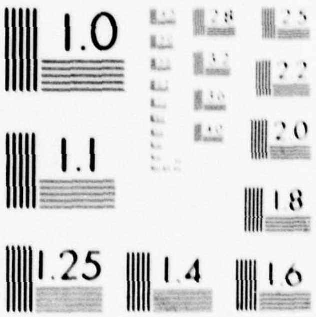
AD-A072 084 NATIONAL AVIATION FACILITIES EXPERIMENTAL CENTER ATL--ETC F/G 17/9
AN EVALUATION OF CERTAIN SELECTED MODIFICATIONS TO THE NATIONAL--ETC(U)
APR 79 R E LEFFERTS

UNCLASSIFIED FAA-NA-79-16 NL

1 OF 1

AD
A072084

END
DATE
FILMED
9-79
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

FAA-NA-79-16

AN EVALUATION OF CERTAIN SELECTED MODIFICATIONS TO THE NATIONAL AIRSPACE SYSTEM BIMODAL TRACKING ALGORITHM

12
B.S.

Robert E. Lefferts

APRIL 1979

NAFEC REPORT

Prepared for

U. S. DEPARTMENT OF TRANSPORTATION
FEDERAL AVIATION ADMINISTRATION
National Aviation Facilities Experimental Center
Atlantic City, New Jersey 08405

Technical Report Documentation Page

1. Report No.
FAA-NA-79-16

2. Government Accession No.

3. Recipient's Catalog No.

~~**4. Title and Subtitle**~~
6
AN EVALUATION OF CERTAIN SELECTED MODIFICATIONS TO THE NATIONAL AIRSPACE SYSTEM BIMODAL TRACKING ALGORITHM

~~**5. Report Date**~~
11
Apr[redacted]79

6. Performing Organization Code
ANA-220

7. Author(s)
10
Robert E. Lefferts

8. Performing Organization Report No.
14
FAA-NA-79-16

~~**9. Performing Organization Name and Address**~~
Federal Aviation Administration
National Aviation Facilities Experimental Center
Atlantic City, New Jersey 08405

~~**10. Work Unit No. (TRAIS)**~~

11. Contract or Grant No.
975-200-10A

12. Sponsoring Agency Name and Address
Federal Aviation Administration
National Aviation Facilities Experimental Center
Atlantic City, New Jersey 08405

12 41p.

13. Type of Report and Period Covered
NAFEC

14. Sponsoring Agency Code
ANA-1

15. Supplementary Notes
Continuation of work reported in FAA-NA-79-15

A072082

16. Abstract

In the design of a practical tracking algorithm, various nonlinearities are introduced which cannot be evaluated using the standard linear theory. For this reason these nonlinearities, which involve the interaction of the correlation region (search area) and the smoothing constants used in tracking, must be evaluated using simulation of the actual algorithm. Various modifications of the standard bimodal tracking algorithm have been considered, including a comparison of fixed versus dynamic search areas, circular versus noncircular search areas, the introduction of a one-scan delay before use of the large search area smoothing constants, and an evaluation of the influence of the search area design probability on the performance of the tracking algorithm. For a straight-line trajectory, the velocity errors observed using the fixed search area design were found to be approximately three times larger than those theoretically obtainable using an alpha-beta tracking algorithm. The use of a dynamic search area with one-scan delay was found to give a considerable improvement in the straight-line performance of the tracking algorithm, while at the same time small reductions were observed in the peak transient errors for maneuvering targets. Since the performance of the tracking algorithm is highly dependent on the size of the search area (which is based on the statistical properties of the radar errors) the search area parameters should be site-dependent so as to allow equivalent performance from all radar sites. The results obtained in this study show there is considerable room for improvement in the performance of the standard bimodal tracking algorithm, and this is true even without considering changes in the smoothing parameters or the use of such features as track-oriented smoothing.

17. Key Words

Tracking	Correlation Regions
α-β Filters	Search Radar
Radar Tracking	
Track-while-scan radars	

18. Distribution Statement
Document is available to the U.S. public through the National Technical Information Service, Springfield, Virginia 22161.

79 07 30 110

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	41	

Form DOT F 1700.7 (8-72)

Reproduction of completed page authorized

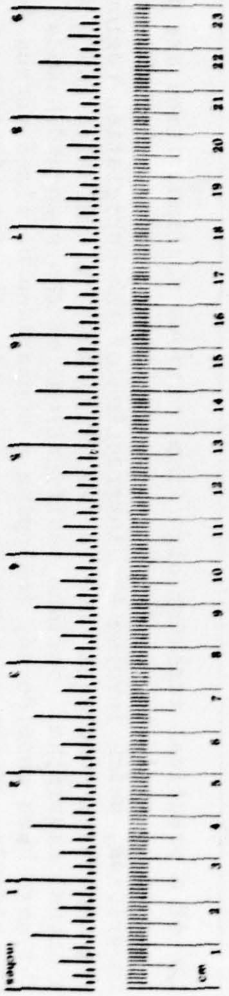
240550

METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in ≡ 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

TABLE OF CONTENTS

Page

LIST OF ILLUSTRATIONS

LIST OF TABLES

EXECUTIVE SUMMARY

The tracking algorithm is of fundamental importance in air traffic control because it performs the function of velocity estimation which is then used to predict the future position of an aircraft. The objective in this study is to determine if there is any room for performance improvement using modifications to the tracking algorithm which do not require significant computational resources for implementation., One of the possible modifications is a dynamic search area, for which a design procedure was developed previously (reference 1), and this modification provides a significant improvement in the tracking algorithm in terms of the ability to discriminate between straight-line and maneuvering targets. The results of this study show that the circular dynamic search area will give a level of tracking performance which is superior to either a non-circular dynamic search area or a circular fixed search area for both straight-line and maneuvering targets, thus confirming that significant tracking improvements can be achieved by modifications to the tracking algorithm which do not require significant computational resources.

