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LINEAR STABILITY OF SELF-SIMILAR FLOW: 5. CONVECTIVE INSTABILITY--ETC(U)

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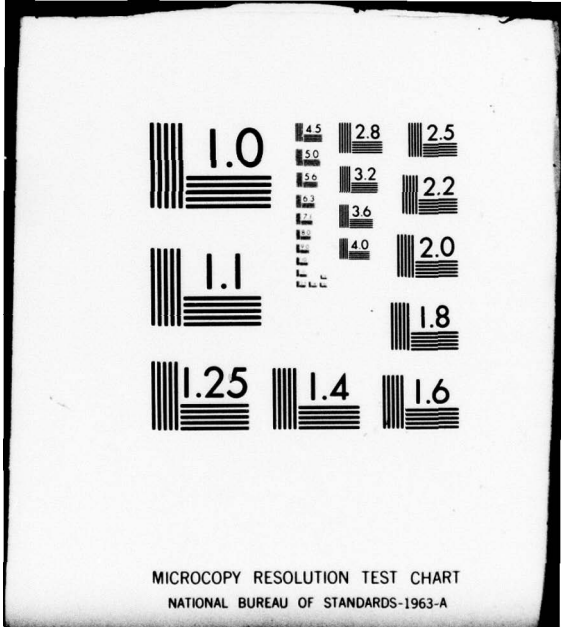
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**Linear Stability Of Self-Similar Flow:
5. Convective Instability In Bounded
Uniform Self-Gravitating Spheres**

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Laboratory for Computational Physics

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A new type of instability, similar to thermal convective instabilities, has been found to occur in self-gravitating spherical clouds uniformly expanding or collapsing in a vacuum. Unlike the Jeans instability, it has a growth rate which increases with zonal mode number l , so that objects of arbitrarily small size are unstable. The modes are compressional and have non-vanishing vorticity. The eigenfunctions peak at the boundary, so the perturbations develop predominantly into surface irregularities. The perturbation amplitudes grow algebraically at first, then at late times become			

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20. ABSTRACT (Continued):

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proportional to the cloud radius $R(t)$. For a collapsing system in which stars are produced by the condensation of diffuse matter the theory predicts fragmentation and enhanced condensation in the peripheral regions.



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LINEAR STABILITY OF SELF-SIMILAR FLOW: 5. CONVECTIVE INSTABILITY IN BOUNDED UNIFORM SELF-GRAVITATING SPHERES

In this letter we discuss a type of fluid instability, the "convective gravitational instability," which can occur in a nonstationary self-gravitating system.

Unlike the Jeans instability (see, e.g., Spitzer, 1978), it does not involve a critical mass or size, and short, rather than long wavelengths, are the most unstable (Book and Bernstein, 1978). It is also distinct from the Rayleigh-Taylor instability, and in a spherical system can arise even when the pressure and density gradients satisfy $p'(r) \rho'(r) > 0$ everywhere. Rather, it is closely related to the convective instability found in a thermally stratified fluid with a temperature profile such that $\mathbf{g} \cdot \nabla T > 0$, where \mathbf{g} is the gravitational acceleration (see Landau and Lifshitz, 1959). This convective instability persists even in the limit of vanishing gravitational interaction, a physical situation discussed by Book (1979). There it was established that the criterion for instability in an ideal polytrope is $s'(r) < 0$ [$s(r)$ is the entropy density]. It can be shown (Book and Bernstein, 1980) that this same inequality governs the onset of instability when self-gravitation is included.

Here we consider the convective gravitational instability of a particular model (Staniukovich, 1949) which turns out to be completely soluble: a sphere of polytropic fluid with $\gamma = 4/3$, having uniform density, and undergoing uniform self-similar expansion (or contraction) under the combined influence of inertial, pressure and Newtonian gravitational forces. As in related stability problems of self-similar flow treated previously by Bernstein and Book (1978) and Book (1978, 1979), we employ Lagrangian variables. This greatly simplifies the analysis, since both unperturbed and perturbed equation become separable.

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As will be seen, this work differs from the many previous treatments of gravitational instability primarily in retaining both finite pressure and finite radius in the model. The instability, which is localized at the periphery of the sphere, is driven by the buoyancy which fluid elements experience in the combined inertial and gravitational force fields. We can therefore expect the instability to give rise to enhanced mixing in the outer layers of the system. This result has clear implications for star formation in contracting proto-galaxies and possibly for stellar evolution as well.

