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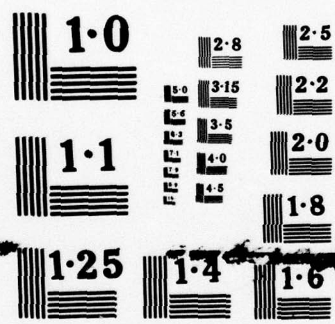

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THE DESIGN OF DIGITAL CONTROLLERS FOR THE C-141 AIRCRAFT USING ENTIRE EIGENSTRUCTURE ASSIGNMENT AND THE DEVELOPMENT OF AN INTER-ACTIVE COMPUTER DESIGN PROGRAM

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Preface

This report is the result of my attempt to design a digital control law for the C-141 aircraft, using the multivariable time-domain technique which achieves entire eigenstructure assignment. Also, an interactive computer program was developed that aids the user in designing the digital control law. The theoretical background that I used extensively came from the work of Professor John D'Azzo and Professor Brian Porter from the University of Salford. Two previous theses by Lt. Stan Larimer and Capt. James Colgate provided assistance in the development of the interactive computer program.

I wish to express my graditude to Professor D'Azzo for his continuous guidance in the development of this thesis and his painstaking effort in reading the thesis for accuracy and completeness. I also want to thank Major Gary Reid and Prof C. H. Houpis for their assistance and suggestions.

Thomas A. Kennedy

Contents

List of Figures

List of Tables

List of Symbols

ω_s	Sampling frequency
ω_c	Highest signal frequency
u	Control input vector
x	System state vector
K	Feedback gain matrix
F	Discretized plant matrix
G	Discretized control input matrix
Q	Controllability matrix
A	Continuous plant matrix
B	Continuous control input matrix
R	Continuous disturbance input matrix
y	System output vector
DR	Discretized disturbance input matrix
d	Disturbance input vector
e^{AT}	State transition matrix
z	Integrator vector
T	Sampling time
λ	Eigenvalue
M	Integrated output matrix
h	Altitude (ft)
V_{TO}	Total linear steady-state velocity (ft/sec)
U_o	Linear steady-state velocity along x-axis (ft/sec)
W_o	Linear steady-state velocity along the z-axis (positive down)
α_o	Steady-state (trim) angle of attack
θ	Pitch angle

w	Linear perturbed velocity along the z-axis
u	Linear perturbed velocity along the x-axis
δ_e	Elevator surface deflection from trim
δ_{sp}	Spoiler surface deflection from trim
δ_{rpm}	Engine speed change
M_w	Dimensional variation of pitching moment with w
M_u	Dimensional variation of pitching moment with speed
M_q	Dimensional variation of pitching moment with pitch rate
$M_{\delta e}$	Dimensional variation of pitching moment with elevator
$M_{\delta sp}$	Dimensional variation of pitching moment with elevator deflection
X_w	Dimensional variation of X_s - force with w
X_u	Dimensional variation of X_s - force with speed
$X_{\delta rpm}$	Dimensional variation of X_s - force with engine speed
$X_{\delta e}$	Dimensional variation of X_s - force with elevator deflection
$X_{\delta sp}$	Dimensional variation of X_s - force with spoiler deflection
Z_w	Dimensional variation of Z_s - force with w
Z_u	Dimensional variation of Z_s - force with speed
Z_q	Dimensional variation of Z_s - force with pitch rate
$Z_{\delta e}$	Dimensional variation of Z_s - force with elevator deflection
$Z_{\delta sp}$	Dimensional variation of Z_s - force with spoiler deflection

Abstract

In this report, a deadbeat tracker and regulator are developed for the C-141 aircraft for three different flight conditions using the method of entire eigenstructure assignment. A heuristic iterative method is presented for determining an "optimal" sampling time for deadbeat controllers by using the controllability matrix.

In order to perform the above designs and to evaluate them through a continuous-time simulation, a computer aided design program is developed. This program is fully interactive and provides complete error protection and abort protection. The program allows the complete design and simulation of regulator, tracker, and disturbance-rejection control systems with full-order observers. In addition, the program permits complete state space analysis of the system being designed, including both discrete and continuous-time responses. A user's manual and programmer's guide are provided for further development of the program.

I. Introduction

The increased flexibilty and miniaturization of the digital computer has made it a valuable asset in the design of modern control systems. In order to make full use of the digital computer as a controller for the feedback control system,a discrete control law methodology must be developed and proven to give desirable results. A design methodology that meets the above criteria is entire eigenstructure assignment. This consists of the assignment of the closed-loop eigenvalue spectrum and the associated set of eigenvectors and generalized eigenvectors.

Background

Most modal control approaches, to date, deal with the assignment of eigenvalues for the closed-loop system (Ref. 1). The ability to assign arbitrary eigenvalues to the closed-loop system allows the designer to match the stability characteristics of the closed-loop system to some desirable model. Therefore, both the model and the system have the same steady-state responses. However, there is no assurance that the transient responses of the closed-loop plant match those of the model. The reason for this is that although the previous modal control approach allows arbitrary assignment of an eigenvalue spectrum to match some model, it does not allow any flexibility in the assignment of associated eigenvectors of the closed-loop system. Thus, the closed-loop system has the desired stability characteristics but does not necessarily have the same transient responses or performance characteristics of the model.

Now, with the entire eigenstructure assignment methodology, it is possible to match both the stability and the performance characteristics of the closed-loop plant to a desired model. Entire eigenstructure assignment is a time domain, multivariable state space control law methodology that assigns both the desired eigenvalue spectrum and associated eigenvectors to the closed-loop system (Ref. 2).

Statement of Problem

The two main goals of this investigation are

(1) Develop an interactive, user oriented computer program that aids in the design of regulator, tracker, and disturbance rejection control systems. All of the designs are performed in the time domain and are based on a multivariable control law methodology utilizing entire eigenstructure assignment. The core of the program is built around the algorithms furnished by Professor Porter of the University of Salford, England and the Air Force Flight Dynamics Lab in the computer program FORTRAC (Ref. 3).

(2) Design a regulator and tracker for the longitudinal axis of the C-141 for three different flight conditions, using the interactive computer package developed in goal 1. All of the control law designs are based on the C-141 having both elevator and spoiler control surfaces. The method of entire eigenstructure assignment is used and all closed-loop eigenvalues are assigned to the origin to produce deadbeat designs, that is, the output response reaches steady-state in a finite numberof sampling periods. The design technique presented in this thesis also produces a ripple-free response in the steady-state for constant inputs.

Approach

Structured design techniques are employed stressing modularity and top-down design in the formation of the computer program. A requirements definition is first developed, stating the need for an iterative approach to the design of a control law. A detailed explanation of the structured design approach used is presented in Chapter IV.

In the design of the control laws, the C-141 is first modeled in the continuous-time domain using the body axes stability derivatives found in Ref. 4. The plant is then discretized using a sampling time that is determined using the heuristic approach formulated in Chapter II. Next, for the tracker design, the plant is augmented with discrete-time integrators to produce a zero steady-state error in the desired output. By transforming the system matrix into the Brunovsky canonical form, the control indices of the system are determined (Ref. 5). This is done so that the minimum possible index of nilpotency of the closed-loop plant matrix can be determined. A deadbeat control law is then synthesized by assigning all eigenvalues to the origin in order to drive the plant to the desired states in p sampling times, where p is the minimum index of nilpotency for the closed-loop system (Ref. 5). Entire eigenstructure assignment is the control law methodology used in the control law synthesis. Since the original plant is a continuous sytem, a continuous-time simulation is performed using the sampled-data control defined above. Thus, the simulation depicts the continuous-time response of the plant, both between and at the sampling times. Chapter III gives a detailed explanation of the above design methodology.

Review of Literature

Most of the theory presented in this thesis is from works by Porter, Bradshaw, and D'Azzo. Porter and Bradshaw's work on nilpotency properties in the design of deadbeat controllers is used to design closed-loop systems that have a finite settling time equal to the size of the minimum permissible polynomial (Ref. 5). The use of integral plus proportional feedback in the design of multivariable tracking and disturbance-rejection systems by Porter and Bradshaw is drawn upon in the design of trackers that have zero steady-state error in the desired output (Ref. 6). Also, D'Azzo's work on entire eigenstructure assignment is used extensively in the design of the trackers and regulators in this thesis (Ref. 2).

Development of the interactive computer program is based on structured design approaches recommended by Yordon (Ref. 7). The actual implementation of the interactive program on the Wright-Patterson CDC-6600/Cyber-74 was aided by previous work by Colgate (Ref. 8) and Larimer (Ref. 9).

II. A Heuristic, Iterative Procedure for Determining An "Optimal" Sampling Time for a Deadbeat Controller

In order to design a discrete control law, the continuous plant must be discretized using an appropriate sampling time, T. A theorem that is often used in determining an appropriate sampling time is Shannon's Sampling Theorem. Shannon's Theorem, in essence, states that the sampling frequency ω_s must satisfy the condition

$$\omega_s > 2\,\omega_c \tag{1}$$

where ω_s is the sampling frequency and ω_c is the highest frequency contained in the signal to be sampled.

However, following Shannon's Sampling Theorem in the selection of a sampling time to discretize a linear multivariable plant does not necessarily guarantee that the discretized plant is controllable even if the original continuous plant is controllable. Another fact that must be considered, when choosing a sampling time, is the "degree of controllability" that the discretized system possesses. In other words, the degree of controllability determines the magnitudes of the feedback gains necessary to drive the plant to the desired state. Since it is well known that high feedback gains result in high overshoots in the transient response, a system that has a "low degree of controllability" is defined in this report as a system that requires large feedback gains to drive the plant to some desired state. Conversely, a system that requires small feedback gains to drive the plant to some desired state is defined here as a system with a "high degree of controllability."

Determination of Degree of Controllability

When state-variable feedback is implemented with the control law

$$u(kT) = K\, x(kT) \tag{2}$$

for a discretized plant represented by

$$x(kT+T) = F\, x(kT) + G\, u(kT), \tag{3}$$

the closed-loop plant is represented by

$$x(kT+T) = (F + GK)\, x(kT) \tag{4}$$

where F is the discretized nxn plant matrix, G is the discretized nxm input matrix, and K is the mxn feedback gain matrix. A necessary condition for the above system to be controllable is that the rank of the controllability matrix be equal to n, the dimension of the plant matrix, where the controllability matrix is defined as

$$Q = [G, FG \dots F^{n-1}G] \tag{5}$$

The above condition determines whether a system is controllable or uncontrollable. However, for a controllable system, it is not possible to gain any insight into the degree of controllability of a system from this approach. For a controllable system the controllability matrix reduces to the following nxn matrix containing n linearly independent vectors

$$\hat{Q} = [g_1, Fg_1, \ldots F^{d-1} g_1, g_2, \ldots, F^{d_2-1} g_2, \ldots, g_m, \ldots F^{d_m-1} g_m] \qquad (6)$$

where $g_1, g_2, \ldots g_m$ are the columns of G and $d_1, d_2, \ldots d_m$ are the control indices of the discretized system (Ref. 10:81). The difference between the two representations of the controllability matrix, Eq. 5 and 6, is that in the representation of Eq. 5 the controllability matrix is square with dimension n only when G is of dimension nxl. But in the representation of Eq. 6, the controllability matrix is always square with order n. Thus, it is possible to take the determinant of the controllability matrix in the form of Eq. 6. Now if the magnitude of the determinant of the controllability matrix, as defined by Eq. 6, is small the system is nearly uncontrollable. This condition implies that one or more of the columns of the controllability matrix are almost linearly dependent. Also, if the magnitude of the determinant is small, then at least one of the eigenvalues must have a small magnitude.

Therefore, the magnitudes of the eigenvalues and size of the determinant, for the matrix of Eq. 6, possess information on the "degree" of independence of the columns of the controllability matrix. This fact is used to determine the "degree of controllability" of a discretized system. Thus, if a system is discretized using two different sampling times, and one discretized system has a larger controllability determinant than the other, the system with the larger determinant is deemed to have a higher degree of controllability than the system with the smaller determinant.

Results

The C-141 model for medium altitude cruise is discretized for eight different sampling times and augmented with discrete integrators as discussed in Chapter VI. The determinant of the controllability matrix and the average absolute value of the control gains are calculated for each sampling time. These values are listed in Table I. All runs are performed with no modification to the null spaces as discussed in Chapter VI. As shown in Table I the results suggest that using the "degree of controllability" to determine an optimum sampling time is more of a qualitative method than an absolute method. However, this assumption is based on the fact that the smaller the feedback gains are, in general, the smaller the transient overshoots are for a system. Entire eigenstructure assignment allows manipulation of the null spaces, and the changing of the null spaces has a direct effect on the size of the feedback gains in the algorithm used to compute the feedback gains.

It is a matter of experience to choose a proper sampling time. In this case it appears that sampling times between 1.50 seconds and 2.0 seconds should lead to good results. But it is not possible to change the null spaces of the systems discretized with sampling times between 0.1 seconds and 1.25 seconds to get feedback gains comparable to those for the other sampling times of 1.5 to 2.0 seconds. Thus it can be concluded that the determinant of the controllability matrix is directly related to the "degree of controllability" of a system. It can also be used as a qualitative method in determining an "optimal" sampling time.

TABLE I

Degree of Controllability Test Results

Sampling Time (Sec)	Determinant of Controllability Matrix	Average Absolute Value of the Control Gains	Largest Value of Gain
.1	-8.6278×10^{-12}	1266.8297	2.184×10^{5}
.5	-2.3234	34.9136	-492.6
.75	-611.4945	10.7710	-135.1
1.0	-11304.8835	4.1481	49.83
1.25	-58031.4352	1.7338	19.79
1.50	-142195.9579	.5177	4.3
1.75	-221437.9541	.6701	7.349
2.0	-263022.6545	1.2263	-14.97

Plant Constraints

Two aspects that must be taken into consideration in choosing an appropriate sampling time for a deadbeat controller are the size of the minimal polynomial of the closed-loop system and the physical constraints of the plant. The size of the minimal polynomial plays a large factor, since the index of nilpotency is equal to the size of the largest Jordan block which can be selected by proper assignment of the eigenvector/generalized eigenvector chains to the closed-loop eigenvalue spectrum (Ref. 2). In this case the transient response given by

$$x(kT) = (F+GK)^L x(0) \quad (7)$$

becomes zero in L sampling times, where L is equal to the size of the minimal polynomial. Since the entire eigenstructure assignment methodology allows the designer to choose the degree of the closed-loop minimum polynomial to be greater than or equal to the largest controllability index, d_1, there is flexibility in choosing the "fastness" or the time required to reach steady-state by the deadbeat controller.

In order to decide on the proper degree of "fastness" for a deadbeat controller, the physical constraints of the plant should be considered. For example, the trackers designed in this thesis for the C-141 are to track an altitude rate step input. If a sampling time of 1.0 seconds is used and, since the minimal polynomial of the closed-loop system is designed to be equal to 4, the aircraft would track the command input in a deadbeat manner in 4 seconds. However, since there is a delay of one sampling period before the aircraft control surfaces receive any control input, the aircraft actually only takes 3 seconds to track the commanded input in a deadbeat manner, from the time of actual control surface deflection.

It is obvious that this response is much too fast for a transport aircraft such as the C-141. There are two possible ways to circumvent this problem. One way is to increase the size of the minimal polynomial of the closed-loop system. The other solution is to use a larger sampling time.

Summary

The following steps are a heuristic guide for choosing a sampling time for a deadbeat digital controller:

1. Determine a sampling time with a "high degree of controllability".
2. Check the lowest degree of the minimal permissible polynomial of the discretized system. This is equal to the largest control index, d_1.
3. Consider the physical constraints of the plant and decide whether the deadbeat controller is too fast or too slow. If the sampling time does not satisfy the above, go back to step one and pick a new sampling time accordingly.

III. Design Methodology Using Entire Eigenstructure Assignment

The design procedure for a digital controller consists of four main steps:

1. Form the discrete-time model of the plant from the continuous-time model.
2. Augment the discretized plant with discrete integrators in the case of a tracker or disturbance rejector design.
3. Perform the control law synthesis in the discrete-time domain.
4. Simulate the continuous-time closed-loop system and evaluate the performance of the digital controller.

These steps are used in this thesis to achieve desirable controllers for both regulator and tracker designs for the C-141 aircraft.

Sampled-Data Transformation

The first step in the design of a digital controller is to discretize the continuous plant depicted in Fig. 1a and described by

$$\dot{x}(t) = A\, x(t) + B\, u(t) + R\, d(t) \tag{8}$$

$$y(t) = C\, x(t)$$

to form the sampled-data system shown in Fig. 1b and represented by

$$x(kT+T) = F(T)\, x(kT) + G(T)\, u(kT) + DR(T)\, d(kT) \tag{9}$$

$$y(kT) = C\, x(kT)$$

where T is the sampling time. This transformation can be accomplished by using Eqs. 10, 11, and 12.

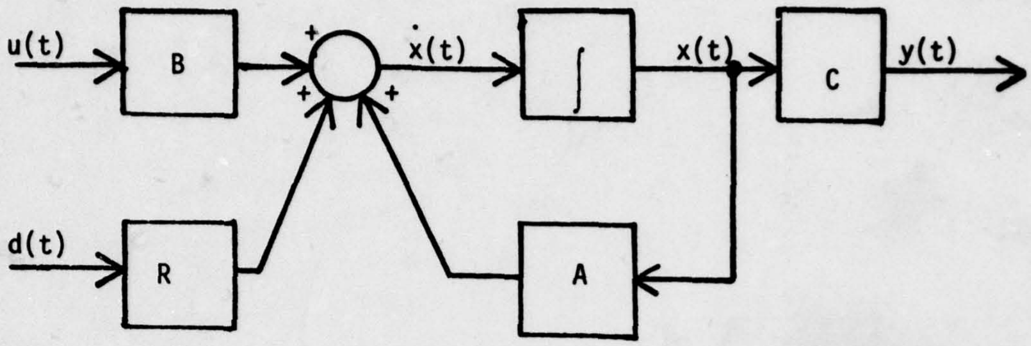


Figure 1a Continuous - Time System

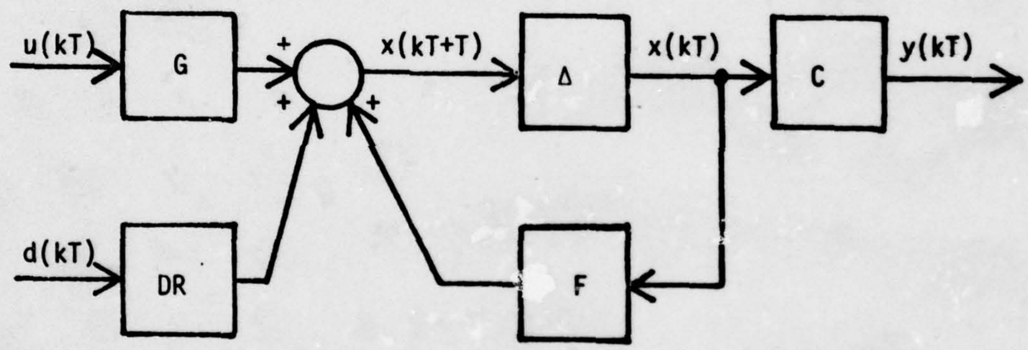


Figure 1b Discretized System

$$F(T) = e^{AT} \tag{10}$$

$$G(T) = \int_0^T e^{At}\, B dt \tag{11}$$

$$DR(T) = \int_0^T e^{At}\, R\, dt \tag{12}$$

where e^{AT} is the state transition matrix. Since the system is assumed linear and time invariant, the B and R matrices are constant and can be taken out of the integral. Thus, all that is left is the determination of e^{AT} and the $\int_0^T e^{At}\, dt$. Two methods are employed in the solution of the above problem in this thesis.

The first method consists of diagonalizing the plant matrix as shown by

$$A = X \; \Lambda \; X^{-1} \tag{13}$$

where X is the modal matrix for the plant. Using the above simplification, the equations for the discrete transformation reduce to

$$F(T) = X\, e^{\Lambda T} X^{-1} \tag{14}$$

$$G(T) = X\left[\int_0^T e^{\Lambda t}\, dt\right] X^{-1}\, B \tag{15}$$

$$DR(T) = X\left[\int_0^T e^{\Lambda t}\, dt\right] X^{-1}\, R \tag{16}$$

Since the modal matrix is constant it can be taken outside the integral. The computation of $e^{\Lambda T}$ and $\int_0^T e^{\Lambda t}\, dt$ is a scaler problem where

$\lambda_i(i=1,2,\ldots,n)$ are the distinct eigenvalues of A which appear on the diagonal of the Λ matrix. This method is not general since a system with multiple eigenvalues produces a singular modal matrix. The above method is used to discretize all continuous plants for the control law synthesis. However, a general method of discretizing the continuous plant containing multiple eigenvalues employs the infinite series form

$$e^{AT} = I + AT + \frac{A^2T^2}{2!} + \ldots. + \frac{A^nT^n}{n!} \tag{17}$$

where I is the identity matrix. The integral of e^{AT} is then calculated using the following infinite series

$$\int_0^T e^{At}\, dt = IT + \frac{AT^2}{2!} + \ldots. + \frac{A^{n-1}T^n}{n!} \tag{18}$$

Both series converge rapidly for a small T and are easy methods for digital implementation. Due to time limitations, this method was used only for the continuous-time simulation.

Augmentation

The use of proportional plus integral state feedback in multivariable control systems results in zero steady-state error for both tracking (Ref. 11) and disturbance-rejection systems (Ref. 12) if the augmentented system remains controllable. Therefore, both of these systems are augmented with discrete time integrators described by

$$Z(kT + T) = Z(kT) + T\,[e'(kT)] \tag{19}$$

where e'(kT) is the error vector that is to be driven to zero in the steady-state. This error is represented by Eq. 20 for the disturbance rejection system,

$$e'(kT) = M\,x(kT) \tag{20}$$

where M is a pxn constant matrix with p less than or equal to the number of controls. This restriction on p is made so that the augmented system is controllable (Ref. 6). Thus, the disturbance rejection system is described by

$$\begin{bmatrix} x(kT+T) \\ z(kT+T) \end{bmatrix} = \begin{bmatrix} F, & 0 \\ TM, & I \end{bmatrix} \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} u(kT) + \begin{bmatrix} DR \\ 0 \end{bmatrix} d(kT) \tag{21}$$

$$y(kT) = [C,\ 0] \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix}$$

It is obvious that any combination of states represented by Mx(kT) is driven to zero in the steady-state.

The error for the tracking problem is

$$e'(kT) = M\,x(kT) - v(kT) \tag{22}$$

where v(kT) is the piecewise constant command vector. The augmented system for the tracker is represented by

$$\begin{bmatrix} x(kT+T) \\ z(kT+T) \end{bmatrix} = \begin{bmatrix} F & , & 0 \\ TM & , & I \end{bmatrix} \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} u(kT) + \begin{bmatrix} 0 \\ E \end{bmatrix} v(kT) \quad (23)$$

$$y(kT) = [C, 0] \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix}$$

where E is a pxp diagonal matrix with $e_{ii} = -T$ and p is the number of commands to be tracked. The vector v(kT) represents the piecewise constant command inputs. In order for the augmented system to be controllable, the number of command inputs to be tracked must be less than or equal to the number of controls (Ref. 6).

Control Law Synthesis

The purpose of entire eigenstructure assignment is to assign, using state feedback, the closed-loop eigenvalue spectrum

$$\sigma(F+GK) = \{\lambda_1, \lambda_2, \ldots, \lambda_n\} \quad (24)$$

and the associated set of eigenvectors

$$\{\chi_1, \chi_2, \ldots, \chi_n\} \quad (25)$$

Since the eigenvalues and the eigenvectors for the closed-loop system are related by (Ref. 2:23)

$$[F-\lambda_i I, G] \begin{bmatrix} \chi_i \\ \omega_i \end{bmatrix} = 0 \quad (i = 1, 2, \ldots, n) \quad (26)$$

where

$$\omega_i = k\chi_i \qquad (i = 1,2,..n) \qquad (27)$$

and

$$\begin{bmatrix} \chi_i \\ \\ \omega_i \end{bmatrix} \qquad (i = 1, 2, \ldots , n) \qquad (28)$$

is a vector that lies in the null space of the matrix

$$S(\lambda_i) = [F-\lambda_i ,G] \qquad (i = 1, 2, \ldots, n) \qquad (29)$$

the feedback matrix K is given by (Ref. 2:17)

$$K = [\omega_i ,\omega_2, \ldots , \omega_n][\chi_1, \chi_2, \ldots , \chi_n]^{-1} \qquad (30)$$

$$= \Omega \, X^{-1}$$

Although no restrictions are placed on the desired eigenvalue spectrum, the ker $S(\lambda_i)$ does place a constraint on the eigenvector that is associated with the assigned eigenvalues. The reason for this is that the ker $S(\lambda_i)$ specifies the null space within which the eigenvector must lie. Also, the eigenvectors, which are the columns of the modal matrix X, must be independent since the inverse of X is used in the calculation of the feedback gain matrix as shown in Eq. 30.

In the case of multiple eigenvalue assignment, as in the design of a deadbeat controller, it may be necessary to generate chains of generalized eigenvectors satisfying the relationship

$$[F-\lambda_i I, G] \begin{bmatrix} \chi_i^{(m_{ji},j)} \\ \omega_i^{(m_{ji},j)} \end{bmatrix} = \chi_i^{(m_{ji}-1,j)} \qquad (31)$$

$$(j = 1, 2, \ldots, k_i; \quad i = 1, 2, \ldots, p)$$

This generates k_i strings of vectors associated with the eigenvalue λ_i, where $\chi_i^{(\ell,j)}$ is the ℓth vector in the jth string which is of length m_{ji} since the ker $S(\lambda_i)$ occupies a subspace of size equal to m, the number of controls, and therefore only m linearily independent eigenvectors may be generated from this null space (Ref 2:23,25).

Equation 31 allows the generation of eigenvector/generalized eigenvector chains from the original null space given by Eq. 29, so that the size of the largest control index, d_1, of the open-loop system may be retained for the closed-loop system. This is possible if (1) the eigenvalue spectrum of the closed-loop system is assigned to the origin, (2) the combined length of the eigenvector/generalized eigenvector chains is set equal to the number of system states, and (3) the largest chain length is set equal to d_1. In this case the outputs are equal to the inputs after a finite number of sampling times, that is, the system is deadbeat. This, allows a deadbeat response to take place in q sampling times, where q is the order of the minimal polynomial of the closed-loop system and is equal to the largest control index of the pair [F,G].

Regulator

The control law to be synthesized using entire eigenstructure assignment for the regulator system is

$$u(kT) = K\,x(kT) \tag{32}$$

where K is the feedback gain matrix that is calculated using Eq. 30. The closed-loop regulator system is then represented by

$$\begin{aligned} x(kT+T) &= (F+GK)\,x(kT) \\ y(kT) &= C\,x(kT) \end{aligned} \tag{33}$$

and is shown in Fig. 2.

Distrubance Rejector

Using the design methodology described above, with the plant now augmented with discrete integrators, the control vector u(kT), for the disturbance rejection system of Fig. 3 with $v(t) = 0$, becomes

$$u(kT) = K_1 x(kT) + K_2\, z(kT) \tag{34}$$

This control law results in the closed-loop system represented by

$$\begin{bmatrix} x(kT+T) \\ z(kT+T) \end{bmatrix} = \begin{bmatrix} F+GK_1, FK_2 \\ TM \quad , I_p \end{bmatrix} \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix} + \begin{bmatrix} DR \\ 0 \end{bmatrix} d(kT)$$

$$y(kT) = [C,\ 0] \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix}$$

This system possesses both proportional and integral control. The closed-loop system is stable if the desired eigenvalue spectrum

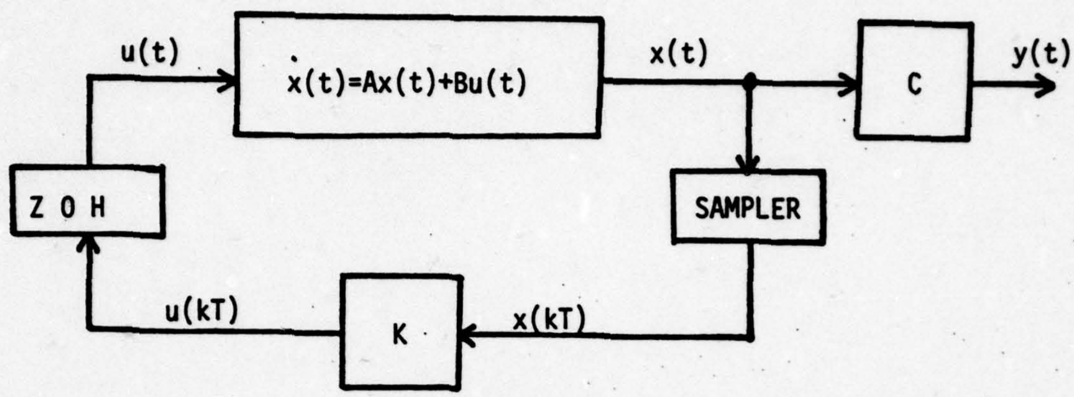


Figure 2. Regulator System

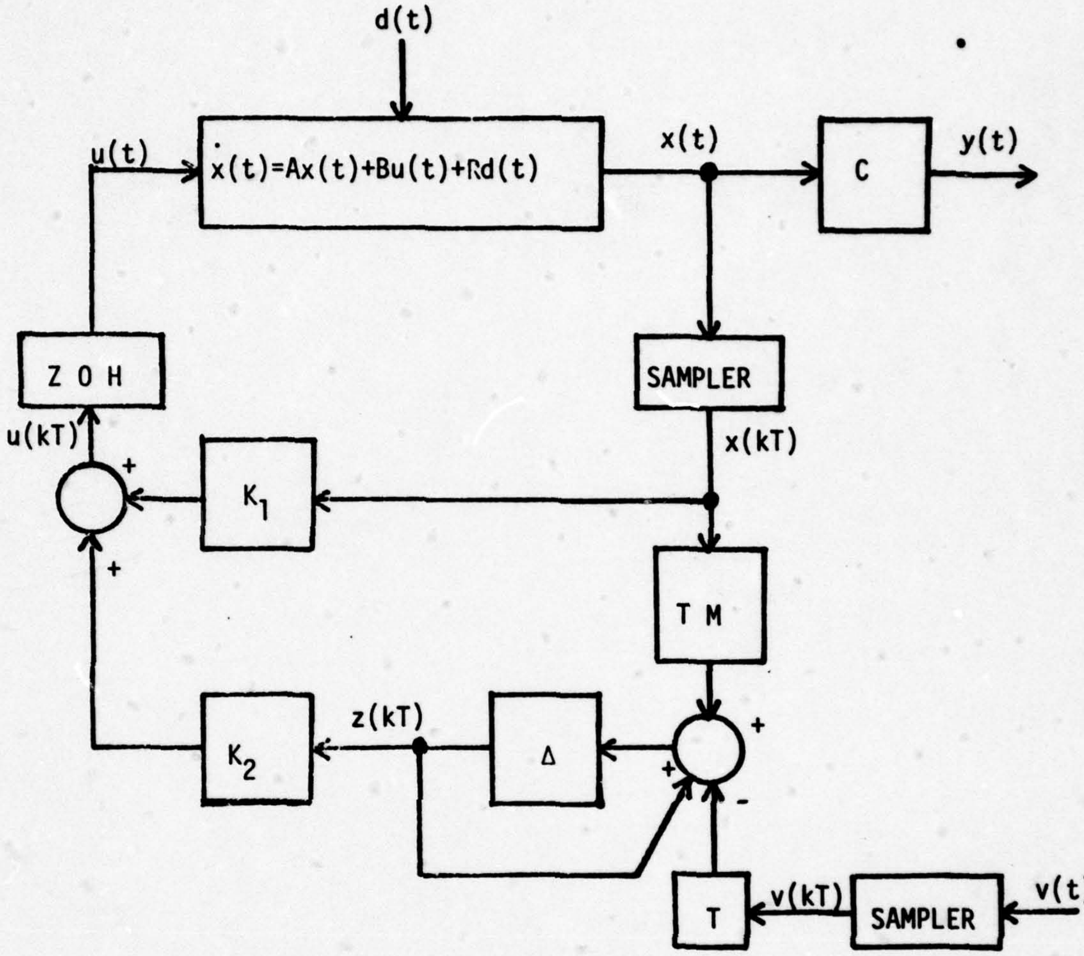


Figure 3. Closed-loop System for Disturbance Rejector (v(t)=0) and for Tracker (d(t)=0)

chosen is inside the unit circle. Therefore,

$$\lim_{t\to\infty} (z(kT+T) - z(kT)) = 0 \quad (36)$$

and as a result, in the steady-state

$$TM\ x(kT) = 0 \quad (37)$$

Thus, if M is equal to the output matrix C, all the outputs, $y(kT) = C\,x(kT)$, are driven to zero in the steady-state.

Tracker

The control law for the tracker is identical to the control law for the disturbance rejector as described by

$$u(kT) = K_1\ x(kT) + K_2\ z\ (kT) \quad (38)$$

and the closed-loop system formed by this control law is

$$\begin{bmatrix} x(kT+T) \\ z(kT+T) \end{bmatrix} = \begin{bmatrix} F+GK_1, & GK_2 \\ TM & , I_p \end{bmatrix} \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix} + \begin{bmatrix} 0 \\ E \end{bmatrix} V(kT)$$

$$y(kT) = [C, 0] \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix} \quad (39)$$

where $v(kT)$ is the discretized command input vector and $E = [e_{ij}]$ is a pxp diagonal matrix with e_{ii}=-T. Also, M is a pxn matrix, where p is less than or equal to m, the number of controls, and n is the number of plant states. When the closed-loop eigenvalues are within the unit circle, it is obvious that

$$\lim_{t\to\infty} [z(kT+T) - z(kT)] = 0 \tag{40}$$

Thus,

$$\lim_{t\to\infty} [TM\, x(kT) + E\, V(kT)] = 0 \tag{41}$$

and therefore the desired combination of states

$$M\, x\, (kT) \tag{42}$$

representing some of the outputs will track the command inputs with zero steady-state error. Figure 3 represents the tracking system.

Continuous Simulation

Since the plant is continuous, it is necessary to perform a continuous-time simulation of the closed-loop system in order to evaluate the effectiveness of the digital control law. This is necessary because the complete closed-loop system is partly discrete and partly continuous. Therefore the system is simulated using the continuous state space representation with piecewise constant control inputs.

In order to determine the continuous-time solution to the state space representation of the linear, time invariant system, a discrete approximation to the exact solution is used. This method is used to discretize the plant, as explained in the preceding section on sampled data transformation, with a sampling time that is less than or equal to TSAMP/12, where TSAMP is the sampling time of the digital computer used in the control law. The state and output responses are obtained from one sampling time to the next with the control input u(kT) held constant. Then the control input is updated at the next sampling time and the process is continued. Therefore, it is possible to determine the system responses between the sampling times.

IV. Development of Initial Program Guidelines

CESA, which stands for Complete EigenStructure Assignment, is an interactive, user oriented computer program that uses the multivariable control law methodology of entire eigenstructure assignment in the design of the following three different control systems: regulator, tracker, and disturbance rejector. This chapter deals with the requirements definition, program design emphasis, and the system constraints that were faced in the early development stages of the computer program. Chapter V describes the actual program structure of CESA. Appendix A consists of a User's Manual for CESA and Appendix B contains a Programmer's Manual to CESA.

Requirements Definition

In order to exploit the advantages of the structured design techniques formulated by Yordon in Ref. 7, a fundamental definition of what the computer program is expected to accomplish is needed. This fundamental definition is called a requirements definition (Ref. 7). The requirements definition for the computer program is

1. The computer package must be interactive and user oriented and allow for an iterative design process.
2. The program must be able to recover from user input errors.
3. A process must exist that allows the user to store and update all input data for some future time.
4. Entire eigenstructure assignment is the control law design methodology used in the design of regulators, trackers, and disturbance rejectors.
5. There must be a method available that discretizes the continuous plant.

6. The plant must be augmented with integrators for the tracking and disturbance rejection systems.
7. The user must have complete freedom in forming the eigenvectors from linear combinations of the basis vectors in the appropriate null space.
8. The controllability and observability of the open-loop system must be checked.
9. The ability to calculate the control indices and the observer indices of the open-loop system must be available.
10. A continuous-time simulation must be included in order to evaluate the control law.
11. A plotting package must be included so that the state response for the continuous-time simulation can be plotted.

Using the above design requirements, a structured chart was derived as shown in Fig. 4. The structured chart of Fig. 4 illustrates the modules of the program and the interconnection between the modules.

Program Design Emphasis

The manner in which a program is physically divided into modules can significantly affect the structural complexity of the resulting program. Two measures that can be used to judge the goodness of the design are coupling and cohesion. These two concepts form the basis of structured design theory.

Coupling, the measure of the strength of interconnections between one module and another, must be kept at a minimum to assure an acceptable level of independence between modules. Thus, if a program has a low level of coupling between modules, debugging or modification of one

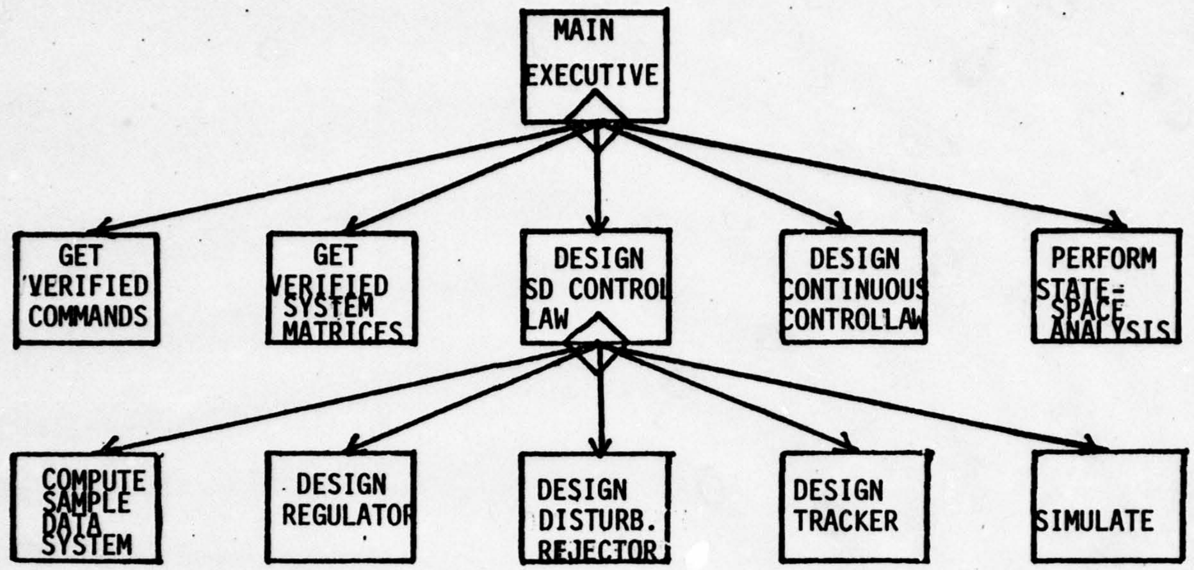


Figure 4. Structured Chart for CESA

module does not affect another module. The level of maintainability, the extent to which a system can be easily corrected when bugs are discovered in a program, and the modifiability, the ability of the system to be changed or enhanced to meet the needs of a user during a program's life time, of a program are both increased as the level of coupling is reduced (Ref 7:76-94). Some factors that influence the level of coupling are the type of connections, the complexity of the interfaces and the type of data that flows between modules. Also, the use of global data, commons, has a dramatic effect on increasing the level of coupling. In order to keep the level of coupling low, the use of global data is kept at a minimum. Also, when the use of global storage is necessary, labeled commons are used instead of blank commons, since labeled commons do not raise the level of coupling as high as blank commons.

Cohesion, the degree of functional relatedness of processing elements within a single module, also has a direct effect on the maintainability and modifiability of a program (Ref 7:95, 125). A high level of cohesion results in a high level of maintainability and modifiability. The level of cohesion is kept high within a module, by putting only functionally related processes within a module. However, when memory could be saved at the expense of a loss of cohesion, the saving of memory outweighs the loss of cohesion.

A top-down design is employed, a design strategy in which the major functions of a program are identified and their implementation expressed in terms of lower-level functions, thus stressing modularity and acceptable levels of both coupling and cohesion. However, as shown above, it was sometimes necessary to make tradeoffs in order to save memory.

Design Constraints

There are two main design constraints, one the computer language that must be used and two, the available core memory. Since the available routines from FORTRAC (Ref. 3) and coded in the FORTRAN IV computer language, FORTRAN IV is used in the coding of CESA. The interactive computer system constrains a program to operate in less than $60{,}000_8$ words of core memory. Therefore, CESA is defined to operate in less than $60{,}000_8$ words of core memory.

V. Program Structure

In order to fit CESA in the $60{,}000_8$ core of memory, it is necessary to use special techniques to save memory space. Two of the main techniques used to conserve memory are overlaying and mass storage. Also, since the program is completely interactive, the user interface is an important part of the program structure. The purpose of this chapter is to discuss how and why overlaying and mass storage are used and to explain the functions of the user interface.

Overlaying

Overlaying is a technique that divides a large program into smaller programs that fit in the memory space available. Whenever one of these smaller programs is needed, it is loaded into central memory by the executive routine. When the executive routine is finished with a smaller program it unloads it and then loads the next program needed. The executive routine is called the main overlay and the other programs are called the primary overlays.

The primary overlays are stored on a mass storage device by the system, so they take up central core only when they are loaded by the main overlay. Thus, as long as the combined memory needed by the main overlay and any primary overlay is less than or equal to $60{,}000_8$ the total program fits on the interactive system.

Blank and labeled commons are used to pass data between overlays. However, to keep coupling at a low level, as mentioned in Chapter IV, labeled commons are used instead of blank commons for communication between overlays in CESA.

A more detailed description of the overlays used in CESA is given in the Programmer's Manual located in Appendix B.

Data Base

The manner in which a program handles data plays a strong part in its efficiency and therefore is an important part of the program structure. Three different types of data storage are used in CESA: local storage, global storage, and mass storage.

The two different types of mass storage used are sequential-access files and random-access files. Each of the above mentioned storage devices have strong points and weak points as pointed out below:

1. Local storage is temporary storage that a program uses to store variables that are needed only during the execution of a particular routine. Once the routine is finished, all variables stored in local storage are lost.

2. Global storage is storage that each routine has access to. On the CDC 6600, labeled commons and blank commons perform this function. Two disadvantages of global storage are:

   a. Memory space is taken up at all times, whether the stored variables are used or not used.

   b. Global storage creates a high degree of coupling between routines as mentioned in Chapter IV.

3. Sequential-access files are mass storage devices that data may be written to or from in a sequential format. An advantage of a sequential file is that it can hold much more information than global storage while taking up the same amount of core.
   In fact, the only core needed for a sequential file is input-output buffer core (202_8-2000_8) Ref (13:16-2).
   A disadvantage of the sequential file is that, to get information stored on the file, the whole file must be read.

