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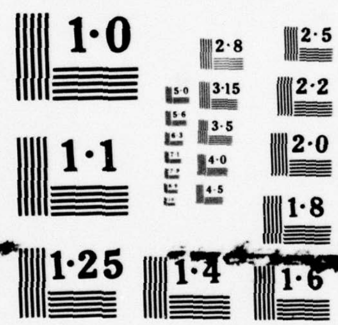

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Robust Sequential Detection of Narrowband Acoustic Signals in Noise

A Paper Presented at the IEEE International Conference on Acoustics, Speech, and Signal Processing, 2-4 April 1979

Roger F. Dwyer
Surface Ship Sonar Systems Department

9 May 1979

NUSC
Naval Underwater Systems Center
Newport, Rhode Island • New London, Connecticut

PREFACE

This technical document was prepared under Project No. A-750-28, "Adaptive Nonparametric Sequential Detection," Principal Investigator, R. F. Dwyer (Code 333); Program Element 61152N, Navy Subproject/Task ZR 0000101, "In-House Laboratory Independent Research," Program Manager, J. H. Probus (MAT 08T1).

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W. A. VonWinkle
Associate Technical Director
for Technology

The author of this document is located at the
New London Laboratory, Naval Underwater Systems Center,
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Introduction

It is well known that impulsive interference can seriously degrade performance of detectors designed with the assumption that Gaussian noise exists. Therefore, it is of interest to develop robust procedures for detecting signals in noise; i.e., it is desired that a sequential detector perform relatively well in a broad class of noise environments. Obtaining optimum detector structures is of special interest in noise environments consisting of an additive mixture of Gaussian and impulsive (large variance) noise. These noise environments occur from reverberation in active sonar and from natural as well as man-made sources in passive sonar.

-- First slide please --

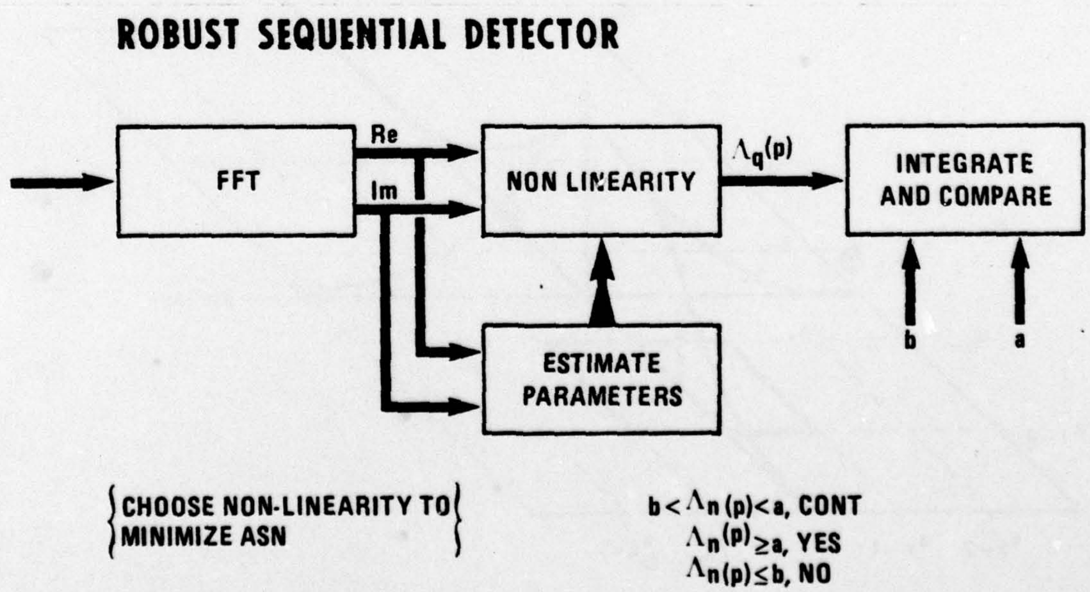
Slide 1

The optimum sequential detector at the output of an FFT can be obtained by forming a loglikelihood ratio. Since the optimum detector must also be adaptive, the loglikelihood ratio is formed based on quantiles and related functions, which can be efficiently estimated. Depending on the signal characteristics, the loglikelihood ratio will be composed of incoherent and coherent components. For any particular noise environment, the nonlinearities for both the real (*Re*) and imaginary (*Im*) parts are chosen to minimize the average sample number.

The output of the nonlinear device is integrated and compared with two thresholds *a* and *b*. The classical sequential decision is made about the presence or absence of the signal, and, in the event that no decision is made, the process is continued until a threshold is reached. The thresholds are functions of the desired false alarm and false dismissal probabilities.

The output *cap lambda of q*, where *q* represents the current FFT output, is assumed independent and identically distributed for all *q*. Also, it is assumed that the signal is weak and that the mean and variance of the output are sufficient to characterize the performance of the robust sequential detector in terms of its operating characteristic function and average sample number.

