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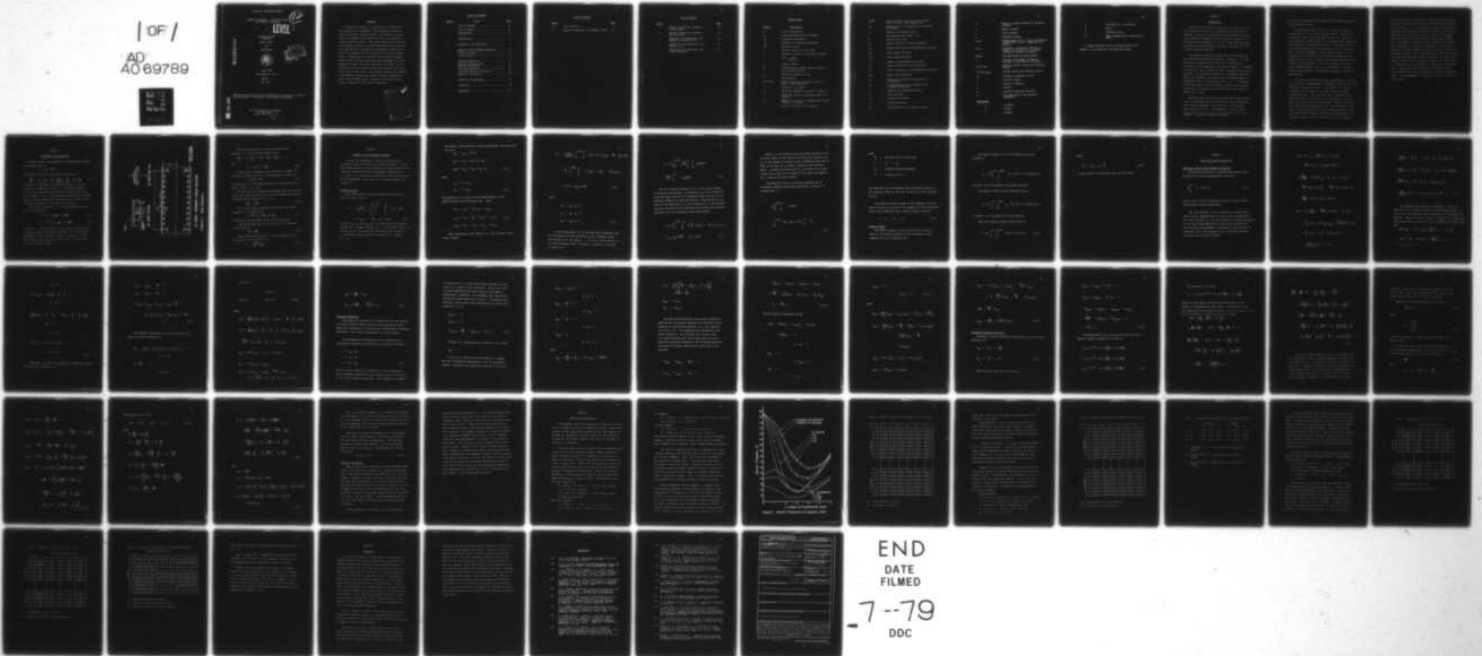
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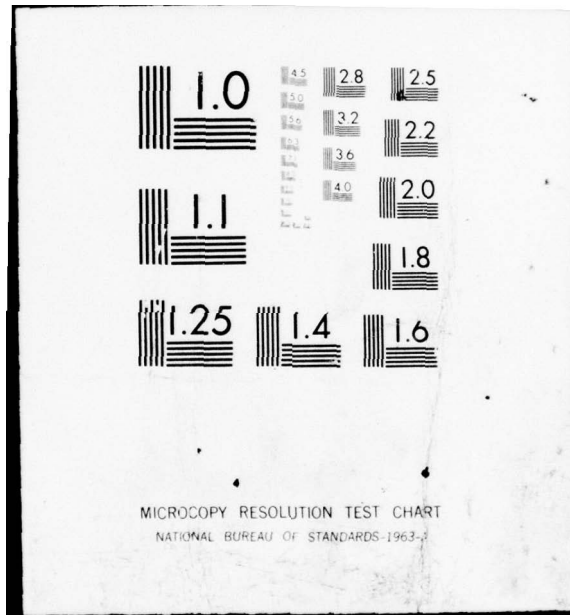
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Dynamics of Submerged Cylindrical Shells  
With Eccentric Stiffening

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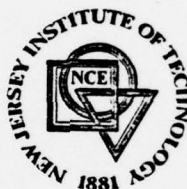
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Kerry Kier-Ten Chu  
(Deceased)

Michael Pappas

and

Harry Herman



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New Jersey Institute of Technology  
323 High Street  
Newark, New Jersey 07102

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ABSTRACT

A theoretical analysis is presented for treating the free vibrations of submerged, ring-stiffened cylindrical shells with simply supported ends. The effects of the eccentric stiffeners are averaged over the thin-walled isotropic cylindrical shell. The energy method is utilized and the frequency equation is derived from Hamilton's Principle. All three degrees of freedom are considered. Numerical results are presented for the frequencies of several example shells. Comparisons with previous theoretical and experimental results indicate reasonably good agreement for shells immersed in water, but poorer agreement in vacuum. The inaccuracies are due to limitations of the Donnell type orthotropic shell theory used. The procedure appears well adapted to preliminary optimization studies of shells immersed in water under minimum natural frequency constraints and for optimal separation of the lower frequencies.

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NOMENCLATURE

<u>Symbol</u>	<u>Description</u>
A	- Fourier coefficient
$A_r$	- Cross-sectional area of stiffener
[A]	- Frequency determinant
$A_{ij}$	- Elements of frequency determinant
{B}	- Amplitude vector
c	- Acoustic velocity in the fluid
D	- Flexural stiffness of isotropic cylinder wall, $\frac{Et^3}{12(1-\mu^2)}$
E	- Young's modulus
$\bar{e}$	- Distance from cylinder middle surface to line on which $\bar{N}_x$ acts
$F_i$	- Defined by equation (3.21)
G	- Shear modulus
$H_n^{(2)}(KR)$	- Hankel function of the second kind, of order n and argument KR
h	- Thickness of cylinder
$h_i$	- Stiffener dimension as shown in Figure 2.1
I	- Moment of inertia of stiffener about its centroid
$I_o$	- Moment of inertia of stiffener about middle surface of cylinder
J	- Torsional constant for stiffener

$K_n(\overline{KR})$	- Modified Bessel function of the second kind, or order $n$ and argument $\overline{KR}$
$K, \overline{K}$	- Coefficient of the equation for potential fluid flow
$L$	- Length of cylindrical shell
$l$	- Ring spacing (see Figure 2.1)
$\overline{M}$	- Mass per unit area
$M_x$	- Moment resultant in axial direction
$M_y$	- Moment resultant in circumferential direction
$M_{xy}$	- Shear moment resultant
$M_{yx}$	- Shear moment resultant
$m$	- Number of longitudinal half-waves
$N_x$	- Stress resultant in axial direction
$N_y$	- Stress resultant in circumferential direction
$N_{xy}$	- Shear stress resultant
$\overline{N}_x$	- Externally applied load resultant in $x$ direction
$\overline{N}_y$	- Circumferential stress resultant due to applied pressure
$n$	- Number of circumferential waves
$P$	- Total pressure
$P_h$	- Hydrostatic pressure
$P_r$	- Radiated pressure
$\overline{P}_r$	- The amplitude of the radial pressure

- R - Radius to middle surface of isotropic cylinder.
- T - Kinetic energy.
- t - Time variable.
- V - Potential energy.
- u, v, w - Displacements in x, y, and z directions, respectively (axial, tangential and radial).
- x, y, z - Orthogonal coordinates defined in Figure 2.1 (x and y lie in middle surface of cylinder or plate).
- U, V, W - The amplitudes of displacement.
- $\bar{z}$  - Distance from middle surface of cylinder to centroid of stiffener.
- $\epsilon_x, \epsilon_y, \gamma_{xy}$  - Membrane normal strains and shearing strain.
- $\epsilon_{xT}, \epsilon_{yT}, \gamma_{xyT}$  - Normal strains and shearing strain.
- $\Lambda_i$  - Defined by equation (4.22).
- $\mu$  - Poisson's ratio.
- $\omega$  - Circular frequency.
- $\rho$  - Density.
- $\phi$  - Velocity potential function.
- $\Phi$  - The amplitude of the velocity potential.