1. INTRODUCTION

In a previous report (reference 1), a procedure was developed for the design of circular correlation regions or search areas which was based on an analysis of the statistical characteristics of the radar and tracking errors. In the present report, which is a continuation of the study of the search area, certain nonlinear interactions between the search area and tracking are considered which cannot be handled by the linear mathematical analysis used previously. As a consequence in this paper, the interaction between the search area and tracking is evaluated by means of a statistical simulation in which actual tracking statistics are used for comparison of the alternatives.

As an example of the type of questions which might be considered, it would be useful to know what should be used for the probability on which the search area design is based. In order to obtain reliable tracking, a very high probability should be used in the design of the search area in order to acquire data in all reasonable circumstances. However, in the case in which an alternative set of smoothing parameters is used for maneuvering targets and the choice between smoothing parameters is based on a radar return being inside or outside of a particular search area, then it might be preferable to use a smaller probability to design this search area so as to switch to the proper smoothing constants as soon as possible in order to follow a maneuver more precisely. Since the choice of smoothing parameters is a nonlinear operation, it cannot be analyzed using the standard analytical methods such as those used previously (reference 1) and must instead be evaluated by simulating the alternative choices and then making a determination based on the performance statistics of the simulation.

2. DESCRIPTION OF THE MONTE CARLO SIMULATION PROCEDURE

2.1 COMPARISON OF RESULTS.

The procedure which will be used in this report is to simulate the operation of a practical tracking algorithm in various scenarios which are typical of the en route air traffic control environment. At each cycle of operation of the tracking algorithm certain selected sample moments are calculated which describe the statistical behavior of the tracking algorithm at that particular cycle. The multiple-time series consisting of the selected sample moments constitute the data base which will be used for the comparison of various alternative formulations of the tracking algorithm.

In each scenario, there is one true track which is perturbed by random errors of the appropriate statistical characteristics corresponding to radar errors and which is repeated a number of times to generate a population of random radar returns for the specified true track. Each of the random radar returns is operated on independently by the tracking algorithm to form an ensemble of tracks for which the sample moments define the statistical characteristics of the simulation output at any particular moment in time.

In all the comparative simulations to be performed, and which will be discussed in later sections, it is important to note that the random number generator was initialized with the same seed, thus generating the exact same sequence of pseudorandom errors. As a consequence, the comparison of the simulation output can be made directly on the basis of the differences observed in the sample moments without the need to resort to statistical hypothesis tests to determine if any significant differences exist. Since the same sequence of errors is used as the input in all cases, any differences which are observed are directly due to the changes in the implementation of the tracking algorithm itself and are not due to random differences in the input sequence. In many cases, it will be found that the differences in the performance measures for different formulations of the tracking algorithm are so insignificant that these differences would most likely not be detected using hypothesis tests. In order to present such differences to the reader, it will be necessary to present tabular listings of the sample data rather than relying on a less-detailed graphical presentation.

2.2 TRACKING ALGORITHM.

The tracking algorithm of interest in this report is the en route tracking algorithm which is a bimodal tracker, since the weighting parameters can take two different sets of nonzero values depending on the deviation between the measured and predicted positions. At any epoch, k, the tracking algorithm is specified by the following equations:

$$\vec{X}_s(k)=\vec{X}_p(k)+\alpha\,\Delta\vec{r}(k) \tag{1}$$

$$\vec{V}_s(k)=\vec{V}_s(k-1)+\beta\,\Delta\vec{r}(k)/T \tag{2}$$

$$\vec{X}_p(k+1)=\vec{X}_s(k)+T\,\vec{V}_s(k) \tag{3}$$

$$\Delta\vec{r}(k)=\vec{X}_m(k)-\vec{X}_p(k) \tag{4}$$

where: $\vec{X}_s(k)$ = Estimated position

$\vec{V}_s(k)$ = Estimated velocity

$\vec{X}_p(k)$ = Predicted position

$\Delta r(k)$ = Track datum deviation

α = Position smoothing constant

β = Velocity smoothing constant

$X_m(k)$ = Position of the radar measurement

T = Scan time or measurement interval.

For simplicity, the problems of operation in a multiple radar site environment and asynchronous operation of the sensor with respect to the tracking algorithm will not be considered. With this simplifying assumption, it can then be assumed that the measurement data, $\vec{X}_m$, are available at a constant rate specified by the time interval T.

The track datum deviation, $\Delta\vec{r}(k)$, is used as the basis for choosing the value of the smoothing constants. The decision process for the smoothing constants can be defined as follows:

$$\alpha = \begin{cases} \alpha_S & \Delta\vec{r} \subset A_S \\ \alpha_L & \Delta\vec{r} \subset A_L \\ 0 & \Delta\vec{r} \not\subset A_L \end{cases} \tag{5}$$

$$\beta = \begin{cases} \beta_S & \Delta\vec{r} \subset A_S \\ \beta_L & \Delta\vec{r} \subset A_L \\ 0 & \Delta\vec{r} \not\subset A_L \end{cases} \tag{6}$$

where A_S and A_L are referred to as the small and large search areas, respectively. It should be noted that A_S is contained within A_L.

The search areas used in practice vary in size and shape according to the type of data being used and the magnitude of the track datum deviation (e.g., see reference 2). The smoothing constants used in the small search area are chosen to give a high degree of noise reduction (via relatively small values of α and β), while the smoothing constants used in the large search areas are chosen on the basis of the transient response characteristics (thus implying larger values of α and β). Various criteria can be used as the basis for choosing the smoothing constants in either search area, but it is usually intended that the small search area smoothing constants be used with straight-line tracks, while the large search area smoothing constants be used with maneuvering targets.

The need for multiple smoothing constants arises because smoothing constants which give a high degree of noise reduction are not satisfactory for following maneuvering targets and vice versa. When a target begins to maneuver, a bias will develop in the predicted position, which is based on the assumption of a constant-velocity, straight-line track, and in most cases this bias will eventually be sufficiently large to cause the tracker to switch to the larger smoothing constants in the large search area which will then remove the bias, thus causing the tracker to revert to the use of the small search area smoothing constants. The magnitude of the bias, which is observed in both position and velocity, is of considerable interest because advanced air traffic control features, such as Conflict Alert, require accurate estimates of future position in order to operate properly, and bias errors can cause significant problems in the operation of such features. As a result of the switching between alternative sets of smoothing constants, the tracking algorithm just described is referred to as a bimodal tracker.

