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MUTUAL RADIATION IMPEDANCE OF PISTONS (OF KA = 0.40) SYMMETRICA--ETC(U)
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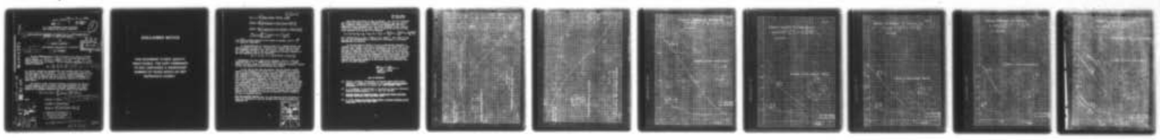

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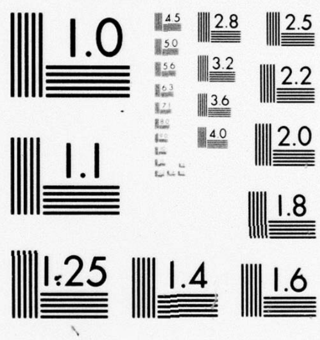

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MUTUAL RADIATION IMPEDANCE OF PISTONS (OF ka = 0.40) SYMMETRICALLY ARRANGED IN A STIFF PLANE BAFFLE

by

Harvey A. Alperin

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Technical memo

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The net radiation force on the i^{th} element of an array of elements can be written $f_i = \sum Z_{ij} V_j$ where Z_{ij} is the mutual radiation impedance between the i^{th} and j^{th} elements ($Z_{ij} = Z_{ji}$) and V_j is the face velocity of the j^{th} element. The net radiation impedance of the i^{th} element is therefore

$$Z_i = \frac{f_i}{V_i} = \sum_j Z_{ij} \frac{V_j}{V_i}$$

If, however, elements are arranged with such symmetry that their velocities are equal then $Z_i = \sum_j Z_{ij}$. In order to investigate quantitatively for this special case, the effects on the net radiation impedance of varying the number of elements and their spacing, several calculations have been made for the case of circular pistons (of fixed ka = 0.40) set in a plane baffle.

The specific mutual radiation impedance z_{12} between two identical circular pistons 1 and 2 of radius a and center distance d in a plane infinite stiff baffle is derived by Pritchard (reference (a)) using a method due to Bouwkamp (reference (b)) and is given by the expression

$$Z_{12} = r_{12} + jX_{12} = \sum_{s=0}^{\infty} \sigma_s(ka) \left(\frac{a}{d}\right)^s J_s^{(2)}(kd)$$

where

$$\sigma_0(ka) = 2 J_1^2(ka)$$

$$\sigma_1(ka) = 2 J_1(ka) J_2(ka)$$

$$\sigma_2(ka) = \frac{3}{2}\left[J_1(ka) J_3(ka) + J_2^2(ka)\right]$$

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$$\sigma_3(ka) = \frac{5}{4}\left[J_1(ka)\,J_4(ka) + 3J_2(ka)\,J_3(ka)\right]$$

$$\sigma_4(ka) = \frac{35}{32}\left[J_1(ka)\,J_5(ka) + 4\,J_2(ka)\,J_4(ka) + 3J_3^2(ka)\right]$$

$$\sigma_5(ka) = \frac{63}{64}\left[J_1(ka)\,J_6(ka) + 5J_2(ka)\,J_5(ka) + 10J_3(ka)\,J_4(ka)\right]$$

$$J_s^{(2)}(kd) \equiv \sqrt{\frac{\pi}{2kd}}\left[J_{s+1/2}(kd) + j(-1)^s J_{-s-1/2}(kd)\right]$$

with $J_n(x)$ the Bessel function of n'th order of argument x and $k = \frac{2\pi}{\lambda}$ (λ being the wavelength).

By calculating z_{12}* for several values of d a smooth function can then be plotted for z_{12} vs d. Actually, what has been done is to plot the convenient quantities r_{12}/r_{11} and x_{12}/x_{11} vs $d/2a$ (for the fixed ka = 0.40) where r_{11} and x_{11} are the resistive and reactive components of the self-impedance given by the expression

$$Z_{11} = r_{11} + jx_{11} = 1 + \frac{1}{ka}\left[-J_1(2ka) + jS_1(2ka)\right]$$

($S_1(2ka)$ denotes the first order Struve function). For ka = 0.40, the resistive part is r_{11} = .07789 and the reactive part is x_{11} = 0.3252. Figure 1 is then a graph of r_{12}/r_{11} and x_{12}/x_{11} as a function of spacing $kd/2ka$ for two such pistons in a plane infinite stiff baffle.

