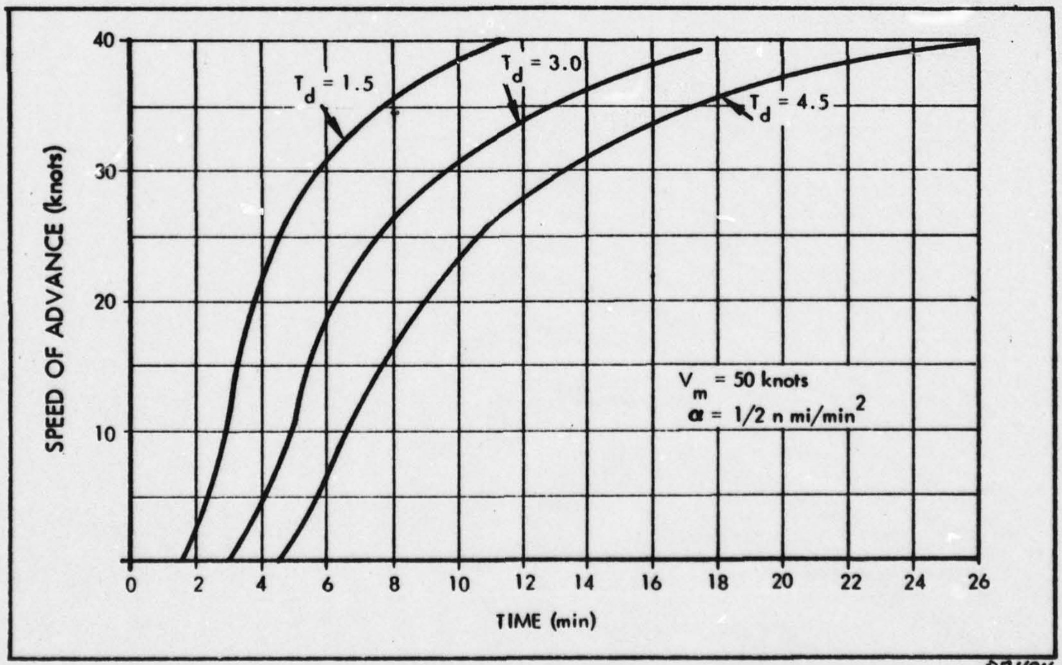

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Figure 8. Hydrofoil Cycle Time as a Function of Sonar Dip Time and Speed of Advance

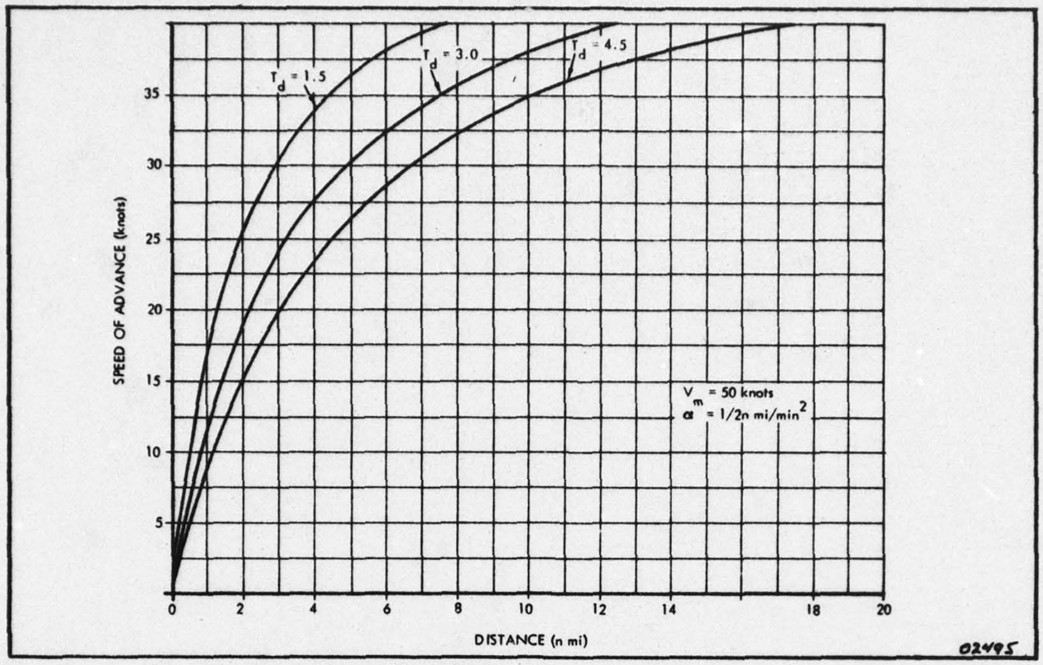

Figure 9. Distance Travelled Per Cycle as a Function of Sonar Dip Time and Speed of Advance

such a way that its relative movement is along a radius of the convergence zone circle. Expressed in other words, the submarine will attempt to pass through the convergence zone annulus on a collision course with the hydrofoil. It may therefore be expected that if the submarine commander is cognizant of the situation and is able, he will steer a collision course on the hydrofoil while passing through the annulus in which he is exposed to detection.

As was mentioned in Paragraph 1.5, the ratio of swept to unswept areas can serve as a measure of effectiveness of a moving hydrofoil search. For cycles employing a "minimum dip" (i.e., where T_d is only the time required for a single "look" at the target), the effectiveness can be expressed as the simple ratio of the width of the searched annulus to the relative distance change between submarine and hydrofoil during one cycle. Define:

w = width of convergence zone annulus

D_r = the relative distance change in one cycle between a hydrofoil and a submarine

E = single hydrofoil effectiveness; specifically, the ratio of the relative distance searched in one cycle to the relative distance closed in one cycle by a hydrofoil and a submarine on collision course.

Then the effectiveness measure of single hydrofoil search is:

$$E(\text{minimum dip}) = \frac{w}{D_r} \tag{13a}$$

An effectiveness of one indicates that the hydrofoil is providing complete search coverage with no overlap in the direction of relative movement. Values of E greater than one indicate that each successive searched area overlaps the previous one; E less than one means that a single hydrofoil must leave gaps between successive search zone in order to maintain its station on the protected force as it advances.

The geometry of the movement between a submarine and a hydrofoil on collision course is shown in Figure 10. A hydrofoil at point o is proceeding in the direction of the course vector $\overline{ov}$. The submarine at point b selects a course $\overline{ou}$ such that the direction of relative movement, $\overline{vu}$, is parallel to the line of bearing between the hydrofoil and submarine, $\overline{bo}$. Since the hydrofoil advances a distance D while the submarine travels a distance UT, the relative distance change between the hydrofoil and the submarine in one cycle is the vector difference between these two quantities. In Figure 10, let

θ = the angle between the relative movement vector and the hydrofoil course vector (angle $\overline{ovu}$)*

then the relative distance change in one cycle is:

$$D_r = D \cos\theta + \sqrt{U^2 T^2 - D^2 \,\mathrm{xin}^2\, \theta} \tag{14}$$

Equation (13a) may now be written:

$$E(\text{minimum dip}) = \frac{w}{D\cos\theta + \sqrt{U^2 T^2 - D^2 \sin^2\theta}}$$

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[*] For the collision course penetration under consideration, θ is also the relative bearing of the submarine from the hydrofoil (angle $\overline{vob}$). For subsequent applications, however, it is preferable to define θ in terms of the vector triangle.

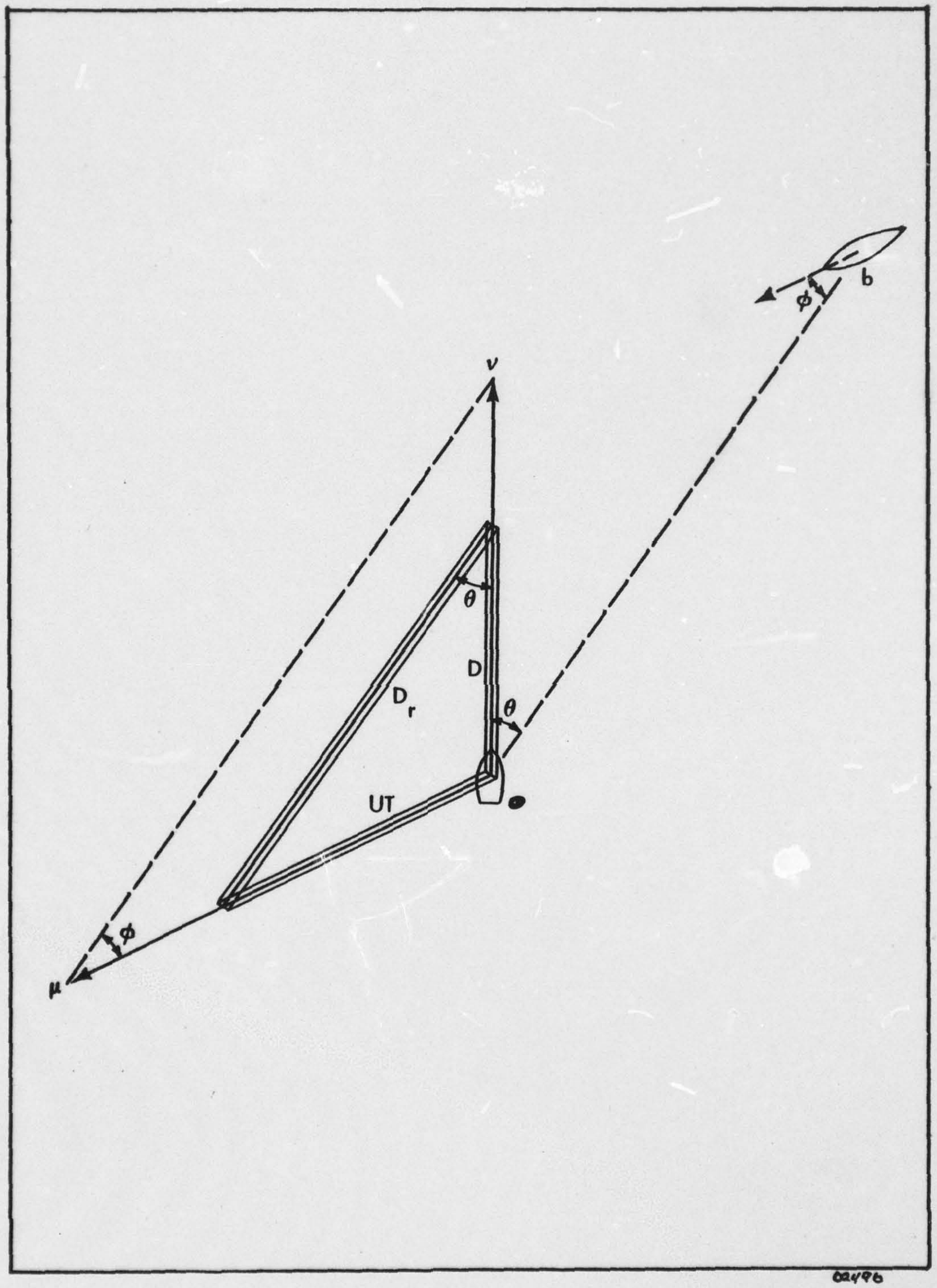

Figure 10. Relative Movement of Hydrofoil and Submarine on Collision Course

or, using the relationship of Equation (5):

$$E(\text{minimum dip}) = \frac{w}{T\left(S\cos\theta + \sqrt{U^2 - S^2\sin^2\theta}\right)} \qquad (13c)$$

Under some circumstances, it may be desirable for a hydrofoil to loiter on each search station—that is, the hydrofoil may choose to continue its sonar search for some time beyond the time necessary for one "look." If the hydrofoil employs this tactic, the search effectiveness changes for two reasons. First, increasing the dip time increases the cycle time, T. From Equation (13c), it is seen that increasing T tends to decrease the effectiveness. Second, the effective width of the searched annulus is increased by virtue of the submarine movement during the search period. During a prolonged search the hydrofoil will have had an opportunity to detect not only all submarines within the width w at the start of the search, but also all submarines that will have moved into the searched sector before the end of the search. Let:

T_{dm} = minimum dip time; that is, the time required to obtain one "look" at the target

ϕ = the angle between the relative movement vector and the submarine course vector (angle $\overline{vuo}$ of Figure 10)

While the hydrofoil is stopped and conducting a prolonged search, the submarine approaches the searched zone with a speed component $U\cos\phi$. The submarines that will have moved into the searched sector during a prolonged search are those that were at the start of the search within a distance $(T_d - T_{dm})\,U\cos\phi$ from the searched area. This is the amount of effective increase of the searched width w. Equation (13c) may now be written in general form.

$$E = \frac{w + \left(T_d - T_{dm}\right) U\cos\phi}{T\left(S\cos\theta + \sqrt{U^2 - S^2\sin^2\theta}\right)} \qquad (15)$$

Equation (15) is unwieldly in its general form. However, in three specific cases of importance, Equation (15) simplifies considerably.

