AD-A069 997 VIRGINIA UNIV CHARLOTTESVILLE DEPT OF MECHANICAL AND--ETC F/G 12/1
ON THE SOLUTION OF SHELLS OF REVOLUTION WITH CUTOUTS.(U)
APR 79 W D PILKEY, I B PARK N00014-75-C-0374

UNCLASSIFIED UVA/525303/MAE79/104 NL

| OF |
AD
A069997

END
DATE
FILMED
7-79
DDC

LEVEL II

AD A069997

ON THE SOLUTION OF SHELLS OF REVOLUTION WITH CUTOUTS

Technical Report

Contract No. N00014-75-C-0374

Office of Naval Research
800 N. Quincy Street
Arlington, Virginia 22217

Attn: Dr. N. Perrone

Submitted by:

W. D. Pilkey

and

I. B. Park

DDC
RECEIVED
JUN 15 1979
C

Department of Mechanical and Aerospace Engineering
RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES
SCHOOL OF ENGINEERING AND APPLIED SCIENCE
UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA

410 696

DDC FILE COPY

This document has been approved for public release and sale; its distribution is unlimited.

Report No. UVA/525303/MAE79/104

April 1979

79 06 13 034

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
14 UVA/525303/MAE79/104

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
6 On the Solution of Shells of Revolution with Cutouts

5. TYPE OF REPORT & PERIOD COVERED
9 Technical Report

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
W. Pilkey and I. B. Park

8. CONTRACT OR GRANT NUMBER(s)
15 N00014-75-C-0374

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Department of Mechanical and Aerospace Engineering
University of Virginia
Charlottesville, Virginia 22901 410 696

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
NR 064-542/4-20-78 (474)

11. CONTROLLING OFFICE NAME AND ADDRESS
Structural Mechanics Program
Office of Naval Research
Arlington, Virginia 22217

12. REPORT DATE
11 April 1979

13. NUMBER OF PAGES

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
10 W. D. / Pilkey I. B. / Park

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
12 11 p.

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Shells
Thin Shells
Shell with Cutouts
Shell of Revolution
Shell with Attachments

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A procedure is outlined for analyzing shells of revolution with nonaxisymmetric geometrical or physical occurrences, e.g. cutouts. Such a cutout means that the shell is no longer a shell of revolution. Advantage is taken of the efficient tools available for analyzing shells of revolution to find the response of a shell which can be modeled as a shell of revolution with such nonaxisymmetric occurrences as cutouts or attachments. In particular, the proposed scheme is a postprocessor for the numerical integration method of shell of revolution analysis. 410 696

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Accession For
NTIS GRA&I
DDC TAB
Unannounced
Justification
By
Distribution/
Availability Codes
Dist Avail and/or special

INTRODUCTION

A procedure for analyzing shells of revolution with nonaxisymmetric geometrical or physical occurrences, e.g. cutouts, is outlined. This means, of course, that the shell is not actually a shell of revolution. The goal is to take advantage of the efficient tools available for analyzing shells of revolution to find the response of a shell which can be modeled as a shell of revolution with such nonaxisymmetric occurrences as cutouts or attachments.

In some stress analysis problems, certain variables can be expanded as Fourier series permitting the unknown Fourier coefficients to be determined such that the governing differential equations and boundary conditions are satisfied. Thus, in a three-dimensional problem the introduction of a Fourier series into the governing differential equations can eliminate one coordinate from the problem. Thereby, a three-dimensional problem can be reduced to a two-dimensional problem. It is in this manner that the circumferential coordinate is frequently eliminated from the shell of revolution governing differential equations. The resulting set of simultaneous ordinary differential equations, rather than the original partial differential equations, are often solved using finite differences (e.g. (1)) or the numerical integration (transfer matrix) method (e.g. (2,3,4) and numerous subsequent papers by the same authors). However, if there is a nonsymmetric cutout or attachment this method can not be applied directly. In this technical note, a combination of the numerical integration method and the finite difference method could be used rather than the finite element method to couple to the numerical integration technique.