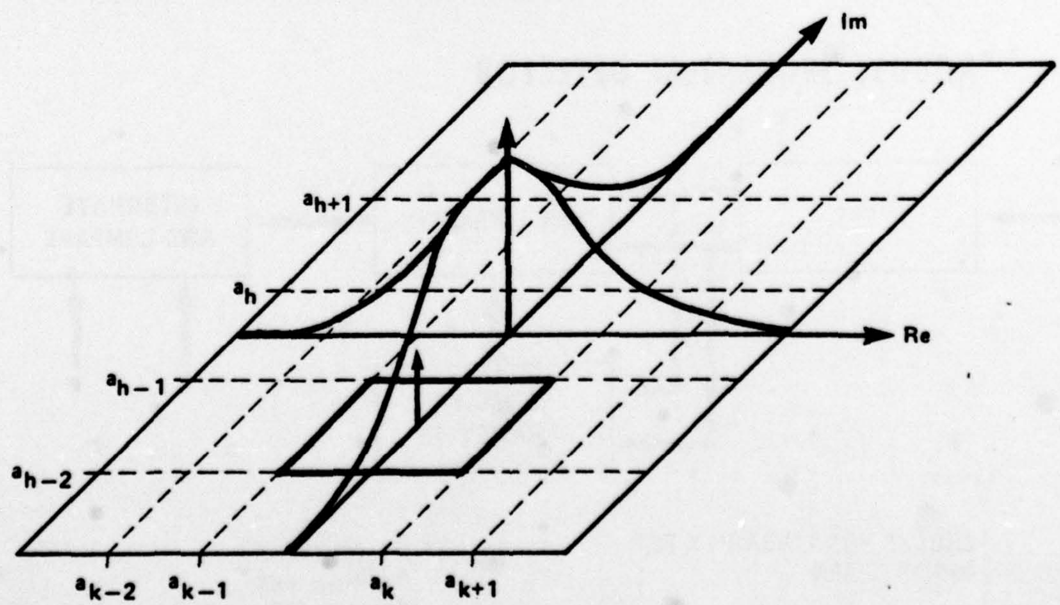
For an incoherent signal and purely Gaussian noise, the locally optimum nonlinearity is a square law device. In mixture distributions containing impulsive noise, the locally optimum nonlinearity is not a square law device. The problem, then, is to choose the nonlinearity in contaminated impulsive noise so that the average sample number is minimized while maintaining the desired error probabilities over the signal-to-noise ratios of interest. This is done by estimating a set of parameters from the unknown noise distribution.



Slide 2

The parameters to be estimated are the quantiles and scores for each frequency component. It is assumed that the noise statistics change slowly over frequency so that a noise estimate is composed of both frequency and time samples. The impulsive interference is assumed to be narrowband compared with the total bandwidth of the FFT. However, it is broad compared with the signal's bandwidth. In other noise models, the interference may have the same bandwidth as the signal. In such cases, a signal-free estimate is assumed prior to detection or the signal characteristics are used to obtain the required noise-only estimate. For example, if the signal has constant but unknown phase, which is usually characterized as being randomly distributed over the interval from 0 to 2 pi, then the signal can be extracted from any noise estimate. The quantiles for both the real and imaginary parts are usually estimated recursively, and, at the same time, the density function (in the case of an incoherent signal) and the slope of the density function (in the case of a coherent signal) are estimated at the quantile locations. The density function and its slope are used to construct weights which are called scores. In this way, under any mixture noise environment, the detection statistic maintains a constant false alarm rate. For example, if a data sample falls between the quantiles *a sub k minus one* and *a sub k* for the real part and between *a sub h minus one* and *a sub h minus two* for the imaginary part, then the appropriate score will be assigned. Note that no squaring or multiplying in the usual sense is done, rather the data falling into an interval is assigned a score from a look-up table. (The look-up table and quantiles are updated from the recursive algorithm.)

