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SOLUTION OF COUPLED RATE EQUATIONS AND PREDICTION OF TIME HISTO--ETC(U)
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SOLUTION OF COUPLED RATE EQUATIONS AND
PREDICTION OF TIME HISTORIES OF EMISSION LINES
IN A RAPIDLY IONIZING PLASMA

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ABSTRACT

The time histories of emission lines from successive ionization stages of impurity atoms in a plasma are predicted by solving the coupled rate equations governing the populations of these ionization stages with the help of a computer program. A detailed description of the program is given. An application of such predictions for the measurement of ionization rate coefficients of ions occurring in a rapidly ionizing plasma is discussed.

I. INTRODUCTION

Knowledge of the lifetime of various ionic stages of impurity atoms in high temperature hydrogen or deuterium plasmas is very important in controlled nuclear fusion research for the estimation of energy losses as well as in understanding, e.g., the physics of the solar corona (flares, etc.). The basic processes governing the ionic lifetimes are collisional ionization and the various recombination processes--radiative, dielectronic, three-body, whose relative importance is determined by the plasma conditions. In order to understand the relative importance of these processes, the plasma conditions could be simulated and the time histories of emission lines from various ionization stages be predicted by a computer program. The purpose of this report is to describe in detail such a computer code.¹ It was used for example, to determine the ionization rate coefficients of various ions using a θ -pinch plasma^{2,3} with added impurities. The principles of such measurements are as described by Hinnov.⁴

In Section II the basic equations and the adopted procedure for their solution are described. In Section III the set up of the program is described. In Section IV the description and listing of the program is given. In Section V sample test data are set up such that analytical solutions can be obtained. A comparison with the computer solution checks the accuracy. In Appendix I plotting subroutines are listed. In Appendix II an application of this program for the determination of ionization rates is discussed using the data from an experiment⁵ as an example.

II. BASIC EQUATIONS

The impurity ions in a transient plasma go successively through the various ionization stages, the degree of ionization lagging behind the corresponding quasistationary corona equilibrium.

The density N_k of ions (K is the nuclear charge) in the total plasma Volume V is determined by the following rate equation:

$$\frac{d(N_k V)}{dt} = V N_{k-1} N I_{k-1} - V N_k N I_k + V N_{k+1} N \alpha_{k+1} - V N_k N \alpha_k \quad (1)$$

where N is the electron density, I_k is the ionization rate coefficient and α_k the recombination rate coefficient. The plasma volume would change with time in a Θ -pinch plasma due to compression or expansion. This variation was accounted for in our experiments by the assumption that particle losses are negligible, which is found to be valid in the operation of our Θ -pinch with a reverse bias field.

$$\frac{d}{dt} (NV) = 0.$$

$$\frac{dV}{dt} = - \frac{V}{N} \frac{dN}{dt} \quad (2)$$

This also assumes small concentrations of impurity ions such that electron production by ionization is negligible. Hence using Eq. 2 in 1

$$\frac{dN_k}{dt} = N_{k-1} N I_{k-1} - N_k N I_k + N_{k+1} N \alpha_{k+1} - N_k N \alpha_k + \frac{1}{N} \frac{dN}{dt} N_k \quad (3)$$

However, for different plasma machines and plasma conditions Eq. (2) could be different which would make the last term in Eq. (3)

to be different. Hence the last term in Eq. (3) is called the source term which should be appropriately modeled.

These coupled rate equations are solved numerically with the measured electron density and temperature given as input.

The following formulae were used for the rate coefficients in Eq. (3):

Ionization: A choice can be made to use one of the following approximations.

1. Semiempirical formula due to Kunze² derived by fitting to the semiempirical predictions of Lotz.⁶ It is referred to here as "S(k)-Lotz & Kunze"

$$S(K) = SA(K) \left[\left(\ln \frac{40(kT)}{E_i} \right)^3 + 40 \right] \frac{kT^{1/2}}{E_i + 3kT} e^{-E_i/kT} \text{ cm}^3/\text{sec} \quad (4)$$

$$SA(K) = 7.5 \times 10^{-8} \frac{\eta_i}{E_i} \quad (5)$$

Here " η_i " is the number of electrons in the i^{th} subshell (e.g. for the $1s^2 2s^2 2p$ configuration η_i is 1 for 2p, 2 for $2s^2$ and 2 for $1s^2$ subshells, and the ionization rate of the ground state is a sum of all these contributions) and E_i is the corresponding ionization energy. Both kT and E_i are in e.v. The constant $SA(K)$ is given as input.

A comparison to another semiempirical formula due to Hinno⁴ was found to yield the same results as Eq. (4).

2. Summers⁷ described the procedure for calculating the ionization rate coefficients using the Exchange Collisional

and Impact Parameter (E.C.I.P.) method due to Burgess.⁸

This procedure is adopted into our program and the ionization rate is referred to as S(K)- Burgess and Summers.

$$S(K) = A(K) X \text{ (Burgess and Summers rate.) } \text{ cm}^3/\text{sec} \quad (6)$$

A(K) is a multiplicative value to change the rate if one wishes. It is given as input. Other input data needed to evaluate Eq. (6) is described in the Comments of Subroutine Main - R/W in Sec. IV.

Recombination: Three processes of recombination are considered. Generally, for a rapidly ionizing plasma recombination rates are negligible. However, one has to check this especially for higher ionization stages. The total recombination rate is referred to as Alpha(K).

1. Radiative recombination: RALPHA

For the evaluation of RALPHA a formula due to Seaton⁹ (Eq. 4.3.35 Ref. 10) was adopted.

$$\text{RALPHA}(R\alpha_k) = 5.20 \cdot 10^{-14} Z(E/kT)^{1/2} (0.429 + 0.5 \ln \frac{E}{kT} + 0.469 (kT/E)^{1/3} \text{ (cm}^3 \text{ sec}^{-1}) \quad (7)$$

where 'Z' is the effective charge of the recombining ion K (giving K-1) and E is the ionization potential of the ion upon recombination. (i.e. K-1)

2. Dielectronic recombination: DALPHA

For the evaluation of DALPHA simple formulae due to Landini and Monsignori Fossi¹¹ are used.

For ions pertaining to H, He, Ne, K to Ni DALPHA (D_{α_k}) =

$$1.60 \times 10^{-10} (Z+1)^2 F(K) (kT)^{-3/2} W(K)^{-1/2} \exp \left[\frac{-0.9134W(K)}{kT} \right] \quad (8)$$

where Z is the charge of the recombining ion, W(K) is the first excitation energy in e.v for the recombining ion and F(K) could be taken as the number of electrons in the outer shell (F(k)=1 for H-like, 2 for He-like, 6 for Ne-like ...)

For ions pertaining to Li to F, Na to Ar, Cu to Kr DALPHA(D_{α_k}) =

$$1.6 \times 10^{-10} (Z+1)^2 F(K) (kT)^{-3/2} \exp \left[\frac{-0.9134 (Z+2.3) W(K)}{3.3 kT} \right] \quad (8)$$

where Z+1 and W(K) are defined as in Eq. 8. F (K) values are taken from Ref. 7.

3. Three body recombination: BALPHA

For the evaluation of three body recombination a formula due to Griem¹² was used.

$$\text{BALPHA } (B_{\alpha_k}) = 1.4 \times 10^{-31} N Z^{-6} \left(\frac{E_1}{kT} \right)^2 \exp \left[\frac{E_1}{kT(n^1+1)^2} \right] \quad (9)$$

where N is the electron density, Z the charge of the ion before recombination and n^1 the quantum number of the Collision limit¹² i.e. the level from which radiative decay is about as probable as collisional excitation to higher levels.

$$n^1 \approx 1.26 \times 10^2 Z^{14/17} N^{-2/17} (E_1/kT)^{-1/17} \exp \left[4 E_1 / 17 (n^1)^3 kT \right] \quad (10)$$

A plot of n^1 is given in Ref. 9.

Having determined the time history of ionic populations, the time histories of their emission lines could be predicted. The emission coefficient of an optically thin allowed line, whose upper level is mostly populated by electron collisions from the ground state is given by

$$\epsilon = \frac{h\nu}{4\pi} N X N_k \text{ ergs} / \text{cm}^3 \text{ sr sec} \quad (11)$$

We assume excited state populations to be small so that N_k represents the ground-state population. Also X is the excitation rate coefficient. If N and X are constant, the time history of the line is identical to the time history of the ion. However, variations in N and X can be accounted for even though the absolute value of X is not known. The time dependence of X is accounted for by the effective Gaunt-factor approximation.

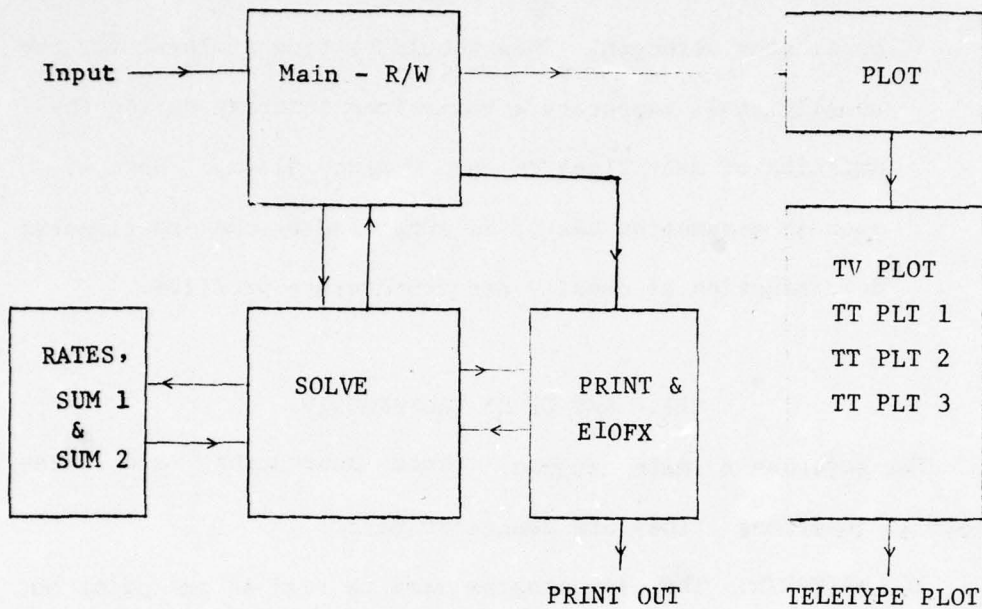
$$X = 1.6 \times 10^{-5} f e^{-\Delta E/kT} (\Delta E)^{-1} (kT)^{-1/2} \quad (12)$$

where ΔE is the excitation potential in ev and 'f' is the oscillator strength. This should be true at least for the usually small temperature variations occurring during the emission of many lines in our Θ -pinch plasma. However, such an assumption has to be supported by the experimental determination of density and temperature profiles.

III. SET UP OF THE PROGRAM

The code has a main program, four subroutines and three external functions. They are described below.

1. MAIN-R/W: The main program used to read in and print out the data, and control the rest of the program.
2. SOLVE: A subroutine that solves the differential equations.
3. RATES: A subroutine that interpolates density and temperature, calculates the rate coefficients and increments in populations at each fine time step 'J' required by SOLVE. External functions SUM1 and SUM2 are for the evaluation of $S(K)$ - Burgess and Summers.
4. PRINT: Print is used to output the calculations of ionic populations calculated in "SOLVE". Print together with external function EIOFX calculate the intensities of spectral lines.
5. PLOT: The intensity as a function of time is stored for teletype plotting.



OUTPUT

The program TV PLOT and the Subroutines TT PLT 1, TT PLT 2, and TT PLT 3 are for teletype plotting the data stored by PLOT.

IV. PROGRAM DETAILS AND LISTING

The following variables are in the common block of each Subroutine.

- J. Index counter counting the fine time steps in the Subroutine SOLVE. When J reaches Jmax the Subroutine PRINT is called and the output is printed. Then J is set back to 3.

Jmax: Given as data in Main-R/W. The maximum value J takes is Jmax.

ND: Number of ions ($\sqrt{\text{ION}}$) for which SOLVE solves the differential equations. It is given as input.

H: Variable representing the step size(time) in SOLVE.

DTmax: Predicted value of H given as input.

Kstep: Counter counting the number of fine steps in the integration program SOLVE.

KSend: Maximum limit "Kstep" could take in the Subroutine SOLVE. It is given as input.

KDOB: Flag. If it becomes -1 the time step H is considered for doubling in SOLVE subroutine.

KHAV1
KHAV2
KHAV3: Flags used in SOLVE to decide whether at a certain time, the step size H is to be made half.

DOB1,
DOB2: Counters used in SOLVE to limit the maximum number of iterations in finding proper H.

T(60): Time at each fine time step J in SOLVE.

EPSI(60): Accumulated error indicating how well the integration is proceeding.

Y(60,18): Population density of each ionization stage at each time step.

YR(60,18): Increments in populations for each time step --i.e. $\frac{dN_k}{dt}$. These are calculated in "RATES".

S(18): Ionization rate Coefficients.

Alpha(18): Total recombination rate Coefficients.

NTIME: Maximum number of coarse time steps for the input data.

TIME (100), DENS (100), TEMP (100): Electron density and temperature data at coarse time steps L=1 to NTIME given as input.

DNE(60), TE(60): Interpolated electron density and temperature data at each five time step J.

Ion: Maximum number of ions for which rate equations are going to be solved.

IonM1: Ion -1.

KION(18): Counter for the number of ions.

SA(18): The coefficients for the ionization rates calculated by the formula of Lotz and Kunze. They are given as input.

SB(18): Values used in Lotz and Kunze formula. (SB is set 3.)

SE(18): Ionization potentials for the ions given as input.

ANPRIM(18): Data for three body recombination in RATES to be given as input.

NAME(18): Identification of the plasma condition.

SFN(18): Oscillator strengths for the lines whose time histories are to be predicted.

SEX(18): Excitation potentials in eV for these lines.

ØØ1: Electron density divided by the sum of all ionic populations at t=0 µsecs.

YMIN: 10^{-4} times the sum of all ionic populations.

YYMIN: 10^{-6} times the sum of all ionic populations.

EN, ED, EH: Constants to check accuracy in SOLVE.

NDT, NAINI: Constants used in the program.

NA: A variable representing the stage of ionization below which populations are less than YMIN.

THI(60, 18): Theoretical intensities stored for plotting. Input data for dielectronic recombination (DALPHA) formulae due to Landini and Fossi.¹¹

F(18): Given in Ref. 11.

W(18): First excitation energy in recombining ion. Given in Ref. 11.

Yes(18): A flag whose value is read in for each ion to decide which formula for DALPHA is to be used.

Flag: A flag decides whether experimental or theoretical intensities are to be plotted.