#### Subscripts

- c - cylinder
- i - Integer
- j - Integer

- r - Stiffening in y direction.
- w - Water.
- A - Prestress state.
- B - Small changes away from prestress state.

A comma indicates partial differentiation with respect to the subscript following the comma.

## CHAPTER 1

### INTRODUCTION

Since the early efforts of Rayleigh [1]<sup>1</sup> and Love [2], the vibration of shells in a vacuum has been extensively analyzed. Typical of recent work on prestressed, eccentrically stiffened, cylindrical shells of finite length are the investigations by McElman, et al. [3], using the orthotropic or smeared stiffener approach and Harari and Baron [4], who consider discrete stiffeners.

The dynamic interaction between shells and fluids has also received considerable attention. Junger [5, 6, 7] was the first to analyze the free and forced vibrations of a cylindrical shell submerged in an acoustic medium. He treated an infinitely long, cylindrical shell utilizing plain strain analysis. The transient response of a submerged, infinitely long, ring-stiffened cylindrical shell has been studied by Herman and Klosner [8] and Lyons, et al. [9]. A submerged cylindrical shell of infinite length, having radial surface motion over a stiffened, finite section has been studied by Paslay, et al. [10].

Recently, some interest has been directed toward optimization of submerged shells with eccentric ring stiffeners under natural frequency constraints and or with dynamic merit criteria [11-13]. Unfortunately, this early work

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<sup>1</sup>Numbers in brackets designate references.

utilizes an invacuo vibration model which yields excessively large frequency values.

The objective of this effort is to develop a more accurate model for such optimal design studies. Donnell type orthotropic theory along the lines of Ref. [3] is used in this preliminary study because of its computational simplicity. Conservation of computational effort is quite important for preliminary optimization investigations, since it is desirable to perform parametric optimal design studies so as to determine the characteristics of optimal designs and since optimization for even a single set of design parameters often requires several hundred to several thousand sets of frequency determinations. Thus, it is desirable to use the simplest model capable of generating reasonable results for such studies.

This report therefore describes a procedure for the natural frequency determination of submerged, eccentrically ring stiffened, cylindrical shells. The effects of eccentric stiffeners are averaged over the thin-walled isotropic cylindrical shell. The case where axial, tangential, and radial motions of the structure are considered as well as where only radial motions are included. Boundary conditions are established and satisfied at the fluid-structure interface, yielding expressions for the radiated pressure. The derivation of the frequency equation is accomplished by defining the Donnell type of nonlinear strain displacements

for the shell and its stiffeners, formulating the potential and kinetic energies of the system, and then applying Hamilton's Principle. A set of appropriate nonlinear equilibrium equations and boundary conditions are derived. The nonlinear equations are used to derive linear equations that govern small vibrations of the system and a three-degree freedom and a one degree of freedom frequency equation are obtained. The frequencies are numerically obtained. For the three degrees of freedom formulation once the values of frequency have been obtained, the corresponding ratios of radial, axial, and tangential amplitudes may be evaluated. The numerical results obtained from the present analysis for several cases of interest are given in Chapter 5. Reasonably good agreement is found with other theoretical investigations and with experimental results for the case of shells immersed in water. Poorer agreement is found for the case of shells in vacuo or air.

## CHAPTER 2

GOVERNING FIELD EQUATIONS

The well known field equation for a homogeneous acoustic fluid medium [14] is

$$\nabla\phi^2 = \frac{1}{c^2} \frac{\partial^2\phi}{t^2}$$

Written out in terms of the notation used herein:

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r} \frac{\partial\phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2\phi}{\partial(y/R)^2} + \frac{\partial^2\phi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2\phi}{\partial t^2} \quad (2.1)$$

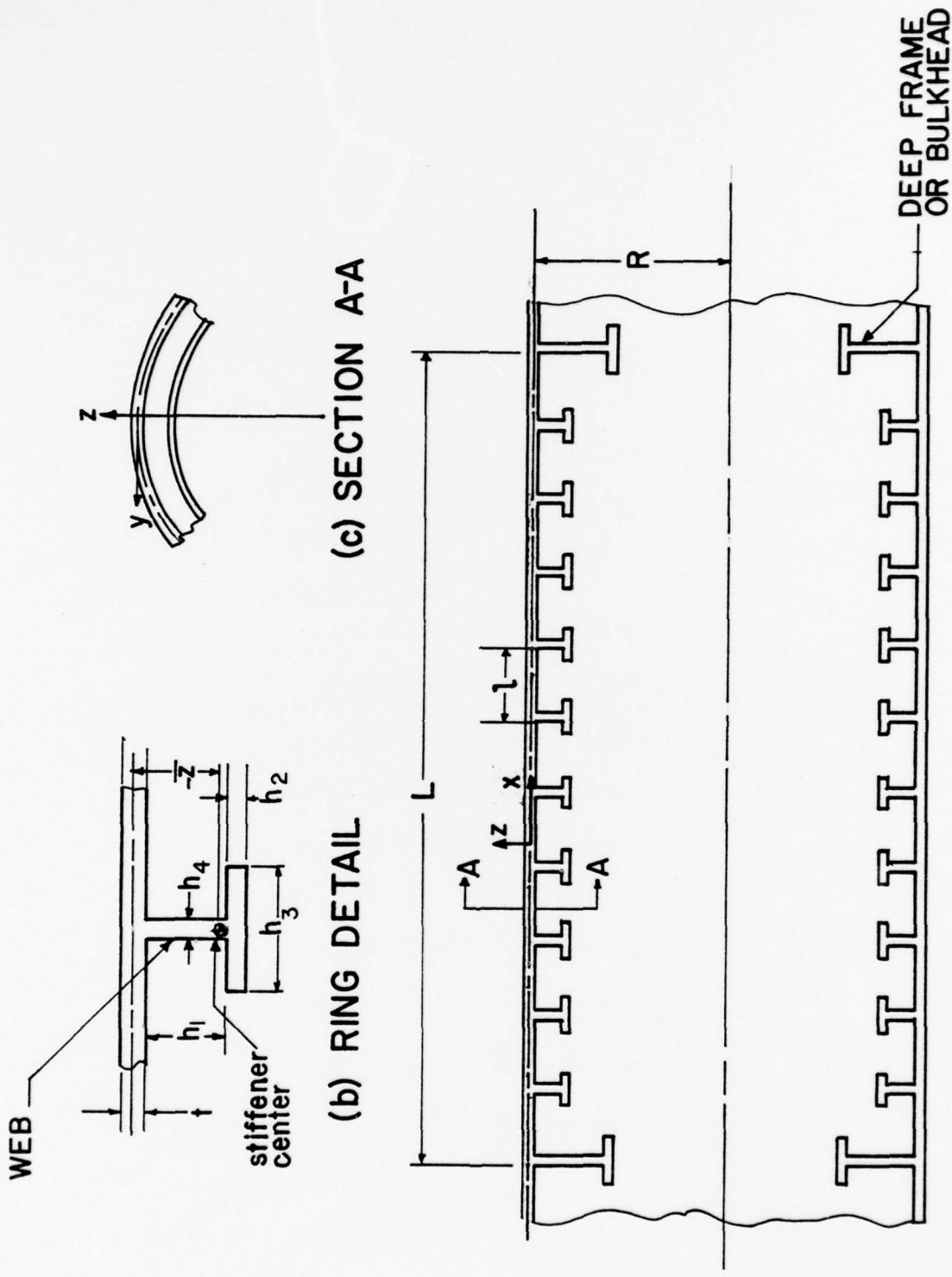
in which  $\phi$  is the velocity potential function,  $c$  is the velocity of sound in the fluid,  $t$  is the time variable,  $x$  is the shell coordinate parallel to the axis of the structure, (Figure 2.1),  $r$  is the variable describing the distance from the shell surface into the fluid, and  $y$  is the circumferential coordinate at the mean shell radius  $R$ .