For the scheme developed here, the numerical integration method is proposed for region I and III of Fig. 1 and the finite element method can be used for region II. To employ the combination of the two methods, we need to transform the field (transfer) matrix developed by numerical integration into a stiffness matrix or the stiffness matrix of the finite element method into a transfer matrix. If the nodal variables in the transfer matrix and the stiffness matrix are of the same type as each other, this can be accomplished using a simple rearrangement of variables (5). However, since for the numerical integration the variables are expanded as a Fourier series, this method produces a state vector containing Fourier coefficients rather than the physical variables (e.g., displacements themselves). Therefore it is necessary to make the variables compatible with the nodal physical variables of the finite element method. This can be accomplished with the following formulations.

FORMULATION

Let $\underline{z}$ be a state vector which includes generalized displacements and generalized forces at a station. Also, let x be the longitudinal coordinate and θ be the circumferential coordinate of a shell of revolution. Then

$$\underline{z} = z(x,\theta) \tag{1}$$

If state vector $\underline{z}(x,\theta)$ is expanded as a Fourier series (6)

$$\underline{z}(x,\theta) = \sum_{m=0}^{\infty} [z_m^c(x)\cos m\theta + z_m^s(x)\sin m\theta] \tag{2}$$

in which $\underline{z}_m^c(X)$ is a vector of coefficients of a Fourier cosine series and $\underline{z}_m^s(x)$ is a vector of coefficients of a Fourier sine series. For the purpose of demonstrating the technique, consider only m=0, 1 and θ=0, θ_1, θ_2. The formulation is readily extended to more general cases, i.e. more terms in the series and more circumferential locations.

Derivation of the stiffness matrix from the transfer matrix. - Let $x_0 = 0$ and x_1 correspond to the longitudinal coordinates of the edge of region I as shown in Fig. 1. If $x = x_0$ and $x = x_1$ are introduced into Eq. 2,

$$\underline{z}(x_0,\theta) = \sum_{m=0}^{\infty} [\underline{z}_m^c(x_0) \cos m\theta + \underline{z}_m^s(x_0) \sin m\theta] \tag{3a}$$

$$\underline{z}(x_1,\theta) = \sum_{m=0}^{\infty} [\underline{z}_m^c(x_1) \cos m\theta + \underline{z}_m^s(x_1) \sin m\theta] \tag{3b}$$

in which $\underline{z}(x_0,\theta)$ is the state vector $\underline{z}$ at $x=x_0$ and $\underline{z}_m^c(x_0)$ is the vector of coefficients of the Fourier cosine series at $x=x_o$, etc. Numerical integration produces a transfer matrix so that

$$\left.\begin{aligned} \underline{z}_m^c(x_1) &= U_m^c\, \underline{z}_m^c(x_o) \\ \underline{z}_m^s(x_1) &= U_m^s\, \underline{z}_m^s(x_o) \end{aligned}\right\} \tag{4}$$

in which U_m^c and U_m^s are transfer matrices for coefficients of the Fourier cosine and sine series, respectively. Then, $\underline{z}(x_1,\theta)$ can be expressed as

$$\underline{z}(x_1,\theta) = \sum_{m=0}^{\infty} [U_m^c\, \underline{z}_m^c(x_o) \cos m\theta + U_m^s\, \underline{z}_m^s(x_o) \sin m\theta] \tag{5}$$

To derive the stiffness matrix, a direct relationship between $\underline{z}(x_0,\theta)$ and $\underline{z}(x_1,\theta)$ is needed. If $m=0$, 1 and $\theta=0$, θ_1, θ_2 are introduced into Eqs. (3a) and (5).