In the case of more than two pistons, with all elements equally spaced on the periphery of a circle (i.e., corresponding to an equilateral triangular arrangement for three elements, square arrangement for four elements and succeedingly higher order polygonal arrangements for more elements), it is obvious that all elements have the same identical sum of mutual radiation impedances, and, therefore, it is intuitively clear (and rigorously provable) that they also have identical velocities, and, consequently, also identical net radiation impedances. In this kind of symmetrical arrangement of identical transducers, identically terminated electrically, the velocities and forces on each transducer face are equal. For this arrangement in polygonal symmetry the radiation impedance has been calculated for 3, 4, 5, 6 and 7 elements and is shown in Figures 2 to 6 (inclusive) in the form of numerical factors convenient for use in transducer design.

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*For the numerical calculations, the half-integral Bessel functions are taken from reference (c) and the integral order Bessel functions from reference (d).

The actual calculation is made as follows: e.g., for the 7-radiator case (Figure 6) there are three independent distances, d_{12}, d_{13}, and d_{14} (d_{ij} = distance between elements i and j) and consequently only three independent mutual impedances since the mutual impedance between elements i and j is a function of d_{ij} only. Hence, we may write for the net radiation impedance z_i of any one of the elements:

$$z_1 = z_2 = z_3 = z_4 = z_5 = z_6 = z_7 = z_{11}\left[1 + 2\frac{z_{12}(kd_{12})}{z_{11}} + 2\frac{z_{13}(kd_{13})}{z_{11}} + 2\frac{z_{14}(kd_{14})}{z_{11}}\right]$$

From geometry $d_{12} \equiv d$, $d_{13} = 1.803\ d$, $d_{14} = 2.245\ d$. The values of z_{12}, z_{13}, z_{14} are then obtained as a function of d from Figure 1, and summed appropriately to obtain z_i.

In the case given in Figure 7 we have a close-packed hexagonal arrangement (one element in the center surrounded by six others) and we note that the center element could not be interchanged with any of the others, and hence it would have a velocity or force (depending on the electrical termination) different from the others. Nevertheless, the total impedance is calculated for equal-velocity transducers, from which follows that this situation cannot be achieved with identical elements identically electrically terminated. (It is also to be noted that even with equal-velocity elements the radiation impedance of the center element is different from that of the outer elements.)

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HARVEY A. ALPERIN
Physicist

LIST OF REFERENCES

(a) Robert L. Prichard, "Directivity of Acoustic Linear Point Arrays, Appendix C", Technical Memorandum No. 21, Harvard Acoustic Research Laboratory, NR-014-903, 15 January 1951 (USL A11/HARV. 8990).

(b) C. J. Bouwkamp, "A Contribution to the Theory of Acoustic Radiation", Phillips Research Report 1, No. 4, 1956, pp. 260-262.

(c) National Bureau of Standards Tables of Spherical Bessel Functions, Columbia University Press, New York, 1947.

(d) E. Cambi, Eleven and Fifteen-Place Tables of Bessel Functions of the First Kind, Dover, New York, 1948.