QUANTILE AND SCORE SELECTION



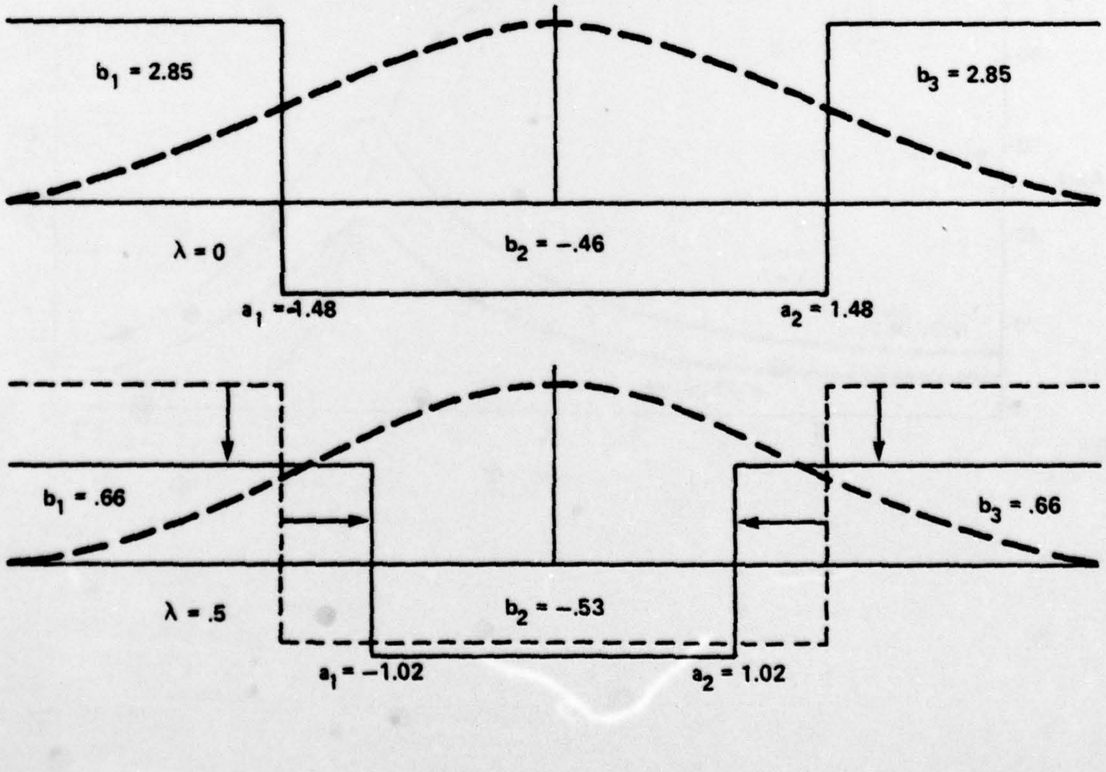
Slide 3

For convenience only, the incoherent signal case will be discussed in the following examples. The optimum weak signal incoherent detector forms a *sum of the squares* of the real and imaginary parts, which is known as a quadratic detector. Here the optimum nonlinearity is approximated using two quantiles and three scores. The distribution was assumed symmetric around zero and the real and imaginary parts identical. Therefore, only one quantile and two scores need be estimated. In applications, the number of quantiles and scores used to form the statistic depends upon the desired performance. In general, the more quantiles and scores used in the robust detector, the closer its performance is to the optimum; however, usually there is little to be gained after 8 or 10 quantiles.

The figure at the top represents the nonlinearity for Gaussian noise of unit variance without impulsive interference, indicated by *lambda equal to zero*, where *lambda* represents the percent of time interference is present during the decision interval. However, the time of occurrence of any impulsive sample is assumed random. The quantiles are located at + and − *1.48* and the scores are *2.85* and *−.46*. As the noise variance increases for the uncontaminated case, the quantiles will spread apart and the scores will adjust so that the detector maintains a constant false alarm rate.

In the lower figure, the noise is contaminated with Gaussian noise with a standard deviation of 3 *occurring 50 percent of the time*. The quantiles are now closer to the center compared to the top figure, and the scores have also been reduced. This means that the optimum statistic should weight the larger data samples less in impulsive interference compared with the uncontaminated case.

OPTIMUM QUANTILES AND SCORES; m=3



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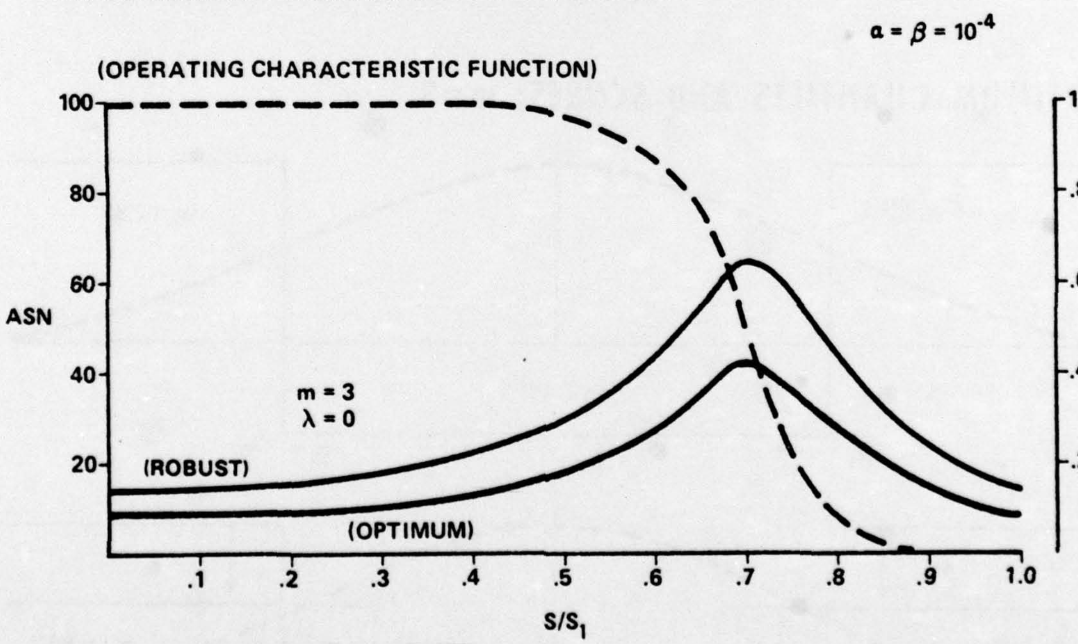
The performance of the robust sequential detector is based on its average sample number and operating characteristic function or on the probability of accepting the signal-free case as a function of the signal-to-noise ratio. Here, the performance is compared with the optimum weak signal incoherent sequential detector. The solid curves compared the average-sample-numbers of the robust and optimum where two quantiles and three scores were used to construct the robust sequential detector. If more quantiles were used, the robust's average sample number would approach the optimum's average sample number. However, the operating characteristic function of both sequential detectors are identical, which means an efficiency comparison based on the ratio of average numbers is meaningful. In this example, there was no contaminating noise. It can be shown that the operating characteristic function can be maintained under impulsive interference by adapting the quantiles and scores for the robust sequential detector and by normalizing the output by dividing by the noise variance for the optimum or conventional sequential detector. So an efficiency comparison is still meaningful under impulsive interference.