Tim: Variable used in "PRINT" to find the fine time steps that correspond to the Coarse time steps.

DIF 2: The difference between fine time step and the coarse time step evaluated in "PRINT".

TTT(60): The fine time steps that are equal to or close to the coarse time steps.

Flag 1: A flag read in input data which decides whether Burgess and Summers or Lotz and Kunze ionization rate coefficient is to be used.

M(I,K): Number of equivalent electrons in shell I of ion K.

H(I,K): Ionization energy in eV of an electron in shell "I" of ion "K". Five shells should be more than enough.

A(K): A factor multiplying Burgess and Summers ionization rate to match time histories.

PROGRAM DETAILS:

1. MAIN - R/W: It reads in the data and manages the rest of sub-routines. The integer variables FLAG, FLAG1 and YES are flags whose

values given as input would make the required calculations. The possible manipulations are described in the comments of the listing of this subroutine. In the appendix II data is set up as an example. The array YI(J,K) is used to read in the experimental time histories. A brief description of the purpose of the following variables is given below.

A SUM: Sum of initial populations of all ions.

ØØ1: Electron density at time $t=0 \mu\text{s}$ divided by A SUM. This is used in PRINT to normalize the calculated populations. This procedure is to check how the effect of compression was taken care of in the subroutine RATES by the variable ABL and how the smooth transfer of populations from one stage to the other is proceeding.

YMin and YYMin: These are used to avoid the accuracy check in SOLVE for ions with negligible populations at a certain time step J.

2. SOLVE: This subroutine solves the coupled differential equations by numerical integration using Adams predictor-corrector methods. A general outline of the working process of this subroutine is described below.

The flag NSTART takes values 1, 2 or 3 given as input data. If NSTART is set to 1, the program, between statement numbers 5 through 19, will try to find a proper time step H for continuing the solution with stability and convergence. An initial value for H (DTMAX) is given as input. Also the density and temperature information has

to be given at one negative time (non physical) for this self starting procedure. This information is only useful for starting the program and does not influence the solution.

If NSTART is set to 2, the initial value for H given as input is used to continue the integration. However, the program tries to adjust "H" to obtain solutions within the accuracy requirements.

If NSTART is set to 3, the difference between coarse time steps 1 and 2 at which density and temperature information was given would be taken as the starting value of H and the program tries to adjust H, if required, to continue integration keeping the required accuracy.

The program had provisions to speed up the calculation by doubling the step size or to retrace back one time step and cut the step size to half and continue the integration within the accuracy requirements. The accuracy checks were done by using the values for EN, EH and ED as 1, .1 and .001 respectively. These values were found to yield good stability, convergence and accuracy in the running of SOLVE. In section V analytical solutions were compared with numerical solutions for a simple two ion system. Further details about the working of the program are in the comments of the listing of SOLVE.

3. RATES: It calculates the ionization and recombination rates, as well as increments in populations at each time step J. The variable ABL takes care of effects of any compression etc. The external functions SUM1 and SUM2 are used to calculate S(K)--Burgess and Summers if desired (i.e. Flag 1.GT.0). Other details are in comments of the listing.

PRINT: This subroutine prints the concentration of each ionization stage at each time step J. It calculates the intensities of emission lines, using the external function EIOFX and prints them.

PLOT: The time histories of emission lines are written on a tape for later plotting.

MAIN - R/W

```
1.  C
2.  C THIS PROGRAM IS FOR SOLVING COUPLED RATE EQUATIONS AND PREDICTING
3.  C TIME HISTORIES OF THE IMPURITY IONS AND THEIR EMISSION LINES IN A
4.  C RAPIDLY IONIZING PLASMA LIKE THAT OF A THETA PINCH.
5.  C DATA NEEDED AS INPUT IS ELECTRON DENSITY AND TEMPERATURE PROFILES
6.  C WITH RESPECT TO TIME AND THE CHARACTERISTIC DATA FOR EVALUATING
7.  C THE VARIOUS RATE COEFFICIENTS--IONIZATION&RECOMBINATION(RADIATIVE,
8.  C THREE BODY&DIELECTRONIC) RATE COEFFICIENTS.
9.  C
10. C     INTEGER TIM,FLAG,FLAG1,YES
11. C     DIMENSION YI(60,18)
12. C     COMMON J,JMAX,ND,H,DTMAX,NDT,KSTEP,KSEND,END,ED,EH,EN,KDOB,
13. C     1      KHAV1,KHAV2,KHAV3,DOB1,DOB2
14. C     COMMON T(60),EPSI(60), Y(60,18),YR(60,18
15. C     1      ),ANPRIM(18)
16. C     COMMON S(18),ALPHA(18),DENS(100),TEMP(100),TIME(100),SA(18),SB(18)
17. C     1      ,SE(18),NAME(8),KION(18),DNE(60),TE(60),SFN(18),SEX(18)
18. C     COMMON ATIME,ION,IONM1,NA,NAINI,OO1,YMIN,YYMIN
19. C     COMMON TH(60,18),TIM,FLAG,F(18),YES(18),W(18),M(5,18),HI(5,18)
20. C     1 ,FLAG1,TTT(60),DIF2,A(18)
21. C
22. C IN THIS PROGRAM COARSE TIME STEPS MEAN THE TIME STEPS AT WHICH
23. C DENSITY AND TEMPERATURE INFORMATION IS GIVEN.
24. C
25. C TIM -IT IS A VARIABLE USED IN THE SUBROUTINE PRINT TO FIND THE
26. C     COARSE TIME STEPS AMONG THE FINE TIME STEPS GENERATED BY THE
27. C     SUBROUTINE SOLVE.
28. C
29. C FLAG IS A FLAG TO INSTRUCT THE COMPUTER WHAT IS TO BE PLOTTED.
30. C 'FLAG' VALUES:
31. C     0 - EXPERIMENTAL INTENSITIES ONLY (PLOT THEM)
32. C     1 - BOTH THEORETICAL AND EXPERIMENTAL INTENSITIES (PLOT BOTH
33. C     OF THEM. (THE THEORETICAL INTENSITIES ARE THE ONES COMPUTED BY
34. C     THE SUBROUTINE SOLVE.) THIS VALUE CAN ONLY BE USED FOR
35. C     REOTE TERMINAL PLOTTING BUT NOT FOR TELETYPE PLOTTING.
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```

36. C           THIS IS THE WAY SUB ROUTINES TTSTORE & PLOTTER ARE CONSTRUC
37. C           -TED.
38. C           2 - THEORETICAL INTENSITIES ONLY (PLOT THEM)
39. C
40. C FLAG1 IS A FLAG TO INSTUCT THE COMPUTER WHICH FORMULAS FOR IONIZATION
41. C ARE TO BE USED.
42. C FLAG1 VALUES:
43. C           0 - SEMI EMPERICAL FORMULA DUE TO LOTZ&KUNZE IS USED.
44. C           1 - SEMI CLASSICAL FORMULAE DUE TO BURGESS&SUMMERS ARE
45. C           USED.TWO SUBROUTINES SUM1&SUM2 DO THIS JOB.
46. C EXPLANATION OF THE INPUT DATA FOR THE RATE COEFFICIENTS IS GIVEN
47. C BELOW.
48. C SA(K) &SB(K) ARE CONSTANTS USED IN LOTZ&KUNZE'S IONIZATION RATE
49. C COEFFICIENT.IF FLAG1 IS NOT SET 0 THEN THESE ARE STILL READ BUT
50. C NEVER USED.
51. C
52. C THE FOLLOWING IS FOR SUMMERS&BURGESS IONIZATION RATE COEFFICIENT.
53. C "M(I,K)"- NO OF EQUIVALENT ELECTRONS IN "I"SHELL OF ION K.
54. C "H(I,K)"- IONIZATION ENERGY IN EV OF AN ELECTRON IN SHELL "I" OF ION
55. C K. FIVE SHELLS SHOULD BE MORE THAN ENOUGH !
56. C
57. C DALPHA IS DIELECTRONIC RECOMBINATION. THE FOLLOWING ARE NEEDED TO
58. C IT'S EVALUATION.
59. C "F" IS INPUT DATA FOR DALPHA.
60. C "W" -FIRST EX ENERGY OF RECOMBINING ION- IN DALPHA.
61. C
62. C           READ(5,1(0)) (NAME(K),K=1,8)
63. C           100 FORMAT(8A4)
64. C           1 READ(5,1(1)) ION,NTIME,NAINI,FLAG,FLAG1
65. C           101 FORMAT(12I6)
66. C           READ(5,1(2))(KION(K),SA(K),SB(K),SE(K),Y(2,K),ANPRIM(K),K=1,ION)
67. C           102 FORMAT(I12,4E12.4,F12.4)
68. C           READ(5,1(3)) JMAX,ND,NDT,KSEND,END,ED,EH, EN,DTMAX
69. C           103 FORMAT(3I4,I8,5E8.1)
70. C           READ(5,1(4)) (TIME(K),TEMP(K),DENS(K),K=1,NTIME)
71. C           104 FORMAT(3E12.4)
72. C           READ(5,1(5)) (SFN(K),SEX(K),K=1,ION)
73. C           105 FORMAT(2F10.5)

```

```

76.      READ(5,106)(YES(K),F(K),W(K),K=1,ION)
77.      106 FORMAT(I3,F7.5,F10.5)
78.      IF (FLAG1.EQ.7) GO TO 107
79.      DO 109 K=1,ION
80.      109 READ(5,130)A(K),(M(I,K),HI(I,K),I=1,5)
81.      130 FORMAT (E10.4,(5(I2,F10.4)))
82.      107 IF (FLAG1.GT. 1) GOTO 120
83.      NT=NTIME-E
84.      DO 110 J = 1,NT
85.      ITT(J)=TIME(J+L)
86.      110 READ(5,115) (YI(J,K),K=1,ION)
87.      115 FORMAT(1EF4.2)
88.      C
89.      C ALL THE NEEDED DATA IS READ BY NOW. IT WAS DONE IN THE FOLLOWING
90.      C ORDER. DATA FOR KUNZE&LOTE, DENSITY & TEMP PROFILES, DATA FOR EXCITATION
91.      C , DALCHA, SUMMERS&BURGESS, & EXPERIMENTAL TIME HISTORIES.
92.      C
93.      DO 900 J=1,NT
94.      900 WRITE(6,116) (YI(J,K),K=1,ION)
95.      116 FORMAT(1EF6.2)
96.      C
97.      C NOW NORMALIZE AND PLOT THE EXPERIMENTAL INTENSITIES
98.      C
99.      CALL PLOTT(YI,1,0)
100.     IF (FLAG1.EQ. 0) STOP
101.     C
102.     C
103.     120 IONN1 = ION - 1
104.     WRITE(6,200)
105.     200 FORMAT(1F1,10X,76H IONIZATION OF IMPURITY IONS IN A PLASMA WITH GI
106.     IVEN DENSITY AND TEMPERATURE ///)
107.     201 FORMAT(10X,10H IMPURITY ,D44//)
108.     WRITE(6,201) (NAME(K),K = 1,8)
109.     WRITE(6,202) ION
110.     202 FORMAT(10X,24H NUMBER OF IONS ION =16//)
111.     IF (FLAG1.EQ.0) GO TO 211
112.     WRITE(6,212)

```

```

112. 312 FORMAT(10X,42H DATA FOR ECIP IONIZATION RATE COEFFICIENT //)
113.     DO 313 K=1,ION
114. 313 WRITE(6,314)(KION(K),(M(I,K),HI(I,K),I=1,5),Y(2,K),ANPRIM(K),A(K))
115. 314 FORMAT(5X,I2,5(I2,E10.4),E10.4,F3.1,F10.6)
116.     WRITE(6,315)
117. 315 FORMAT(///10X," THE IONIZATION RATE COEFFICIENTS ARE CALCULATED
118. 1BY THE FORMULA"//10X," S = A * SEMICLASSICAL RATE COEFFICIENT "
119. 2///10X," THE FORMULA USED FOR TOTAL RECOMBINATION:"//
120. 3 10X, 85H ALPHA = 5.2E-14*Z*SQRT(E(I-1)/T) * (0.429 + 0.5*LN(E(I-
121. 41)/T) + 0.469*CBRT(T/E(I-1)) + /15X,75H1.4E-31 * N * (NPRIME/Z)*
122. 5*6 * (E(I-1)/T)**2 * EXP(E(I-1)/(T*(NPRIME+1)**2)),2H +,/15X ,
123. 6" DIELECTRONIC RECOMBINATION(LANDINI&FOSSI SOLAR PHYSICS 20,322P
124. 7 (1971))"///
125. 811X,"FOR THE EXCITATION WE USE G(EFF) APPROXIMATION"//10X," X= 1.5
126. 98E-05*FN*P*(EXP-EX/T)/(EX*SQRT(T))"////)
127.     GO TO 316
128. 211 WRITE(6,203)
129. 203 FORMAT(10X," CHARACTERISTIC DATA FOR THE IONS "///15X,
130. 1 4H ION,5X,2H A,10X,2H B,10X7H E (EV),10X,"INITIAL DENSITY",15X,
131. 2 6HNPRIME//)
132.     WRITE(6,204) (KION(L),SA(L),SB(L),SE(L),Y(2,L),ANPRIM(L),L=1,ION)
133. 204 FORMAT(15X,I3,1X,3E12.4,12X,E12.4,10X,F12.4)
134.     WRITE(6,205)
135. 205 FORMAT(///10X," THE IONIZATION RATE COEFFICIENTS ARE CALCULATED
136. 1BY THE FORMULA"//10X," S = A*((LN(40*T/E)**3)+40)*SQRT(T)*EXP(-E
137. 2/T)/(E+BT)"//10X," THE FORMULA USED FOR TOTAL RECOMBINATION:"//
138. 3 10X, 85H ALPHA = 5.2E-14*Z*SQRT(E(I-1)/T) * (0.429 + 0.5*LN(E(I-
139. 41)/T) + 0.469*CBRT(T/E(I-1)) + /15X,75H1.4E-31 * N * (NPRIME/Z)*
140. 5*6 * (E(I-1)/T)**2 * EXP(E(I-1)/(T*(NPRIME+1)**2)),2H +,/15X ,
141. 6" DIELECTRONIC RECOMBINATION(LANDINI&FOSSI-SOLAR PHY.20(1971))"///
142. 711X,"FOR THE EXCITATION WE USE G(EFF) APPROXIMATION"//10X," X= 1.5
143. 88E-05*FN*P*(EXP-EX/T)/(EX*SQRT(T))"////)
144. 316 WRITE(6,208) JMAX,ND,NDT,KSEND,END,ED,EH,EN,DTMAX
145. 208 FORMAT(10X,"CONSTANTS FOR THE INTEGRATION PROGRAM "///
146. 1 10X,7H JMAX =14,5X,5X,5H ND =14,5X,6H NDT =14,
147. 25X,8H KSEND =18//10X,6H END =E8.1,5X,5H ED =E8.1,5X,5H EH =E8.1,
148. 3 5X,5H EN =E8.1,5X,8H DTMAX =E8.4//)
149.     WRITE(6,214)

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150.      214 FORMAT(//10X,36H DATA FOR DIELECTRONIC RECOMBINATION//,
151.          *15X,3HION,3X,7HFORMULA,3X,7H F.OR.N,8X,6H W(EV)//)
152.          WRITE(6,215)(KION(K),YES(K),F(K),W(K),K=1,ION)
153.      215 FORMAT(15X,I3,7X,I3,7X,F3.0,2X,F10.5)
154.          WRITE(6,206)
155.      206 FORMAT(1H1,10X,"INPUT DENSITY & TEMPERATURE PROFILES ARE:"//)
156.          1 15X,5H TIME,3X,11H TEMP (EV) ,16H DENSITY(CM**-3)//)
157.          WRITE(6,207) (TIME(K),TEMP(K),DENS(K),K=1,NTIME)
158.      207 FORMAT(10X,3E12.4)
159.          WRITE(6,212)
160.      212 FORMAT(//10X," DATA FOR THE EXCITATION OF THE IONS "//15X,4H ION
161.          1,5X,3H FN,8X,6HEX(EV)//)
162.          WRITE(6,213) (KION(K),SFN(K),SEX(K),K=1,ION)
163.      213 FORMAT(15X,I3,1X,F12.5,1X,F12.3)
164.          WRITE(6,210)
165.      210 FORMAT(1H1)
166.      C
167.      C
168.          ASUM = 0.0
169.          DO 301 K = 1,ION
170.      301  ASUM = ASUM + Y(2,K)
171.          OOI = DENS(2)/ASUM
172.          YMIN = 1.0E-04*ASUM
173.          YYMIN = 1.0E-06*ASUM
174.          T(2) = 0.0
175.          NA = MAINI
176.          TIM = 3
177.          CALL SOLVE(1,NEND)
178.          CALL PRINT
179.          WRITE(6,209) NEND
180.      209 FORMAT(1H0,50X,7H NEND =15//10X, "HURRAY!!!!!!SUCCESSFUL RUN !!!")
181.          IF(NEND.GT.1) GO TO 500
182.      C
183.      C THE FOLLOWING PART IS FOR PLOTTING:
184.      C
185.          DO 370 K=1,ION
186.          DO 360 I=3,NTIME
187.      360  TNI(I-2,K)=TNI(I,K)