We start with the equations describing ideal hydrodynamic motion of a self-gravitating fluid:

$$\dot{\rho} + \mathbf{v} \cdot \nabla \rho = 0; \quad (1)$$

$$\rho \dot{\mathbf{v}} + \nabla p + \rho \nabla \phi = 0; \quad (2)$$

$$(\rho \rho^{-\gamma}) = 0; \quad (3)$$

$$\nabla^2 \phi = 4\pi G \rho. \quad (4)$$

Here ρ is mass density, \mathbf{v} is velocity, p is pressure, and ϕ is the gravitational potential. Dots are used to denote total (convective) derivatives; γ is the ratio of specific heats and G is the gravitational constant. For a self-similar flow of the type known as uniform or homogeneous, the radial position R at time t of an arbitrary fluid element whose initial position was r is taken to satisfy

$$R = rf(t), \quad (5)$$

where $f(0) = 1$. Equation (1) then yields

$$\rho(r,t) = \rho_0(r) f^{-3}. \quad (6)$$

Then from Eq. (3),

$$p(r,t) = s(r) \rho^\gamma = s(r) \rho_0^\gamma f^{-3\gamma}, \quad (7)$$

where the entropy function $s(r)$ is arbitrary. In this paper we will assume that initially the mass is distributed uniformly inside a sphere of finite radius with density ρ_0 . This spherical cloud is surrounded by vacuum, so that $\rho(r) = 0 = \rho(r)$ for $r > r_0$. Equation (4) can be integrated to give

$$\phi(r,t) = 2\pi G \rho_0^2 / 3f. \quad (8)$$

Substitution of Eqs. (5)-(8) in Eq. (2) yields

$$r\rho_0 f^2 \ddot{f} + \frac{d}{dr}(s\rho_0 \dot{f}) f^{-3\gamma+4} + \frac{4\pi G\rho_0}{r^2} \int_0^r \rho_0 r'^2 dr = 0. \quad (9)$$

Equation (9) can be solved by separation of variables, yielding

$$\rho(r,t) = P_0 (1 - r^2/r_0^2) f^{-3\gamma}, \quad (10)$$

where $P_0 = s(0)\rho_0 \dot{f}$.

The time dependence is then determined by

$$f^2 \ddot{f} - \frac{2P_0}{\rho_0 r_0^2} f^{4-3\gamma} + \frac{4\pi G\rho_0}{3} = 0. \quad (11)$$

In analyzing (11), we find that the character of the solutions depends on the magnitude of γ , the relative magnitude of self-gravitational and pressure forces, and how large \dot{f} is in the initial ($f = 1$) state. To simplify the analysis, we set $\gamma = 4/3$, obtaining the solution first found by Staniukovich (1949). One can dispense with either this assumption or that of uniform density; the analysis of the general cases, which is somewhat involved, will be given elsewhere (Book and Bernstein, 1980). It is convenient to define

$$\beta = \frac{3P_0}{2\pi G\rho_0^2 r_0^2}, \quad (12)$$

and to introduce the characteristic time, defined by

$$\tau^{-2} = 4\pi G\rho_0/3. \quad (13)$$

Equation (11) then becomes

$$\tau^2 f^2 \ddot{f} = \beta - 1. \quad (14)$$

The first integral of (14) can now be written

$$\frac{\tau^2}{2} (\dot{f}^2 - \dot{f}_0^2) = (\beta - 1) (1 - f^{-1}), \quad (15)$$

where \dot{f}_0 is the value of $\dot{f}(t)$ at $t = 0$. Following Sedov (1959), we distinguish three types of motion. In type I motions, $\dot{f} \neq 0$ for all f , $0 \leq f < \infty$. Trajectories of Type I for which $f \rightarrow 0$ as $f \rightarrow \infty$ become possible if $\beta < 1$:

$$\frac{1}{2} \tau^2 \dot{f}^2 = (1 - \beta) f^{-1}. \quad (16)$$

When there is a turning point, we take the zero of time there:

$$\frac{1}{2} \tau^2 \dot{f}^2 = (1 - \beta) (f^{-1} - 1), \quad (17)$$

which yields solutions of Type II for $\beta < 1$ and Type III for $\beta > 1$.