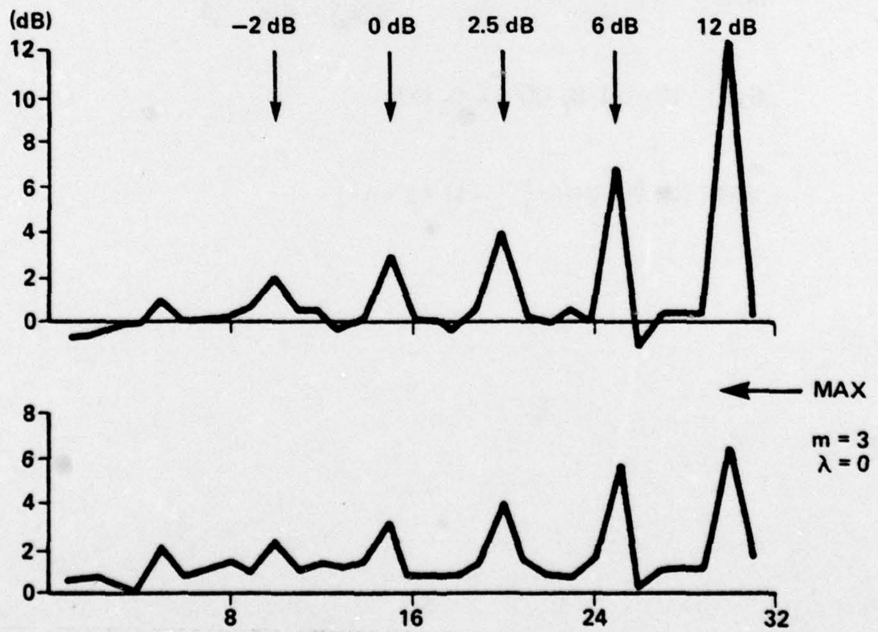
PERFORMANCE OF ROBUST AND OPTIMUM WEAK SIGNAL INCOHERENT SEQUENTIAL DETECTORS



Slide 5

Here the spectrum for a 64 pt FFT is shown using a 100 sample average. The upper figure represents the conventional magnitude squared spectrum using 36 bits of information. The lower figure is the output of the robust detector using two quantiles and 3 scores. Five different signal-to-noise ratios were injected from – 2 dB to 12 dB. The lower signal-to-noise ratios appear at about the same level in dB for both figures. As the signal level is increased, the spectrum based on quantiles will reach a maximum value. All signal values beyond that point will appear at the same level. By using more quantiles and scores, the robust spectrum will approach the upper spectrum.

SPECTRUM OF 64 PT. FFT; 100 SAMPLES



Slide 6

The weak signal performance measure will be based on the ratio of the average sample numbers of the conventional and robust sequential detector. As the signal-to-noise ratio approaches zero for fixed error probabilities, the performance measure is called the asymptotic relative efficiency or *ARE*. The *ARE* compares the noise variance at the output of the quadratic sequential detector to the variance estimate based on quantiles and scores for the robust sequential detector, which will maintain the desired operating characteristic function.

The probability of falling into an interval is given by *cap G of X*. The optimum choice of quantiles and scores for fixed m could be obtained by maximizing the ARE under any particular conditions. A more practical technique fixes cap G of X under all conditions. Then, the quantiles and scores are adapted under this constraint. The noise variance is given in the lower equation. It is a function of the contaminated interference indicated by *lambda*.