```

```

188.      370  CONTINUE
189.          DO 371 I=3,NTIME
190.      371  TTT(I-2)=TTT(I)
191.          NTT=NTIME-2
192.          DO 372 K=1,ION
193.          DO 373 I=1,NTT
194.          WRITE(6,350) TTT(I),THI(I,K),K
195.      350  FORMAT(10X,'TTT=',E12.4,5X,'THI(I,K)=',E12.4,5X,'K=',I3/)
196.      373  CONTINUE
197.      372  CONTINUE
198.  C   IN ABOVE WHAT WAS DONE IS THAT 'TTT(I) & THI(I)' ARE MADE TO START
199.  C   FROM I=1. ORIGINALLY I(TIM) WAS SET=3 TO AVOID THE FIRST TWO COARSE
200.  C   TIME STEPS
201.  C   NOW NORMALIZE AND PLOT THE THEORETICAL INTENSITIES STORED IN 'THI'
202.  C
203.          IF (FLAG1.EQ.0) GO TO 400
204.          CALL PLOTT(THI,2,1)
205.          GO TO 500
206.      400  CALL PLOTT(THI,2,2)
207.  C
208.  C   THE CALCULATIONS & PLOTTING ARE OVER .
209.  C
210.      500  STOP
211.          END

```

```

1.      SUBROUTINE SOLVE(NSTART,NEND)
2.      C
3.      C INTEGRATION PROGRAM FOR THE COUPLED 1-ORDER DIFFERENTIAL EQUATIONS
4.      C NUMERICAL INTEGRATION DONE BY " PREDICTOR CORRECTOR " METHODS OF ADA
5.      C MS & MOULTON TYPE. THIS PROGRAM HAS THE OPTION FOR SELF STARTING. IT
6.      C VARIES THE STEP SIZE-H TO OBTAIN THE REQUIRED ACCURACY.
7.      C
8.      COMMON J,JMAX,ND,H,DTMAX,NDT,KSTEP,KSEND,END,ED,EH,EN,KDOB,
9.      1      KHAV1,KHAV2,KHAV3,DOB1,DOB2
10.     COMMON T(60),EPSI(60),Y(60,18),YR(60,18
11.     1      ),ANPRIM(18)
12.     COMMON S(18),ALPHA(18),DENS(100),TEMP(100),TIME(100),SA(18),SB(18)
13.     1      ,SE(18),NAME(8),KION(18),DNE(60),TE(60),SFN(18),SEX(18)
14.     COMMON ATIME,ION,IONM1,NA,NAINI,OO1,YMIN,YYMIN
15.     COMMON THI(60,18),TIM,FLAG,F(18),YES(18),W(18),M(5,18),HI(5,18)
16.     !      ,FLAG1,TTT(60),DIF2,A(18)
17.     C
18.     INTEGER DOB1,DOB2,MXDOB
19.     INTEGER END
20.     C
21.     C COEFFICIENTS FOR THE PREDICTOR CORRECTOR FORMULAE.
22.     C
23.     MXDOB=2**25
24.     CF1 = 2./3.
25.     CF2 = 4./3.
26.     CF3 = 1./8.
27.     CF4 = 1./12.
28.     CF5 = 5./12.
29.     CF6 = 23./12.
30.     CF7 = 1./24.
31.     CF8 = 5./24.
32.     CF9 = 9./24.
33.     CF10=19./24.
34.     NSTART = NSTART
35.     2      GO TO(5,6,4),NSTART

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36.  C
37.  C   SET THE VALUES AT THE THREE INITIAL POINTS.
38.  C
39.      4   H=T(2)-T(1)
40.      IF(H) 101,100,101
41.  100   NEND=6
42.      WRITE(6,600)
43.  600   FORMAT(1X,' EXIT DUE TO TIMESTEP H AT #4 OR #101 ')
44.      RETURN
45.  101   IF(ABS(T(3)-T(2)-H)-ABS(H)*.01)102,102,100
46.  102   N1=NDT
47.      Z1=ABS(H)
48.      NDT=0
49.  105   IF(Z1-ABS(DTMAX))103,104,104
50.  103   NDT=NDT+1
51.      Z1=Z1*2.
52.      GO TO 105
53.  104   DOB1=MXDCB/2**NDT
54.      NDT=N1
55.      J = 1
56.      CALL RATES
57.      J=2
58.      CALL RATES
59.      J = 3
60.      CALL RATES
61.      EPSI(2) = 0.
62.      KSTEP = 3
63.      GO TO 3
64.  C
65.  C PROGRAM STARTS HERE IF NTART=1.
66.  C
67.      5   H = DTMAX * 2. **(-NDT)
68.      DOB1 = MXDOB/2**NDT
69.      J = 2
70.  C
71.  C INITIALIZE
72.  C
73.      CALL RATES

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74.          DO 13  N = NA,ND
75.          Y (1,N) = Y (2,N)
76.          YR(1,N) = YR(2,N)
77.          Y (3,N) = Y (2,N)
78.          13  YR(3,N) = YR(2,N)
79.          C
80.          C ITERATION OF THE ADAMS MOULTON K=2 CORRECTOR.IT DOES 5 ITERATIONS HERE
81.          C IF THE CONVERGENCE IS NOT FAST ENOUGH TO MEET THE ACCURACY CHECK, ITCU
82.          C TS THE TIMESTEP BY HALF AND TRIES AGAIN FOR FIVE ITERATIONS. THIS GOES
83.          C ON UNTIL DOB1 BECOMES ZERO THAT IS NDT TIMES.
84.          C
85.          C
86.          12  T(1) = T(2) - H
87.          T(3) = T(2) + H
88.          IT1 = 0
89.          IT2 = -1
90.          24  ZE = 0.
91.          Z1 = EN * H
92.          DO 18 N = NA,ND
93.          Z2 = CF1 * YR(2,N)
94.          Y(1,N) = Y(2,N) - (CF5 * YR(1,N) + Z2 - CF4 * YR(3,N)) * H
95.          ZY =      Y(2,N) + (CF5 * YR(3,N) + Z2 - CF4 * YR(1,N)) * H
96.          C
97.          C THIS "IF" OMITTS THE ACCURACY CHECK FOR SMALL Y(J,N), (E.G. ZERO)
98.          C
99.          IF( Y(1,N) - YYMIN) 18,18,220
100.         220  Z2 = ABS(ZY - Y(3,N))/(ABS(Y(2,N)) + (ABS(YR(1,N))+ABS(YR
101.         1   (2,N)) + ABS(YR(3,N))))* Z1)
102.         ZE = AMAX1(ZE,Z2)
103.         18  Y(3,N) = ZY
104.         IF (IT2) 19,20,20
105.         C
106.         C THE FOLLOWING "IF" IS ACCURACY CHECK.
107.         C
108.         20  IF(ZE - EH/16.) 21,21,22
109.         601 FORMAT(1X,'ZE=',F10.5/)
110.         22  IT1 = IT1 + 1
111.         WPITE(6,601) ZE

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112.          IF(IT1 - 5) 19,23,23
113.      19      J = 1
114.          CALL RATES
115.          J = 3
116.          CALL RATES
117.          IT2 = 1
118.          GO TO 24
119.      23      DOB1=DOB1/2
120.          H = 0.5 * H
121.          WRITE (6,602)
122.      602     FORMAT(1X, 'ITERATION DID NOT SUCCEED.SO TRY HALF THE TIME STEP'
123.          1 /)
124.          IF (DOB1.NE.0) GO TO 12
125.      25      NEND = 3
126.          WRITE(6,700)
127.      700     FORMAT(1X, 'EXIT AT #25-SOLVE.INPUT HMAX IS TOO LARGE ')
128.          RETURN
129.      603     FORMAT(1X, 'BY NOW IT GOT HOLD OF THREE POINTS'/)
130.      C
131.      C TRY THE NEXT POINT--T(4).
132.      C *****
133.      21      KSTEP = 1
134.          WRITE(6,601) ZE
135.          WRITE(6,603)
136.          DOB2=DOB1
137.          J = 3
138.      3      EPSI(3)=0.
139.          KDOB=1
140.          KPRINT = 1
141.          KHAV1 = 1
142.          KHAV2 = 1
143.      C
144.      C NORMALCASE OF INTEGRATION.
145.      C
146.      604     FORMAT(1X, ' NORMAL CASE ')
147.      6      IF(KHAV1) 33,34,34
148.      33     IF(KHAV3) 35,29,29
149.      29     KHAV1 = 1

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150.          KHAV2 = 1
151.          WRITE(6,604)
152.          GO TO 34
153.          35    KHAV3 = KHAV3 + 1
154.          34    IF(J -JFAX) 36,37,37
155.          C
156.          C RESET PRINT
157.          C
158.          37    IF(KPRINT) 26,27,27
159.          27    CALL PRINT
160.          26    KPRINT = 1
161.          J1 = JMAX - 2
162.          NJ = 1
163.          DO 38 J = J1,JMAX
164.          T(NJ) = T(J)
165.          EPSI(NJ) = EPSI(J)
166.          DO 43 N = NA,ND
167.          Y(NJ,N) = Y(J,N)
168.          43    YR(NJ,N) = YR(J,N)
169.          38    NJ = NJ + 1
170.          J = 3
171.          C
172.          C EXTRAPOLATION: PREDICT AT THE NEXT TIME STEP WITH ADAMS K=2 PREDICTOR
173.          C
174.          C      EXTRAPOLATION
175.          C
176.          36    T(J + 1) = T(J) + H
177.          WRITE(6,605)
178.          605  FORMAT(1X,' NORMALCASE. EXTRAPOLATION.')
179.          IF (H-DTMAX)47,48,48
180.          47  DOB2=DOB2+DOB1
181.          48    DO 55 N = NA,ND
182.          55    Y(J+1,N) = Y(J,N) + (CF5*YR(J-2,N)-CF2*YR(J-1,N)+CF6*YR(J,N))*H
183.          J = J + 1
184.          CALL RATES
185.          C
186.          C INTERPOLATION : CORRECTION WITH ADAMS K=3 CORRECTOR.
187.          C

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188.         ZE = C.C
189.         WRITE(6,606)
190.         606 FORMAT(1X,' INTERPOLATION.')
191.         Z1 = EN * H
192.         DO 61 N = NA,ND
193.         ZY = Y(J-1,N)+(CF7*YR(J-3,N)-CF8*YR(J-2,N)+CF10*YR(J-1,N)+CF9*
194.         1   YR(J,N))*H
195.         C THIS 'IF' OMITTS THE ACCURACY CHECK FOR SMALL Y ( E.G. ZERO)
196.         IF(Y(J,N) - YYMIN) 61,61,211
197.         211  Z2 = ABS(ZY-Y(J,N))/(ABS(ZY)+(ABS(YR(J,N))+ABS(YR(J-1,N))+
198.         1   ABS(YR(J-2,N))+ABS(YR(J-3,N)))) * Z1)
199.         ZE = AMAX1(ZE,Z2)
200.         61   Y(J,N) = ZY
201.         EPSI(J) = 0.5 * EPSI(J-1) + ZE
202.         KSTEP = KSTEP + 1
203.         WRITE(6,607)KSTEP,END,J,EPSI(J)
204.         607 FORMAT(1X,' KSTEP= ',I5,' END ',F10.5,' J ',I5,' EPSI ',F10.5/)
205.         IF (KSTEP - KSEND) 62, 63, 63
206.         63   NEND = 2
207.         WRITE(6,710)
208.         710 FORMAT(1X,'///' NUMBER OF STEPS (KSTEP) BECAME GT THAN LIMIT
209.         1 KSEND')
210.         RETURN
211.         62   IF (END)          64,56 ,56
212.         64   NEND = 1
213.         RETURN
214.         56   IF (KSTEP-4) 49,40,40
215.         49   IF (EPSI(J) - EH/8.) 6,6,23
216.         C
217.         C CONTINUE INTEGRATION *****
218.         C
219.         40   IF (H - DTMAX)    67, 70, 70
220.         67   IF (KCOB)        68, 69, 69
221.         69   KCOB = KCOB - 1
222.         GO TO 70
223.         C
224.         C #71-TO-#79 BELOW ARE FOR DOUBLING THE STEP SIZE. #6-TO-#49 IN ABOVE
225.         C FOR STEP SIZE KEPT SAME. #80-TO-#95 ARE TO CUT THE STEP SIZE BY HALF.

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226. C
227. 68 IF (EPSI(J) - ED) 71, 70, 70
228. 71 Z=DOB1.AND.DOB2
229. C
230. C .AND. IS A MANIPULATIVE OPERATOR. Z IS ZERO AS DOB1 IS NOT=DOB2(#47)
231. C
232. IF (Z) 6, 72, 6
233. 72 NJ = J
234. IF (J - 4) 73, 73, 74
235. 73 J = JMAX - 1
236. KPRINT = -1
237. GO TO 2E
238. 74 J = J - 2
239. 28 T(J - 1) = T( J )
240. T( J ) = T( NJ )
241. EPSI(J-1) = EPSI(J)
242. EPSI(J) = EPSI(NJ)
243. DO 79 N = NA,ND
244. Y (J-1,N) = Y (J,N)
245. Y(J,N) = Y (NJ,N)
246. YR(J-1,N) = YR(J,N)
247. 79 YR(J,N) = YR(NJ,N)
248. DOB1=DOB1*2
249. H = 2.*H
250. KDOB = 1
251. KSTEP = KSTEP-2
252. GO TO 6
253. C
254. C HALF THE TIME STEP.
255. C
256. 70 IF (EPSI(J)-EH) 6,6,80
257. 80 IF (K+AV2) 81,82,82
258. 81 NEND = 5
259. WRITE(6,73C)
260. 730 FORMAT(1X,///' THE ACCUMALATED ERROR IS GT THAN EH EVEN IN THE
261. 1 RECALCULATION OF TWO EARLIER TIME STEPS ! ')
262. RETURN
263. 82 IF (K+AV1) 83,83,84

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264.      84      KHAV1 = -1
265.          KHAV3 = -2
266.          DOB2=DOB 2-DOB1
267.          DOB1=DOB 1/2
268.          H = 0.5 * H
269.          IF(DOB1.NE.0) GO TO 86
270.      85      NEND = 4
271.          WRITE(6,740)
272.      740 FORMAT(1X,///' EXIT AT #85 AS DOB1 BECAME ZERO.I.E TOO MANY ITERAT
273.          IONS.INPLT HMAX IS TOO LARGE. ')
274.          RETURN
275.      83      KHAV2 = -1
276.          DOB2=DOB 2-DOB1*3
277.          KSTEP = KSTEP-2
278.          IF (KSTEP-2) 96,96,30
279.      96      KSTEP = 4
280.      30      J = J - 2
281.          IF(J-2) 31,31,86
282.      31      J = JMAX - 1
283.          KPRINT = -1
284.      C
285.      86      T(J) = T(J - 1)
286.          EPSI(J) = EPSI(J - 1)
287.          DO 32 N = NA,ND
288.          Y (J,N) = Y (J -1,N)
289.      32      YR(J,N) = YR(J -1,N)
290.          T(J - 1) = T(J) - H
291.          EPSI(J - 1) = EH
292.          Z = 0.5 * H
293.          DO 95 N=NA,ND
294.      95      Y(J - 1,N) = (Y(J,N)+Y(J-2,N)-(YR(J,N)-YR(J-2,N))*Z)*0.5
295.          J = J - 1
296.          CALL RATES
297.          J = J + 1
298.          GO TO 6
299.          END

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```

1.      SUBROUTINE RATES
2.      C
3.      C RATE COEFFICIENTS AND INCREMENTS IN POPULATIONS YR(J,K) AT EACH TIME
4.      C STEP J ARE CALCULATED.
5.      C
6.      INTEGER YES, FLAG1
7.      COMMON J, JMAX, ND, H, DTMAX, NDT, KSTEP, KSEND, END, ED, EH, EN, KDOB,
8.      1      KHAV1, KHAV2, KHAV3, DOB1, DOB2
9.      COMMON T(60), EPSI(60), Y(60,18), YR(60,18
10.     1      ), ANPRIM(18)
11.     COMMON S(18), ALPHA(18), DENS(100), TEMP(100), TIME(100), SA(18), SB(18)
12.     1      , SE(18), NAME(8), KION(18), DNE(60), TE(60), SFN(18), SEX(18)
13.     COMMON NTIME, ION, IONM1, NA, NAINI, O01, YMIN, YYMIN
14.     COMMON THI(60,18), TIM, FLAG, F(18), YES(18), W(18), M(5,18), HI(5,18)
15.     !      , FLAG1, TTT(60), DIF2, A(18)
16.     C
17.     IF( T(J) + DTMAX - TIME(NTIME)) 6,7,7
18.     7      END = -1.0
19.     C
20.     C INTERPOLATION OF DENSITY AND TEMPERATURE
21.     C
22.     6 DO 1 L = 1, NTIME
23.     IF( T(J) - TIME(L) ) 3,2,1
24.     1 CONTINUE
25.     2      DNE(J) = DENS(L)
26.     TE(J) = TEMP(L)
27.     ABL = ( DENS(L+1) - DENS(L-1) ) / ( (TIME(L+1) - TIME(L-1)) * DENS(L) )
28.     GO TO 4
29.     3      DT = TIME(L) - TIME(L-1)
30.     RDT = ( T(J) - TIME(L-1) ) / DT
31.     DIFFD = DENS(L) - DENS(L-1)
32.     DNE(J) = DENS(L-1) + DIFFD * RDT
33.     TE(J) = TEMP(L-1) + ( TEMP(L) - TEMP(L-1) ) * RDT
34.     ABL = DIFFD / ( DT * DNE(J) )
35.     C

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36.  C  CALCULATION OF IONIZATION&RECOMBINATION RATE COEFFICIENTS.
37.  C
38.      XSE = 0.0
39.      XNPRIM = 0.0
40.  4  DO 25 K = NA,ION
41.      IF (FLAG1.EQ.0)GO TO 26
42.  C
43.  C  FLAG1 # 0 . EVALUATE &USE ECIP IONIZATION RATE - BURGESS&SUMMERS.
44.  C
45.      S(K)=COEF(K,TE(J),M(1,K),HI(1,K),M(2,K),HI(2,K),M(3,K),HI(3,K),
46.      1M(4,K),HI(4,K),M(5,K),HI(5,K))*A(K)
47.      GOTO 30
48.  26  AAA = 40.0 * TE(J) / SE(K)
49.      IF(AAA-1.0) 20,20,21
50.      20  AAA = 1.0
51.  C
52.  C  FLAG1=0.EVALUATE & USE SEMIEMPERICAL IONIZATION RATE - LOTZ&KUNZE.
53.  C
54.  21  S(K) = SA(K)*((ALOG( AAA
55.      1  XP(-SE(K)/TE(J)))/(SE(K) + SB(K)*TE(J))
56.  30  IF(XSE) 9,9,10
57.      10  IF(XNPRIM) 12,12,11
58.  C
59.  C  EVALUATE THREEBODY RECOMBINATION "BALPHA" -DUCHS&GRIEM,PHYS.FLUIDS
60.  C  (1966)VOL 9.
61.  C
62.      11  BBB = XSE/(TE(J)*(XNPRIM+1.0)*(XNPRIM+1.0))
63.      IF(BBB-25.0) 19,19,18
64.      18  BBB = 25.0
65.      19  BALPHA = 1.4E-31*DNE(J)*((XNPRIM/(FLOAT(K)-1.0))**6) * ((XSE/TE(J)
66.      1  )**2) * EXP(BBB)
67.      GO TO 13
68.      12  BALPHA = 0.0
69.  C
70.  C  NOW EVALUATE DIELECTRONIC RECOMBINATION "DALPHA" -LANDINI&FOSSI.
71.  C
72.      IF(YES(K).EQ.1)GO TO 14
73.  C

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74.  C   YES=1 MEANS ION IS H,HE,NE,K-NI LIKE ION (RECOMBINING)
75.  C
76.      GO TO 15
77.      14  DALPHA=1.6CE-10*((FLOAT(K))*2)*F(K)*SQRT(W(K))*EXP(-0.9134*
78.          :W(K)/TE(J))/SQRT(TE(J)**3)
79.  C
80.  C IN 14 F(K) IS NO OF ELECTRONS IN THE OUTER SHELL.K IS ION.EG.CV.
81.  C
82.      GO TO 13
83.  C
84.  C IF YES#1 RECCMBINING ION IS LI-F,NA-AR LIKE ION.F(K)IS GIVEN BY
85.  C LONDINI&FOSSI PAPER TABLE 2.
86.  C
87.      15  DALPHA=1.60E-10*((FLOAT(K))*2)*F(K)*EXP(-0.9134*(FLOAT(K)+1.3)*W
88.          :(K)/(3.3*TE(J)))/SQRT(TE(J)**3)
89.  C
90.  C   EVALUATE RADIATIVE RECOMBINATION "RALPHA"
91.  C
92.  C
93.      13  RALPHA=5.20E-14*(FLOAT(K)-1.0)*SQRT(XSE/TE(J))*(0.429+
94.          1  0.5 *ALCG(XSE/TE(J))+0.469*CBRT(TE(J)/XSE))
95.          ALPHA(K)=BALPHA+DALPHA+RALPHA
96.      GO TO 5
97.      9  ALPHA(K)=0.0
98.      5  XNPRIM=ANPRIM(K)
99.          WRITE(6,17)K,TE(J),BALPHA,DALPHA,RALPHA,ALPHA(K),S(K)
100.      17  FORMAT(1X,I2,6E10.4)
101.      25  XSE = SE(K)
102.  C
103.  C   TEST FOR NEGATIVE DENSITIES.
104.  C
105.      DO 50 K = NA,ION
106.          IF ( Y(J,K)) 51,50,50
107.      51          Y(J,K) = 0.0
108.      50 CONTINUE
109.  C
110.  C   INCREMENTS IN POPULATIONS FOR USE IN SOLVE..-PREDICTOR
111.  C   -CORRECTOR METHOD OF SOLVING.