4. Random-access files are mass storage devices which differ from sequential files in that information is written to a random-access file in a random manner. The main disadvantage of the random file is the I/O time required to read or write from a random access file.

Local storage is used in CESA as temporary scratch registers in different routines. Although global storage takes up memory and increases the coupling between modules, it is used since reading and writing to global storage takes much less time than reading and writing to mass storage. However, since labeled commons increase coupling at a lower degree than blank commons, they are used instead of blank commons.

Three sequential-access files are used in CESA as storage devices. Character strings are stored in one sequential file as part of the input data verification process in CESA. Another sequential file serves the purpose of a backup storage device for CESA's labeled commons. The use of this file also allows the user to store all input data for some future use. This cuts down on data input time. The third sequential file used by CESA is the answer file. At the user's command, all of the program output is written to this file, which then can be disposed to the system printer after termination of the program.

Random-access files serve two main purposes in CESA. One purpose is to satisfy the need for storage of the numerous matrices that are used in the entire eigenstructure assignment algorithm. Twenty-nine two-dimensional matrices, ranging in size from 14x14 to 33x23, are stored in one random access file. The second purpose that the random-access file serves is to store the results of the simulation that must be made available to both the printing routine and the plotting routine.

User Interface

In the requirements definition in Chapter IV, it is stated that an interactive user interface is needed to perform program control, data manipulation, and error protection and recovery. The manner in which these functions are implemented is described in the following sections.

Program Control Interface. The program control interface for CESA is modeled after the program control interface that is used in TOTAL, an interactive control design package Ref. (9). The user's interface is in the form of option numbers and commands with which the user controls the program. Options are selected by the user so that certain computational functions of the program are performed. An input data verification routine determines that the input is an option number and then passes data to the main executive which then selects the proper routine to accomplish the computation selected. Commands allow the user to set mode control switches. For example, if the command ANSWER, ON is entered, all program output is written to a sequential-access file called ANSWER until the command ANSWER, OFF is entered. In the case of a command input, the input data verification routine deciphers the alphanumeric input and then sets a mode switch accordingly. Appendix B, Programmer's Manual, gives a detailed explanation of options and commands.

Data Manipulation. The data manipulation interface allows the user direct access to the global storage of CESA which contains the continuous and discrete versions of the system plant. Either of these two system discriptions may be printed out or changed upon request. In order to have one of the plant matrices printed out, the name of the matrix is typed in under the option mode. Any value stored in a plant matrix can be changed by equating that storage location to a numerical value.

The user also has access to 29 matrices that are stored in a random-access file. To determine the contents of these matrices,the user types the name of the matrix to be printed. For a listing of these matrices and more detailed information on the data manipulation interface, see Appendicies A and B.

Error Protection and Recovers

Three levels of error protection and recovery are provided by CESA. The first level verifies that all input data is of the proper format and, if it is not, requires the user to reenter data from the last correct piece of data. In the case of numeric input, the second level checks that the numeric input is within the proper range and, if it is not, the user is required to enter a proper value. Since it is impossible to preceive every possible problem that a program can encounter due to a user input error or a program error, it is impossible to completely debug a large program without it being in full use for some trial time. In order to spare the user any significant problems that may be encountered by a system abort, a CDC system recovery routine is included in the CESA computer package. Thus, if a mode error is encountered, the program recovers and no data is lost. This last level is a catch all and is not intended to be the main error protection and recovery routine. Rather it is a device that is used to prevent any unnecessary hardship to the user due to a program flaw. More information on the three levels of error protection and recovery can be found in Appendix B, the programmer's guide.

VI. Deadbeat Control Law Synthesis Using Entire Eigenstructure Assignment

In this Chapter, two deadbeat controllers, one a regulator and the other a tracker, are developed for the C-141 aircraft for three different flight conditions-landing, low altitude cruise, and medium altitude cruise. The strength of the multivariable design methodology, entire eigensturcture assignment, in the design of a deadbeat controller for a multi-input, multi-output system is exhibited by the addition of spoilers and speed control to the C-141 aircraft. All of the estimated spoiler stability derivatives and all of the longitudinal stability derivatives are taken from Ref. 4. The mathematical modeling of the C-141 aircraft is done in the body axes since all aircraft sensor signals are generated in body axes coordinates. A continuous-time simulation is performed on each of the closed-loop systems to determine the effectiveness of the deadbeat controllers.

C-141 Model

The longitudinal equations of motion used for the C-141 model are

$$\ddot{\theta} = M_{\dot{w}} \dot{w} + M_w w + M_u u + M_q \dot{\theta} + M_{\delta_e} \delta_e + M_{\delta_{sp}} \delta_{sp} \tag{43}$$

$$\dot{w} = Z_u u + Z_w w + U_o \dot{\theta} + Z_{\delta_e} \delta_e + Z_{\delta_{sp}} \delta_{sp} \tag{44}$$

$$\dot{u} = X_u u + X_w w - g\theta + X_{\delta_{sp}} \delta_{sp} + X_{\delta_e} \delta_e - W_o \dot{\theta} + X_{\delta_{rpm}} \delta_{rpm} \tag{45}$$

$$\dot{h} = -w + U_o \theta \tag{46}$$

$$U_o = V_{TO} \cos \alpha_o \tag{47}$$

$$W_o = V_{TO} \sin \alpha_o \tag{48}$$

All of the stability derivatives in the equations are with respect to the body axes. Since Ref. 4 supplied only the spoiler stability derivatives with respect to the stability axes, the following equations from Ref. 14 are used to compute the spoiler stability axes derivatives with respect to the body axes

$$(X_{\delta_{sp}})_b = X_{\delta_{sp}} \cos \alpha_o - Z_{\delta_{sp}} \sin \alpha_o \tag{49}$$

$$(Z_{\delta_{sp}})_b = Z_{\delta_{sp}} \cos \alpha_o + X_{\delta_{sp}} \sin \alpha_o \tag{50}$$

$$(M_{\delta_{sp}})_b = M_{\delta_{sp}} \tag{51}$$

The transfer functions for the elevator and spoiler actuator-servo combinations are modeled as

$$\frac{\delta_e}{e_i} = \frac{10}{s+10} \tag{52}$$

$$\frac{\delta_{sp}}{e_{sp}} = \frac{4}{s+4}$$

and the transfer function representing engine lag is

$$\frac{\delta_{rpm}}{\delta T} = \frac{2}{s+2} \tag{53}$$

In order to express the dynamics of the C-141 aircraft in state variable form, the following state equations are used

$$x(t) = A\, x(t) + B\, u(t) \tag{54}$$

where

$$
x(t) \begin{bmatrix} \dot{\theta}(t) \\ w(t) \\ u(t) \\ \theta(t) \\ \delta_e(t) \\ \delta_{sp}(t) \\ \delta_{rpm}(t) \end{bmatrix}, \quad
B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 10 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad
u(t) = \begin{bmatrix} e_i \\ e_{sp} \\ \delta T \end{bmatrix}
$$

$$
A = \begin{bmatrix}
(M_{\dot{w}} \cdot U_o) + M_q, & (M_{\dot{w}} \cdot Z_w) + M_w, & (M_{\dot{w}} \cdot Z_u) + M_u, & 0 & ,(M_{\dot{w}} \cdot Z_{\delta_e}) + M_{\delta e}, & (M_{\dot{w}} \cdot Z_{\delta sp}) + M_{\delta sp}, & 0 \\
U_o & , Z_w & , Z_u & , 0 & , Z_{\delta e} & , Z_{\delta sp} & , 0 \\
-W_o & , X_w & , X_u & , -g & , X_{\delta e} & , X_{\delta sp} & , X_{\delta rpm} \\
1 & , 0 & , 0 & , 0 & , 0 & , 0 & , 0 \\
0 & , 0 & , 0 & , 0 & , -10 & , 0 & , 0 \\
0 & , 0 & , 0 & , 0 & , 0 & , -4 & , 0 \\
0 & , 0 & , 0 & , 0 & , 0 & , 0 & , -2
\end{bmatrix}
$$

The above system is discretized with a sampling time of 2 seconds, which is selected according to the approach mentioned in Chapter II, and the discretized system is described by

$$x(2k+2) = F\,x(2k) + G\,u(2k) \tag{56}$$

All of the coefficients that are used in the modeling of the C-141 aircraft for the three different flight conditions are given in Table II (Ref 4:178, 197).

Regulator Design

A deadbeat regulator is designed for the longitudinal axes of the C-141 aircraft for three different flight conditions - landing, low altitude cruise, and medium altitude cruise. In order for the system to exhibit a ripple-free response in a finite number of sampling periods, all of the eigenvalues of the discretized closed-loop system are assigned to the origin. The sampling time used in all cases is 2 seconds.

Landing

A regulator is designed for the landing flight condition. Substituting the values of the stability derivatives into the state equations, Eqs. 54 and 55, the continuous plant matrix becomes

TABLE II

C-141 Longitudinal Dimensional Derivatives in Body Axes

PARAMETER	LANDING	LOW ALT CRUISE	MED ALT CRUISE	PARAMETER	LANDING	LOW ALT CRUISE	MED ALT CRUISE
$M_{\dot{w}}$	-.00108	-.000498	-.000279	X_w	.141	.0296	.0314
M_w	-.00567	-.008117	-.00608	X_u	-.0234	-.00917	-.00545
M_u	-.000159	-.000308	.000415	X_{δ_e}	.554	-.247	.133
M_{δ_e}	-.710	-3.97	-3.51	V_{TO}	200	545	660
$M_{\delta_{sp}}$	.240	1.04	.791	V_o	199.9	544.98	659.9
Z_w	-.260	-.118	-.0591	g	32.18	32.18	32.18
Z_{δ_e}	-5.18	-30.8	-27.19	α_o	6.1°	- .460°	.280°
$Z_{\delta_{sp}}$	32.4	13.521	10.29	X_T	-7.16	-2.6	-2.11

$$A = \begin{bmatrix} -.8797 & -.446E{-2} & .439E{-3} & 0 & .7093 & .2051 & 0 \\ 199.9 & -.5623 & -.2602 & 0 & -5.176 & 32.36 & 0 \\ -21.37 & .1408 & -.0234 & -32.19 & .5535 & -7.735 & -7.162 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{bmatrix} \tag{57}$$

and the difference equation for a sampling time of 2 seconds is

$$x(2k+2) = F\,x(2k) + G\,u(2k) \tag{58}$$

where

$$F = \begin{bmatrix} -.1224 & -.0009 & ..0010 & -.0350 & .0081 & -.0148 & -.0028 \\ 52.01 & -.06157 & -.1399 & 7.7960 & -4.010 & 3.615 & .5091 \\ -24.79 & .2471 & .8741 & -61.10 & 1.623 & -1.097 & -3.210 \\ .5935 & -.0028 & .0011 & .9759 & -.0414 & .0096 & -.2492 \\ 0 & 0 & 0 & 0 & .2061E{-08} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .0003 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .0183 \end{bmatrix} \tag{59}$$

$$G = \begin{bmatrix} -.4141, & .0386, & -.005 \\ -93.66, & 50.59, & 1.226 \\ 18.52, & -10.94, & -10.39 \\ -.6352, & -.0996, & -.0029 \\ 1, & 0, & 0 \\ 0, & .9997, & 0 \\ 0, & 0, & .9817 \end{bmatrix} \tag{60}$$

All of the eigenvalues of the closed-loop system are assigned to the origin so that the flight control system exhibits a deadbeat response. The control indices of the matrix pair (F,G) are the set of integers {3,2,2}. In order for the closed-loop system to achieve a ripple-free response in three sampling periods, three eigenvector/generalized eigenvector chains of lengths three, two, and two are generated. Thus, using the algorithm from Ref. 2

$$\ker S(0) = \ker[F,G] = \text{span}\left\{ \begin{bmatrix} .0136 \\ 1.0 \\ 0 \\ -.0034 \\ 0 \\ -.1871 \\ .1056 \\ 0 \\ .00006 \\ -.0020 \end{bmatrix}, \begin{bmatrix} -.0018 \\ 0 \\ 1.0 \\ .0005 \\ 0 \\ .0257 \\ .2863 \\ 0 \\ -.8617E-05 \\ -.0053 \end{bmatrix}, \begin{bmatrix} .0922 \\ 0 \\ 0 \\ -.0114 \\ 1 \\ -.2054 \\ .0858 \\ 0 \\ .00007 \\ -.0016 \end{bmatrix} \right\} \tag{61}$$

The vectors selected from the null space of S(0) are

$$\begin{bmatrix} \chi_o^{(1,1)} \\ \omega_o^{(1,1)} \end{bmatrix} = \begin{bmatrix} .0136 \\ 1.0 \\ 0 \\ -.0034 \\ 0 \\ -.1871 \\ .1056 \\ 0 \\ .00006 \\ -.0020 \end{bmatrix} , \quad (62)$$

$$\begin{bmatrix} \chi_o^{(1,2)} \\ \omega_o^{(1,2)} \end{bmatrix} = \begin{bmatrix} -.0018 \\ 0 \\ 1.0 \\ .0005 \\ 0 \\ .0257 \\ .2863 \\ 0 \\ -.8617E-05 \\ -.0053 \end{bmatrix} , \quad (63)$$

and

$$\begin{bmatrix} \chi_o(1,3) \\ \omega_o(1,3) \end{bmatrix} = \begin{bmatrix} .0922 \\ 0 \\ 0 \\ -.0114 \\ 1 \\ -.2054 \\ .0858 \\ 0 \\ .00007 \\ -.0016 \end{bmatrix} , \tag{64}$$

The first set of generalized vectors, generated by using the method described in Chapter III from Ref.2, are

$$\begin{bmatrix} \chi_o(2,1) \\ \omega_o(2,1) \end{bmatrix} = \begin{bmatrix} .7280 \\ 1.0 \\ 0 \\ -.3407 \\ 0 \\ -7.438 \\ 4.004 \\ 0 \\ -.1846 \\ .0328 \end{bmatrix} \tag{65}$$

$$\begin{bmatrix} \chi_o^{(2,2)} \\ \omega_o^{(2,2)} \end{bmatrix} = \begin{bmatrix} -.1045 \\ 0 \\ 1.0 \\ .0451 \\ 0 \\ 1.212 \\ -1.634 \\ 0 \\ .0253 \\ .3221 \end{bmatrix} , \qquad (66)$$

and

$$\begin{bmatrix} \chi_o(2,3) \\ \omega_o(2,3) \end{bmatrix} = \begin{bmatrix} 10.60 \\ 0 \\ 0 \\ -4.421 \\ 1.0 \\ -120.9 \\ 53.36 \\ 1.0 \\ -.1649 \\ -.9081 \end{bmatrix} , \qquad (67)$$

Continuing the chain for the last generalized vector, yields the vector

$$\begin{bmatrix} \chi_o^{(3,1)} \\ \\ \omega_o^{(3,1)} \end{bmatrix} = \begin{bmatrix} 28.76 \\ 1.0 \\ 0 \\ -13.65 \\ 0 \\ -304.7 \\ 163.7 \\ 0 \\ -7.338 \\ 1.024 \end{bmatrix} \qquad (68)$$

Using Eq. 30, the resulting state-feedback matrix is

$$K = [\,\omega_o^{(1,1)}, \omega_o^{(2,1)}, \omega_o^{(3,1)}, \omega_o^{(1,2)}, \omega_o^{(2,2)}, \omega_o^{(1,3)}, \omega_o^{(2,3)}\,]$$

$$x\ [\,\chi_o^{(1,1)}, \chi_o^{(2,1)}, \chi_o^{(3,1)}, \chi_o^{(1,2)}, \chi_o^{(2,2)}, \chi_o^{(1,3)}, \chi_o^{(2,3)}\,]^{-1} \qquad (69)$$

$$= \begin{bmatrix} .6049, -.0049, .0023, 1.471, -.0409, -.0117, -.0054 \\ 1.151, -.0089, .0051, 3.607, -.0712, -.0360, -.0130 \\ -2.008, .0197, .0623, -5.793, .1254, -.0592, -.2344 \end{bmatrix} \qquad (70)$$

and the resulting closed-loop matrix is

F + GK =

$$\begin{bmatrix} -.3185, & .0007, & .0001, & -.4762, & .0217, & -.0111, & .00007 \\ 51.12, & -.0241, & -.0187, & 45.41, & -3.621, & 2.821, & .0662 \\ -5.310, & .0484, & .2133, & -13.13, & .3398, & -.3054, & -.7307 \\ .3298, & -.0006, & -.00002, & .4176, & -.0229, & .0137, & .0003 \\ .6049, & -.0049, & .0022, & 1.471, & -.0409, & -.0117, & -.0054 \\ 1.150, & -.0089, & .0051, & 3.606, & -.0711, & -.0356, & -.0130 \\ -1.972, & .0193, & .06114, & -5.687, & .1232, & -.0581, & -.2118 \end{bmatrix} \quad (71)$$

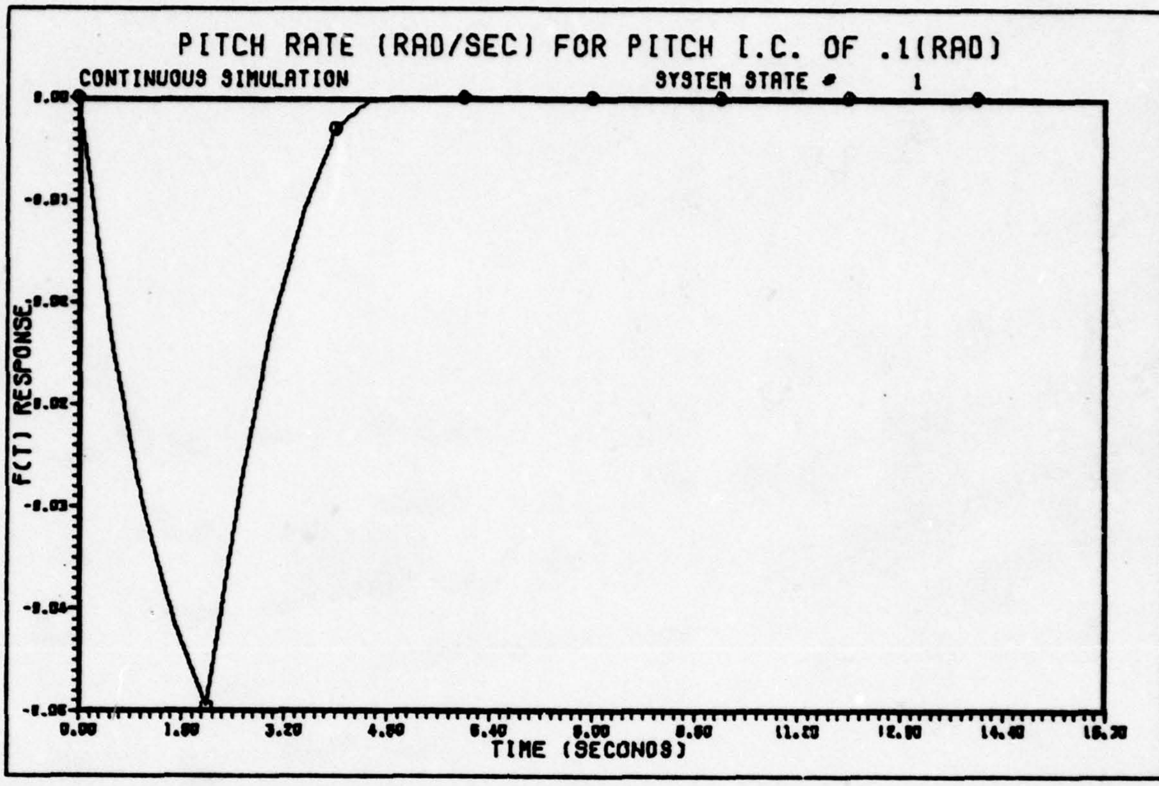
All of the eigenvalues of the closed-loop system are located at the origin and the size of the minimal polynomial is three as required by the assignment of the associated eigenvector/generalized eigenvector chains from Eq. 62 through Eq. 68. A continuous-time simulation of the closed-loop system is performed and responses are plotted in Fig. 5 for the initial condition

$$X(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (72)$$

which represents a 0.1 radian (5.72 degrees) initial condition for the pitch angle. The maximum change of angle of attack is 0.0233 radian (1.34 deg) and the maximum change in velocity along the body vertical axis is -2.031 ft/sec. The maximum control values are a 0.1471 radian (8.4 degrees) elevator deflection, a 0.3541 radian (20.7 degrees) spoiler deflection and a -0.5687 RPM change for the engines. As shown in Fig. 5, the closed-loop system achieves a ripple-free steady-state response in three sampling periods. Thus, the C-141 aircraft exhibits good transient responses and the ability to return to an equilibrium state in a finite length of time.

<u>Low Altitude Cruise</u>

A regulator is designed in this case for the low altitude cruise condition. Equation C-2 gives the continuous plant matrix formed from the derivatives from Table II and Eqs. 54 and 55 and the difference equation for a sampling time of 2 seconds takes the form of Eq. 56. The discretized system matrices are given in Appendix C in Eq. C-3. The control invariants for this system are the set of integers {3,2,2} and the set of choosen vector/generalized vector chains are given by


PITCH RATE (RAD/SEC) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 1
F(T) RESPONSE
TIME (SECONDS)

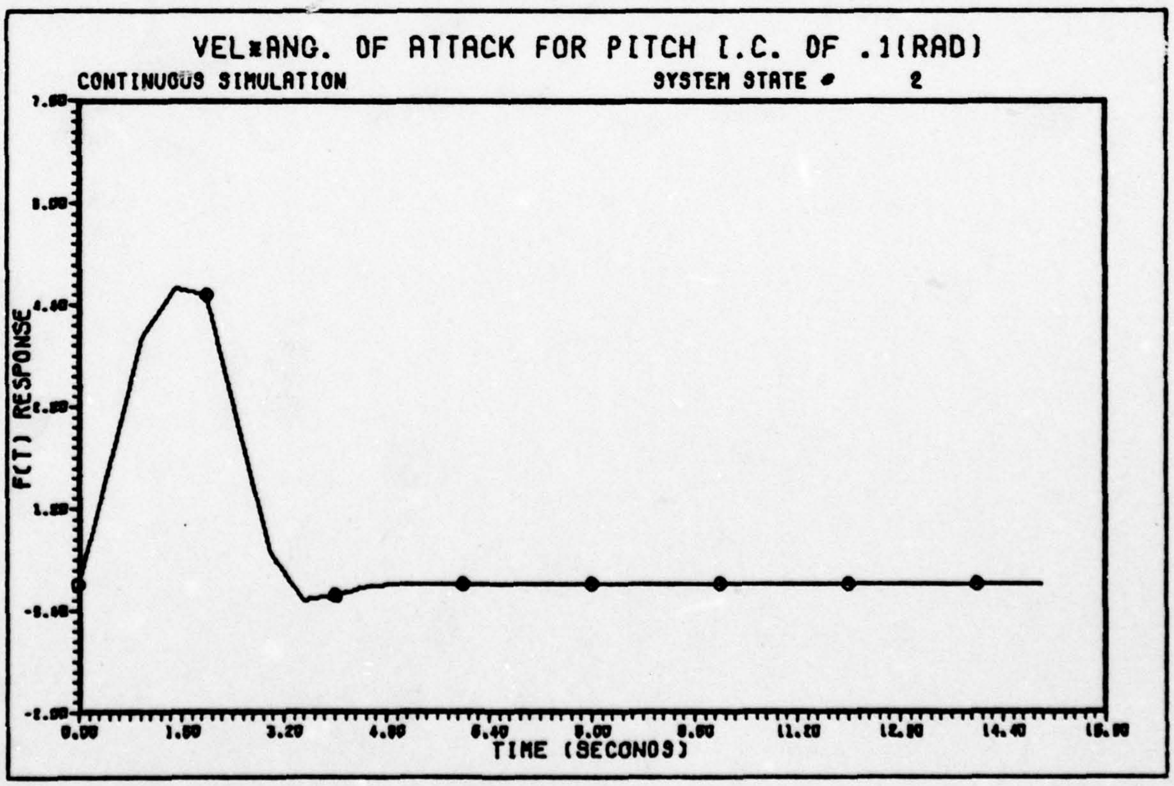
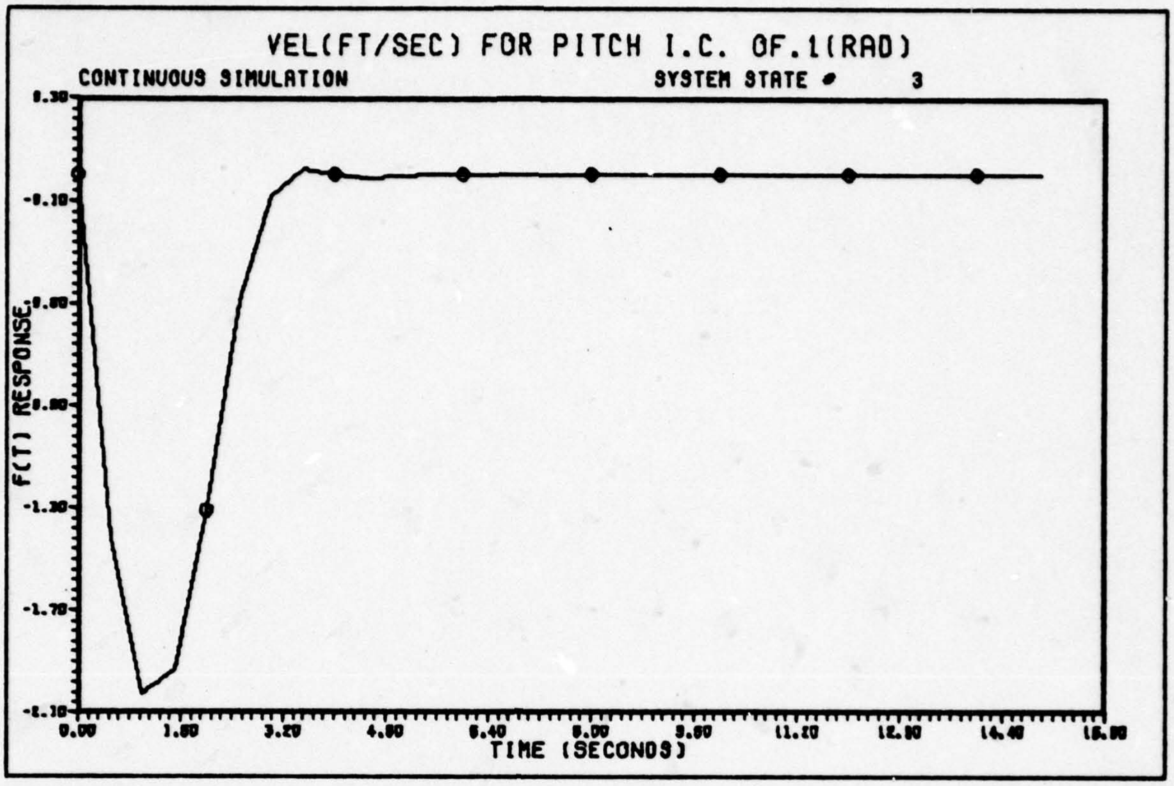
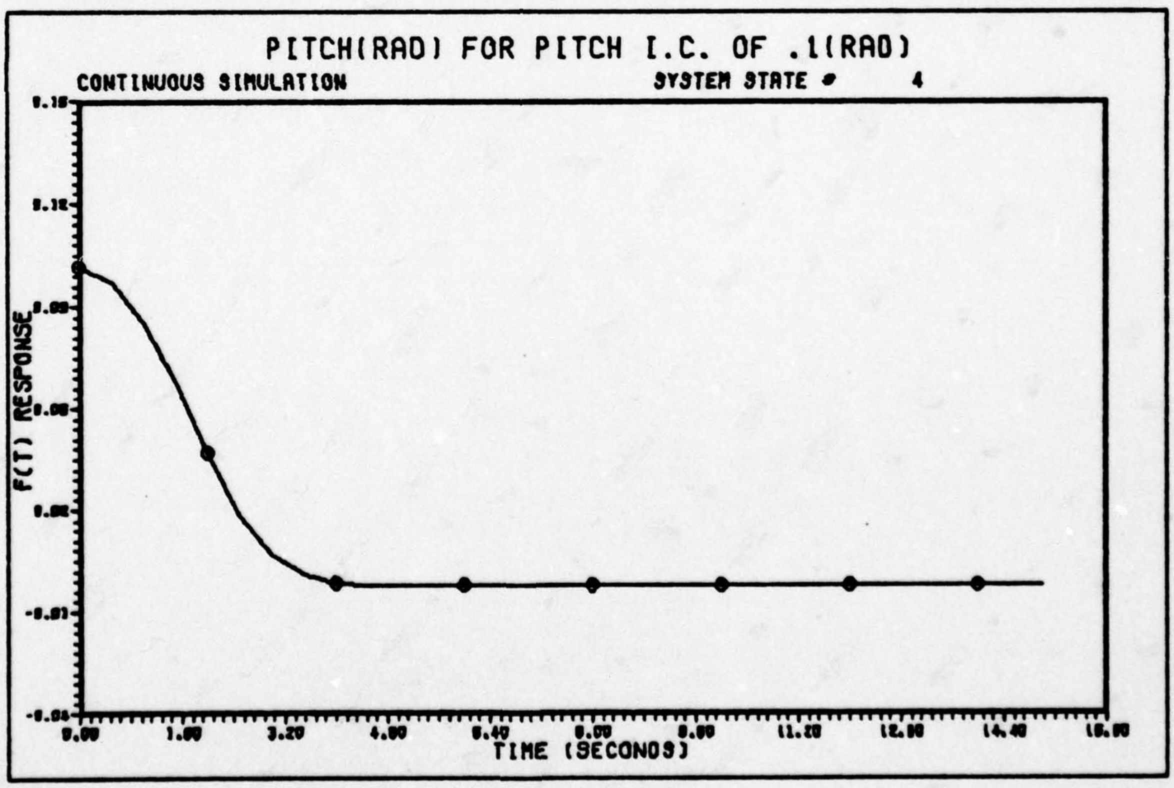

VEL*ANG. OF ATTACK FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 2
F(T) RESPONSE
TIME (SECONDS)

Figure 5a

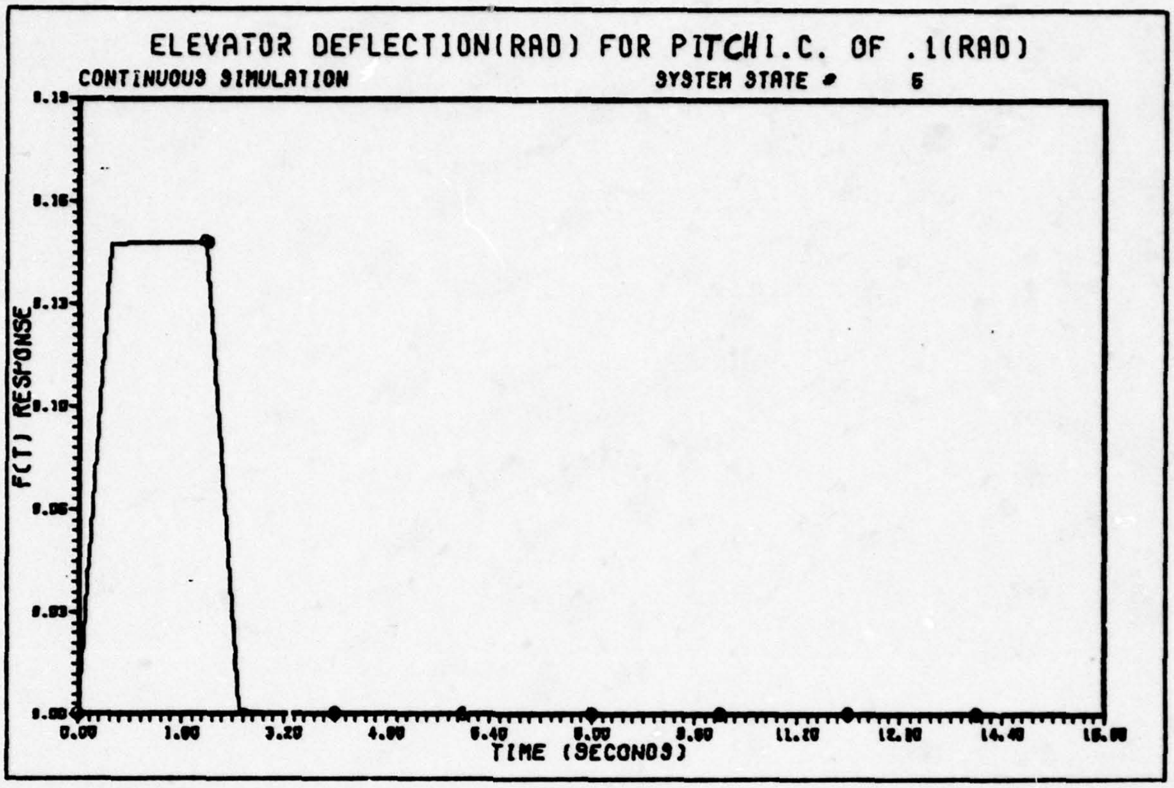


VEL(FT/SEC) FOR PITCH I.C. OF.1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 3
F(T) RESPONSE
0.30
-0.10
-1.30
-1.70
-2.10
0.00
1.60
3.20
4.80
6.40
8.00
9.60
11.20
12.80
14.40
16.00
TIME (SECONDS)



PITCH(RAD) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 4
F(T) RESPONSE
0.15
0.12
-0.04
0.00
1.60
3.20
4.80
6.40
8.00
9.60
11.20
12.80
14.40
16.00
TIME (SECONDS)

Figure 5b


ELEVATOR DEFLECTION(RAD) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 5
F(T) RESPONSE
TIME (SECONDS)

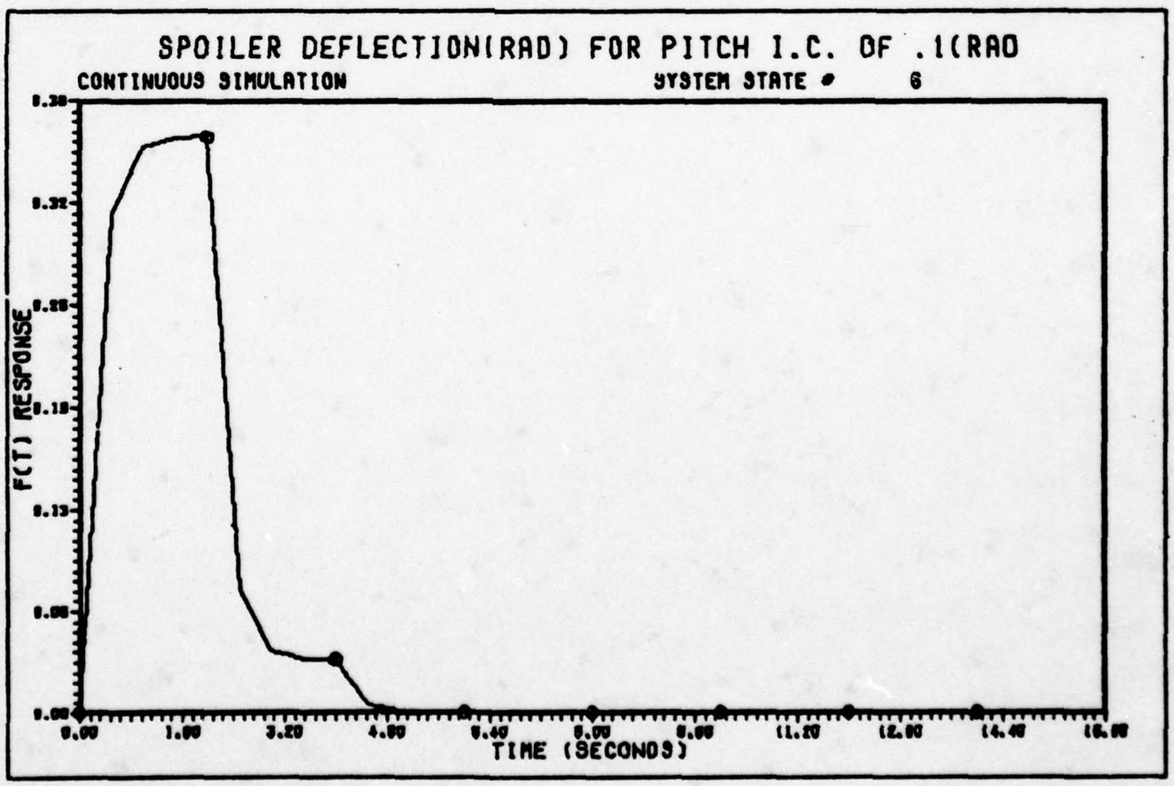

SPOILER DEFLECTION(RAD) FOR PITCH I.C. OF .1(RAD
CONTINUOUS SIMULATION
SYSTEM STATE # 6
F(T) RESPONSE
TIME (SECONDS)

Figure 5c

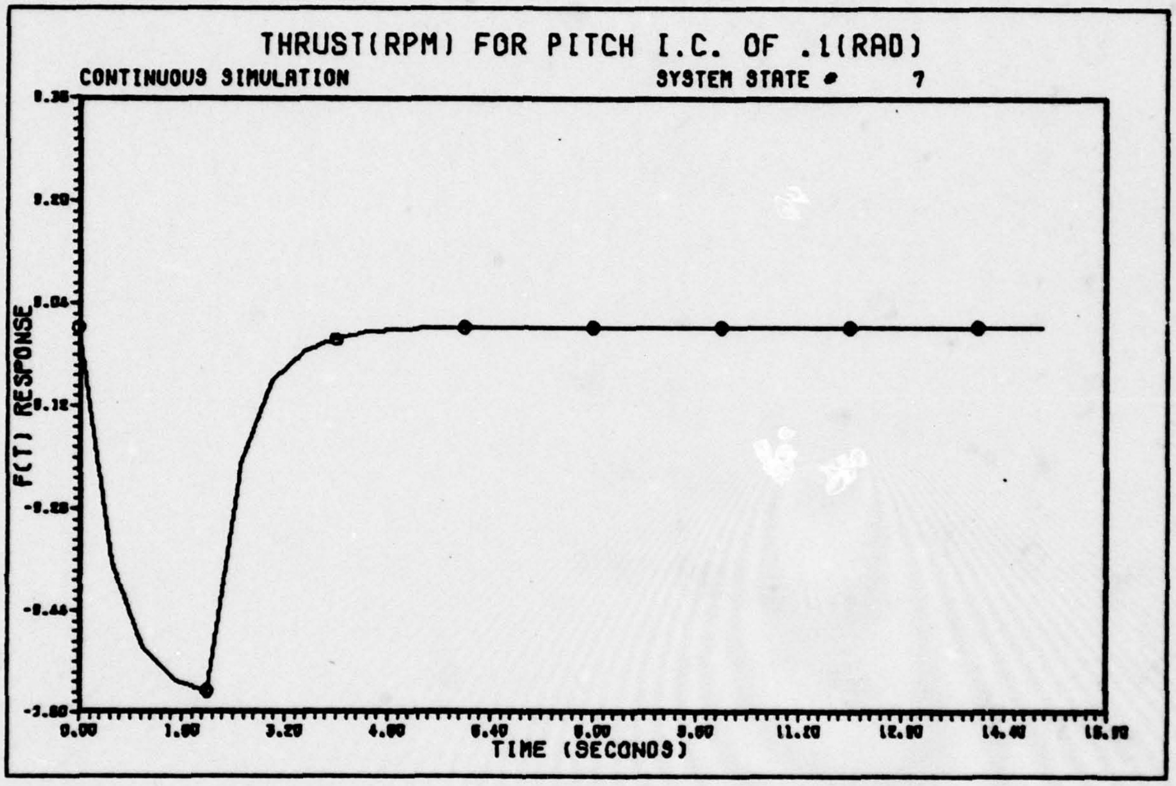

THRUST(RPM) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 7
F(T) RESPONSE
TIME (SECONDS)
3.20
4.80
6.40
8.00
11.20
14.40

Figure 5d

$$
\begin{bmatrix} \chi_i \\ \omega_i \end{bmatrix} =
\left[\begin{array}{ccccccc}
-.1200 & 10.74 & -.2334 & 20.19 & -.2136 & 18.13 & 174.4 \\
-.800 & -.8000 & -.1 & -.1 & .655 & .655 & .655 \\
1.100 & 1.100 & .75 & .75 & .6 & .6 & .6 \\
.269E\text{-}2 & -2.556 & .0072 & -4.84 & .0076 & -4.365 & 22.06 \\
-.4000 & -.400 & -.75 & -.75 & -.6725 & -.6725 & -.6725 \\
-.2118 & .1836 & -.3693 & .9959 & -.3162 & 1.252 & -1290 \\
1.038 & 22.62 & .8297 & 46.40 & .6692 & 41.99 & 175.8 \\
\hline
0 & -.4 & 0 & -.75 & 0 & -.6725 & -.6725 \\
.711E\text{-}4 & .2120 & .123E\text{-}3 & -.3697 & .106E\text{-}3 & -.3167 & 1.685 \\
-.0194 & .6349 & -.0155 & -.0206 & -.0125 & -.1017 & 39.49
\end{array}\right] \tag{73}
$$

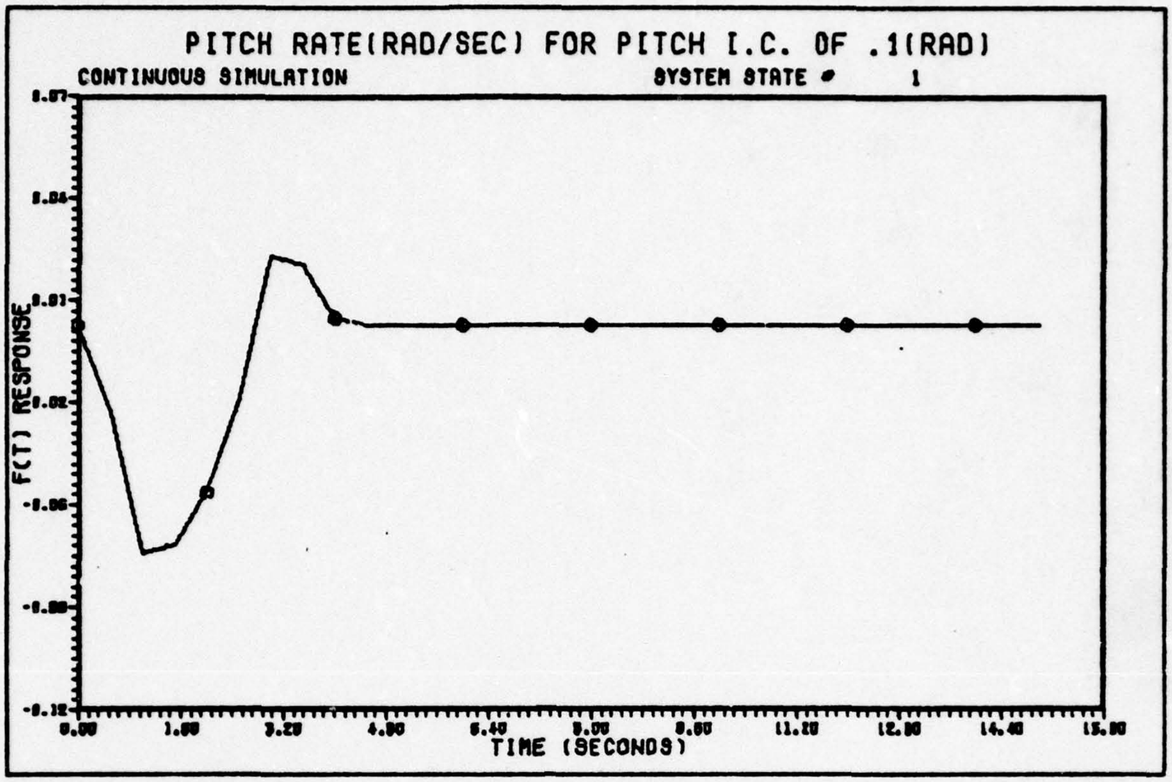
The resulting feedback gain matrix is

$$
K = \left[\omega_o^{(1,1)}, \omega_o^{(2,1)}, \omega_o^{(1,2)}, \omega_o^{(2,2)}, \omega_o^{(1,3)}, \omega_o^{(2,3)}, \omega_o^{(3,3)}\right]
$$
$$
\times \left[\chi^{(1,1)}, \chi^{(2,1)}, \chi^{(1,2)}, \chi^{(2,2)}, \chi^{(1,3)}, \chi^{(2,3)}, \chi^{(3,3)}\right]^{-1} \tag{74}
$$

$$
= \begin{bmatrix}
-.2686 & .952E\text{-}3 & -.249E\text{-}3 & -.9896 & 1.0 & -.0527 & .276E\text{-}3 \\
-1.541 & .701E\text{-}2 & -.136E\text{-}2 & -6.494 & .5740 & .3208 & .1408E\text{-}2 \\
-.4302 & .301E\text{-}2 & .1942 & -4.248 & .1238 & .1959 & -.2532
\end{bmatrix} \tag{75}
$$

With the resulting closed-loop matrix found in Appendix C, Eq. C-8.