First, in the head-on case where the hydrofoil and the submarine approach an opposite course, $\theta = \phi = 0$, and the effectiveness is

$$E(\text{head-on case}) = E_h = \frac{w + U\left(T_d - T_{dm}\right)}{T(S + U)} \qquad (16)$$

Second, where the submarine attempts to overtake the hydrofoil from directly astern, $\theta = \pi$ and $\phi = 0$, and the effectiveness is

$$E(\text{stern case}) = E_s = \frac{w + U\left(T_d - T_{dm}\right)}{T(U - S)} \qquad (17)$$

For the submarine to penetrate from astern, the submarine speed, U, must be greater than the speed of the surface force, S. Negative solutions of Equation (17) have no meaning other than to indicate situations where it is impossible for the submarine to overtake its target.

The third case of significance occurs when the submarine approaches the hydrofoil from abeam, in such manner that the relative movement line is perpendicular to the direction of advance of the surface force. In this case,

$\theta = \frac{\pi}{2}$ and $\cos \phi = \frac{\sqrt{U^2 - S^2}}{U}$; and the effectiveness is

$$E(\text{beam case}) = E_b = \frac{w + \left(T_d - T_{dm}\right)\sqrt{U^2 - S^2}}{T\sqrt{U^2 - S^2}} \qquad (18)$$

Where the hydrofoil dips for only the minimum time, Equations (16) to (18) simplify further, with the right-hand term of numerator dropping out.

The effectiveness measures, Equations (16) to (18) involve many independent parameters—specifically w, T_{dm}, V_m, α, T_d, U and S. For a meaningful display of the interrelation of the parameters, some of them must be fixed at representative values. By specifying the convergence zone, w and T_{dm} may be held at the values indicated in Paragraph 1.2. The "nominal hydrofoil" described in Paragraph 1.5 enables V_m and α to be fixed. For the minimum dip cycle, the three effectiveness measures may then be examined with regard to the submarine speed and the speed of advance of the protected force. This is done in Figure 11 to 13.

The influence of increasing the dip time is shown in Figure 14, where the submarine speed is held constant at 20 knots in order to examine the effectiveness of various dip times as a function of speed of the protected force.

1.7 MINIMUM DIP TIME VERSUS PROLONGED DIP

In the situation represented by Figure 14, the search effectiveness is seen to decrease significantly as the dip period increases. It is therefore pertinent to examine the relative advantages of minimum cycle versus prolonged dipping. As a first step, it is helpful to consider the approximate solution obtained by neglecting the effects of hydrofoil acceleration and deceleration, in order to ascertain the basic interrelations of the parameters involved. From Equation (6) and the definition of T , an expression for T in terms of constant hydrofoil velocity V may be obtained.

$$T = T_d + T_t = T_d + \frac{ST_d}{V-S} = \frac{T_d V}{V-S} \qquad (19)$$

Combining Equation (19) with the expression for hydrofoil effectiveness in the head-on case, Equation (16),

$$E_h = \left(\frac{V - S}{U + S}\right)\left(\frac{w + U\left|T_d - T_{dm}\right|}{T_d V}\right) \qquad (20)$$

Now let $T_{d1} = T_{dm}$ and let $T_{d2} = kT_{dm}$ $(k > 1)$

so that T_{d1} represents the minimum dip time and T_{d2} represents some dip time greater than the minimum time. Solving for the values of effectiveness E_1 and E_2 corresponding to T_{d1} and T_{d2} :