```

```

112.   C
113.       IF (NA-1) 70,70,71
114.   70   YR(J,1)=-DNE(J)*(Y(J,1)*S(1)-Y(J,2)*ALPHA(2))+ABL*Y(J,1)
115.       NAA = 2
116.       GO TO 72
117.   71   NAA = NA
118.       ALPHA(NA)=C.0
119.       S(NA - 1) = 0.0
120.   72   DO 8 K = NAA,IONM1
121.       YR(J,K)=DNE(J)*(Y(J,K-1)*S(K-1)+Y(J,K+1)*ALPHA(K+1)
122.   1       -Y(J,K)*(S(K)+ALPHA(K))) + ABL*Y(J,K)
123.   8     CONTINUE
124.       K = ION
125.       YR(J,K)=DNE(J)*(Y(J,K-1)*S(K-1)-Y(J,K)*ALPHA(K)) + ABL*Y(J,K)
126.       RETURN
127.       END

```

SUM1

```

1.  C
2.  C THIS IS EVALUATION OF E.C.I.P IONIZATION RATE COEFFICIENT USING
3.  C SEMI-CLASSICAL THEORY OF A.BURGESS.
4.  C
5.  C THE FOLLOWING IS COMPUTER FORTRAN TRANSLATION DONE BY M.BLAHA
6.  C OF THE CALCLATION PROCEDURE DESCRIBED BY SUMMERS IN THE APPELTON
7.  C LABORATORY REPORT I.M.367 (1974) PAGES 586.
8.  C INCORPORATION OF THIS PROGRAM IN TO THE CODE FOR RAPIDLY IONIZING
9.  C PLASMAS WAS DONE BY RAJU DATLA.
10. C
11.     FUNCTION COEF(K,TT,I ,CHI1,J ,CHI2,L ,CHI7,
12.     1  M ,CHI4,NA,CHI5)
13.     DIMENSION X(10),OM(10),TEMP(10),N(5),CHI(5)
14. C
15. C "X"&"OM" ARE "X(K)"&"OMEGA(K)" IN PAGE 5.THEY ARE THE NODES AND
16. C WEIGHTS OF AN N-POINT GAUSS-LAGUERRE QUADRATURE.
17. C "N(I) & CHI(I)" ARE NO OF EQUIVALENT ELECTRONS IN "I" SHELL&
18. C IONIZATION ENERGY OF AN ELECTRON IN THAT SHELL.
19. C
20.     DATA X/C.137793,0.729455,1.808343,3.401434,5.552496,8.330153,
21.     1  11.843785,16.279258,21.996586,29.920697/, OM/0.308441,
22.     2  0.401120,0.218068,0.620875E-01,0.950152E-02,0.753008E-03,
23.     3  0.282592E-04,0.424931E-06,0.183956E-08,0.991183E-12/
24. C
25. C
26.     Z=K
27. C
28. C
29.     N(1)=I
30.     N(2)=J
31.     N(3)=L
32.     N(4)=M
33.     N(5)=NA
34.     CHI(1)=CHI1
35.     CHI(2)=CHI2

```

```

36.      CHI(3)=CHI3
37.      CHI(4)=CHI4
38.      CHI(5)=CHI5
39.      DO 7 I=1,5
40.      IF (N(I).EQ.0) GO TO 7
41.      CHI(I)=CHI(I)/13.606
42.      7  CONTINUE
43.      IT=1
44.      TEMP(1)=IT
45.      COEF=0.0
46.      DO 11 NE=1,5
47.      IF (N(NE).EQ.0) GO TO 11
48.      SUM=0.0
49.      AN=Z/SQRT(CHI(NE))
50.      C
51.      C  "AN" IS "NU" IN PAGE 6.
52.      C
53.      DO 12 K=1,10
54.      EPS=TEMP(IT)*X(K)/13.606+CHI(NE)
55.      C
56.      C  2EPS" IS SYMBOL EPSILON IN PAGE 5.
57.      C
58.      B=(EPS-CHI(NE))/CHI(NE)
59.      C1=B-(B+1)*ALOG(B+1)/(B+2)
60.      C2=1.0-1.0/(B+1)**3
61.      Q=C1/(B+2)+0.65343*C2*YP(Z,AN,B)/AN
62.      C
63.      C  "YP" IS THE FUNCTION Y(Delta) IN PAGE 6.THIS IS EVALUATED IN SUM2
64.      C  AS A SEPARATE FUNCTION.
65.      C
66.      Q=4*Q/((B+1)*CHI(NE)**2)
67.      SUM=SUM+EPS*Q*OM(K)
68.      12  CONTINUE
69.      A=13.606*CHI(NE)/TEMP(IT)
70.      COEF=COEF+N(NE)*SUM*EXP(-A)
71.      11  CONTINUE
72.      COEF=21.7203E-09*COEF*SQRT(13.606/TEMP(IT))
73.      C