The velocity potential,  $\phi$ , and the radiated pressure,  $P_r$  can be written as

$$\phi = e^{i\omega t} \phi \cos \frac{ny}{R} \sin \frac{m\pi x}{L} \quad (2.2)$$

$$P_r = e^{i\omega t} \bar{P}_r \cos \frac{ny}{R} \sin \frac{m\pi x}{L} \quad (2.3)$$

in which  $\phi$  is the amplitude of the velocity potential,  $\bar{P}_r$  is the amplitude of the radial pressure,  $\omega$  is the circular frequency,  $m$  is the number of axial half waves,  $R$  is the mean radius of the cylindrical shell, and  $L$  is the length of the cylindrical shell.



(c) SHELL SEGMENT CROSS-SECTION

Figure.2.1 Shell Geometry

Substituting equation (2.2) into the governing field equation (2.1) for the fluid medium yields:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \left( \frac{\omega^2}{c^2} - \frac{n^2}{r^2} - \frac{m^2 \pi^2}{L^2} \right) \phi = 0$$

Let

$$K^2 = \left( \frac{\omega}{c} \right)^2 - \left( \frac{m\pi}{L} \right)^2 \quad (2.4)$$

If the fluid is assumed to be infinitely extended, the solution of equation (2.4) for an outgoing wave is given by

$$\phi = AH_n^{(2)}(Kr) \quad (2.5)$$

in which  $H_n^{(2)}$  is the Hankel function of the second kind of order  $n$ ,  $H_n^{(2)} = J_n - iY_n$ .

The constant  $A$  is evaluated by ensuring that the velocity of the shell and the velocity of the fluid medium are equal at the shell-fluid interface,  $r = R$ , i.e.

$$\frac{\partial w}{\partial t} = \frac{\partial \phi}{\partial r} \quad (2.6)$$

For a shell simply supported at each end, the radial deflection is assumed to be

$$w = \bar{W} e^{i\omega t} \sin \frac{m\pi x}{L} \cos \frac{ny}{R} \quad (2.7)$$

in which  $\bar{W}$  = the amplitude of the radial deflection.

Substituting equations (2.2) and (2.7) into equation (2.6) results in

$$i\omega \bar{W} = \frac{\partial \phi}{\partial r} \quad (2.8)$$

Upon substituting equation (2.5) into equation (2.8), the constant  $A$  can be written as

$$A = \frac{i\omega \bar{W}}{KH_n^{(2)}(KR)} \quad (2.9)$$

## CHAPTER 3

ENERGY OF THE STIFFENED STRUCTURE

Consider an axisymmetric structure consisting of a cylindrical shell with reinforcing ring stiffeners. In the cylindrical portion of the structure, the displacements of the shell are defined by the three orthogonal components  $u$ ,  $v$ ,  $w$ , in the  $x$ ,  $y$ ,  $z$  directions respectively, which are functions of the coordinates  $x$  and  $y$  only at the middle surface (see Figure 2.1).

Potential Energy

The strain energy of the unstiffened thin-walled isotropic cylinder [15] is

$$V_c = \frac{E}{2(1-\mu^2)} \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^{2\pi R} \int_0^L (\epsilon_{xT}^2 + \epsilon_{yT}^2 + 2\mu\epsilon_{xT}\epsilon_{yT} + \frac{1-\mu}{2} \gamma_{xyT}^2) dx dy dz \quad (3.1)$$

in which  $\epsilon_{xT}$ ,  $\epsilon_{yT}$  and  $\gamma_{xyT}$  are the total normal and shearing strains,  $E$  is Young's modulus, and  $\mu$  is Poisson's ratio. A cylinder may be considered thin walled when the thickness  $h$  is sufficiently small compared to the radius  $R$ . Usually,  $R/h > 60$ .

The Donnell type nonlinear strain displacement relations used here are

$$\begin{aligned}\varepsilon_{xT} &= u_{T,x} + \frac{1}{2} w_{,x}^2 \\ \varepsilon_{yT} &= v_{T,y} + w/R + \frac{1}{2} w_{,y}^2 \\ \gamma_{xyT} &= u_{T,y} + v_{T,x} + w_{,x} w_{,y}\end{aligned}\quad (3.2)$$

where

$$\begin{aligned}u_T &= u - zw_{,x} \\ v_T &= v - zw_{,y}\end{aligned}\quad (3.3)$$

The quantities  $u, v,$  and  $w$  are the displacements of the middle surface of the cylinder wall. Thus,

$$\begin{aligned}\varepsilon_{xT} &= u_{,x} + \frac{1}{2} w_{,x}^2 - zw_{,xx} \\ \varepsilon_{yT} &= v_{,y} + \frac{w}{R} + \frac{1}{2} w_{,y}^2 - zw_{,yy} \\ \gamma_{xyT} &= u_{,y} + v_{,x} + w_{,x} - 2zw_{,xy}\end{aligned}\quad (3.4)$$

After integration with respect to  $z,$  the cylinder strain energy becomes

$$\begin{aligned}
V_c = & \frac{Eh}{2(1-\mu^2)} \int_0^{2\pi R} \int_0^L (\epsilon_x^2 + \epsilon_y^2 + 2\mu\epsilon_x\epsilon_y + \frac{1-\mu}{2} \gamma_{xy}^2) dx dy \\
& + \frac{D}{2} \int_0^{2\pi R} \int_0^L [ w_{,xx}^2 + w_{,yy}^2 - 2\mu w_{,xx}w_{,yy} \\
& + 2(1-\mu) w_{,xy}^2 ] dx dy \tag{3.5}
\end{aligned}$$

where

$$\begin{aligned}
\epsilon_x &= \epsilon_{xT} \Big|_{z=0} \\
\epsilon_y &= \epsilon_{yT} \Big|_{z=0} \\
\gamma_{xy} &= \gamma_{xyT} \Big|_{z=0} \tag{3.6}
\end{aligned}$$

If the displacement in the cylinder and stiffening rings are continuous and the properties of the stiffening rings are averaged over the spacing  $\lambda$ , the total strain energy [12] for the stiffening rings of spacing  $\lambda$  attached to the shell is found to be

$$\begin{aligned}
 V_r = & \frac{1}{l} \int_0^{2\pi R} \left( \frac{E_r}{2} \int_0^L \int_{A_r} \epsilon_{xT}^2 dA_r dx + \right. \\
 & \left. \frac{G_r J_r}{2} \int_0^a w_{,xy}^2 dx \right) dy \quad (3.7)
 \end{aligned}$$

The first term in equation (3.7) is the strain energy of extension and bending. The quantity  $dA_r$  is an element of cross-sectional area of the stiffening ring, and  $G_r J_r$  is the twisting stiffness of the ring section. Substitution of the first of the equations (3.4) into equation (3.7) and integrating over the area of the stiffening ring yields the following expression for the stiffening ring strain energy

$$\begin{aligned}
 V_r = & \frac{1}{l} \int_0^{2\pi R} \int_0^L \left[ \frac{E_r}{2} \left( A_r \epsilon_y^2 - 2\bar{z}_r A_r \epsilon_y w_{,yy} + \right. \right. \\
 & \left. \left. I_{or} w_{,yy} \right) + \frac{G_r J_r}{2} w_{,xy}^2 \right] dx dy \quad [3.8]
 \end{aligned}$$

Here  $\bar{z}_r$  is the distance from the middle surface of the isotropic shell to the centroid of ring cross-section, and  $I_{or}$  is the moment of inertia of the stiffening ring with respect to an axis in the middle surface of the isotropic shell. It should be noted that  $\bar{z}_r$  is positive for a stiffening ring on the outer surface of the shell and negative for a ring on the inner surface.

The potential energy of external pressure and an externally applied axial load resultant  $\bar{N}_x$  (positive in compression) is

$$V_P = \int_0^{2\pi R} \int_0^L P w dx dy + \int_0^{2\pi R} \left[ \bar{N}_x u_T |_{z = \bar{e}} \right] \Big|_0^L dy$$

(3.9)

where

$\bar{N}_x$  = function of  $P_h$  in this case

$P$  =  $P_h$  +  $P_r$

$P_h$  = constant external pressure

$P_r$  = radiated pressure

The quantity  $\bar{e}$  is the distance from the middle surface of the isotropic shell to the line on which the load resultant  $\bar{N}_x$  acts.

The potential strain energy of the combined structure can be written as the sum of the energies of the cylindrical shell, the stiffening rings, and the loads as follows:

$$V = V_c + V_r + V_p \quad (3.10)$$

### Kinetic Energy

The kinetic energy of the system can be written in terms of the kinetic energies of the cylindrical shell segments and the stiffening rings.