The continuous simulation uses the same initial conditions as given in Eq. 72. The results as depicted in Fig. 6 show that the closed-loop system reaches a steady-state ripple-free response in three sampling periods (6 seconds)


PITCH RATE(RAD/SEC) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 1
F(T) RESPONSE
TIME (SECONDS)

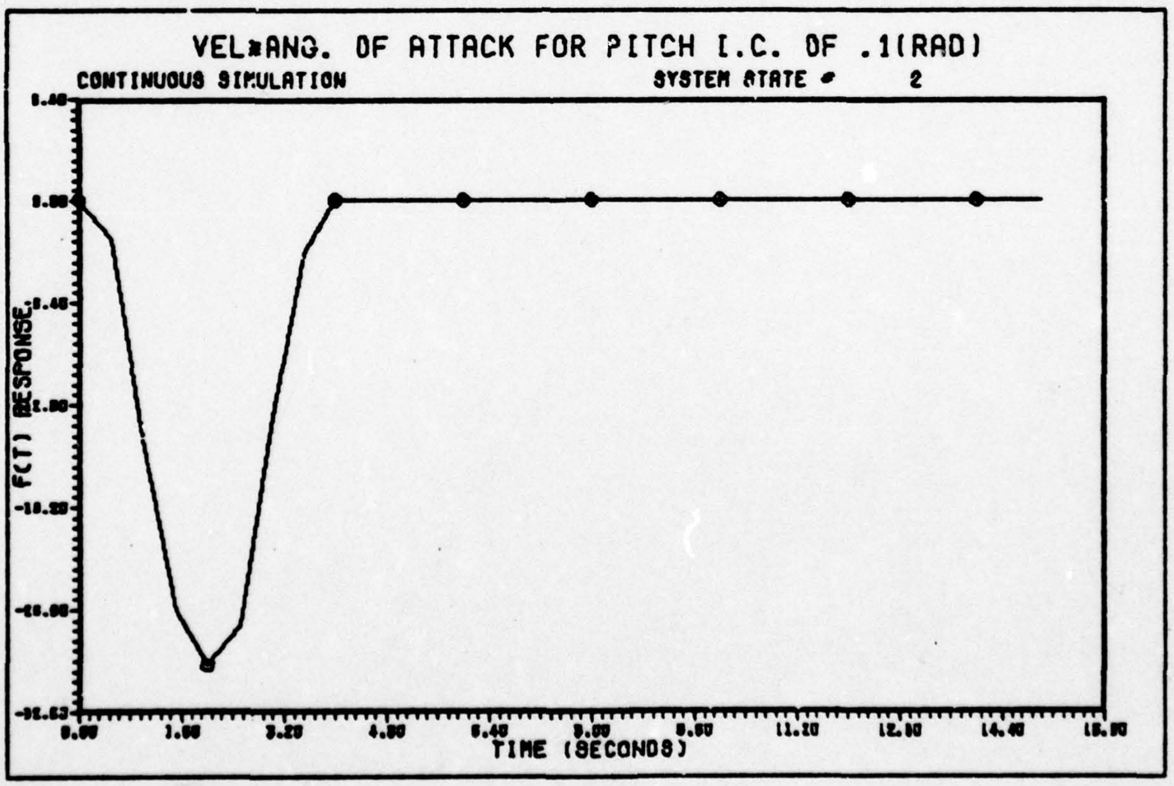
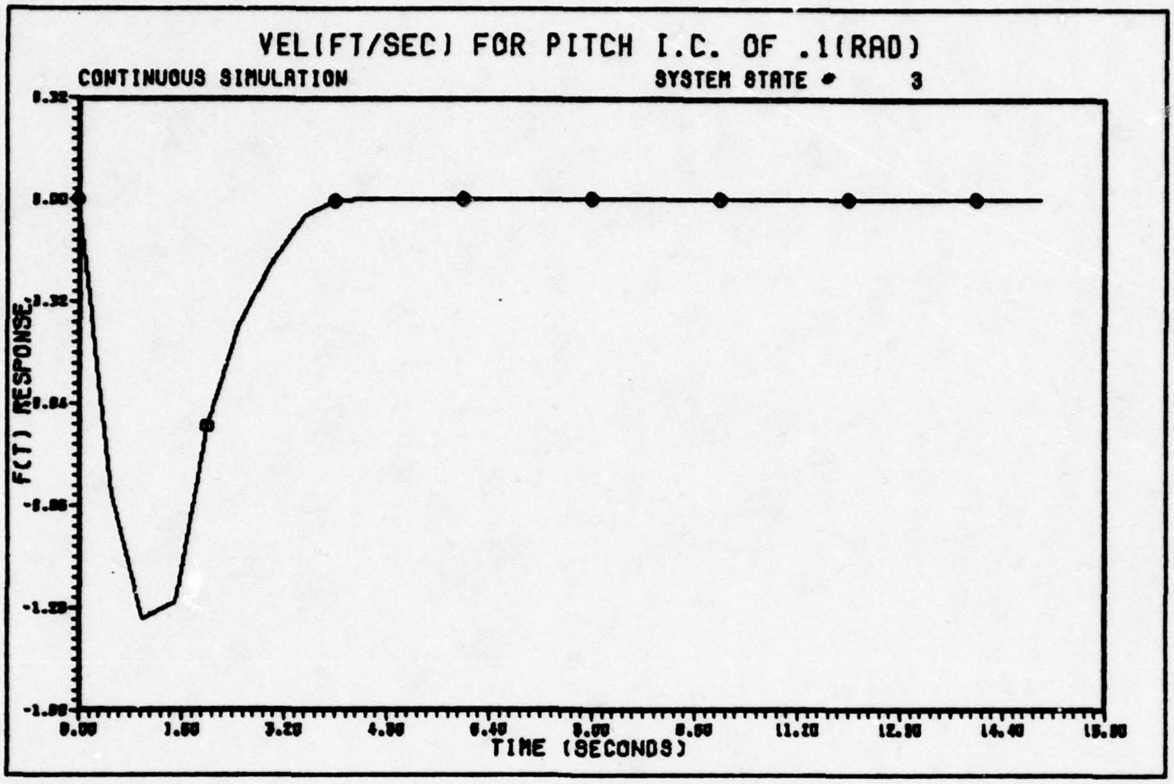

VEL*ANG. OF ATTACK FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 2
F(T) RESPONSE
TIME (SECONDS)

Figure 6a


VEL(FT/SEC) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 3
F(T) RESPONSE
TIME (SECONDS)

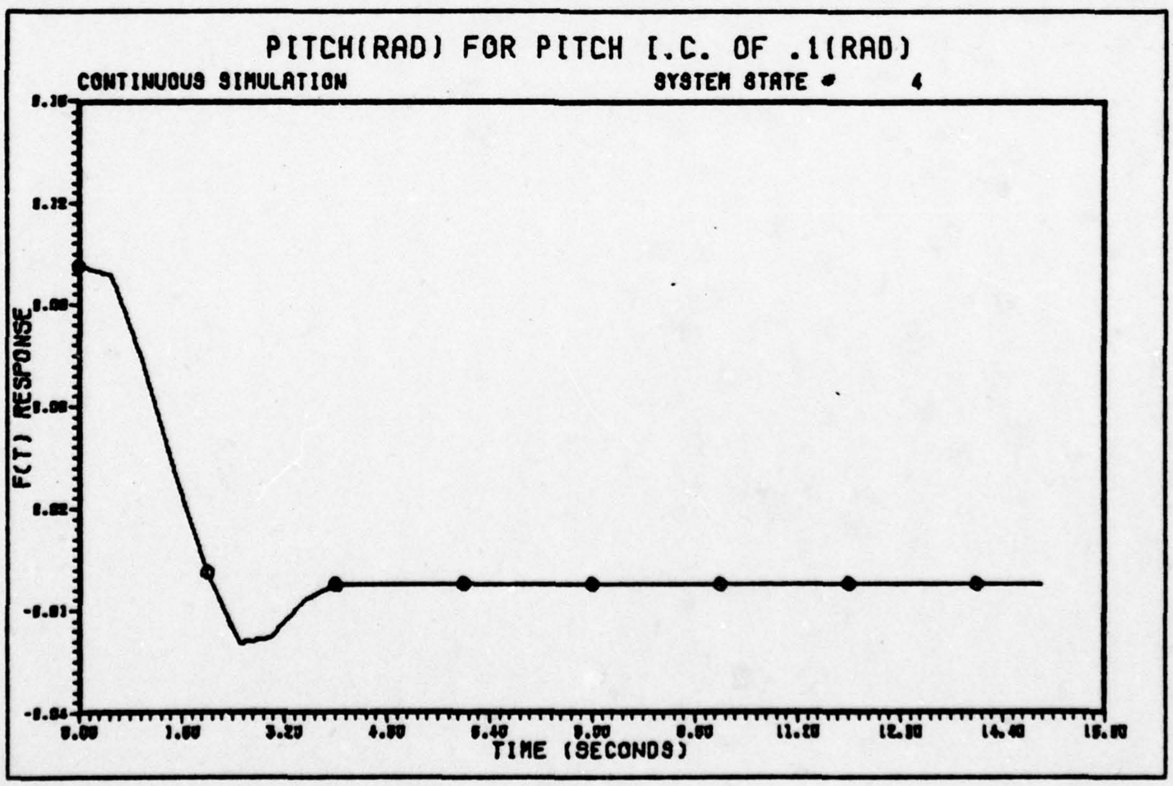
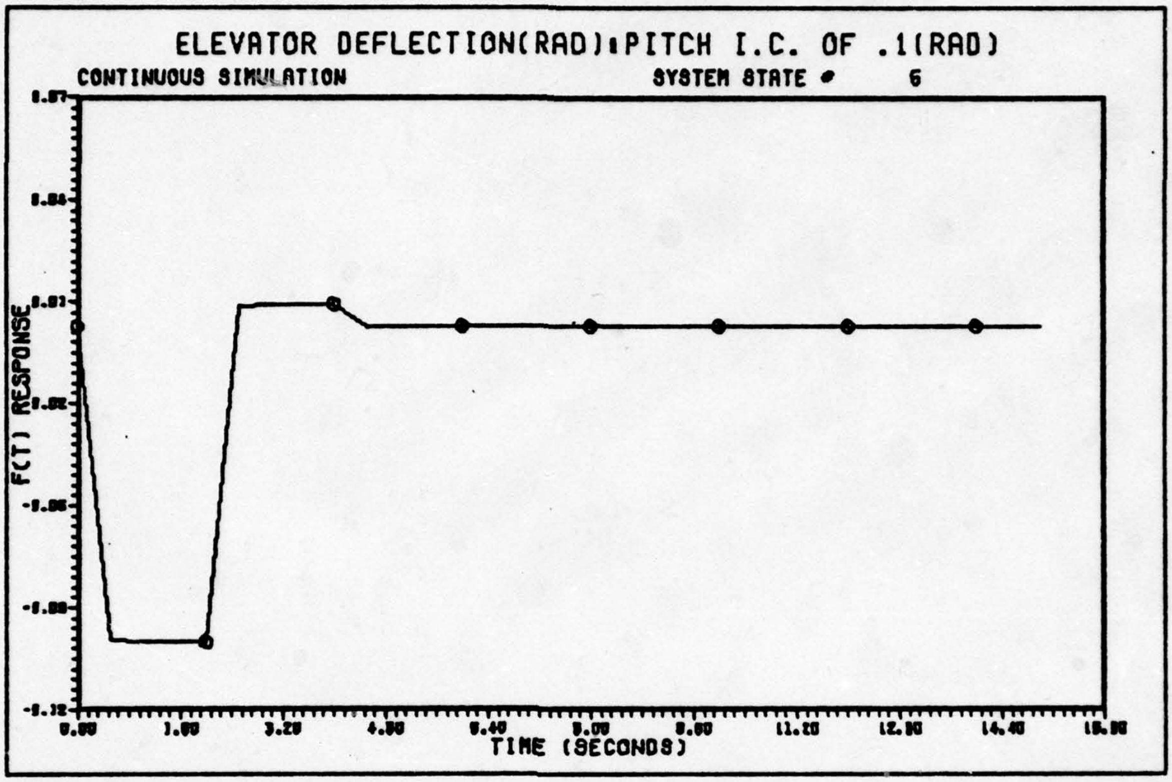

PITCH(RAD) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 4
F(T) RESPONSE
TIME (SECONDS)

Figure 6b


ELEVATOR DEFLECTION(RAD):PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 6
F(T) RESPONSE
TIME (SECONDS)
1.60
3.20
4.80
11.20
12.80
14.40

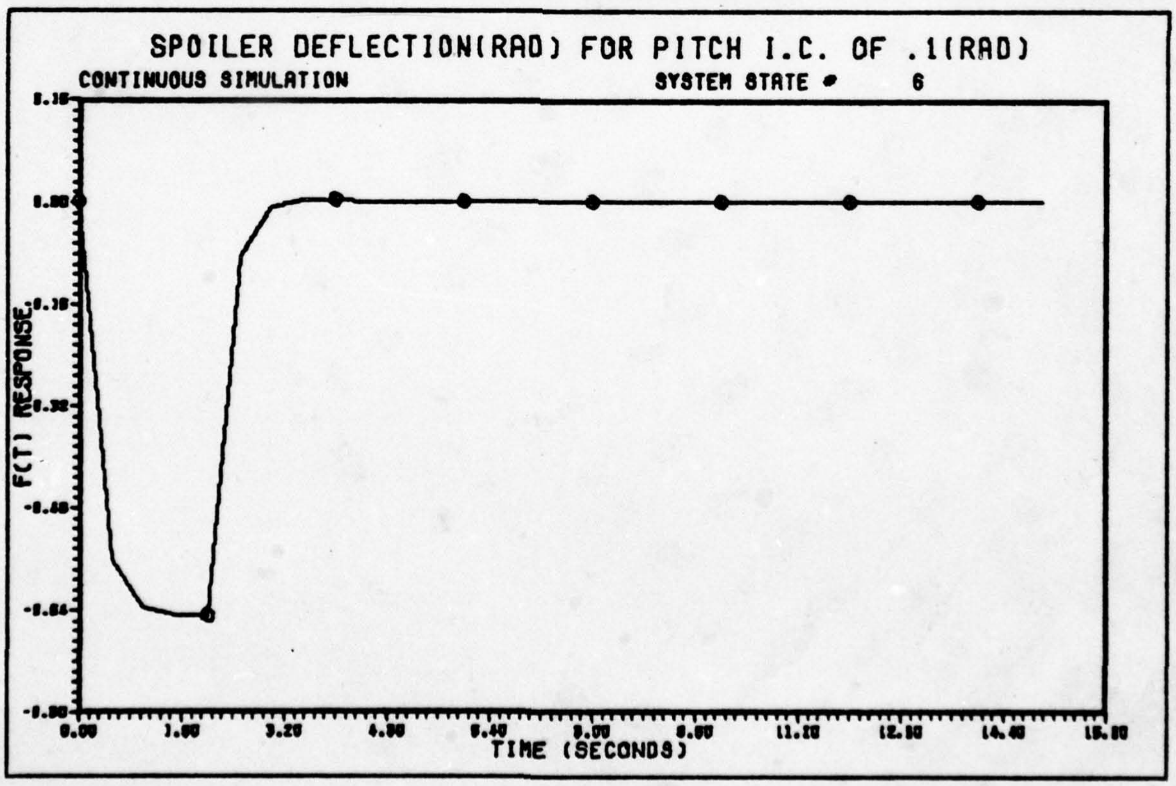
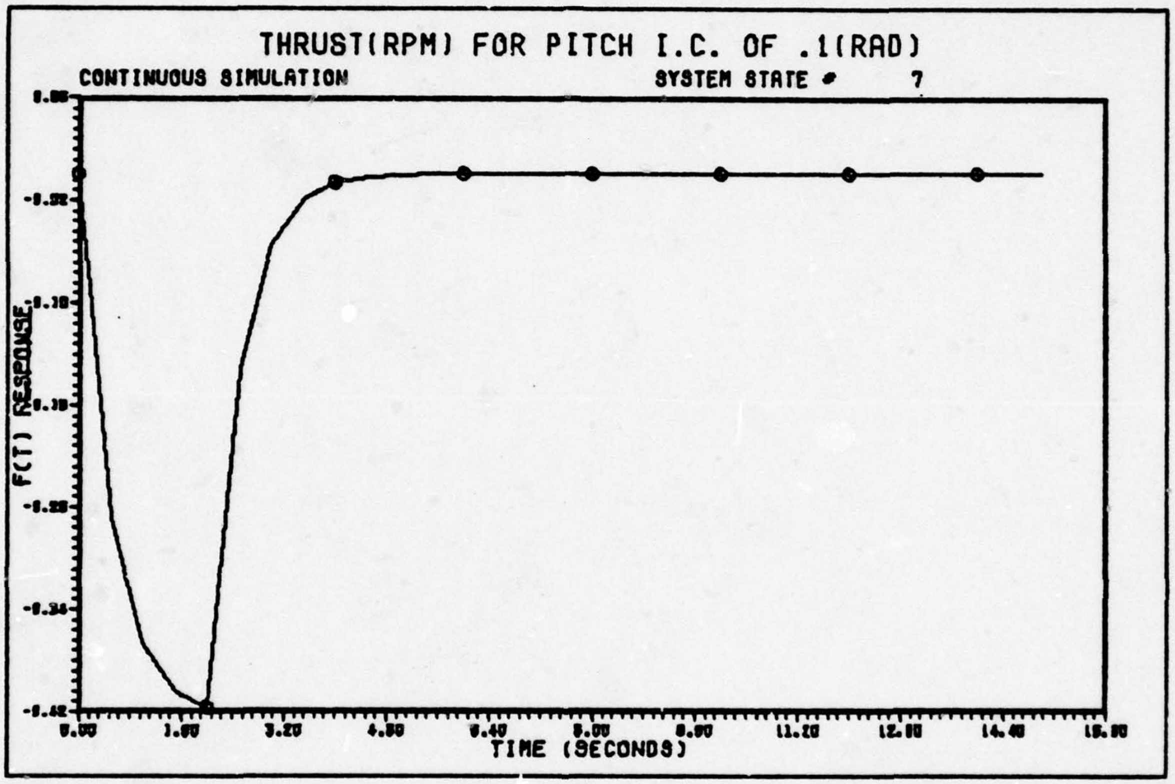

SPOILER DEFLECTION(RAD) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 6
F(T) RESPONSE
TIME (SECONDS)
1.60
3.20
4.80
11.20
12.80
14.40

Figure 6c



THRUST(RPM) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 7
F(T) RESPONSE
TIME (SECONDS)
1.60
3.20
4.80
11.20
14.40

Figure 6d

with a maximum elevator deflection of -.0989 radian (-5.67 degrees) and a maximum spoiler deflection of -.6492 radian (-37.2 degrees). The maximum change in engine RPM is -0.417 RPM. Thus, the closed-loop system exhibits a good transient response with the ability to bring the aircraft back to an equilibrium condition with acceptable control surface deflections and engine speed changes.

Medium Altitude Cruise

The stability derivatives for the medium altitude cruise condition from Table II and Eqs. 54 and 55 are used to set up the state equations for the aircraft. The resulting continuous plant matrix is given in Appendix C, Eq. C-4. This system is discretized with a sampling time of 2 seconds, resulting in the discrete state equation, Eq 56. The discretized system matrices are given in Appendix C, Eq. C-5. The control indices for this system are the set of integers {3,2,2}. Below is the matrix representing the vector/generalized vector chains generated from the original null space of the matrix, S(0).

$$
\begin{bmatrix} X_o \\ \\ \omega_o \end{bmatrix} =
\left[\begin{array}{ccccccc}
-.533E-2, & .0615, & .285E-3, & -.117E-2, & .2516, & -16.58, & 69.16 \\
1.0, & 1.0, & 0, & 0, & 0, & 0, & 0 \\
.0125, & .804E-2, & .9726, & 5.182, & .3675, & 30.18, & 275.9 \\
.133E-2, & -.0176, & -.528E-4, & .128E-5, & -.571E-2, & 2.759, & -29.07 \\
0, & 0, & 0, & 0, & 1, & 1, & 1 \\
.0264, & .1058, & -.172E-2, & -.0118, & .6424, & 30.89, & 451.7 \\
0, & 0, & 1, & 1, & 0, & 0, & 0 \\
\hline
0, & 0, & 0, & 0, & -.206E-8, & 1, & 1 \\
-.887E-5, & .0264, & .601E-6, & -.179E-2, & -.215E-3, & .6323, & 30.69 \\
0 & 0 & -.0187, & 1, & 0, & 0, & 0
\end{array}\right] \quad (76)
$$

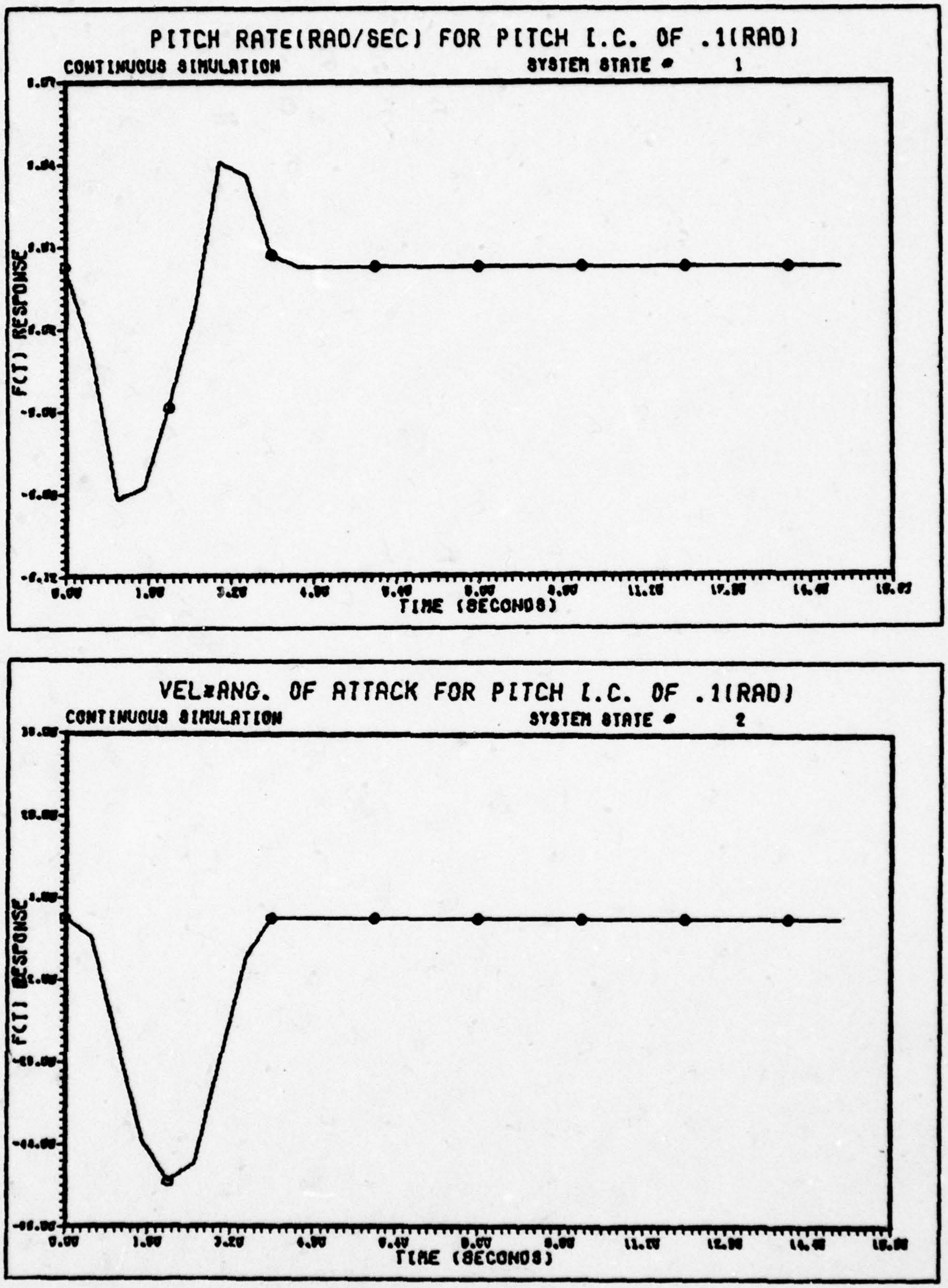
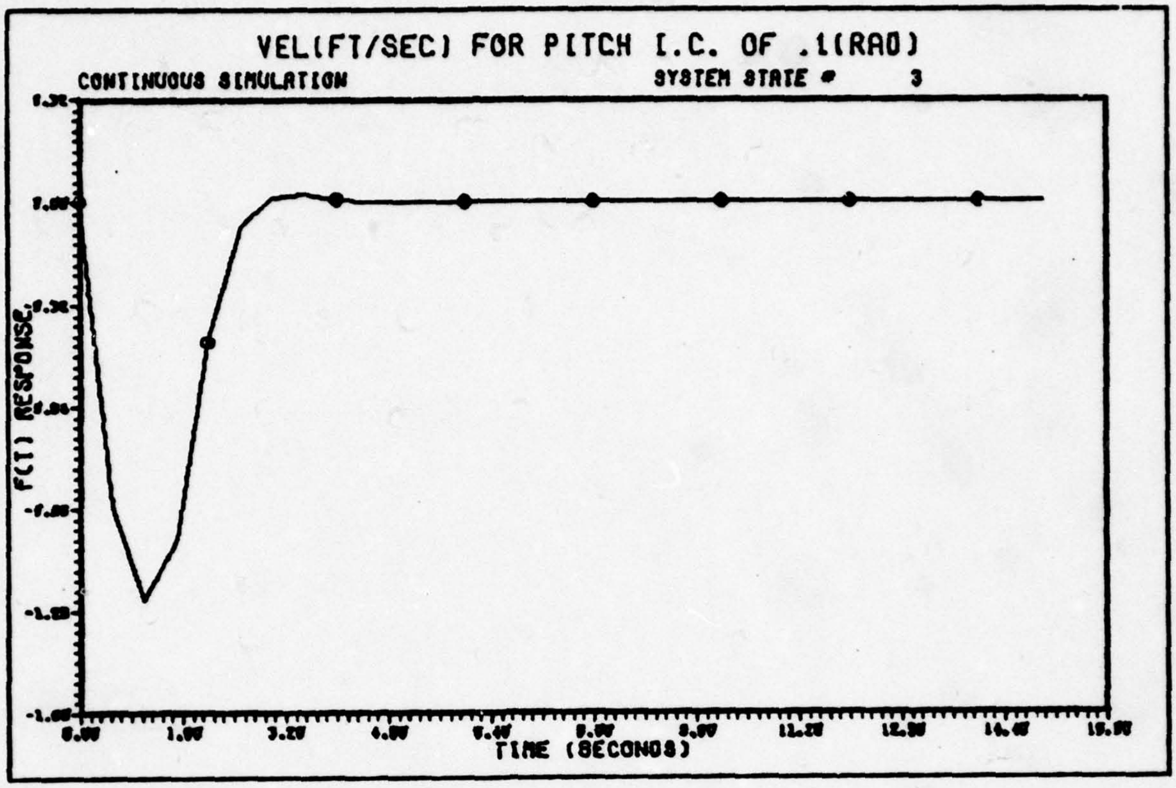

PITCH RATE(RAD/SEC) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 1
F(T) RESPONSE
TIME (SECONDS)
VEL*ANG. OF ATTACK FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 2
F(T) RESPONSE
TIME (SECONDS)

Figure 7a


VEL(FT/SEC) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 3
F(T) RESPONSE
TIME (SECONDS)

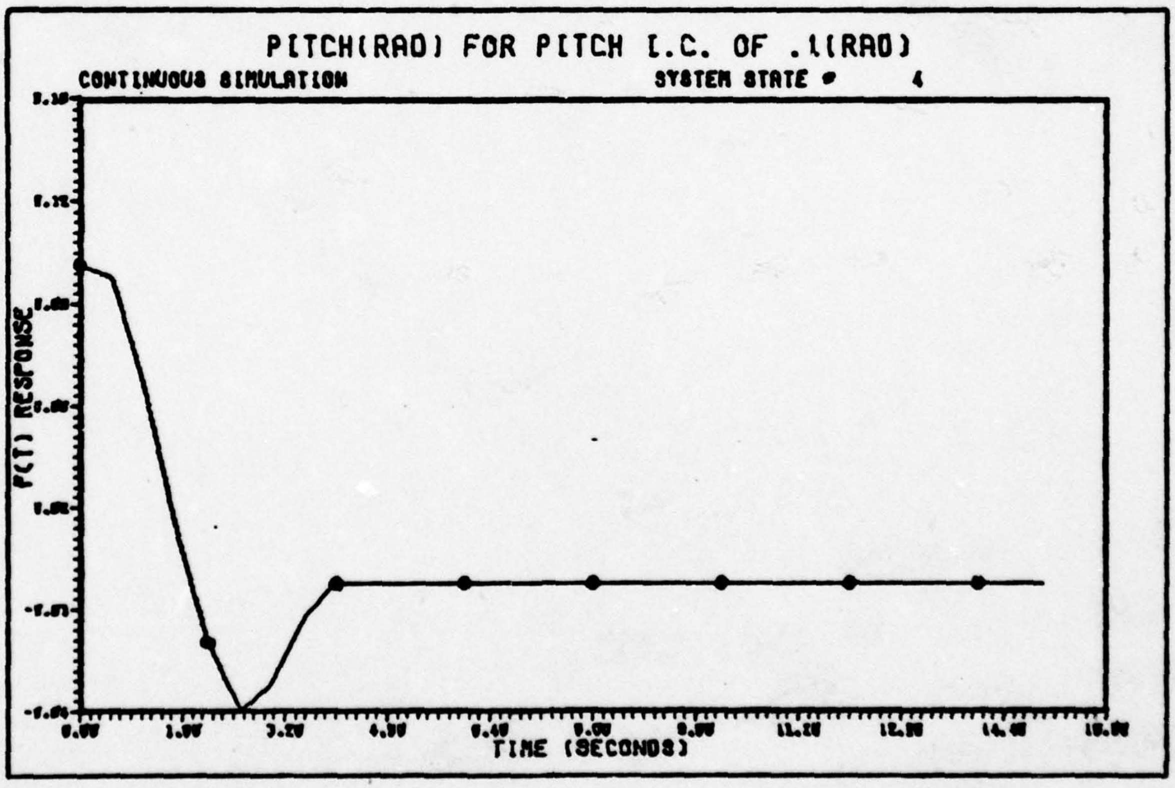
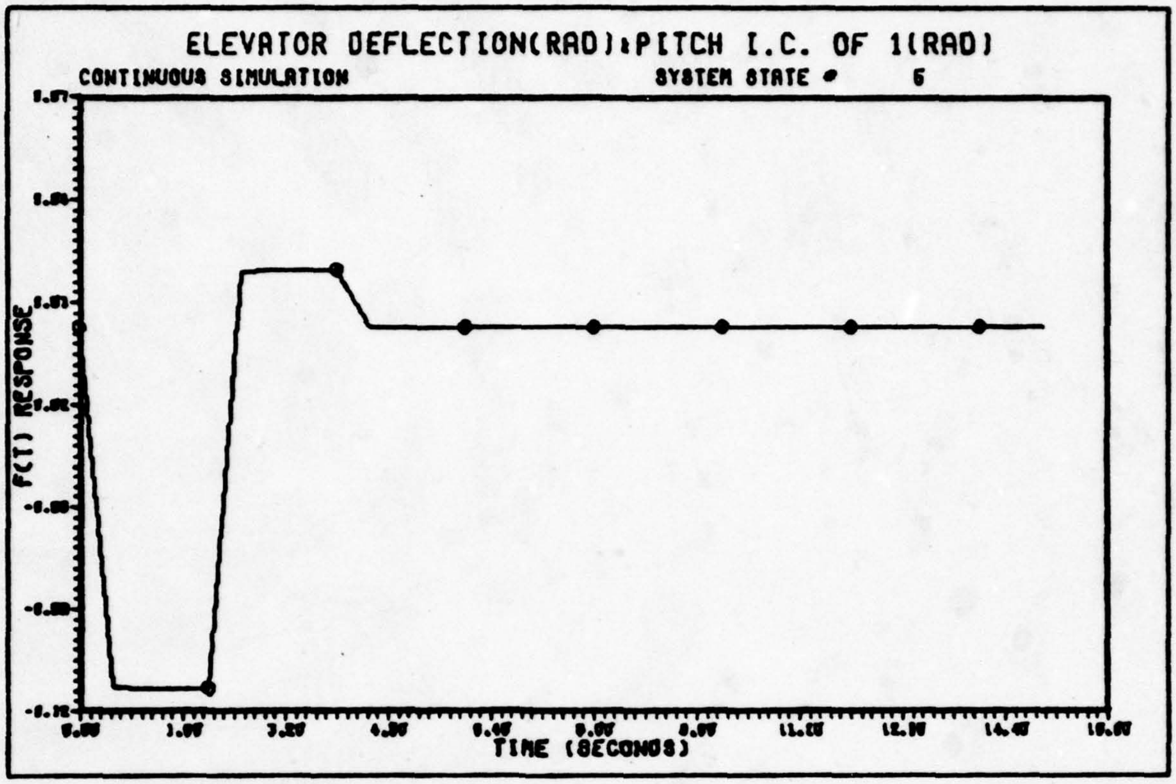

PITCH(RAD) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 4
F(T) RESPONSE
TIME (SECONDS)

Figure 7b


ELEVATOR DEFLECTION(RAD):PITCH I.C. OF 1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 6
F(T) RESPONSE
TIME (SECONDS)

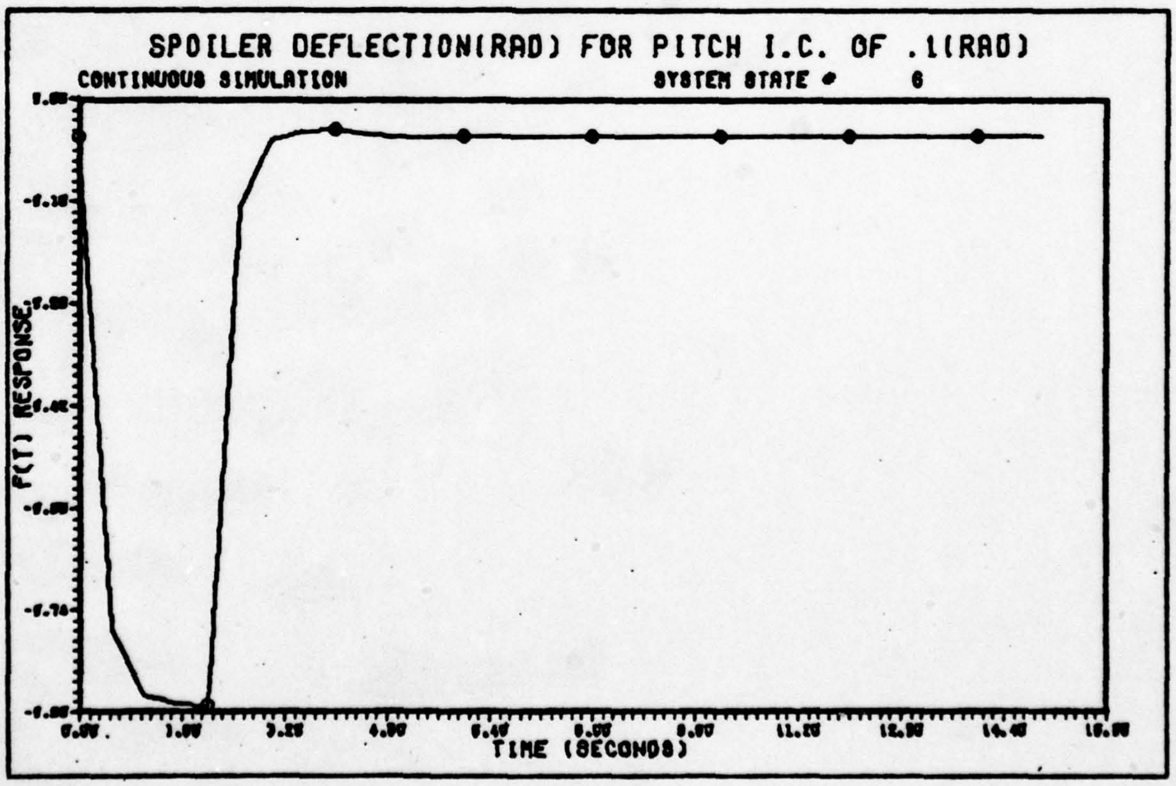

SPOILER DEFLECTION(RAD) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 6
F(T) RESPONSE
TIME (SECONDS)

Figure 7c

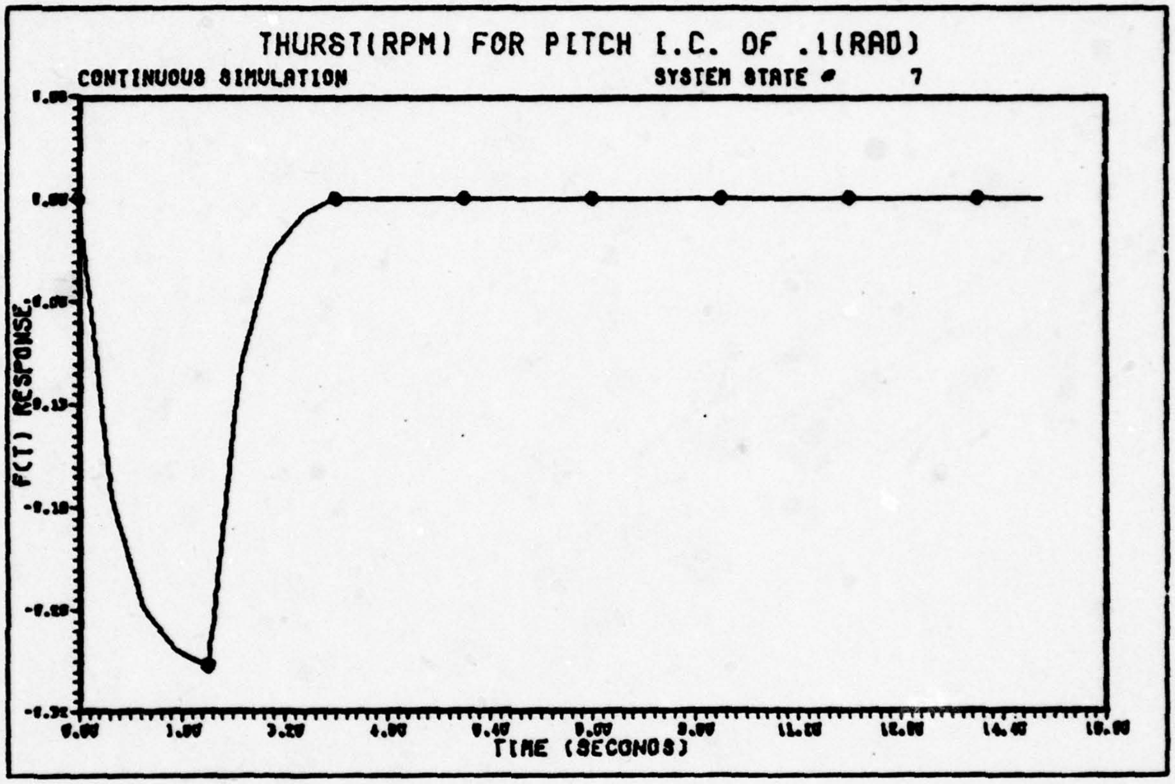

THURST(RPM) FOR PITCH I.C. OF .1(RAD)
CONTINUOUS SIMULATION
SYSTEM STATE # 7
F(T) RESPONSE
TIME (SECONDS)

Figure 7d

The resulting control gain matrix is

$$K = \left[\omega_o^{(1,1)}, \omega_o^{(2,1)}, \omega_o^{(1,2)}, \omega_o^{(2,2)}, \omega_o^{(1,3)}, \omega_o^{(2,3)}, \omega_o^{(3,3)} \right] \tag{77}$$

$$\times \left[\chi_o^{(1,1)}, \chi_o^{(2,1)}, \chi_o^{(1,2)}, \chi_o^{(2,2)}, \chi_o^{(1,3)}, \chi_o^{(2,3)}, \chi_o^{(3,3)} \right]^{-1}$$

$$= \begin{bmatrix} -.2919, .639E-3, -.148E-3, -1.131, .0837, -.0260, .121E-3 \\ -1.891, .753E-2, -.148E-2, -8.911, .5643, -.2163, .112E-3 \\ -.5236, .498E-2, .2412, -2.966, .1922, -.2585, -.2538 \end{bmatrix} \tag{78}$$

The closed-loop matrix using the feedback gain matrix of Eq. 78 is given in Appendix C, Eq. C-9. Using the same initial conditions as in Eq. 72, the continuous-time simulation was obtained with the results shown in Fig. 7. Figure 7 clearly shows that the closed-loop system reaches a deadbeat response in 3 sampling periods (6 seconds). The maximum elevator deflection is -0.1131 radian (-6.48 degrees) and the maximum spoiler deflection is -.8908 radian (-51.03 degrees). The maximum change in engine RPM is -0.2912 RPM. The controlled aircraft has an acceptable transient response and the desired ability to return to an equilibrium condition with reasonable control surface deflections.