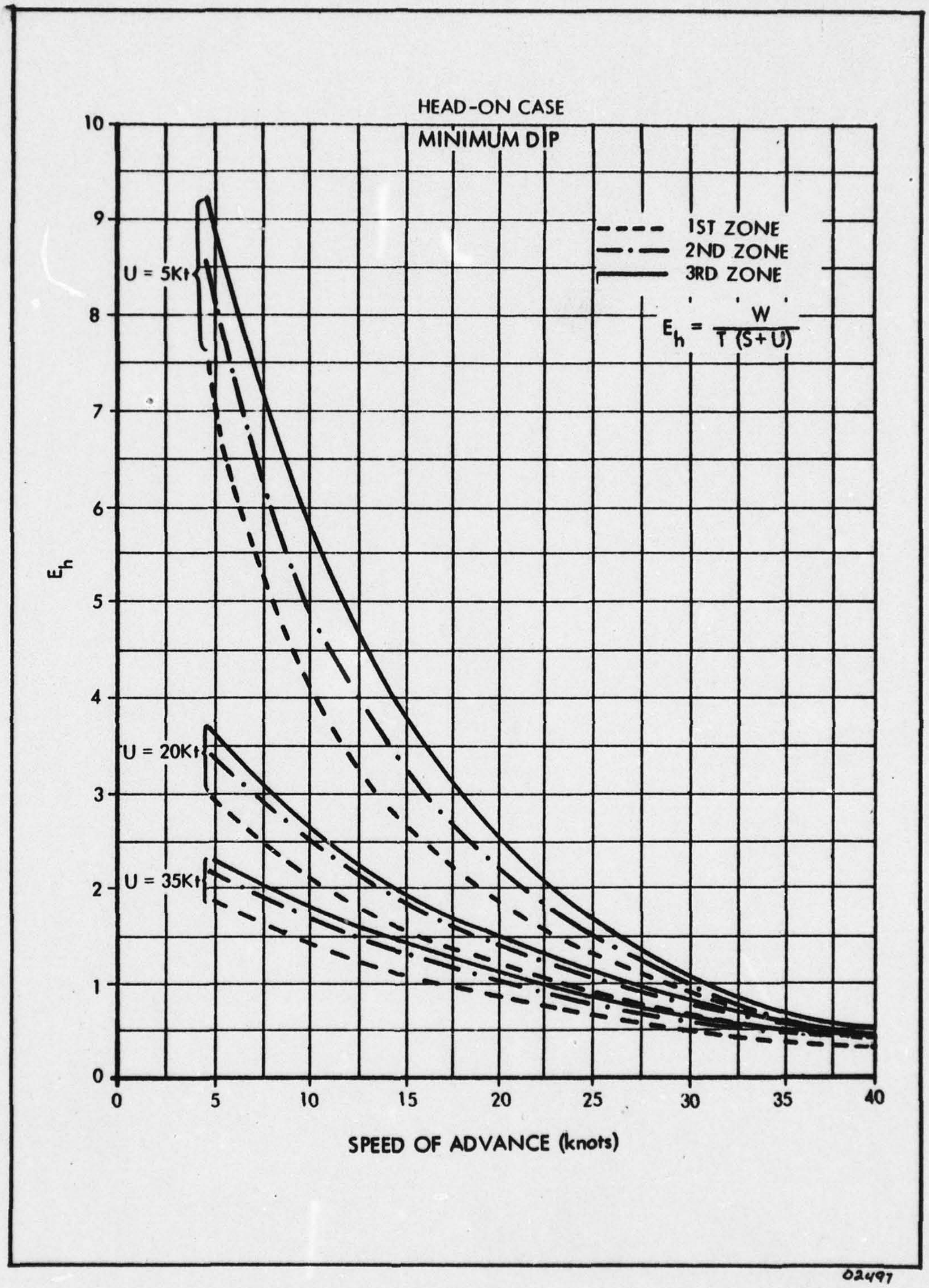

Figure 11. Single Hydrofoil Effectiveness - Head-On Case

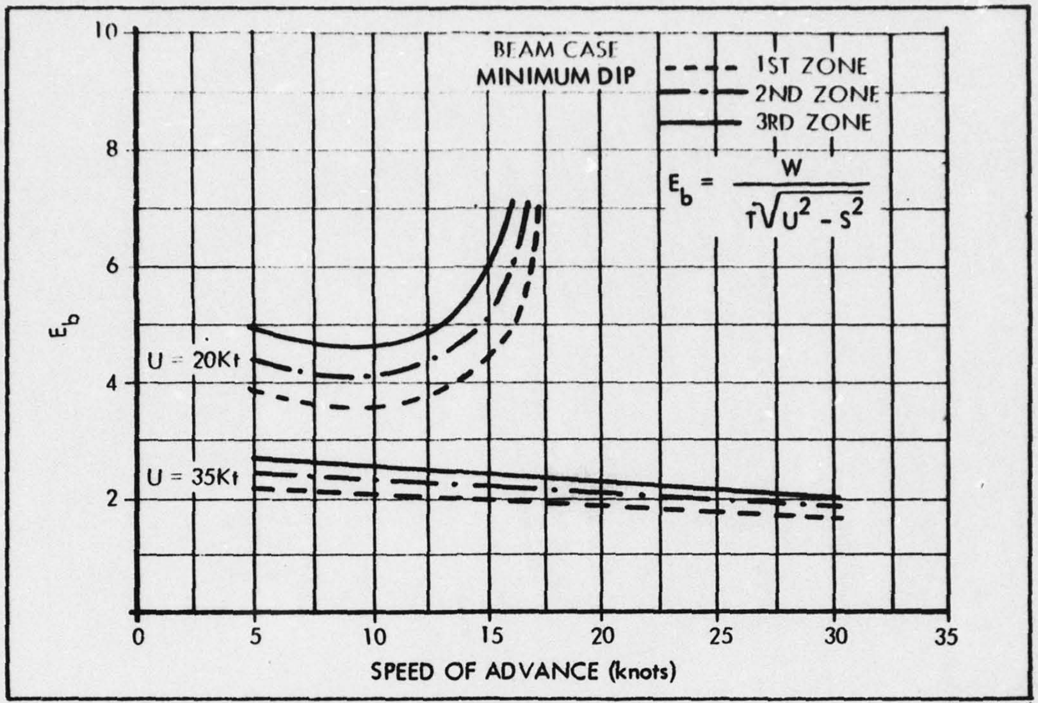

Figure 12. Single Hydrofoil Effectiveness - Beam Case

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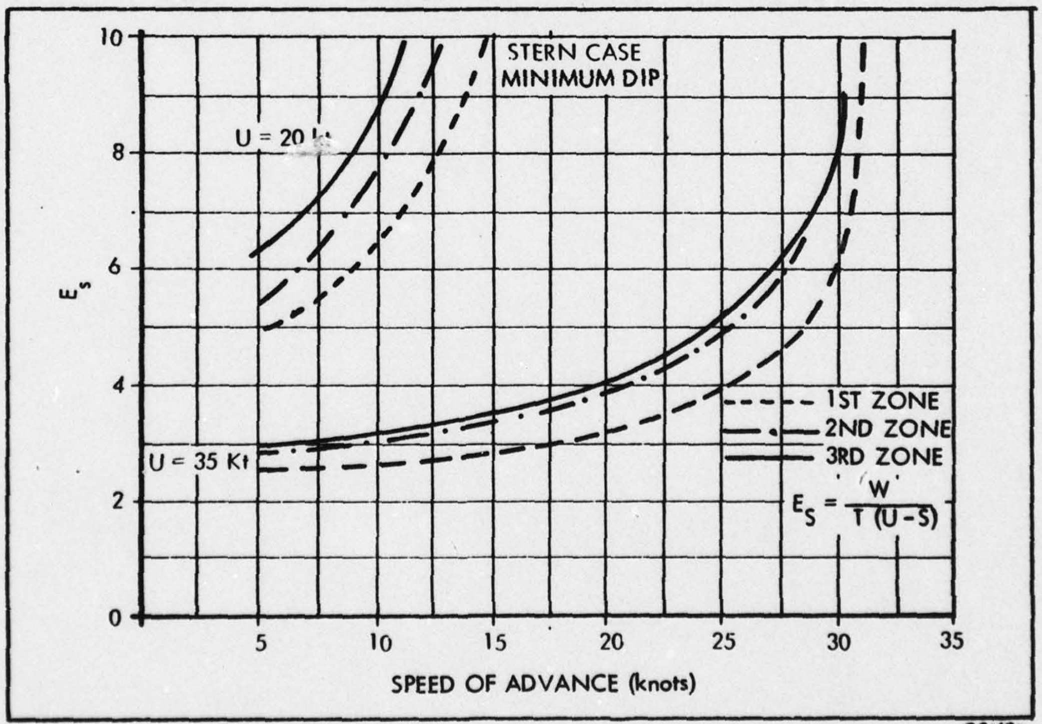

Figure 13. Single Hydrofoil Effectiveness - Stern Case

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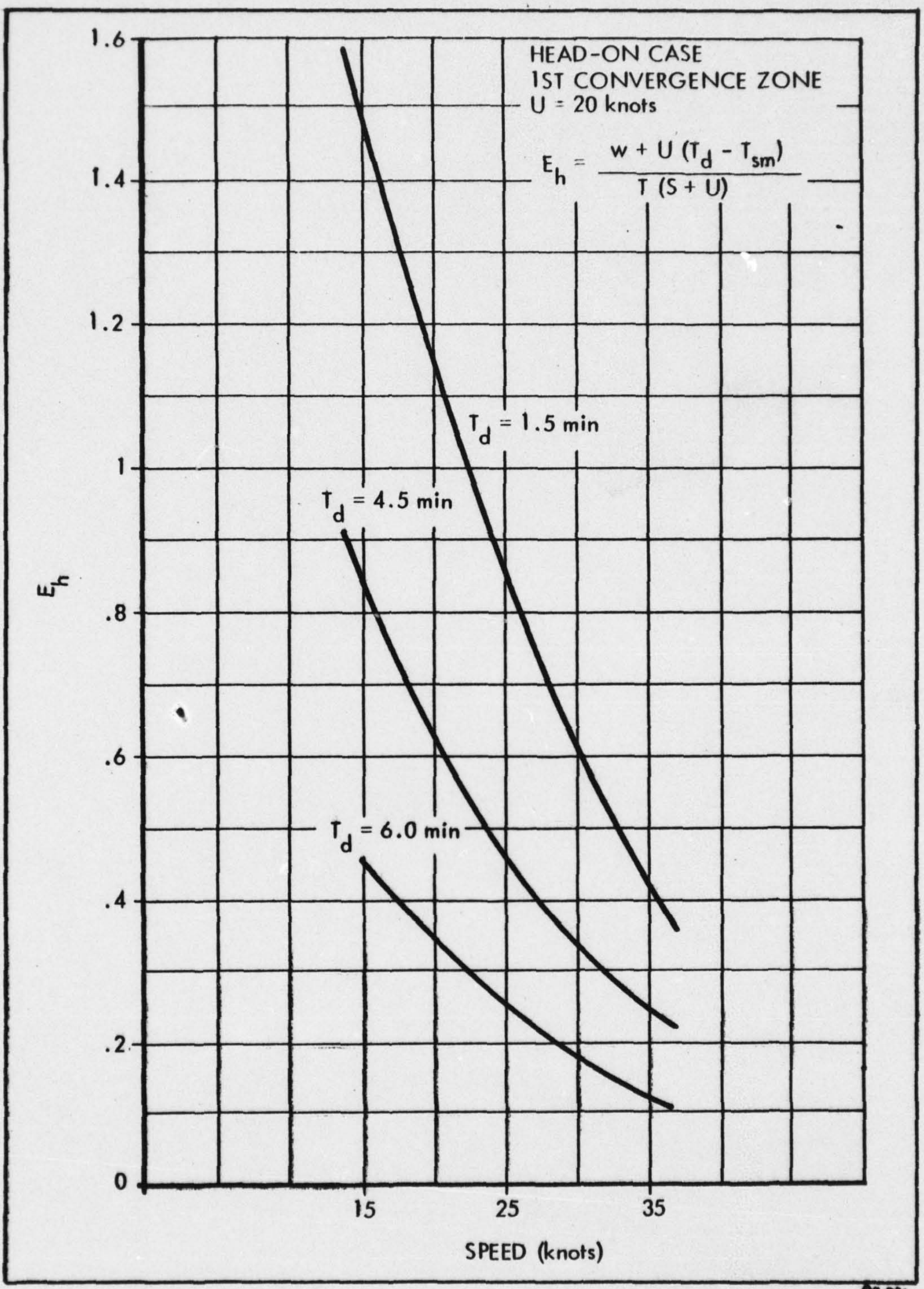

Figure 14. Single Hydrofoil Effectiveness for Prolonged Dip Cycles

$$E_1 = \left(\frac{V - S}{U + S}\right) \frac{w}{T_{dm} V} \tag{21}$$

and

$$E_2 = \left(\frac{V - S}{U + S}\right)\left(\frac{w + UT_{dm}(k-1)}{kT_{dm} V}\right) . \tag{22}$$

In order to compare the efficiency of minimum search versus prolonged search, Equation (22) will be subtracted from Equation (21) . Positive values of $E_1 - E_2$ will indicate situations where the minimum dip is most efficient. Negative values of $E_1 - E_2$ will represent cases where prolonged dipping is more effective. Subtracting,

$$E_1 - E_2 = \frac{(k - 1)(V - S)(w - UT_{dm})}{(U + S)\, kVT_{dm}} \tag{23}$$

The denominator of the right hand side of Equation (23), and the first term of the numerator, will always be positive. The second term of the numerator can be negative only if the speed of the protected force is greater than the hydrofoil speed—a meaningless situation insofar as this model is concerned. Negative values of the last term of the numerator conceivably can occur, and will occur if the submarine speed U is greater than the quantity w/T_{dm} .

Solutions to the beam and stern cases show that submarine speed constrains the usefulness of prolonged dipping equally or more severely in the latter cases. From this it may be concluded that (subject to the assumption of negligible acceleration effect) prolonged dipping is profitable only if the submarine speed is greater than w/T_{dm}.

Consider next the more precise model where the effects of acceleration and deceleration are taken into account. When the hydrofoil attains maximum speed, the cycle time is:

$$T = \frac{T_d V_m + V_m^2 / 2\alpha}{V_m - S} \tag{24}$$

and the effectiveness in the head-on case is:

$$E_h = \left(\frac{V_m - S}{T_d V_m + V_m^2/2\alpha}\right)\left(\frac{w + U\ T_d - T_{dm}}{U + S}\right) \qquad (25)$$

Solving for the difference in effectiveness between the minimum dip and the prolonged dip cycles,

$$E_1 - E_2 = \frac{\left(V_m - S\right) T_{dm} (k-1) \left(w - UT_{dm} - UV_m/2\alpha\right)}{(U + S)\ V_m \left(kT_{dm} + V_m\ 2\alpha\right)\left(T_{dm} + V_m/2\alpha\right)} \qquad (26)$$

As in the preceding case, the last term of the numerator is the only term that can cause the value of $E_1 - E_2$ to be negative. Similar solutions to the beam and stern cases show that prolonged dipping can be profitable only when

$$U \geq \frac{w}{T_{dm} + V_m/2\alpha} \qquad (27)$$

For the "nominal hydrofoil" described earlier, Equation (27) indicates that prolonged dipping can be preferable only if the submarine speed is greater than 75 knots. The minimum dip cycle may therefore be expected to yield the highest search efficiency within the realm of practical submarine and hydrofoil speeds. Obviously, the duration of the hydrofoil cycle may be dictated by a requirement other than tactical efficiency—as, for example, if frequent stopping and starting fatigues the crew excessively, or causes unacceptable engine wear. Nonetheless, in the remainder of this study, the minimum dip cycle will be used as a standard for the establishment of screen requirements. Subsequent study may determine cost in additional units that must be paid to operate under a policy of less than maximum tactical efficiency.

1.8 HYDROFOIL SCREEN EFFECTIVENESS

To proceed from single hydrofoil effectiveness to overall screen effectiveness in any rigourous manner would require an elaborate computational program involving a high speed computer. Such an effort is beyond the scope of this study, and it is necessary at this point to make certain gross assumptions:

A. The probability of detection, given that a submarine is within the convergence zone annulus when the hydrofoil "looks", is a constant less than or equal to one. This is analogous to applying the definite range law to the solid-area type search.

B. The probability of detection is not increased when there is an area of overlap on two successive "looks" by the search element. This may be rationalized under the hypothesis that the event of detection or non-detection is governed primarily by the presence or absence of the convergence zone sonar propagation path. Hence, search from successive dip positions is subject to essentially the same conditions that determined success or failure on the previous look.

C. The increase in probability of detection resulting from overlapping coverage by adjacent screen stations is negligible. (Section 1.11 provides a methodology whereby this assumption may be removed.)

The screening concept described in Section 1.4 enables a given radius of antisubmarine coverage to be obtained with the minimum screen force. For the minimum screen to detect a submarine outside of the specified radius of protection, it is necessary that detection take place as the submarine penetrates the convergence zone annulus for the first time. The submarine must ultimately pass through the annulus twice, but the second passage will generally be at a range less than the prescribed minimum range from the protected force. Consequently, only the first detection opportunity—the inward penetration through the outer half of the annulus—is considered in determining screen effectiveness.

The probability of detection of a penetrating submarine can be expressed in terms of the single look probability and the search coverage effectiveness. Let:

P = probability of detection of a submarine penetrating a convergence zone annulus;

P_S = probability that a submarine is detected, given that the submarine is within the convergence zone annulus when the hydrofoil "looks";

P_L = probability that a penetrating submarine is within the convergence zone annulus during at least one "look" by a hydrofoil.