The kinetic energy of the unstiffened thin-walled cylinder is

$$T_c = \frac{1}{2} \int_0^L \int_0^{2\pi R} \rho_c h (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dx dy \quad (3.11)$$

in which  $\rho_c$  is the density of cylinder material.

The kinetic energy of the stiffening ring is

$$T_r = \frac{1}{2} \int_0^L \int_0^{2\pi R} \rho_r \frac{A_r}{l} (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dx dy \quad (3.12)$$

in which  $\rho_r$  is the density of ring material.

The total kinetic energy of the system is

$$T = \frac{1}{2} \int_0^L \int_0^{2\pi R} M (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dx dy \quad (3.13)$$

where

$$\bar{M} = \rho_c h + \rho_r \frac{A_r}{l} \quad (3.14)$$

is the average or distributed mass per unit area.

## CHAPTER 4

FREQUENCY EQUATION DERIVATIONEquation of Motion and Boundary Conditions

The partial differential equations and boundary conditions are derived from Hamilton's principle

$$\delta \int_{t_1}^{t_2} [T - V] dt = 0 \quad (4.1)$$

where T and V are the structural system's kinetic energy and potential energy respectively.

The three motions of this conservative system from a given initial configuration to a given final configuration in a time interval  $(t_1, t_2)$  are obtained by allowing variation of the three displacements  $\delta u$ ,  $\delta v$  and  $\delta w$  to be arbitrary and utilizing the fundamental technique of the calculus of variations [16]. From equation (4.1), the three equations of motion are derived as follows:

$$u_{,xx} + \frac{\mu}{R} w_{,x} + \frac{1-\mu}{2} (u_{,xy} + w_{,x} w_{,xy})$$

$$+ \frac{1+\mu}{2} (v_{,xy} + w_{,y} w_{,xy}) - \bar{M}\ddot{u} = 0$$

$$1 + \left[ \frac{E_r A_r}{Eh l} (1-\mu^2) \right] (v_{,xx} + \frac{1}{R} w_{,y} + w_{,y} w_{,yy})$$

$$+ \frac{1+\mu}{2} (u_{,xy} + w_{,x} w_{,xy}) + \frac{1-\mu}{2} (v_{,xy} + w_{,xx} w_{,y})$$

$$- \frac{E_r A_r}{Eh l} (1-\mu^2) \bar{z}_r w_{,yy} - \bar{M}\ddot{v} = 0$$

$$D\nabla^4 w + \frac{1}{R} \left( \frac{Eh}{1-\mu^2} + \frac{E_r A_r}{l} \right) (v_{,y} + \frac{w}{R} + \frac{1}{2} w_{,y}^2)$$

$$+ \frac{E_r}{l} (I_r + \bar{z}_r^2 A_r) w_{,yyyy} - \frac{\bar{z}_r E_r A_r}{l} (v_{,yyyy}$$

$$+ \frac{2}{R} w_{,yy} + w_{,yy} + w_{,y} w_{,yyy}) +$$

$$+ \frac{Eh}{(1-\mu^2)R} (u_{,x} + \frac{1}{2} w_{,x})$$

$$\begin{aligned}
& + \frac{Eh}{1-\mu^2} \left[ u_{,x} + \frac{1}{2} w_{,x}^2 + \mu \left( v_{,y} + \frac{w}{R} + \frac{1}{2} w_{,y} \right) \right] w_{,xx} \\
& - \frac{Eh}{1-\mu^2} \left[ v_{,y} + \frac{w}{R} + \frac{1}{2} w_{,y}^2 + \mu u_{,x} + \frac{1}{2} w_{,x}^2 \right] w_{,yy} \\
& - \frac{E_r A_r}{i} \left( v_{,y} + \frac{w}{R} + \frac{1}{2} w_{,x} - \bar{z}_r w_{,yy} \right) w_{,yy} \\
& - 2 \frac{Eh}{2(1+\mu)} \left( v_{,y} + v_{,x} w_{,x} w_{,y} \right) w_{,xy} - \bar{M} \ddot{w} = 0 \quad (4.2)
\end{aligned}$$

The necessary and sufficient conditions to cause the equation (4.1) to vanish during the variational process (3.1) are to zero the remainder part in addition to the equation of motions. Then the natural boundary conditions are established. These conditions at each end of the stiffened cylindrical shell are:

$$\begin{aligned}
& D \left[ w_{,xxx} + (2-\mu) w_{,xyy} \right] - \frac{Eh}{1-\mu^2} \left[ u_{,x} + \frac{1}{2} w_{,x}^2 \right. \\
& \left. + \mu \left( v_{,y} + \frac{w}{R} + w_{,x}^2 \right) \right] w_{,x} - \frac{Eh}{2(1+\mu)} (u_{,y} + v_{,x} \\
& + w_{,x} w_{,y}) w_{,y} = 0
\end{aligned}$$

$$\text{or } w = 0$$

$$D (w_{,xx} + \mu w_{,yy}) - \bar{N}_x \bar{e} = 0$$

$$\text{or } w_{,x} = 0$$

$$\frac{Et}{1-\mu^2} \left[ u_{,x} + \frac{1}{2} w_{,x}^2 + \mu \left( v_{,y} + \frac{w}{R} + \frac{1}{2} w_{,y}^2 \right) \right]$$

$$+ \bar{N}_x = 0$$

$$\text{or } u = 0$$

$$Gt (u_{,y} + v_{,x} + w_{,x} w_{,y}) = 0$$

$$\text{or } v = 0$$

(4.3)

Equations (4.2) may be conveniently rewritten in terms of the stress resultants as

$$N_{x,x} + N_{xy,y} - \bar{M}\ddot{u} = 0$$

$$N_{y,y} + N_{xy,x} - \bar{M}\ddot{v} = 0$$

$$\begin{aligned}
 & - M_{x,xx} + M_{xy,xy} - M_{yx,xy} - M_{y,yy} + \frac{N_y}{R} \\
 & - N_x w_{,xx} - N_y w_{,yy} - 2N_{xy} w_{,xy} + P - \bar{M}\ddot{w} \\
 & = 0 \qquad (4.4)
 \end{aligned}$$

The boundary conditions (4.3) may be rewritten in terms of stress resultants as

$$M_{x,x} - (M_{xy,y} - M_{yx,y}) + N_x w_{,x} + N_{xy} w_{,y} = 0$$

$$\text{or } w = 0$$

$$M_x + \bar{N}_x \bar{e} = 0$$

$$\text{or } w_{,x} = 0$$

$$N_x + \bar{N}_x = 0$$

$$\text{or } u = 0$$

$$N_{xy} = 0$$

$$\text{or } v = 0$$

(4.5)

where

$$N_x = \frac{Eh}{1-\mu^2} \left[ u_{,x} + \frac{1}{2} w_{,x}^2 + \mu \left( v_{,y} + \frac{w}{R} + \frac{1}{2} w_{,y}^2 \right) \right]$$

$$N_y = \frac{Eh}{1-\mu^2} \left[ v_{,y} + \frac{w}{R} + \frac{1}{2} w_{,y}^2 + \mu \left( u_{,x} + \frac{1}{2} w_{,x}^2 \right) \right]$$

$$+ \frac{E_r A_r}{l} \left( v_{,y} + \frac{w}{R} + \frac{1}{2} w_{,y}^2 - \bar{z}_r w_{,yy} \right)$$

$$N_{xy} = Gh \left( v_{,y} + v_{,x} + w_{,x} w_{,y} \right)$$

$$M_x = -D \left( w_{,xx} + \mu w_{,yy} \right)$$

$$M_y = -D \left( w_{,yy} + \mu w_{,xx} \right) - \frac{E_r I_r}{l} w_{,yy} \\ + \bar{z}_r \frac{E_r A_r}{l} \left( v_{,y} + \frac{w}{R} - \frac{1}{2} w_{,y}^2 - \bar{z}_r w_{,yy} \right)$$

$$M_{xy} = \frac{Gh^3}{6} w_{,xy}$$

$$M_{yx} = -\left(\frac{Gh^3}{6} + \frac{G_r J_r}{l}\right) w_{,xy} \quad (4.6)$$

### Vibration Equations

The equations of motion (4.4) derived in the last section are used to obtain linear equations which govern the small amplitude vibrations of a prestressed, eccentrically stiffened, cylindrical shell which is submerged in a fluid medium.