$$\begin{Bmatrix} \underline{z}(x_o,0) \\ \underline{z}(x_o,\theta_1) \\ \underline{z}(x_o,\theta_2) \end{Bmatrix} = \begin{bmatrix} I & I & 0 \\ I & I\cos\theta_1 & I\sin\theta_1 \\ I & I\cos\theta_2 & I\sin\theta_2 \end{bmatrix} \begin{Bmatrix} \underline{z}_0^c(x_o) \\ \underline{z}_1^c(x_o) \\ \underline{z}_1^s(x_o) \end{Bmatrix} = [T_1] \begin{Bmatrix} \underline{z}_0^c(x_o) \\ \underline{z}_1^c(x_o) \\ \underline{z}_1^s(x_o) \end{Bmatrix} \tag{6a}$$

$$\left\{\begin{matrix} \underline{z}(x_1,0) \\ \underline{z}(x_1,\theta_1) \\ \underline{z}(x_1,\theta_2) \end{matrix}\right\} = \begin{bmatrix} U_0^c & U_1^c & 0 \\ U_0^c & U_1^c\cos\theta_1 & U_1^s\sin\theta_1 \\ U_0^c & U_1^c\cos\theta_2 & U_1^s\sin\theta_2 \end{bmatrix} \left\{\begin{matrix} \underline{z}_0^c(x_0) \\ \underline{z}_1^c(x_0) \\ \underline{z}_1^s(x_0) \end{matrix}\right\} \equiv [T_2] \left\{\begin{matrix} \underline{z}_0^c(x_0) \\ \underline{z}_1^c(x_0) \\ \underline{z}_1^s(x_0) \end{matrix}\right\} \qquad (6b)$$

in which I = [I] is an identity matrix, 0 = [0] is a null matrix, and I $\cos\theta$ = [I] $\cos\theta_1$, etc. From Eqs. 6a and 6b,

$$\left\{\begin{matrix} \underline{z}(x_0,0) \\ \underline{z}(x_0,\theta_1) \\ \underline{z}(x_0,\theta_2) \end{matrix}\right\} = [T_1]\,[T_2]^{-1} \left\{\begin{matrix} \underline{z}(x_1,0) \\ \underline{z}(x_1,\theta_1) \\ \underline{z}(x_1,\theta_2) \end{matrix}\right\} \qquad (7)$$

Rearrangement of the state variables to express the generalized forces in terms of the generalized displacements gives the desired global stiffness matrix. If m = 0,1,2, ---, n and θ = 0, θ_1, θ_2, ---, θ_{2n}, $[T_1]$ and $[T_2]$ are square matrices. However, if other combinations of m and θ are used, $[T_1]$ and $[T_2]$ will be rectangular matrices. In such cases, the pseudoinverse can be used to obtain the final equation such as Eq. 7.

The stiffness matrix for region III in Fig. 1 can be obtained in a fashion similar to that employed above for region I. Then, the stiffness matrices for region I and III are combined with the stiffness matrix for region II which is obtained from the finite element formulation. Finally the resulting equation is solved for the unknown displacement variables.

<u>Derivation of the transfer matrix from the stiffness matrix.</u> - It was shown in Ref. 7 that a transfer matrix in terms of boundary state vectors at boundaries x = x_1 and x = x_2 in Fig. 1 could be derived from given a stiffness matrix by condensing out the intermediate state vectors between the boundaries. That is, the following relation can be formed.

$$\begin{Bmatrix} \underline{z}(x_2,0) \\ \underline{z}(x_2,\theta_1) \\ \underline{z}(x_2,\theta_2) \\ 1 \end{Bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{Bmatrix} \underline{z}(x_1,0) \\ \underline{z}(x_1,\theta_1) \\ \underline{z}(x_1,\theta_2) \\ 1 \end{Bmatrix} \tag{8}$$

in which
$$[A] = \left[\begin{array}{c|c} - - - - - - - - - - & - \\ - - - - - - - - - - & - \\ - - - - - - - - - - & - \\ 0 & 1 \end{array}\right]$$

The next task is to express the state vectors in terms of the Fourier coefficients. For this purpose, use the following relationships.