Then,

$$P = P_S P_L \tag{28}$$

By assumption A., P_S, the probability that the hydrofoil "sees" the submarine, is a constant. P_L, the probability that the hydrofoil "looks" at the submarine, will be related in paragraphs 1.9 and 1.10 to the single hydrofoil measure of effectiveness, **E**.

1.9 SCREEN REQUIREMENTS—COLLISION COURSE PENETRATIONS

The effectiveness of a single hydrofoil searching for a submarine that penetrates the detection zone on collision course has been defined to be **E**. The effectiveness of the search operation at any screen station can be made greater than **E** by increasing the number of hydrofoils per station. With proper coordination, the distance searched per cycle increases linearly with the number of hydrofoils assigned to the station.

Let: m = the number of hydrofoils assigned to a screen station

Then, by assumption B., the probability that at least one hydrofoil "looks" at the submarine is:

$$P_L = \begin{cases} mE & (mE < 1) \\ 1 & (mE \geq 1) \end{cases} \tag{29}$$

The value of **E** varies with the direction from which the submarine approaches, and the general expression for **E** (Equation 15) is not readily integrable with respect to the direction angle. It is therefore necessary to approximate further to obtain a manageable representation of the screen for overall effectiveness computations.

Consider a circular hydrofoil screen divided into quadrants as shown in Figure 15. The collision course penetrations from all directions

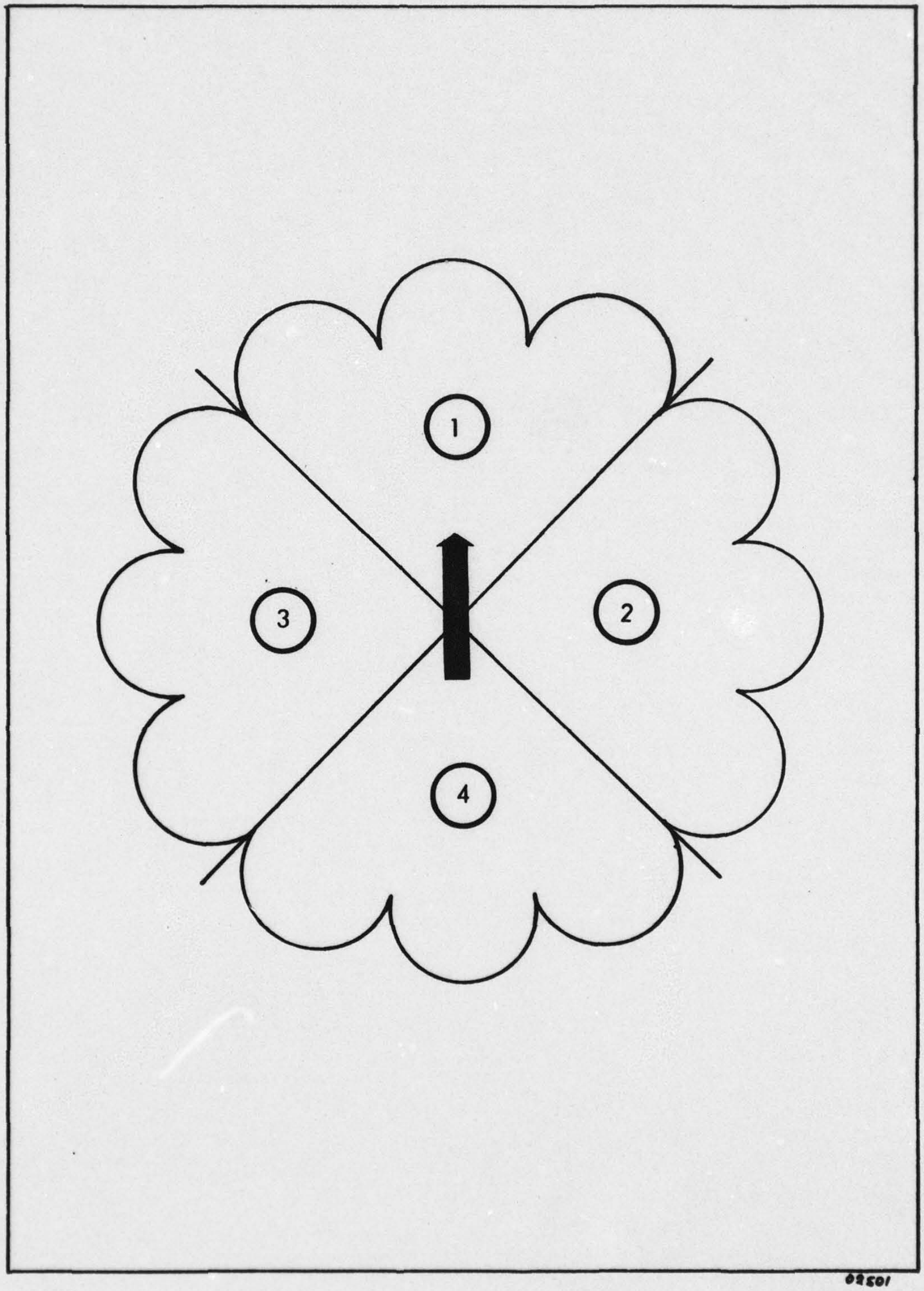

Figure 15. Hydrofoil Screen Quadrants

may be approximated by assuming that those in Sector 1 are head-on cases, those in Sectors 2 and 3 are beam cases, and those in Sector 4 are stern cases. The single hydrofoil effectiveness applicable to Sector 1, then is E_h (Equation 16); for Sectors 2 and 3, it is E_b (Equation 18); and for Sector 4, E_s (Equation 17).

From Equations (28) and (29):

$$P = P_s mE \qquad (mE < 1) \tag{30}$$

$$= P_s \qquad (mE \geq 1)$$

To achieve the maximum probability of detection $P = P_s$ in each sector with the minimum number of hydrofoils, m for that sector must be the least integer larger than $\frac{1}{E}$. The range of values of m to meet this requirement have been examined in some detail. The results are summarized in Table 2.

Most significantly, Table 2 shows that for foreseeable tactical situations, only one hydrofoil per station will be required for the beam and stern sectors. In the ahead sector, up to three and possibly four hydrofoils per station may be required in some circumstances. Hydrofoils per station requirements are slightly lower for sonars searching in the more distant convergence zones.

The relative force requirements for different tactical situations may be compared in terms of the average number of hydrofoils required per station. Let:

$\overline{m}$ = average number of hydrofoils per station required to achieve $P = P_S$

For the "nominal hydrofoil" of Section 1.5, typical values of $\overline{m}$ have been averaged on the basis of the quadrant model of Figure 15. Table 3 summarizes the values of $\overline{m}$ in terms of speed of advance and submarine speed. Where an entry shows two values of $\overline{m}$, the lower number refers to search with third convergence zone sonar, and the higher to search with first convergence zone sonar.

Table 2

Number of Hydrofoils per Station, m, Required to
"Nominal Hydrofoil" (V_m = 50 knots, α = 1/2 n
Collision Course Penetrations

(m is the least integer $\geq \frac{1}{E}$)

Table 2 a. Sector 1; 1st Convergence Zone (T_d = 1.5 min, w = 3 n.mi.)

Speed of Advance S(knots)	Submarine Speed U(knots)	No. of H/F m
15	U < 40	1
25	U < 20	1
25	U > 20	2
35	U < 11	2
35	11< U < 34.5	3
35	U> 34.5	4

Table 2b. Sectors 2 and 3; 1st Convergence Zone (T_d = 1.5 min. w = 3 n.mi.)

Speed of Advance S(knots)	Submarine Speed U(knots)	No. of H/F m
15	U < 57	1
25	U < 42.5	1
35	U < 42	1

Table 2c. Sector 4; 1st Convergence Zone (T_d = 1.5 min, w = 3 n.mi.)

Speed of Advance S(knots)	Submarine Speed U(knots)	No. of H/F m
15	U < 69	1
25	U < 56	1
35	U < 57.5	1

Table 2d. Sector 1; 2nd Convergence Zone (T_d = 3 min, w =

Speed of Advance S(knots)	Submarine Speed U(knots)
15	U < 52
25	U < 22.6
25	U > 22.6
35	U < 21.8
35	U > 21.8

Table 2e. Sectors 2 and 3; 2nd Convergence Zone (T_d = 3 min,

Speed of Advance S(knots)	Submarine Speed U(knots)
15	U < 69
25	U < 52
35	U < 54.5

Table 2f. Sector 4; 2nd Convergence Zone (T_d = 3 min, w = 6

Speed of Advance S(knots)	Submarine Speed U(knots)	No.
15	U < 82	
25	U < 71	
35	U < 64	

2

Table 2

Iydrofoils per Station, m, Required to Achieve $P = P_S$ with the ydrofoil" (V_m = 50 knots, α = 1/2 n. mi./min^2) against urse Penetrations

(m is the least integer $\geq \frac{1}{E}$)

Table 2d. Sector 1; 2nd Convergence Zone (T_d = 3 min, w = 6 n.mi.)

peed of Advance S(knots)	Submarine Speed U(knots)	No. of H/F m
15	U < 52	1
25	U < 22.6	1
25	U > 22.6	2
35	U < 21.8	2
35	U > 21.8	3

Table 2e. Sectors 2 and 3; 2nd Convergence Zone (T_d = 3 min, w = 6 n.mi.)

peed of Advance S(knots)	Submarine Speed U(knots)	No. of H/F m
15	U < 69	1
25	U < 52	1
35	U < 54.5	1

able 2f. Sector 4; 2nd Convergence Zone (T_d = 3 min, w = 6 n.mi.)

eed of Advance S(knots)	Submarine Speed U(knots)	No. of H/F m
15	U < 82	1
25	U < 71	1
35	U < 64	1

Table 2 g. Sector 1; 3rd Convergence Zone (T_d = 4.5 min, w = 9 n.mi.)

Speed of Advance S(knots)	Submarine Speed U(knots)	No. of H/F m
15	U < 55	1
25	U < 25.3	1
25	U > 25.3	2
35	U < 25.5	2
35	U > 25.5	3

Table 2 h. Sectors 2 and 3; 3rd Convergence Zone (T_d = 4.5 min, w = 9

Speed of Advance S(knots)	Submarine Speed U(knots)	No. of H/ m
15	U < 73	1
25	U < 58	1
35	U < 46	1

Table 2 i. Sector 4; 3rd Convergence Zone (T_d = 4.5 min, w = 9 n.mi.)

Speed of Advance S(knots)	Submarine Speed U(knots)	No. of H, m
15	U < 86	1
25	U < 75	1
35	U < 65	1

$\overline{m}$, Average Number of "Nominal Hydrofoils" (V_m = 50 kn., = 1/2 n.mi./min^2) per Station Required to Achieve Overall Screen Effectiveness of P_S

	Submarine Speed		
Speed of Advance	U = 15 kn.	U = 25 kn.	U = 35 kn.
S = 15 kn.	1	1	1
S = 25 kn.	1	1 - 1.25	1.25
S = 35 kn.	1.25 - 1.50	1.25 - 1.50	1.50 - 1.75

Table 3.

1.10 SCREEN REQUIREMENTS—RANDOM PENETRATIONS

It was pointed out in Section 1.6 that the submarine minimizes its exposure to detection by penetrating the convergence zone annulus on a collision course with the hydrofoil. Under some circumstances, however, the submarine may not be able to take advantage of this least risk tactic. It is therefore appropriate to examine situations where the submarine penetrates other than in a radial direction.