Regulator Design Conclusions

In all three regulator designs, entire eigenstructure assignment proves to be a powerful tool in the design of time-optimal regulators for the longitudinal axis of the C-141 aircraft. Using a sampling time of 2 seconds and an initial pitch angle of 0.1 radian (5.73 degrees),

...ulators of all three designs return the aircraft to an equilibrium condition within 3 sampling periods (6 seconds). This result is possible since the length of the eigenvector/generalized eigenvector chains are set equal to the control indices {3,2,2}. Table III summarizes some of the salient results of the continuous-time simulation for the three regulator designs.

Tracker Design

In this section three time-optimal tracking systems are designed for the longitudinal axis of the C-141 aircraft. One tracker is designed for each of the three flight conditions-landing, low altitude cruise, and medium altitude cruise. Discrete-time integrators and comparators are added to the discretized system, as discussed in Chapter III, so that the aircraft can track a command input with zero steady-state error. The aircraft is modeled by Eqs. 43 thru 48.and is set up in state-space form according to Eqs 54 and 55 using the stability derivatives from Table II.

The plant in this section is augmented in order to provide tracking of an altitude rate command, where altitude rate is described by

$$\dot{h} = -w + U_0 \theta \tag{79}$$

It is also desired to keep the forward velocity and the spoiler deflection at the equilibrium values when the aircraft reaches the steady-state flight condition. Therefore, in all three tracker designs the discretized plant is augmented with three discrete-time integrators and comparators as defined by the equation

TABLE III

Regulator Characteristics for the Longitudinal Dynamics of the C-141 Aircraft for an Initial Condition of 0.1 radian for Pitch Angle

PARAMETER	LANDING	LOW ALTITUDE CRUISE	MEDIUM ALTITUDE CRUISE
$\|\delta_e\|_{max}$(rad)	0.1471	0.0989	0.1131
$\|\delta_{sp}\|_{max}$(rad)	0.3541	0.6492	0.8889
$\|\delta_{rpm}\|_{max}$(rpm)	0.5687	0.4170	0.2912
$\|\dot{\theta}\|_{max}$(rad)	0.0476	0.0716	0.0905
$\|\alpha\|_{max}$(rad)	0.0233	0.0534	0.0777
$\|u\|_{max}$(ft/sec)	2.031	1.317	1.244
Settling Time (Sec)	6	6	6

$$z(kT+T) = z(kT) + T \begin{bmatrix} y_1(kT) \\ y_2(kT) \\ y_3(kT) \end{bmatrix} - T \begin{bmatrix} v_1(kT) \\ v_2(kT) \\ v_3(kT) \end{bmatrix} \quad (80)$$

where the outputs to be tracked are

$$\begin{bmatrix} y_1(kT) \\ y_2(kT) \\ y_3(kT) \end{bmatrix} = Mx(kT) = \begin{bmatrix} 0,-1,0,U_0,0,0,0 \\ 0,0,1,\ 0\ ,0,0,0 \\ 0,0,0,\ 0\ ,0,1,0 \end{bmatrix} x(kT) \quad (81)$$

and the command inputs are

$$\begin{bmatrix} v_1(kT) \\ v_2(1T) \\ v_3(kT) \end{bmatrix} = \begin{bmatrix} \dot{h}(kT) \text{ command} \\ u(kT) \text{ command} \\ \delta_{sp}(kT) \text{ command} \end{bmatrix} \quad (82)$$

with

$$\dot{h}(kT) = 12 \text{ radians/second} \quad (83)$$

$$u(kT) = 0 \quad (84)$$

$$\delta_{sp}(kT) = 0 \quad (85)$$

The augmentation of Eq. 80 to the discretized plant equation results in the composite discretized input and output equations

$$\begin{bmatrix} x(kT+T) \\ z(kT+T) \end{bmatrix} = \begin{bmatrix} F\ , & 0 \\ TM, & I_p \end{bmatrix} \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} u(kT) + \begin{bmatrix} 0 \\ E \end{bmatrix} v(kT)\ , \tag{86}$$

and

$$y(kT) = [C,0] \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix} \tag{87}$$

and now the augmented plant, input, and output matrices are

$$\bar{F} = \begin{bmatrix} F & , & 0 \\ TM & , & I_p \end{bmatrix}\ , \tag{88}$$

$$\bar{G} = \begin{bmatrix} G \\ 0 \end{bmatrix}\ , \tag{89}$$

$$\bar{E} = \begin{bmatrix} 0 \\ E \end{bmatrix}\ , \tag{90}$$

and

$$\bar{C} = [C,0] \tag{91}$$

As shown in Chapter III, the eigenvalues of the composite system can be assigned arbitraily if and only if the unaugmented discretized plant is controllable and the number of discrete-time integrators, p, is less than or equal to the number of controls m.

<u>Landing</u>

The continuous plant matrix for the landing condition is given by Eq. 57. The discretized form of the state equation for a sampling interval of 2 seconds for the landing condition is given by Eqs. 58, 59, and 60.

Using Eqs 79 thru 90, the augmented system matrices are

$$\bar{F} = \begin{bmatrix} -.1224 & -.937E-3 & .955E-3 & -.0351 & .807E-2 & -.0148 & -.282E-2 & 0 & 0 & 0 \\ 52.01 & -.0616 & -.1399 & 7.796 & -4.010 & 3.615 & .5091 & 0 & 0 & 0 \\ -24.79 & .2471 & .8741 & -61.10 & 1.623 & -1.097 & -3.210 & 0 & 0 & 0 \\ .5935 & -.277E-2 & .109E-2 & .9759 & -.0414 & .965E-2 & -.249E-2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .206E-8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .336E-3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .0183 & 0 & 0 & 0 \\ 0 & -2.0 & 0 & 399.7 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 \end{bmatrix} \qquad (92)$$

$$\bar{G} = \begin{bmatrix} -.4141 & .0386 & -.498E-2 \\ -93.66 & 50.59 & 1.226 \\ 18.52 & -10.94 & -10.39 \\ .6352 & .0997 & -.28E-2 \\ 1.0 & 0 & 0 \\ 0 & .9997 & 0 \\ 0 & 0 & .9817 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (93)$$

$$\bar{E} = \begin{bmatrix} 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ -2, & 0, & 0 \\ 0, & -2, & 0 \\ 0, & 0, & -2 \end{bmatrix} \tag{94}$$

Since the pair $(\bar{F},\bar{G})$ is controllable and it is desired that the closed-loop system exhibit a time-optimal response, all the eigenvalues of the closed-loop system are assigned to the origin by implementing the control law

$$u(kT) = K_1\, x(kT) + K_2\, z(kT) \tag{95}$$

The control indices of the pair $(\bar{F},\bar{G})$ are the set of integers {4,3,3}. Therefore, by selecting a set of eigenvector/generalized eigenvector chains with lengths equal to the control indices , it is possible to achieve a ripple-free steady-state response for all states in 4 sampling periods (8 seconds). The null space of $S(0) = [\bar{F},\bar{G}]$, using the algorithm from Ref. 2 is

$$\ker S(0) = \text{span}\left\{ \begin{bmatrix} -.0742 \\ 1.0 \\ 0 \\ .751E-2 \\ -.9520 \\ .851E-2 \\ .0239 \\ -1.0 \\ 0 \\ -.017 \\ 0 \\ 0 \\ -.447E-3 \end{bmatrix}, \begin{bmatrix} -.941E-3 \\ 0 \\ -.5 \\ 0 \\ -.0201 \\ -.871E-2 \\ -.1449 \\ 0 \\ 1.0 \\ .0174 \\ 0 \\ .292E-5 \\ .270E-2 \end{bmatrix}, \begin{bmatrix} -.0337 \\ 1.0 \\ 0 \\ .250E-2 \\ -.5133 \\ -.0816 \\ .0616 \\ 1.0 \\ 0 \\ .1632 \\ 0 \\ .273E-4 \\ -.115E-2 \end{bmatrix} \right\} \quad (96)$$

The vectors selected from the ker S(0) are

$$\begin{bmatrix} \chi_o^{(1,1)} \\ \omega_o^{(1,1)} \end{bmatrix} = \begin{bmatrix} -.0742 \\ 1.0 \\ .751E-2 \\ -.9520 \\ .851E-2 \\ .0239 \\ -1.0 \\ 0 \\ -.017 \\ 0 \\ 0 \\ -.447E-3 \end{bmatrix}, \quad (97)$$

$$\begin{bmatrix} \chi_o^{(1,2)} \\ \omega_o^{(1,2)} \end{bmatrix} = \begin{bmatrix} -.941E-3 \\ 0 \\ -.5 \\ 0 \\ -.0201 \\ -.871E-2 \\ -.1449 \\ 0 \\ 1.0 \\ .0174 \\ 0 \\ .292E-5 \\ .270E-2 \end{bmatrix} \qquad (98)$$

and

$$\begin{bmatrix} \chi_o^{(3,1)} \\ \omega_o^{(1,3)} \end{bmatrix} = \begin{bmatrix} -.0337 \\ 1.0 \\ 0 \\ .250E-2 \\ -.5133 \\ -.0816 \\ ..0616 \\ 1.0 \\ 0 \\ .\ 1632 \\ 0 \\ 273E-4 \\ -.115E-2 \end{bmatrix} \qquad (99)$$

and the first generalized vectors, generated by using the algorithm from Ref. 2, are

$$\begin{bmatrix} \chi_o^{(2,1)} \\ \\ \omega_o^{(2,1)} \end{bmatrix} = \begin{bmatrix} 21.86 \\ 1.0 \\ 0 \\ .05 \\ 337.7 \\ 38.08 \\ -17.75 \\ -1 \\ 0 \\ -76.17 \\ -\ .9520 \\ -\ .427E-2 \\ .3555 \end{bmatrix} , \tag{100}$$

$$\begin{bmatrix} \chi_o^{(2,2)} \\ \\ \omega_o^{(2,2)} \end{bmatrix} = \begin{bmatrix} .3768 \\ 0 \\ 0 \\ 0 \\ 5.824 \\ .6398 \\ .3874 \\ 0 \\ 1.0 \\ -1.262 \\ -\ .0201 \\ -\ .892E-2 \\ -\ .1548 \end{bmatrix} \tag{101}$$

and

$$\begin{bmatrix} \chi_o^{(2,3)} \\ \\ \omega_o^{(2,3)} \end{bmatrix} = \begin{bmatrix} 10.84 \\ 1.0 \\ 0 \\ .005 \\ 168.1 \\ 19.97 \\ -8.923 \\ 1.0 \\ 0 \\ -39.78 \\ -.5133 \\ -.0883 \\ .2292 \end{bmatrix} \qquad (102)$$

The second set of generalized vectors, generated by using the algorithm from Ref. 2, are

$$\begin{bmatrix} \chi_o^{(3,1)} \\ \omega_o^{(3,1)} \end{bmatrix} = \begin{bmatrix} -7358 \\ 1.0 \\ 0 \\ .005 \\ -.114E+6 \\ -.132E+5 \\ 5927 \\ -1.0 \\ 0 \\ .2623E+5 \\ 337.7 \\ 42.50 \\ -128.7 \end{bmatrix} \tag{103}$$

$$\begin{bmatrix} \chi_o^{(3,2)} \\ \omega_o^{(3,2)} \end{bmatrix} = \begin{bmatrix} -127.0 \\ 0 \\ 0 \\ 0 \\ -1966 \\ -266.5 \\ 99.86 \\ 0 \\ 1.0 \\ 451.8 \\ 5.824 \\ .7160 \\ -1.468 \end{bmatrix} \tag{104}$$

and

$$\begin{bmatrix} \chi_o^{(3,3)} \\ \omega_o^{(3,3)} \end{bmatrix} = \begin{bmatrix} -3654 \\ 1.0 \\ 0 \\ .005 \\ -.566E+5 \\ =6543 \\ 2943 \\ 1.0 \\ 0 \\ .131E+5 \\ 168.1 \\ 22.17 \\ -64.00 \end{bmatrix} \qquad (105)$$

Continuing the chain of generalized vectors, using the algorithm from Ref. 2, the third generalized vector selected is

$$
\begin{bmatrix} \chi_o^{(4,2)} \\ \omega_o^{(4,2)} \end{bmatrix} = \begin{bmatrix} .428E+5 \\ 0 \\ 0 \\ 0 \\ .663E+6 \\ .766E+5 \\ -.345E+5 \\ 0 \\ 1 \\ -.153E+6 \\ -1966 \\ -252.6 \\ 744.6 \end{bmatrix} . \qquad (106)
$$

The required feedback gain matrix is

$$
K = [K_1, K_2] =
$$

$$
[\omega_1^{(1,1)}, \omega_o^{(2,1)}, \omega_o^{(3,1)}, \omega_o^{(1,2)}, \omega_o^{(2,2)}, \omega_o^{(3,2)}, \omega_o^{(4,2)}, \omega_o^{(1,3)}, \omega_o^{(2,3)}, \omega_o^{(3,3)}]
$$

$$
\times \quad [\chi_o^{(1,1)}, \chi_o^{(2,1)}, \chi_o^{(3,1)}, \chi_o^{(1,2)}, \chi_o^{(2,2)}, \chi_o^{(3,2)}, \chi_o^{(4,2)}, \chi_o^{(1,3)}, \chi_o^{(2,3)}, \chi_o^{(3,3)}]^{-1} \qquad (107)
$$

$$
= \begin{bmatrix} 1.098, & -.0159, & -.952E-4, & 4.343, & -0724, & .2213, & -.0062, & .394E-2, & -.151E-2, & .1186 \\ 1.133, & -.0305, & .215E-2, & 7.824, & -.0706, & -.3126, & -.842E-2, & .011, & -.744E-3, & -.1421 \\ -.2065, & .132E-2, & .1483, & .7421, & .0112, & 1.294, & -.2996, & .387E-2, & .0325, & .7022 \end{bmatrix}
$$

and the resulting closed-loop matrix is

$[\bar{F}+\bar{G}K] =$

$$
\begin{bmatrix}
-.5321 & .447E-2 & .338E-3 & -1.535 & .0353 & -.1250 & .902E-3 & -.123E-3 & .433E-3 & -.058 \\
6.310 & -.110 & .1597 & -2.218 & -.7890 & -31.34 & .2940 & .1938 & .1433 & -17.44 \\
-14.72 & .2719 & -.6923 & -73.99 & .9388 & -7.02 & -.1184 & -.0879 & -.3576 & -3.545 \\
.993E-2 & .430E-2 & .938E-3 & -1.005 & -.250E-2 & -.1658 & .145E-2 & -.142E-2 & .789E-3 & -.0915 \\
1.098 & -.0159 & -.952E-4 & 4.343 & -.0724 & .2213 & -.618E-2 & .394E-2 & -.151E-2 & .1186 \\
1.133 & -.0304 & .215E-2 & 7.822 & -.0706 & -.3122 & -.842E-2 & .0110 & -.744E-3 & -.1421 \\
-.207 & .129E-2 & .1456 & .7285 & .0110 & 1.270 & .2758 & .379E-2 & .0319 & .6894 \\
0 & -2.0 & 0 & 399.7 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 2.0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1
\end{bmatrix}
\quad (108)
$$

All of the eigenvalues of the closed-loop system are at the origin and, since the lengths of the eigenvector/generalized eigenvector chains are equal to the control indices of the pair $[\overline{F},\overline{G}]$, the order of the minimal polynomial for the closed-loop system is equal to 4. The state responses of the continuous simulation, with a command input vector as described by Eq. 82 thru Eq. 85., are plotted in Fig. 8. The altitude rate command and the altitude rate response are plotted in Fig. 9. It is clearly seen, from the plots, that the closed-loop system tracks the command input vector in 4 sampling periods (8 seconds). The plots also show that the maximum elevator deflection is -.0946 radian (-5.42 degrees) and the maximum spoiler deflection is -.2648 radian (-15.17 degrees). In the steady-state the change in forward velocity (u) is zero and the spoiler control surface returns to its equilibrium position. Therefore, the designed control system for the C-141 aircraft tracks the command vector with desirable state responses and control surface deflections.

Low Altitude Cruise

Equation C-2 gives the continuous plant matrix and Eq. C-5 gives the discretized form of the state equations for a sampling interval of 2 seconds for the low altitude cruise condition. The augmented system matrices are formed using Eqs. 80 thru 91 and are given by Eq. C-6. In order for the closed-loop system to exhibit a ripple-free response, the eigenvalue spectrum for the closed-loop system is assigned to the origin using the control law described by Eq. 95. The control invariants of the pair $[\overline{F},\overline{G}]$ are the set of integers {4,3,3}. Since a time-optimal system response is desired, the lengths of the eigenvector/generalized eigenvector chains are set equal to the control indices.. The vector/generalized vector chains selected from the null space of S(0), using the

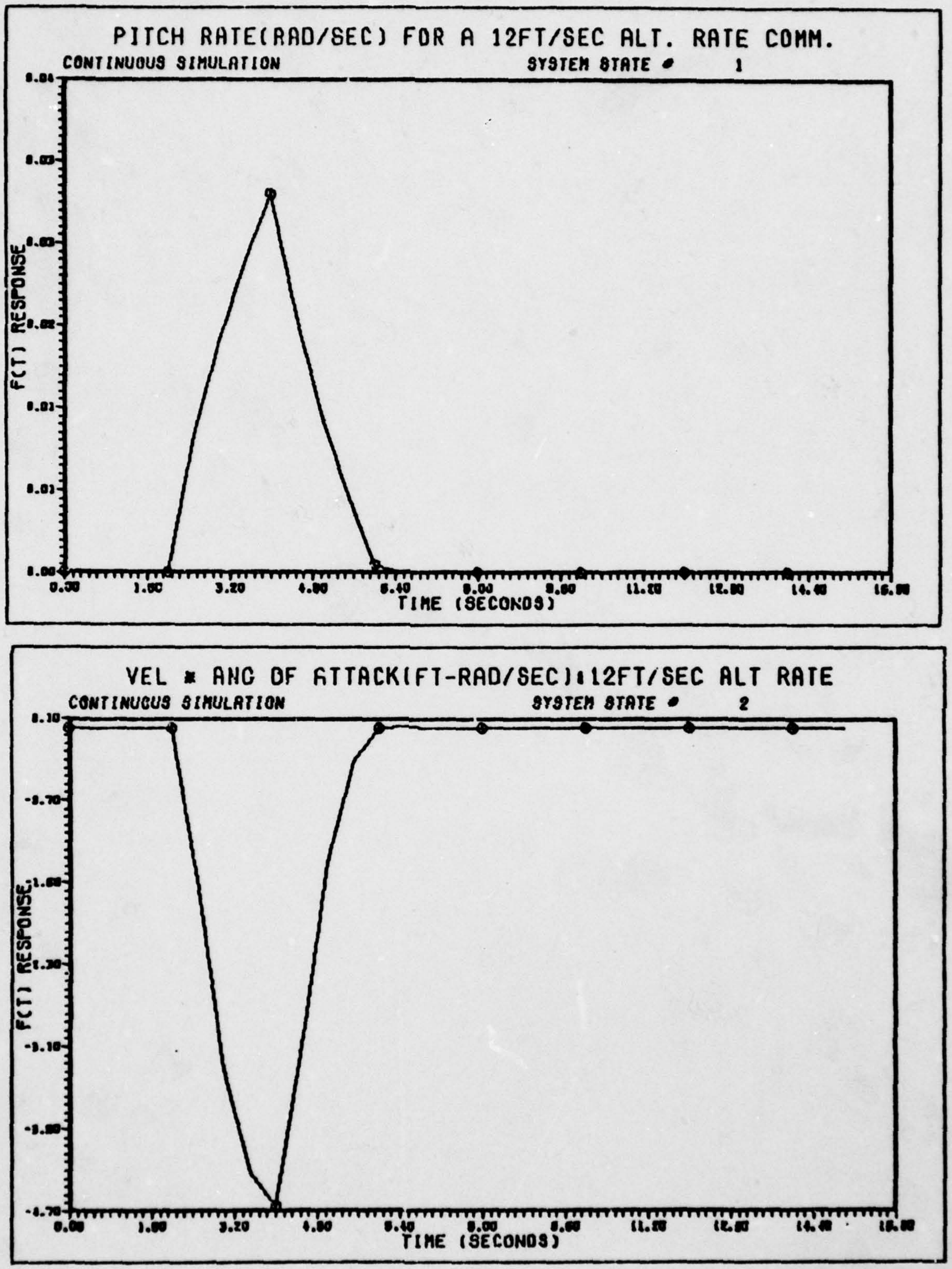
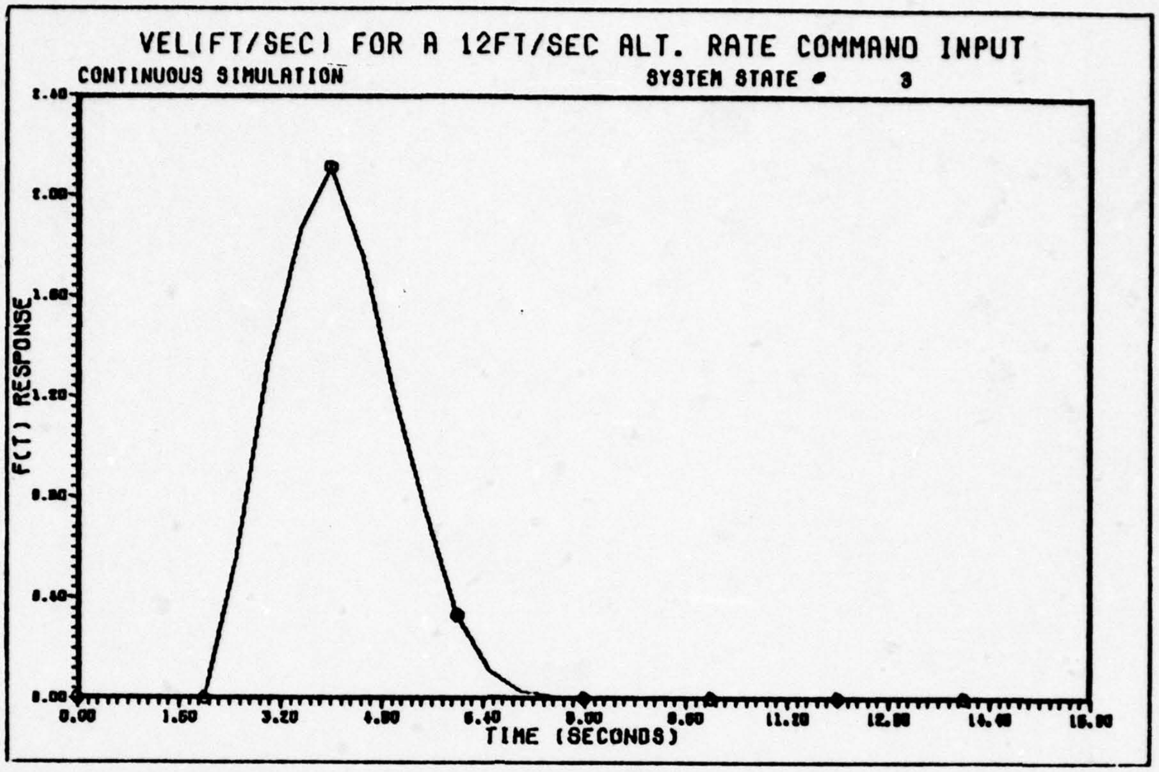

PITCH RATE(RAD/SEC) FOR A 12FT/SEC ALT. RATE COMM.
CONTINUOUS SIMULATION
SYSTEM STATE # 1
F(T) RESPONSE
TIME (SECONDS)
VEL * ANG OF ATTACK(FT-RAD/SEC):12FT/SEC ALT RATE
CONTINUOUS SIMULATION
SYSTEM STATE # 2
F(T) RESPONSE
TIME (SECONDS)

Figure 8a


VEL(FT/SEC) FOR A 12FT/SEC ALT. RATE COMMAND INPUT
CONTINUOUS SIMULATION
SYSTEM STATE # 3
F(T) RESPONSE
TIME (SECONDS)

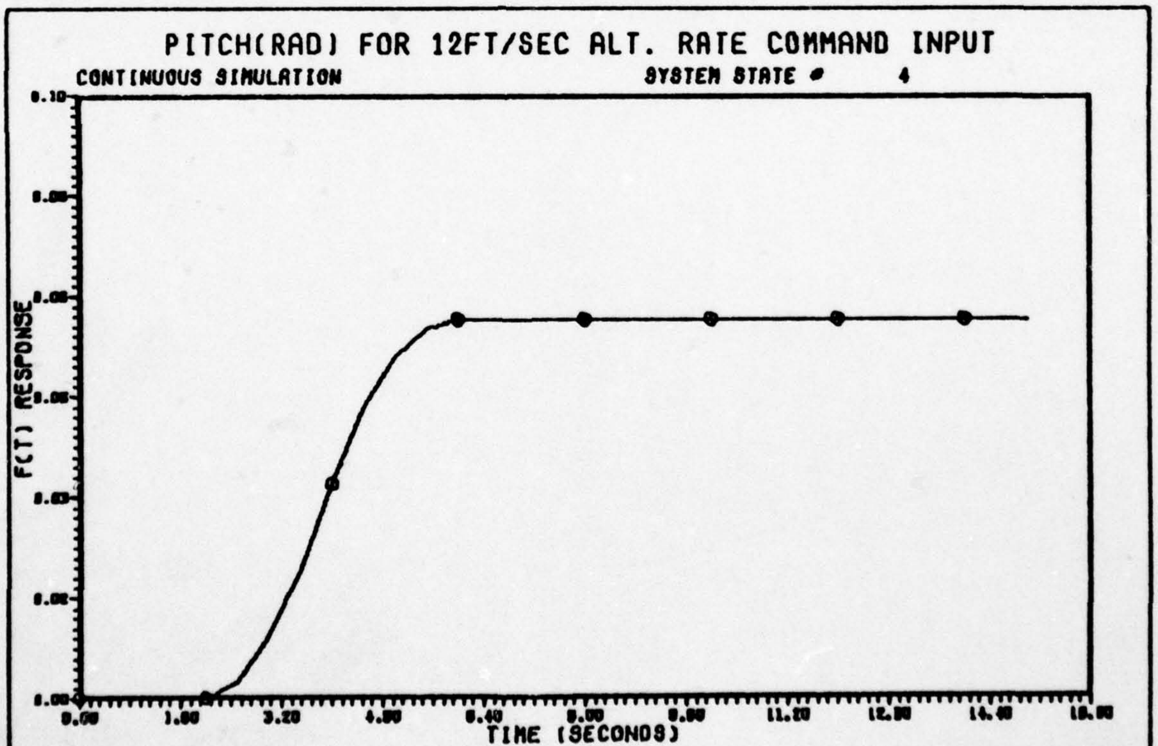
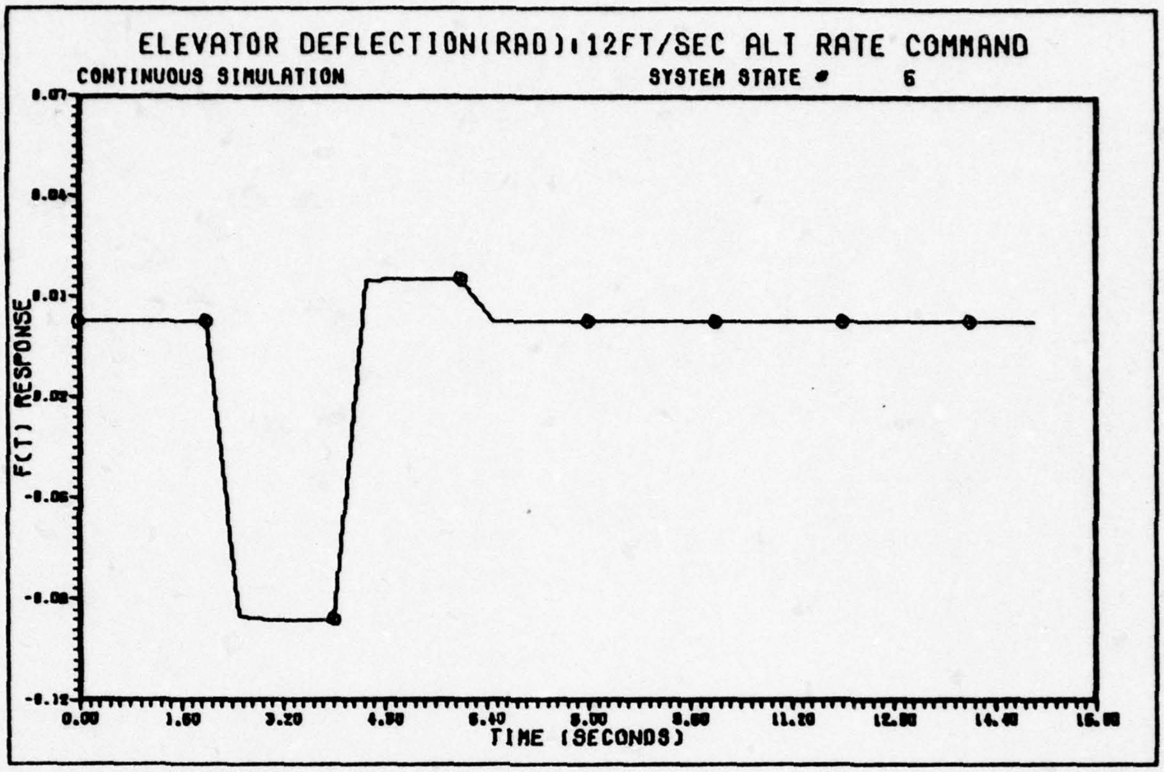

PITCH(RAD) FOR 12FT/SEC ALT. RATE COMMAND INPUT
CONTINUOUS SIMULATION
SYSTEM STATE # 4
F(T) RESPONSE
TIME (SECONDS)

Figure 8b


ELEVATOR DEFLECTION(RAD): 12FT/SEC ALT RATE COMMAND
CONTINUOUS SIMULATION
SYSTEM STATE # 6
F(T) RESPONSE
TIME (SECONDS)
0.00
1.60
3.20
4.80
6.40
8.00
11.20
12.80
14.40
16.00

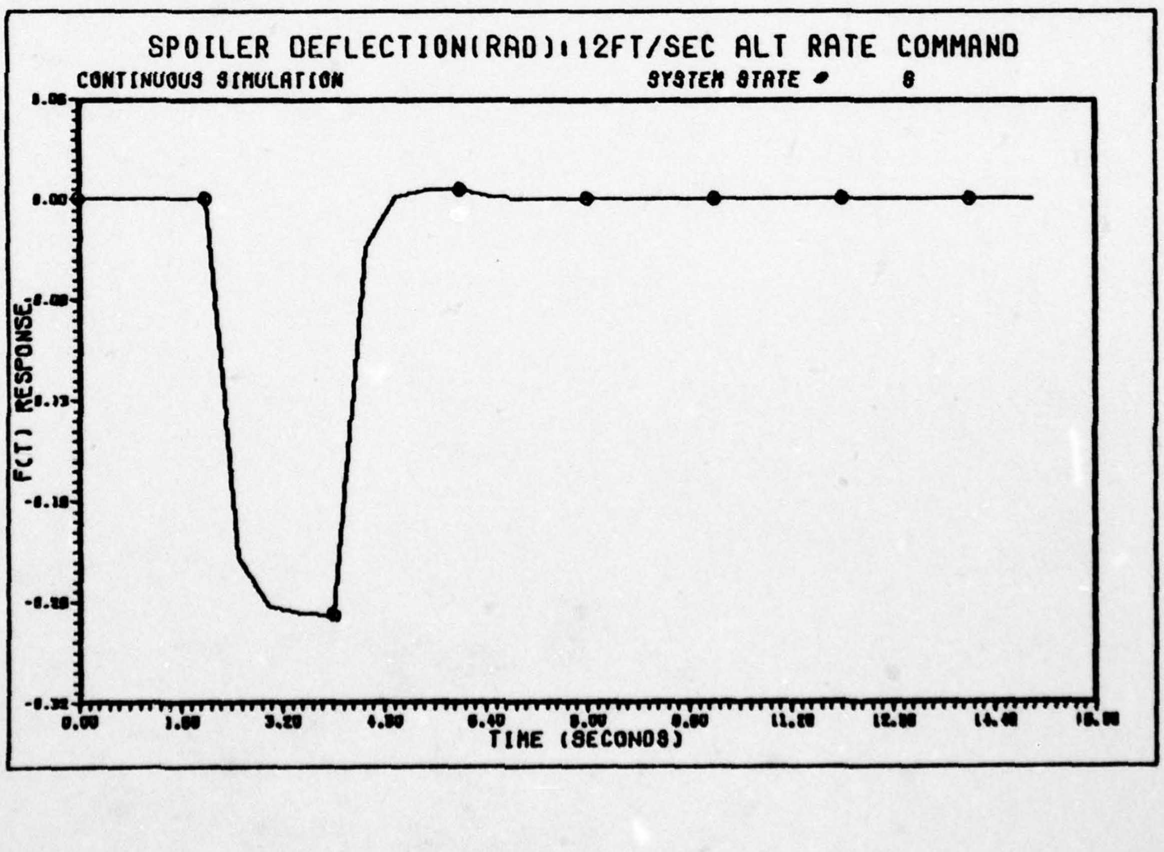

SPOILER DEFLECTION(RAD): 12FT/SEC ALT RATE COMMAND
CONTINUOUS SIMULATION
SYSTEM STATE # 8
F(T) RESPONSE
TIME (SECONDS)

Figure 8c

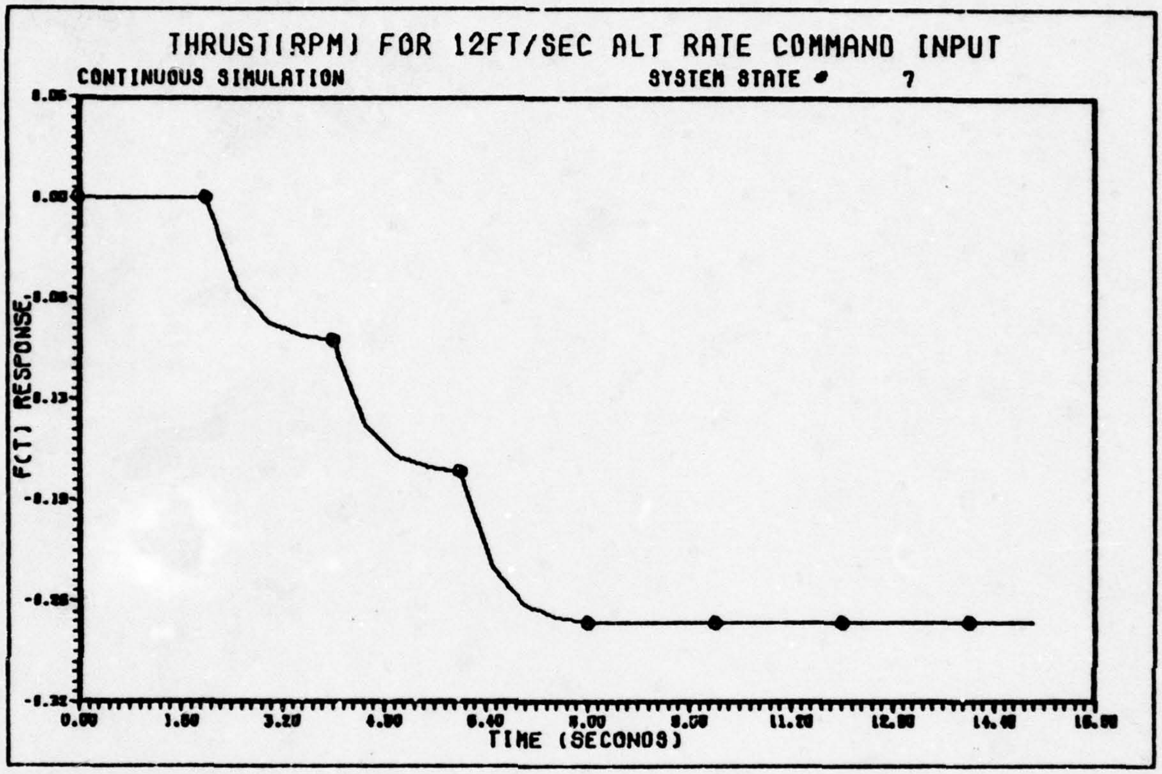

THRUST(RPM) FOR 12FT/SEC ALT RATE COMMAND INPUT
CONTINUOUS SIMULATION
SYSTEM STATE # 7
F(T) RESPONSE
TIME (SECONDS)
0.00

Figure 8d

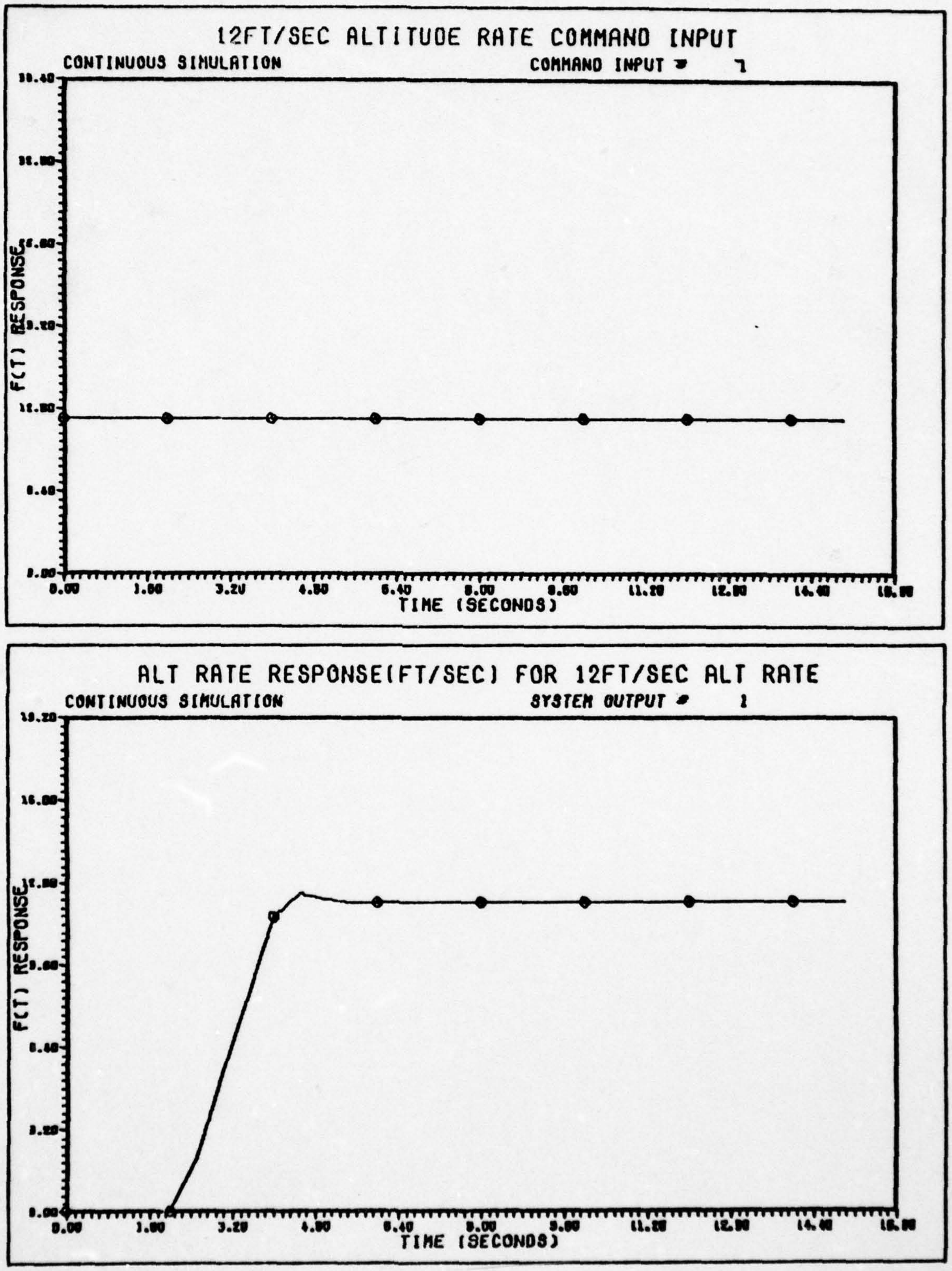

12FT/SEC ALTITUDE RATE COMMAND INPUT
CONTINUOUS SIMULATION
COMMAND INPUT # 1
F(T) RESPONSE
TIME (SECONDS)
ALT RATE RESPONSE(FT/SEC) FOR 12FT/SEC ALT RATE
CONTINUOUS SIMULATION
SYSTEM OUTPUT # 1
F(T) RESPONSE
TIME (SECONDS)

Figure 9

algorithms from Ref. 2, are

$$\begin{bmatrix} X_i \\ \omega_i \end{bmatrix} =$$

-.0453	-23.38	-.150E+5	-.971E+7	.0125	9.337
1.0	1.0	1.0	1.0	1.0	1.0
-.5	0	0	0	-.5	0
.275E-3	.184E-2	.184E-2	.184E-2	.918E-3	.184E-2
-.1263	-86.04	.558E+5	-.360E+8	.059	34.94
-.0419	-41.44	-.274E+5	-.177E+8	.0489	17.48
-.3996	33.78	-.209E+5	.135E+8	-.4505	-11.34
-1.0	-1.0	-1.0	-1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0
.0837	82.96	.549E+5	.355E+8	-.0978	-35.06

———————————————————————————

0	-.1263	-86.04	-.558E+5	0	.059
.141E-4	-.028	-32.26	-.215E+5	-.164E-4	.0431
.746E-2	-1.037	-354.8	-.230E+6	.841E-2	-.2473

(continued)

6063.	.0192	8.764	5601
1.0	-1.0	-1.0	-1.0
0	-.5	0	0
.184E-2	.184E-2	-.184E-2	.184E-2
.225E+5	.0435	31.96	.208E+5
.111E+5	.113E-2	15.08	.102E+5
-8432.	-.4069	-10.01	-7781
1.0	0	0	0
1.0	1.0	1.0	1.0
-.222E+5	-.225E-2	-30.16	.204E+5