WEAK SIGNAL PERFORMANCE MEASURE

$$ARE = \frac{VAR(|X|^2)}{4} \sum_{k=1}^{m} \frac{\left(g'(a_k) - g'(a_{k-1})\right)^2}{G(a_k) - G(a_{k-1})}$$

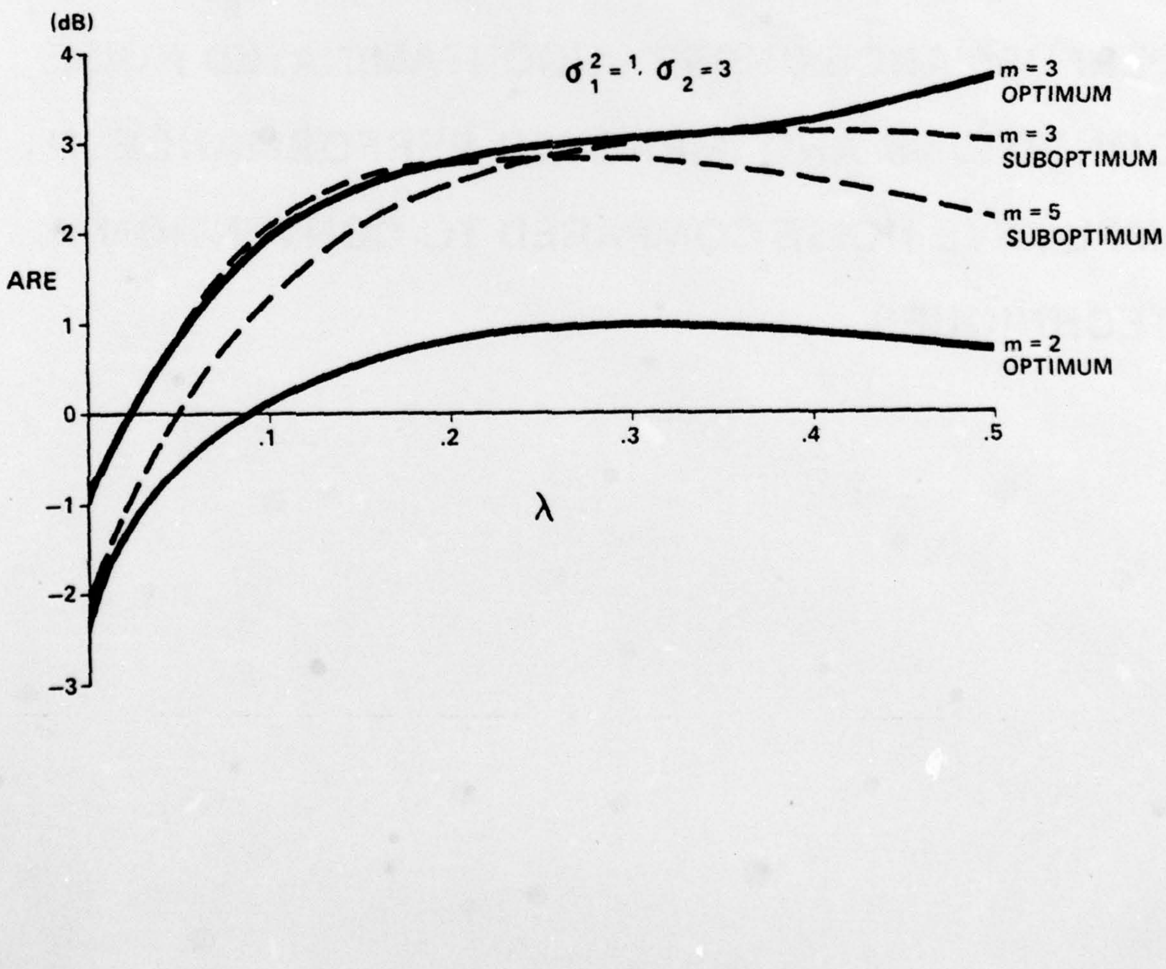
$$G(X) = (1-\lambda)\, G_1(X) + \lambda\, G_2(X)$$

$$VAR(|X|^2) = 2 + \left((\sigma_2^2)^2 - 1\right)(\lambda + \lambda^2)$$

Slide 7

Here the performance of the robust and conventional sequential detectors are compared based on the ARE calculation. The results are given in dB, which is defined as ten log of the square root of the ARE. Therefore, the results can be interpreted in terms of signal-to-noise ratio. The horizontal axis gives the percentage of time impulsive interference occurred. Everything above 0 dB represents signal-to-noise ratio improvement over the conventional detector. The solid curves represent the performance of the optimum robust detector for *m equal to two and m equal to three.* The broken curves give the performance when the interval probabilities were fixed. For *lambda equal to zero*, the conventional sequential detector is more efficient; however, this can be overcome by using more quantiles. As *lambda* increases, the robust detector becomes more efficient. This improvement can be very large depending on the severity of the impulsive interference.

WEAK SIGNAL PERFORMANCE IN IMPULSIVE NOISE



Slide 8

Robust processing provides near optimum performance under uncontaminated noise conditions and improved performance in impulsive noise compared to the conventional techniques.

CONCLUSION

ROBUST PROCESSING PROVIDES NEAR OPTIMUM PERFORMANCE UNDER UNCONTAMINATED NOISE CONDITIONS AND IMPROVED PERFORMANCE IN IMPULSIVE NOISE COMPARED TO CONVENTIONAL TECHNIQUES.