In the case in which the track datum, $\vec{X}_m(k)$, falls outside the large search area, the use of a smoothing constant of zero implies that the data are not used at all because the track datum deviation is so large that it is unlikely that the radar return under consideration is actually from the target being tracked. The sources of false targets are of no concern in the present report. It is possible, however, that in some cases the target can maneuver with such a high acceleration that the true target return falls outside the large search area. In these cases, the association between the track and the target data will be broken, and the track will be lost. The design of the large search area to avoid such situations is discussed in the previous report (reference 1).

2.3 PERFORMANCE STATISTICS.

The sample moments which are used for the comparison of the tracking simulations are obtained in the following manner. The sensor measurements are assumed to consist of the sequences $\{\rho_T(k)\}$ and $\{\theta_T(k)\}$ where $\rho_T(k)$ and $\theta_T(k)$ are the true range and azimuth, respectively, of the target at time epoch k. In the simulation program, the sensor measurements are corrupted by errors, $\Delta\rho(k)$ and $\Delta\theta(k)$, which are generated in such a manner that the sequences $\{\Delta\rho(k)\}$ and $\{\Delta\theta(k)\}$ are each white and stochastically independent of one another. The simulated track data sequence $\{\rho_T(k)+\Delta\rho(k)\}$ and $\{\theta_T(k)+\Delta\theta(k)\}$ are transformed to a Cartesian coordinate system to give the simulated radar measurement sequence $\{\vec{X}_m(k)\}$ which is input to a digital filter defined by equations (1) to (4). In order to determine what constitutes the typical behavior of the tracking algorithm, the experiment of generating a random track data sequence for use as input to the tracking algorithm is repeated to generate an ensemble of responses, each of which is stochastically independent of one another. At each epoch, k, there are various sample moments (ensemble averages) which can be used to characterize the performance of the tracking algorithm for the true track being used as the basis for the simulation.

For each of the N simulated radar measurement sequences, the output of the digital filter consists of the sequences

$$\{X_s(k, i), Y_s(k, i)\}$$

$$\{V_x(k, i), V_y(k, i)\}$$

$$\{X_p(k, i), Y_p(k, i)\}$$

where X_s, Y_s = estimated position components

V_x, V_y = estimated velocity components

X_p, Y_p = predicted position components

and $i=1,\ldots,N$ where i is the population index and N is the sample size. Denoting the true position and velocity of the target as $\{X_T(k), Y_T(k)\}$ and $\{V_{Tx}(k), V_{Ty}(k)\}$, respectively, the following sequences of sample moments will be defined as the performance measures:

(a) Radial position error

$$r_e(k) = \frac{1}{N}\sum_{i=1}^{N} \sqrt{(X_s(k,\ i)-X_T(k))^2 + (Y_s(k,\ i)-Y_T(k))^2} \qquad (7)$$

(b) Radial velocity error

$$v_e(k) = \frac{1}{N}\sum_{i=1}^{N} \sqrt{(V_x(k,\ i)-V_{Tx}(k))^2 + (V_y(k,\ i)-V_{Ty}(k))^2} \qquad (8)$$

(c) Mean speed

$$\bar{v}(k) = \frac{1}{N}\sum_{i=1}^{N} \sqrt{V_x^2(k,\ i) + V_y^2(k,\ i)} \qquad (9)$$

(d) Mean heading error

$$\bar{\theta}(k) = \frac{1}{N}\sum_{i=1}^{N} \left[\begin{aligned} &\tan^{-1}(V_x(k,\ i)/V_y(k,\ i)) \\ &\qquad -\tan^{-1}(V_{Tx}(k)/V_{Ty}(k)) \end{aligned} \right] \qquad (10)$$

It is quite obvious that these are not the only performance measures which could be defined on the simulation output; however, it is thought that these measures are sufficient to characterize the simulation output for purposes of comparison. It should be noted that particular emphasis is being placed on the velocity output of the tracking algorithm because the prediction of future position required for advanced air traffic control features is so heavily dependent on accurate velocity estimates. In a later section, some additional performance measures will be defined which are of particular interest in the case of straight-line tracks. The same performance measures have been used in other studies of tracking performance. It must be emphasized, however, that it is not possible to make direct comparisons between the performance measures computed in this study and those found elsewhere because in many cases the results presented in other reports are for a multiple-site environment with asynchronous operation of the tracking algorithm with respect to the sensor, which is not the case in the present study. For this reason, discrepancies may exist between the results presented in this study and those presented elsewhere. Specifically, the peak errors in speed and heading observed in this study may be less than those found elsewhere.

2.4 STANDARD SCENARIOS AND PARAMETER VALUES.

In the evaluation of the modifications to the tracking algorithm, it is necessary to maintain a standard set of conditions which is used as a baseline against which to compare the change in performance due to the particular modification under consideration. The standard simulation parameters and scenarios are described in tables 1 and 2. Any deviation from these standard conditions will be explicitly noted in the section where the results of the deviation are discussed. For the purpose of this report, the various scenarios listed in table 2 are considered to be sufficient for evaluation of the tracking algorithm modifications under consideration, but are not to be considered as representing any particular field environment, but rather are representative of what might be encountered in any field environment.

It should also be noted that the tracking simulation procedure being used in this study (single radar site with fixed data interval and random errors initialized with the same seed number) is a very sensitive test bed for evaluation of minute differences in performance. In a more realistic environment (multiple radar sites with timing errors, fixed point computations, and nonrepeatable error sequences), it may not even be possible to detect any differences in performance for some of the modifications being examined in this report. For this reason, a simplified simulation procedure is to be preferred in some cases because a highly realistic, but complicated, simulation may not provide a sufficiently stable test bed for the detection of minute differences in performance. One additional point which should be noted is that all the simulations performed for this report were done with the master plane used as the coordinate system; i.e., it is implicitly assumed that all targets at different altitudes have been mapped onto a common coordinate system using an appropriate projection technique so that altitude variations can be eliminated from consideration.