$$\begin{Bmatrix} \underline{z}(x_1,0) \\ \underline{z}(x_1,\theta) \\ \underline{z}(x_1,\theta_2) \\ 1 \end{Bmatrix} = \begin{bmatrix} I & I & I & 0 \\ I & I\cos\theta_1 & I\sin\theta_1 & 0 \\ I & I\cos\theta_2 & I\sin\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \underline{z}_0^c(x_1) \\ \underline{z}_1^c(x_1) \\ \underline{z}_1^s(x_1) \\ 1 \end{Bmatrix} \equiv [B] \begin{Bmatrix} \underline{z}_0^c(x_1) \\ \underline{z}_1^c(x_1) \\ \underline{z}_1^s(x_1) \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} \underline{z}(x_2,0) \\ \underline{z}(x_2,\theta_1) \\ \underline{z}(x_2,\theta_2) \\ 1 \end{Bmatrix} = \begin{bmatrix} I & I & I & 0 \\ I & I\cos\theta_1 & I\sin\theta_1 & 0 \\ I & I\cos\theta_2 & I\sin\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \underline{z}_0^c(x_2) \\ \underline{z}_1^c(x_2) \\ \underline{z}_1^s(x_2) \\ 1 \end{Bmatrix} \equiv [B] \begin{Bmatrix} \underline{z}_0^c(x_2) \\ \underline{z}_1^c(x_2) \\ \underline{z}_1^s(x_2) \\ 1 \end{Bmatrix} \tag{9}$$

Introducing Eq. 9 into Eq. 8,

$$[B] \begin{Bmatrix} \underline{z}_0^c(x_2) \\ \underline{z}_1^c(x_2) \\ \underline{z}_1^s(x_2) \\ 1 \end{Bmatrix} = [A]\,[B] \begin{Bmatrix} \underline{z}_0^c(x_1) \\ \underline{z}_1^c(x_1) \\ \underline{z}_1^s(x_1) \\ 1 \end{Bmatrix} \tag{10}$$

Then, the desired transfer matrix for the Fourier coefficients is

$$\begin{Bmatrix} \underline{z}_0^c(x_2) \\ \underline{z}_1^c(x_2) \\ \underline{z}_1^s(x_2) \\ 1 \end{Bmatrix} = [B]^{-1}\,[A]\,[B] \begin{Bmatrix} \underline{z}_0^c(x_1) \\ \underline{z}_1^c(x_1) \\ \underline{z}_1^s(x_1) \\ 1 \end{Bmatrix} \equiv [U_{II}] \begin{Bmatrix} \underline{z}_0^c(x_1) \\ \underline{z}_1^c(x_1) \\ \underline{z}_1^s(x_1) \\ 1 \end{Bmatrix} \qquad (11)$$

in which $[U_{II}]$ is the transfer matrix for region II.

Combine $[U_{II}]$ with $[U_I]$ and $[U_{III}]$ which are the transfer matrices for regions I and III respectively.

$$\begin{Bmatrix} \underline{z}_0^c(x_3) \\ \underline{z}_1^c(x_3) \\ \underline{z}_1^s(x_3) \\ 1 \end{Bmatrix} = [U_{III}]\,[U_{II}]\,[U_I] \begin{Bmatrix} \underline{z}_0^c(x_o) \\ \underline{z}_0^c(x_o) \\ \underline{z}_1^s(x_o) \\ 1 \end{Bmatrix} \equiv [U] \begin{Bmatrix} \underline{z}_0^c(x_o) \\ \underline{z}_0^c(x_o) \\ \underline{z}_1^s(x_o) \\ 1 \end{Bmatrix} \qquad (12)$$

Upon introduction of the boundary conditions, Eq. (12) can be solved for the remaining unknown variables.

ACKNOWLEDGEMENTS

This work was supported by the Office of Naval Research.

REFERENCES

1. Bushnell, D., "Stress, Buckling, and Vibration of Prismatic Shells", American Institute of Aeronautics and Astronautics Journal, Vol. 9, No. 10, Oct. 1971, pp. 2004-2013.

2. Goldberg, J.E., Bogdanoff, J., and Marcus, L.," On the Calculation of the Axisymmetric Modes and Frequencies of Conical Shells", Journal of the Acoustical Society of America, Vol. 32, No. 6, June, 1960, pp. 738-742.

3. Kalnins, A., "Free Vibration of Rotationally Symmetric Shells", Journal of Applied Mechanics, ASME, Paper No 64-APM-33.

4. Cohen, G.A., "Computer Analysis of Asymmetric Buckling of Ring Stiffened Orthotropic Shells of Revolution", American Institute of Aeronautics and Astronautics Journal, Vol. 6, No. 1, Jan. 1968 pp. 141-149.