Proceedings Reprint

Robust Sequential Detection of Narrowband Acoustic Signals in Noise

ROBUST SEQUENTIAL DETECTION OF NARROWBAND ACOUSTIC SIGNALS IN NOISE

Roger F. Dwyer

Naval Underwater Systems Center
New London, CT 06320

ABSTRACT

Given only the outputs from a discrete Fourier transform (DFT) a robust sequential detector (RSD) for weak signals is derived based on m-interval partitioning. A loglikelihood ratio at the output of the DFT is formulated assuming knowledge of a set of quantiles and scores of the unknown and possible changing noise distribution. The quantiles and scores are chosen to minimize the average time to a decision under practical implementation constraints.

INTRODUCTION

Often only the outputs of a DFT are available for signal detection and usually the constraints needed to insure Gaussianity (1),(2) are only approximately met in practice. In cases where the input noise is corrupted by impulsive or large variance noise, the performance of detectors designed assuming Gaussian statistics can degrade significantly (3). In the following discussion the noise will be assumed to consist of an additive mixture of Gaussian and impulsive noise (4),(5),

$$F(x)=(1-\lambda)F_1(x)+\lambda F_2(x) \qquad (1)$$

where F_1 and F_2 are the distribution functions of Gaussian and impulsive noise respectively, and, $0\leq\lambda\leq 1$, represents the average amount of time impulsive noise is present over the decision interval.

THE LOGLIKELIHOOD RATIO

For the set of complex numbers

$$\left\{ x(qM+i) \right\}_{i=0}^{M-1}, \quad q=0,1,\ldots,n,$$

where M is finite and n is random, the DFT is defined as

$$X_q(p)=\frac{1}{\sqrt{M}}\sum_{i=0}^{M-1} x(qM+i)\exp(-j2\pi ip/M) \qquad (2)$$

$$= s_q^c(p)+N_q^c(p) + j\left(s_q^s(p)+N_q^s(p) \right)$$

$$= X_q^c(p)+j\, X_q^s(p) \qquad p=0,1,\ldots,M-1$$

where $s_q^c(p)$ and $s_q^s(p)$ are the in-phase and quadrature component estimates of the signal at the p^{th} frequency over the DFT length M respectively, and,

$N_q^c(p)$ and $N_q^s(p)$ are the in-phase and quadrature components of the i.i.d. noise samples for, p=0,1,...,M-1.

The detection problem may be stated in terms of a hypothesis test

$$H_0: \prod_{q=0}^{n} G\left(X_q^c(p), X_q^s(p) \right)$$

$$H_1: \prod_{q=0}^{n} G\left(X_q^c(p)-s_q^c(p), X_q^s(p)-s_q^s(p) \right) \qquad p=0,1,\ldots,M-1$$

where G is the joint cdf of the noise output.

Robust Sequential Detector

The robust sequential detector (RSD) (6) requires the specification of 2*(m-1) quantiles (a_k^c; a_h^s) k=1,...,m-1, h=1,...,m-1, for each frequency component, p, where $a_k^c=G_c^{-1}(\epsilon_k)$, $a_h^s=G_s^{-1}(\epsilon_h)$ and G_c, G_s are the marginal cdf's of G and ϵ_k, ϵ_h are chosen under a suitable criteria. The loglikelihood ratio based on the quantiles becomes

$$\Lambda(\tilde{X})=\ln\left\{ \frac{\prod_{q=0}^{n}\prod_{k=1}^{m}\prod_{h=1}^{m}\left[H(a_k^c-s_q^c, a_h^s-s_q^s)\right]^{n_{khq}}}{\psi} \right.$$

2

$$\prod_{q=0}^{n} \prod_{k=1}^{m} \prod_{h=1}^{m} \left[H(a_k^c, a_h^s) \right]^{n_{khq}} \Bigg| \quad (3)$$

where $H(a_k^c, a_h^s) = G(a_k^c, a_h^s) - G(a_{k-1}^c, a_h^s) - G(a_k^c, a_{h-1}^s) + G(a_{k-1}^c, a_{h-1}^s)$, $\langle\cdot\rangle_\psi$ represents averaging over the signal phase, n_{khq} indicates which interval the DFT output falls into, and the dependence on p has been left off.

<u>Weak Signal Approximation</u>

For weak signals $H(a_k^c - s_q^c, a_h^{\hat{}} - s_q^s)$ can be expanded in a Taylor series and if the in-phase and quadrature components are independent equation 3 reduces to

$$\Lambda(\tilde{X}) = \sum_{q=0}^{n} \left[1/2 \langle (s_q^c)^2 \rangle_\psi \sum_{k=1}^{m} b_k' \, n_{kq} + 1/2 \langle (s_q^s)^2 \rangle_\psi * \sum_{h=1}^{m} b_h' \, n_{hq} - B_1 \right] + 1/2 \sum_{q \neq r}^{N_0} \sum^{N_0} \left[\langle s_q^c s_r^c \rangle_\psi * \sum_k^m \sum_h^m b_k b_h n_{kq} n_{hq} + \langle s_q^s s_r^s \rangle_\psi \sum_k^m \sum_h^m b_k b_h n_{kq} n_{hq} - B_2 \right] \quad (4)$$