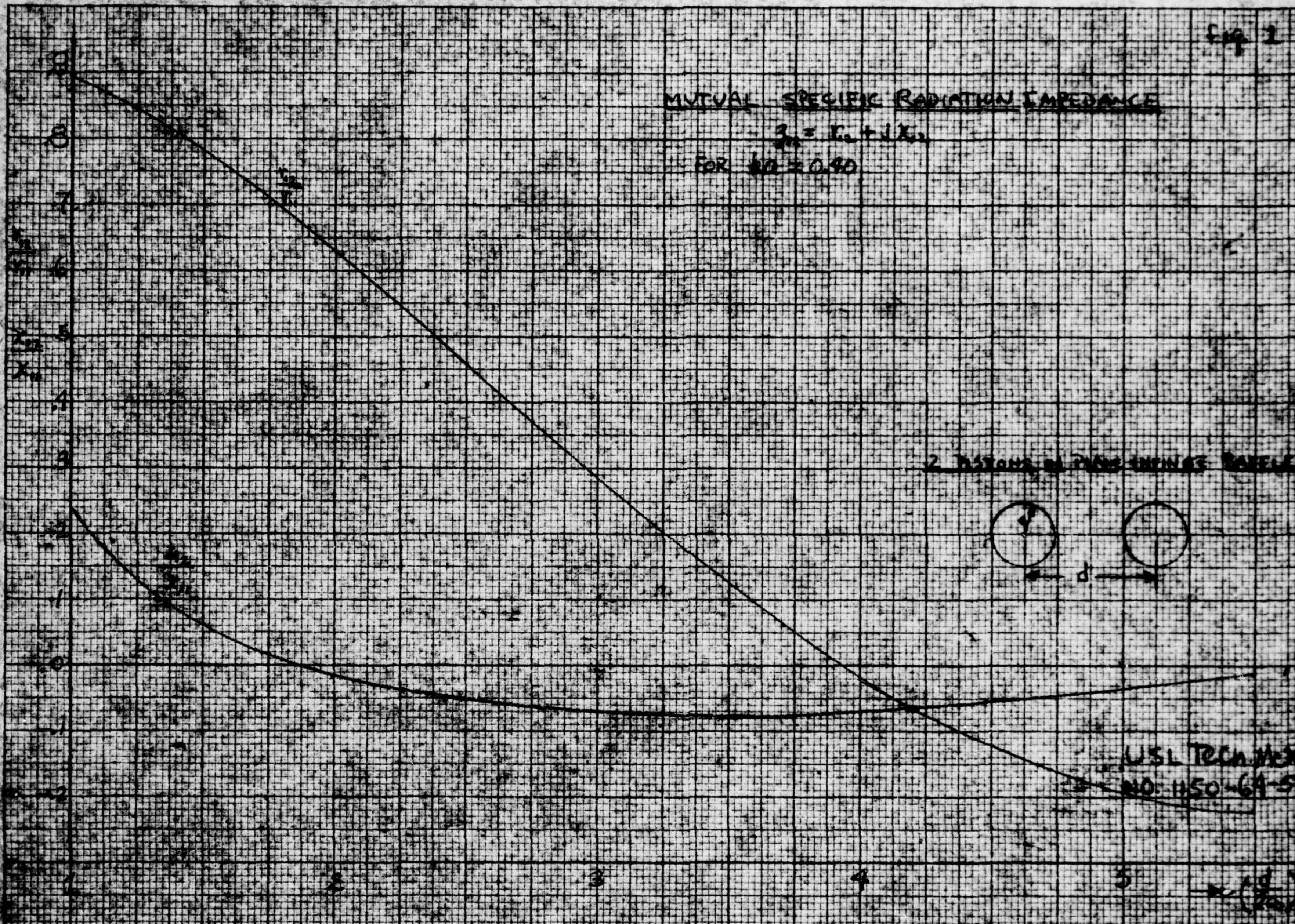
Fig 1
MUTUAL SPECIFIC RADIATION IMPEDANCE