———————————————————————————

34.94	0	.0435	31.96
13.77	-.378E-6	-.393E-2	11.66
145.8	.759E-2	-.2278	135.0

(109)

and the resulting feedback gain matrix is

$$K = [K_1, K_2] =$$

$$\left[\omega_o^{(1,1)}, \omega_o^{(2,1)}, \omega_o^{(3,1)}, \omega_o^{(4,1)}, \omega_o^{(1,2)}, \omega_o^{(2,2)}, \omega_o^{(3,2)}, \omega_o^{(1,3)}, \omega_o^{(2,3)}, \omega_o^{(3,3)}\right]$$

x

$$\left[X_o^{(1,1)}, X_o^{(2,1)}, X_o^{(3,1)}, X_o^{(4,1)}, X_o^{(1,2)}, X_o^{(2,2)}, X_o^{(3,2)}, X_o^{(1,3)}, X_o^{(2,3)}, X_o^{(3,3)}\right]$$

$$= \begin{bmatrix} .0284, .163E-2, -.309E-3, -.8049, -.894E-2, -.2810, .412E-4, -.973E-3, -.146E-3, -.1432 \\ -.5904, .0156, -.124E-2, -9.308, .2309, -1.359, .473E-3, -.622E02, -.388E-3, -.6063 \\ -.9142, .016, .4366, 4.370, .2740, .8829, -.3072, .0102, .988E-1, .5791 \end{bmatrix} \quad (110)$$

and the closed-loop matrix formed with this feedback matrix is given in Eq. C-9. All of the eigenvalues are at the origin and the closed-loop system has a Jordan Canonical form with one block of size 4 and two blocks of size 3. Therefore, the order of the minimal polynomial of the system is equal to 4. The state responses of the continuous simulation, with a command input vector given by Eqs 82 thru 85, are plotted in Fig. 10. The altitude rate command input and the aircraft's altitude rate response are plotted in Fig. 11. Effective tracking of the command input vector is achieved in 4 sampling periods (8 seconds) as demonstrated by the plots. Also, the plots show that the maximum elevator deflection is .0234 radian

AD-A069 192 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OHIO SCH--ETC F/G 1/2
THE DESIGN OF DIGITAL CONTROLLERS FOR THE C-141 AIRCRAFT USING --ETC(U)
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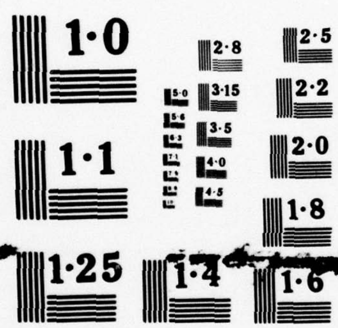
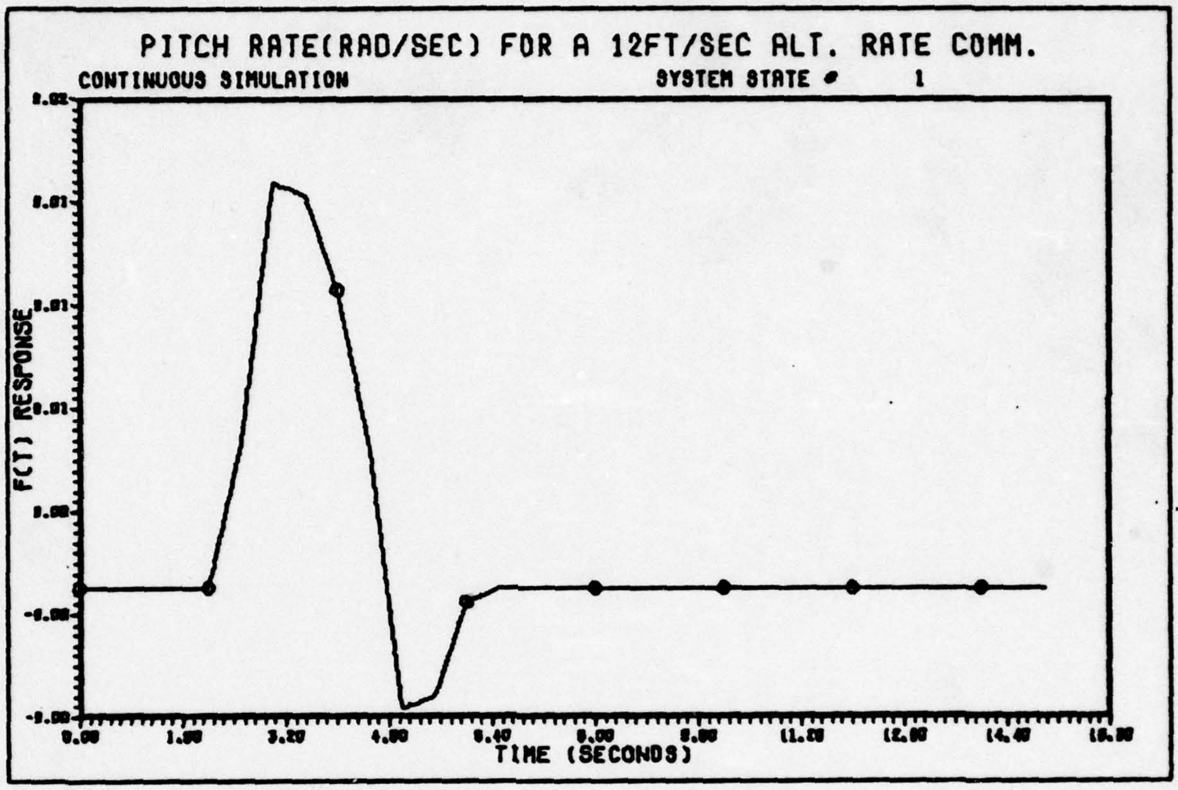


NATIONAL BUREAU OF STANDARDS
MICROCOPY RESOLUTION TEST CHART


PITCH RATE(RAD/SEC) FOR A 12FT/SEC ALT. RATE COMM.
CONTINUOUS SIMULATION
SYSTEM STATE # 1
F(T) RESPONSE
TIME (SECONDS)

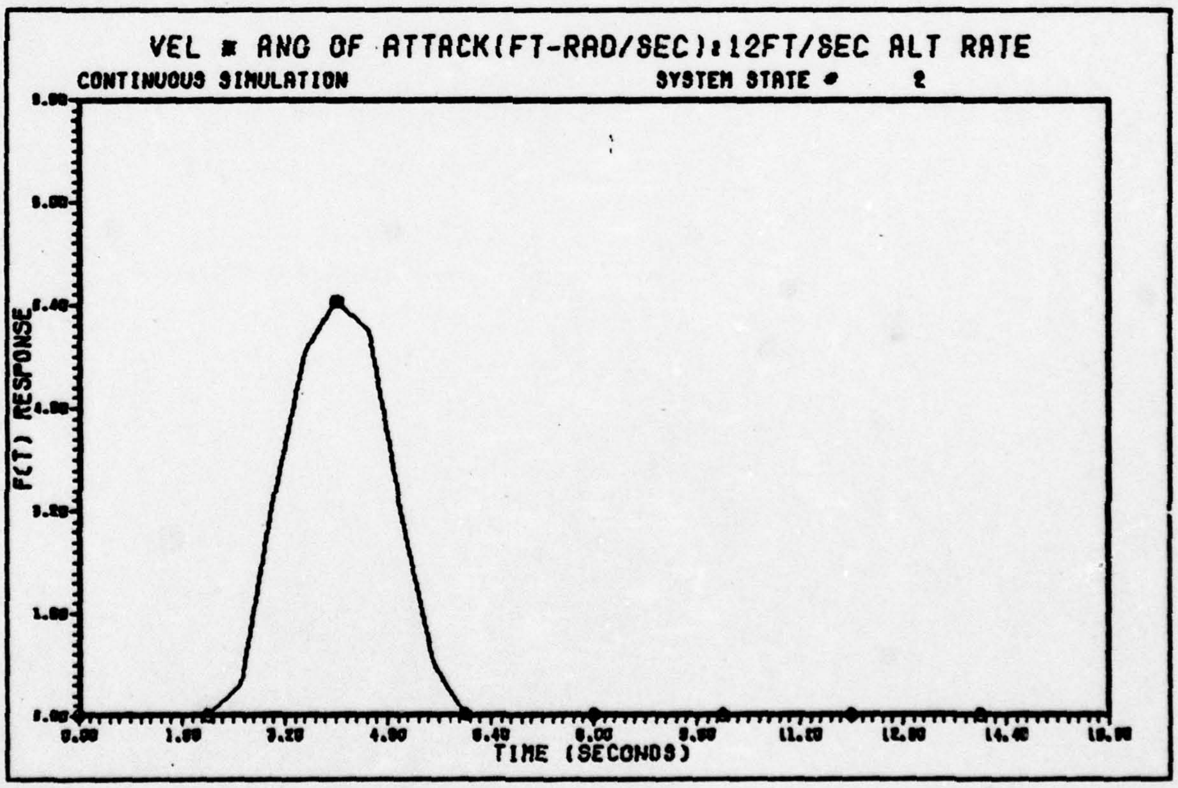

VEL * ANG OF ATTACK(FT-RAD/SEC):12FT/SEC ALT RATE
CONTINUOUS SIMULATION
SYSTEM STATE # 2
F(T) RESPONSE
TIME (SECONDS)

Figure 10a

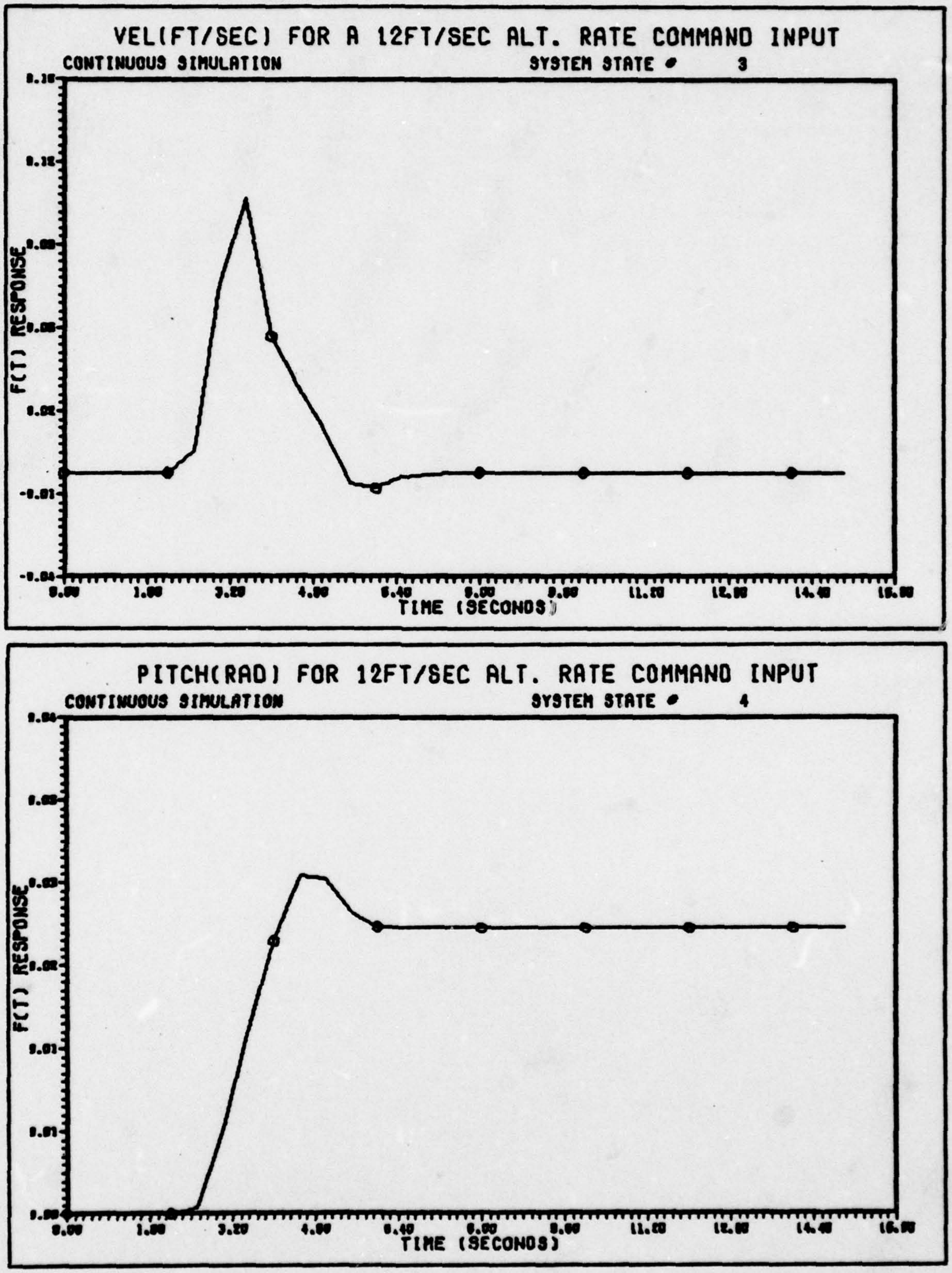
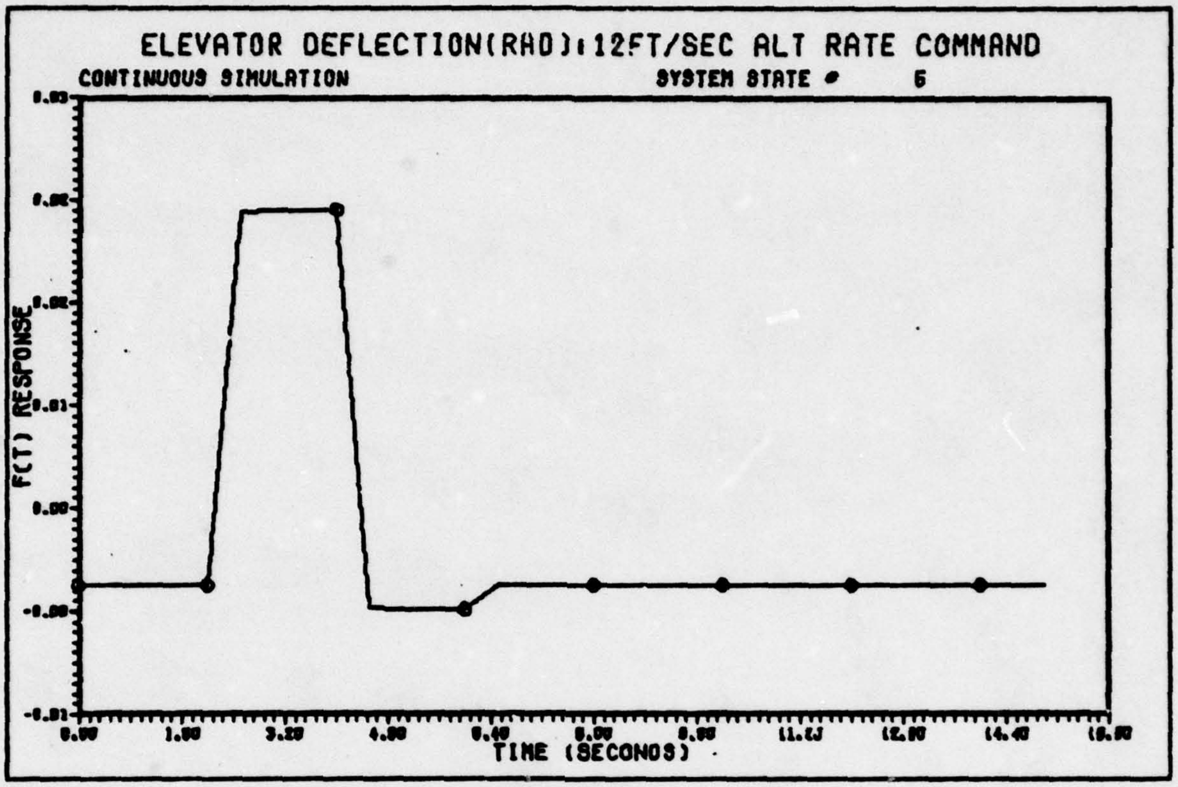

VEL(FT/SEC) FOR A 12FT/SEC ALT. RATE COMMAND INPUT
CONTINUOUS SIMULATION
SYSTEM STATE
3
F(T) RESPONSE
TIME (SECONDS)
PITCH(RAD) FOR 12FT/SEC ALT. RATE COMMAND INPUT
CONTINUOUS SIMULATION
SYSTEM STATE
4
F(T) RESPONSE
TIME (SECONDS)

Figure 10b


ELEVATOR DEFLECTION(RAD):12FT/SEC ALT RATE COMMAND
CONTINUOUS SIMULATION
SYSTEM STATE # 5
F(T) RESPONSE
TIME (SECONDS)

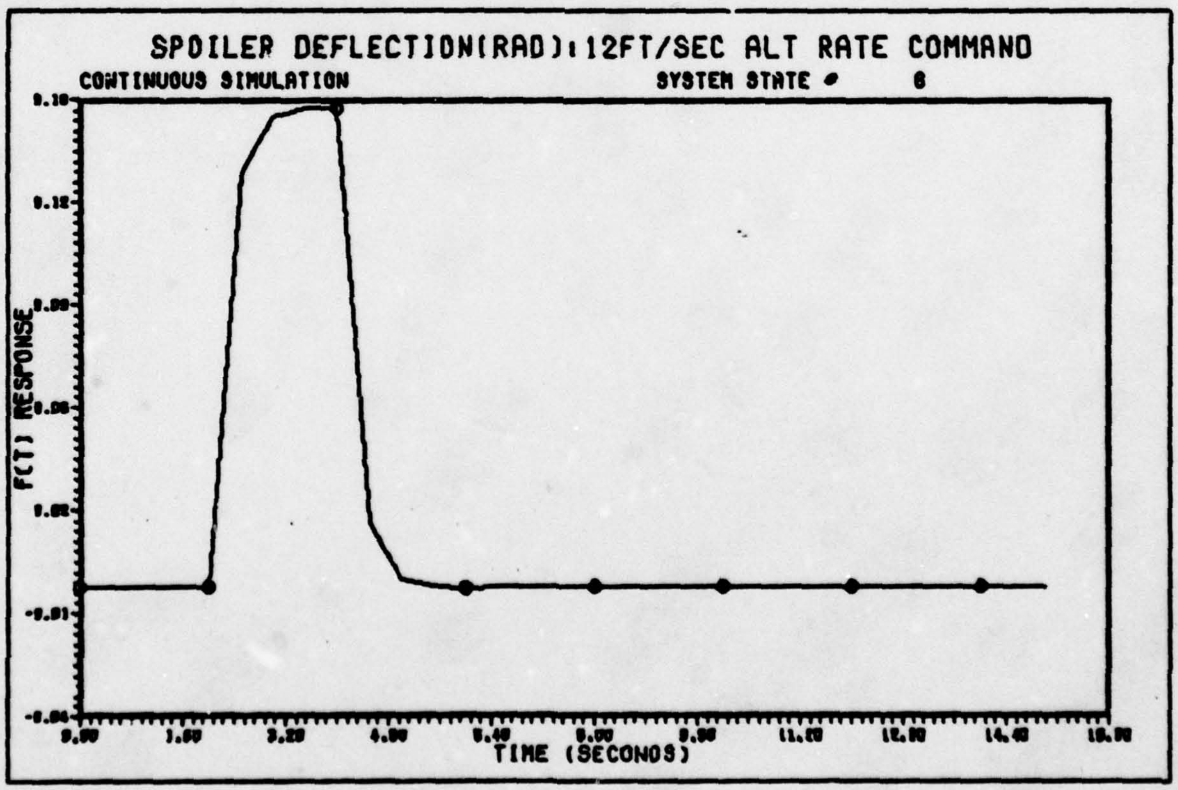

SPOILER DEFLECTION(RAD):12FT/SEC ALT RATE COMMAND
CONTINUOUS SIMULATION
SYSTEM STATE # 6
F(T) RESPONSE
TIME (SECONDS)

Figure 10c

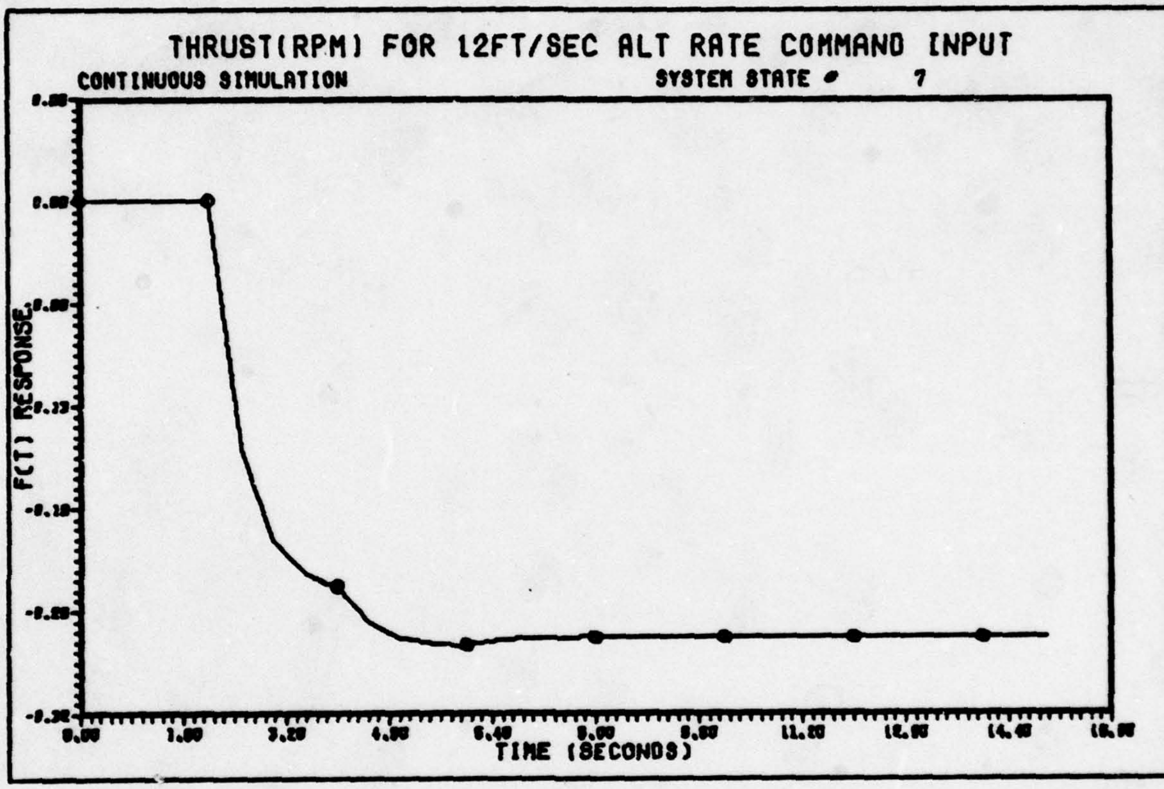

THRUST(RPM) FOR 12FT/SEC ALT RATE COMMAND INPUT
CONTINUOUS SIMULATION
SYSTEM STATE # 7
F(T) RESPONSE
TIME (SECONDS)

Figure 10d

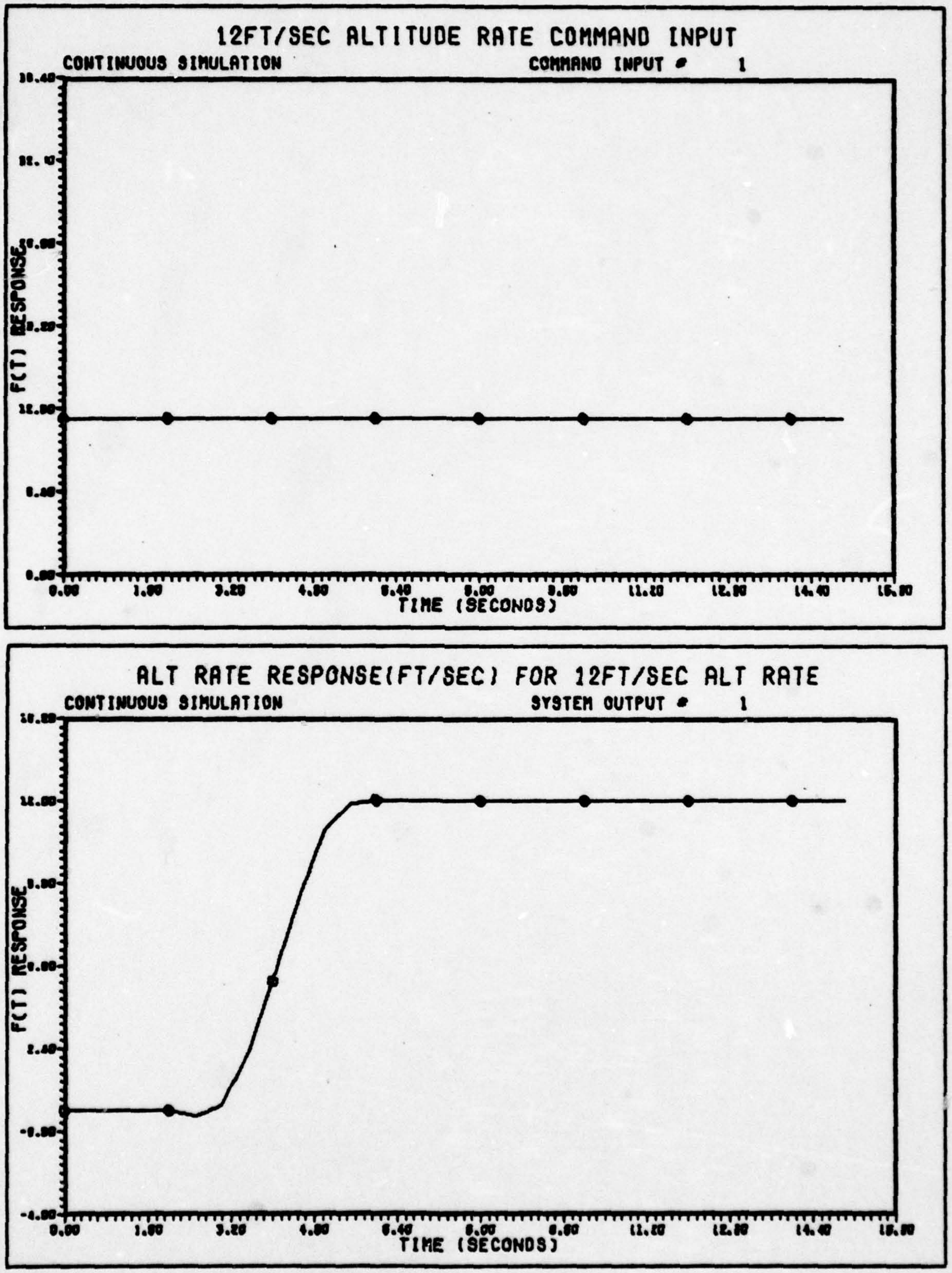

12FT/SEC ALTITUDE RATE COMMAND INPUT
CONTINUOUS SIMULATION
COMMAND INPUT # 1
F(T) RESPONSE
TIME (SECONDS)
ALT RATE RESPONSE(FT/SEC) FOR 12FT/SEC ALT RATE
CONTINUOUS SIMULATION
SYSTEM OUTPUT # 1
F(T) RESPONSE
TIME (SECONDS)

Figure 11

(1.34 degrees) and the maximum spoiler deflection is .1492 radian (8.55 degrees). In the steady-state the change in forward veloctiy (u) is zero and the spoiler control surface returns to its equilibrium position. Thus, the controlled C-141 aircraft has the ability to track the command input vector with desirable transient responses and control surface deflections.

Medium Altitude Cruise

The continuous plant matrix for the medium altitude cruise condition is given by Eq C-4 and the sampled-data state equation for the medium altitude cruise condition is given by Eq. C-5. Equation C-7 gives the augmented sampled-data equations that are formed using Eqs. 80 thru 91. Since it is desired that the closed-loop system have a deadbeat response, the closed-loop eigenvalues are all assigned to the origin. The control indices of the pair $[\bar{F},\bar{G}]$ are the set of integers {4,3,3}. In order to achieve a time-optimal closed-loop system response, the lengths of the eigenvector/generalized eigenvector chains are set equal to the control invariants. The vector/generalized vector chains generated from the null space of S(0), using the algorithms from Ref. 2, are

$$\begin{bmatrix} \chi_i \\ \omega_i \end{bmatrix} =$$

-.562E-2,-1.576 ,-738.3 ,-.350E+6, .0412 , 16.60 ,

.5 , .5 , .5 , .5 ,-.5 ,- .5 ,

-.5 , 0 , 0 ,0 ,-.5 , 0 ,

.758E-3, .758E-3, .758E-3, .758E-3,-.152E-2,- .758E-3,

-.0112 ,-7.140 ,-3411. ,-.162E+7,..1538 , 76.43 ,

.697E-2,-4.875 ,-2416. ,-.115E+7, .0866 , 53.75 ,

-.5163 , 5.270 , 1491. , .711E+6,-.5658 ,-31.14 ,

0 , 0 , 0 ,0 ,1.0 , 1.0 ,

1.0 , 1.0 , 1.0 ,1.0 ,1.0 , 1.0 ,

-.0140 , 9.737 , 4893. , .230E+7,-.1732 ,-107.7 ,

---

0 ,-.0112 ,- 7.140 ,-3411 ,0 , .1538 ,

-.234E-5, .861E-2 ,- 4.066 ,-2033 ,-.291E-4, .0686 ,

.963E-2,-.6243 ,- 22.45 ,-.118E+5, .0106 , .467E-2,

$$
\left.\begin{array}{llll}
7853 & ,\ .772E-2, & 2.571 & ,1202 \\
-.5 & ,-.5 & ,-\ .5 & ,-\ .5 \\
0 & ,-.5 & ,\ 0 & ,\ 0 \\
-.758E-3, & -.758E-3, & -\ .758E-3, & -\ .758E-3 \\
.363E+5, & .0207 & ,\ 11.72 & ,5585 \\
.257E+5, & .998E-3, & 8.113 & ,3957 \\
-.160E+5, & -.5155 & ,\ -2.829 & ,-2461 \\
1.0 & ,0 & ,\ 0 & ,\ 0 \\
1.0 & ,1.0 & ,\ 1.0 & ,\ 1.0 \\
-.515E+5, & -.199E-2, & -16.23 & ,-7930 \\
\hline
76.43 & ,0 & ,\ .0207 & ,11.72 \\
45.14 & ,-.335E-6, & -.172E-2, & 6.788 \\
266.0 & ,\ .962E-2, & -.4723 & ,43.04
\end{array}\right] \qquad (111)
$$

Thus, the feedback gain matrix is

$$K = [K_1, K_2] =$$

$$\left[\omega_o^{(1,1)}, \omega_o^{(2,1)}, \omega_o^{(3,1)}, \omega_o^{(4,1)}, \omega_o^{(1,2)}, \omega_o^{(2,2)}, \omega_o^{(3,2)}, \omega_o^{(1,3)}, \omega_o^{(2,3)}, \omega_o^{(3,3)}\right]$$

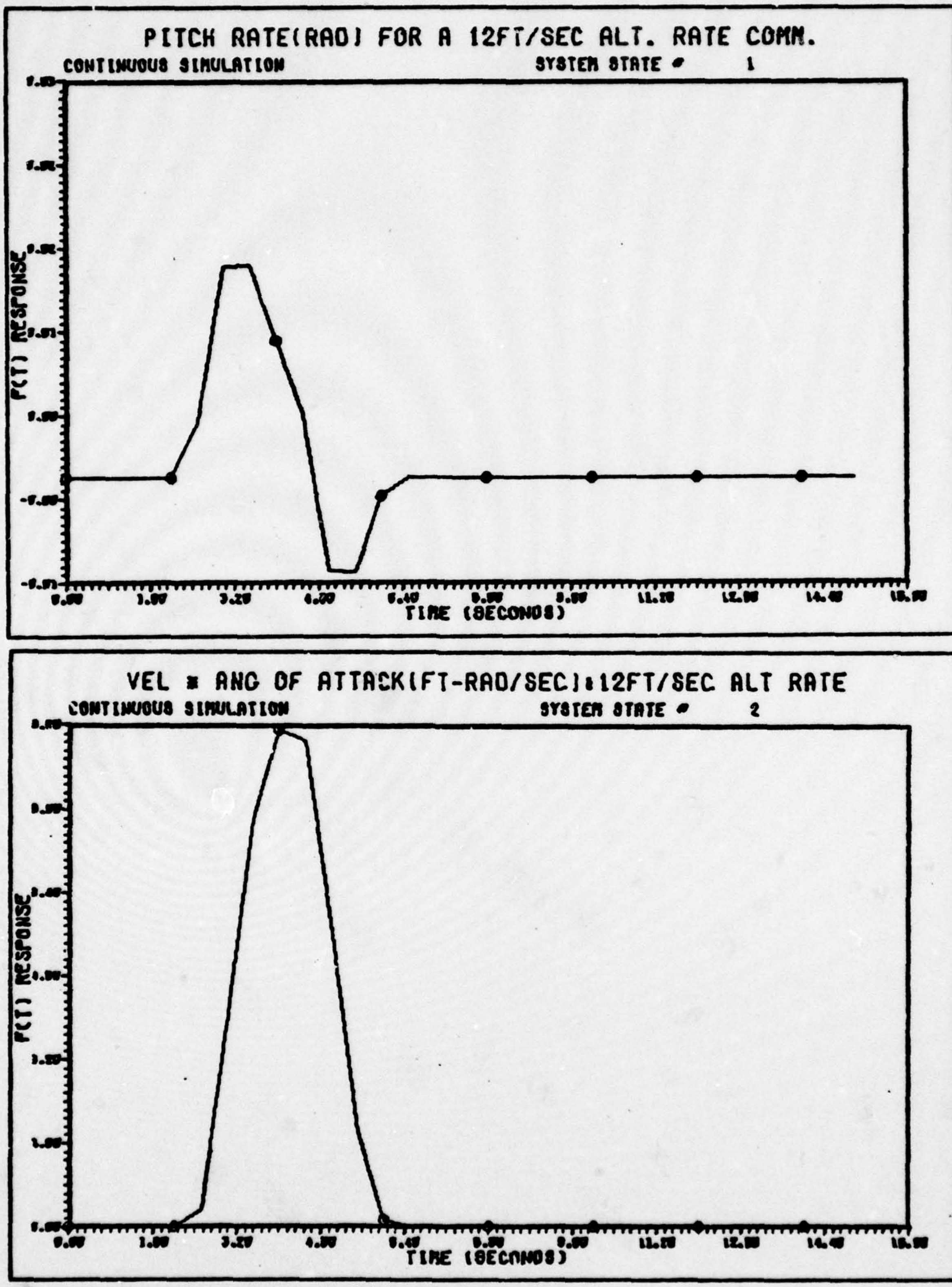
$$\times$$

$$\left[X_o^{(1,1)}, X_o^{(2,1)}, X_o^{(3,1)}, X_o^{(4,1)}, X_o^{(1,2)}, X_o^{(2,2)}, X_o^{(3,2)}, X_o^{(3,2)}, X_o^{(1,3)}, X_o^{(2,3)}, X_o^{(3,3)}\right]^{-1}$$

$$
= \begin{bmatrix} -.0602, & .226E-2, & -.355E-3, & -1.501, & .0225, & -.2513, & .685E-5, & -.118E-2 \\ -1.199, & .0224, & -.192E-2, & -15.85, & .4126, & -1.397, & .557E-3, & -.826E-2, \\ -.9898, & -.993E-2, & .5396, & 4.626, & .2798, & .6740, & -.3077, & .808E-2, \end{bmatrix}
$$

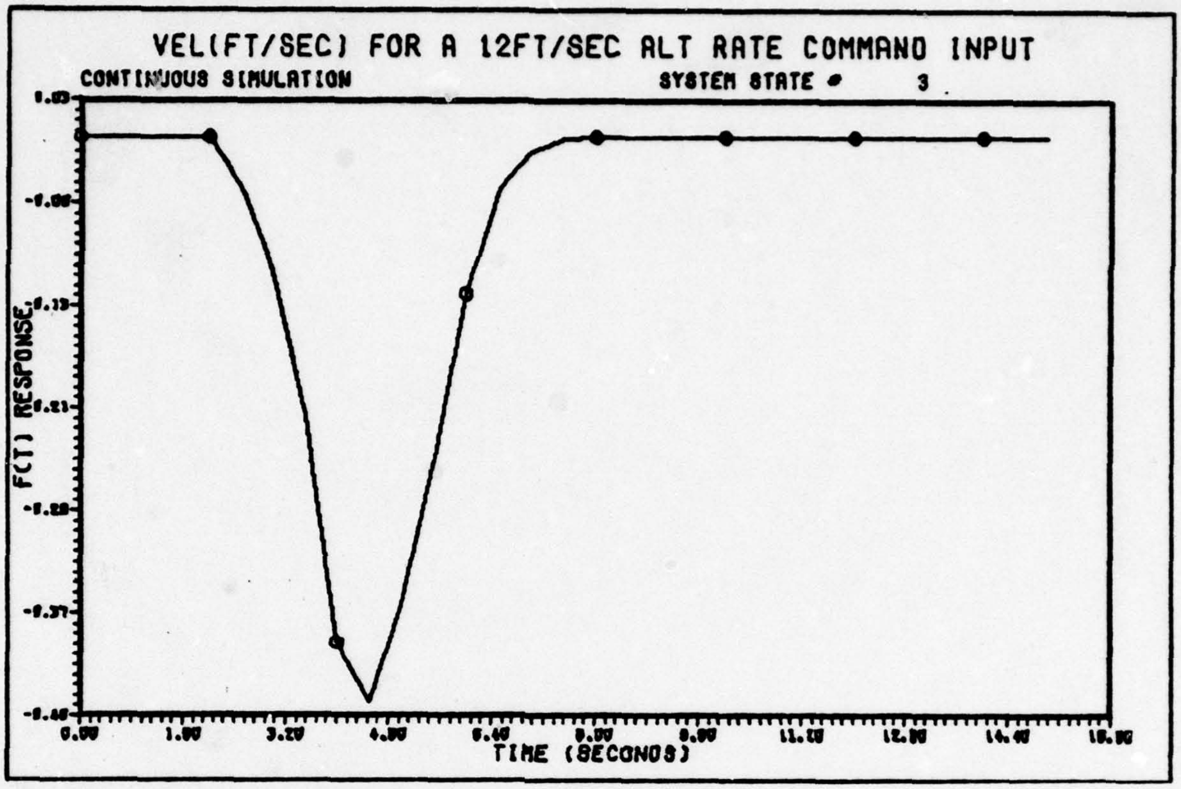
$$
\begin{bmatrix} -.174E-3, & -.1202 \\ -.516E-3, & -.5904 \\ .1215, & .4727 \end{bmatrix} \quad (112)
$$

and the resulting closed-loop matrix is given in Eq. C-11. All of the eigenvalues of the closed-loop system are at the origin and the closed-loop system has a Jordan Canonical form with one block of size 4 and two blocks of size 3, so the order of the minimal polynomial of the system is equal to 4. The state responses of the continuous-time simulation, with a command input vector given by Eqs. 82 thru 85, are plotted in Fig. 12 and the altitude rate command input and the altitude rate response are plotted in Fig. 13. As demonstrated by the plots, the closed-loop system tracks the command input vector in 4 sampling periods (8 seconds). Also, it is shown that the maximum elevator deflection is 0.0284 radian (1.62 degrees) and the maximum spoiler deflection is 0.1981 radian (11.35 degrees). In the steady-state the change in forward velocity (u) is zero and the spoiler control surface returns to its equilibrium position. The C-141 aircraft exhibits good transient response and the desired ability to track the command input vector with reasonable values of control surface deflections.