The geometric model illustrated in Figure 10 was devised to analyze collision course penetrations, but it is readily applicable to other submarine tactics. The vector diagram of Figure 10 actually describes the relative movement of a penetration at any point by any submarine with velocity vector $\overline{ou}$. Figure 16 illustrates the general nature of the vector diagram, with typical penetrations represented by submarines at positions b_1 to b_5.

The expression for D_r (Equation 14) holds in the general case, but the collision course effectiveness measure E (Equation 15) no longer

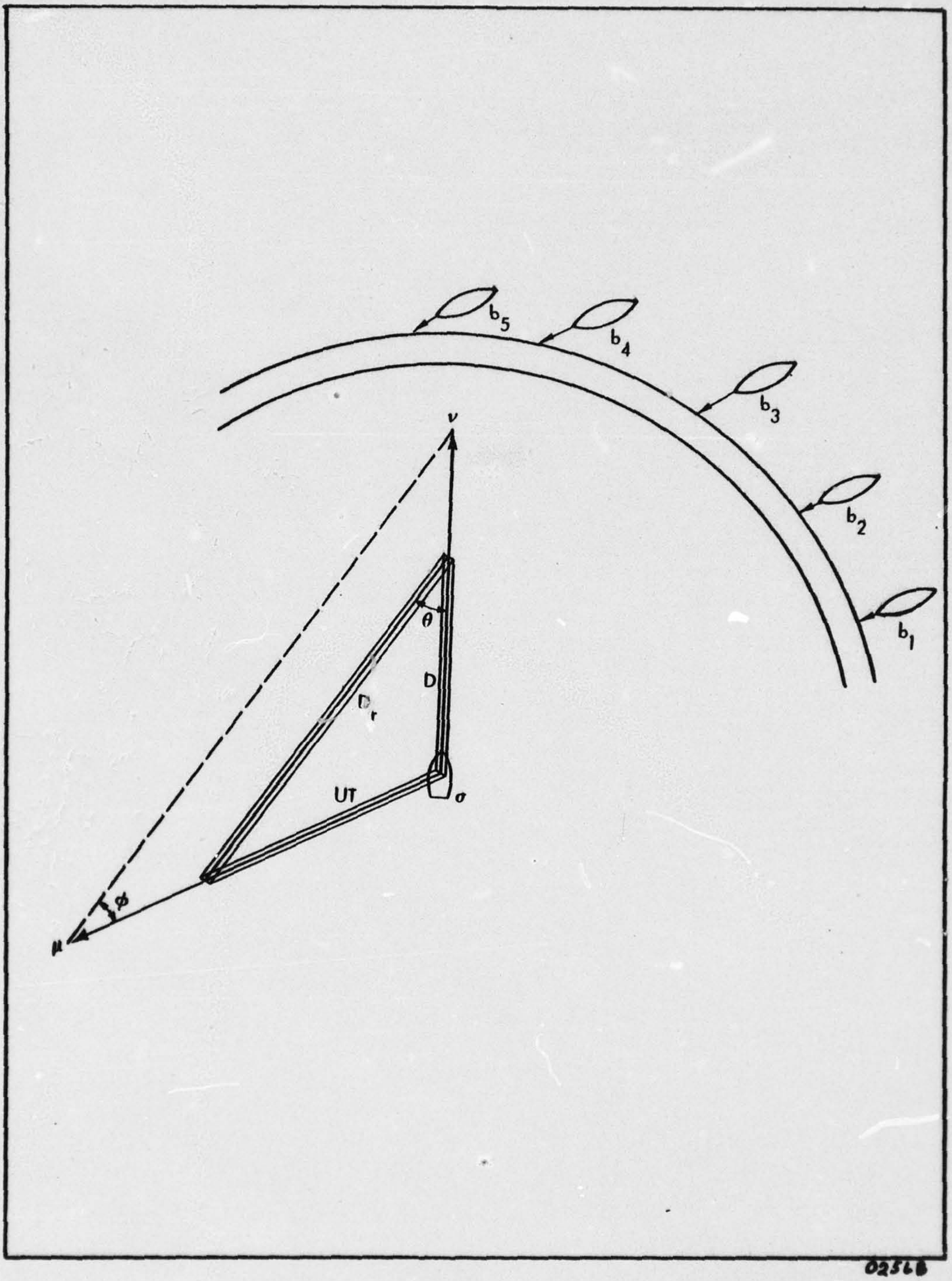

Figure 16. Generalized Representation of Relative Movement Between Hydrofoils and Penetrating Submarine

applies because the submarine may pass obliquely through the convergence zone annulus. For the general case of random penetrations, it is convenient to describe the hydrofoil search effectiveness in terms of an area measure rather than the linear measure upon which the effectiveness against radial penetrations was based.

Define:

a = the region searched for the first time by a hydrofoil during one cycle

A = the unsearched region left by a hydrofoil between two successive "looks"

E' = single hydrofoil effectiveness against random penetrations; specifically, the ratio of searched area to area advanced per cycle.

By definition:

$$E' = \frac{a}{A + a} \tag{31}$$

Figure 17 describes the geometry of area search by a hydrofoil employing the "minimum dip" cycle. The unsearched region of one cycle, A, is the shaded crescent shaped area. The region searched, a, is the portion of the annulus ahead of the unsearched area. The dimensions of these areas are expressed in terms of the previously defined quantities:

R_c: the inner radius of the convergence zone annulus

w: the width of the convergence zone annulus

D_r: the relative distance change between the hydrofoil and the submarine in one cycle.

Inspection of Figure 17 reveals that the total area advanced in one cycle (A + a) can be approximated by the rectangle defined by the outer diameter of the convergence zone annulus and the relative distance traversed in one cycle:

$$(A + a) \simeq 2D_r (R_c + w) \tag{32a}$$

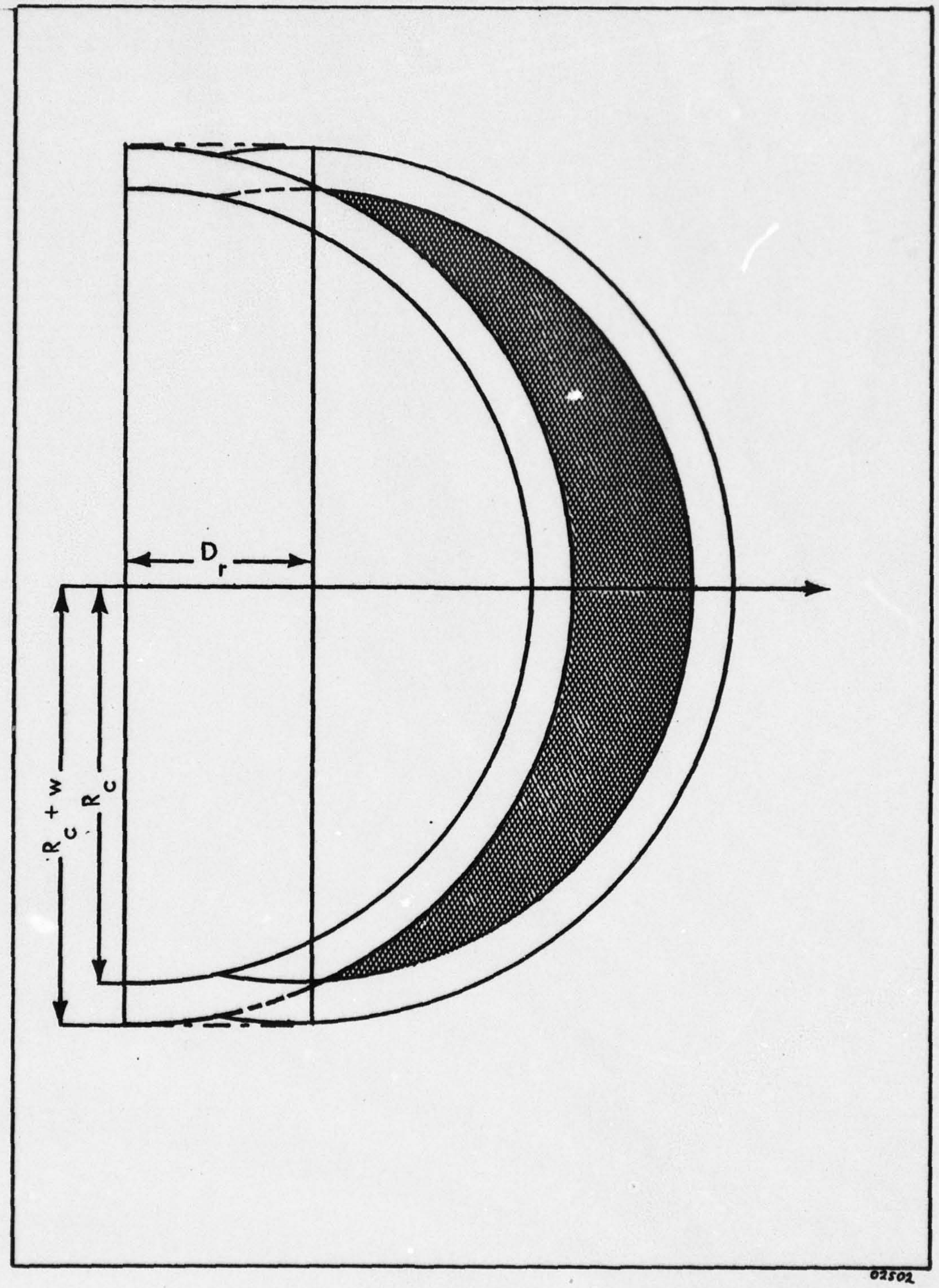

Figure 17. Search Geometry of an Advancing Hydrofoil

In Section 1.2 typical values of the dimensions of the various convergence zones were cited. The width and the radius of the zone in each case bore the approximate relation to each other:

$$w \simeq \frac{R_c}{10} \qquad (33)$$

By the definition of E in Equation (13a), an expression for D_r is obtained:

$$D_r = \frac{w}{E} \simeq \frac{R_c}{10E} \qquad (34)$$

With the help of Equations (33) and (34), Equation (32a) may be rewritten:

$$(A + a) \simeq \frac{0.22 R_c^2}{E} \qquad (32b)$$

The unsearched area, A, can be evaluated by integration. Performing this integration, and substituting from Equations (33) and (34) where appropriate, yields:

$$A \simeq R_c^2 \left\{ \left(\frac{21E^2+1}{20E}\right)\sqrt{1.21 - \left(\frac{21E^2+1}{20E}\right)^2} - \left(\frac{21E^2-1}{20E}\right)\sqrt{1 - \left(\frac{21E^2-1}{20E}\right)^2} \right.$$

$$\left. + 1.21 \sin^{-1}\left(\frac{21E^2+1}{22E}\right) - \sin^{-1}\left(\frac{21E^2-1}{20E}\right) - (0.21)\frac{\pi}{2} \right\} \qquad (35a)$$

Let the symbol f(E) represent that portion of Equation (35a) contained within the brackets.

Equation (35a) may then be abbreviated:

$$A \simeq R_c^2 \cdot f(E) \qquad (35b)$$

From Equations (31), (32b) and (35b):

$$E' \simeq \frac{0.22 - E \cdot f(E)}{0.22} \qquad (36)$$

For very low values of E (i.e. $D_r >> w$) the approximation of Equation (32a) does not hold. However, for reasonable combinations of D_r and w (e.g. $E = \frac{w}{D_r} > 0.2$), Equation (36) provides acceptable accuracy for converting between the linear and the area effectiveness measures. A specific illustrative case is worked out by another method in Section II for comparative purposes.