The deformations associated with the vibration of a prestressed cylinder are divided into two parts as follows:

$$u = u_A + u_B$$

$$v = v_A + v_B$$

$$w = w_A + w_B \quad (4.7)$$

The first part, denoted by subscript A, is an axisymmetric static prestress deformation which occurs prior to excitation at one of the natural frequencies. The second part, denoted

by subscript B, is a small additional deformation which occurs as a result of the excitation. Since the A subscripted quantities are static they are axisymmetric deformations; therefore, the terms  $\bar{M}\ddot{u}_A$ ,  $\bar{M}\ddot{v}_A$ ,  $\bar{M}\ddot{w}_A$  and all derivatives with respect to y vanish. The equilibrium equations which govern these deformations are found from equations (4.4) as

$$N_{xA,x} = 0$$

$$N_{xyA,x} = 0$$

$$-M_{xA,xx} + \frac{N_{yA}}{R} - N_{xA}w_{A,xx} + P_h = 0 \quad (4.8)$$

If there is no applied shear, equation (4.8) yields

$$N_{xyA} = 0$$

If we let all derivatives with respect to y equal zero for axisymmetric deformation, a set of appropriate boundary conditions are found from equations (4.5) to be

$$M_{xA,x} + N_{xA} w_A = 0$$

$$\text{or } w_A = 0$$

$$M_{xA} + N_x \bar{e} = 0$$

$$\text{or } w_{A,x} = 0$$

$$N_{xA} + \bar{N}_x = 0$$

$$\text{or } u_A = 0$$

$$N_{xyA} = 0$$

$$\text{or } v_A = 0$$

where

$$N_{xA} = \frac{Et}{1-\mu^2} \left[ u_{A,x} + \frac{1}{2} w_{A,x}^2 + \mu \left( \frac{w_A}{R} \right) \right]$$

$$N_{yA} = \frac{Et}{1-\mu^2} \left[ \frac{w_A}{R} + \mu \left( u_{A,x} + \frac{1}{2} w_{A,x}^2 \right) \right] + \frac{E_r A_r}{l} \frac{w_A}{R}$$

$$N_{xyA} = G w_{A,x}$$

$$M_{xA} = -D w_{A,xx}$$

The equilibrium equations and boundary conditions which govern the dynamic deformations (subscript B) are obtained by substituting equation (4.7) into equation (4.4) and (4.5). By eliminating the axisymmetric prestress equations, and retaining only linear terms, i.e. neglecting the high order terms under the small amplitude vibration assumption, the following equations governing the dynamic deformations (subscript B) are obtained:

$$N_{xB,x} + N_{xyB,y} - \bar{M}\ddot{u}_B = 0$$

$$N_{yB,y} + N_{xyB,x} - M\ddot{v}_B = 0$$

$$\begin{aligned}
& - M_{xB,xx} + M_{xyB,xy} - M_{yxB,xy} - M_{yB,yy} \\
& + \frac{N_{yB}}{R} - N_{xA} w_{B,xx} - N_{xB} w_{A,xx} - N_{yA} w_{B,yy} \\
& + P_r - \bar{M} \dot{w}_B = 0 \qquad (4.11)
\end{aligned}$$

and the boundary conditions become

$$\begin{aligned}
M_{xB,x} - (M_{xyB,y} - M_{yxB,y}) + N_{xA} w_{B,x} \\
+ N_{xB} w_{A,x} = 0
\end{aligned}$$

$$\text{or } w_B = 0$$

$$M_{xB} = 0$$

$$\text{or } w_{B,x} = 0$$

$$N_{xB} = 0$$

$$\text{or } u_B = 0$$

$$N_{xyB} = 0$$

$$\text{or } v_B = 0 \quad (4.12)$$

where

$$N_{xB} = \frac{Et}{1-\mu^2} \left[ u_{B,x} + w_{A,x} w_{B,x} + \mu \left( v_{B,y} + \frac{w_B}{R} \right) \right]$$

$$\begin{aligned} N_{yB} = \frac{Et}{1-\mu^2} & \left[ v_{B,y} + \frac{w_B}{R} + \mu (u_{B,x} + w_{A,x} w_{B,x}) \right] \\ & + \frac{E_r A_r}{l} \left( v_{B,y} + \frac{w_B}{R} \right. \\ & \left. - \bar{z}_r w_{B,yy} \right) \end{aligned}$$

$$N_{xyB} = Gh (u_{B,x} + v_{B,x} + w_{A,x} w_{B,y})$$

$$M_{xB} = -D (w_{B,xx} + \mu w_{B,yy})$$

$$M_{yB} = -D (w_{B,yy} + \mu w_{B,xx}) - \frac{E_r I_r}{l} w_{B,yy} \\ + \bar{z}_r \frac{E_r A_r}{l} \left( v_{B,y} + \frac{w_B}{R} - \bar{z}_r w_{B,yy} \right)$$

$$M_{xyB} = \frac{Gh^3}{6} w_{B,xy}$$

$$M_{yxB} = - \left( \frac{Gh^3}{6} + \frac{G_r J_r}{l} \right) w_{B,xy} \quad (4.13)$$

#### Frequency Equation Derivation

Assuming a constant prestress deformation  $w_A$ , the solution equations are

$$N_{xA} = - \bar{N}_x = - \frac{PR}{2}$$

$$N_{yA} = - \bar{N}_y = - PR \quad (4.14)$$

Equation (4.11) may now be written as

$$N_{xB,x} + N_{xyB,y} - \bar{M} \ddot{u}_B = 0$$

$$N_{yB,y} + N_{xyB,x} - \bar{M} \ddot{v}_B = 0$$

$$- M_{xB,xx} + M_{xyB,xy} - M_{yxB,xy} - M_{yB,yy}$$

$$+ \frac{N_{yB}}{R} + \bar{N}_x w_{B,xx} + \bar{N}_y w_{B,yy} + P_r$$

$$- \bar{M} \ddot{w}_B = 0 \quad (4.15)$$

The displacement  $u_B$ ,  $v_B$  and  $w_B$  which satisfy simple support boundary conditions are given as

$$u_B = U e^{i\omega t} \cos\left(\frac{m\pi x}{L}\right) \sin\left(\frac{ny}{R}\right)$$

$$v_B = V e^{i\omega t} \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{ny}{R}\right)$$

$$w_B = W e^{i\omega t} \sin\left(\frac{m\pi x}{L}\right) \cos\left(\frac{ny}{R}\right) \quad (4.16)$$

The equation (2.11) gives

$$P_r = \rho_w \omega^2 f(\omega) e^{i\omega t} \bar{W} \sin\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi y}{R}\right)$$

where  $m$  is the number of axial half-waves and  $n$  is the number of circumferential full waves. If equations (4.16) are substituted into equation (4.15), taking into account (4.13), and the immediately preceding equation, it yields

$$\begin{aligned} & - \left( M\omega^2 + \frac{Eh}{1-\mu^2} \frac{m^2\pi^2}{L} + Gh \frac{n^2}{R} \right) \bar{U} \\ & + \frac{mn\pi}{LR} \left( \frac{\mu Eh}{1-\mu} + Gh \right) \bar{V} - \frac{\mu Eh}{1-\mu^2} \frac{m\pi}{LR} \bar{W} = 0 \end{aligned}$$

$$\begin{aligned} & \frac{mn\pi}{LR} \left( \frac{\mu Eh}{1-\mu} + Gh \right) \bar{U} - \left( M\omega^2 + \frac{Eh}{1-\mu^2} \frac{n^2}{R^2} \right. \\ & \left. + \frac{E_r A_r}{l} \frac{n^2}{R^2} + Gh \frac{m^2\pi^2}{L^2} \right) \bar{V} - \frac{n}{R^2} \left( \frac{Eh}{1-\mu^2} \right. \\ & \left. + \frac{E_r A_r}{l} + \frac{E_r A_r \bar{z}_r n^2}{lR} \right) \bar{W} = 0 \end{aligned}$$

$$\begin{aligned}
& \frac{\mu E h}{1-\mu^2} \frac{\mu m \pi}{R L} \bar{U} - \frac{n}{R^2} \left( \frac{E h}{1-\mu^2} + \frac{E_r A_r}{l} \right. \\
& \left. + \frac{E_r A_r z_r n}{l R} \right) \bar{V} - \left[ D \left[ \left( \frac{m \pi}{L} \right)^4 + \left( \frac{n}{R} \right)^4 + 2 \mu \left( \frac{m n}{L R} \right)^2 \right] \right. \\
& \left. + \left( \frac{G h^3}{3} + \frac{G_r J_r}{l} \right) \left( \frac{m n \pi}{L R} \right)^2 + \frac{E_r I_r}{l} \left( \frac{n}{R} \right)^4 \right. \\
& \left. + \frac{E_r A_r}{l R^2} \left[ 1 + \bar{z}_r \left( \frac{2 n}{R} \right)^2 + \bar{z}_r \frac{n^4}{R^2} \right] + \frac{E h}{1-\mu^2} \frac{1}{R^2} \right. \\
& \left. - N_x \left( \frac{m \pi}{L} \right)^2 - N_y \left( \frac{n}{R} \right)^2 - \left[ M + \rho_w f(\omega) \right] \omega^2 \right] \bar{W} = 0
\end{aligned}$$