```

74. C IN C.G.S $(2 * \text{ALPHA} * C / \text{SQRT}(\text{PI})) * \text{PI} * (A ** 2) = 21.7203E-09$

75. C

76. RETURN

77. END

SUM2

```

1.  C
2.  C   "YP" IS A FUNCTION REFERENCED IN COEF. SO
3.  C   E.C.I.P IONIZATION RATE COEFFICIENT EVALUATION CONTINUED.
4.  C
5.      FUNCTION YP(Z,AN,B)
6.      DIMENSION AA(20),BB(20)
7.  C
8.  C   THE FOLLOWING IS DATA FOR AA&BB WHICH ARE A(J) & B(J) IN PAGE 6
9.  C   AND TABLE 15.
10. C
11.      DATA AA/2.3916,1.6742,1.2583,0.9738,0.7656,0.6078,0.48561,0.38976,
12.      1  0.31388,0.25342,0.20501,0.16610,0.13476,0.10944,0.08896,
13.      2  0.07237,0.058903,.047971,0.039086,0.031860/,BB/1.0091,
14.      4  0.3015,0.1314,0.0763,0.0504,0.03561,0.02634,0.01997,0.01542,
15.      5  0.01205,0.950E-02,0.757E-02,0.602E-02,0.484E-02,0.389E-02,
16.      6  0.3123E-02,0.2535E-02,0.2047E-02,0.1659E-02,0.1344E-02/
17.      A=SQRT(B+1.0)
18.      D=(Z/AN)*((5*AN**2+1)/(4*Z)+2*AN**2*A/(Z*Z*(B+2)))/(A+SQRT(B))
19.  C
20. C   "D" IS "DELTA" IN PAGE 6.
21. C
22.      IF (D.LE.0.1) GO TO 1
23.      IF (D.GE.2.0) GO TO 2
24.      I=10*D
25.      R=10*D-I
26.      S=I+1-10*D
27.      YP=R*AA(I+1)+S*AA(I)+((R**3-R)*BB(I+1)+(S**3-S)*BB(I))/6.0
28.      RETURN
29.      1 A=ALOG(1.1229/D)
30.      YP=A+0.25*D*D*(1.0-2*A*A)
31.      RETURN
32.      2 YP=1.5707963*EXP(-2*D)*(1.0+0.25/D-0.09375/D**2+0.0703125/D**3
33.      1  -0.0472/D**4)
34.      RETURN
35.      END

```

```

1.      SUBROUTINE PRINT
2.      C
3.      C SUBROUTINE TO PRINT THE THEORETICAL INTENSITIES AND IONIC POPULATIONS
4.      C COMPUTED BY 'SOLVE'.
5.      C
6.      INTEGER TIM
7.      COMMON J,JMAX,ND,H,DTMAX,NDT,KSTEP,KSEND,END,ED,EH,EN,KDOB,
8.      1      KHAV1,KHAV2,KHAV3,DOB1,DOB2
9.      COMMON T(60),EPSI(60),Y(60,18),YR(60,18
10.     1      ),ANPRIM(18)
11.     COMMON S(18),ALPHA(18),DENS(100),TEMP(100),TIME(100),SA(18),SB(18)
12.     1      ,SE(18),NAME(8),KION(18),DNE(60),TE(60),SFN(18),SEX(18)
13.     COMMON  NTIME,ION,IONM1,NA,NAINI,OO1,YMIN,YYMIN
14.     COMMON  THI(60,18),TIM,FLAG,F(18),YES(18),W(18),M(5,18),HI(5,18)
15.     !      ,FLAG1,TTT(60),DIF2,A(18)
16.     DIMENSION CONC(18),XINT(18),ATOTAL(100)
17.     C
18.     IF(ION - 9) 1,1,2
19.     C
20.     1 WRITE(6,201) (KION(L),L=1,9)
21.     200 FORMAT(E12.4,9F11.4,F10.4,3X,I2)
22.     201 FORMAT(1F1,12H  TIME (SEC),4X,5H ION I2,4X,5H ION I2,4X,5H ION I2,
23.     1      4X,5H ION I2,4X,5H ION I2,4X,5H ION I2,4X,5H ION I2,4X,5H ION I
24.     22,4X,5H ION I2,4X,3HSUM,5X,2HNA///)
25.     301 FORMAT(1F1,12H  TIME (SEC),8X,5H INT I2,5X,5H INT I2,5X,5H INT I2,
26.     1      5X,5H INT I2,5X,5H INT I2,5X,5H INT I2,5X,5H INT I2,5X,5H INT I
27.     22,5X,5H INT I2///)
28.     401 FORMAT(1F1,12H  TIME (SEC),4X,5H ION I2,4X,5H ION I2,4X,5H ION I2,
29.     1      4X,5H ION I2,4X,5H ION I2,4X,5H ION I2,4X,5H ION I2,4X,5H ION I
30.     22,5X,5H ION I2///)
31.     DO 3 L = 4,J
32.     O1 = OO1/DNE(L)
33.     TOTAL = C.O
34.     DO 6 K=1,9
35.     CONC(K) = O1*Y(L,K)

```

```

36.      6 TOTAL = TOTAL + CONC(K)
37.      3 WRITE(6,200) T(L),(CONC(K),K=1,9),TOTAL,NA
38.      NC = 1
39.      NCM = 10N
40.      NCN = 9
41.      GO TO 95C
42.      2 WRITE(6,401) (KION(K),K=1,9)
43.      DO 4 L = 4,J
44.      O1 = 001/DNE(L)
45.      TOTAL = C.C
46.      DO 7 K=1,9
47.      CONC(K) = O1*Y(L,K)
48.      7 TOTAL = TOTAL + CONC(K)
49.      ATOTAL(L) = TOTAL
50.      4 WRITE(6,200) T(L),(CONC(K),K=1,9)
51.      WRITE(6,201) (KION(K),K=10,18)
52.      DO 5 L = 4,J
53.      O1 = 001/DNE(L)
54.      BTOTAL = ATOTAL(L)
55.      DO 8 K=10,18
56.      CONC(K) = O1*Y(L,K)
57.      8 BTOTAL = BTOTAL + CONC(K)
58.      5 WRITE(6,200) T(L),(CONC(K),K=10,18),BTOTAL,NA
59.      NC = 1
60.      NCM = 9
61.      NCN = 9
62.      950 DO 603 K=NC,NCM
63.      IF(SFN(K)) 603,603,601
64.      603 CONTINUE
65.      GO TO 602
66.      601 WRITE(6,301) (KION(L),L=NC,NCM)
67.      L=4
68.      302 IF(L.GT.J)GO TO 602
69.      DO 303 K=NC,NCM
70.      IF(SFN(K)) 304,304,305
71.      304 XINT(K) = C.0
72.      GO TO 303
73.      305 V = SEX(K)/TE(L)

```

```

74.          IF(V=0.07) 306,306,307
75.      306 PP = 0.325*EI(V)
76.          GO TO 308
77.      307 PP = 0.2+0.035/V
78.      308 XINT(K) = 2.01E-25*Y(L,K)*DNE(L)*SFN(K)*EXP(-V)*PP/SQRT(TE(L))
79.      303 CONTINUE
80.  C
81.  C IF THE FINE TIME STEP 'T(L)' IS EQUAL TO ONE OF THE COARSE TIME
82.  C STEP THEN REMEMBER IT IN 'THI' FOR LATER PLOTTING IN THE MAIN PROGRAM.
83.  C
84.          IF(TIME(TIM).GT.T(L))GO TO 98
85.      91 DIF1=ABS(TIME(TIM)-T(L))
86.          T1=T(L)
87.          GO TO 96
88.      98 IF (TIM.LT.NTIME)GO TO 99
89.          IF (L.EQ.J)GO TO 91
90.      99 DIF2=ABS(TIME(TIM)-T(L))
91.          DO 100 K=1,ION
92.      100 THI(TIM,K)=XINT(K)
93.          T2=T(L)
94.          WRITE(6,309) T(L),(XINT(K),K=NC,NCM)
95.      102 L=L+1
96.          GO TO 302
97.      96 IF (DIF1.GT.DIF2) GO TO 90
98.          DO 92 K=1,ION
99.      92 THI(TIM,K)=XINT(K)
100.         TTT(TIM)=T1
101.      94 TIM=TIM+1
102.         IF(TIM.GT.NTIME)GO TO 102
103.         GO TO 302
104.      90 TTT(TIM)=T2
105.         GO TO 94
106.      309 FORMAT(E12.4,5X,9E12.4)
107.      602 IF(ION-NCN)550,550,350
108.      350 NC = 10
109.          NCM = ION
110.          NCN = 18
111.          GO TO 950

```

```

112.  C
113.  C AS TIME GOES BY THE LOWER IONS WILL HAVE NO POPULATIONS
114.  C SO THOSE IONS ARE NOT PURSUED ANY MORE IN SOLVE. (#550TO#82 MAKE THIS H/
115.  C
116.  550 JA = J - 2
117.      NAB = NA
118.      DO 59 K = 1,ION
119.      KK=K
120.      IF(Y(JA,K) - YMIN) 59,60,60
121.  59 CONTINUE
122.  60 NA = KK
123.      IF(NAB - NA) 78,82,78
124.  78 NAM = NA - 1
125.      IF(NAM) 82,82,81
126.  81 DO 69 JJ = 1,J
127.      DO 79 KK = 1,NAM
128.      Y(JJ,KK) = 1.0
129.      YR(JJ,KK) = 0.0
130.  79 CONTINUE
131.      EPSI(JJ) = 0.0
132.  69 CONTINUE
133.  82 RETURN
134.      END

```

```
1.      FUNCTION EI(X)
2.      C      COMPUTES VALUE OF -EI(-X) WHERE X IS +VE AND LESS THAN 0.1
3.      Y=X-ALOG(1.781072*X)
4.      TERM=X
5.      DO 1 I=2,4
6.      A=FLOAT(I)
7.      TERM=TERM*(-X)*(A-1.0)/(A*A)
8.      1 Y=Y+TERM
9.      EI=Y
10.     RETURN
11.     END
```

PLOT

```

1.      SUBROUTINE PLOTT(INTE,PFLAG,PFLAG1)
2.      C
3.      C "PLOTT" NORMALIZES DATA FOR INTENSITIES AND STORES.SO TELETYPE
4.      C PLOTTER ROUTINES COULD BE USED FOR PLOTTING AT THE TELETYPE.
5.      C
6.      INTEGER FFLAG,PFLAG1
7.      C
8.      C PFLAG &PFLAG1 ARE COUNTERS TO KNOW WHAT IS PLOTTED
9.      C
10.     REAL MAX,INTE
11.     COMMON J,JMAX,ND,H,DTMAX,NDT,KSTEP,KSEND,END,ED,EH,EN,KDOB,
12.     1      KHAV1,KHAV2,KHAV3,DOB1,DOB2
13.     COMMON T(60),EPSI(60),Y(60,18),YR(60,18
14.     1      ),ANPRIM(18)
15.     COMMON S(18),ALPHA(18),DENS(100),TEMP(100),TIME(100),SA(18),SB(18)
16.     1      ,SE(18),NAME(8),KION(18),DNE(60),TE(60),SFN(18),SEX(18)
17.     COMMON NTIME,ION,IONM1,NA,NAINI,OO1,YMIN,YYMIN
18.     COMMON THI(60,18),TIM,FLAG,F(18),YES(18),W(18),M(5,18),HI(5,18)
19.     ! ,FLAG1,TTT(60),DIF2,A(18)
20.     DIMENSION INTE(60,18)
21.     C
22.     C
23.     C
24.     NT=NTIME-2
25.     DO 13 K = 1,ION
26.     C FIRST FIND INENSITY MAXIMUM "MAX" FOR THE ION AND USE IT FOR
27.     C NORMALIZATION
28.     MAX=0.0
29.     DO 11 I = 1,NT
30.     11 IF(INTE(I,K).GT.MAX)MAX=INTE(I,K)
31.     C IF THE MAX IS 0.0 SKIP THAT ION FOR PLOTTING.
32.     IF(ABS(MAX).LT.1E-10) GO TO 13
33.     DO 12 I=1,NT
34.     INTE(I,K)=INTE(I,K)/MAX
35.     12 IF(ABS(INTE(I,K)).LT.1E-02)INTE(I,K)=0.0

```

```
36.      13 CONTINUE
37.          WRITE(7)ION,NT,((TTT(I),INTE(I,K),I=1,NT),K=1,ION),PFLAG,
38.          1(KION(L),SA(L),SE(L),A(L),L=1,ION),PFLAG1
39.      7 RETURN
40.      END
```

V. ACCURACY CHECK

A simple system of two ionic stages is considered. The density and temperature are kept constant. Recombination rates are neglected. The coupled equations are

$$\frac{dN_1}{dt} = -N_1 NI_1 \quad (11)$$

$$\frac{dN_2}{dt} = N_1 NI_1 - N_2 NI_2 \quad (12)$$

The solutions are

$$\frac{N_1}{N_0} = e^{-NI_1 t} \quad ; \quad \frac{N_2}{N_0} = \frac{I_1}{I_1 - I_2} \left[e^{-NI_2 t} - e^{-NI_1 t} \right]$$

The neutral and singly ionized boron in a plasma of constant density ($N = 5 \times 10^{14} / \text{cm}^3$) and temperature ($T = 250 \text{ eV}$) are considered.

The ionization rates for these ions are taken to be

$$I_1 = .7239 \times 10^{-9} \text{ cm}^3/\text{sec} \text{ and } I_2 = .2344 \times 10^{-7} \text{ cm}^3/\text{sec}.$$

A comparison of the results from the analytical solutions given above and the numerical solutions obtained with the code, is shown in Table 1. The maximum difference is less than one percent.

(PTG)

Time (μsec)	N_1/N_0		N_2/N_0	
	Analytical	Code	Analytical	Code
.025	.4045	.4046	.4986	.5049
.05	.1637	.1639	.5735	.5807
.075	.0662	.0664	.5095	.5158
.1	.0268	.0269	.4131	.4183
.125	.0108	.0109	.3216	.3256
.15	.0044	.0044	.2453	.2484

Table 1. Comparison between analytical and numerical solutions.

APPENDIX

1 > Plotting on the teletype:

The program TVPLOT together with the subroutines ¹³ TTPLT1, TTPLTZ and TTPLT3 are listed below. The plotting is done on the teletype and only the time histories of the emission lines from the last four ionization stages could be plotted. Examples of the plots are given in Appendix II.