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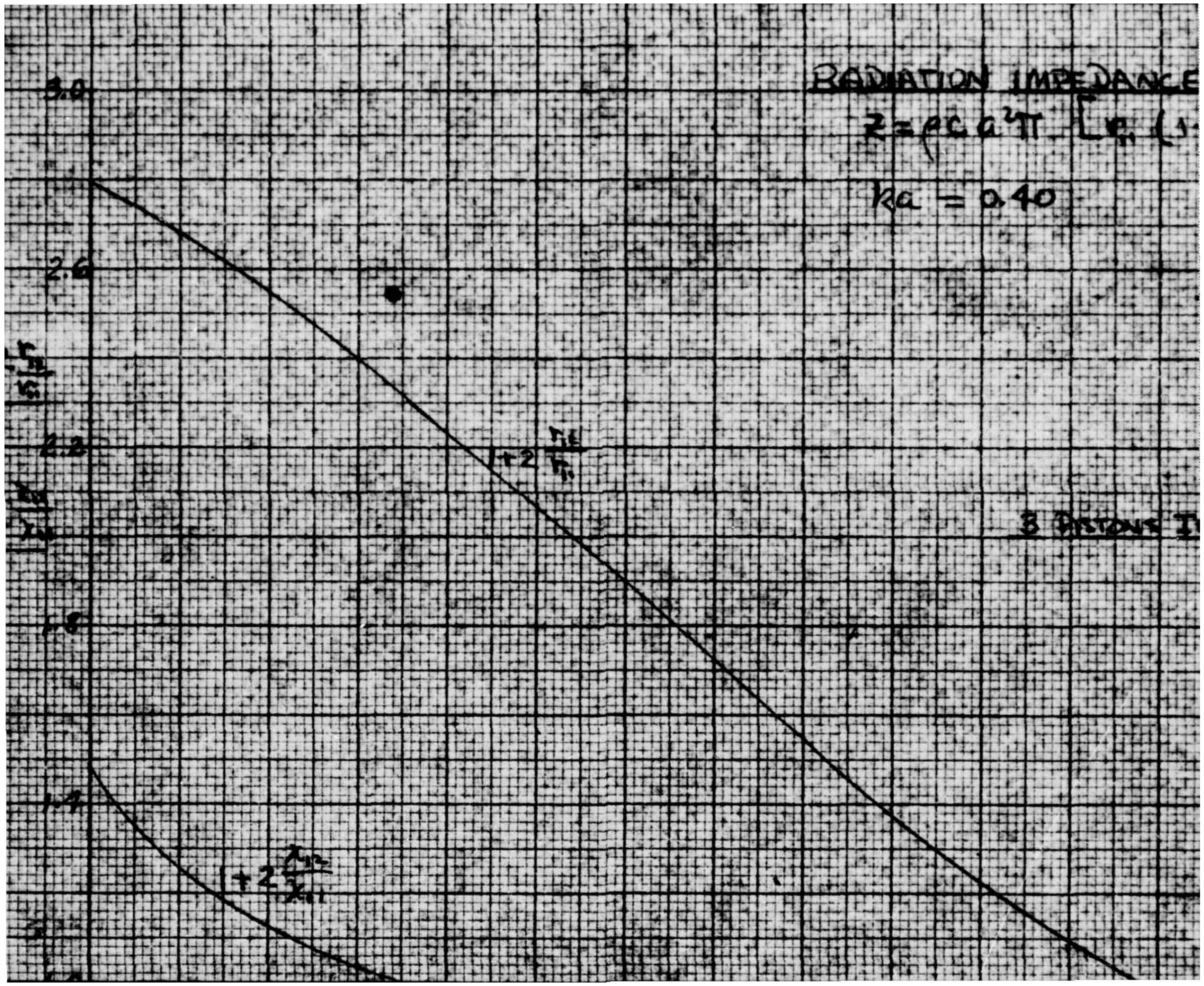
NO. 1150-64-56