where $\langle s_q^c \rangle = \langle s_q^s \rangle = 0, n_{khq} = n_{kq} n_{hq}$,

b_k', b_k, b_h', b_h are called scores, N_0 represents the coherence time of the signal, $\langle s_q s_r \rangle = 0$ if $|q-r| > N_0$, and B_1 and B_2 are constants chosen to insure that the desired typeI and type II errors (α, β) are maintained. If the quantiles and scores are not known they may be estimated by an efficient recursive procedure (7), (8), and(9). The estimation of parameters in many applications are less demanding than indicated above since the statistics over the frequency components usually change only slightly and the number of quantiles and scores used can be kept small with high performance efficiency. The scores are given by

$$b_v' = \frac{g'(a_v) - g'(a_{v-1})}{G(a_v) - G(a_{v-1})} \; , \; b_v = \frac{g(a_v) - g(a_{v-1})}{G(a_v) - G(a_{v-1})} \qquad v = k, h$$

$g(.), g'(.)$ are the pdf and its derivative respectively associated with the cdf $G(.)$ and evaluated at the quantile locations.

PERFORMANCE OF RSD

The performance measures, probability of detection and average sample size, $E(n/\theta_1, \theta)$ (ie, the average number of DFT's needed to reach a decision) can be obtained from Wald's fundamental identity (10). However as shown in (6) the essential weak signal performance measure, called relative efficiency, is based on the ratio of two sequential detectors average sample size satisfying regularity conditions (6) for the same α and β,

$$RE = \frac{E_1(n/\theta_1 \theta)}{E_2(n/\theta_1 \theta)}$$

where θ_1, θ are the designed and actual signal-to-noise ratio respectively. As $(\theta_1 \to 0, \theta \to 0)$ where θ/θ_1 is finite then $n \to \infty$ and the sequential detector satisfies Wald's conditions (10) exactly. In (6) it was shown that under these conditions RE approaches the asymptotic relative efficiency (ARE).

<u>Example : Incoherent Detection</u>

In many practical cases the signal is incoherent, $N_0 = 1$ in equation 4, and the optimum small signal detector under Gaussian noise is a quadratic detector (11). Let the noise at the output of the DFT be composed of a additive mixture of Gaussian N(0,1) and large variance Gaussian $N(0, \sigma^2)$ noise. It can be shown by the characteristic function method that if the noise at the input to the DFT is an additive mixture the output will also be an additive mixture of noise with the variance modified by a term which can be set to one for convenience without effecting the relative results. The RSD will be compared with a sequential quadratic detector operating at the output of a DFT under the same conditions. The ARE can be shown to be given by

$$ARE = var(|X_q|^2)/4 \sum_{v=1}^{m} \frac{\left[(1-\lambda) g_1' + \lambda \, g_2' \right]^2}{(1-\lambda) G_1 + \lambda \, G_2} \quad (5)$$

where $g_z' = g_z'(a_v) - g_z'(a_{v-1})$, $G_z = G_z(a_v) - G_z(a_{v-1})$, $z = 1,2$

$G_1(.) = N(0,1)$ and $G_2(.) = N(0, \sigma^2)$. For any m the quantiles (a_v) are chosen to maximize equation 5. A consistent set of equations to maximize equation 5 can be obtained from the relationship

$\frac{\partial}{\partial a_v}(ARE) = 0$, $v = 1, \ldots, m$. However in order to solve for the optimum quantiles a computer solution is needed. Tables 1 and 2 give the results for the optimum quantiles and scores for m= 2 and 3, $\sigma^2 = 9$, and $0 \le \lambda \le .5$. For m=2, which corresponds to estimating only one quantile, the ARE is .3 when $\lambda=0$, however the ARE can be greater than one when $\lambda>0$. For m=3, or estimating 2 quantiles the ARE increases subtantially when $\lambda>0$. In practice the quantiles must be estimated from the noise and the quantiles estimated are usually suboptimum. Tables 3,4, and 5 give the suboptimum quantiles and scores, and the results for the ARE when the quantiles are estimated from the equation (12)

$$Q(v,m) = (1-\lambda)G_1 + \lambda G_2 = (v-1/2)/(m-1) \;,\; m > 2$$

for m=2 Q(v,m) was set to 1/4. The results show that the quantiles can be estimated under practical conditions and used to improve performance in impulsive noise and in purely Gaussian noise for $m \geq 11$, ARE >.9, and as m increases further the $\overline{ARE} \to 1$, which is an important property of robust sequential detectors. There should be more powerful techniques for estimating quantiles in impulsive noise then the one used above. Also, it may not be possible to estimate the quantiles under the same impulsive noise conditions existing during the decision interval. Then a reduced probability space as discribed in (6) will help to maintain the desired performance levels.

CONCLUSION

A robust sequential detector was derived at the output of a DFT which was based on estimating quantiles and related functions and forming a loglikelihood ratio. This structure has the advantage of not requiring knowledge of the functional form of the distribution and is able to adapt to changing noise fields. It was shown that improved performance in terms of ARE were possible over conventional detectors operating under impulsive noise.