PITCH RATE(RAD) FOR A 12FT/SEC ALT. RATE COMM.
CONTINUOUS SIMULATION
SYSTEM STATE # 1
F(T) RESPONSE
TIME (SECONDS)
VEL * ANG OF ATTACK(FT-RAD/SEC):12FT/SEC ALT RATE
CONTINUOUS SIMULATION
SYSTEM STATE # 2
F(T) RESPONSE
TIME (SECONDS)

Figure 12a


VEL(FT/SEC) FOR A 12FT/SEC ALT RATE COMMAND INPUT
CONTINUOUS SIMULATION
SYSTEM STATE # 3
F(T) RESPONSE
TIME (SECONDS)

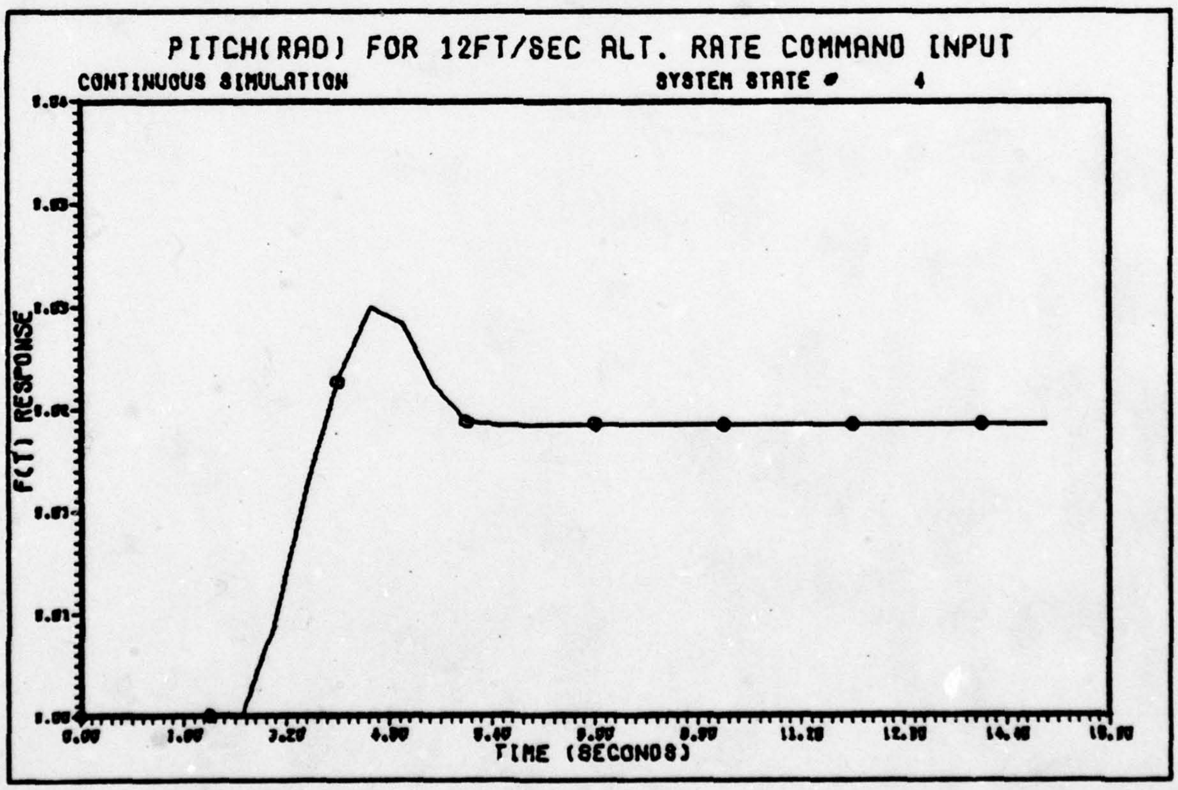

PITCH(RAD) FOR 12FT/SEC ALT. RATE COMMAND INPUT
CONTINUOUS SIMULATION
SYSTEM STATE # 4
F(T) RESPONSE
TIME (SECONDS)

Figure 12b

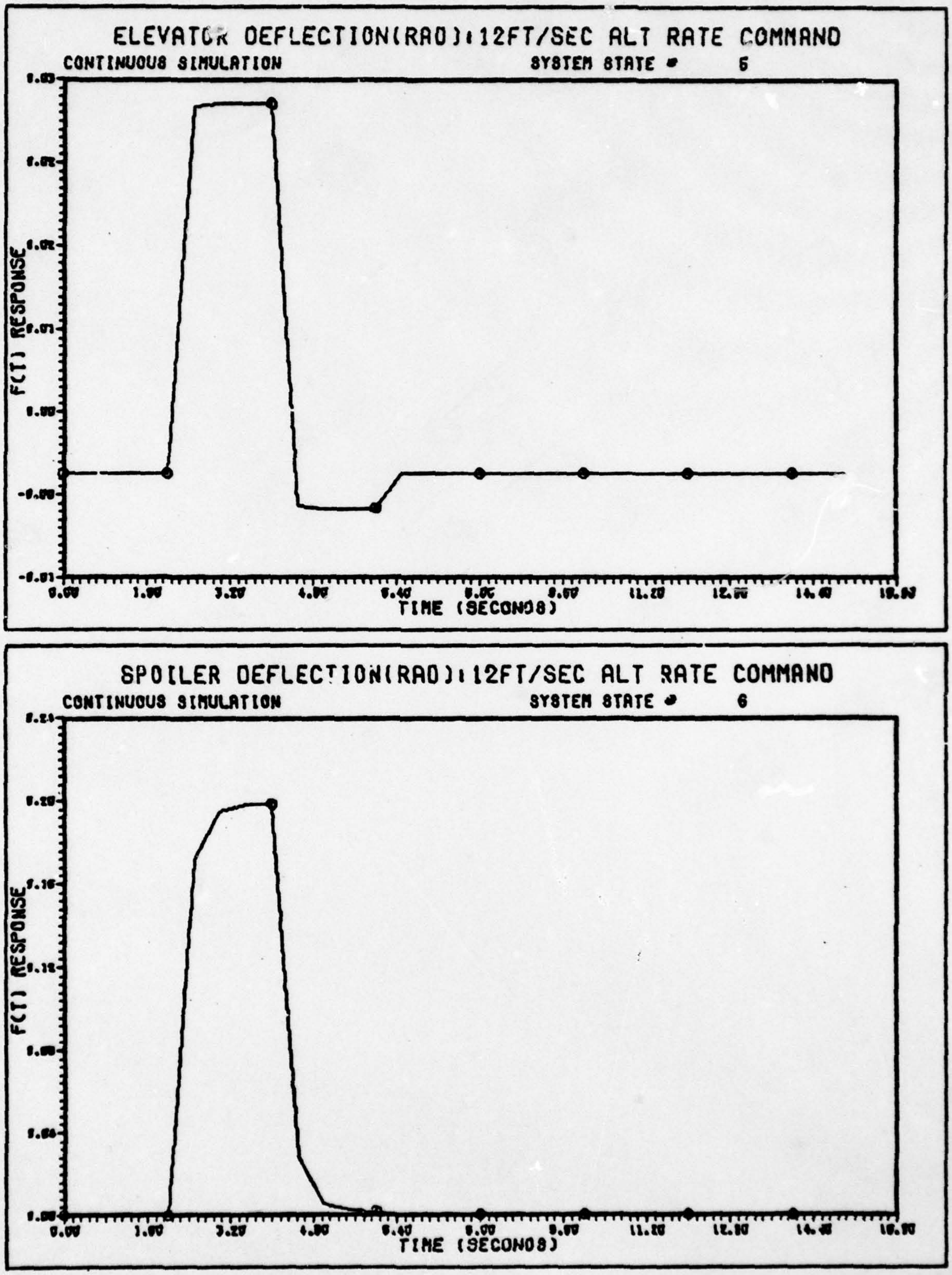

ELEVATOR DEFLECTION(RAD): 12FT/SEC ALT RATE COMMAND
CONTINUOUS SIMULATION
SYSTEM STATE # 6
F(T) RESPONSE
TIME (SECONDS)
SPOILER DEFLECTION(RAD): 12FT/SEC ALT RATE COMMAND
CONTINUOUS SIMULATION
SYSTEM STATE # 6
F(T) RESPONSE
TIME (SECONDS)

Figure 12c

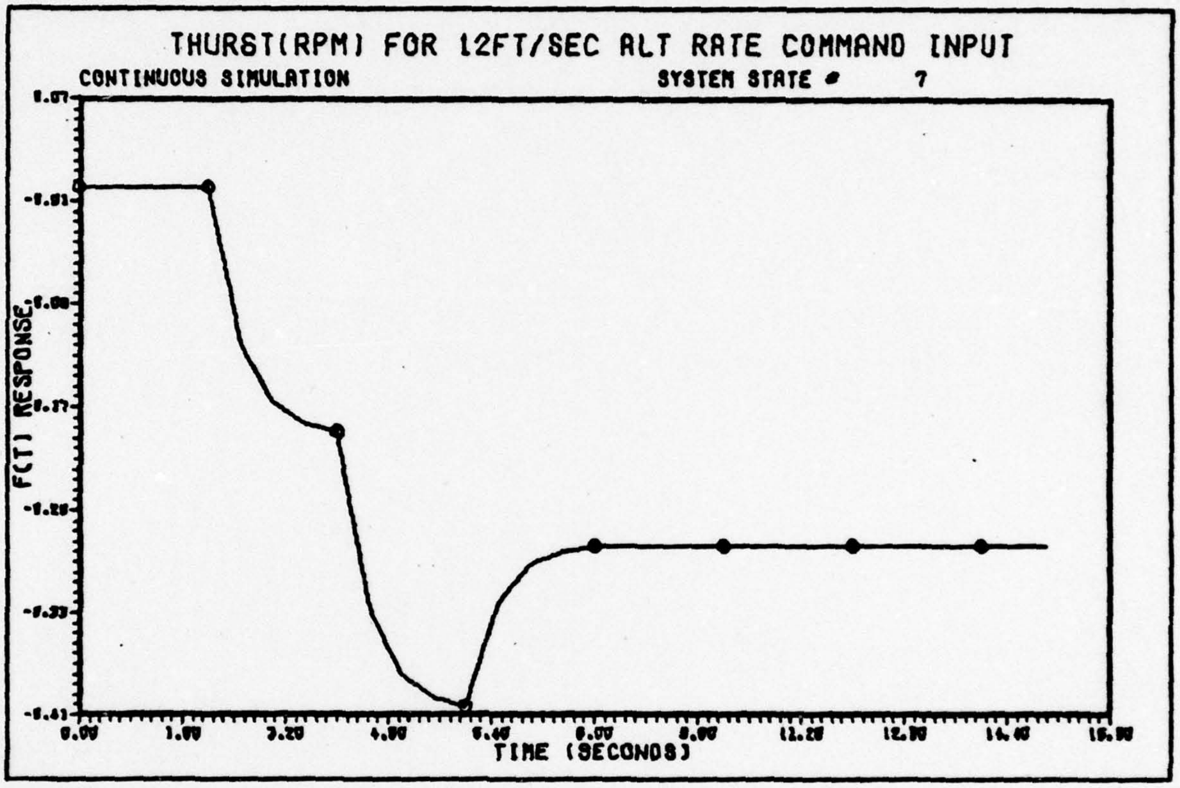

THURST(RPM) FOR 12FT/SEC ALT RATE COMMAND INPUT
CONTINUOUS SIMULATION
SYSTEM STATE # 7
F(T) RESPONSE
TIME (SECONDS)

Figure 12d

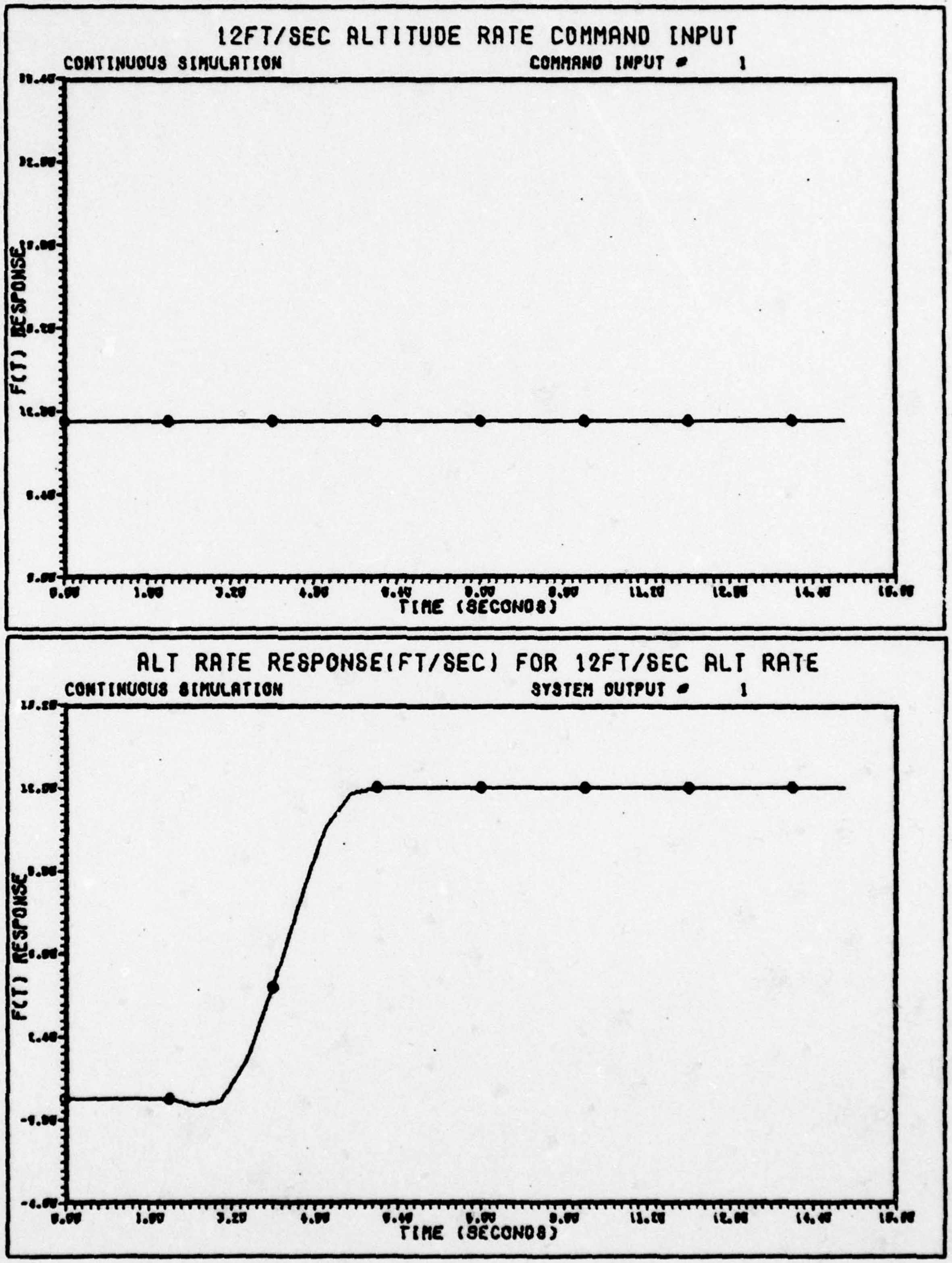

12FT/SEC ALTITUDE RATE COMMAND INPUT
CONTINUOUS SIMULATION
COMMAND INPUT # 1
F(T) RESPONSE
TIME (SECONDS)
ALT RATE RESPONSE(FT/SEC) FOR 12FT/SEC ALT RATE
CONTINUOUS SIMULATION
SYSTEM OUTPUT # 1
F(T) RESPONSE
TIME (SECONDS)

Figure 13

Tracker Design Conclusions

The entire eigenstructure assignment methodology and the addition of integral control to the closed-loop system is a powerful method for the design of digital flight controllers as demonstrated by the three time-optimal altitude rate tracker designs for the C-141 aircraft. In all three designs a sampling time of 2 seconds is used and, since the lengths of the eigenvector/generalized eigenvectors chains are set equal to the control indices {4,3,3} of the pair $[\bar{F},\bar{G}]$, the aircraft tracks the command input vector with a deadbeat, ripple-free response in 4 sampling periods (8 seconds). Table IV depicts some of the salient values of the system responses of the continuous-time simulation of all three designs.

TABLE IV

Tracker Characteristics for the Longitudinal Dynamics of the C-141 Aircraft for the Command Inputs

$\dot{h}$ = 12 ft/sec, u = 0 ft/sec

and

δ_{sp} = 0 radian

PARAMETER	LANDING	LOW ALTITUDE CRUISE	MEDIUM ALTITUDE CRUISE
$\lvert\delta_e\rvert_{max}$(rad)	.0133	.0234	.0284
$\lvert\delta_{sp}\rvert_{max}$(rad)	.2648	.1492	.1981
$\lvert\delta_{rpm}\rvert_{max}$(rpm)	.2697	.2773	.4039
$\lvert\dot{\theta}\rvert_{max}$ ((rad/sec)	.0294	.0157	.0163
$\lvert\alpha\rvert_{max}$ (rad)	.0232	.0118	.0144
$\lvert u\rvert_{max}$ (ft/sec)	2.110	.1059	.4404
$\lvert\theta\rvert_{max}$ (rad)	.0600	.0261	.0255
$\lvert\dot{h}\rvert_{max}$ (ft/sec)	12.35	12.01	12.0
$\lvert\delta_e\rvert_{ss}$ (rad)	0	0	0
$\lvert\delta_{sp}\rvert_{ss}$ (rad)	0	0	0
$\lvert\delta_{rpm}\rvert_{ss}$ (rpm)	.2697	.2725	.2786
$\lvert\dot{\theta}\rvert_{ss}$ (rad/sec)	0	0	0
$\lvert\theta\rvert_{ss}$(rad)	.0600	.0220	.0181
$\lvert\alpha\rvert_{ss}$ (rad)	0	0	0
$\lvert u\rvert_{ss}$(ft/sec)	0	0	0
$\lvert\dot{h}\rvert_{ss}$ (ft/sec)	12	12	12
Settling Time (sec)	8	8	8

VII. Conclusions and Recommendations

Conclusions

The combined use of the sampling time selection procedure, presented in Chapter II, and the method of entire eigenstructure assignment provides a powerful, time domain approach for the design of deadbeat controllers for multi-input, multi-output systems. The distinct relationship between system controllability and sampling time selection is an important factor in the selection of a suitable sampling time for the digital controller. Also, as mentioned in Chapter II, system constraints must be considered in the design of deadbeat controllers. In the case of tracker designs, the use of discrete integrators and comparators forces the closed-loop system to track a constant command input with a deadbeat, ripple-free response.

CESA, the interactive computer package for the design of digital controllers using the entire eigenstructure assignment methodology, meets all of the design requirements mentioned in Chapter III. It allows the user to completely analyze the structure of the state space representation of the plant and also to analyze the discrete control law for the closed-loop system. Since the program is totally interactive, the user can use an iterative design approach in the selection of an "optimum" control law. This attribute is considerably important, for it is usually necessary to repeat the synthesis procedure for a number of selected eigenvector/generalized eigenvector chains and to evaluate the resulting control law thru a continuous-time simulation which is a part of the program.

Both the regulator and tracker designs for the longitudinal dynamics of the C-141 aircraft are time-optimal with all control values within system limits. All of the tracker designs track the command input vector in a number of sampling periods with zero steady-state error. The designs were evaluated using a continuous-time simulation, where the plant is continuous with a digital computer as the controller. Plots of all of the design responses are included in the thesis.

Recommendations

The use of entire eigenstructure assignment and the proper selection of a sampling time for the design of deadbeat regulators and trackers are investigated in this thesis. Also, an interactive computer program is developed to aid in the design procedure. However, there is much more room for further work in all of the above areas. Some suggestions for further work in these areas are

1. The development of specific guidelines in the selection of the eigenvectors and generalized eigenvectors from the null space of $S(\lambda_i)$.
2. The modification of CESA by adding routines that determine the multivariable zeros of a system. This will allow further investigation into the affects of sampling time on system controllability, since the selection of a sampling time directly affects the position of the zeros of the discretized system. Also, in order to have a minimum phase control law the zeros of the discretized system must be inside the unit circle.
3. The addition of routines to CESA that allow the assignment of complex eigenvalues, using entire eigenstructure assignment. These routines are presently available in updated versions of FORTRAC (Ref. 3).

4. The addition of routines to CESA that allow for the design of linear multivariable discrete-time systems incorporating error-actuated dynamic controllers (Ref..17). These routines are presently being added to FORTRAC (Ref. 3).
5. The investigation of the use of the method of entire eigenstructure assignment in the rejection of random disturbances, such as a gust rejection analysis for an aircraft.
6. The investigation of tracking a C* input as discussed in Appendix D.

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APPENDIX A

USER'S MANUAL FOR CESA

MARCH 1979

CONTENTS

Overview of CESA

CESA, which stands for Complete EigenStructure Assignment is an interactive computer program that was created to aid the user in designing a state-space control law for a multi-input, multi-output system using the method of entire eigenstructure assignment. The following overview describes some of the highlights of the program.

*CESA aids in the design of three different control systems:

- A regulator is designed using the method of entire eigenstructure assignment. A full-order observer may also be designed if the plant states are inaccessable.
- A disturbance rejector is designed by augmenting the plant with integrators and comparators and then determining a control law for the augmented system using the method of entire eigenstructure assignment. If the plant states are inaccessable, a full-order observer may be developed.
- A tracker is designed by augmenting the plant with integrators and comparators and then determining the control law for the augmented system using the method of entire eigenstructure assignment. A full-order observer may be designed if the plant states are inaccessable.

*All of the control law designs may be evaluated by a continuous-time simulation of the closed-loop system. Calcomp plots of the system state and output response are available.

*A complete state-space analysis of a desired system may be performed.

*System matrices can be entered in either continuous or discrete form. The continuous system may be discretized using any desired sampling time.

*All input data can be saved on a memory file for future use. This prevents the laborous task of inputting the same data whenever the same system is under study.

*The program performs complete error detection and diagnostics, thus preventing the loss of input data due to abnormal termination.

1. Introduction to CESA

CESA, which stands for Complete EigenStructure Assignment, is an interactive computer program package that uses the method of entire eigenstructure assignmentin the design of control laws for multi-input, multi-output systems. The computer package contains sixty options and some special commands that give the user an iterative approach method in the design of control laws for regulators, disturbance rejectors, and trackers. The state-space representation of the system can be entered in either its continuous or discretized form. Once the system model is entered, a complete state-space analysis of the system model can be performed. A continuous-time simulation is available to evaluate the designed control laws, and Calcomp plots of the system state and output responses are available. The following should give the user all the necessary information for optimal use of the computer program package.

1.1 CESA'S Input Modes

CESA has two input modes in which it requests the user to input information: OPTION and DATA. Each mode has its own restrictions on allowable inputs and its own method of requesting information.

OPTION Mode. The OPTION mode is the main command mode of CESA and its prompt message is

OPTION>

Once the program is in this mode, the user is allowed to select any option, perform any command, modify any system matrix, list any system matrix, or list any design matrix. This mode is the main input mode of CESA since it allows the user complete access to the program and data files.

Multiple commands or options may be typed on one line. However all input items must be separated by a blank or a comma.

DATA Mode. CESA enters into the DATA mode when information is needed to perform an option. A particular prompt message is printed out for the user depending upon the information needed. There are two types of DATA modes. In one type of data mode the program requests a yes or no reply, and one of the two must by entered. The other type of DATA mode requires a numerical input and, unlike the other DATA mode, a user abort to the OPTION mode may be made from this data mode by entering a "$".

1.2 CESA Options

CESA contains sixty options that allow the user versatility in the design and analysis of control laws for regulators, disturbance rejectors, and trackers. The options are grouped together into four sets as follows:

- 0-15: Matrix Input Options
- 16-29: State-Space Analysis Options
- 30-45: Control Law Design Options
- 46-59: Simulation Options

The first option in each group lists the options of that group. There are also two auxilary options used in CESA. Option 60 updates all data in a backup memory file and option 100 gives a brief description of CESA. A complete list of the CESA options is given at the end of this manual.

1.3 CESA Matrices

CESA distinguishes between two different groups of matrices. One group, the inputtedsystem matrices, may be modified or listed anytime during the program execution. However, the second group, the resulting design matrices, may not be modified but they can be listed at any time during the program execution. Also, only the inputted system matrices are saved on a memory file that can be fed into CESA's data banks at a latter time. The following is a brief description of the matrices in CESA.

Inputted System Matrices

AMAT	--	Continuous plant matrix
BMAT	--	Continuous control input matrix
CMAT	--	Output matrix
CDMAT	--	Integrated Output matrix
RMAT	--	Continuous disturbance input matrix
ASDMAT	--	Sampled-data plant matrix
BSDMAT	--	Sampled-data control input matrix
RSDMAT	--	Sampled-data disturbance input matrix
EMAT	--	Sampled-data command input matrix

Resulting Design Matrices

AUGAMAT	--	Augmented plant matrix
AUGBMAT	--	Augmented control input matrix
AUGRMAT	--	Augmented disturbance input matrix
AUGEMAT	--	Augmented command input matrix
AUGCMAT	--	Augmented output matrix
QMAT	--	Initial matrix set up to determine the subspace of ker $S(\lambda_i)$
QTRNMAT	--	Matrix containing the null spaces of $S(\lambda_i)$
QCHMAT	--	Matrix containing the changed null space vectors
NULLMAT	--	Matrix containing just the null space vectors of $S(\lambda_i)$
VMAT	--	Matrix containing the selected eigenvectors and generalized eigenvectors $[\chi_i]$
WMAT	--	Matrix containing the remaining part of the selected vectors and generalized vectors from the null space of $S(\lambda_i)$. $[\omega_i]$
CONLAW	--	Feedback gain matrix: WMAT * VMAT^{-1} $[\Omega X^{-1}]$
QOBMAT	--	Initial matrix set up to determine the subspace of ker $S(\lambda_i)$ for the observer design
QOBCHAN	--	Matrix containing the changed null spaces of $S(\lambda_i)$ for the observer
OBNULSP	--	Null space of $S(\lambda_i)$ for the observer design.
OBVMAT	--	Matrix containing the selected eigenvectors and generalized eigenvectors for the observer design
OBWMAT	--	Matrix containing the remaining part of the selected vectors and generalized vectors from the null space of $S(\lambda_i)$ for the observer design

OBTRAN	--	Matrix containing the null spaces of $S(\lambda_i)$ for the observer design
ADCMAT	--	[A+LC] where A is the plant matrix, L is the observer feedback gain matrix, and C is the output matrix
CLMAT	--	Closed-Loop Matrix
ACOMP	--	Composite plant matrix
BCOMP	--	Composite control input matrix
ECOMP	--	Composite command input matrix
RCOMP	--	Composite disturbance input matrix
CNLCOMP	--	Composite control law matrix

2. Complete Description of CESA Options

In order to exploit CESA's assets in an optimum manner, it is necessary to have an understanding of the 60 options available in CESA. This manual is intended to provide all of the information needed to accomplish the above. Each of the following options described may be selected by simply typing in the option number while the program is in the option mode.

2.1 Matrix Input Options

Since a control law design can not be started without knowledge of the plant model, it is necessary to enter the state-space representation of the system for which the control law is to be designed. Therefore, CESA's matrix input options are an integral part of the program.

CESA allows the user to input either the continuous state-space representaion of the system

$$x(t) = AMAT\ x(t) + BMAT\ u(t) + RMAT\ d(t) \qquad (A-1)$$

$$y(t) = CMAT\ x(t)$$

$$\hat{y}(t) = CDMAT\ x(t)$$

where

AMAT -- Continuous plant matrix

BMAT -- Continuous control input matrix

RMAT -- Continuous disturbance input matrix

CMAT -- System output matrix

CDMAT -- Integrated system output matrix

or the discretized version of the state-space representation

$$x(kT+T) = ASDMAT\ x(kT) + BSDMAT\ u(kT) + RSDMAT\ d(kT)$$

$$y(kT) = CMAT\ x(kT) \qquad \text{(A-2)}$$

$$\hat{y}(kT) = CDMAT\ x(kT)$$

where

ASDMAT -- Sampled-data plant matrix

BASMAT -- Sampled-data control input matrix

RSDMAT -- Sampled-data disturbance input matrix

CMAT -- System output matrix

CDMAT -- Integrated output matrix

<u>Option 0</u>: <u>List options</u>

This option gives a list of all the matrix input options (0-15).

<u>Option 1</u>: <u>Recover all Data from MEMORY File</u>

During the execution of CESA, the user may update the MEMORY file with new data (Option 60), and at the termination of the program the MEMORY file is filled with data representing continuous and discrete versions of the system under study. Option 1 allows the user to recover all the data stored in this file. Thus, the user can terminate CESA, save the MEMORY file, and at a latter date restart the program, using option 1 to recover all data from the MEMORY file.

Options 2-6 Continuous System Matrix Input Options

These options allow the user to input AMAT, BMAT, RMAT, CMAT and CDMAT. All of these input options have identical formats. For example, if the user wishes to enter the command input matrix, BMAT, he would use option 3.

```
OPTION >3
INPUT OF ( BMAT) MATRIX:
ENTER MATRIX SIZE: ROWS,COLUMNS>  3,2
ENTER 3 ELEMENTS PER COLUMN:
COLUMN 1 > 0,1,0
COLUMN 2 > 0,0,1

  COL >   1           2
ROW
 1      0.          0.
 2      1.000       0.
 3      0.          1.000
```

And if he wants to enter the integrated output matrix, CDMAT, he would use option 6.

```
OPTION >6
INPUT OF ( CDMAT ) MATRIX:
ENTER MATRIX SIZE: ROWS,COLUMNS>  2,3
ENTER 3 ELEMENTS PER ROW:
ROW 1 > 1,1,0
ROW 2 > 0,0,1

  COL >   1           2           3
ROW
 1      1.000       1.000       0.
 2      0.          0.          1.000
```

Note that in the first example there are fewer columns than rows, and the user is requested to enter BMAT by columns. In the second example the number of columns is greater than the number of rows and the user is requested to enter CDMAT by rows.

Option 7 Help User Set Up Continuous State-Space Model of System

This option is used to input all the matrices needed for the continuous state-space representation. It first requests the user to input the number of states, number of control inputs, the number of disturbances and the number of outputs. Then the program determines the size of the matrices and requests the values for the elements of each matrix as in options 2 to 6. Also, the user is asked if there are any outputs that are to be integrated for either tracking or disturbance rejection. And if so, the number of integrated outputs are requested by the computer. Then the user is asked to input the elements of CDMAT as in options 2 to 6. The following is a typical example of option 7.

```
OPTION >7

THE STATE EQUATIONS TO BE ENTERED HAVE THE FORM:

  XDOT(T)=( AMAT )X(T) + ( BMAT )U(T) + ( RMAT )D(T)
    Y(T)=( CMAT )X(T)

AND (CDMAT) * X(T) ARE THE OUTPUTS TO BE PROTECTED
FROM DISTURBANCES OR TO TRACK COMMAND INPUTS

AND  X IS A VECTOR OF N STATE VARIABLES
     U IS A VECTOR OF M INPUTS
     D IS A VECTOR OF K DISTURBANCES
     Y IS A VECTOR OF L OUTPUTS

ENTER NO. OF STATES,INPUTS,OUTPUTS,DISTURBANCES >  3,2,2,2
ENTER AMAT WITH 3 ROWS AND 3 COLUMNS.

ENTER 3 ELEMENTS PER ROW:
ROW 1 > 0,1,0
ROW 2 > 0,0,1
ROW 3 > -6,-11,-6
```

COL >	1	2	3
ROW			
1	0.	1.000	0.
2	0.	0.	1.000
3	-6.000	-11.00	-6.000

ENTER BMAT WITH 3 ROWS AND 2 COLUMNS.

ENTER 3 ELEMENTS PER COLUMN:
COLUMN 1 > 0,1,0
COLUMN 2 > 0,0,1

COL >	1	2
ROW		
1	0.	0.
2	1.000	0.
3	0.	1.000

ENTER CMAT WITH 2 ROWS AND 3 COLUMNS.

ENTER 3 ELEMENTS PER ROW:
ROW 1 > 1,1,0
ROW 2 > 0,0,1

COL >	1	2	3
ROW			
1	1.000	1.000	0.
2	0.	0.	1.000

IS THERE A DISTURBANCE MATRIX,RMAT,--YES OR NO? > Y
ENTER RMAT WITH 3 ROWS AND 2 COLUMNS.

ENTER 3 ELEMENTS PER COLUMN:
COLUMN 1 > 1,0,0
COLUMN 2 > 0,1,0

COL >	1	2
ROW		
1	1.000	0.
2	0.	1.000
3	0.	0.

```
ARE THERE ANY OUTPUTS TO BE INTEGRATED EITHER FOR
DISTURBANCE REJECTION OR COMMAND INPUT TRACKING
ENTER YES OR NO ---> YES
NUMBER OF CONTROLS FOR SYSTEM EQUALS 2
THE NO. OF INTEGRATED OUTPUTS MUST BE LESS THAN OR EQUAL
TO THE NUMBER OF CONTROLS.
ENTER THE NUMBER OF INTEGRATED OUTPUTS ---> 2
 ENTER CDMAT WITH 2 ROWS AND 3 COLUMNS.

ENTER 3 ELEMENTS PER ROW:
ROW 1 > 1,1,0
ROW 2 > 0,0,1

  COL >   1           2           3
ROW
 1       1.000       1.000       0.
 2       0.          0.          1.000


THE STATE SPACE REPRESENTATION IS COMPLETE.
```

Options 8 - 10 Sampled-Data System Matrix Input Options

Options 8 through 10 allow the user to input ASDMAT, BSDMAT and RSDMAT. All of these options have the identical formats as options 2-6.

Option 11 Help User Set Up Sampled-Data State-Space Represenation of System

Option 11 is identical to option 7 except that in this case the sampled-data system representation is entered. The following is a typical example of option 11.

```
 OPTION >11

 THE EQUATIONS TO BE ENTERED HAVE THE FORM:
       X(K+1) = (ASDMAT)X(K) + (BSDMAT)U(K) + (RSDMAT)D(K)
         Y(K) = ( CMAT )X(K)
 AND (CDMAT) * X(K) ARE THE OUTPUTS TO BE PROTECTED
 FROM DISTURBANCES OR TO TRACK COMMAND INPUTS

 AND   X IS A VECTOR OF N STATE VARIABLES
       U IS A VECTOR OF M INPUTS
       D IS A VECTOR OF K DISTURBANCES
       Y IS A VECTOR OF L OUTPUTS

 ENTER NO. OF STATES,INPUTS,OUTPUTS,DISTURBANCES > 3,2,2,2
```

```
ENTER ASDMAT WITH 3 ROWS AND 3 COLUMNS.

ENTER 3 ELEMENTS PER ROW:
ROW 1 > .7474,.4530,.0735
ROW 2 > -.5774,-.4611,.0121
ROW 3 > -.0723,-.5735,-.1334

  COL >   1            2            3
ROW
 1       .7474        .4530        .7350E-01
 2      -.5774       -.4611        .1210E-01
 3      -.7230E-01   -.5735       -.1334


ENTER BSDMAT WITH 3 ROWS AND 2 COLUMNS.

ENTER 3 ELEMENTS PER COLUMN:
COLUMN 1 > .3261,.4530,-1.061
COLUMN 2 > .0421,.0735,.0121

  COL >   1            2
ROW
 1       .3261        .4210E-01
 2       .4530        .7350E-01
 3      -1.061        .1210E-01


ENTER CMAT WITH    2ROWS AND    3 COLUMNS.

ENTER 3 ELEMENTS PER ROW:
ROW 1 > 1,1,0
ROW 2 > 0,0,1

  COL >   1            2            3
ROW
 1       1.000        1.000        0.
 2       0.           0.           1.000

IS THERE A DISTURBANCE MATRIX,RMAT,--YES OR NO? > YES
ENTER RSDMAT WITH 3 ROWS AND 2 COLUMNS.

ENTER 3 ELEMENTS PER COLUMN:
COLUMN 1 > .9161,-.2526,-.4410
COLUMN 2 > .3261,.4530,-1.0610

  COL >   1            2
ROW
 1       .9161        .3261
 2      -.2526        .4530
 3      -.4410       -1.061
```

```
ARE THERE ANY OUTPUTS TO BE INTEGRATED FOR DISTURBANCE
REJECTION OR TO TRACK COMMAND INPUTS?
 ENTER YES OR NO YES
NUMBER OF CONTROLS EQUALS 2
NO. OF INTEGRATED OUTPUTS MUST BE LESS THAN OR EQUAL
TO THE NUMBER OF CONTROLS. ENTER THE NUMBER OF
INTEGRATED OUTPUTS----> 2
 ENTER CDMAT WITH 2 ROWS AND 3 COLUMNS.

ENTER 3 ELEMENTS PER ROW:
ROW 1 > 1,1,0
ROW 2 > 0,0,1

  COL >   1          2          3
ROW
 1       1.000      1.000      0.
 2       0.         0.         1.000


THE STATE-SPACE REPRESENTATION IS COMPLETE.
```

Options 12 - 15 Reserve Options

These options have been set aside in case future modification of CESA requires the input of additional matrices.

2.2 State-Space Analysis Options

In order to design a control law for a system or to evaluate a system's responses, it is necessary to analyse the state-space representation of the system. Options 17 through 29 perform different operations on the state-space matrices of the system.

Option 16 List Options

Option 16 lists all of the state-space analysis options.

Option 17 Compute Eigenstructure of AMAT

This option evaluates the eigenvalues of AMAT and determines the modal matrix of AMAT using Eispack routines (Ref. 14).

Option 18 Check Controllability of Continuous System

The controllability of the pair [AMAT, BMAT] is checked by this option.

Option 19 Check Observability of Continuous System

This option checks the observability of the pair [AMAT, CMAT].

Option 20 Compute Sampled-Data System

Option 20 computes the sampled-data representation of the system under study for some user inputted sampling time, T. The procedure uses the fact that

$$\text{ASDMAT} = \exp\ (\text{AMAT*T}) = Xe^{\Lambda T}X^{-1} \tag{A-3}$$

$$\text{BSDMAT} = \left(X \int_0^T e^{\Lambda\tau}\, d\tau\ X^{-1}\right)*(\text{BMAT}) \tag{A-4}$$

$$\text{RSDMAT} = \left(X \int_0^T e^{\Lambda\tau}\, d\tau\ X^{-1}\right)*(\text{RMAT}) \tag{A-5}$$

where T is the sampling time, X is the modal matrix for AMAT, and Λ is a diagonal matrix containing the eigenvalues of AMAT along its diagonal.

NOTE: Since the inverse modal matrix is used to compute the state transition matrix (e^{AT}), the sampled-data representation of a matrix with multiple eigenvalues cannot be computed using this option.

Option 21 Compute Eigenstructure of ASDMAT

This option evaluates the eigenvalues of ASDMAT and determines the modal matrix of ASDMAT using Eispack routines (Ref. 14).

Option 22 Check Controllability of the Sampled-Data System

Option 22 checks the controllability of the pair [ASDMAT, BSDMAT].

Option 23 Check Observability of the Sampled-Data System

This option determines whether the pair [ASDMAT, CMAT] is observable.

Option 24 Compute the Control Indices of the Sampled-Data System

Option 24 computes the control indices of the pair [ASDMAT, BMAT] by transforming the system into the Brunovsky cannonical form (Ref. 5), and the counting the size of the system subblocks. These are equal to the control indices of pair [ASDMAT, BMAT]. Also, using the control indices, the regular vectors of the controllability matrix are used to form an nxn "reduced" controllability matrix, where n is the number of states in the system and a regular vector is a linearly independent vector. The determinant and the eigenvalues of the "reduced" controllability matrix are evaluated to determine the "level of controllability" of the system.

Option 25 Compute the Observer Indices of the Sampled-Data Systems

This option transforms the pair [ASDMAT', CMAT'] into the Brunovsky cannonical form and then counts the sizes of the transformed system's subblocks which are equal to the observer indices of the system.

Option 26 Compute the Controllability of the Augmented System

The controllability of the pair [AUGAMAT, AUGBMAT] is computed by this option.

Option 27 Compute the Observability of the Augmented System

This option evaluates the observability of the pair [AUGMAT', AUGCMAT'].

Option 28 Compute the Control Indices of the Agumented System

This option is identical to option 24 except AUGAMAT replaces ASDMAT and AUGBMAT replaces BSDMAT.

Option 29 Compute the Observer Indices of the Augmented System

This option is identical to option 25 except AUGAMAT replaces ASDMAT and AUGCMAT replaces CMAT.

2.3 Control Law Calculation Options

CESA allows the user to design three different control laws: regulator; disturbance rejector; and tracker. In cases where the states are inaccessable, full-order observers may be designed. The method of entire eigenstructure assignment is used in all of the control law developments. Therefore, the user can assign both the eigenvalue spectrum and the associated eigenvector/generalized eigenvectors to the closed-loop system.

Options 31 thru 36 aid the user in developing continuous control laws for the above systems. Options 38 thru 43 allow the user to design discrete control laws for regulator, disturbance rejector and tracker systems.

Option 30 List Options

This option lists options 30 thru 45.

Option 31 Design Continuous Regulator

*NOTE: This option has not been completed at this writing.

Option 32 Design Continuous Regulator with Full-Order Observer

*NOTE: This option has not been completed at this writing.

Option 33 Design Continuous Disturbance Rejector

*NOTE: This option has not been completed at this writing.

Option 34 Design Continuous Disturbance Rejector with Full-Order Observer

*NOTE: This option has not been completed at this writing.

Option 35 Design Continuous Tracker

*NOTE: This option has not been completed at this writing.

Option 36 Design Continuous Tracker with Full-Order Observer

*NOTE: This option has not been completed at this writing.

Option 37 Reserve Option

Option 38 Design Sampled-Data Regulator

This option uses the entire eigenstructure assignment methodology in the design of the digital control law

$$u(kT) = K_1 \, x(kT) \tag{A-6}$$

where T is the sampling time, and K_1 is the state feedback matrix. The program accomplishes this in the following five steps:

1. Discretizes the continuous system, if needed, according to a user inputted sampling time, T.

2. Performs a complete state-space analysis on the system upon user request.

3. Allows the user to assign distinct real eigenvalues or multiple real eigenvalues and the associated eigenvector/generalized eigenvectors to the closed-loop system.

4. Determines the feedback gain matrix K_1 according to

$$\begin{aligned} K_1 &= [\omega_1, \omega_2, \ldots, \omega_n] \, [x_1, x_2, \ldots, x_n]^{-1} \\ &= \Omega \, X^{-1} \end{aligned} \tag{A-7}$$

where

$$\begin{bmatrix} \chi_i \\ \omega_i \end{bmatrix} \quad \varepsilon \quad \ker \; [ASDMAT - \lambda_i I, \; BSDMAT] \tag{A-8}$$

5. Forms the closed-loop matrix CLMAT where

$$CLMAT = [ASDMAT + BSDMAT^{*}K_1] \tag{A-9}$$

Option 39 Design Sampled-Data Regulator with Full-Order Observer

This option allows the user to design a digital controller for a regulator when the system states are inaccessable and the system is observable. Thus, for a linear multivariable sampled-data system governed by the discrete-time representation

$$x(kT+T) = ASDMAT\, x(kT) + BSDMAT\; u(kT) \tag{A-10}$$

and

$$y(kT) = CMAT\; x(kT) \tag{A-11}$$

with the states x(kT) observable in the output y(kT), it is possible to design a discrete-time observer which satisfies the matrix difference equation

$$\hat{x}(kT+T) = ASDMAT\; \hat{x}(kT) + BSDMAT\; u(kT) + L[C\hat{x}(kT)-y(kT)] \tag{A-12}$$

where $\hat{x}(kT)$ is the observer state vector (Ref. 5). Subtracting Eq. A-12 from Eq. A-10, using Eq. A-11, and designating the observer error states x(kT) by

$$\tilde{x}(kT) = x(kT) - \hat{x}(kT) \tag{A-13}$$

yields the observer error state equation

$$\tilde{x}(kT+T) = (ASDMAT + L*CMAT)\tilde{x}(kT) \qquad \text{(A-14)}$$

If the L matrix is chosen so that the eigenvalues of the matrix [ASDMAT+L*CMAT] are assigned within the unit circle, then

$$\lim_{k\to\infty} \tilde{x}(kT) = \lim_{k\to\infty} (ASDMAT + L*CMAT)^k = 0 \qquad \text{(A-15)}$$

for any initial condition.

Therefore, as $k\to\infty$, the observer states are equal to the sampled-data plant states and the observer states $\hat{x}(kT)$ may be used instead of the inaccessable plant states x(kT) in the control law

$$u(kT) = k\,\hat{x}(kT) \qquad \text{(A-16)}$$

The composite closed-loop system with the discrete-time observer and the control law of Eq. A-16 is represented by

$$\begin{bmatrix} x(kT+T) \\ x(kT+T) \end{bmatrix} = \begin{bmatrix} ASDMAT+BSDMAT*K_1 & ,-BSDMAT*K_1 \\ 0 & ,ASDMAT+L*CMAT \end{bmatrix} \begin{bmatrix} x(kT) \\ \tilde{x}(kT) \end{bmatrix} \qquad \text{(A-17)}$$

The program allows the user to determine both the K_1 matrix and the L matrix according to the procedure of option 38. Then it forms the composite system given by Eq. A-17 where

$$CLMAT = \begin{bmatrix} ASDMAT + BSDMAT*K_1 & ,-BSDMAT*K_1 \\ 0 & ,ASDMAT+L*CMAT \end{bmatrix} \qquad \text{(A-18)}$$

Option 40 Design Sampled-Data Disturbance Rejector

This option allows the user to design a digital control law of the form

$$u(kT) = K_1\ x(kT) + K_2\ z(kT) \qquad \text{(A-19)}$$

for the augmented discrete-time system

$$\begin{bmatrix} x(kT+T) \\ z(kT+T) \end{bmatrix} = \begin{bmatrix} ASDMAT, & 0 \\ TM, & I_p \end{bmatrix} \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix} + \begin{bmatrix} BSDMAT \\ 0 \end{bmatrix} u(kT) + \begin{bmatrix} RSDMAT \\ 0 \end{bmatrix} d(kT)$$

(A-20)

where z(kT) are the discrete-time integrators that are augmented to the system and the p chosen outputs are

$$M\ x(kT) \qquad \text{(A-20)}$$

where p must be less than or equal to m, the number of controls, in order to maintain controllability. This system rejects the piecewise-constant disturbance input vector d(kT) (Ref. 11). Once the augmented system is set up, the program procedes as in option 38 to form the closed-loop matrix

$$CLMAT = \begin{bmatrix} ASDMAT + BSDMAT*K_1, & BSDMAT * K_2 \\ TM & , \quad I_p \end{bmatrix} \qquad \text{(A-21)}$$

Option 41 Design Sampled-Data Disturbance Rejector with Full-Order Observer

This option allows the user to design a control law that rejects piecewise-constant disturbance inputs, d(kT), for a controllable and observable system with inaccessable states. An observer is designed for the system as in option 39. Then the control law used is

$$u(kT) = K_1\, x(kT) + K_2\, z(kT) \tag{A-22}$$

The composite closed-loop system for the disturbance rejector is

$$\begin{bmatrix} x(kT+T) \\ z(kT+T) \\ \tilde{x}(kT+T) \end{bmatrix} \begin{bmatrix} ASDMAT+BSDMAT*K_1, & BSDMAT*K_2, & -BSDMAT*K_1 \\ TM, & I_p, & 0 \\ 0 & 0 & ,ASDMAT+L*CMAT \end{bmatrix} \begin{bmatrix} x(kT) \\ z(kT) \\ \tilde{x}(kT) \end{bmatrix} +$$

$$\begin{bmatrix} RSDMAT \\ 0 \\ 0 \end{bmatrix} d(kT) \tag{A-23}$$

Option 42 Design Sampled-Data Tracker

Option 42 is identical to option 40 except that there is a command input instead of a disturbance input. Also, in this case the addition of discrete-time integrators and comparators forces the system to track the p piecewise-constant command inputs in the steady-state if the eigenvalues of the closed-loop system are assigned within the unit circle.