The interrelation between E and E' by Equation (36) is graphed in Figure 18. It is seen from the figure that E' is asymptotic to unity as E approaches one, and, for practical purposes, is equal to unity for values of E greater than 0.9. Hence it may be said that when coverage against collision course penetrations is 0.9, coverage against random penetrations is essentially unity. Although the linear measure E is defined for values greater than one, this region is not of interest because E' has realized its boundary for lesser values of E.

Since E' is the expected value of a variable effectiveness against penetrations at different points, E' does not increase linearly as the number of hydrofoils per station increases. Subject to this limitation, the remarks in Section 1.9 relative to the effectiveness E against collision course penetrations are generally applicable to the effectiveness E' against random penetrations.

1.11 INCREASE IN PROBABILITY OF DETECTION WITH OVERLAPPING COVERAGE

In the discussion thus far, it has been assumed that no increase in probability of detection accrues from overlap of coverage between adjacent hydrofoils. Under the screening concepts described in Section 1.4, the screen units are stationed to minimize overlap at the desired distance of coverage; hence the assumption is not unreasonable. However, should there be an abundance of screen units,

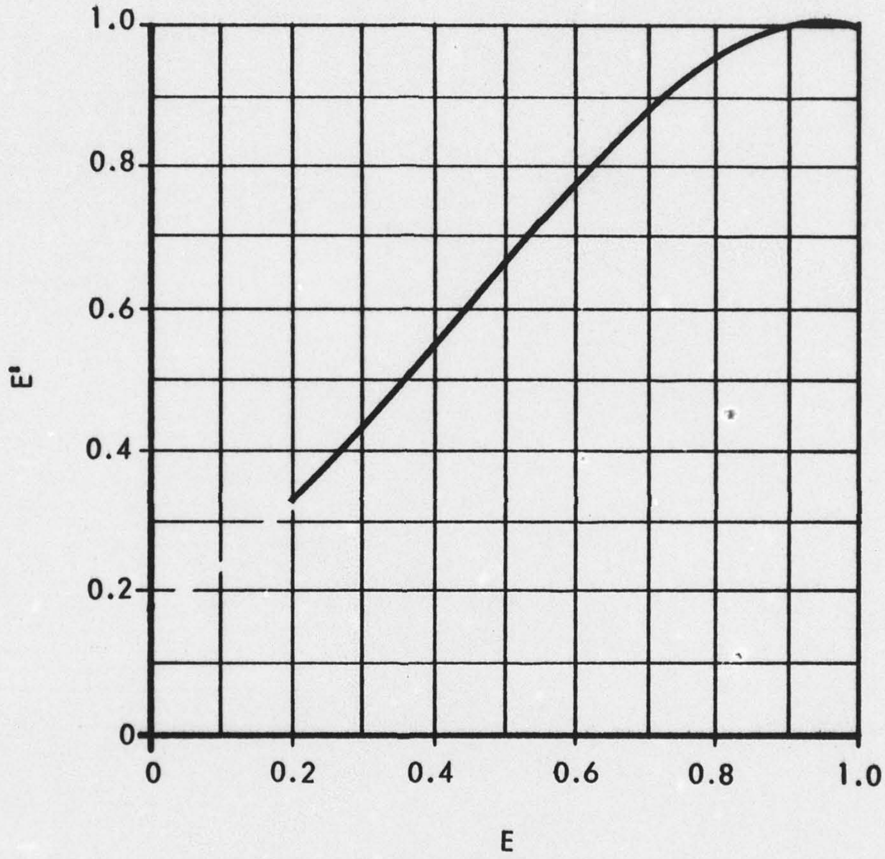

Figure 18. Random Penetration Effectiveness, E', Versus Collision Course Penetration Effectiveness E

or for other reasons should the search zones of the screen units overlap, the detection probability will change for two reasons. First, the density of search between two adjacent screen units will be greater, increasing the probability of "looking". Second, in part of the searched area, the submarine will be "looked" at by two hydrofoils; the probability of "seeing" will therefore increase. Assumption (b) of Section 1.8—that the probability of "seeing" is not increased when a single hydrofoil "looks" a second time at the submarine—will continue to apply; hence it is necessary to consider only the areas that each hydrofoil searches for the first time.

The overlapping search condition will be examined in terms of an illustrative case. Consider the situation where submarines penetrate at random points from ahead of the screen on course opposite to the hydrofoil direction of advance. Figure 19a depicts the annular detection zones of three screen elements (H_1, H_2 and H_3) stationed at intervals of d in a line normal to the direction of advance. It may be noted from Figure 19 a that the limiting hydrofoil spacing under which overlap will occur is $d = 2(R_c + w)$. At this distance, the outer radii of the detection zones of adjacent hydrofoils are tangent. For $d = (R_c + w)$, the outer radii of the detection zones of hydrofoils H_1 and H_3 are tangent; if the spacing is closer than $(R_c + w)$, there is multiple overlap of the detection zones of all three hydrofoils. Attention will be directed to the range of values of d between $2(R_c + w)$ and $(R_c + w)$ or, using the approximation of Equation (33), for $0.45 < \frac{R_c}{d} < 0.9$.

Because of the symmetry of the screen geometry, only a segment of the screen of length d (the area between the dashed lines of Figure 19 a) needs to be considered. The hydrofoil search pattern with this subarea is shown in Figure 19 b. Define:

b_1 = the region searched for the first time by one hydrofoil during one cycle

b_2 = the region searched for the first time by the second hydrofoil during one cycle

b_{12} = the area searched for the first time by both hydrofoils during one cycle

B = the area advanced by the hydrofoil screen in one cycle

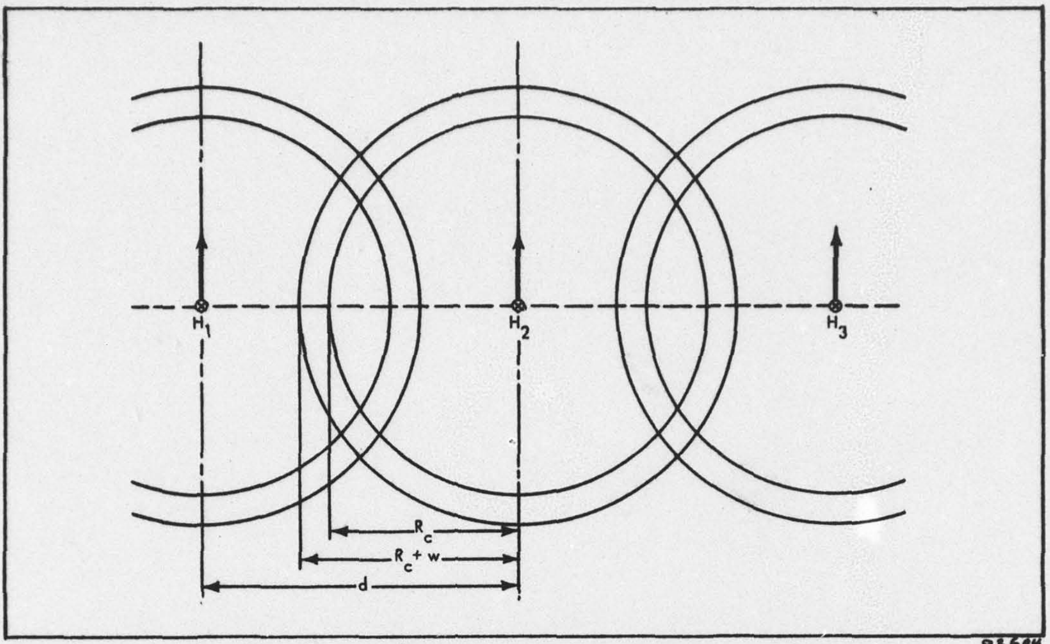

Figure 19a. Single Cycle Search Pattern by a Line of Hydrofoils with Overlapping Coverage

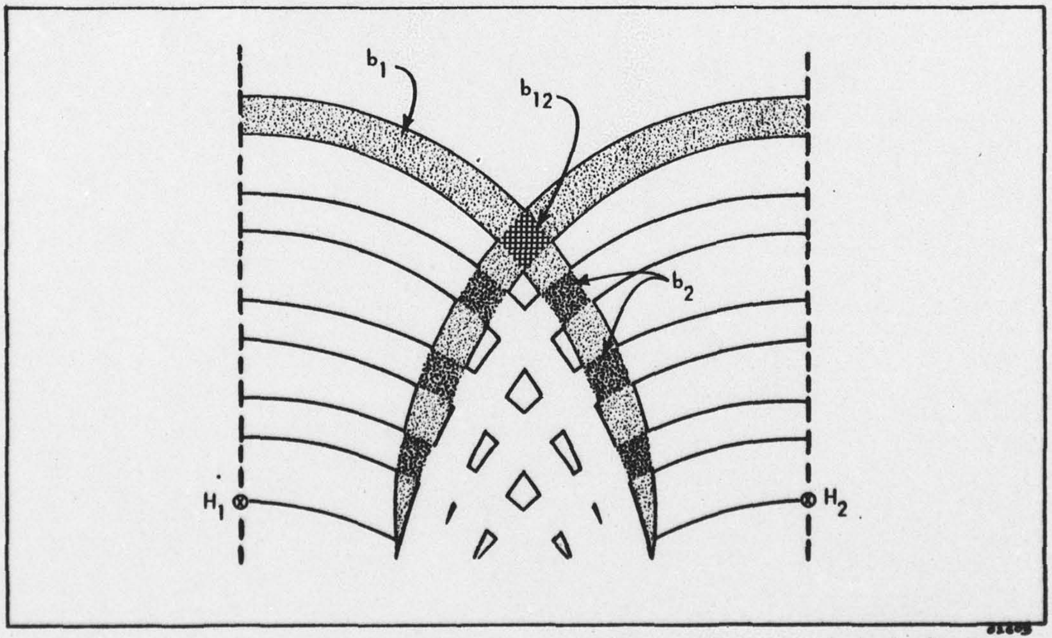

Figure 19b. Cumulative Search Pattern by Two Adjacent Hydrofoils with Overlapping Coverage

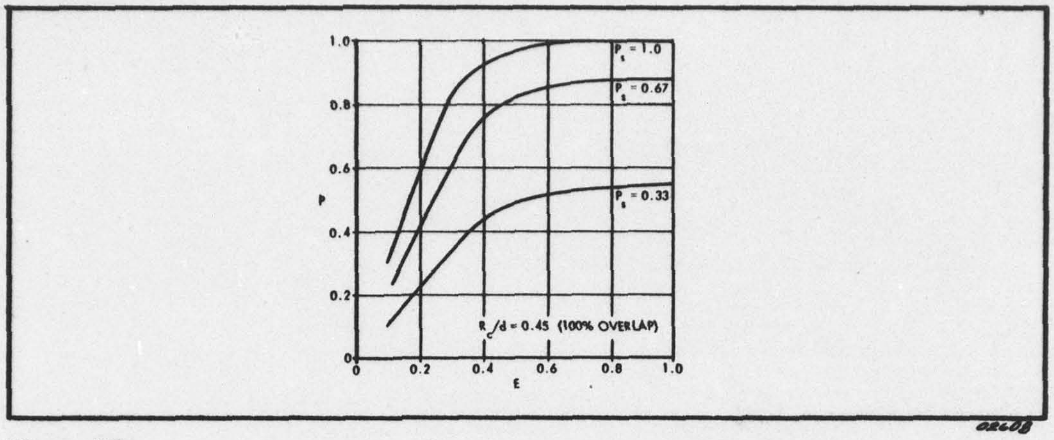

Figure 20a.