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TABLE 1

m= 2 OPTIMUM

λ	ARE	Scores	Quantiles
0	.3	$b_1' = 3.2$	$a_1 = -1.57$
		$b_2' = -.19$	
.1	1.05	$b_1' = 1.76$	$a_1 = -1.36$
		$b_2' = -.22$	
.2	1.42	$b_1' = 1.23$	$a_1 = -1.26$
		$b_2' = -.22$	
.3	1.55	$b_1' = .92$	$a_1 = -1.22$
		$b_2' = -.2$	
.4	1.53	$b_1' = .707$	$a_1 = -1.2$
		$b_2' = -.18$	
.5	1.36	$b_1' = .53$	$a_1 = -1.13$
		$b_2' = -.17$	

TABLE 2

m=3 OPTIMUM

λ	ARE	Scores	Quantials
0	.65	$b_1' = 2.58, b_2' = -.46$	$a_1 = -1.48$
		$b_3 = 2.58$	$a_2 = 1.48$
.1	2.42	$b_1' = 1.6, b_2' = -.56$	$a_1 = -1.25$
		$= b_3$	$a_2 = -a_1$
.2	3.5	$b_1' = 1.12, b_2' = -.59$	$a_1 = -1.13$
		$= b_3$	$a_2 = -a_1$
.3	4.1	$b_1' = .84, b_2' = -.58$	$a_1 = -1.1$
		$= b_3$	$a_2 = -a_1$

4

λ	ARE	Scores	Quantiles
.4	4.27	$b_1'=.64, b_2'=-.57$	$a_1=-1.04$
		$b_3'=.64$	$a_2=1.04$
.5	5.68	$b_1'=.66, b_2'=-.53$	$a_1=-1.02$
		$b_3'=.66$	$a_2=1.02$

TABLE 3

m=2 SUBOPTIMUM

λ	ARE	Scores	Quantiles
0	.18	$b_1'=.89, b_2'=-.29$	$a_1=-.69$
.1	.61	$b_1'=.84, b_2'=-.27$	$a_1=-.75$
.2	1.01	$b_1'=.76, b_2'=-.25$	$a_1=-.81$
.3	1.31	$b_1'=.7, b_2'=-.23$	$a_1=-.89$
.4	1.44	$b_1'=.62, b_2'=-.2$	$a_1=-.99$
.5	1.35	$b_1'=.52, b_2'=-.17$	$a_1=-1.11$

TABLE 4

m=3 SUBOPTIMUM

λ	ARE	Scores	Quantiles
0	.34	$b_1'=.89, b_2'=-.89$	$a_1=-.69$
		$b_3'=.89$	$a_2=.69$
.1	1.8	$b_1'=.84, b_2'=-.84$	$a_1=-.75$
		$b_3'=.84$,	$a_2=.75$
.2	3.03	$b_1'=.77, b_2'=-.77$	$a_1=-.81$
		$b_3'=.77$	$a_2=.81$
.3	3.91	$b_1'=.7, b_2'=-.7$	$a_1=-.89$
		$b_3'=.7$	$a_2=.89$
.4	4.25	$b_1'=.62, b_2'=-.62$	$a_1=-.99$
		$b_3'=.62$	$a_2=.99$
.5	4.0	$b_1'=.52, b_2'=-.52$	$a_1=-1.11$
		$b_3'=.52$	$a_2=1.11$

TABLE 5

m=5 SUBOPTIMUM

λ	ARE	Scores	Quantiles
0	.63	$b_1'=2, b_2'=-.5$	$a_1=-1.17, a_2=-.33$
		$b_3'=-1, b_4'=-.5$	$a_3=.33, a_4=1.17$
		$b_5'=2$	
.1	2.58	$b_1'=1.68, b_2'=-.32$	$a_1=-1.3, a_2=-.36$
		$b_3'=-1, b_4'=-.32$	$a_3=.36, a_4=1.3$
		$b_5'=1.68$	
.2	3.63	$b_1'=1.36, b_2'=-.2$	$a_1=-1.46, a_2=-.38$
		$b_3'=-.96, b_4'=-.2$	$a_3=.38, a_4=1.46$
		$b_5'=1.36$	
.3	3.57	$b_1'=1.01, b_2'=-.1$	$a_1=-1.67, a_2=-.42$
		$b_3'=-.83, b_4'=-.1$	$a_3=.42, a_4=1.67$
		$b_5'=1.01$	
.4	3.16	$b_1'=.67, b_2'=.06$	$a_1=-1.93, a_2=-.46$
		$b_3'=-.78, b_4'=.06$	$a_3=.46, a_4=1.93$
		$b_5'=.67$	
.5	2.87	$b_1'=.41, b_2'=.17$	$a_1=-2.2, a_2=-.5$
		$b_3'=-.71, b_4'=.17$	$a_3=.5, a_4=2.24$
		$b_5'=.41$	

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