TVPLOT

```

1.      INTEGER FFLAG,PFLAG1
2.      REAL INTE,Y1,Y2,Y3,Y4
3.      DIMENSION INTE(60,18),TIMEX(60),Y1(60),Y2(60),Y3(60),Y4(60),
4.      *KION(18),SA(18),SE(18),A(18)
5.      READ(7)ICN,NT,((TIMEX(I),INTE(I,K),I=1,NT),K=1,ION),PFLAG,
6.      1(KION(L),SA(L),SE(L),A(L),L=1,ION),PFLAG1
7.      DO 6 I=1,NT
8.      K=ION
9.      Y1(I)=INTE(I,K)
10.     K=K-1
11.     Y2(I)=INTE(I,K)
12.     K=K-1
13.     Y3(I)=INTE(I,K)
14.     K=K-1
15.     Y4(I)=INTE(I,K)
16.     6 CONTINUE
17.     CALL TTPLOT(TIMEX,NT,30,50,Y1,Y2,Y3,Y4)
18.     C
19.     C IF 'PFLAG' IS SET TO '1' THEN THE TITLE*
20.     C 'EXPERIMENTAL TIME EVOLUTION OF IONS IN A PLASMA'
21.     C SHOULD BE WRITTEN
22.     C
23.     IF(PFLAG.GT.1)GO TO 100
24.     WRITE(6,50)
25.     50 FORMAT(2X,11X,'EXPERIMENTAL TIME HISTORIES OF EMISSION LINES FROM
26.     1SUCCESSIVE IONIZATION STAGES IN A PLASMA ')
27.     GO TO 50(
28.     C
29.     C IF 'PFLAG' IS SET TO '2' THEN THE TITLE:
30.     C 'THEORETICAL TIME EVOLUTION OF IONS IN A PLASMA'
31.     C SHOULD BE WRITTEN
32.     C
33.     100 IF (PFLAG .NE. 2) GOTO 500
34.     IF (PFLAG1.EQ.1)GO TO 300
35.     WRITE(6,200)

```

```

36.      200 FORMAT(/12X,'PREDICTED TIME HISTORIES OF EMISSION LINES '/12X,
37.          1'LOTZ&KUNZE IONIZATION RATE COEFFICIENT WAS USED '/15X,4H ION,5X
38.          2,2H A,10X,7H E (EV))
39.          WRITE(6,210)(KION(L),SA(L),SE(L),L=1,ION)
40.      210 FORMAT(15X,I3,1X,E12.4,1X,F12.4)
41.          GO TO 500
42.      300 WRITE(6,310)
43.      310 FORMAT(2X, //2X,'PREDICTED TIME HISTORIES OF EMISSION LINES '/2X,
44.          1'E C I P IONIZATION RATE COEFFICIENT (BURGESS&SUMMERS) WAS USED
45.          2'/15X,4H ION,5X,2H A,10X,7H E (EV))
46.          WRITE(6,320)(KION(L),A(L),SE(L),L=1,ION)
47.      320 FORMAT(15X,I3,1X,F10.4,1X,F12.4)
48.      500  END

```

TTPLT1

```

1.  C
2.  C THIS SUBROUTINE SCALES AND DRAWS PRINTER PLOTS.
3.  C THE SCALING ASSUMES A TELETYPE WIDTH OF 60 LINES FOR THE GRAPH
4.  C AND 10 LINES FOR THE AXIS LABELS--DIFFERENT VALUES CAN BE
5.  C CONJURED UP IF DESIRED.
6.  C
7.  C-----SUROUTINE BY B.K.REID NOV.1970-----
8.  C
9.      SUBROUTINE RPLOTR(X,NPTS,LENGTH,WIDTH,NY)
10.     DIMENSION YY(1500),X(NPTS),AX(2),LINE(63),MARK(4),NMARK(4)
11.     DIMENSION YMIN(5),YMAX(5),XX(35),YLB(10)
12.     INTEGER WIDTH,WID,H
13.     WID=MIN(WIDTH,60)
14.     NAX=WID//10
15.     WID=10*NAX
16.     DATA MARK/'-','+','*','=','/
17.     DATA NMARK/'1','2','3','4'/
18.     DATA AX/'-----','----+'/
19.  C
20.  C ROUND THE WIDTH UP TO A MULTIPLE OF 10 ROWS.
21.  C
22.     NAXL=(LENGTH-1)//5
23.     LEN=1+5*NAXL
24.  C
25.  C ROUND THE ELEMENT UP TO A MULTIPLE OF 5 COLUMNS
26.  C
27.     (I=1,1,I.GT.5,YMIN(I)=1E37,YMAX(I)=-1E37)
28.     DO 10 I=1,NPTS
29.  C
30.  C WE SCAN THROUGH THE DATA GETTING MINIMUM AND MAXIMUM VALUES
31.  C
32.     YMAX(1)=MAX(YMAX(1),X(I))
33.     YMIN(1)=MIN(YMIN(1),X(I))
34.     DO 10 J=1,NY
35.     YVAL=YQY(I,J)

```

```

36.      YMAX(J+1)=MAX(YMAX(J+1),YVAL)
37.      YMIN(J+1)=MIN(YMIN(J+1),YVAL)
38.      10  CONTINUE
39.      NS=NY+1
40.      DO 101 L=2,NS
41.      CALL TSCALE(YMIN(L),YMAX(L),YLB,NAX)
42.      PRINT 899,MARK(L-1),(YLB(H),H=1,NAX+1)
43.      899  FORMAT(1X,A1,8X,6G10.4)
44.      C
45.      C CONVERT MAXIMUM TO SPAN
46.      C
47.      101  CONTINUE
48.      CALL TSCALE(YMIN(1),YMAX(1),XX,NAXL)
49.      DO 100 L=1,NS
50.      YMAX(L)=YMAX(L)-YMIN(L)
51.      IF(YMAX(L).EQ.0) YMAX(L)=1
52.      100  CONTINUE
53.      YY(1)=0 @ INITIAL LINK IN COLUMN ZERO.
54.      IXY=1 @ INITIAL STORE POINTER
55.      C
56.      C THE DATA HAS BEEN SCALED. WE NOW WIFFLE THROUGH IT A SECOND
57.      C TIME AND SET UP THE PLOT ARRAY
58.      C
59.      C
60.      DO 1001 IXA=1,NPTS
61.      XV=X(IXA)
62.      MY=1+LEN*(XV-YMIN(1))/YMAX(1)
63.      DO 1001 L=1,NY
64.      M=WID*(YGY(IXA,L)-YMIN(L+1))/YMAX(L+1)
65.      C
66.      C WE ARE GOING TO SEARCH THE PLOTTED-POINT LIST FOR A PLACE TO
67.      C PUT THIS ONE. THE DATA FORMAT IS:
68.      C
69.      C .....
70.      C . . . . .
71.      C . SYMBOL . COLUMN . ROW (X-AXIS) NO. . FORWARD LINK BY ROW .
72.      C . . . . .
73.      C .....

```

```

74.  C
75.  C
76.      JX=1
77.      LMMY=MY+(M.LS.12)+(L.LS.18)
78.  1002  CONTINUE
79.      KX=FLD(24,12,YY(JX)) @ FORWARD LINK
80.      IF(KX.EQ.0) GO TO 1003
81.      IF(LMMY.EQ.FLD(0,24,YY(KX))) GO TO 1001
82.      IF(FLD(12,12,YY(KX)).GT.MY) GO TO 1003
83.      JX=KX
84.      GO TO 1002
85.  1003  CONTINUE
86.  C
87.  C WE HAVE FOUND THE CORRECT SPOT IN THE CHAIN FOR THIS
88.  C ITEM--OPEN THE LINK AND INSERT IT.
89.  C
90.      IXY=IXY+1
91.      FLD(0,24,YY(IXY))=LMMY
92.      FLD(24,12,YY(IXY))=KX
93.      FLD(24,12,YY(JX))=IXY
94.  1001  CONTINUE
95.      WID=WID+1
96.      CALL MIMTPL(YY,IXY,XX,LEN)
97.      RETURN
98.      SUBROUTINE MIMTPL(Y,NXY,X,NX)
99.  C
100.  C THIS SUBROUTINE DRAWS A PLOT ON A TELETYPE FOR MIMIC
101.  C
102.  C Y IS THE ARRAY OF VALUES, INTEGER, RANGE 1-WID
103.  C MY IS THE NUMBER OF Y VALUES
104.  C X IS THE ARRAY OF X VALUES
105.  C NX IS THE NUMBER OF X VALUES, I.E. THE GRAPH LENGTH.
106.  C
107.  C
108.      DIMENSION X(NX)
109.      INTEGER Y
110.      REAL X
111.      DIMENSION Y(1500)

```

```

112.      PARAMETER NMOD=5
113.      PRINT 8100,(AX,H=1,NAX)
114. 8100  FORMAT(16X,1H+,6(A6,A4))
115.      IX=FLD(24,12,Y(1)) @ LINK TO SMALLEST FIRST
116.      DO 10 I=1,NX
117.      IXA=I//5
118.      DO 15 H=1,WID
119.      C
120.      C CLEAR WORKING BUFFER TO ZERO FOR EACH LINE
121.      C
122.      15  LINE(H)=C
123. 809    CONTINUE
124.      KK=FLD(6,6,Y(IX))+1
125.      JJ=FLD(6,6,Y(IX))
126.      MY=FLD(12,12,Y(IX))
127.      IF(MY.NE.I) GO TO 20
128.      IF(LINE(KK)) 50,,60
129.      C
130.      C IF THIS IS THE FIRST POINT HERE, JUST STORE IT
131.      C
132.      LINE(KK)=JJ
133.      GO TO 21
134. 60     CONTINUE
135.      LINE(KK)=-2
136.      C
137.      C IF THERE IS ANOTHER POINT HERE, RECORD THAT
138.      C
139.      GO TO 21
140. 50     CONTINUE
141.      C
142.      C AT LEAST 2 OTHER POINTS WERE ON THAT SPOT
143.      C
144.      LINE(KK)=LINE(KK)-1
145. 21     CONTINUE
146.      IX=FLD(24,12,Y(IX))
147.      GO TO 809
148. 20     CONTINUE
149.      DO 40 J=1,WID

```

```
150.      L=LINE(J)
151.      M=ABS(L)
152.      IF(L.GT.0) LINE(J)=MARK(L)
153.      IF(L.LT.0) LINE(J)=NMARK(M)
154.      IF(L.EQ.0) LINE(J)=' '
155.  40    CONTINUE
156.      IF(MOD(I+4,NMOD).EQ.0) PRINT 8900,X(IXA),(LINE(H),H=1,WID)
157.      IF(MOD(I+4,NMOD).NE.0) PRINT 8901,(LINE(H),H=1,WID)
158.  10    CONTINUE
159.      PRINT 8100,(AX,H=1,NAX)
160.      RETURN
161.  8900  FORMAT(G14.4,2H I,61A1)
162.  8901  FORMAT(15X,1H1,61A1)
163.      END
164.      END
```


36.	TTPLOT*	L,U	AO,4,X11	. COUNT THE ARGUMENTS
37.		LXI,L	AO,1	. ONE AT A TIME
38.		TZ,H1	O,AO	. IF IT'S NOT AN ARGUMENT
39.		TP,X+1	O,AO	. OR AN ALTERNATE RETURN
40.		JMGI	AO,\$-2	. THEN STOP. ELSE GO.
41.		AH	AO,(-1,1)	. COMPUTE RETURN ADDRESS
42.		S,H2	AO,RTN	. TO GO HOME AGAIN
43.		AN,U	AO,5,X11	. COMPUTE # OF ARGS
44.		S	AO,NY	. SAVE FOR A RAINY DAY
45.		LXI,L	X11,4,AO	. LOAD INCREMENT FOR WBACK VALUE
46.		S	X11,WB+1	. SAVE INDEX 11 FOR 'NEXT' CALL
47.		DL	AO,0,X11	. GET X AND NX
48.		DS	AO,CALL	
49.		DL	AO,2,X11	. GET LENGTH AND WIDTH
50.		DS	AO,CALL+2	.
51.		LMJ	X11,RPLOTR	. CALL FORTRAN DRIVE PROG
52.	CALL	RES	4	. SPACE FOR X,NPTS,LENGTH,WIDTH
53.		+	NY	. NUMBER OF Y VALUES
54.		+	33,WB	. WALKBACK
55.	RTN	J	O	. RETURN WORD
56.	WB	'TTPLOT'		
57.		RES	1.	
58.	NY	RES	1	.
59.	.			
60.	.			
61.	.			
62.	.	FUNCTION TO RETURN N TH OBSERVATION OF K TH Y		
63.	.			
64.	.			
65.	.	CALLING SEQUENCE: X=YQY(N,K)		
66.	.			
67.	.	THE ALTERNATE RETURN IS TAKEN IF THERE IS NO SUCH VALUE		
68.	.			
69.	.			
70.	YQY*	L	AO,*1,X11	. K
71.		L	A3,WB+1	. RESTORE CALLING INDEX
72.		A,U	A3,3,AO	. FIND THE ARGUMENT
73.		L,H2	A1,,A3	. A1 HAS ARGUMENT BASE ADDR

74.	A	A1,*0,X11	. FIND THE N+1 TH ENTRY
75.	AN,U	A1,1	. AND TEMPER IT DOWN TO N
76.	L	A0,,A1	. GET THE VALUE OF IT
77.	J	3,X11.	
78.	END		

TTPLT3

```

1.      SUBROUTINE TSCALE(VMIN,VMAX,VSCA,NAXIS)
2.      DIMENSION VSCA(10)
3.      LOGICAL NEG
4.      PARAMETER NSCF=12
5.      C
6.      C
7.      C SUBROUTINE TO SCALE A PLOT.
8.      C
9.      C THE INPUT VARIABLES
10.     C VMIN AND VMAX CONTAIN THE OBSERVED VALUES OF MINIMUM/MAXIMUM
11.     C VALUES OF THE VARIABLE TO BE SCALED.  ON OUTPUT, THEY WILL
12.     C CONTAIN THE ADJUSTED VALUES TO BE USED FOR 'NICE' NUMBERS
13.     C ALONG THE AXIS OF THE PLOT.
14.     C
15.     C VSCA IS THE ARRAY WHICH WILL CONTAIN AXIS LABELS.  NAXIS IS
16.     C THE NUMBER OF AXIS BLOCKS, SO THAT THERE WILL BE 'NAXIS+1'
17.     C ENTRIES MADE IN VSCA.
18.     C
19.     DIMENSION XSCF(NSCF)/0.,1.,1.25,1.5,2.,3.,4.,5.,6.,7.5,8.,10./
20.     NLOOP=0  & LOOP PREVENTER
21.     D      OMIN=VMIN
22.     D      OMAX=VMAX
23.     650    SCF=(VMAX-VMIN)/NAXIS
24.     *24.    DX1=0.
25.     ZERO=VMIN
26.     IF(SCF.EG.0) GO TO 695
27.     XNC=ALOG10(SCF)
28.     INC=XNC
29.     IF(XNC.LE.0.) INC=INC-1
30.     IF(INC.GT.0) XNC10=10.**INC
31.     IF(INC.LE.0) XNC10=- (10.**ABS(INC))
32.     DX=PSCALE(SCF,-XNC10)
33.     DO 651 ISC=1,NSCF-1
34.     651   IF(DX.GT.XSCF(ISC)) DX1=XSCF(ISC+1)
35.     C