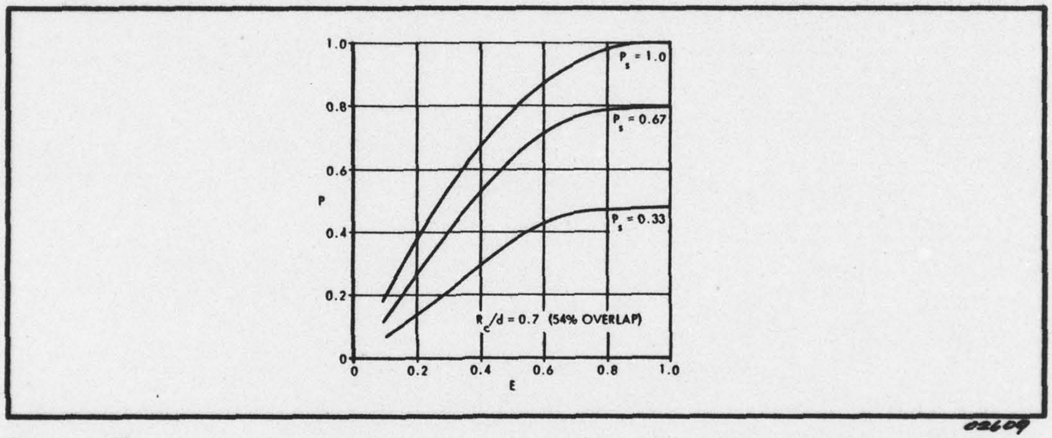

Figure 20b.

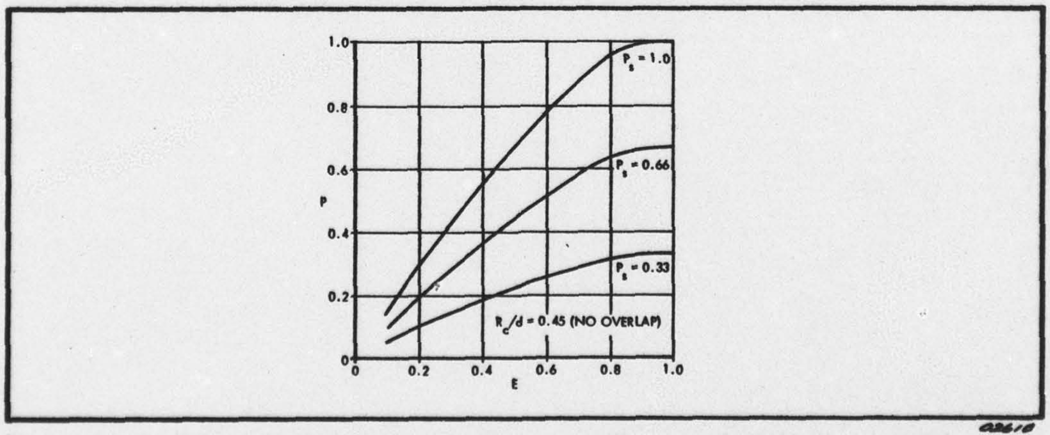

Figure 20c. Over-all Probability of Detection by a Hydrofoil Screen with Overlapping Search Coverage

P_S has previously been defined as the single hydrofoil conditional probability of detection, given that the hydrofoil "looks" at the submarine. The total probability of detection by the screen P, is then:

$$P = P_S \frac{b_1}{B} + (1 - P_S)(P_S) \frac{b_2}{B} + \left[1 - (1 - P)^2\right] \frac{b_{12}}{B} \qquad (37)$$

As before, the area advanced in one cycle, B, can be approximated by the area of the rectangle of dimensions d and D_r. With Equation (33), Equation (37) may be rewritten:

$$P = \frac{10 P_S E}{R_c d} \left[(b_1 + b_2 + 2b_{12}) - P_S(b_2 + b_{12})\right] \qquad (38)$$

The areas b_1, b_2 and b_3 can be described in terms of the independent variables of d, R_c and E; but the mathematics necessary to derive an analytical solution are formidable. Graphical methods have therefore been employed to compute the overall probability of detection for representative combinations of $\frac{R_c}{d}$, E, and P_S. The situations where $\frac{R_c}{d} = 0.9$ (100 % overlap of search coverage by adjacent hydrofoils), where $\frac{R_c}{d} = 0.7$ (54% overlap) and $\frac{R_c}{d} = 0.45$ (no overlap) are shown in Figures 20a, 20b, and 20c respectively. It is seen in each case that P, initially zero, increases with increasing values of the linear effectiveness measure E, ultimately becoming asymptotic at some level determined by the single hydrofoil probability of "seeing", P_S, the hydrofoil spacing, d, and the convergence zone radius R_c.

SECTION II

INTERACTION BETWEEN ENEMY SUBMARINES AND HYDROFOILS ESCORTING A TASK FORCE

2.1 EFFECTS OF SUBMARINE TACTICS ON GENERALIZED SEARCH AND SCREEN METHODS

This section examines the influence of various submarine tactics on antisubmarine screening effectiveness. Both the convergence zone sonar screens described in Section I, and the conventional sonar screens discussed in the Boeing comparison study are considered.

In considering the effect of enemy submarine tactics, it must first be assumed that the objective of the submarine is to attack the force being screened, and that the submarine will attempt to remain undetected prior to delivering the attack. In so doing, it would be expected that the submarine can gain a certain amount of tactical intelligence about the task force and the screen to plan his penetration to the attack point. Conversely, it would be expected that the task force commander would have sufficient intelligence of enemy submarine capabilities to dispose his screen at distances greater than maximum submarine weapon range.

Thus, the submarine will be required to penetrate the screen undetected to attack and must employ approach tactics that will serve to minimize the probabilities of his being detected.

Against screening with convergence zone sonar, the submarine is confronted with an active mode of transmission where running silent contributes nothing and running deep cannot evade the path of sonar propagation. He must pass through the ring and his alternative tactics are:

1. To expose minimum silhouette while passing through the zone to reduce the probability of detection per ping.

2. To penetrate the zone radially and at maximum speed to minimize his time of exposure.

3. To parallel the surface formation course and speed while remaining just outside the detection zone, and then attempt to dash through while the escorts are foiling from one search position to another.

Inasmuch as this study has not dealt with specific sonar equipments, reduction in probability of detection due to minimum target aspect can only be illustrated and this will be shown later in the discussion of conventional sonar coverage (See Figure 22).

As regards the use of maximum submarine speed to penetrate the zone, the relative movement between the submarine, the screen, and the task force has been analyzed. It was shown that in employing speed to close the task force, the submarine can achieve the greatest relative movement, and hence the best speed advantage, by penetrating the screen from ahead of the task force on opposite course. The analysis has further shown that in the case of the "minimum dip" cycle, where the number of escorts per station afford an effectiveness of one (1), the submarine cannot penetrate at any speed without falling into the zone of detection of at least one escort during at least one "look," and therefore his detectability is then related to the performance of the sonar equipment per se. Where the number of escorts is not sufficient to provide unity effectiveness, the submarine then has some probability of passing through the screen without actually being exposed to detection. Within the realm of practical situations, this is possible only in penetration from ahead, because as shown, one escort per station is sufficient for unity effectiveness against beam or astern penetration.

To illustrate the advantage of radial penetration over random penetration from ahead, a typical screening disposition was taken and an exposure ratio was computed graphically (Figure 21). The exposure ratio is defined as the ratio of the length of path in the zone in the head-on case to the length of path of a submarine penetrating randomly from ahead on parallel opposite course from the task force. The following values of the determinants were chosen:

number of escort stations $(n) = 5$

radius to the first convergence zone $(R_c) = 30$ n. mi.

width of the convergence zone $(w) = 3$ n. mi.

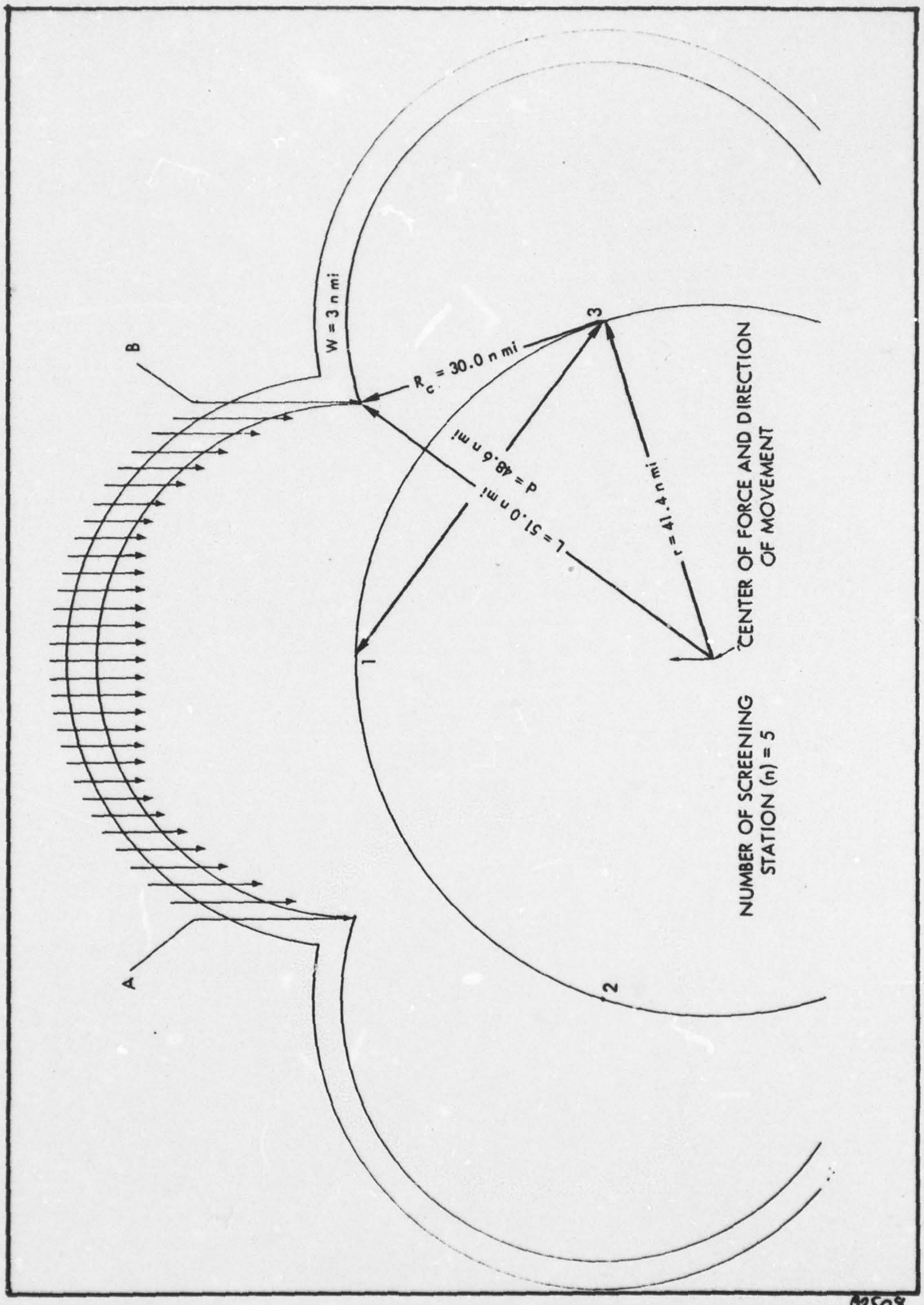

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Figure 21. Submarine Exposure Penetrating Between A and B