We return to Eqs. (1)-(4) and make use of the results obtained by Bernstein and Book (1978) and Book (1978, 1979) in deriving linearized equations for the evolution of a small perturbation about the basic state solutions. The perturbed displacement ξ satisfies the linearized form of Eq. (2),

$$\rho \ddot{\xi} + \rho_1 \ddot{\mathbf{R}} = -\nabla_{\mathbf{R}} p_1 + (\nabla_{\mathbf{R}} \xi) \cdot \nabla_{\mathbf{R}} p - \rho \nabla_{\mathbf{R}} \phi_1 + \rho (\nabla_{\mathbf{R}} \xi) \cdot \nabla_{\mathbf{R}} \phi - \rho_1 \nabla_{\mathbf{R}} \phi, \quad (18)$$

where first-order quantities are distinguished by the subscript 1. Here the gradient operator $\nabla_{\mathbf{R}}$ is defined with respect to the unperturbed coordinate \mathbf{R} . The perturbed density is

$$\rho_1 = -\rho \nabla_{\mathbf{R}} \cdot \xi \quad (19)$$

and the perturbed pressure is

$$p_1 = -\gamma p \nabla_{\mathbf{R}} \cdot \xi. \quad (20)$$

Using Eqs. (19) and (20) and the unperturbed forms of Eqs. (2) and (4), we rewrite Eq. (18) as

$$\rho \ddot{\xi} + (\nabla_{\mathbf{R}} p) \nabla_{\mathbf{R}} \cdot \xi - \gamma \nabla_{\mathbf{R}} (p \nabla_{\mathbf{R}} \cdot \xi) + \rho (\nabla_{\mathbf{R}} \xi) \cdot \ddot{\mathbf{R}} + \rho \nabla_{\mathbf{R}} \phi_1 = 0. \quad (21)$$

We find an equation for ϕ_1 by substituting Eq. (8) in Eq. (4) and collecting first order terms:

$$\nabla_{\mathbf{R}}^2 \phi_1 - \nabla_{\mathbf{R}} \cdot [(\nabla_{\mathbf{R}} \xi) \cdot \nabla_{\mathbf{R}} \phi] - \nabla_{\mathbf{R}} \xi \cdot \nabla_{\mathbf{R}} \nabla_{\mathbf{R}} \phi = 4\pi G \rho_1. \quad (22)$$

At this point it is convenient to use Eq. (5) to rewrite the equations in terms of \mathbf{r} instead of \mathbf{R} and then to introduce the notation $\nabla \equiv \nabla_{\mathbf{r}}$.

We can eliminate ϕ_1 from Eq. (21) either by operating with the curl or by taking a divergence and substituting from Eq. (22). The results are given by

$$f^3 \ddot{\sigma} - \frac{2\beta}{3} \nabla^2 \left[(1 - r^2)\sigma \right] + 2(1 + \beta)\sigma + \beta\omega \quad (23)$$

and

$$f^3 \ddot{\omega} = (\beta - 1)\omega + \beta(2\mathbf{r} \cdot \nabla\sigma + \mathbf{r}\mathbf{r}:\nabla\nabla\sigma - r^2\nabla^2\sigma), \quad (24)$$

where r is expressed in units of r_0 and t is units of τ , and we have defined

$$\sigma = \nabla \cdot \underline{\xi}; \quad (25)$$

$$\omega = \mathbf{r} \cdot \nabla \times \nabla \times \underline{\xi}. \quad (26)$$

We seek separable solutions of the form

$$\sigma(\mathbf{r}, t) = S(r) Y_{lm}(\theta, \phi) T(t) \quad (27)$$

$$\omega(\mathbf{r}, t) = W(r) Y_{lm}(\theta, \phi) T(t). \quad (28)$$

Eliminating W , we find

$$f^3 \ddot{T} = (\beta - 1)\mu T, \quad (29)$$

and

$$r^{-2} \left[\{r^2[(1 - r^2)S]'\}' - l(l + 1)(1 - r^2)S \right] + KS = 0. \quad (30)$$

Here

$$K = \frac{3}{2\beta(1 - \beta)} \left[\mu(1 - \beta)^2 + 2(1 - \beta^2) - \frac{\beta^2 l(l + 1)}{\mu - 1} \right], \quad (31)$$

and μ is a separation constant which determines the time behavior. Let

$$z(r) = r(1 - r^2)S. \quad (32)$$

Then Eq. (30) can be written

$$z'' + \left[\frac{K}{1 - r^2} - \frac{l(l + 1)}{r^2} \right] z = 0. \quad (33)$$

Since p_1 must be finite at $r = 0$ and vanish at $r = 1$, the boundary conditions appropriate to Eq. (33) are $z(0) = z(1) = 0$.