The control law designed for the sampled-data tracker is

$$u(kT) = K_1\, x(kT) + K_2\, z(kT) \tag{A-24}$$

and the composite system formed using the control law of Eq. A-24 is

$$\begin{bmatrix} x(kT+T) \\ z(kT+T) \end{bmatrix} = \begin{bmatrix} ASDMAT+BSDMAT*K_1, & BDMAT*K_2 \\ TM & , \ I_p \end{bmatrix} \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix} + \begin{bmatrix} 0 \\ EMAT \end{bmatrix} v(kT) \qquad \text{(A-25)}$$

where EMAT is a pxp diagonal matrix with -T along the diagonal and v(kT) is the piecewise constant command input,

Option 43 Design Sampled-Data Tracker with Full-Order Observer

Option 43 is used when the discretized states, x(kT), of the system are inaccessable but are observable in the output vector y(kT). In this case, a full-order observer is designed for the system as in option 38. Also, the digital control law becomes

$$u(kT) = K_1 \hat{x}(kT) + K_2 z(kT) \qquad \text{(A-26)}$$

and the composite closed-loop system is

$$\begin{bmatrix} x(kT+T) \\ z(kT+T) \\ \tilde{x}(kT+T) \end{bmatrix} = \begin{bmatrix} ASDMAT+BSDMAT*K_1, & BSDMAT*K_2, & -BSDMAT*K_1 \\ TM & , \ I_p & , \ 0 \\ 0 & , \ 0 & , ASDMAT+L*CMAT \end{bmatrix} \begin{bmatrix} x(kT) \\ z(kT) \\ \tilde{x}(kT) \end{bmatrix} + \begin{bmatrix} 0 \\ EMAT \\ 0 \end{bmatrix} v(kT)$$

(A-27)

Options 44 and 45 Reserve Option

2.4 Simulation Options

In order to evaluate the discrete controllers designed in options 28 thru 43, it is necessary to perform a continuous-time simulation on the closed-loop system with the control input, u(kT), held constant over each sampling period of the digital controller. Since it is necessary to determine the system state responses and outputs between sampling times, options 47 thru 52 discretize the continuous system with a sampling time less than or equal to T/12, where T is the sampling time of the digital controller. Then the following recursive formula is used

$$x(kT_s + T_s) + ASDMAT_{Ts}\, x(kT_s) + BSDMAT_{Ts} u(kT_s) + RSDMAT_{Ts}\, d(kT_s) \qquad \text{(A-28)}$$

where $T_s \leq T/12$ and $u(kT_s)$ and $d(kT_s)$ are held constant over the sampling interval T. Options 53 to 58 are discrete simulations and give the state and output responses only at the sampling interval, T. Thus, depending on the discrete simulation choosen, the recursive formula given by one of the following equations is used: Eq. A-10; Eq A-19; Eq. A-20; Eq A-23; Eq. A-25; or Eq. A-27.

Option 46 List Options

This option lists options 46-60

Option 47 Continuous-Time Simulation of Regulator with Discrete-Time Control Law

Option 47 performs a continuous-time simulation of a regulator with a discrete-time control law given by

$$u(kT) = K_1\, x(kT) \qquad \text{(A-29)}$$

Option 48 Continuous-Time Simulation of Regulator with Discrete-Time Control Law and Full-Order Observer

This option performs a continuous-time simulation of a regulator with a discrete-time full-order observer and a digital control law given by

$$u(kT) = K_1 \hat{x}(kT) \tag{A-30}$$

Option 49 Continuous-Time Simulation of Disturbance Rejector with Discrete-Time Control Law

A continuous-time simulation of a disturbance rejector with a discrete-time control law given by

$$u(kT) = K_1 x(kT) + K_2 z(kT) \tag{A-31}$$

is performed.

Option 50 Continuous-Time Simulation of Disturbance Rejector with Discrete-Time Control Law and Full-Order Observer

This option performs a continuous-time simulation of a disturbance rejector with a discrete-time full-order observer and a digital control law given by

$$u(kT) = K_1 \hat{x}(kT) + K_2 z(kT) \tag{A-32}$$

Option 51 Continuous-Time Simulation of Tracker with Discrete-Time Controller

Option 51 performs a continuous-time simulation of a tracker with a discrete-time control law given by

$$u(kT) + K_1 x(KT) + K_2 z(kT) \tag{A-33}$$

Option 52 Continuous-Time Simulation of Tracker with Discrete-Time Controller and Full-Order Observer

This option performs a continuous-time simulation of a tracker with a discrete-time full-order observer and a digital control law given by

$$u(kT) = K_1 \hat{x}(kT) + K_2 z(kT) \quad \text{(A-34)}$$

Option 53 Discrete-Simulation of Regulator

This option uses the recursive formula of Eq. A-10 to perform a discrete simulation of a regulator with a digital control law given by Eq. A-29.

Option 54 Discrete Simulation of Regulator with Full-Order Observer

A discrete simulation of a regulator with a full-order observer is performed, using the recursive formula given by Eq. A-17 and a digital control law given by Eq. A-30.

Option 55 Discrete Simulation of Disturbance Rejector

Option 55 performs a discrete simulation of a disturbance rejector, using the recursive formula given by Eq. A-20 and a digital control law given by Eq. A-31.

Option 56 Discrete Simulation of Disturbance Rejector with Full-Order Observer

This option performs a discrete simulation of a disturbance rejector with a full-order observer, using the recursive formula given by Eq. A-23 and a digital control law described by Eq. A-32.

Option 57 Discrete Simulation of Tracker

A discrete simulation is performed on a tracker using the recursive formula described by Eq. A-25 and a digital control law given by Eq. A-33.

Option 58 Discrete Simulation of Tracker with Full-Order Observer

This option performs a discrete simulation on a tracker with full-order observer, using the recursive formula described by Eq. A-27 and a control law described by Eq. A-26.

Option 59 Calcomp Plot of Simulation

This option allows the user to get Calcomp plots of the system state or output responses for any of the above simulations. The Calcomp plots are stored in a local file called PLOT which the user can later dispose to the plotter after CESA has been terminated.

The user is first asked which simulation option is to be plotted. Then, after that simulation has been completed and the responses printed out, the user is asked whether he wants the outputs plotted, the states plotted, the command inputs plotted or the disturbance inputs plotted. After the user has selected the type of response to be plotted, he is asked the number of the state, output, command or disturbance to be plotted. At this point the user is given the opportunity to control the physical size of the plot. Normally with FACTOR=1, the plot is 6x9 inches. Making FACTOR = 2 double the size of the plot. Setting FACTOR = 0.5 reduces the plot to half size. Finally the user is asked to input a plot title to be drawn above the plot. The title may be up to 50 characters long. A guide is printed to aid the user as shown below.

```
>(---------ENTER TITLE (50 CHARACTERS MAX)-----------)<
>EXAMPLE TRACKER OUTPUT #1 RESPONSE
 EXAMPLE TRACKER OUTPUT #1 RESPONSE
```

The program then allows the user to plot other responses or to terminate the option. An example plot with FACTOR=1 is shown in Fig. A-2 (see page 152).

2.4 Miscellaneous Options

This option allows the user to store all the system matrices on the file MEMORY during the option. This is a precautionary measure in case, for some unexpected reason, the program terminates abnormally.

Option 100 CESA Information

Option 100 gives a brief description of CESA and a group listing of options.

3.0 Special Commands

At the present time CESA has only two special commands; STOP; and ANSWER. These commands may be typed only in the option mode.

STOP

This command automatically updates the local file MEMORY, and prints out messages notifying the user of any local files that have been created during execution of the program. Then CESA is terminated.

ANSWER, OFF

When this command is typed in, all information is printed at the user's terminal.

ANSWER, ON

This command causes all program output, except prompt messages to the user, to be sent to a local file called ANSWER. When CESA is terminated this file can be routed to the line printer.

4.0 CESA OPTIONS

COMMAND - ATTACH, CESA, ID=AFIT

COMMAND - CESA

WELCOME TO CESA -- VERSION 1.0

TYPE 100 FOR HELP

TYPE STOP TO END PROGRAM

OPTION > 100

CESA IS AN INTERATIVE COMPUTER-AIDED DESIGN PROGRAM FOR DIGITAL & CONTINUOUS CONTROL LAW DEVELOPMENT USING COMPLETE EIGENSTRUCTURE ASSIGNMENT. IT CONTAINS 60 OPTIONS THAT ARE DIVIDED INTO 4 GROUPS AS FOLLOWS. OPTIONS 0, 16, 30, and 46 GIVE LISTINGS OF THE OPTIONS IN THESE RESPECTIVE GROUPS.

0-15: MATRIX INPUT OPTIONS

16-29: STATE-SPACE ANALYSIS OPTIONS

30-45: CONTROL LAW DESIGN OPTIONS

46-60: SIMULATION OPTIONS

OPTION> 0, 16, 30, 46

(0-15) MATRIX INPUT OPTIONS

0 *LIST OPTIONS

1 *RECOVER ALL DATA FROM FILE MEMORY

2 *AMAT CONTINUOUS PLANT MATRIX

3 *BMAT CONTINUOUS CONTROL INPUT MATRIX

4 *CMAT SYSTEM OUTPUT MATRIX

5 *RMAT CONTINUOUS DISTURBANCE INPUT MATRIX

6 *CDMAT SYSTEM INTEGRATED OUTPUT MATRIX

7 *HELP USER SET UP CONTINUOUS STATE-SPACE MODEL OF SYSTEM

8 *ASDMAT SAMPLED-DATA SYSTEM MATRIX

9 *BSDMAT SAMPLED-DATA CONTROL INPUT MATRIX

10	*RSDMAT SAMPLED-DATA DISTURBANCE INPUT MATRIX
11	*HELP USER SET UP SAMPLED-DATA STATE-SPACE MODEL OF SYSTEM
12	*RESERVED OPTION
13	*RESERVED OPTION
14	*RESERVED OPTION
15	*RESERVED OPTION
(16-29)	STATE-SPACE ANALYSIS OPTIONS
16	*LIST OPTIONS
17	*COMPUTE EIGENSTRUCTURE OF AMAT
18	*CHECK CONTROLLABILITY OF CONTINUOUS SYSTEM
19	*CHECK OBSERVABILITY OF CONTINUOUS SYSTEM
20	*COMPUTE SAMPLED-DATA SYSTEM
21	*COMPUTE EIGENSTRUCTURE OF ASDMAT
22	*CHECK CONTROLLABILITY OF SAMPLED-DATA SYSTEM
23	*CHECK OBSERVABILITY OF SAMPLED-DATA SYSTEM
24	*COMPUTE CONTROL INDICES OF SAMPLED-DATA SYSTEM
25	*COMPUTE OBSERVER INDICES OF SAMPLED-DATA SYSTEM
26	*COMPUTE CONTROLLABILITY OF AUGMENTED SYSTEM
27	*COMPUTE OBSERVABILITY OF AUGMENTED SYSTEM
28	*COMPUTE CONTROL INDICES OF AUGMENTED SYSTEM
29	*COMPUTE OBSERVER INDICES OF AUGMENTED SYSTEM
(30-45)	CONTROL LAW DESIGN OPTIONS
30	*LIST OPTIONS
31	*DESIGN CONTINUOUS REGULATOR
32	*DESIGN CONTINUOUS REGULATOR WITH FULL-ORDER OBSERVER
33	*DESIGN CONTINUOUS DISTURBANCE REJECTOR
34	*DESIGN CONTINUOUS DISTURBANCE REJECTOR WITH FULL-ORDER OBSERVER
35	*DESIGN CONTINUOUS TRACKER

36 *DESIGN CONTINUOUS TRACKER WITH FULL-ORDER OBSERVER

37 *RESERVED OPTION

38 *DESIGN SAMPLED-DATA REGULATOR

39 *DESIGN SAMPLED-DATA REGULATOR WITH FULL-ORDER OBSERVER

40 *DESIGN SAMPLED-DATA DISTURBANCE REJECTOR

41 *DESIGN SAMPLED-DATA DISTURBANCE REJECTOR WITH FULL-ORDER OBSERVER

42 *DESIGN SAMPLED-DATA TRACKER

43 *DESIGN SMAPLED-DATA TRACKER WITH FULL-ORDER OBSERVER

44 *RESERVED OPTION

45 *RESERVED OPTION

(46-60) SIMULATION ROUTINES

46 *LIST OPTIONS

47 *CONTINUOUS-TIME SIMULATION OF REGULATOR WITH DISCRETE-TIME CONTROL LAW

48 *CONTINUOUS-TIME SIMULATION OF REGULATOR WITH FULL-ORDER OBSERVER AND DISCRETE-TIME CONTROL LAW

49 *CONTINUOUS-TIME SIMULATION OF DISTURBANCE REJECTOR WITH DISCRETE-TIME CONTROL LAW

50 *CONTINUOUS-TIME SIMULATION OF DISTURBANCE REJECTOR WITH DISCRETE-TIME CONTROL LAW AND FULL-ORDER OBSERVER

51 *CONTINUOUS-TIME SIMULATION OF TRACKER WITH DISCRETE-TIME CONTROL LAW

52 *CONTINUOUS-TIME SIMULATION OF TRACKER WITH FULL-ORDER OBSERVER AND DISCRETE-TIME CONTROL LAW

53 *DISCRETE-TIME SIMULATION OF REGULATOR

54 *DISCRETE-TIME SIMULATION OF REGULATOR WITH FULL-ORDER OBSERVER

55 *DISCRETE-TIME SIMULATION OF DISTURBANCE REJECTOR

56 *DISCRETE-TIME SIMULATION OF DISTURBANCE REJECTOR WITH FULL-ORDER OBSERVER

57 *DISCRETE-TIME SIMULATION OF TRACKER

58 *DISCRETE-TIME SIMULATION OF TRACKER WITH FULL-ORDER OBSERVER

59 *CALCOMP PLOT OF SIMULATION

60 *UPDATE

5.0 Tracker Design

In this section a digital control law is developed for the continuous system that is entered in the example for option 7. Options 20, 42, 59, and 57 are used to design the tracker. The following is the actual computer printout of the design and simulation. Also, the two Calcomp plots that are set up during the program are included at the end of the simulation.

OPTION 20

OPEN LOOP EIGENVALUES

REAL	IMAGINARY
-1.000	0.
-2.000	0.
-3.000	0.

REAL PART OF MODAL MATRIX OF AMAT

COL > ROW	1	2	3
1	.1741	.2667	.6731
2	-.1741	-.5733	-2.019
3	.1741	1.067	6.058

IMAG PART OF MODAL MATRIX OF A

COL > ROW	1	2	3
1	0.	0.	0.
2	0.	0.	0.
3	0.	0.	0.

ENTER SAMPLING TIME > 1
TSAMP= 1.

Z-PLANE EIGENVALUES

	REAL PART	IMAG. PART
1	.3679	0.
2	.1353	0.
3	.4979E-01	0.

IS THERE A DISTURBANCE MATRIX(RMAT)?--YES OR NO > Y

ASDMAT

COL > ROW	1	2	3
1	.7474	.4530	.7350E-01
2	-.4410	-.6104E-01	.1205E-01
3	-.7350E-01	-.5735	-.1334

BSDMAT

COL > ROW	1	2
1	.3261	.4210E-01
2	.4530	.7350E-01
3	-1.061	.1205E-01

RSDMAT

COL > ROW	1	2
1	.9161	.3261
2	-.2526	.4530
3	-.4410	-1.061

```
OPTION >42
 HAS SAMPLED DATA SYSTEM BEEN ENTERED
ENTER YES OR NO ---> YES
T = 1.
ENTER THE NUMBER OF INTEGRATIONS DESIRED FOR EACH OF
THE 2 OUTPUTS THAT ARE TO TRACK THE 2
COMMAND INPUTS--->1,1
THE NUMBER OF COMMAND INPUTS EQUALS 2
```

AUGMENTED SAMPLED MATRIX A:AUGAMAT

COL > ROW	1	2	3	4	5
1	.7474	.4530	.7350E-01	0.	0.
2	-.4410	-.6106E-01	.1205E-01	0.	0.
3	-.7230E-01	-.5735	-.1334	0.	0.
4	1.000	1.000	0.	1.000	0.
5	0.	0.	1.000	0.	1.000

AUGMENTED SAMPLED MATRIX B:AUGBMAT

COL > ROW	1	2
1	.3261	.4210E-01
2	.4530	.7350E-01
3	-1.061	.1205E-01
4	0.	0.
5	0.	0.

AUGMENTED SAMPLED MATRIX E:AUGEMAT

COL > ROW	1	2
1	0.	0.
2	0.	0.
3	0.	0.
4	-1.000	0.
5	0.	-1.000

```
DO YOU WANT TO CHECK THE CONTROLLABILITY AND THE
CONTROL INDICIES OF THE AUGMENTED SYSTEM
ENTER YES OR NO ---> YES
 EIGENVALUES OF AUGMENTED SYSTEM(AUGAMAT)
   REAL PART      IMAGINARY PART

 .100000E+01  0.

 .100000E+01  0.

 .497871E-01  0.

 .135335E+00  0.

 .367879E+00  0.

    SYSTEM IS CONTROLLABLE

DO YOU WISH TO HAVE THE BRUNOVSKY CANONICAL FORM OF THE
SYSTEM PRINTED OUT ? ENTER YES OR NO: Y
```

SYSTEM PLANT MATRIX IN BRUNOVSKY CANONICAL FORM

COL >	1	2	3	4	5
ROW					
1	0.	1.000	0.	0.	0.
2	0.	0.	1.000	0.	0.
3	.1090	-.7023	1.593	-.1542E-01	.1542E-01
4	0.	0.	0.	0.	1.000
5	-.4458	-.2174	.6632	.4031E-01	.9597

SYSTEM CONTROL MATRIX IN BRUNOVSKY CANONICAL FORM

COL >	1	2
ROW		
1	0.	0.
2	0.	0.
3	1.000	.1622
4	0.	0.
5	0.	1.000

CONTROLLABILITY INDICIES

N 1 = 3
N 2 = 2

THE DETERMINANT OF THE CONTROLLABILITY MATRIX = -.003394449713119

EIGENVALUES OF CONTROLLABILITY MATRIX

REAL PART	IMAGINARY PART
-.982012E+00	0.
-.512270E-01	.458447E+00
-.512270E-01	-.458447E+00
.127380E+00	.423788E-02
.127380E+00	-.423788E-02

```
***A TRAMP = 1.
ENTER THE # OF DIFFERENT EIGENVALUES TO BE ASSIGNED
TO THE CLOSED-LOOP SYSTEM....> 1
ENTER THE EIGENVALUES TO BE ASSIGNED TO THE CLOSED LOOP
SYSTEM ---> 0.0
()
```

EIGENVALUE= 0.0000

QMAT

COL >	1	2	3	4	5
ROW					
1	.7474	.4530	.7350E-01	0.	0.
2	-.4410	-.4106E-01	.1205E-01	0.	0.
3	-.7230E-01	-.5735	-.1334	0.	0.
4	1.000	1.000	0.	1.000	0.
5	0.	0.	1.000	0.	1.000
6	1.000	0.	0.	0.	0.
7	0.	1.000	0.	0.	0.
8	0.	0.	1.000	0.	0.
9	0.	0.	0.	1.000	0.
10	0.	0.	0.	0.	1.000
11	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.

COL >	6	7
ROW		
1	.3261	.4210E-01
2	.4530	.7350E-01
3	-1.061	.1205E-01
4	0.	0.
5	0.	0.
6	0.	0.
7	0.	0.
8	0.	0.
9	0.	0.
10	0.	0.
11	1.000	0.
12	0.	1.000

DIMENSION OF NULL SPACE=   2

QTRAN

COL >	1	2	3	4	5
ROW					
1	1.000	0.	0.	0.	0.
2	0.	1.000	0.	0.	0.
3	0.	0.	1.000	0.	0.
4	0.	0.	0.	1.000	0.
5	0.	0.	0.	0.	1.000
6	3.352	3.033	2.325	0.	0.
7	-5.170	-7.946	-4.981	0.	0.
8	0.	0.	0.	0.	0.
9	1.818	4.913	2.656	1.000	0.
10	0.	0.	0.	0.	1.000
11	2.566	4.088	1.592	0.	0.
12	0.	0.	0.	0.	0.

COL >	6	7
ROW		
1	0.	0.
2	0.	0.
3	0.	0.
4	0.	0.
5	0.	0.
6	.2714E-01	-.3920
7	-.1086	.8617
8	1.000	0.
9	.1614	-.4697
10	-1.000	0.
11	-.2560E-01	-.4277
12	0.	1.000

DO YOU WISH TO CHANGE THE POSITIONS OF THE NULL SPACE
VECTORS? ENTER YES OR NO---> YES
ENTER THE COLUMN NUMBERS OF THE TWO NULL VECTORS TO BE
INTERCHANGED---> 6,7
DO YOU WANT TO SWITCH ANY MORE NULL SPACE VECTORS?
ENTER YES OR NO---> NO

QCHMAT

COL >	1	2	3	4	5
ROW					
1	1.000	0.	0.	0.	0.
2	0.	1.000	0.	0.	0.
3	0.	0.	1.000	0.	0.
4	0.	0.	0.	1.000	0.
5	0.	0.	0.	0.	1.000
6	3.352	3.033	2.325	0.	0.
7	-5.170	-7.946	-4.981	0.	0.
8	0.	0.	0.	0.	0.
9	1.818	4.913	2.656	1.000	0.
10	0.	0.	0.	0.	1.000
11	2.566	4.088	1.592	0.	0.
12	0.	0.	0.	0.	0.

COL >	6	7
ROW		
1	0.	0.
2	0.	0.
3	0.	0.
4	0.	0.
5	0.	0.
6	-.3920	.2714E-01
7	.8617	-.1086
8	0.	1.000
9	-.4697	.1614
10	0.	-1.000
11	-.4277	-.2560E-01
12	1.000	0.

```
DO YOU WANT TO CHANGE THE NULL SPACE VECTORS?
ENTER YES OR NO----> YES
TWO NULL SPACE VECTORS ARE CHANGED AT A TIME ACCORDING TO
THE FOLLOWING EQUATIONS:
NOTE P(I,J)=Q(I,J) INITIALLY

Q(I,MV1)=P(I,MV1) + C1 * P(I,MV2)

Q(I,MV2)=P(I,MV1) + C2 * P(I,MV2)

WHERE: MV1 $ MV2 ARE THE COLUMN NUMBERS OF THE NULL
 SPACE VECTORS TO BE CHANGED.

       C1 $ C2 ARE ARBITRARY CONSTANTS
ENTER MV1,MV2,C1,C2 6,7,-1,1
```

GENMAT

COL > ROW	1	2	3	4	5
1	1.000	0.	0.	0.	0.
2	0.	1.000	0.	0.	0.
3	0.	0.	1.000	0.	0.
4	0.	0.	0.	1.000	0.
5	0.	0.	0.	0.	1.000
6	3.352	3.033	2.325	0.	0.
7	-5.170	-7.946	-4.581	0.	0.
8	0.	0.	0.	0.	0.
9	1.818	4.943	2.656	1.000	0.
10	0.	0.	0.	0.	1.000
11	2.566	4.088	1.592	0.	0.
12	0.	0.	0.	0.	0.

COL > ROW	6	7
1	0.	0.
2	0.	0.
3	0.	0.
4	0.	0.
5	0.	0.
6	-.4192	-.3649
7	1.050	.6731
8	-1.000	1.000
9	-.6311	-.3082
10	1.000	-1.000
11	-.4021	-.4533
12	1.000	1.000

```
DO YOU WANT TO CHANGE THE NULL SPACE VECTORS?
ENTER YES OR NO----> NO
ENTER THE SIZE OF THE EIGENVECTOR/GENERALIZED
EIGENVECTOR CHAIN TO BE CALCULATED FROM EACH
OF THE 2 NULL SPACES ---> 3,2
```

NULL SPACE

COL >	1	2	3	4	5
ROW					
1	-.4192	1.050	-1.000	-.6311	1.000
2	-.3649	.6731	1.000	-.3082	-1.000

COL >	6	7
ROW		
1	-.4021	1.000
2	-.4533	1.000

CHECK

COL >	1	2	3	4	5
ROW					
1	.1776E-14	.3329E-14	.1066E-13	-.3553E-14	0.
2	.2665E-14	.3553E-14	.3553E-14	0.	0.

DO YOU WISH TO ADD A NULL SPACE VECTOR TO A GENERALIZED
VECTOR ? ENTER YES OR NO-----> NO

GENERALIZED EIGENVECTOR

COL >	1	2	3	4	5
ROW					
1	-.9642	-.1464	-1.000	.4795	2.000
2	2.778	-7.770	1.000	4.684	-2.000

COL >	6	7
ROW		
1	1.224	1.000
2	2.934	1.000

GENERALIZED EIGENVECTOR CHECK

COL >	1	2	3	4	5
ROW					
1	-.4192	1.050	-1.000	-.6311	1.000
2	-.3649	.6731	1.000	-.3082	-1.000

DO YOU WISH TO ADD A NULL SPACE VECTOR TO A GENERALIZED
VECTOR ? ENTER YES OR NO-----> NO

GENERALIZED EIGENVECTOR

ROW \ COL >	1	2	3	4	5
1	-6.420	12.18	-1.000	-5.280	3.000
2	-12.29	43.07	1.000	-26.09	-3.000

ROW \ COL >	6	7
1	-5.067	1.000
2	-23.50	1.000

GENERALIZED EIGENVECTOR CHECK

ROW \ COL >	1	2	3	4	5
1	-.9642	-.1464	-1.000	.4795	2.000
2	2.778	-7.770	1.000	4.684	-2.000

VHAT

ROW \ COL >	1	2	3	4	5
1	-.4192	-.9642	-6.420	-.3649	2.778
2	1.050	-.1464	12.18	.6731	-7.770
3	-1.000	-1.000	-1.000	1.000	1.000
4	-.6311	.4795	-5.280	-.3082	4.684
5	1.000	2.000	3.000	-1.000	-2.000

MATRIX VHAT INVERSE

ROW \ COL >	1	2	3	4	5
1	-4.163	-3.143	-3.740	-3.357	-3.302
2	-1.128	-1.156	.3244	-1.193	.3203
3	.4356	.6985	.2555E-01	.9179	.5234E-01
4	-4.598	-3.842	-1.765	-4.275	-2.356
5	-.2569	.2305	-.6245	.6374	-.5720

SYSTEM CONTROL LAW: CONLAW

ROW \ COL >	1	2	3	4	5
1	-.5890	-1.281	.7270	-.9477	.0252
2	-9.710	-7.222	-5.780	-7.275	-5.056

THE AVERAGE ABSOLUTE CONTROLLER GAIN =4.021304531912
ARE YOU SATISFIED WITH THIS CONTROL LAW ?
ENTER YES OR NO---->
YES

THE DISCRETE COMPOSITE MATRIX:ACOMP

ROW \ COL >	1	2	3	4	5
1	.7474	.4530	.7350E-01	0.	0.
2	-.4410	-.6104E-01	.1205E-01	0.	0.
3	-.7230E-01	-.5735	-.1334	0.	0.
4	1.000	1.000	0.	1.000	0.
5	0.	0.	1.000	0.	1.000

THE DISCRETE COMPOSITE MATRIX:BCOMP

COL >	1	2
ROW		
1	.3261	.4210E-01
2	.4530	.7250E-01
3	-1.061	.1200E-01
4	0.	0.
5	0.	0.

THE DISCRETE COMP. CONTROL LAW:CNLCOMP

COL >	1	2	3	4	5
ROW					
1	-.5890	-1.281	.7270	-.9477	.8252
2	-9.710	-7.222	-5.780	-7.275	-5.856

BCOMP * CNLCOMP (BK)

COL >	1	2	3	4	5
ROW					
1	-.6008	-.7217	-.6241E-02	-.6153	.2257E-01
2	-.9805	-1.111	-.9543E-01	-.9640	-.5655E-01
3	.5079	1.272	-.8411	.9179	-.9462
4	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.

CLOSED LOOP MATRIX:CLMAT

COL >	1	2	3	4	5
ROW					
1	.1466	-.2686	.6726E-01	-.6153	.2257E-01
2	-1.422	-1.172	-.8338E-01	-.9640	-.5655E-01
3	.4356	.6985	-.9744	.9179	-.9462
4	1.000	1.000	0.	1.000	0.
5	0.	0.	1.000	0.	1.000

CLOSED LOOP EIGENVALUES

REAL PART	IMAGINARYPART
-.151077E-04	.261681E-04
-.151077E-04	-.261681E-04
.302155E-04	0.
.497330E-13	.587841E-07
.497300E-13	-.587841E-07

ENTER THE POWER TO WHICH THE CLOSED LOOP MATRIX SHOULD BE RAISED3

CLOSED LOOP MATRIX: CLMAT X 3

COL >	1	2	3	4	5
ROW					
1	-.3553E-14	-.7105E-14	-.7772E-14	0.	-.1144E-13
2	.8082E-13	.1030E-12	-.2220E-14	.9592E-13	.8438E-14
3	-.1137E-12	-.1296E-12	.1743E-13	-.1421E-12	.6217E-14
4	-.6750E-13	-.7016E-13	.6217E-14	-.6750E-13	.4441E-15
5	.7105E-13	.8882E-13	-.1332E-13	.8171E-13	-.3775E-14

```
OPTION >59
IT IS POSSIBLE TO SPEED UP THE FOLLOWING SIMULATION
BY ONLY STORING THE MEASURED OUTPUTS AND THE
DISTURBANCES OR COMMANDS DEPENDING ON THE OPTION
SELECTED.  DO YOU WISH TO HAVE ONLY THE MEASURED
OUTPUTS AND EITHER THE DISTURBANCES OR COMMANDS
PRINTED OUT.  ENTER YES OR NO---> NO
ENTER THE SIMULATION OPTION NUMBER TO BE PLOTTED
 ---->:57
 ENTER THE FINAL TIME FOR THE SIMULATION WHERE THE
 FINAL TIME (TF) IS LESS THAN (<) TSAMP * 200 10
ENTER THE INITIAL CONDITIONS OF THE 3 PLANT STATES
 ----> 0,0,0


INITIAL CONDITIONS OF PLANT


0.        0.        0.
ENTER THE INITIAL CONDITIONS OF THE 2
     INTEGRATOR STATES
 -----> 0,0


     THE INITIAL CONDITIONS OF THE INTEGRATORS


0.        0.
SAMPLING INTERVAL FOR CONTINUOUS SIMULATION(TDEL)= .08333333333333
 ENTER THE TOTAL NUMBER OF COMMAND CHANGES
 BETWEEN 1 AND 10 --->1
ENTER THE TYPE OF INPUT COMMAND  1-STEP
                                 2-RAMP
FOR COMMAND 1 AT INTERVAL 1 ---> 1
ENTER THE MAGNITUDE OF THE STEP INPUT--->1
ENTER THE TYPE OF INPUT COMMAND  1-STEP
                                 2-RAMP
FOR COMMAND 2 AT INTERVAL 1 ---> 1
ENTER THE MAGNITUDE OF THE STEP INPUT--->2
ENTER ONE OF THE SUBOPTIONS FOR THE DESIRED
SIMULATION - 1-DISCRETE
             2-CONTINUOUS
-----> 2


   CONTINUOUS SIMULATION

 THE SIMULATION IS COMPLETE. THE FOLLOWING VALUES CAN
 BE PRINTED OUT:STATES,COMMANDS,MEASURED OUTPUTS,
 DISTURBANCES,INTEGRATORS,OBSERVER,CONTROLS,OBSERVER
 ERROR. YOU WILL BE REQUESTED TO PICK OUT ONE OR MORE
 OF THE ABOVE ACCORDING TO THE SYSTEM SIMULATED.

STATES: ENTER YES OR NO-->
YES
```

TIME STATES

0.	0.	0.	0.
.2500	0.	0.	0.
.5000	0.	0.	0.
.7500	0.	0.	0.
1.000	0.	0.	0.
1.250	.1325E-01	.2005	2.340
1.500	.1170	.6247	2.263
1.750	.3154	.9346	1.598
2.000	.5701	1.077	.9744
2.250	.7919	.6957	.9277
2.500	.9229	.3677	1.297
2.750	.9842	.1394	1.692
3.000	1.000	.9023E-12	2.000
3.250	1.000	-.3162E-12	2.000
3.500	1.000	-.7843E-12	2.000
3.750	1.000	-.8740E-12	2.000
4.000	1.000	-.8029E-12	2.000
4.250	1.000	-.2007E-12	2.000
4.500	1.000	.6395E-13	2.000
4.750	1.000	.2593E-12	2.000
5.000	1.000	.3544E-12	2.000
5.250	1.000	.2656E-12	2.000
5.500	1.000	.1801E-12	2.000
5.750	1.000	.1359E-12	2.000
6.000	1.000	.1066E-12	2.000
6.250	1.000	.1421E-12	2.000
6.500	1.000	.1750E-12	2.000
6.750	1.000	.1972E-12	2.000
7.000	1.000	.2096E-12	2.000
7.250	1.000	.1945E-12	2.000
7.500	1.000	.1821E-12	2.000
7.750	1.000	.1741E-12	2.000
8.000	1.000	.1739E-12	2.000
8.250	1.000	.1776E-12	2.000
8.500	1.000	.1794E-12	2.000
8.750	1.000	.1767E-12	2.000
9.000	1.000	.1750E-12	2.000

COMMANDS: ENTER YES OR NO--->
YES

TIME COMMANDS

0.	1.000	2.000
.2500	1.000	2.000
.5000	1.000	2.000
.7500	1.000	2.000
1.000	1.000	2.000
1.250	1.000	2.000
1.500	1.000	2.000
1.750	1.000	2.000
2.000	1.000	2.000
2.250	1.000	2.000
2.500	1.000	2.000
2.750	1.000	2.000
3.000	1.000	2.000
3.250	1.000	2.000
3.500	1.000	2.000
3.750	1.000	2.000
4.000	1.000	2.000
4.250	1.000	2.000
4.500	1.000	2.000
4.750	1.000	2.000
5.000	1.000	2.000
5.250	1.000	2.000
5.500	1.000	2.000
5.750	1.000	2.000
6.000	1.000	2.000
6.250	1.000	2.000
6.500	1.000	2.000
6.750	1.000	2.000
7.000	1.000	2.000
7.250	1.000	2.000
7.500	1.000	2.000
7.750	1.000	2.000
8.000	1.000	2.000
8.250	1.000	2.000
8.500	1.000	2.000
8.750	1.000	2.000
9.000	1.000	2.000

INTEGRATORS: ENTER YES OR NO--->
.YES

TIME	INTEGRATORS	
0.	0.	0.
.2500	0.	0.
.5000	0.	0.
.7500	0.	0.
1.000	-1.000	-2.000
1.250	-1.000	-2.000
1.500	-1.000	-2.000
1.750	-1.000	-2.000
2.000	-2.000	-4.000
2.250	-2.000	-4.000
2.500	-2.000	-4.000
2.750	-2.000	-4.000
3.000	-1.353	-5.026
3.250	-1.353	-5.026
3.500	-1.353	-5.026
3.750	-1.353	-5.026
4.000	-1.353	-5.026
4.250	-1.353	-5.026
4.500	-1.353	-5.026
4.750	-1.353	-5.026
5.000	-1.353	-5.026
5.250	-1.353	-5.026
5.500	-1.353	-5.026
5.750	-1.353	-5.026
6.000	-1.353	-5.026
6.250	-1.353	-5.026
6.500	-1.353	-5.026
6.750	-1.353	-5.026
7.000	-1.353	-5.026
7.250	-1.353	-5.026
7.500	-1.353	-5.026
7.750	-1.353	-5.026
8.000	-1.353	-5.026
8.250	-1.353	-5.026
8.500	-1.353	-5.026
8.750	-1.353	-5.026
9.000	-1.353	-5.026

INTEGRATED OUTPUTS: ENTER YES OR NO--->
.YES

TIME	INTEGRATED OUTPUTS	
0.	0.	0.
.2500	0.	0.
.5000	0.	0.
.7500	0.	0.
1.000	0.	0.
1.250	.2138	2.340
1.500	.7417	2.263
1.750	1.250	1.598
2.000	1.647	.9744
2.250	1.408	.9277
2.500	1.291	1.297
2.750	1.124	1.692
3.000	1.000	2.000
3.250	1.000	2.000
3.500	1.000	2.000
3.750	1.000	2.000
4.000	1.000	2.000
4.250	1.000	2.000
4.500	1.000	2.000
4.750	1.000	2.000
5.000	1.000	2.000
5.250	1.000	2.000
5.500	1.000	2.000
5.750	1.000	2.000
6.000	1.000	2.000
6.250	1.000	2.000
6.500	1.000	2.000
6.750	1.000	2.000
7.000	1.000	2.000
7.250	1.000	2.000
7.500	1.000	2.000
7.750	1.000	2.000
8.000	1.000	2.000
8.250	1.000	2.000
8.500	1.000	2.000
8.750	1.000	2.000
9.000	1.000	2.000

MEASURED OUTPUTS: ENTER YES OR NO--->
YES

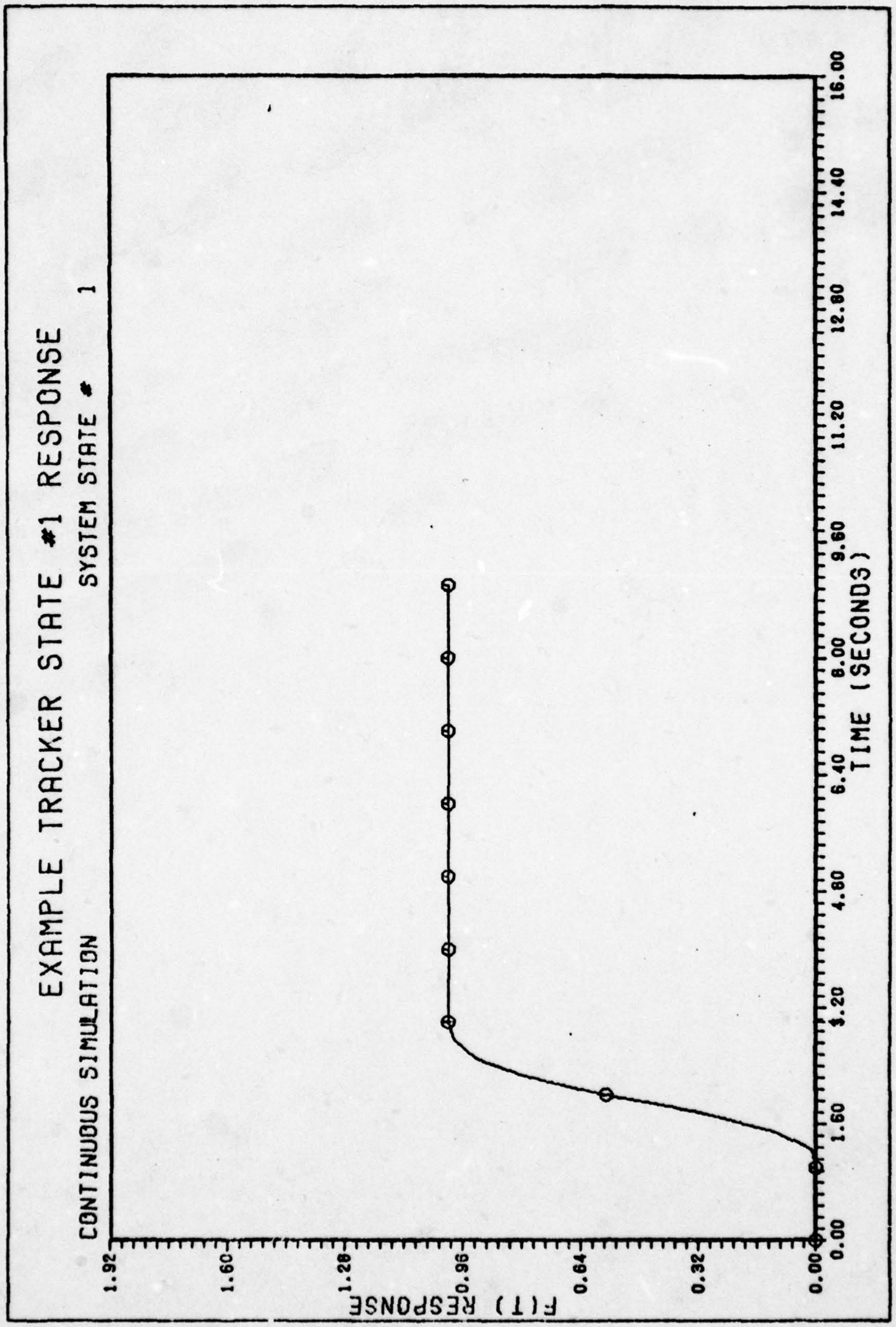
TIME	MEASURED OUTPUTS	
0.	0.	0.
.2500	0.	0.
.5000	0.	0.
.7500	0.	0.
1.000	0.	0.
1.250	.2138	2.340
1.500	.7417	2.263
1.750	1.250	1.598
2.000	1.647	.9744
2.250	1.408	.9277
2.500	1.291	1.297
2.750	1.124	1.692
3.000	1.000	2.000
3.250	1.000	2.000
3.500	1.000	2.000
3.750	1.000	2.000
4.000	1.000	2.000
4.250	1.000	2.000
4.500	1.000	2.000
4.750	1.000	2.000
5.000	1.000	2.000
5.250	1.000	2.000
5.500	1.000	2.000
5.750	1.000	2.000
6.000	1.000	2.000
6.250	1.000	2.000
6.500	1.000	2.000
6.750	1.000	2.000
7.000	1.000	2.000
7.250	1.000	2.000
7.500	1.000	2.000
7.750	1.000	2.000
8.000	1.000	2.000
8.250	1.000	2.000
8.500	1.000	2.000
8.750	1.000	2.000
9.000	1.000	2.000