```

```

36. C A PROBABLE SCALE FACTOR HAS BEEN COMPUTED, AND IT IS 'DX1'
37. C WE NOW MUST ASSIGN A 'ZERO' TO THE SCALE, AND SEE IF THE
38. C ASSIGNMENT OF A ROUNDED ZERO IS GOING TO CHANGE THE SCALE
39. C FACTOR.
40. C
41.     IF(VMIN.EQ.0) GO TO 675
42.     ABMIN=10(C.C*PSCALE(VMIN,-XNC10)
43. C
44. C SCALE THE ZERO TO THE SAME ORDER OF MAGNITUDE AS THE
45. C INCREMENT, BUT MULTIPLY BY 100 SO THAT .75 AND .25 AND SO
46. C ON MAY BE KEPT IN INTEGER CONVERSION.
47. C
48.     JBMIN=ABS(ABMIN)
49.     NEG=ABMIN.LT.C
50.     LAST3=MOD(JBMIN,1000) @ GET LAST 3 DIGITS
51.     IF(LAST3.LT.0)LAST3=LAST3+1000
52.     IDELT=DX1*100.0 @ GET 3-DIGIT INCREMENT
53.     DO 652 ISC=0,8
54.     LTEST=IDELT*ISC
55.     IF(LTEST.GT.LAST3) GO TO 653
56. C
57. C BACK OFF ON THE LAST THREE DIGITS UNTIL WE COME TO THE
58. C FIRST SPOT WHICH IS A MULTIPLE OF THE SCALE FACTOR, INCLUDING 0.
59. C
60. 652 CONTINUE
61. 653 CONTINUE
62.     IF(.NOT.NEG) LTEST=LTEST-IDELT
63. C
64. C USE THE LARGEST PREVIOUS VALUE FOR POSITIVE SCALING, BUT
65. C USE THIS VALUE FOR NEGATIVE SCALING.
66. C
67.     JBMIN=JBMIN-LAST3
68.     ABMIN=(JBMIN+LTEST)/100.
69.     ZERO=PSCALE(ABMIN,XNC10)
70.     VMIN=SIGN(ZERO,VMIN)
71. 675 CONTINUE
72.     DX=DX1
73.     DX1=NAXIS*DX1

```

```

74.          DX1=PSCALE(DX1,XNC10)
75.          SPAN=VMIN+DX1
76.          C
77.          C THE SHENANEGANS ABOVE ARE INSERTED TO PREVENT A FLOATING-PT
78.          C TRUNCATION IN CASES WHERE THE SCALE FACTOR IS LESS THAN ONE
79.          C AND NOT EXACTLY REPRESENTABLE IN FLOATING POINT.  FOR EXAMPLE,
80.          C IF THE EVENTUAL INTERVAL IS .1, THEN MULTIPLYING NAXIS BY
81.          C 0.1 WILL NOT GIVE THE SAME ANSWER AS MULTIPLYING NAXIS BY 1
82.          C AND THEN DIVIDING BY 10.  THIS PRECISION IS IMPORTANT.
83.          C
84.          IF(SPAN.GE.VMAX) GO TO 649
85.          NLOOP=NLOOP+1
86.          IF(NLOOP.LE.1) GO TO 650
87.          PRINT 124
88.          124  FORMAT(' ERROR IN PLOT SCALING ROUTINE:')
89.          D123  FORMAT(1X,A6,1X,G12.5,013)
90.          D125  FORMAT(1X,A6,1X,I12)
91.          D     PRINT 123,'SPAN',SPAN,SPAN
92.          D     PRINT 123,'VMAX',OMAX,OMAX
93.          D     PRINT 123,'VMIN',OMIN,OMIN
94.          D     PRINT 123,'ZERO',ZERO,ZERO
95.          D     PRINT 123,'XNC10',XNC10,XNC10
96.          D     PRINT 125,'LAST3',LAST3
97.          D     PRINT 125,'IDELT',IDELT
98.          D     PRINT 125,'LTEST',LTEST
99.          D     PRINT 125,'JBMIN',JBMIN
100.         D     PRINT 123,'DX1',DX1,DX1
101.         649  CONTINUE
102.         VMAX=SPAN
103.         695  CONTINUE
104.         DX=PSCALE(DX,XNC10)
105.         DO 655 IH=1,NAXIS+1
106.         VSCA(IH)=VMIN+(IH-1)*DX
107.         655  CONTINUE
108.         RETURN
109.         C
110.         C
111.         C FUNCTION TO SCALE BY EXPONENTIAL SCALE FACTOR

```

112. C
113. FUNCTION PSCALE(MANTIS,CHARAC)
114. REAL MANTIS
115. IF(CHARAC.GT.0) PSCALE=MANTIS*CHARAC
116. IF(CHARAC.LT.0) PSCALE=MANTIS/ABS(CHARAC)
117. RETURN
118. END
119. END

APPENDIX II

Determination of ionization rate coefficients:

One possible use of this code as mentioned earlier, is to determine ionization rate coefficients. This involves changing the ionization rate coefficients in the coupled rate equations until the predicted time histories match with the experimentally observed time histories. What is thus measured is therefore an effective ionization rate, i.e. a sum of the ionization rates from the ground state, including inner-shell ionization, and all the populated excited states. A detailed description of this method was given in Ref. 2. Another useful result in matching the time histories is that the percentage composition of the various ionization stages at any instant of time would also be predicted.

In using the procedure for the determination of ionization rates the following points are to be carefully considered.

1.) As we are theoretically predicting the time histories we should carefully evaluate how critically all these predictions depend on initial conditions i.e., degree of ionization, electron density and temperature at early times. As we know, it is very difficult to be accurate about such initial conditions. So the earlier ionization stages may not be considered for measurements. The ionization rate coefficients of these ions could be adjusted until the decay of an emission line from at least one ion prior to the ion under consideration would match with the experiment. This should also make the early part of the emission from the ion under consideration match with the experiment. Thus the initial conditions could be fixed for the ions

under consideration.

2.) The electron density N and the temperature T should be spatially non-varying so that ionization proceeds uniformly in the plasma under observation and the modelling of the plasma in Eq. (3) would be perfect.

3.) Also, the electron density and temperature at the time of occurrence of the ions under consideration should ideally be non-varying in time for a good determination of ionization rates.¹ However, some variations of N and T can be accounted for. As was discussed in Ref. 2, the first three terms in Eq. (7), Ref. 2 should not dominate the time history.

4.) It is important to select and obtain the time histories of emission lines connecting to the ground state which are not self-absorbed.

As an example, the determination of the ionization rate coefficient for C V (He-like carbon) is considered from an experiment on the 15 KJ θ -pinch.⁵ The density and temperature profiles for this plasma condition obtained by the laser scattering technique are given in Fig. 1. Experimentally observed time histories of emission lines from C IV, C V, and C VI are given in Fig. 2. The predicted time histories that are matched to the experimental ones are given in Fig. 3 and Fig. 4 using Lotz and Kunze ionization rate coefficients and Burgess and Summers ionization rate coefficients, respectively, which yielded the same results. The effective ionization rate coefficient thus determined was $0.8 \times 10^{-10} \text{ cm}^3/\text{sec}$ at an electron tempera-

ture of 200 ev and a density of $4.95 \times 10^{15} / \text{cm}^3$. The corresponding estimate for the theoretical ionization rate, both by semiempirical and semiclassical methods after including the effect of the metastable state ($1s2p \ ^3S$) ionization agrees with the experiment. The matching of time histories was sensitive enough to show a significant difference if the C V ionization rate is varied by $\pm 25\%$.

DATA INPUT TO THE COMPUTER*

CARBON, 11 MTØRR

6	37	1	2	1			
	1	0.53E-08		3.00E 00	1.13E 01	5.00E 07	
	2	0.20E-09		3.00E 00	2.44E 01	1.00E 00	
	3	0.75E-09		3.00E 00	4.79E 01	1.00E 00	
	4	0.13E-09		3.00E 00	6.45E 01	1.00E 00	
	5	5.00E-10		3.00E 00	3.92E 02	1.00E 00	
	6	2.04E-10		3.00E 00	4.90E 02	1.00E 00	
50	6	2	10000	1.0E+00	1.0E-03	1.0E-01	1.0E+00 2.5E-03

DATA
for Lotz &
Kunze Ioniza-
tion rates.

-0.10E-06	2.50E 01	0.00E 00
0.00E 00	2.50E 01	0.50E 15
0.10E-06	3.50E 01	1.00E 15
0.20E-06	3.50E 01	1.50E 15
0.30E-06	3.50E 01	1.50E 15
0.40E-06	3.50E 01	1.90E 15
0.50E-06	0.90E 02	1.90E 15
0.60E-06	1.05E 02	1.90E 15
0.70E-06	1.16E 02	1.90E 15
0.80E-06	1.23E 02	2.00E 15
0.90E-06	1.32E 02	2.50E 15
1.00E-06	1.40E 02	3.00E 15
1.10E-06	1.62E 02	3.50E 15
1.20E-06	1.88E 02	4.00E 15
1.30E-06	2.20E 02	4.20E 15
1.40E-06	2.20E 02	4.25E 15
1.50E-06	2.20E 02	4.35E 15
1.60E-06	2.20E 02	4.35E 15
1.70E-06	2.20E 02	4.35E 15
1.80E-06	2.20E 02	4.35E 15
1.90E-06	2.19E 02	4.35E 15
2.00E-06	2.18E 02	4.30E 15
2.10E-06	2.17E 02	4.25E 15
2.20E-06	2.15E 02	4.25E 15
2.30E-06	2.14E 02	4.20E 15
2.40E-06	2.13E 02	4.15E 15
2.50E-06	2.11E 02	4.10E 15
2.60E-06	2.06E 02	4.05E 15
2.70E-06	2.00E 02	4.00E 15
2.80E-06	1.85E 02	3.95E 15
2.90E-06	1.82E 02	3.80E 15
3.00E-06	1.60E 02	3.65E 15
3.10E-06	1.46E 02	3.50E 15
3.20E-06	1.32E 02	3.35E 15
3.30E-06	1.19E 02	3.05E 15
3.40E-06	1.06E 02	2.70E 15
3.50E-06	0.96E 02	2.30E 15

Experimentally obtained
electron temperature and
electron density profiles

0	0
0	0

DATA for excitation

0.2	39.68
0.65	307.89
1.0	367.5

10	9.3	
10	12.7	
3	8.0	
1	2	307.9
1	1	367.0

DATA FOR
dielectronic
recombination

.400 E+00 2.8298 E+01 2.1293 E+02

(P.T.O)

* * For details, see-read statements in MAIN-R/W.

.400 E+00 2.6293 E+01 2.1293 E-02
 .140 E+00 1.24376E+02 2.31397E+02
 .400 E+00 2.47364E+02
 .020 E+01 1.64476E+02
 .150 E+01 2.39199E+03
 .180 E+01 1.4398 E+03

DATA for Burgess and Summers
 (E.C.I.P.) ionization rates.

01
 05
 11
 19
 27
 34 05 00
 37 095
 39 14
 40 17
 39 245 15
 37 50 2
 32 325 3
 23 35 10
 23 35 145
 19 39 19
 15 40 23
 12 40 265
 10 39 30
 6 33 33
 6 37 35
 5 35 36
 4 33 38
 3 30 395
 2 265 40
 1 235 40
 1 20 40
 13 375
 15 345
 13 315
 11 27
 9 23

Digitized data for the
 experimental time his-
 tories of emission lines
 from C IV, C V and C VI.

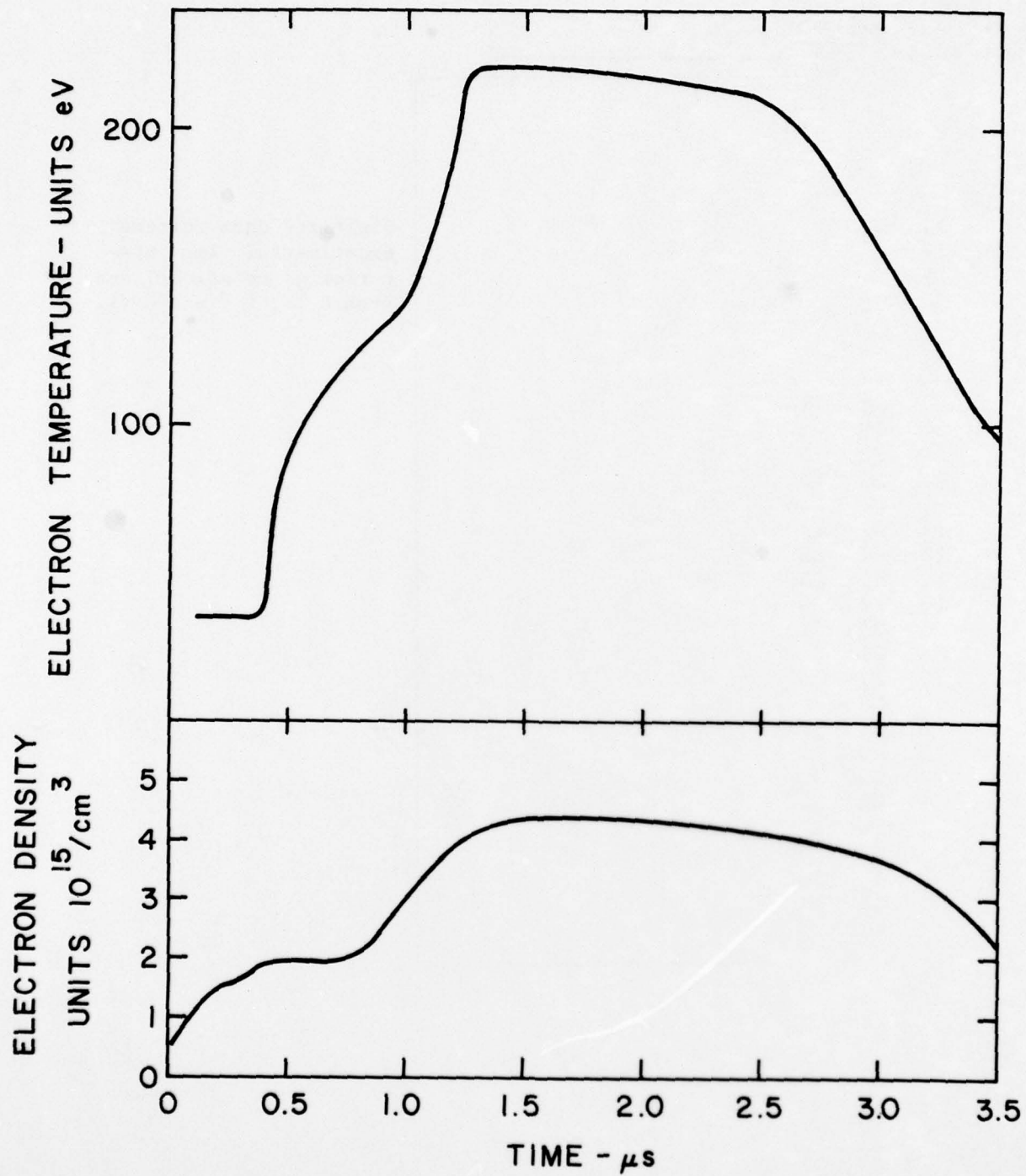


Fig.1 ELECTRON DENSITY AND TEMPERATURE PROFILES
(2% CARBON IN 11 mTorr H_2 CASE)