RADIATION IMPEDANCE
ka = 0.40
3 PISTONS
3.0
2.6
2.2
1.8
1.4

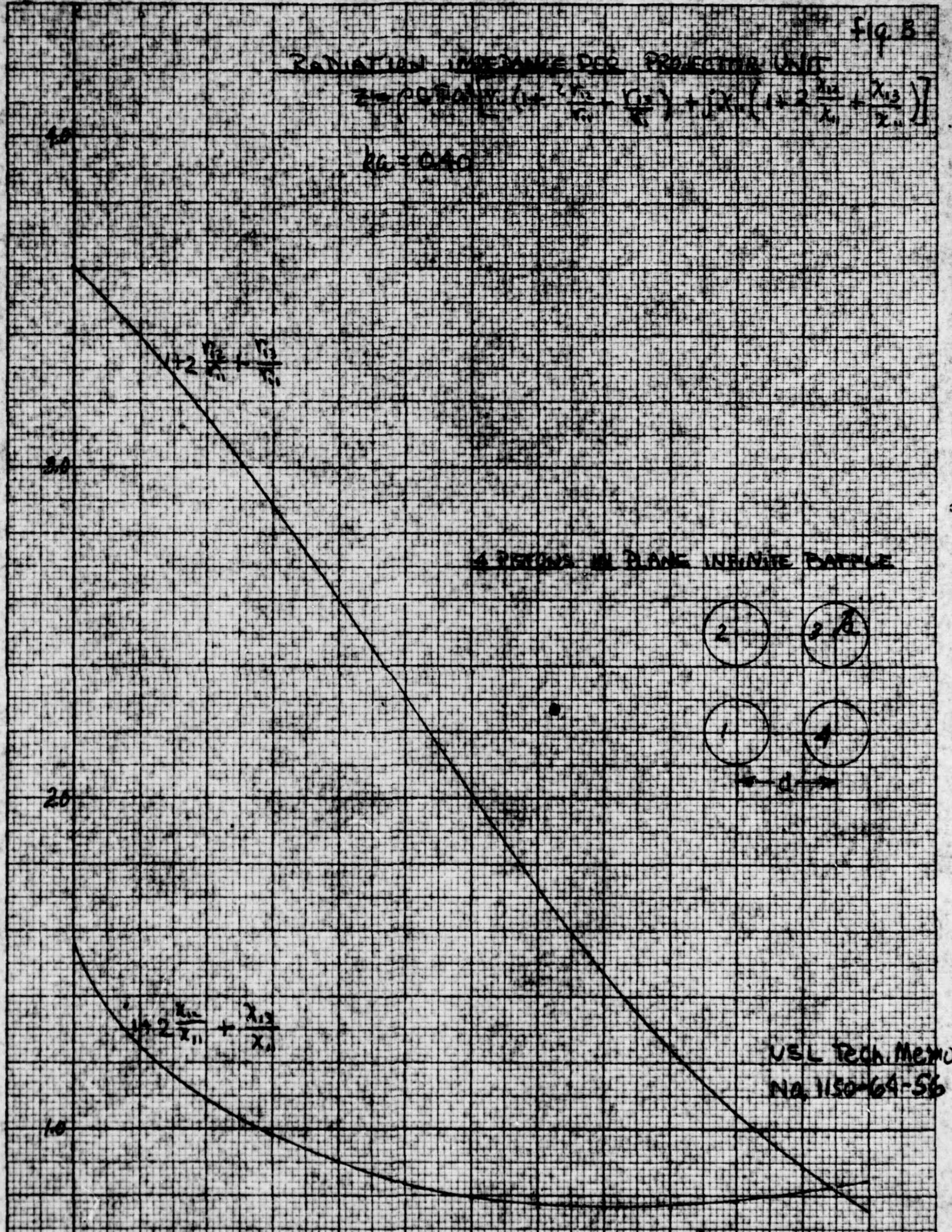


fig. 8
RADIATION IMPEDANCE PER PROJECTOR UNIT
$Z = \rho c \pi a^2 \left[r_{11}\left(1 + \frac{2r_{12}}{r_{11}} + \frac{r_{13}}{r_{11}}\right) + jx_{11}\left(1 + 2\frac{x_{12}}{x_{11}} + \frac{x_{13}}{x_{11}}\right)\right]$
ka = 0.40
4.0
3.0
2.0
1.0
$1 + 2\frac{r_{12}}{r_{11}} + \frac{r_{13}}{r_{11}}$
$1 + 2\frac{x_{12}}{x_{11}} + \frac{x_{13}}{x_{11}}$
4 PISTONS IN PLANE INFINITE BAFFLE
2
3
1
4
d
1
2
3
4
$\left(\frac{d}{2a}\right)$
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fig. 4

RADIATION IMPEDANCE PER PROJECTOR UNIT

$$Z = \rho c a^2 \pi \left[r_{11} \sum_{n=1}^{5} \frac{r_{1n}}{r_{11}} + j\chi_{11} \sum_{n=1}^{5} \frac{\chi_{1n}}{\chi_{11}} \right]$$