(4.17)

The last of these equations contains a term involving  $f(\omega)$  which may be real or complex, depending on whether the  $K^2$  of Equation (2.4) is negative or positive, respectively. When  $f(\omega)$  is complex, the real part may be considered as the "added mass" effect of the fluid and the imaginary part a form of damping associated with energy loss to the fluid. For the range of frequencies of interest here for shells

immersed in water,  $K^2$ , will usually be negative, and thus  $f(\omega)$  is real. Thus, for the purpose of this study, a frequency equation where  $f(\omega)$  is real is developed.

Equation (4.17) may conveniently be written as

$$[A] \{B\} = 0 \quad (4.18)$$

where

$$\{B\} = \begin{Bmatrix} \bar{U} \\ \bar{V} \\ \bar{W} \end{Bmatrix} \quad (4.19)$$

This is not, however, a linear eigenvalue problem, since  $\omega$  occurs in  $f(\omega)$ .

For the case where  $f(\omega)$  is real, the determinant of the displacement coefficients, when set equal to zero, yields a sixth-order equation of the circular frequency

$$[A] = 0 \quad (4.20)$$

where

$$A_{12} = A_{21} = \frac{mn\pi}{LR} \left( Gh + \frac{\mu Eh}{1-\mu^2} \right)$$

$$A_{13} = A_{31} = \frac{\mu E h}{1 - \mu^2} \frac{m \pi}{LR}$$

$$A_{23} = A_{32} = - \frac{n}{R^2} \left[ \frac{E h}{1 - \mu^2} + \frac{E_r A_r}{l} \left( 1 + \bar{z}_r \frac{n^2}{R^2} \right) \right]$$

$$A_{11} = - \left( \bar{M} \omega^2 + \frac{E h}{1 - \mu^2} \frac{m^2 \pi^2}{L^2} + G h \frac{n^2}{R^2} \right)$$

$$A_{22} = - \left( \bar{M} \omega^2 + \frac{E h}{1 - \mu^2} \frac{n^2}{R^2} + \frac{E_r A_r}{l} \frac{n^2}{R^2} + G h \frac{m^2 \pi^2}{L^2} \right)$$

$$A_{33} = \left[ \bar{M} + \rho_w f(\omega) \right] \omega^2 - \left[ D \left[ \left( \frac{m \pi}{L} \right)^4 + \left( \frac{n}{R} \right)^4 + 2 \mu \left( \frac{m n}{LR} \right)^2 \right] \right.$$

$$+ \left( \frac{G h^3}{3} + \frac{G_r J_r}{l} \right) \left( \frac{m n \pi}{LR} \right)^2 + \frac{E_r J_r}{l} \left( \frac{n}{R} \right)^4$$

$$+ \frac{E_r A_r}{l R^2} \left[ 1 + \bar{z}_r \left( \frac{2 n^2}{R} + \bar{z}_r \frac{n^4}{R^2} \right) \right]$$

$$+ \frac{E h}{1 - \mu^2} \frac{1}{R^2} - N_x \left( \frac{m \pi}{L} \right)^2 - N_y \left( \frac{n}{R} \right)^2 \left. \right] \quad (4.21)$$

The result of  $[A] = 0$  is

$$\Lambda_3 \omega^6 + \Lambda_2 \omega^4 + \Lambda_1 \omega^2 + \Lambda_0 = 0 \quad (4.22)$$

where

$$F_1 = \left[ \bar{M} + \rho_w f(\omega) \right]$$

$$F_2 = \frac{Eh}{1-\mu^2} \frac{m^2 \pi^2}{L^2} + Gh \frac{n^2}{R^2}$$

$$F_3 = \left( \frac{Eh}{1-\mu^2} + \frac{E_r A_r}{l} \right) \frac{n^2}{R^2} + Gh \frac{m^2 \pi^2}{L^2}$$

$$F_4 = A_{12} = \left( Gh + \mu \frac{Eh}{1-\mu^2} \right) \frac{mn\pi}{LR}$$

$$F_5 = A_{23} = - \left[ \frac{Eh}{1-\mu^2} + \frac{E_r A_r}{l} \left( 1 + \bar{z}_r \frac{n^2}{R} \right) \right] \frac{n}{R^2}$$

$$F_6 = A_{13} = \frac{\mu Eh}{1-\mu^2} \frac{m\pi}{LR}$$

$$\begin{aligned}
F_7 = & D \left[ \left( \frac{m\pi}{L} \right)^4 + \left( \frac{n}{R} \right)^4 + 2\mu \left( \frac{m\pi}{LR} \right)^2 \right] \\
& + \left( \frac{Gh^3}{3} + \frac{G_r J_r}{I} \right) \left( \frac{m\pi}{LR} \right)^2 + \frac{E_r I_r}{I} \left( \frac{n}{R} \right)^4 \\
& + \frac{E_r A_r}{I R^2} \left[ 1 + \bar{z}_r \left( \frac{2n^2}{R} + \bar{z}_r \frac{n^4}{R^2} \right) \right] \\
& + \frac{Eh}{1-\mu^2} \frac{1}{R^2} - N_x \left( \frac{m\pi}{L} \right)^2 - N_y \left( \frac{n}{R} \right)^2
\end{aligned} \tag{4.23}$$

and

$$\begin{aligned}
\Lambda_3 &= F_1 M^2 \\
\Lambda_2 &= M \left[ F_1 (F_2 + F_3) - M F_7 \right] \\
\Lambda_1 &= F_1 \left[ F_2 F_3 - (F_4)^2 \right] - M \left[ (F_2 + F_3) F_7 - (F_5)^2 - (F_6)^2 \right] \\
\Lambda_0 &= \left[ (F_4)^2 - F_2 F_3 \right] F_7 + (F_5)^2 F_2 + (F_6)^2 F_3 \\
&+ 2F_4 F_5 F_6
\end{aligned}$$

(4.24)

After the natural frequency,  $\omega$ , is obtained by equating the determinant [A] to zero and substitution of the the value of  $\omega$  into equation (4.18), the non-trivial solution will provide the amplitude ratios from the three algebraic equations of the coefficients of  $U$ ,  $V$  AND  $W$ .

Where only one degree of freedom is to be considered, only the radial surface motion of the system is permitted and other surface motions are neglected. The frequency equation is derived from equation (4.20) by equating  $M\omega^2$  terms to zero in  $A_{11}$  and  $A_{22}$ . This equation has the following form:

$$\Lambda_1' \omega_0^2 + \Lambda' = 0 \quad (4.25)$$

#### Numerical Procedures

Both the three and one degree of freedom frequency equations (4.22) and (4.25), respectively, must be solved numerically since some coefficients are functions of  $\omega$ . For this report, a sequence of values of  $\omega$  were substituted into the left hand side of equation (4.22) and a bisection method employing the second rule [17] was used to locate a root when a change in the sign of the left hand side occurred. This technique was used to investigate the possibility of multiple real roots. None were found. A similar procedure was used for the single degree of freedom equation for programming convenience.

Such procedures are, of course, too computationally

wasteful for optimization work. In the optimal design situation, one almost always has a very close, or may obtain a reasonably close, estimate of the value of  $\omega$  satisfying equation (4.18) or (4.25). Thus, one may effectively use successive substitutions to solve these equations in relatively few iterations. For example, equation (4.18) may be treated as an eigenvalue problem, where the  $f(\omega)$  term is computed using the close approximation, to obtain a still closer approximation. This procedure would be repeated until convergence occurred. Equation (4.25) could be solved similarly. Thus, although the computational effort associated with frequency determination of submerged structures would be several times greater than those *invacuo*, the amount of effort would still be low enough for preliminary optimization studies, since the *invacuo* studies were not particularly computationally demanding [12], particularly if the one degree of freedom equation yields satisfactory accuracy.