3. SIMULATION RESULTS

The simulation results, in terms of performance measures discussed previously, for the various alternative formulations of the tracking algorithm will be given in the following sections. Since the differences observed are, in many cases, extremely minute, it will be necessary to present the results in a tabular form rather than a more desirable graphical presentation.

3.1 VARIATION IN THE SIZE AND SHAPE OF THE SEARCH AREA.

Several variations of both the size and the shape of the search area have been examined in the past. The search area definitions specified in table 1 are referred to as a dynamic search area (reference 3) because the sizes are a function of both the distance to the radar and the velocity of the target, but the shape of the region is circular in all cases. The parameters used in the search area definitions in table 1 are based on a previous analysis (reference 1). Since one of the functions of the correlation regions is to determine which smoothing constants are applicable to the particular situation

TABLE 1. NOMINAL SIMULATION PARAMETERS

Population sample size, N:	250
Total simulation duration, N_s:	50 scans
Scan time, T:	10 seconds
Blip/scan ratio:	1.0
Radar range standard deviation, σ_ρ	0.125 nmi
Radar azimuth standard deviation, σ_θ	3 ACP* (4096 ACP/2π radians)
Search area design probability:	0.95
A_S, Small search area	
Shape	Circular
Radius, r_S	$r_S=\max(0.29,0.0102\rho)$ $\rho = \sqrt{X_s^2+Y_s^2}$
A_L, Large Search Area	
Shape	Circular
Radius, r_L	$r_L=r_S+VT$ $V= \sqrt{V_x^2+V_y^2}$
Smoothing parameters	
Small search area	$\alpha_S=0.3125$ $\beta_S=0.046875$
Large search area	$\alpha_L=0.5$ $\beta_L=0.15625$
Program precision	Floating-point
Programming language	JOVIAL and FORTRAN

---

* Azimuth change pulses

TABLE 2. NOMINAL SCENARIO PARAMETERS

Radar position:	X=100, Y=0 nmi	
Start of turns:	20th scan	
Location of start of turn:	X=100, Y=60 nmi	
90° Turn		
Turn rate	3°/second	
Initial conditions		
1. Position	X=89.44	Y=60.0
Velocity	$V_x=200$	$V_y=0$
2. Position	X=100.0	Y=49.44
Velocity	$V_x=0$	$V_y=200$
100° Turn		
Turn rate	2°/second	
Initial conditions		
1. Position	X=78.88	Y=60.0
Velocity	$V_x=400$	$V_y=0$
2. Position	X=100.0	Y=38.88
Velocity	$V_x=0$	$V_y=400$

detected (i.e., straight-line smoothing constants for small search area data and maneuvering smoothing constants for large search area data), it is apparent that the search areas should be designed to make an accurate distinction between the two cases. In order to perform this function properly, it has been suggested that a noncircular search area would be more appropriate, since this could more closely match the statistical distribution of the radar errors.

Several noncircular search areas have been proposed (reference 2), but all are generally based on the concept that, since the error distribution is a bivariate Gaussian distribution, the small search areas should be elliptical (reference 4) or some approximation to an ellipse. If the shape of the small search area were to be taken as an equidensity contour of the probability density function, then this would provide a more accurate discrimination between straight-line and maneuvering targets and so would lead to reduced tracking errors in the maneuvering case by switching to the large search area smoothing constants more rapidly than in the case of a circular search area.

In order to test the conjecture that a noncircular small search area would result in lower tracking errors by detecting maneuvers more rapidly, a series of simulations was run in which the same sequence of random errors was used with the only difference being the shape of the small search area. In order to perform the comparison on approximately the same basis, the probability (0.95) used to design the search areas was the same in both cases. For the noncircular small search area, a polar differential element was used, and since the range and azimuth errors are independent, the size in each dimension was taken as the centered region which contains $\sqrt{0.95}$ of the total distribution or 2.24σ. Although it might be thought that the noncircular search area defined using the polar differential element with dimensions determined in this manner would be an exact equivalent of the circular search area (on an equiprobability basis) this is not the case, and the reason for this will be discussed later. The results of these simulations, expressed in terms of the performance measures defined in Section 2.3 and for the scenarios in Section 2.4, are given in table 3.

In order to determine whether the aspect of the track with respect to the radar was of any importance, simulations were run in both the x and y directions. Since only the results during the maneuver are of interest, the performance measures are given for several scans following the start of the maneuvers. The simulation had already run for 20 scans before the start of the maneuver so that any initiation transients will have been eliminated by this time. Since the differences between the circular and noncircular search areas were found to be second-order effects, it was necessary to present the results in a tabular form, because no differences would be observed in a graphical presentation. Note that the maneuver only exists for scans one to five for the 400-knot results and from scans one to three for the 200-knot results. Also, the mean heading error is negative in all cases, indicating a lagging relationship with the true heading.