$ka = 0.40$

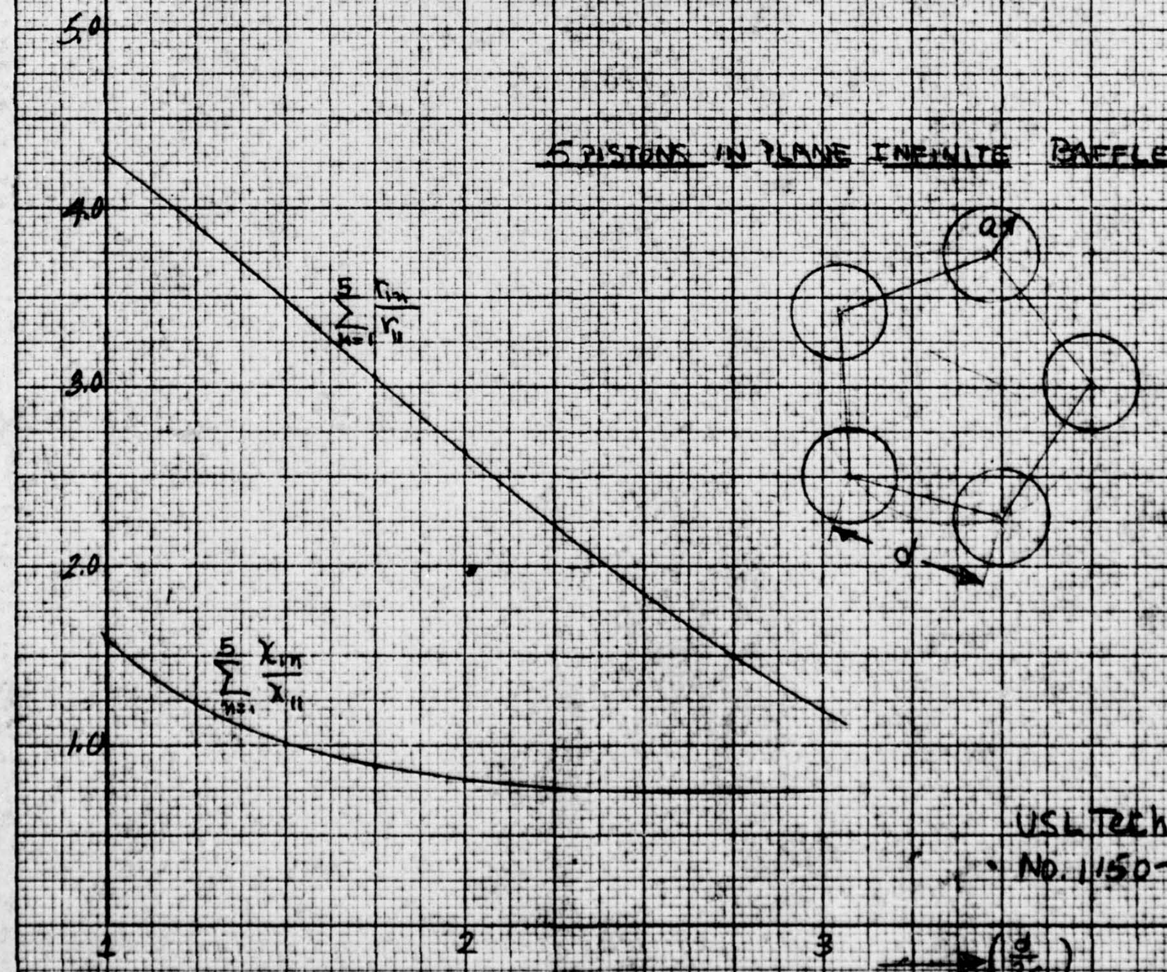

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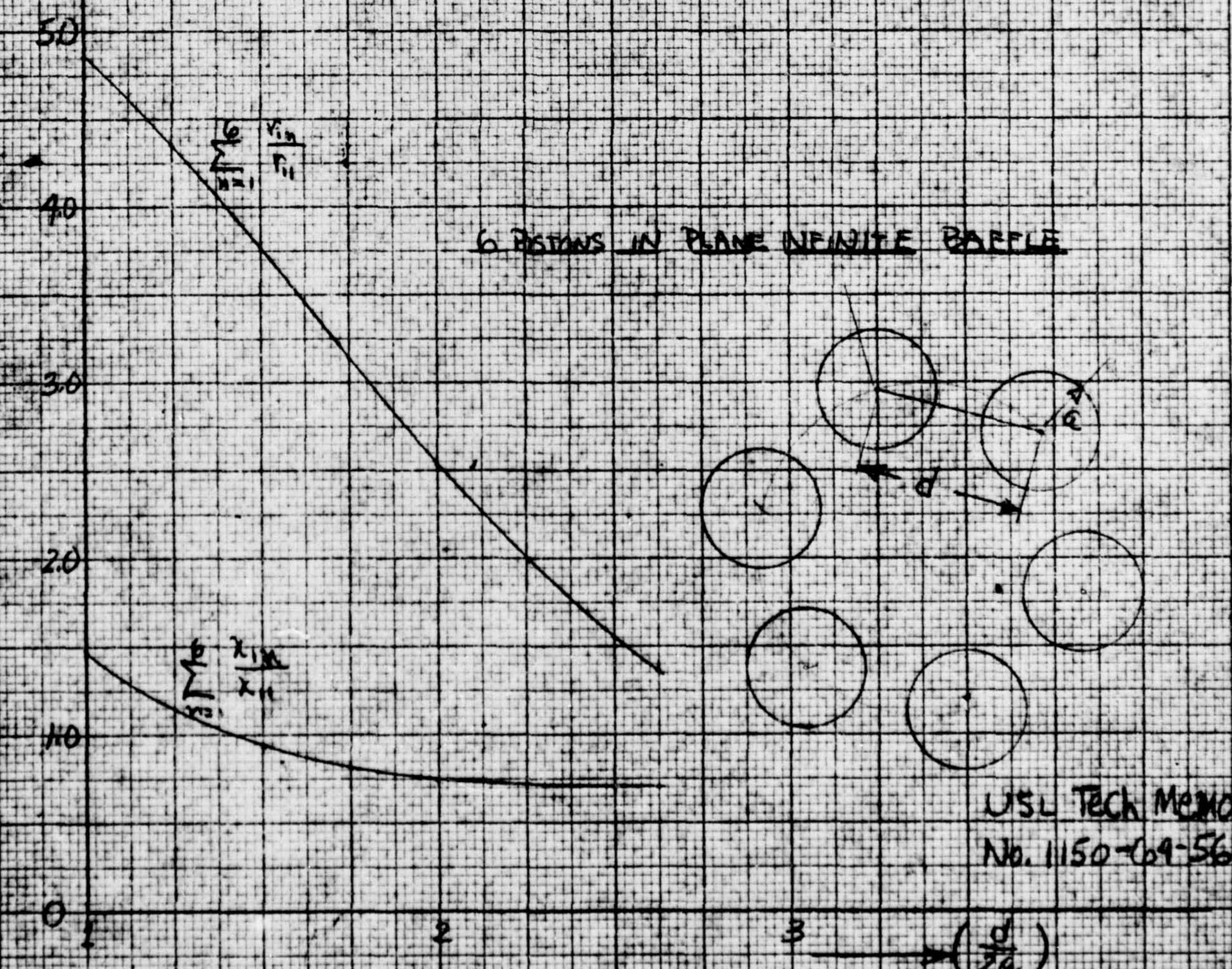
fig. 5