CONTROLS : ENTER YES OR NO--->
.YES

TIME	CONTROLS	
0.	0.	0.
.2500	0.	0.
.5000	0.	0.
.7500	0.	0.
1.000	-.7027	18.99
1.250	-.7027	18.99
1.500	-.7027	18.99
1.750	-.7027	18.99
2.000	-2.412	19.03
2.250	-2.412	19.03
2.500	-2.412	19.03
2.750	-2.412	19.03
3.000	-2.000	18.00
3.250	-2.000	18.00
3.500	-2.000	18.00
3.750	-2.000	18.00
4.000	-2.000	18.00
4.250	-2.000	18.00
4.500	-2.000	18.00
4.750	-2.000	18.00
5.000	-2.000	18.00
5.250	-2.000	18.00
5.500	-2.000	18.00
5.750	-2.000	18.00
6.000	-2.000	18.00
6.250	-2.000	18.00
6.500	-2.000	18.00
6.750	-2.000	18.00
7.000	-2.000	18.00
7.250	-2.000	18.00
7.500	-2.000	18.00
7.750	-2.000	18.00
8.000	-2.000	18.00
8.250	-2.000	18.00
8.500	-2.000	18.00
8.750	-2.000	18.00
9.000	-2.000	18.00

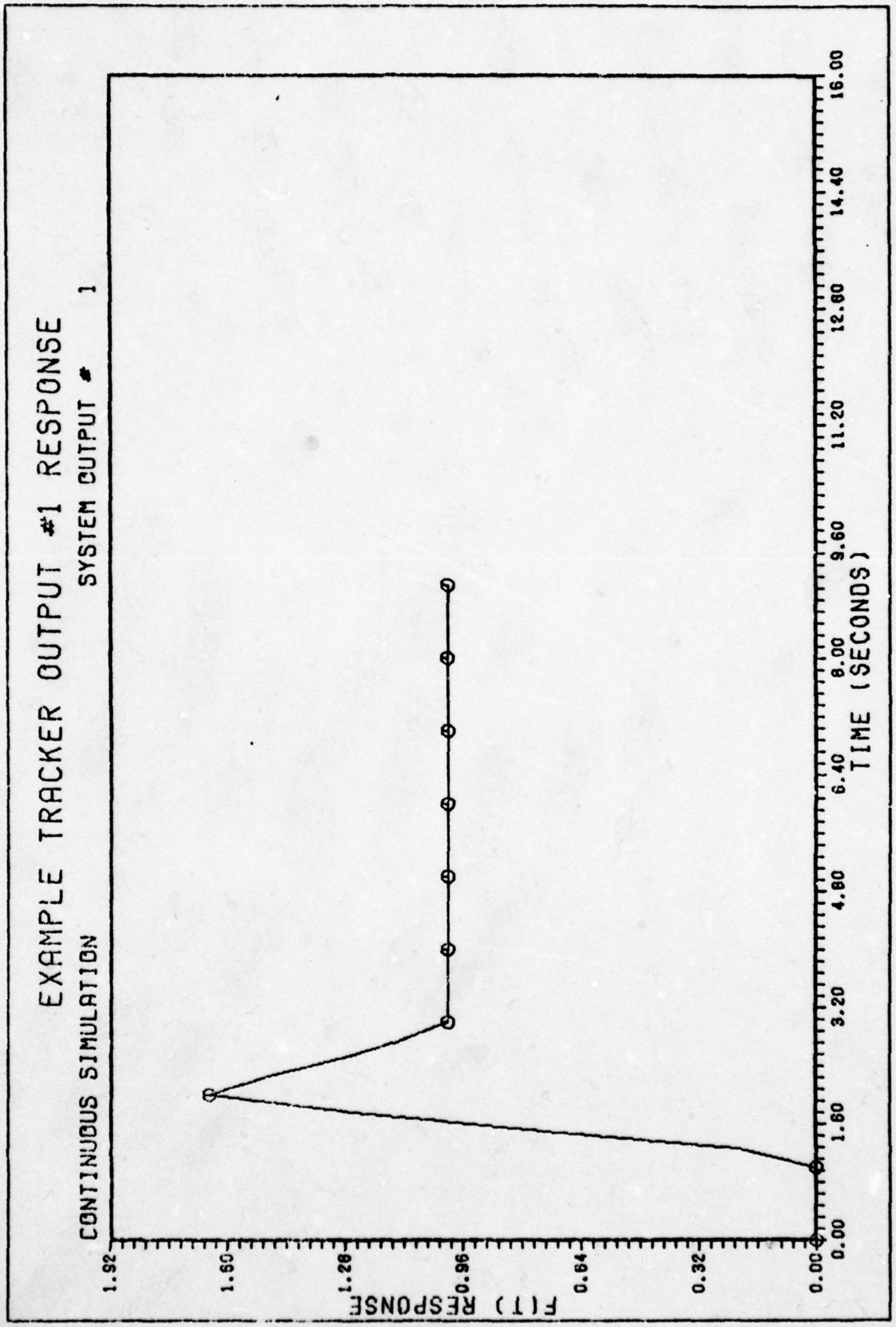
```
ENTER THE NUMBER CORRESPONDING TO ONE OF THE FOLLOWING
TO BE PLOTTED:  1-COMMAND INPUT (V)
                2-DISTURBANCE INPUT
                3-OUTPUT (Y)
                4-SYSTEM STATE
------> 4
THERE ARE 7 SYSTEM STATES. ENTER THE NUMBER OF THE
SYSTEM STATE TO BE PLOTTED---> 1
THE NORMAL VALUE FOR THE MAGNIFICATION FACTOR
IS 1.0 . ENTER THE DESIRED VALUE FOR THE
MAGNIFICATION FACTOR---> 1
>(-------ENTER TITLE (50 CHARACTERS MAX)-----------)<
>EXAMPLE TRACKER STATE #1 RESPONSE
  EXAMPLE TRACKER STATE #1 RESPONSE
DO YOU WANT ANOTHER STATE PLOT ?
ENTER YES OR NO---> NO
DO YOU WANT ANYMORE PLOTS ?
ENTER YES OR NO--->
 YES
ENTER THE NUMBER CORRESPONDING TO ONE OF THE FOLLOWING
TO BE PLOTTED:  1-COMMAND INPUT (V)
                2-DISTURBANCE INPUT
                3-OUTPUT (Y)
                4-SYSTEM STATE
------> 3
THERE ARE 2 SYSTEM OUTPUTS. ENTER THE NUMBER OF THE
SYSTEM OUTPUT TO BE PLOTTED--->1
THE NORMAL VALUE FOR THE MAGNIFICATION FACTOR
IS 1.0 . ENTER THE DESIRED VALUE FOR THE
MAGNIFICATION FACTOR---> 1
>(-------ENTER TITLE (50 CHARACTERS MAX)-----------)<
> EXAMPLE TRACKER OUTPUT #1 RESPONSE
  EXAMPLE TRACKER OUTPUT #1 RESPONSE
DO YOU WANT ANOTHER SYSTEM OUTPUT PLOT ?
ENTER YES OR NO---->NO
DO YOU WANT ANYMORE PLOTS ?
ENTER YES OR NO--->
 NO

OPTION >STOP
ALL INFO IN CESA HAS BEEN SAVED IN LOCAL FILE MEMORY.
     STOP
```



EXAMPLE TRACKER STATE #1 RESPONSE
CONTINUOUS SIMULATION
SYSTEM STATE # 1
F(T) RESPONSE
0.00
0.32
0.64
0.96
1.28
1.60
1.92
0.00
1.60
3.20
4.80
6.40
8.00
9.60
11.20
12.80
14.40
16.00
TIME (SECONDS)

Figure A-1



EXAMPLE TRACKER OUTPUT #1 RESPONSE
CONTINUOUS SIMULATION
SYSTEM OUTPUT # 1
F(T) RESPONSE
TIME (SECONDS)
0.00
1.60
3.20
4.80
6.40
8.00
9.60
11.20
12.80
14.40
16.00
0.00
0.32
0.64
0.96
1.28
1.60
1.92

Figure A-2

APPENDIX B

PROGRAMMER'S MANUAL FOR CESA

MARCH 1979

Appendix B

1. Introduction

This manual provides documentation to assist in future modifications and additions to CESA. This manual provides a description of the overlay structure, subroutines, and mass storage files of CESA that is essential to any further alteration of the program.

The reader is assumed to have a working knowledge of the external operations of this program. A study of the Users Manual for CESA should provide the necessary background to understand the internal parts of the program presented in this manual.

2. Description of Overall Structure

The purpose of creating CESA is to provide the user with an iterative design tool for the design of control laws for regulators, disturbance rejectors, and trackers. Therefore, it is necessary that the program have the capabilities of designing the control laws and evaluating the system responses. Also, the user must have the ability to redesign an unacceptable control law without having to completely reenter all pertinant data. Thus, it is necessary to store all information and for the program to be interactive.

In order for CESA to fullfill these requirements, it is necessary to use special space saving techniques so that CESA can fit within the $60k_8$ of memory that is available on INTERCOM. The two space saving techniques employed in CESA's development are: overlays and mass storage.

2.1 How Overlays Work

Overlays are used to reduce the storage required for a large program by splitting the program into modules. Each module is a separate program and all the modules are linked together through a main executive overlay. The main executive overlay is always in core and controls the program segments called the primary overlays. When a primary overlay is needed, it is called into central memory by the main executive overlay. Only one primary overlay may be in central memory with the executive. However, each primary overlay may control a group of overlays called secondary overlays. It is possible to have the executive overlay, one primary, and one secondary overlay in core at one time.

Data is passed between overlays through common statements or mass storage files. In order for data stored in a common to remain accessable throughout the program execution, the common statement must be in the main executive overlay.

The CDC FORTRAN REFERENCE MANUAL and the CDC LOADER REFERENCE MANUAL contain more detailed information on overlays.

2.2 CESA's Overlay Structure

One executive overlay, 14 primary overlays, and 16 secondary overlays are used in CESA.

The main executive overlay initializes and stores the data in common blocks. Whenever an overlay is needed, it is called by the main executive overlay. A decision making routine in the main executive overlay decides when a particular primary overlay is to be called. This overlay is discussed further in Section 3.

The primary overlays, secondary overlays, and associated subroutines perform all the actual calculations in CESA. Each primary overlay is responsible for a main operation that may require calling up secondary overlays and subroutines to complete the operation. All subroutines are packed behind their associated overlay so that they take up memory only when their associated overlay is in main core. Primary and secondary overlays are discussed in Section 4 and Subroutines are discussed in Section 5.

2.3 CESA's Random Access Mass Storage Files

CESA uses two random-access mass storage files: MEMAUX; and SIMMAS. These random-access files allow the program, through the use of special read and write statements, to store data anywhere on that file and then, at a latter time, to recover the data.

The MEMAUX file stores all matrices used in the program other than the unaugmented continuous and discrete system matrices. The random-access files use an index to keep track of the storage spaces used on the file. Table B1 lists the name and the index number for the matrices stored on the MEMAUX file. Also listed in Table B1 is the location on the one-dimensional index array, INMASS(I), that keeps track of the matrix location and the locations on the two-dimensional array, MATMEM(I,J), that keeps track of the current size of the matrices.

The SIMMAS file is used to store all simulation responses. SIMMIN(I) is the index array used to keep track of the location of the different system responses. Table B2 gives the location spaces for the system responses.

TABLE B2

System Response Location On SIMMAS File

INDEX NUMBER	MATRIX	RESPONSE	SIMMIN(I)
1	Y	States	SIMMIN(1)
2	YC	Outputs	SIMMIN(2)
3	YCDT	Integrated Outputs	SIMMIN(3)
4	YD	Disturbances	SIMMIN(4)
5	YV	Commands	SIMMIN(5)
6	ROE	Observer Error	SIMMIN(6)
7	ROB	Observer States	SIMMIN(7)
8	TI	Integrator	SIMMIN(8)
9	R	Controls	SIMMIN(9)

More information on random-access mass storage files may be obtained from the CDC FORTRAN REFERENCE MANUAL.

3.0 Description of CESA's Main Executive Overlay

CESA's main executive overlay controls the calling of primary overlays and therefore, also controls the flow of data to these routines. Also, CESA's common storage locations are in this main overlay. Thus, the main overlay is the most important overlay in CESA and should be thoroughly understood before any major modifications to CESA are performed.

3.1 The Program Statement

The program statement is important because it controls how much buffer length is allocated to reading and writing from the specified tapes used in the program. Thus, if additional memory is needed somewhere else in the program, the buffer length for different tapes can be reduced. However, there is a tradeoff between reducing the buffer length and increasing Input/Output time (I/O). For example, the Input/Output buffer length for the ANSWER file is set to 500_8 since there is a large amount of information sent to this file, while the seldom used DOODLE file's buffer length is set to 100_8. Another point to remember is that there are no restrictions on the amount of I/O time that is used on INTERCOM. Therefore, the only penalty incurred for increasing I/O time is a slow program response. Additional information on program statements is available in the CDC FORTRAN REFERENCE MANUAL.

3.2 CESA's Common Data Base

Labeled commons are used in CESA for communication between overlays. However, any information or data that must be stored in a labeled common for the duration of the program, must be stored in a labeled common in the executive overlay. Otherwise it will be lost when the particular overlay it originates in is returned from central memory.

3.3 Description of the Main Executive Overlay

CESA's main executive overlay controls the selection of primary overlays. This section is intended to describe how CESA performs this selection and other miscellaneous operations.

Upon execution of CESA all variables located in the common data base are initialized. Also, the two mass storage files MEMAUX and MEMSIM are opened. CESA, then enters the option mode at statement 11110 and calls program READER (overlay 6) to receive and interpret the user's input. READER compiles the user's input, stores the commands in coded form in the array MCOMM, sets the MPT pointer to the first command, and returns control to the executive overlay at statement number 11111.

Statement 11111 is the main control node for the program. All the primary overlays return control to this point. From this node, the program determines where control should be transferred next. CESA uses a three level sorter to accomplish this. A detailed description of this three level sorter may be found in Ref. 9.

4. Description of CESA's Overlays.

The source listing of CESA's primary overlays contains comment statements which are intended to make the programs self-explanatory to the user. This section contains an overview of each of the overlays.

4.1 Overlay (1,0) -- Program Update

Program UPDATE performs two main functions: writes information contained in CESA's common data base to the local file MEMORY, and reads information on the MEMORY file into the common data base.

Upon selection of option 60, UPDATE is called by the main executive overlay. The MEMORY file is rewound and pertainent variables that are stored in CESA's common data base are written to the file. A number code, 979, is written to the tape so that the MEMORY file can be identified.

Control is returned to the main executive overlay when this task is completed.

A reverse procedure takes place when option 1 is selected. The MEMORY file is rewound and tested for the proper number code, 979. If this code is not correct, an error message is printed, otherwise the information on the file is read into the common data base.

When the command STOP is encountered, UPDATE performs option 91 and terminates CESA.

4.2 Overlay (2,0) -- Program MATRIX

Overlay (2,0) is a short executive overlay which calls two secondary overlays which are needed to aid the user in entering the system matrices.

If a continuous system matrix is to be entered, program MATRIX calls program MATCON (Overlay (2,1)). Also, if a discrete system matrix is to be entered, program MATSD (Overlay (2,2)) is called. Sections 4.3 and 4.4 describe these two secondary overlays.

4.3 Overlay (2,1) -- Program MATCON

Program MATCON is responsible for executing options that deal with inputting continuous system matrices. It uses subroutine MATIN which is discussed in Section 5.

4.4 Overlay (2,2) -- Program MATSD

This program is used when an option is selected to input a sampled-data system matrix. MATSD uses subroutine MATIN, which is duscussed in section 5.

4.5 Overlay (4,0) -- Program INDICIE

Program INDICIE is a short executive routine that determines the controllability and observability of a system and transfers control to two secondary overlays is the control or observer indices of a system are to be calculated. Program TRANSFAB (Overlay (4,1)) is called to determine the control indices of a system, and program TRANSFAC (Overlay (4,2)) is called to determine the observer indices of a system.

4.6 Overlay (4,1) -- Program TRANFAB

Overlay (4,1) calculates the control indices of a system by transforming the system into a canonical form and then counting the size of the system's sub-blocks. Also, the determinant of the controllability matrix is determined. Program TRANFAB uses subroutines copy, MATECHO, EIGRF, SETQUP, UREFORM, APLFORM, BRUNFORM, SETQDWN, IDENT,MMPY, AND LINV3F which are all discussed in Section 5.

4.7 Overlay (4,2) -- Program TRANFAC

This program evaluates the observer indices of a system. TRANFAC uses subroutines COPY, TRANPOS, SETQUP, UREFORM, APLFORM, BRUNFRM, SETQDWN, AND MATECHO.

4.8 Overlay (5,0) -- Program SIMU

Program SIMU is an executive routine that sets up both the discrete and continuous-time simulations for the designed closed-loop system with digital control laws. It calls DISSIM (Overlay (5,1)) to perform the discrete simulation and it calls CONSIM (Overlay (5,2)) to perform the continuous-time simulation. After the simulation has been completed and control returned to SIMU, program PRNTSIM is called to list the results.

4.9 Overlay (5,1) -- Program DISSIM

Overlay (5,1) performs the discrete simulation using a sampled-data recursive formula representation of the closed-loop system.

4.10 Overlay (5,2) -- Program CONSIM

This program performs a continuous-time simulation on the closed-loop system as described in the USER'S MANUAL for CESA.

4.11 Overlay (5,3) -- Program PRNTSIM

PRNTSIM lists all the results of the closed-loop simulation.

4.12 Overlay (6,0) -- Program READER

Program READER verifies and codes all input that is entered in the option mode. Reference 9 gives a detailed description of READER.

4.13 Overlay (7,0) -- Program DECODER

Program DECODER is used to decode the MCOMM and DATM arrays set up by READER. Ref. 9 gives a detailed description of this routine.

4.14 Overlay (8,0) -- Program Copy

This program is not being used presently in CESA.

4.15 Overlay (9,0) -- Program LISTOPT

Program LISTOPT contains the print statements that list the options of CESA.

4.16 Overlay (10,0) -- Program SDREG

Program SDREG is the main routine used to perform the sampled-data regulator design options. It calls on the two secondary overlays, NSAB1 and DISCLS1, to calculate the control law and to set up the closed-loop system.

4.17 Overlay (10,1) -- Program NSAB1

This program contains the coded algorithm for entire eigenstructure assignment. It allows the user flexibility in assigning eigenvalues and the associated eigenvectors. This program is called to calculate the state feedback gain matrix and also to calculate the observer gain matrix.

4.18 Overlay (10,2) -- Program DISCLS1

Overlay (10,2) sets up the closed-loop system for the designed control law.

4.19 Overlay (11,0) -- Program SDDREJ

Program SDDREJ is the executive routine used to calculate the digital control law for the disturbance-rejector system. It calls program NSAB and program DISCLS to determine the digital control law and set up the closed-loop system.

4.20 Overlay (11,1) -- Program NSAB

Program NSAB is identical to program NSAB1.

4.21 Overlay (11,2) -- Program DISCLS

Overlay (11,2) is identical to program DISCLS1.

4.22 Overlay (12,0) -- Program SDTRAC

Program SDTRAC is the executive routine used to calculate the digital control law for the tracker system. Program NSAB2 and program DISCLS 2 are called by SDTRAC to determine the digital control law and to form the closed-loop system.

4.23 Overlay (12,1) -- Program NSAB2

This program is identical to program NSAB1.

4.24 Overlay (12.2) -- Program DISCLS2

Overlay (12,2) is identical to program DISCLS1.

4.25 Overlay (13,0) -- Program DISCRETE

Overlay (13,0) is the overlay that checks which type of digital control law is to be calculated and calls the appropriate overlay accordingly. Also, this routine checks whether the sampled-data system has been entered and, if it has not been entered, it calls program SAMPLE.

4.26 Overlay (15,0) -- Program SAMPLE

Program SAMPLE is the primary overlay that is called to discretize the continuous system. It calls the two secondary overlays, EIGVAV and EIGSTR, to form the sampled-data system. Also, this overlay performs the options 17 and 21.

4.27 Overlay (15,1) -- Program EIGVAV

Overlay (15,0) calculates the eigenvalues and eigenvectors of the plant matrix.

4.28 Overlay (15,2) -- Program EIGSTR

Overlay (15,2) sets up the inverse modal matrix.

4.29 Overlay (16,0) -- Program PLOTTS

Program PLOTTS is used when option 59 is selected. It sets up the arrays to be plotted from the simulation responses. Once it sets up the arrays it calls the secondary overlay, CALPLOT, to plot the responses.

4.30 Overlay (16,1) -- Program CALPLOT

The secondary overlay CALPLOT uses Calcomp routines to plot the data sent to it by program PLOTTS.

5.0 Description of Subprograms

An indepth description of many of the subroutines used in CESA may be found in Refs. 3, 9 and 15. TABLE B-1 lists these subroutines and the references where they may be found. Descriptions of the subroutines used in CESA that are not found in the preceding references are found in the following sections.

TABLE B-1

Subroutine Reference List

SUBROUTINE	REFERENCE	SUBROUTINE	REFERENCE
MADD	TOTAL(Ref 9)	SETQDWN	FORTRAC(Ref 3)
INVERT	TOTAL(Ref 9)	CINDS	FORTRAC(Ref 3)
READN	TOTAL(Ref 9)	APLFORM	FORTRAC(Ref 3)
TRANPOS	TOTAL(Ref 9)	UREFORM	FORTRAC(Ref 3)
MMPY	TOTAL(Ref 9)	SETQUP	FORTRAC(Ref 3)
MATECHO	TOTAL(Ref 9)	FULLOB	FORTRAC(Ref 3)
IDENT	TOTAL(Ref 9)	AUGMAT	FORTRAC(Ref 3)
MATIN	TOTAL(Ref 9)	M3CMAT	FORTRAC(Ref 3)
LISTER	TOTAL(Ref 9)	ADDMATX	FORTRAC(Ref 3)
TWODV	TOTAL(Ref 9)	DE01CKF	FORTRAC(Ref 3)
MODIFY	TOTAL(Ref 9)	SUBMATX	FORTRAC(Ref 3)
TITLES	TOTAL(Ref 9)	STIMEST	FORTRAC(Ref 3)
CONVERT	TOTAL(Ref 9)	STSTATE	FORTRAC(Ref 3)
SUB1	FORTRAC(Ref 3)	SAMRES	FORTRAC(Ref 3)
SUB2	FORTRAC(Ref 3)	EIGRF	IMSL(Ref 18)
UFORM	FORTRAC(Ref 3)	LINV3F	IMSL(Ref 18)
CHEROW	FORTRAC(Ref 3)	BALBNC	EISPACK(Ref 15)
ACHEROW	FORTRAC(Ref 3)	ELMHES	EISPACK(Ref 15)
SUB1000	FORTRAC(Ref 3)	ELTRAN	EISPACK(Ref 15)
SUB2000	FORTRAC(Ref 3)	HQR2	EISPACK(Ref 15)
SUB3000	FORTRAC(Ref 3)	BALBANC	EISPACK(Ref 15)
BRUNFRM	FORTRAC(Ref 3)		

5.1 Subroutine AVGAIN

This subroutine determines the average absolute magnitude of the feedback gain matrix

Calling Sequence

CALL AVGAIN (XK,M2,M1,NKX,MKX)

Definition of Symbols

XK = Feedback gain matrix

M2 = Number of rows of XK

M1 = Number of columns of XK

NKX = Number of rows of XK dimensioned in main routine

MKX = Number of columns of XK dimensioned in main routine

5.2 Subroutine FACTORL

This subroutine calculates the factorial of a number.

Calling Sequence

CALL FACTORL (N, NFACT, IFAIL)

Definition of Symbols

N = Integer for which factorial is to be calculated.

NFACT = N!

IFAIL = Error Flag . IFAIL equals 1 if N is within proper ranges. IFAIL equals 2 if $N \leq 0$

5.3 Subroutine EXPAT

The subroutine EXPAT calculates the state transition matrix e^{AT} and the $\int_0^T e^{At}\,dt$. This routine is used in discretizing a continuous linear system.

Calling Sequence

CALL EXPAT (N,A,DEL,EA,EAINT,IFAIL)

Definition of Symbols

N = Size of matrix A

A - Plant matrix

DEL = Basic time unit, T.

$EA = e^{AT}$

$$EAINT = \int_0^T e^{At}\, dt$$

Computational Algorithm

EXPAT first computes EAINT according to

$$EAINT = I*T + \frac{AT^2}{2!} + \frac{A^2T^3}{3!} + \ldots + A^{N-1}\frac{T^N}{N!}$$

where the series continues until it converges with an error less than or equal to 10^{-7}. EA is then obtained as

$$EA = I + A*\ EAINT.$$

Appendix C

Matrices for the Three Different Flight Conditions

This Appendix contains some of the continuous discretized system matrices for the regulator and tracker design of Chapter V according to the following difference equation, for a sampling time T = 2 seconds

$$x(2k+2) = F\,x(2k) + G\,u(2k) \qquad \text{(C-1)}$$

For both the low altitude cruise condition regulator and tracker the continuous plant from Eq. 55 is

$$A = \begin{bmatrix} 1.6754, & -.755E-2, & -.250E-3, & 0 & ,-3.954 & ,1.033 & ,0 \\ 544.9 & ,-1.135 & ,0.1179, & 0 & ,-30.77 & ,13.5212, & 0 \\ 4.374, & .02961 & ,-.917E-2, & -32.18, & -.2470, & -2.592 & ,-2.6 \\ 1 & ,0 & ,0 & ,0 & ,0 & ,0 & ,0 \\ 0 & ,0 & ,0 & ,0 & ,-10 & ,0 & ,0 \\ 0 & ,0 & ,0 & ,0 & ,0 & ,-4 & ,0 \\ 0 & ,0 & ,0 & ,0 & ,0 & ,0 & -2 \end{bmatrix} \qquad \text{(C-2)}$$

and the discretized plant matrices are

$$F = \begin{bmatrix} -.0325, & .0002, & .0001, & -.0035, & .0189 & ,-.0180, & -.0001 \\ -11.98 & ,-.0477, & -.0555, & 3.364 & ,4.399 & ,-1.203, & .0749 \\ -11.23 & ,.0632, & .9777, & -63.68 & ,3.983 & ,-2.928 & ,-1.255 \\ .1789, & -.0013, & .0001, & .9984, & -.0685 & ,0 & ,0 \\ 0 & ,0 & ,0 & ,0 & ,.206E-08, & 0 & ,0 \\ 0 & ,0 & ,0 & ,0 & ,0 & ,.0003, & 0 \\ 0 & ,0 & ,0 & ,0 & ,0 & ,0 & ,.0183 \end{bmatrix} \qquad \text{(C-3)}$$

$$
G = \begin{bmatrix}
-.6849, & .1844, & -.0002 \\
-397.7 & ,105.8 & , .1968 \\
28.36 & ,-10.49 & ,-3.890 \\
-1.712, & .4087, & -.48E\text{-}04 \\
1 & , 0 & , 0 \\
0 & , .9997, & 0 \\
0 & , 0 & , .9817
\end{bmatrix}
$$

Using the stability derivatives from Table II and Eq. 55, the continuous plant matrix for both the regulator and tracker for the medium altitude cruise condition becomes

$$
A = \begin{bmatrix}
1.175, & -.586E\text{-}2, & .431E\text{-}3, & 0 & ,-3.506, & .788, & 0 \\
659.9 & ,-.7686 & ,-.0591 & , 0 & ,-27.19 & ,10.289, & 0 \\
-3.22 & , .0314 & ,-.546E\text{-}2, & -32.18, & .1326, & -2.11 & ,-2.10 \\
1 & ,0 & ,0 & , 0 & , 0 & , 0 & , 0 \\
0 & ,0 & ,0 & , 0 & ,-10 & , 0 & , 0 \\
0 & ,0 & ,0 & , 0 & , 0 & ,-4 & , 0 \\
0 & ,0 & ,0 & , 0 & , 0 & , 0 & ,-2
\end{bmatrix} \qquad \text{(C-4)}
$$

The discretized plant matrices for both the regulator and tracker for the medium altitude cruise condition are

$$F = \begin{bmatrix} -.0941, & .0003, & .0001, & -.0105, & .0430 & ,-.0303, & -.0002 \\ -34.01 & ,-.1110, & .0545, & -1.827 & ,10.19 & ,-2.547 & ,-.0493 \\ -11.30 & , .0738, & .9803, & -63.83 & , 3.606 & ,-2.320 & ,-1.016 \\ .1328, & -.0014, & .0003, & .9904, & -.0471 & , .0299, & -.0003 \\ 0 & ,0 & ,0 & , 0 & , .206E-8, & 0 & , 0 \\ 0 & ,0 & ,0 & , 0 & , 0 & , .0003, & 0 \\ 0 & ,0 & ,0 & , 0 & , 0 & , 0 & , .0183 \end{bmatrix} \quad \text{(C-5)}$$

$$G = \begin{bmatrix} -.4705, & .1198, & -.0005 \\ -574.6 & ,130.3 & ,-.0699 \\ 33.70 & , -9.901 & ,-3.149 \\ -1.636 & , .3397, & -.0004 \\ 1.0 & , 0 & , 0 \\ 0 & , .9997, & 0 \\ 0 & , 0 & , .9817 \end{bmatrix}$$

In the case of the tracker designs the discretized plant is augmented with integrators. The augumented system for low altitude cruise is

$$\overline{F} = \begin{bmatrix} -.0325, & .0002, & .0001, & -.0349, & .0189, & -.0180, & -.0001, & 0, & 0, & 0 \\ -11.98, & -.0478, & -.0555, & 3.364, & 4.399, & -1.203, & .0749, & 0, & 0, & 0 \\ -11.23, & .0632, & .9777, & -63.68, & 3.983, & -2.928, & -1.255, & 0, & 0, & 0 \\ .1789, & -.0013, & .0001, & .9984, & .0685, & .0461, & -.0001, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & .206E-8, & 0, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & .0003, & 0, & 0, & 0, & 0 \\ 0, & 0, & 0, & 0, & 0, & 0, & .0183, & 0, & 0, & 0 \\ 0, & -2, & 0, & 1090, & 0, & 0, & 0, & 1, & 0, & 0 \\ 0, & 0, & 2, & 0, & 0, & 0, & 0, & 0, & 1, & 0 \\ 0, & 0, & 0, & 0, & 0, & 2, & 0, & 0, & 0, & 1 \end{bmatrix}$$

$$\overline{G} = \begin{bmatrix} -.6849, & .1844, & -.0002 \\ -397.7, & 105.8, & .1968 \\ 28.36, & -10.49, & -3.890 \\ -1.712, & .4087, & -.479E-4 \\ 1, & 0, & 0 \\ 0, & .9997, & 0 \\ 0, & 0, & .9817 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \end{bmatrix} \qquad (C-6)$$

$$
\bar{E} = \begin{bmatrix} 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ -2, & 0, & 0 \\ 0, & -2, & 0 \\ 0, & 0, & -2 \end{bmatrix}
$$

The augmented system for the medium altitude condition is

$$
\bar{F} = \begin{bmatrix}
-.0944, & .0003, & .0001, & -.0105, & .0430, & -.0303, & -.0002, & 0, & 0, & 0 \\
-34.01, & -.1110, & .0545, & -1.827, & 10.19, & -2.547, & -.0493, & 0, & 0, & 0 \\
-11.30, & .0738, & .9803, & -63.83, & 3.606, & -2.320, & -1.016, & 0, & 0, & 0 \\
.1328, & -.0014, & .0003, & .9904, & -.0471, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & .206E-8, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & .0003, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & .0183, & 0, & 0, & 0 \\
0, & -2, & 0, & .1320, & 0, & 0, & 0, & 1.0, & 0, & 0 \\
0, & 0, & 2, & 0, & 0, & 0, & 0, & 0, & 1.0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 2, & 0, & 0, & 1.0
\end{bmatrix}
$$

$$\bar{G} = \begin{bmatrix} -.4705, & .1198, & -.0005 \\ -574.6 & ,130.31 & ,-.0699 \\ 33.70 & ,-9.901 & ,-3.149 \\ -1.636 & ,.3397, & -.0004 \\ 1.0 & ,0 & ,0 \\ 0 & ,.9997, & 0 \\ 0 & ,0 & ,.9817 \\ 0 & ,0 & ,0 \\ 0 & ,0 & ,0 \\ 0 & ,0 & ,0 \\ 0 & ,0 & ,0 \end{bmatrix} \qquad \text{(C-7)}$$

$$\bar{E} = \begin{bmatrix} 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \\ -2, & 0, & 0 \\ 0, & -2, & 0 \\ 0, & 0, & -2 \end{bmatrix}$$

The closed-loop matrix for the low altitude cruise regulator is

$$
[F+GK]=\begin{bmatrix}
-.1326 & .0008 & .217E-5 & -.5224 & .0563 & -.0410 & -.152E-4 \\
-68.20 & .3153 & -.0620 & -290.8 & 25.38 & -14.20 & .0643 \\
-1.014 & .0050 & .2293 & -7.118 & .3158 & -.2971 & -.2770 \\
.0091 & -.0001 & -.299E-4 & .0386 & -.0050 & .0053 & .367E-4 \\
-.2686 & .952E-3 & -.2486E-3 & -.9896 & .0999 & -.0527 & .276E-3 \\
-1.540 & .0070 & -.0014 & -6.492 & .5739 & -.3202 & .0014 \\
-.4223 & .0030 & .1907 & -4.170 & .1216 & -1923 & -.2302
\end{bmatrix} \tag{C-8}
$$

and the closed-loop matrix for the mediam altitude regulator is

$$
[F+GK]=\begin{bmatrix}
-.1831 & .0009 & -.925E-4 & .5445 & .0712 & -.0438 & .349E-4 \\
-112.6 & .5014 & -.0698 & -512.7 & 35.58 & -15.79 & .0445 \\
-.7602 & .0052 & .2303 & -4.368 & -.2353 & -.2391 & -.2244 \\
-.0319 & .0001 & -.234E-4 & -.1856 & .0076 & -.956E-3 & .201E-4 \\
-.2919 & .639E-3 & -.148E-3 & -1.131 & .0837 & -.0259 & .120E-3 \\
-1.891 & .0075 & -.0015 & -8.908 & .5641 & -.2159 & .0011 \\
-.5140 & .0049 & .2368 & -2.912 & .1887 & -.2538 & -.2308
\end{bmatrix} \tag{C-9}
$$

In the case of the tracker designs, the closed-loop matrix for the low altitude tracking system is

$$
[\bar{F}+GK] = \left[\begin{array}{cccccccccc}
.1607, & .0019, & .266E-4, & -1.169, & .0675, & -.0762, & -.183E-4, & -.481E-3, & .126E-4, & -.0138 \\
-85.87, & .9554, & .0218, & -660.1, & 32.43, & -33.0, & .0481, & -.2685, & .0363, & -7.061 \\
-.6766, & .0077, & -.7164, & -5.899, & .2413, & -.0802, & -.0637, & -.0022, & -.3845, & .0441 \\
-.1109, & .0023, & .108E-3, & -1.428, & .0412, & -.0282, & .591E-4, & -.875E-3, & .859E-4, & .0026 \\
.0283, & .0016, & -.309E-3, & -.8049, & -.0089, & -.2810, & .412E-4, & -.973E-3, & -.146E-3, & -.1432 \\
-.5901, & .0156, & -.0012, & -9.305, & .2308, & -1.358, & .473E-3, & -.622E-2, & -.388E-3, & -.6061 \\
-.8975, & -.0157, & .4286, & 4.290, & .2690, & .8667, & -.2833, & .01004, & .0970, & .5685 \\
0 & -2.0 & 0 & 1090 & 0 & 0 & 0 & 1.0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0
\end{array}\right] \quad \text{(C-10)}
$$

and the closed loop system for the medium altitude tracking system is

- .2090	.0019	-.202E-3	- 1.205	.0817	- .0797	.491E-4	-.436E-3	-.421E-4	- .0144
-155.6	1.518	-.0296	-1204	50.98	-42.20	.0408	-.3954	.0244	-7.882
1.665	- .0411	-.7120	27.94	-.6013	9215	-.0527	.0164	-.3833	.3065
- .1758	.0025	.529E-4	-1.939	.0562	- .0337	.3648E-4	-.869E-3	.646E-4	- .0041
- .0602	.0023	-.355E-3	-1.501	.0225	- .2513	.685E-5	-.0012	-.174E-3	- .1202
- 1.199	.0224	-.0019	-15.84	.4125	- 1.396	.557E-3	-.0083	-.516E-3	- .5902
- .9716	- .0097	.5298	4.541	.2747	.6617	-.2838	.0079	.1193	.4640
0	-2.0	0	1320	0.	0	0	1	0	0
0	0	2.0	0	0	0	0	0	1	0
0	0	0	0	0	2.0	0	0	0	1

(C-11)

Appendix D

C* Criterion

In order to determine a meaningful performance criterion for a particular control system it is necessary to determine the types and levels of handling qualities that are required by a pilot to accomplish a mission. A particular criterion that was developed to specify short period handling qualities in terms of aircraft parameters that are familiar to the pilot is the C* criterion (Ref:16).

A study was done in this thesis to determine if a deadbeat controller could be designed to track a C* commanded input. However the augmented tracking system had a low degree of controllability and the C* tracker design was dropped. Time limitation prevented an examination of the multivariable zeros of the system to determine their location with respect to the unit circle. In order to provide some background for this problem, the C* equations used for the three sets of trackers are included. There was no speed control for the C* tracker designs.

The C* equation used for all three tracker designs was

$$C^* = \Delta n_{zp} + k_2 q \qquad \text{(D-1)}$$

where the normal acceleration of the pilot,

Δn_{zp} in units of g's is

$$\Delta n_{zp} = (\dot{w} - U_o\dot{\theta} - \ell x\,\ddot{\theta})/32.18 \,, \qquad \text{(D-2)}$$

ℓx is the distance of the pilot from the aircraft's center of gravity,
q is the aircraft's pitch rate, and

$$k_2 = C_2 V_{co} \tag{D-3}$$

where C_2 is a dimensional constant given in Table D1 and V_{co} is the crossover velocity as defined in Ref. 16. Using Eqs. D-1 and D-2 and Table D1 with

$$V_{co} = 350 \text{ ft/sec} \tag{D-4}$$

the C* equation for the three flight condition was: for the landing condition

$$C^* = -.2809u - .3243w + 27.93\delta e + 22.72\delta_{sp} + 47.45\dot{\theta} \tag{D-5}$$

for the low altitude cruise condition

$$C^* = -.003298u - .0242w + 4.818\delta e - 1.089\delta_{sp} + 2.663\dot{\theta} \tag{D-6}$$

and for the medium altitude cruise condition

$$C^* = -.00247u - .01534w + 4.272\delta e - .8329\delta_{sp} + 1.9363\dot{\theta} \tag{D-7}$$

Table D-1

Units

C*	Δn_{zp}	q	C_2	
			Value	Units
g's	g's	rad/sec	$\frac{1}{32.2}$	$\frac{\text{g's-sec}^2}{\text{ft}}$
		deg/sec		
ft/sec^2	ft/sec^2	rad/deg	1.0	radians
		deg/sec	$\frac{1}{57.3}$	$\frac{\text{rad}}{\text{deg}}$

Vita

Thomas A. Kennedy was born on 6 May 1955 in Trenton, New Jersey. He graduated from Saint Anthony's High School in June, 1973. In September 1973 he entered Rutgers University College of Engineering on a full AFROTC scholarship. He received a Bachelor of Science Degree in Electrical Engineering and a commission in the United States Air Force in May 1977. From May 1977 until September 1977 he worked for Personnel Products, an affiliate of Johnson and Johnson Inc., as a research engineer. While there, he completely automated an absorbancy test procedure and supervised the construction of the first prototype machine to perform the test. He entered the School of Engineering, Air Force Institute of Technology, in September 1977. His next assignment is with the Space and Missile Systems Organization (SAMSO), Air Force Systems Command, Los Angeles, California.

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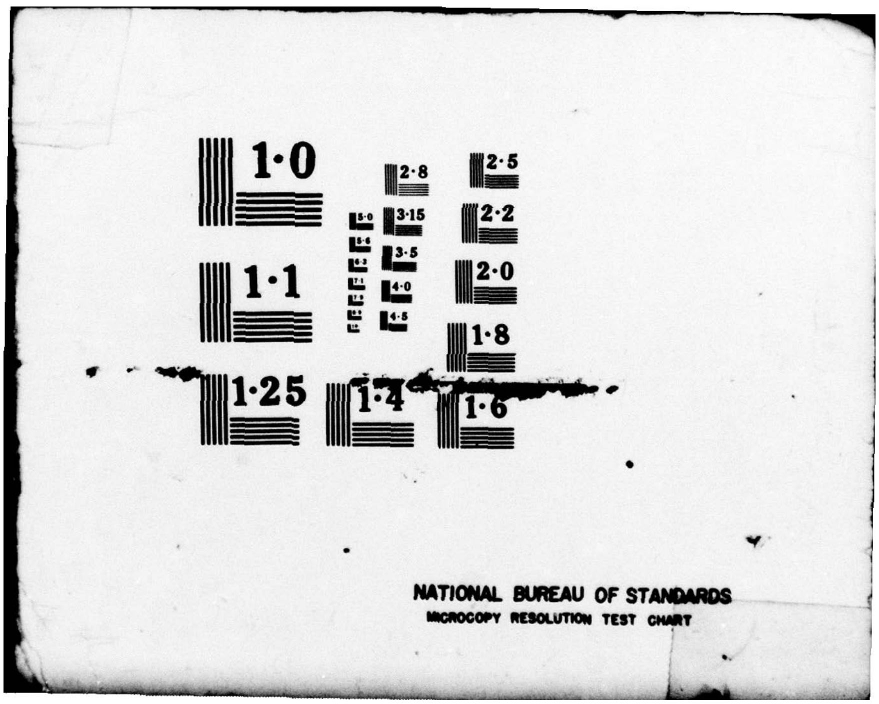
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Eigenvalue	Observers
Eigenvectors	Discrete Control Law
Eigenstructure Assignment	Time Domain Control Methods
State Variable Feedback	Interactive Computer Program
	Continuous Simulation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this report, a deadbeat tracker and regulator are developed for the C-141 aircraft for three different flight conditions using the method of entire eigenstructure assignment. A heuristic iterative method is presented for determining an "optimal" sampling time for deadbeat controllers by using the controllability matrix.
In order to perform the above designs and to evaluate them through a continuous-time simulation, a computer aided design program is developed. This program

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allows the complete design and simulation of regulator, tracker, and disturbance rejection control systems with full-order observers. In addition, the program permits complete state-space analysis of the system being designed, including both discrete and continuous-time responses. A user's manual and programmer's guide are provided for further development of the program.