If one multiplies Eq. (33) by z , there results after an integration by parts

$$K = \frac{\int_0^1 dr \left[\left(\frac{dz}{dr} \right)^2 + \frac{l(l+1)z^2}{r^2} \right]}{\int_0^1 \frac{dr z^2}{1-r^2}} > 0. \quad (34)$$

Note that if we differentiate Eq. (31) with respect to μ for fixed l , it follows that

$$(1-\beta)^{-1} \frac{dK}{d\mu} = \frac{3}{2\beta} \left[1 + \frac{\beta^2 l(l+1)}{(1-\beta)^2 (\mu-1)^2} \right] > 0. \quad (35)$$

Anticipating a result, proved below, that the rate of growth of an unstable mode satisfying Eq. (29) is greater, the nearer μ ($\beta - 1$) approaches $+\infty$, we see that for a given l maximum growth is obtained for the mode with the *smallest* value of K .

Regularity of z at $r = 0$ implies via Eq. (33) that for small r , $z(r) \sim r^{l+1}$. Since the equation is invariant under $r \rightarrow -r$, one can seek a solution in the form

$$z = \sum_{n=0}^{\infty} c_n r^{2n+l+1}, \quad (36)$$

where the $\{c_n\}$ are constant. The recursion relation is readily seen to be

$$c_{n+1} = - \frac{K - 4n(n+l+1/2)}{4(n+1)(n+l+3/2)} c_n. \quad (37)$$

If K is chosen such that the series (36) terminates, a solution for which $z(1) = 0$ can be obtained [since in Eq. (33) the second term would otherwise diverge]. The condition for this to occur is

$$K = 4N(N+l+1/2), \quad (38)$$

$N = 1, 2, \dots$. The smallest eigenvalue is clearly $K = 4l + 6$, corresponding to $N = 1$. The associated eigenfunction is

$$z(r) = r^{l+1} (1-r^2), \quad (39)$$

or equivalently, $S(r) = r^l$.

Taking $K = 4l + 6$ in (31) and solving for μ , we get

$$\mu - 1 = -\frac{3}{2} - \frac{4/\beta}{3(\beta - 1)} \pm \left\{ \left[\frac{3}{2} + \frac{4/\beta}{3(\beta - 1)} \right]^2 + \frac{l(l+1)\beta^2}{(\beta - 1)^2} \right\}^{1/2} \quad (40)$$

With the appropriate sign, $(\mu - 1)(\beta - 1) > 0$ for all $l > 0$. This is the condition for instability, as will now be shown.

The usual definition of instability is inadequate when applied to nonsteady states, since the time dependence $T(t)$ of the perturbations is in general not exponential. It is appropriate to call a mode stable (unstable) if the ratio $\mathcal{Q}(t) = |T(t)/f(t)|$ of the perturbation amplitude to that of the basic state vanishes (diverges) as $t \rightarrow \infty$. Note that this definition is useful both when $f \rightarrow 0$ and when $f \rightarrow \infty$ for large t . Moreover, it does not preclude the possibility that, e.g., in an "unstable" system $\mathcal{Q}(t)$ decreases for a finite time. Because the period during which \dot{f} is effectively nonzero is finite, $\mathcal{Q}(\infty)$ may be finite. If this happens, the pertinent question becomes that of finding how much relative amplification occurs, i.e., of evaluating $\mathcal{Q}(\infty)/\mathcal{Q}(0)$.

If $(\beta - 1)\mu \gg 1$ in Eq. (29), one solution grows and the other damps approximately exponentially for $f \approx 1$, experiencing roughly $|\mu|^{1/2}$ e-foldings. Since $f \approx 1$ holds only for times $|t| \lesssim 1/2$, this is a very crude measure of the real time amplification. Fortunately, it is possible to solve Eq. (29) exactly. This is easily accomplished by using f instead of t as the independent variable.