RADIATION IMPEDANCE PER PROJECTOR UNIT

$$z = \rho c \pi a^2 \left[r_{11} \sum_{n=1}^{6} \frac{r_{1n}}{r_{11}} + i \chi_{11} \sum_{n=1}^{6} \frac{\chi_{1n}}{\chi_{11}} \right]$$

$ka = 0.40$



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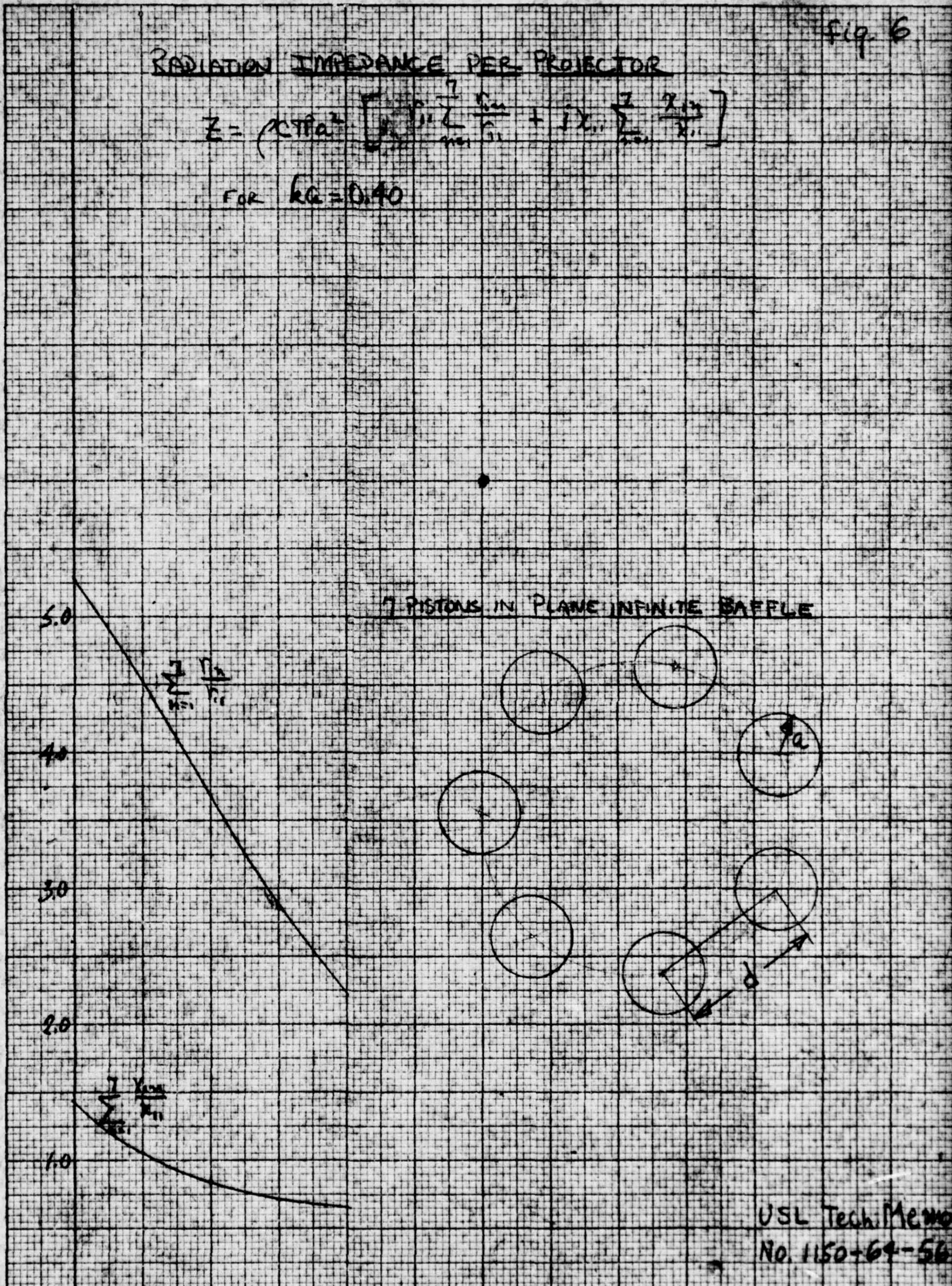