## CHAPTER 5

## RESULTS AND COMPARISONS

Five example shells were analyzed in vacuum and in water so as to examine the effect of immersion in water, examine the accuracy of the formulation described herein, and compare the accuracy of the Donnell type and Flugge type shell theories. In addition, for several examples the effect of stiffener eccentricity is examined.

The first group consists of three shells developed in the studies of Ref [11] and design examples using a minimum frequency constraint from Ref [12] immersed in sea water at a depth of 1,000 feet. They will be referred to as examples 1, 2 and 3, respectively. These shells are analyzed in water and invacuo, where for the latter the effects of the hydrostatic pressure of immersion are considered. Shells with outside stiffeners are also analyzed in addition to the inside stiffened configurations of Refs [11,12]. The parameters used for these studies are

$$R = 198 \text{ in.}, L = 594 \text{ in.}, \mu = 0.33, E = 30 \times 10^6 \text{ psi},$$

$$\rho_c = 7.33 \times 10^{-4} \text{ slug/in}^3$$

$$\rho_w = 0.969 \times 10^{-4} \text{ slug/in}^3, c = 60,000 \text{ in/sec, Depth of immersion 1000 feet}$$

Where for example 1

$$h = 1.2056 \text{ in.}, r = 30.17 \text{ in.}, h_1 = 11.02 \text{ in.},$$

$$h_2 = 0.3071 \text{ in.}, h_3 = 10.363 \text{ in.}, h_4 = 0.2373 \text{ in.}$$

for example 2

$$h = 1.2216 \text{ in.}, \quad r_1 = 33.85 \text{ in.}, \quad h_1 = 20.72 \text{ in.}, \quad h_2 = 0.4653 \text{ in.},$$

$$h_3 = 17.55 \text{ in.}, \quad h_4 = 0.3950 \text{ in.}$$

and for example 3

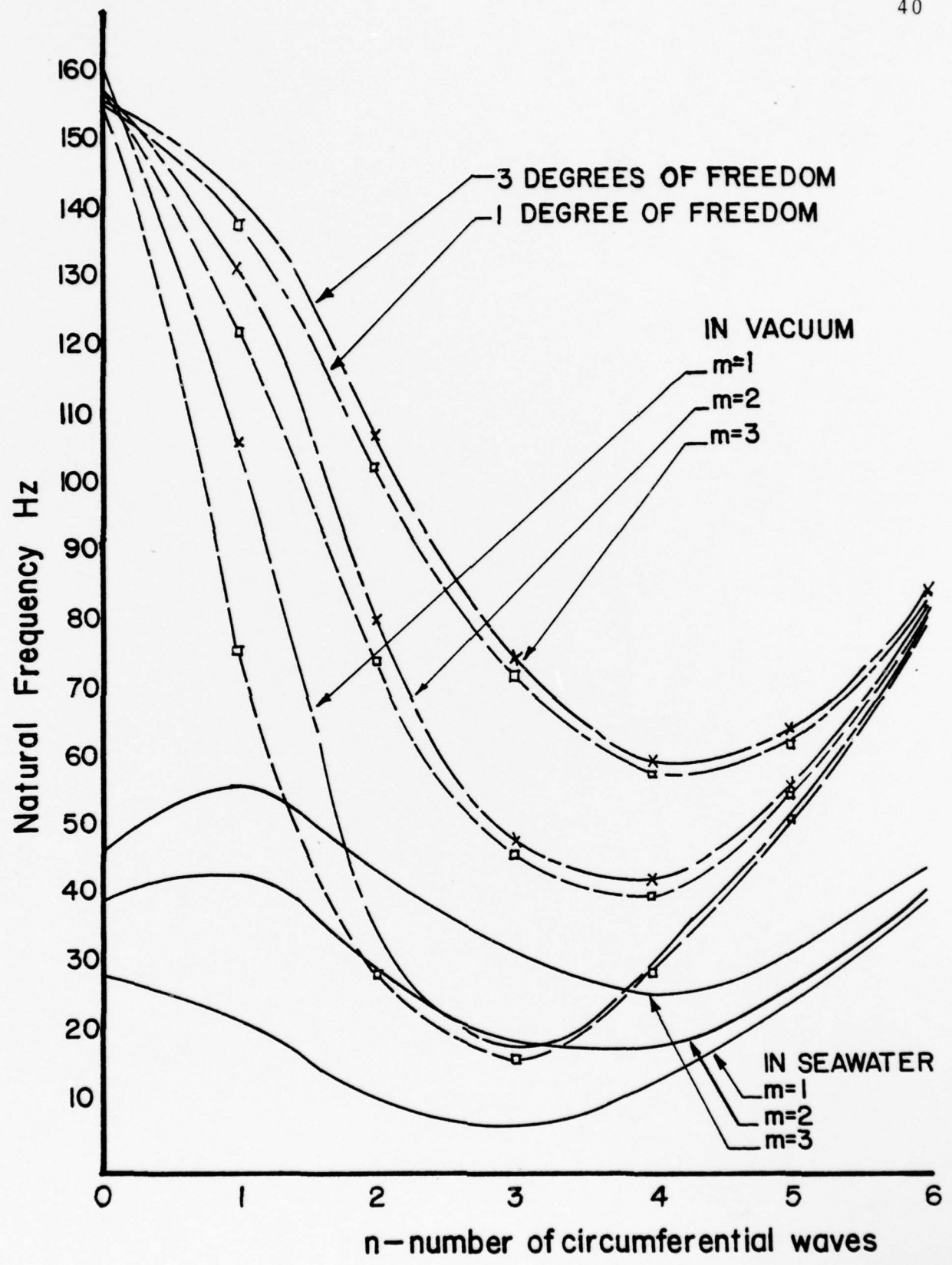
$$h = 1.1623 \text{ in.}, \quad r_1 = 16.71 \text{ in.}, \quad h_1 = 9.411 \text{ in.},$$

$$h_2 = 0.1832 \text{ in.}, \quad h_3 = 5.937 \text{ in.}, \quad h_4 = 0.2010 \text{ in.}$$

These shells differ primarily in the nature of their stiffeners. Example 2 employs relatively large, widely spaced stiffeners, while Example 3 uses relatively small, closely spaced stiffeners.

The effect of immersion in water, of ignoring the  $u$  and  $v$  motion components and stiffener eccentricity are very similar for all these examples. Thus, only the results of Example 1 are presented in tabular form. It may be seen from Table 1 and Figure 5.1 that the natural frequency characteristics of the invacuo model for shells immersed in water is grossly inaccurate, particularly with respect to frequency separation which is much less in water. Thus, the invacuo model does not seem appropriate for optimal frequency separation studies.

Other differences are also apparent. Although there is a substantial difference between frequencies computed using three degrees of freedom [Eq. (4.22)] and one degree of freedom [Eq. (4.25)] with the invacuo model, particularly where  $n = 1$ , this difference is for practical purposes negligible for shells considering fluid interaction effects of water. The largest difference for shells immersed in water also occurred  $n = 1$ . However, here the difference was typically only about



Figure,5-1 Natural Frequencies of Example I Shell

Table 1. Natural Frequencies Hz Example 1-Inside Frames

m		1		2		3	
n	(1)	(2)	(1)	(2)	(1)	(2)	
	In seawater	0	29.4	29.4	39.5	39.5	46.8
1		23.0	22.4	43.9	43.5	56.7	56.5
2		10.3	10.2	29.3	28.9	44.6	44.4
3		6.96	6.90	19.8	19.6	33.4	33.2
4		13.14	13.08	18.8	18.7	28.1	28.0
5		24.8	24.7	27.2	27.1	32.1	32.0
6		40.4	40.3	41.9	41.8	44.6	44.5
In vacuum	0	162	158	159	158	158	157
	1	107	76.8	133	123	142	139
	2	34.3	29.5	81.7	74.9	108	104
	3	18.6	17.6	49.1	46.6	76.1	73.4
	4	30.8	30.0	42.4	41.2	60.0	58.6
	5	53.1	52.2	56.8	55.9	64.5	63.6
	6	80.8	80.0	82.2	81.4	85.2	84.4

(1) Three degrees of freedom

(2) One degree of freedom

3 per cent. Thus, only one degree of freedom need be considered for such shells.