For Type I motions Eq. (29) becomes by virtue of Eq. (16)

$$2f^2 \frac{d^2 T}{df^2} - f \frac{dT}{df} + \mu T = 0. \quad (41)$$

Hence $T(t)$ is a linear combination of the two solutions

$$T_{\pm}(t) = f^{(3 \pm \Delta)/4}, \quad (42)$$

where $\Delta = (9 - 8\mu)^{1/2}$. For $\mu < 9/8$, Δ is real and T_+ grows more rapidly than T_- with increasing f . If $\mu < 1$,

$$T_+/f = f^{(-3 + \Delta)/4} = \mathcal{Q}_+(t) \quad (43)$$

diverges as $f \rightarrow \infty$, while

$$T_-/f = f^{-(3+\Delta)/4} = \mathcal{Q}_-(t) \quad (44)$$

vanishes. For outward motion, evidently perturbations with a component proportional to T_+ blow up at late times faster than the unperturbed radius, and consequently satisfy our condition for instability. If $\mu > 1$, on the other hand, the perturbations are stable. Thus the compressible modes given by Eq. (29) with μ determined from Eq. (40) can be stable or unstable, depending on which branch is taken. For inward motions, \mathcal{Q}_- grows more rapidly than \mathcal{Q}_+ . Both diverge when $\mu > 9/8$, albeit slowly. This divergence has been related by Book (1978) to the fact that $\gamma < 5/3$. The explanation is essentially geometrical, and the limiting behavior can be derived by invoking conservation of the action of a standing wave on the collapsing cloud. When $\mu = 1$, $T_+ = f$. When $\mu < 1$, T_+ is well behaved as $f \rightarrow 0$, but T_- is unstable. A disturbance initialized at any instant of time with spatial dependence $\sim r^\mu Y_{lm}$ blows up as the cloud collapses.

For Type II motions we find that as a function of $x = 1 - f^{-1}$, T satisfies the hypergeometric equation

$$x(1-x)y'' + [c - (a+b+1)x]y' - aby = 0 \quad (45)$$

with

$$\left. \begin{array}{l} a \\ b \end{array} \right\} = \frac{1}{4}(3 \pm \Delta); \quad (46a, b)$$

$$c = \frac{1}{2}. \quad (46c)$$

Writing the two independent solutions

$$\mathcal{F}(t) = {}_2F_1\left[\frac{1}{4}(3 + \Delta), \frac{1}{4}(3 - \Delta); \frac{1}{2}; 1 - f^{-1}\right] \quad (47)$$

$$\mathcal{G}(t) = [2(f^{-1} - 1)]^{1/2} {}_2F_1\left[\frac{1}{4}(5 + \Delta), \frac{1}{4}(5 - \Delta); \frac{3}{2}; 1 - f^{-1}\right] \quad (48)$$

we note that

$$\mathcal{F}(0) = \dot{\mathcal{G}}(0) = 1; \quad (49)$$

$$\dot{\mathcal{F}}(0) = \mathcal{G}(0) = 0, \quad (50)$$

so that in terms of the initial values,

$$T(t) = T(0) \mathcal{F}(t) + \dot{T}(0) \mathcal{G}(t). \quad (51)$$

As $f \rightarrow 0$

$$\begin{aligned} \mathcal{F}(t) &\approx \frac{\sqrt{\pi} \Gamma(-\Delta/2) f^{(3+\Delta)/4}}{\Gamma(3/4 - \Delta/4) \Gamma(-1/4 - \Delta/4)} + \frac{\sqrt{\pi} \Gamma(\Delta/2) f^{(3-\Delta)/4}}{\Gamma(3/4 + \Delta/4) \Gamma(-1/4 + \Delta/4)} \\ &= F_+^{(0)} T_+(t) + F_-^{(0)} T_-(t) \end{aligned} \quad (52)$$

and

$$\begin{aligned} \mathcal{G}(t) &\approx \frac{\sqrt{\pi/2} \Gamma(-\Delta/2) f^{(3+\Delta)/4}}{\Gamma(3/4 - \Delta/4) \Gamma(1/4 - \Delta/4)} + \frac{\sqrt{\pi/2} \Gamma(\Delta/2) f^{(3-\Delta)/4}}{\Gamma(5/4 + \Delta/4) \Gamma(1/4 + \Delta/4)} \\ &= G_+^{(0)} T_+(-t) + G_-^{(0)} T_-(t). \end{aligned} \quad (53)$$