TABLE 3. SIMULATION RESULTS FOR COMPARISON OF CIRCULAR AND NONCIRCULAR SEARCH AREAS

	Scans Since Start of Turn	V_x=400		V_y=400		V_x=200		V_y=200	
		Circle	Non-circular	Circle	Non-circular	Circle	Non-circular	Circle	Non-circular
Radial Position Error	1	0.2012	0.1991	0.1847	0.1897	0.1809	0.1825	0.1715	0.1772
	2	0.3994	0.3752	0.3863	0.3848	0.3381	0.3029	0.3153	0.3181
	3	0.6165	0.5971	0.6097	0.6039	0.4785	0.4470	0.4734	0.4616
	4	0.8010	0.7878	0.7929	0.7876	0.5410	0.5181	0.5377	0.5235
	5	0.9319	0.9245	0.9318	0.9279	0.5058	0.4891	0.5109	0.4964
	6	0.9357	0.9323	0.9476	0.9450	0.4255	0.4051	0.4436	0.4181
	7	0.8000	0.7989	0.8130	0.8116	0.3414	0.3164	0.3625	0.3221
	8	0.6287	0.6288	0.6298	0.6293	0.2920	0.2672	0.3059	0.2626
Radial Velocity Error	1	135.5	132.9	132.4	132.6	101.4	100.2	99.0	99.4
	2	235.5	230.5	235.5	234.8	178.3	170.5	175.7	175.2
	3	297.8	295.2	298.1	296.8	215.0	210.9	216.3	213.1
	4	331.9	331.0	331.2	330.4	158.0	155.8	158.2	155.6
	5	346.3	346.2	346.2	345.9	105.7	104.5	106.6	104.5
	6	243.5	243.8	242.2	244.1	65.3	63.1	67.9	63.6
	7	154.9	155.2	155.2	155.3	38.8	35.9	42.3	35.1
	8	89.1	89.4	89.0	89.1	27.4	25.1	30.3	24.2
Mean Speed	1	399.2	398.7	400.3	400.1	199.1	198.8	200.5	200.3
	2	393.4	392.7	395.4	394.4	193.0	192.6	196.3	194.8
	3	379.2	379.4	380.6	379.5	176.5	177.7	180.9	178.0
	4	361.9	362.3	361.6	361.2	164.6	166.2	166.5	165.0
	5	345.4	345.6	345.5	345.5	166.0	167.1	167.1	166.5
	6	342.5	342.2	342.4	342.4	174.9	176.8	176.6	176.8
	7	357.7	357.2	356.2	356.3	185.0	188.5	187.4	188.1
	8	378.0	377.5	378.7	378.7	191.2	195.9	196.8	196.9
Mean Heading Error (All Negative)	1	19.41	19.02	19.04	19.06	29.13	28.78	28.63	28.73
	2	34.43	33.69	34.44	34.37	53.69	51.16	52.64	52.71
	3	44.76	44.33	44.83	44.67	69.12	67.34	69.10	68.47
	4	51.27	51.10	51.28	51.16	49.84	48.92	50.17	49.40
	5	54.69	54.67	54.77	54.71	31.35	30.96	32.01	31.25
	6	37.22	37.27	37.36	37.35	18.07	17.45	19.02	17.51
	7	22.65	22.69	22.63	22.65	9.92	9.23	11.06	8.63
	8	12.65	12.68	12.41	12.43	5.76	5.27	6.79	4.09

Using the results presented in table 3, the following observations were made:

(a) The peak radial position error and the peak mean speed error occur on the scan after the turn has been completed.

(b) The peak radial velocity error and the peak mean heading error occur on the last scan of the turn.

(c) For the radial position error no significant differences were observed between the results obtained using a circular search area and those obtained using a noncircular search area; however, on a relative basis, the differences were larger for the slower moving targets.

(d) During the maneuver, the maximum difference in the radial velocity error (comparing the circular and noncircular cases) occurs on the second scan of the turn, and the differences are more pronounced for trajectories in the x direction than for those in the y direction.

(e) In terms of the differences observed at the scans where the peak errors occurred, it was found that in all cases the differences between the noncircular and circular cases were insignificant. It was also noted that the maximum difference in the comparison does not occur at the point of maximum error.

(f) In the case of the performance differences for the mean speed for the circular and noncircular search areas, it was found that in most cases these differences were less than 1.5 knots, while for the mean heading error the performance differences were less than 1° in most cases.

The only differences in performance between the circular and noncircular search areas which could reasonably be attributed to a more rapid switching of the smoothing constants are those observed in the radial velocity error on the second scan of the maneuver in the case of tracks with velocity components in the x-direction. In the situation just described, the crosstrack deviations would be in the radial direction, which is the smaller dimension of the small search area at this distance. The fact that some differences between the performance for circular and noncircular search areas were observed indicates that in some cases a noncircular search area can detect a maneuver one scan earlier than a circular search area, but the tracking improvements which are achieved by this early detection are so insignificant that the heavy computational burden which would be imposed by the use of a noncircular search area is unjustified with respect to the performance improvement obtained.

It was noted previously that the circular and noncircular search areas are not exactly equivalent with respect to the design probability. The reason for this is the fact that the influence of tracking errors was included in the design of the circular search area but was not included in the design of the noncircular search area which was based solely on the sensor errors. In a previous report, (reference 1), it was found that the variance of the errors on which the search area is based is 28 percent larger than that of the sensor itself. Assuming that this increase holds for each of the orthogonal errors (range and azimuth) used to design the noncircular search area, then the standard deviation in each case should be 13 percent larger than that actually used. Assuming this to be the case, the design probability for the noncircular search area was actually on the order of 0.90 rather than 0.95; however, if this is true, then the performance increases are actually due to two effects; namely, the noncircular search area and the reduction in the design probability which would yield smaller search areas that would result in better maneuver detection. Therefore, if the noncircular search area had been designed using the same design probability as the circular search area, the noncircular search area would have been slightly larger, thus giving even less of a performance improvement than that observed in table 3, and since this improvement was insignificant, the basic conclusion reached previously is still valid; namely that the use of a noncircular search is unjustified.

The operational air traffic control computer program has used fixed search areas rather than the dynamic search area approach described above. Typically, the size of the small search area was taken as 1.0 nmi, and the large search area was 4.0 nmi. For completeness, two additional simulations were run to illustrate the performance differences between fixed and dynamic search areas, and these results are given in table 4 along with the corresponding results from table 3 which are included for easy comparison. The results for the circular dynamic search areas in table 3 will, in most cases, constitute the baseline against which other tracking modifications will be compared. The results presented in table 4 show the following:

(a) In almost all cases, the performance measures show that the fixed search area results in greater errors than the dynamic search area; however, the magnitude of the difference is only marginally significant.

(b) In all cases, the peak errors were greater using the fixed search area than those observed using the dynamic search area.

(c) On a relative basis, the difference in the errors between the fixed and dynamic search areas was greater for the slower moving target than for the higher velocity target.

While the differences between the performance results in table 4 for the fixed and dynamic search areas are what might be referred to as marginally significant, it must be noted that, since azimuthal errors have greater significance with increasing range from the sensor, the performance differences between fixed and dynamic search areas will vary according to the distance from the radar and the direction of the track. Since it is desirable to maintain the same level of performance throughout the radar coverage area, the dynamic search area, which is the means by which this goal is achieved, has a greater significance than the results presented in table 4 indicate. Suffice it to say that the efficacy of the dynamic search area has been demonstrated elsewhere (reference 3) so that it is not necessary to consider the fixed search area to any greater extent with the exception of the results given in Section 3.5.