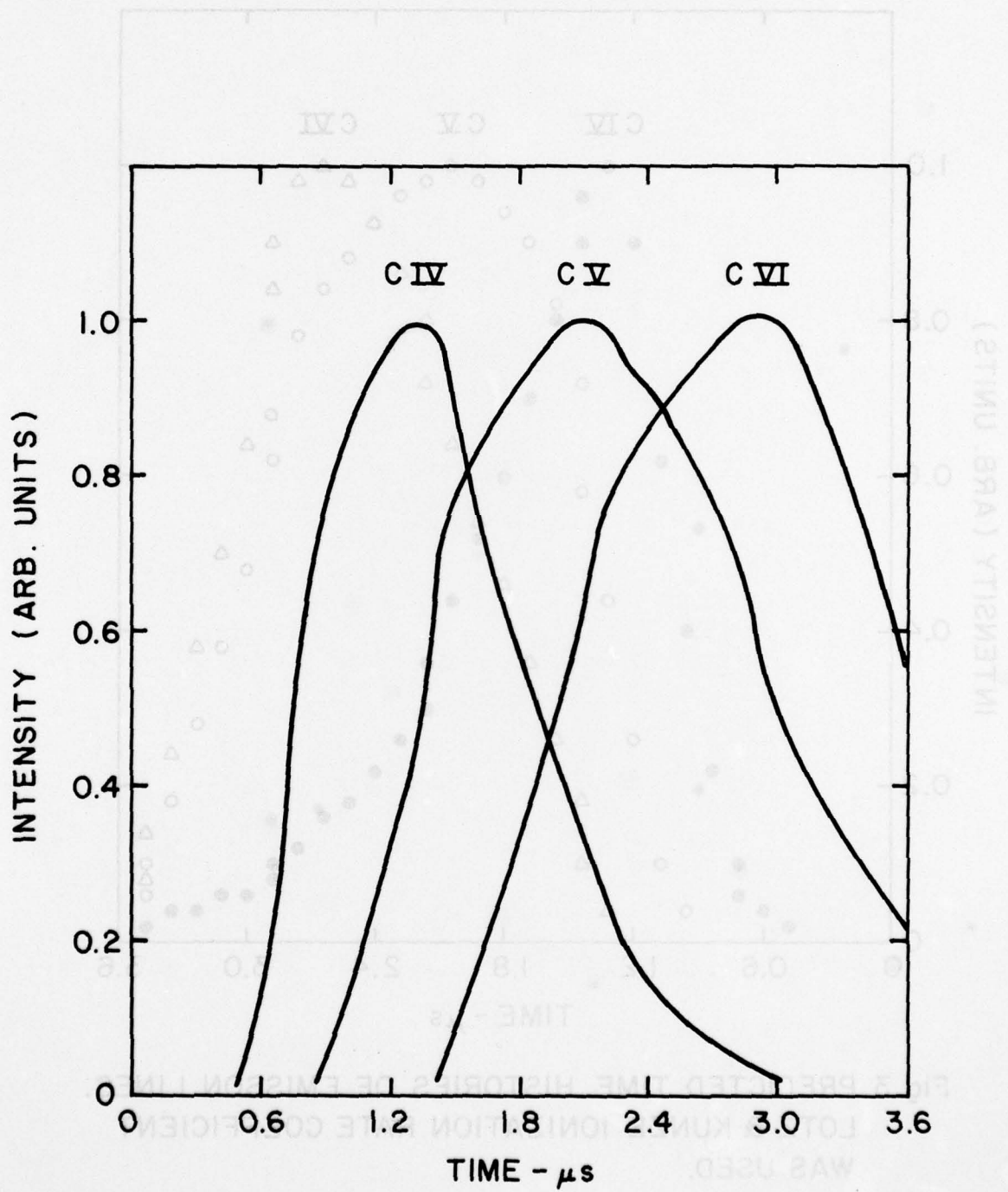


Fig. 2 EXPERIMENTAL TIME HISTORIES OF EMISSION LINES FROM SUCCESSIVE IONIZATION STAGES IN A PLASMA

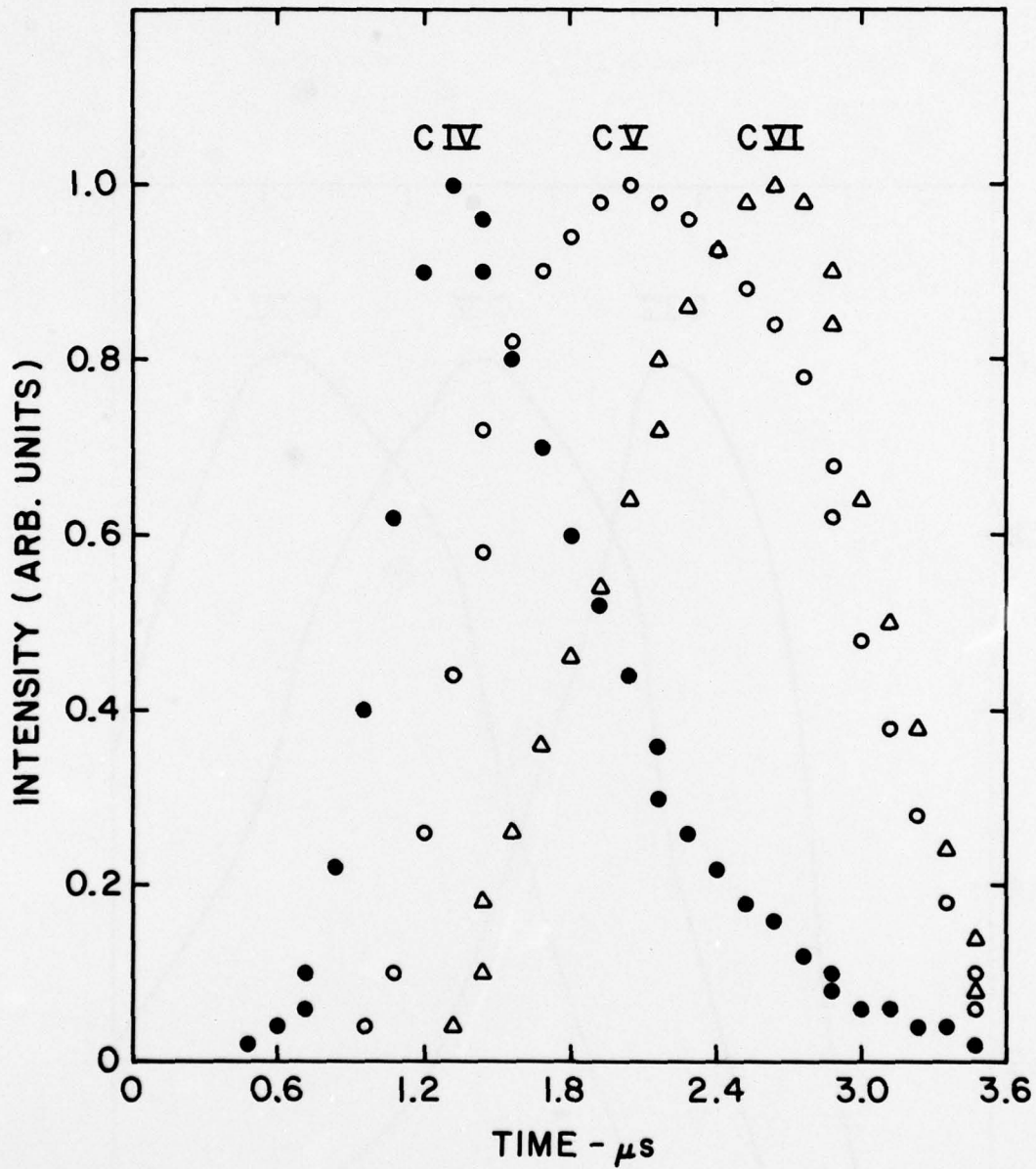


Fig. 3 PREDICTED TIME HISTORIES OF EMISSION LINES. LOTZ & KUNZE IONIZATION RATE COEFFICIENT WAS USED.

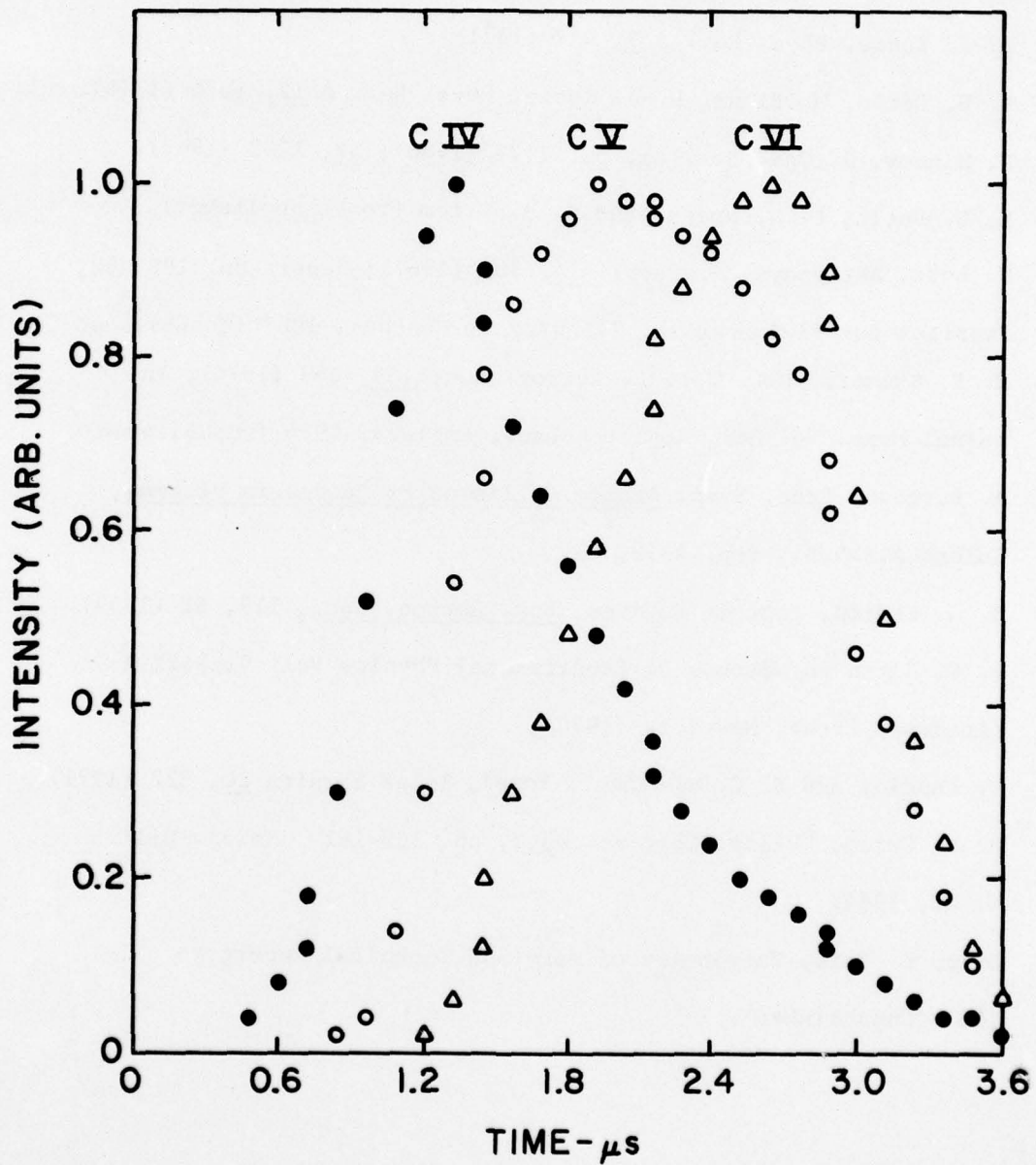


Fig. 4 PREDICTED TIME HISTORIES OF EMISSION LINES, ECIP IONIZATION RATE COEFFICIENT (BURGESS & SUMMERS) WAS USED.

BIBLIOGRAPHY

1. D. Düchs and H. R. Griem, *Phys. Fluids* 9, 1099 (1966).
2. H.-J. Kunze, *Phys. Rev. A* 3, 937 (1971).
3. R. U. Datla, M. Blaha, H.-J. Kunze, *Phys. Rev. A* 12, 1076 (1975).
4. E. Hinnov, *J. Opt. Soc. Am.* 56, 1179 (1966); 57, 1392 (1967).
5. R. U. Datla, L. J. Nugent and H. R. Griem (to be published).
6. W. Lotz, *Astrophys. J. Suppl.* 14, 207 (1967); Report No. IPP1/62, Institut für Plasmaphysik, Garching bei München, 1967 (Unpublished).
7. H. P. Summers, *Mon. Not. R. Astron. Soc.* 169, 663 (1974); Internal Memo. No. 367, Appleton Lab., England, 1974 (Unpublished).
8. A. Burgess, Proc. Symp. Atomic Collision Processes in Plasmas, Culham A.E.R.E., Rep. 4818, 63.
9. M. J. Seaton, *Monthly Notices, Roy. Astron. Soc.*, 119, 81 (1959).
10. R. C. Elton in *Methods of Experimental Physics* Vol. 9, Part A (Academic Press, New York (1970)).
11. M. Landini and B. C. Monsignori Fossi, *Solar Physics* 20, 322 (1971).
12. H. R. Griem, "Plasma Spectroscopy", pp. 160-162. McGraw-Hill, N. Y., 1964.
13. Brian K. Reid, University of Maryland Technical Report No. 71-108 (Unpublished).

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