When $\mu > 9/8$, Δ is imaginary. The second term in each of the above equations is the complex conjugate of the first, and all terms oscillate for small f , diverging as $f^{-1/4}$. When $\mu < 9/8$, Δ is real and the second term in both equations dominates as $f \rightarrow 0$. For $\mu = 1$, $\mathcal{F}(t) = f(t)$. Note that the dependence on f in Eqs. (52)-(53) is asymptotically identical with that of the exact solutions T_{\pm} given in Eq. (42) for Type I motions.

For Type III motions the solutions \mathcal{F} and \mathcal{G} are still valid, with a , b and c as defined in Eq. (46). We are interested now in the stability of perturbations on a cloud expanding outward ($f \rightarrow \infty$). The appropriate asymptotic forms are

$$\mathcal{F}(t) \sim \frac{\sqrt{\pi} f}{\Gamma\left(\frac{3+\Delta}{4}\right) \Gamma\left(\frac{3-\Delta}{4}\right)} = F^{(\infty)} f \quad (54)$$

$$\mathcal{G}(t) \sim \frac{(\pi/2)^{1/2} f}{\Gamma\left(\frac{5+\Delta}{4}\right) \Gamma\left(\frac{5-\Delta}{4}\right)} = G^{(\infty)} f. \quad (55)$$

The coefficients $F^{(\infty)}$ and $G^{(\infty)}$ increase monotonically with μ . Although the amplification for both Eq. (54) and Eq. (55) remains finite, it can be made arbitrarily large as $\mu \rightarrow \infty$. We thus see that the modes undergo a growth which can be catastrophically large.

If we examine the reverse trajectory (the cloud collapsing from infinity, turning at $f = 1$, and rebounding) we find a clearcut instability. The solution of Eq. (29) having unit amplitude as $t \rightarrow -\infty$ is

$$\mathcal{K}(t) = F \left(\frac{3 + \Delta}{4}, \frac{3 - \Delta}{4}; 2; f^{-1} \right) \rightarrow H^{(\infty)} = \frac{\sqrt{\pi}}{\Gamma \left(\frac{5 + \Delta}{4} \right) \Gamma \left(\frac{5 - \Delta}{4} \right)} \quad (56)$$

as $f \rightarrow 1$. Hence the relative amplification $\mathcal{Q}(0)/\mathcal{Q}(-\infty)$ is infinite. If the perturbation is initialized when the cloud is at some large but finite radius $f(-t_0)$, the relative amplification at $t = 0$ of mode l is proportional to $f(-t_0) H^{(\infty)}(l)$, which increases with both l and β .

By choosing an appropriate linear combination of the two independent solutions of Eq. (29), a solution can be found for motions of Type I, II or III which decreases initially, then grows. This justifies our choice of the *asymptotic* relative amplification as a criterion for instability.

In conclusion, we note that the preconditions for instability, viz., that the outermost layers of the system be cooler than the core [$s(r) < 0$], is not at all implausible. In astrophysical contexts, it might result from surface radiation or from nucleosynthesis in the dense core. Even when $\beta \ll 1$, short-wavelength surface perturbations would grow rapidly, leading to mixing, condensation of small masses and development of fine-scale structure there.

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REFERENCES

1. Bernstein, I. B. and Book, D. L., 1978, *Ap. J.* **225**, 633.
2. Book, D. L., 1978, *Phys. Rev. Lett.* **41**, 1551.
3. Book, D. L. 1979, *J. Fluid Mech.* (to be published).
4. Book, D. L. and Bernstein, I. B., 1978, *Bull. AAS* **10**, 437.
5. Book, D. L. and Bernstein, I. B., 1980, *Ap. J.* (to be published).
6. Landau, L. D. and Lifshitz, E. M. , 1959, *Fluid Mechanics* (Addison-Wesley, Reading, MA), p. 8.
7. Sedov, L. I., 1959, *Similarity and Dimensional Methods in Mechanics* (Academic Press, New York), pp. 271-281.
8. Staniukovich, K. P., 1949, *C. R. Acad. Sci. U.R.S.S.*, **64**, 467.