TABLE 4. SIMULATION RESULTS FOR COMPARISON OF FIXED VERSUS DYNAMIC SEARCH AREAS

	Scans Since Start of Turn	V_x=400		V_x=200	
		Dynamic Search Area	Fixed Search Area	Dynamic Search Area	Fixed Search Area
Radial Position Error	1	0.2012	0.1880	0.1809	0.1655
	2	0.3994	0.4879	0.3381	0.3747
	3	0.6165	0.6939	0.4785	0.5848
	4	0.8010	0.8539	0.5410	0.6476
	5	0.9319	0.9628	0.5058	0.6255
	6	0.9357	0.9512	0.4255	0.5658
	7	0.8000	0.8096	0.3414	0.4882
	8	0.6287	0.6685	0.2920	0.4369
Radial Velocity Error	1	135.5	135.9	101.4	101.4
	2	235.5	256.4	178.3	188.6
	3	297.8	309.5	215.0	234.5
	4	331.9	336.7	158.0	174.0
	5	346.3	347.3	105.7	122.6
	6	243.5	243.2	65.3	85.3
	7	154.9	154.7	38.8	58.5
	8	89.1	95.8	27.4	44.4
Mean Speed	1	399.2	399.3	199.1	199.3
	2	393.4	395.8	193.0	195.8
	3	379.2	378.3	176.5	179.8
	4	361.9	360.0	164.6	161.7
	5	345.4	344.3	166.0	159.5
	6	342.5	342.9	174.9	165.7
	7	357.7	359.0	185.0	174.2
	8	378.0	376.1	191.2	180.4
Mean Heading Error (All Negative)	1	19.41	19.56	29.13	29.35
	2	34.43	37.54	53.69	56.81
	3	44.76	46.70	69.12	75.85
	4	51.27	52.20	49.84	56.05
	5	54.69	54.94	31.35	37.43
	6	37.22	37.16	18.07	24.45
	7	22.65	22.64	9.92	15.75
	8	12.65	13.68	5.76	11.38

3.2 DELAY IN THE USE OF LARGE SEARCH AREA SMOOTHING CONSTANTS.

Several previous studies (references 5-7) have shown that there is some advantage in using different smoothing parameters for the first large search area return instead of the normal set of smoothing parameters used in the large search area. The justification for giving special consideration to the first large search area return is the possibility that the return may be spurious. Spurious radar returns can arise in many different ways depending on the type of radar being considered. For search radar data there is the possibility of radar returns from weather clutter rather than aircraft returns, while for beacon systems there are false returns due to reflections and also valid beacon returns from targets using the same beacon code (or nondiscrete codes) which may be incorrectly used in the tracking process. In the case of a straight-line track, even for a valid large search area return for the target of interest, the use of the large search area smoothing constants may result in an undesirable perturbation of the track. For these reasons, there is considerable merit in treating the first large search area return in a different manner than subsequent large search area returns. The alternatives which might be considered for the first large search area return range from discarding the return altogether (because it is "probably" a false return) to applying a very high weighting factor (because it is "probably" the start of a maneuver). The evaluation of these alternatives is dependent on the degree to which a large search area return is thought to represent a maneuvering target or is thought to represent a false return and the penalties associated with an incorrect decision. Generally, the use of an erroneous data point or application of the large search area smoothing constants to a straight-line track are viewed as far more serious errors than the very slightly higher bias errors which result from waiting one scan to apply the large search area smoothing constants. Results which will be presented in a later section will illustrate the specific performance penalties associated with the incorrect use of the large search area smoothing constants in the case of a straight-line track.

For the purposes of this study, a compromise between the requirement to eliminate false targets and the requirement to minimize the bias in a maneuver will be made; namely, the first large search area return will be weighted with the small search area smoothing constants. In all cases when a delay in the use of the large search area smoothing constants is specified, this implies that the first large search area return is weighted with the small search area smoothing constants. Since the considerations which determine the parameter to be used on the first large search area return are more of an operational and procedural nature (in the case of false targets) rather than being quantifiable in a mathematical sense, it is not possible to formulate the parameter choice decision into any of the standard optimization problem formulations.

The results of the simulations in which the use of the large search area smoothing constants is delayed one scan and the small search area smoothing constants are used for the first large search area return are given in table 5. The baseline results, from table 3, are also given in table 5 under the columns marked "No Delay." The results in table 5 show the following:

TABLE 5. SIMULATION RESULTS FOR COMPARISON OF A DELAY IN THE USE OF LARGE SEARCH AREA SMOOTHING CONSTANTS

	Scans Since Start of Turn	V_x=400		V_y=400		V_x=200		V_y=200	
		No Delay	Delay	No Delay	Delay	No Delay	Delay	No Delay	Delay
Radial Position Error	1	0.2012	0.1900	0.1847	0.1787	0.1809	0.1676	0.1715	0.1595
	2	0.3994	0.4857	0.3863	0.4862	0.3381	0.3735	0.3153	0.3691
	3	0.6165	0.7356	0.6097	0.7952	0.4785	0.6093	0.4734	0.6167
	4	0.8010	0.8883	0.7929	0.9326	0.5410	0.6420	0.5377	0.6583
	5	0.9319	0.9868	0.9318	1.0190	0.5058	0.5741	0.5109	0.5900
	6	0.9357	0.9658	0.9476	0.9954	0.4255	0.4642	0.4436	0.4882
	7	0.8000	0.8140	0.8130	0.8365	0.3414	0.3687	0.3625	0.4001
	8	0.6287	0.6336	0.6298	0.6384	0.2920	0.3189	0.3059	0.3430
Radial Velocity Error	1	135.5	135.9	132.4	136.4	101.4	101.3	99.0	101.8
	2	235.5	256.0	235.5	257.4	178.3	188.2	157.7	188.7
	3	297.8	318.7	298.1	331.3	215.0	240.8	216.3	242.4
	4	331.9	341.8	331.2	348.1	158.0	172.1	158.2	175.4
	5	346.3	349.4	346.2	352.0	105.7	111.6	106.6	114.1
	6	243.5	243.8	242.2	245.2	65.3	66.0	67.9	69.1
	7	154.9	153.9	155.2	154.1	38.8	39.6	42.3	44.0
	8	89.1	88.0	89.0	87.5	27.4	30.1	30.3	33.6
Mean Speed	1	399.2	398.9	400.3	399.8	199.1	198.9	200.5	199.9
	2	393.4	396.4	395.4	396.9	193.0	196.2	196.3	196.8
	3	379.2	381.1	380.6	383.5	176.5	184.3	180.9	185.1
	4	361.9	360.1	361.6	358.8	164.6	163.6	166.5	164.0
	5	345.4	343.6	345.5	342.2	166.0	163.3	167.1	163.2
	6	342.5	342.5	342.4	341.8	174.9	175.0	176.6	175.4
	7	357.7	359.4	356.2	358.6	185.0	185.7	187.4	187.4
	8	378.0	380.6	378.7	382.5	191.2	191.1	196.8	195.7
Mean Heading Error (All Negative)	1	19.41	19.55	19.04	19.63	29.13	29.35	28.63	29.50
	2	34.43	37.49	34.44	37.69	53.69	56.67	52.64	56.78
	3	44.76	48.04	44.83	49.97	69.12	77.41	69.10	77.95
	4	51.27	53.07	51.28	54.33	49.84	55.15	50.17	56.66
	5	54.69	55.36	54.77	56.00	31.35	33.39	32.01	34.70
	6	37.22	37.26	37.36	37.54	18.07	18.31	19.02	19.35
	7	22.65	22.51	22.63	22.48	9.92	10.27	11.06	11.71
	8	12.65	12.53	12.41	12.24	5.76	6.71	6.79	7.86

(a) In almost all cases, the performance measure results show that when delay is introduced the errors are greater than when no delay is used; however, the magnitude of the differences is only marginally significant even in the worst case.

(b) In all cases, the peak errors were greater when delay was introduced; however, in some cases, the peak error occurred one scan earlier (radial position error for 400-knot targets), while in other cases the peak error occurred one scan later (mean speed for 200-knot targets).

(c) On a relative basis, the differences in the errors between the delay and no delay simulation results are generally greater for the slower moving targets than for the higher velocity targets.

(d) In a few cases (usually in scans one and two or scans seven and eight), the performance of the tracking algorithm with delay was slightly better than that obtained without delay, but the magnitude of the improvement was insignificant.

The results obtained in these simulations generally show the expected trends; namely, that the bias errors in the case where a delay in the use of the large search area smoothing parameters is introduced are larger than the case where no delay occurs; however, the differences are considered to be marginally significant.

As will be shown in a later section, there is considerable justification to accept the slightly larger bias errors which are found in maneuvers when delay is introduced because of the performance improvement of the bimodal tracking algorithm for straight-line tracks. In no case are any of the slight additional errors due to delay thought to be significant, and, if necessary, the same level of performance could probably be obtained with delay as without delay by changing the values of the smoothing constants used. In fact, the use of delay, with its implicit ability to more reliably descriminate maneuvers from spurious returns, may allow the use of larger smoothing constants than would otherwise be considered acceptable.

Examining some of the additional simulation results, it was noted that the number of large search area returns actually smoothed with the large search area smoothing constants was significantly reduced when delay was introduced, as would be expected. The actual number of returns observed in each of the simulations is given in table 6, and in the case where delay was used, it is also noted how many of the large search area (LSA) returns were actually smoothed with the large search area smoothing parameters. The magnitude of the reduction between the total number of large search area returns and the number smoothed with the large search area parameters in the case where delay was used indicates that there must be a considerable number of cases where the large search area returns occur individually or in pairs in order to achieve the reductions observed. In the case of the 200-knot velocity simulation, the number of large search area returns actually smoothed with the large search area parameters was on the order of half the total number of large search area returns. As would be expected, the number of large search area returns is greater for the higher velocity target, which indicates a greater position bias, as illustrated by the higher values for the radial position error.

TABLE 6. NUMBER OF LARGE SEARCH AREA RETURNS OBSERVED UNDER VARIOUS CONDITIONS

	$V_x=400$	$V_y=400$	$V_x=200$	$V_y=200$
No Delay	3105	2864	2197	2114
With Delay				
LSA Parameters Used	1971	1836	1124	1080
Total LSA	2997	2761	2151	2071
Total Returns (LSA and SSA)	12500	12500	12500	12500

One other point which should be noted is that the total number of large search area returns observed when delay is used is less than the total number observed without the delay. This reduction indicates that several percent of the large search area returns observed when no delay is introduced are probably caused by the erroneous application of the large search area smoothing constants, since it would be expected that the higher bias would result in an increase in the number of large search area returns in a maneuver. This effect will be examined further in the section dealing with the performance of the tracking algorithm for straight-line tracks.

3.3 VARIATION OF THE PROBABILITY USED TO DESIGN THE SMALL SEARCH AREA.

In all the simulations used for the results presented in previous sections in which a dynamic search area was used, the design probability used to specify the size of the small search area was 0.95. The design probability specifies the theoretical probability that a valid return will be in the small search area for a target flying in a straight-line constant velocity trajectory. The choice of the probability used in the design to this point (0.95) was rather arbitrary, with the prime consideration being given to reliably acquiring the valid data in a high percentage of cases. An alternative consideration must now be examined; namely, for the bimodal tracking algorithm to switch to the proper smoothing constants, it would be desirable to make the small search area as small as possible in order to detect a maneuver before a significant bias can develop. Since the goals of reliable data acquisition and rapid maneuver detection are antithetical, one can only be achieved at the expense of the other. In order to determine the magnitude of the effect on the tracking biases, the probability used to design the small search area will be reduced. Since the size of the large search area is also dependent on the size of the small search area, any reduction in the small search area will also cause a reduction in the size of the large search area; however, the magnitude of the reduction is negligible, in most cases, compared to the velocity-time product (see table 1). In addition, the large search area was designed on a worst case basis, which should assure that the slight reduction in the size should be of no consequence.