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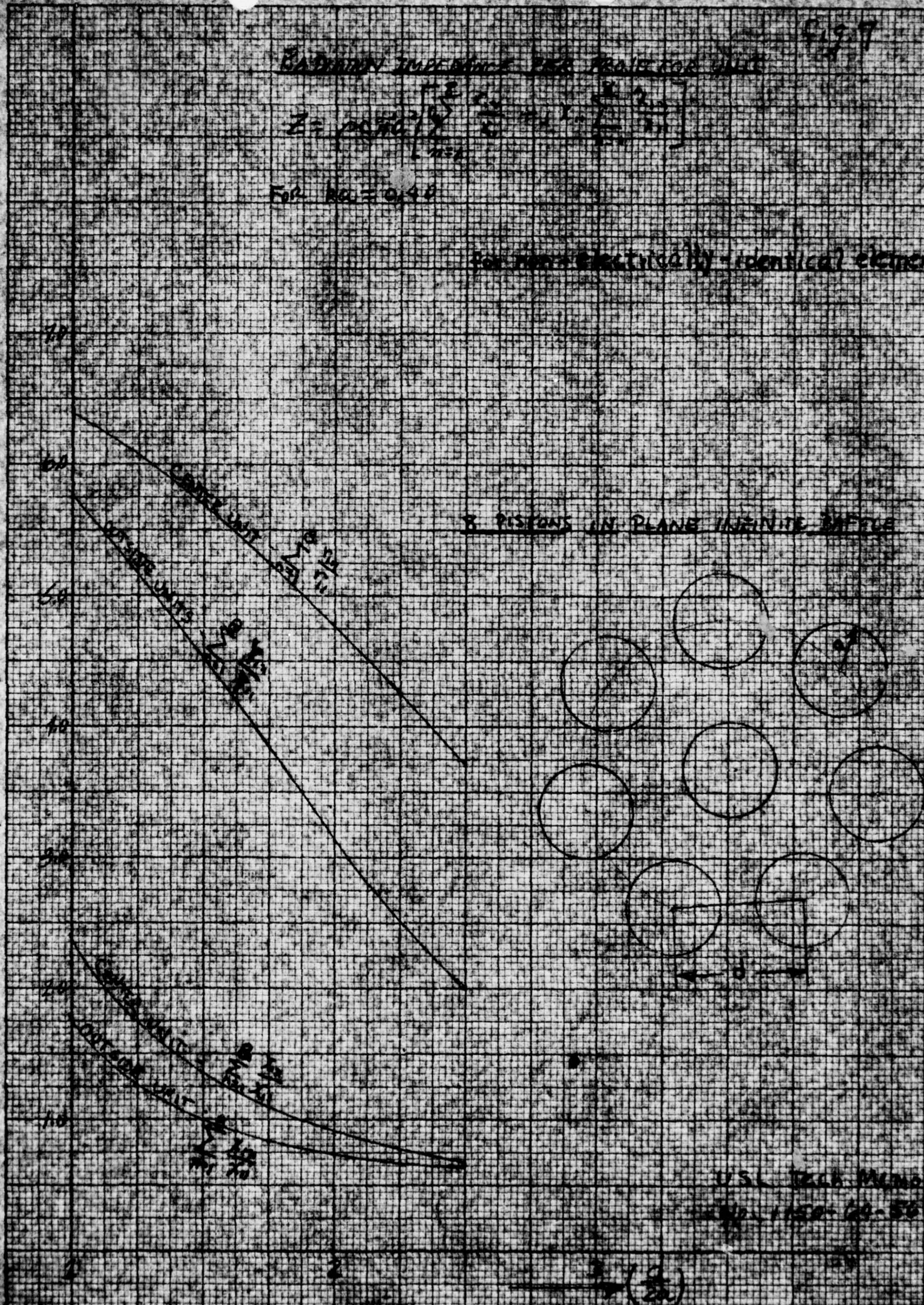
Fig. 7
for non-electrically-identical elements
8 PISTONS IN PLANE INFINITE BAFFLE
CENTER UNIT
OUTSIDE UNITS
CENTER UNIT
OUTSIDE UNIT
7.0
6.0
5.0
4.0
3.0
2.0
1.0
1
2
3
4
d
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