Comparing Tables 1 and 2, it may be seen that stiffer eccentricity has little effect except at the lower frequencies where outside stiffeners produced lower values. Thus, for optimization under a minimum frequency constraint inside stiffeners seem more effective.

A comparison of Donnell vs. Flugge theory for Examples 1 and 2 is given in Table 3. It may be seen that Donnell theory yields substantially higher values for the lower in-vacuo frequencies. As will be seen from Example 5, this difference represents a greater inaccuracy on the part of the Donnell theory at these frequencies.

Example 4 is used to provide some correlation between the present analysis and experimental results for shells immersed in water. Unfortunately, the differences in end conditions cloud the comparison somewhat. Still, this comparison is useful since for such shells the effect of end condition is not great. A comparison with experiments in air is also provided.

For this example;

$$R = 20.25 \text{ in.}, L = 60.75 \text{ in.}, \mu = 0.3, E = 30 \times 10^6 \text{ psi},$$

$$\rho_c = 7.77 \times 10^{-4} \text{ slug/in}^3, \rho_w = 0.934 \times 10^{-4} \text{ slug/in}^3,$$

$$c = 60,000 \text{ in/sec}, \text{ Depth of Immersion } 5 \text{ feet.}$$

$$h = .177 \text{ in.}, l = 3.375 \text{ in.}, h_1 = 1.783 \text{ in.},$$

$$h_2 = 0, h_3 = 0, h_4 = 0.181 \text{ in.}$$

Table 2. Natural Frequencies Hz Example 1-Outside Frames

m		1		2		3	
n		(1)	(2)	(1)	(2)	(1)	(2)
In seawater	0	29.4	29.4	39.5	39.5	46.8	46.8
	1	23.2	22.5	44.3	43.1	57.4	57.1
	2	10.2	10.0	30.3	28.8	46.4	46.0
	3	5.85	5.79	20.5	20.3	36.0	35.7
	4	12.1	12.1	18.4	18.3	30.3	30.1
	5	24.2	24.0	26.2	26.0	32.8	32.6
	6	40.0	39.8	40.8	40.6	44.2	43.9
In vacuum	0	162	158	159	158	158	157
	1	109	77.4	134	125	144	141
	2	34.5	29.2	85.3	77.4	113	108
	3	15.8	14.2	51.5	48.2	82.7	79.1
	4	29.0	27.8	42.0	40.2	65.3	62.9
	5	52.3	50.8	55.2	53.6	66.7	64.8
	6	80.7	79.0	80.9	79.1	85.3	83.4

(1) Three degrees of freedom analysis

(2) One degree of freedom analysis

Table 3. Comparison of Frequencies with Method of Ref [11]

	Example 1			Example 2		
	(1)	(2)	(3)	(1)	(2)	(3)
$\omega_1$ - Hz	12.03	17.6	18.62	28.37	36.70	41.50
$\omega_2$ - Hz	22.90	29.52	30.81	51.96	58.58	60.88
$\omega_3$ - Hz	30.30	30.00	34.27	51.96	68.17	70.55

- (1) Bronowicki et al. Flugge theory three degrees of freedom
- (2) Present analysis. Donnell theory one degree of freedom
- (3) Present analysis. Donnell theory three degrees of freedom

It may be seen from Table 4 that the one degree of freedom model used here provides reasonable agreement with experiment. Considering three degrees of freedom produces slightly poorer results due to the inaccuracy inherent in the Donnell theory for shells of such parameters. Agreement is better for shells in water than in air. This is, of course, expected since the inaccuracies in the Donnell theory affect only the shell stiffener terms and thus this improvement in accuracy in water indicates that the fluid-structure interaction model yields good results.

The final example is used to provide a more extended comparison between Donnell and Flugge theories. The parameters for this study are:

$$R = 10.2 \text{ in.}, L = 30.0 \text{ in.}, \mu = 0.33, E = 10 \times 10^6 \text{ psi},$$

$$\rho_c = 2.54 \times 10^{-4} \text{ slug/in}^3, \rho_w = 9.69 \times 10^{-5} \text{ slug/in}^3,$$

$$c = 60,000 \text{ in/sec.}, \text{ Depth of Immersion } 0 \text{ feet.}$$

$$h = 0.330 \text{ in.}, l = 5.00 \text{ in.}, h_1 = 1.00 \text{ in.}, h_2 = h_3 = 0,$$

$$h_4 = 0.375 \text{ in.}$$

The results for the invacuo study are compared in Table 5 with those given by Harari and Baron [4]. It may be seen that at the lower frequencies the difference is again quite pronounced, although proportionally less than in Examples 1 and 2, where stability effects contribute substantially and thus Donnell inaccuracies are compounded. Divergence between theories increases with increasing  $m$  and decreases with increasing  $n$ . Unfortunately, at larger  $n$ , where agreement be-

Table 4. Comparison with Paisley et al. [10]

		Natural Frequencies Hz			
m		1		2	
n		(1)	(2)	(1)	(2)
In water	2	128/134	147	414	509
	3	204	218	382	428
	4	373	395	466	512
	5	599	657	655	733
	6	860	999	902	1046
In air	2	262/270	322	-	921
	3	369/383	424	-	738
	4	650	719	760	809
	5	990	1127	1036	1089

(1) Experimental results from [10]

(2) Present one degree of freedom analysis

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Table 5. Comparison with Experiment and Harari and Baron  
Natural Frequencies Hz

m	1			2			3		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
0	-	-	-	-	-	-	3020	3105	3096
1	1232	1133	1526	-	-	-	-	-	-
2	627	640	756	-	-	-	-	-	-
3	787	832	922	1190	1194	1381	1602	1650	1923
4	1310	1432	1483	1503	1575	1764	1806	1826	2183
5	1938	2253	2253	2059	2331	2483	2276	2474	2839
6	2594	3276	3205	-	-	-	2602	3424	3753
7	3179	4466	4334	-	-	-	-	-	-
8	3728	5853	5637	-	-	-	-	-	-

- In Vacuum
- (1) Experimental results Ref [18]
  - (2) Harari and Baron orthotropic theory [4]
  - (3) Present one degree of freedom analysis

tween theories is good, orthotropic theory produces poor results [4].

Thus it appears that, although the present method gives reasonably good results for shells immersed in water, use of the Flugge theory could significantly improve accuracy.

Example 4 was also examined for effects of stiffener eccentricity and ignoring the  $u$  and  $v$  motions as was Example 5, which was also analyzed for frequencies immersed in water. Results produced followed closely those of Examples 1-3 with regard to these effects except that the effect of stiffener placement and use of only one degree of freedom were even less significant in Examples 4 and 5.

## CHAPTER 6

CONCLUSION

The interim procedure for calculation of frequencies of submerged shells described here seems quite adequate for preliminary optimization studies of shells immersed in water. The simplicity of the shell model and the fact that only one degree of freedom need be considered allows rapid evaluation of frequency behavior, an important asset for optimization, which usually requires several hundred sets of such evaluations for the determination of one optimal design. For all modes of interest here the radiated pressure was real and thus the procedure readily allows consideration of the fluid interaction effect. Thus, the existing method seems well adapted for preliminary optimization under a minimum frequency constraint or for preliminary optimal frequency separation problems involving the lower frequencies.

Use of a Flugge-type shell theory would improve accuracy and make the procedure useful for design purposes for such problems. Extension to treat cases where the radiated pressure is complex is also straightforward.

The primary limitation of this method even with the above extensions in treating the optimal frequency separation problem is the possible presence of active inter-ring vibration modes at the optimum [13]. Realistic treatment in such

circumstances requires use of discrete theory [4], [19] and its associated large computational effort. Even this extension is probably tractable in light of current optimization developments [20,21]. The other important limitation results from the fact that the end effects of fluid interaction with the shell are ignored. These effects would play an important role in frequencies associated with primarily axial models, although these fluid effects should be less important here than they are for primarily axial modes. The inclusion of these effects is a formidable problem particularly if an approach considering these effects is to be used for the purposes of optimization. Thus, the prospect of early development of reasonably accurate optimization procedures for primarily axial frequency separation seems remote.

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