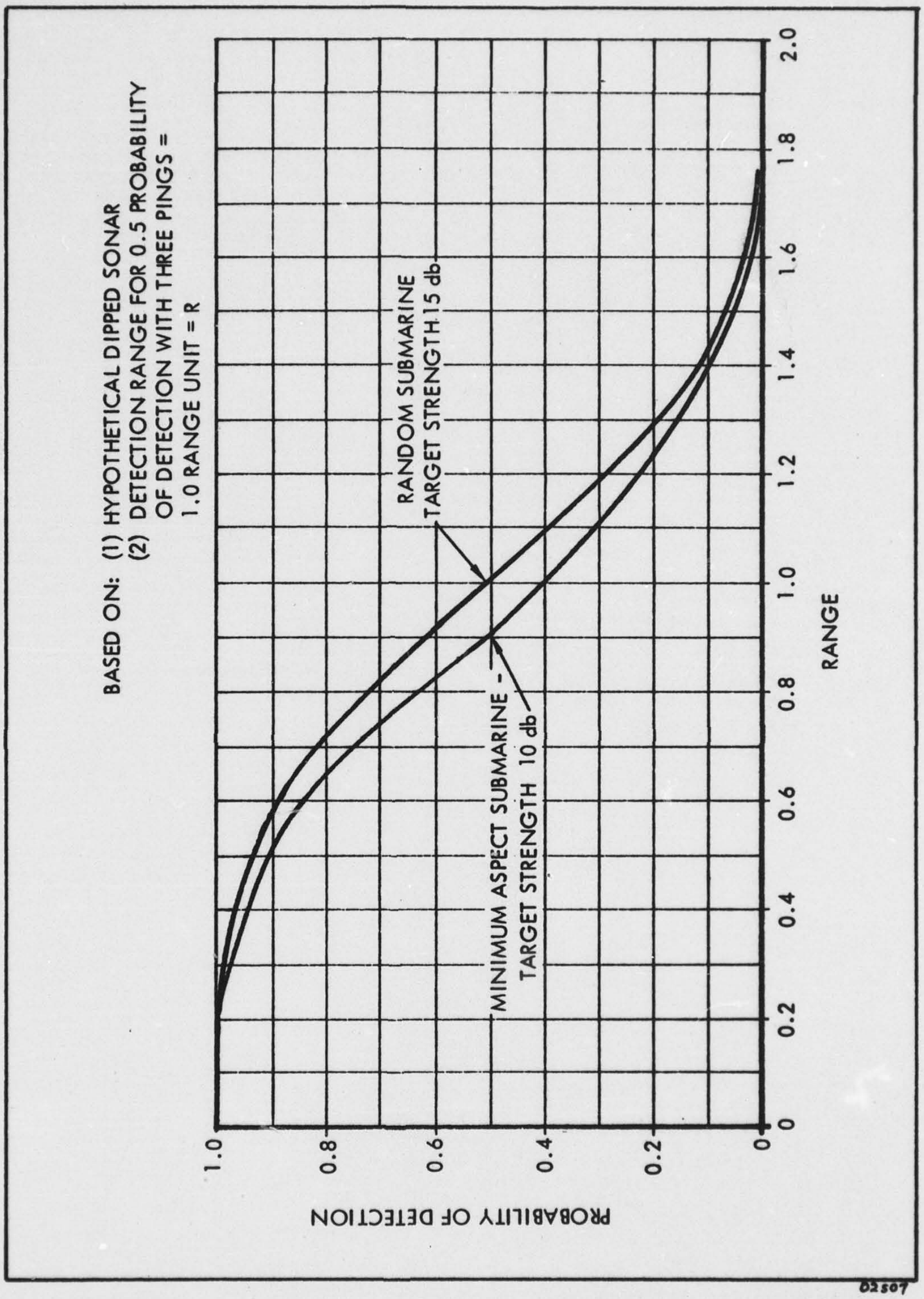

Figure 22. Sonar Detection Versus Normalized Range for Three Pings

distance from task force center for sonar coverage (L) = 1.70 R_c = 51 n. mi.

radius to escort stations (r) = 1.38 R_c = 41.4 n. mi.

distance between escort stations (d) = 1.62 R_c = 48.6 n. mi.

Taking the No. 1 screening station, the penetration distances were measured from equally spaced positions ahead through the convergence zone in the sector between A and B. Totalling and averaging the paths shows that the exposure ratio is 1.5 for a random submarine penetration from ahead on parallel opposite course as compared to a head-on approach.

Against a hydrofoil screen employing conventional sonar search (surface channel or bottom bounce), submarine penetration tactics would be different from those previously discussed. In approaching for attack, the submarine would attempt either to:

1. Penetrate the screen midway between two adjacent escorts, using maximum speed in order to minimize the duration of exposure.

2. Vary his heading during penetration, so as to present minimum target aspect to the hydrofoil each time it conducts sonar search.

The effect of minimum target silhouette is illustrated in Figure 22, using the probability of detection curve vs. normalized range for a random submarine target taken from the Boeing study, and superimposing a similar curve for a minimum aspect target. In making this comparison, a target strength of 15 db was taken for the random target and 10 db for the minimum aspect. It shows, for example, that the probability of detection at the range R = 1.0 is reduced from 0.5 to 0.4 by the minimum target aspect.

From the Boeing study (p. 29-30) it may be deduced that with conventional sonar coverage, the submarine has the shortest distance to travel in evading the hydrofoil screen between intermittent search (i. e., while escorts are foiling) by penetrating at the mid-point between adjacent escorts. It is not considered necessary to develop further mathematical relationships to show that submarine penetration can be countered (given a fixed hydrofoil V_m and a specific sonar capability) by varying the spacing between adjacent escorts and the distance from one search position to the next. Certainly, there should be sufficient forces to provide overlap in sonar coverage between adjacent escorts and there must be such overlap of coverage between successive searches to preclude the submarine from evading the relative zone of coverage (i. e., relative to submarine movement). This must be adjusted by the tactical commander accordingly for the anticipated maximum submarine speed and the number of screen units available.

It is further obvious that greater design sonar performance and maximum hydrofoil speed will allow more coverage and hence a smaller number of hydrofoil escorts required.

2.2 EFFECTS OF SUBMARINE CONTACT DEVELOPMENT

It is difficult to treat the effect of contact development upon the screening requirements of a task force without reference to a specific localization and attack capability. Nevertheless, certain important factors can be focused upon in a general qualitative discussion which may be of value in any subsequent quantitative analysis of more well-defined concepts.

A first consideration toward minimizing the disruption of the screen by development of submarine contact is to have a quick-reacting, flexible localization and attack capability. This will serve to reduce the requirement time-wise for screening escorts to be out of screen position while taking part in the attack phase of the operation. This could be accomplished by either the screening escorts having such an attack capability, or in the protecting force having additional units with attack capability, presumably other hydrofoil craft or aircraft. Screening hydrofoils will be deployed around the task force at a distance such that submarines will be detected at a range from the force center that exceeds enemy weapon range, thereby permitting development of the contact before the submarine can attack.

As a defensive tactic following submarine contact, the task force can change course, speed and disposition. It has been shown in the foregoing analysis that achievement of levels of effectiveness of unity or greater is relatively easy in the beam or astern cases, and in actual operational practice, the threat from abeam and astern has always been less; the most critical cases are with submarines approaching from ahead. When a submarine contact is initially established, the tactical commander normally would change the course of the formation away from the contact and increase speed if possible, to stay out of weapon range until the submarine has been successfully attacked.

Where escorts would be equipped with convergence zone sonar, the escort making initial contact of a submarine should then maneuver to maintain the contact and keep the submarine in "the ring." If the initial contact is made in an outer convergence zone, one tactic might be to have the escort foil toward the submarine while attack units are closing, to reacquire the contact in an inner zone to insure better contact for vectoring in the attacking units. Actual sonar capability and the nature of the attack units will influence such a doctrine, but generally, it would appear more advisable to maintain the contact within the zone of initial detection rather than risk failure of reaquiring contact.

In the case of a screen unit maneuvering to maintain a contact to develop the attack, it would be possible for the tactical commander to alter the course of the formation and the disposition of the escorts as well as to adjust speed of advance to retain a maximum level of screen effectiveness with the remaining escorts.

As an example, consider a screen formed in the five (5) station disposition shown in Figure 21, with two (2) escorts per station on stations 1, 2 and 3 and one (1) on stations 4 and 5. This would insure that all hydrofoils in the screen have an effectiveness greater than one for a task force speed of 25 knots and an enemy submarine speed of 35 knots. A submarine contact from ahead made in the first convergence zone by an escort on station 2, would be put astern of the task force by changing course 120° to the right. The force could be slowed to 15 knots, allowing one escort per station to provide the desired effectiveness and the additional three (3) escorts could be employed in maintaining the contact and delivering the attack. Such a tactic would not involve a reforming of the task force and screen disposition, merely a simultaneous change of course by each unit.

In this example, a 50 knot hydrofoil from station 1 would, by coincidence, have approximately 50 miles to foil to reach an "on top" attack position, taking about one hour from the time of contact to the delivery of attack. This undesirably long attack delivery time indicates the advantage of a quick-reacting attack capability. An antisubmarine rocket with a range and kill radius commensurate with obtainable sonar range and target location accuracy could permit escorts to attack immediately after the submarine contact is verified. This would considerably reduce the necessary contact-holding time and the evasive maneuvering time for the task force until the threat is eliminated.

If the surface force includes aircraft carriers or VTOL aircraft, another approach for quick attack would be to have attack aircraft airborne at all times. In this manner, a high speed attack vehicle would be immediately ready to be vectored by an escort making contact to the datum for localization and attack.

In the case of where hydrofoil escorts are equipped with conventional sonar the criteria for protection of a task force would not differ and the effects of contact development would involve the same considerations. Sonar detection ranges would be shorter and perhaps more accurate, facilitating a quicker attack. A greater number of escorts would be required to protect any given force against a specific enemy weapon range. However, these escorts would be closer together providing for more rapid mutual support in developing the contact. In addition, because conventional sonar provides coverage of a solid circular area, the escort making contact is not as severely constrained in maneuvering to hold the contact as in the case of the convergence zone sonar "ring." A single escort making contact is free to close for attack without undue risk of losing the target. In many situations this could be accomplished without leaving a "hole" in the screen; and where this is not the case, a "back-up" escort could be more quickly deployed to fill the screen because of the shorter distances involved. Maneuvering of the task force to minimize the effect of the submarine contact would be subject to the same doctrinal considerations as previously cited.

One general conclusion might be made from this discussion of contact development. Reaction time from initial contact to attack vitally affects the integrity of the screen and the protection of the task force against other possible submarine penetrations. Screening with convergence zone sonar allows effective coverage with fewer number of escorts but undoubtedly would require more sophisticated attack systems. Conventional sonar coverage,on the other hand, would require a greater number of escorts but would allow more flexibility in developing the contact and simpler attack systems for quick reaction.

Very Good