5. Pestel, E.C. and Leckie, F.A., Matrix Methods in Elastomechanics, McGraw-Hill, 1963.

6. Pilkey, W.D. and Chang, P.Y., Modern Formulas for Statics and Dynamics, McGraw-Hill, 1978.

7. Pilkey, W.D., Haviland, J.K., and Chang, P.Y., "The Analysis of Line Structures by Transfer Matrices Derived from Finite Elements", Journal of Ship Research, Vol. 19, No. 1, March 1975, pp. 57-61.

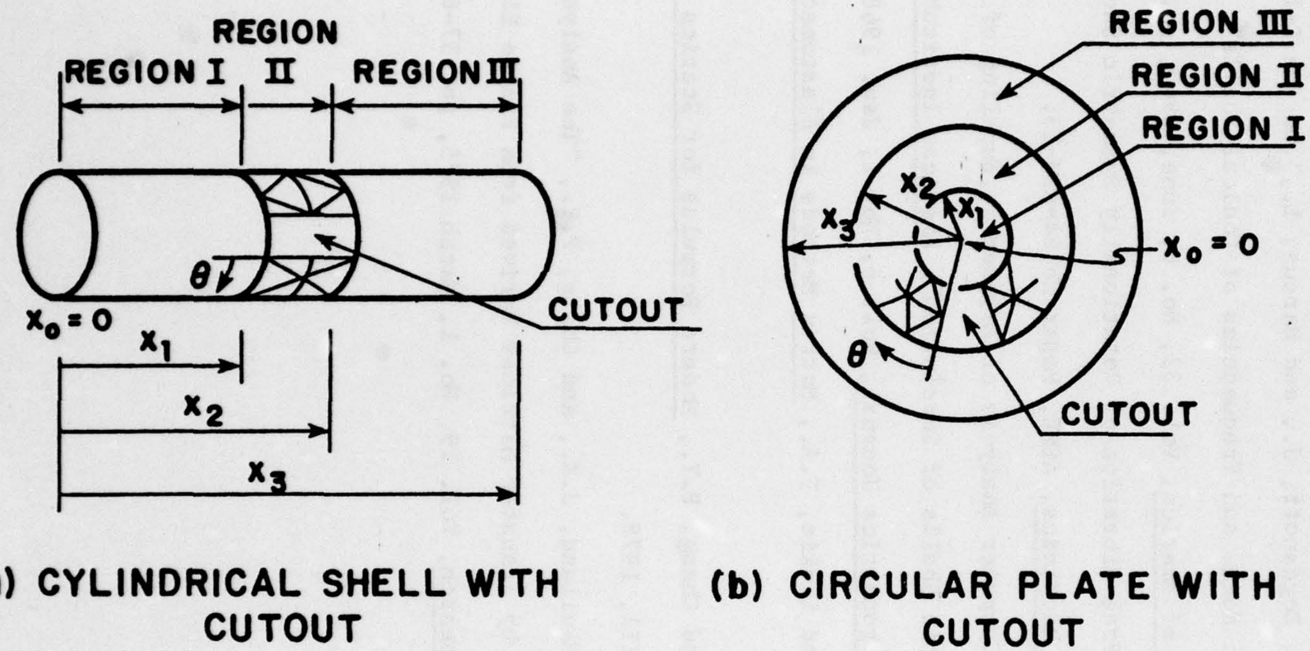

Fig. 1. - Solid of Revolution with Cutout

DISTRIBUTION LIST

No. of Copies	
1 - 6	Office of Naval Research 800 N. Quincy Street Arlington, Virginia 22217
1	Dr. N. Perrone Office of Naval Research 800 N. Quincy Street Arlington, Virginia 22217
1	ONRRR John Hopkins University Room 358, Garland Hall 34th and Charles Street Baltimore, Maryland 21218 Attn: C. Richard Main
1	K. A. Fischer
1	W. D. Pilkey
1	I. B. Park
1	L. S. Fletcher
2	E. H. Pancake Science/Technology Information Center Clark Hall
1	RLES Files
1	Each entry in ONR Structural Mechanics Contract Research Program Distribution List