The final basis for the choice of the probability used to design the small search area must be the tracking performance in maneuvers, so a quantitative measure of the change in tracking performance versus search area size must be obtained. From an operational viewpoint, it would be possible to determine if the small search area were too large simply by noting the number of large search area returns obtained on straight-line tracks, since some should be observed if the size of the small search area has been chosen properly. In order to design the small search area on an analytical basis, the numerical procedure used in the previous report (reference 1) was used to calculate the two parameters needed to define the small search area as a function of range: the minimum value and the slope, or rate of increase with range, and these two parameters are given in figures 1 and 2 as functions of the radar range and azimuth errors for various design probabilities. The two search area parameters are primarily a function of two variables, but in order to simplify the presentation of the results one variable was held constant while the other one was varied instead of varying both simultaneoulsy. This approach can be used because at short ranges the minimum search area size is primarily a function of the range errors and is independent of range, while at large ranges the search area size is primarily determined by the azimuthal errors so the size is specified by the rate of increase with range. As a consequence, the minimum search area size was calculated using the radius obtained from the results for a distance of 10 nmi, while the slope was calculated from the results at 200 nmi. The approach just described was satisfactory for the range of values under consideration except at very low values of the range error standard deviation (<0.06) where the azimuthal errors become of greater significance and cause the deviation from a linear relationship shown in figure 1. The results in figures 1 and 2 are also dependent on other parameters not discussed in the present report (such as the smoothing parameters), so these results cannot be generalized to other situations without considering the effect of the variations with the other parameters, but these variations should be second-order effects.

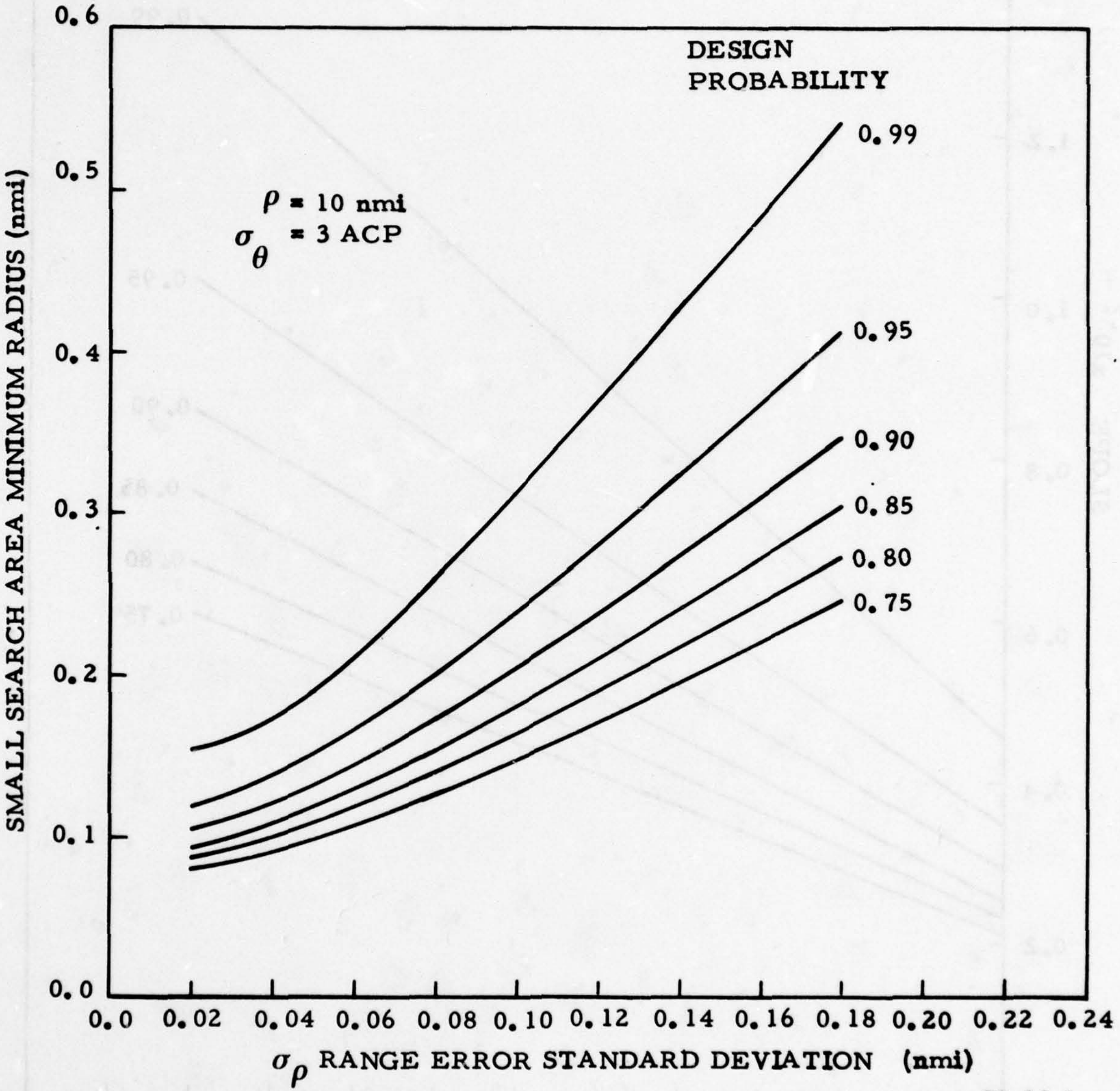
The data in figures 1 and 2 were used to define a small search area with a design probability of 0.75, and a series of simulations was then run to determine the impact of this change on the performance of the tracking algorithm. The results of these simulations are given in table 7, along with the baseline results for a design probability of 0.95, and the results presented show the following:

(a) As in other cases, the impact of the reduction in the design probability was found to be greater for the slower moving targets.

(b) The peak errors occur at the same scan for both design probabilities.

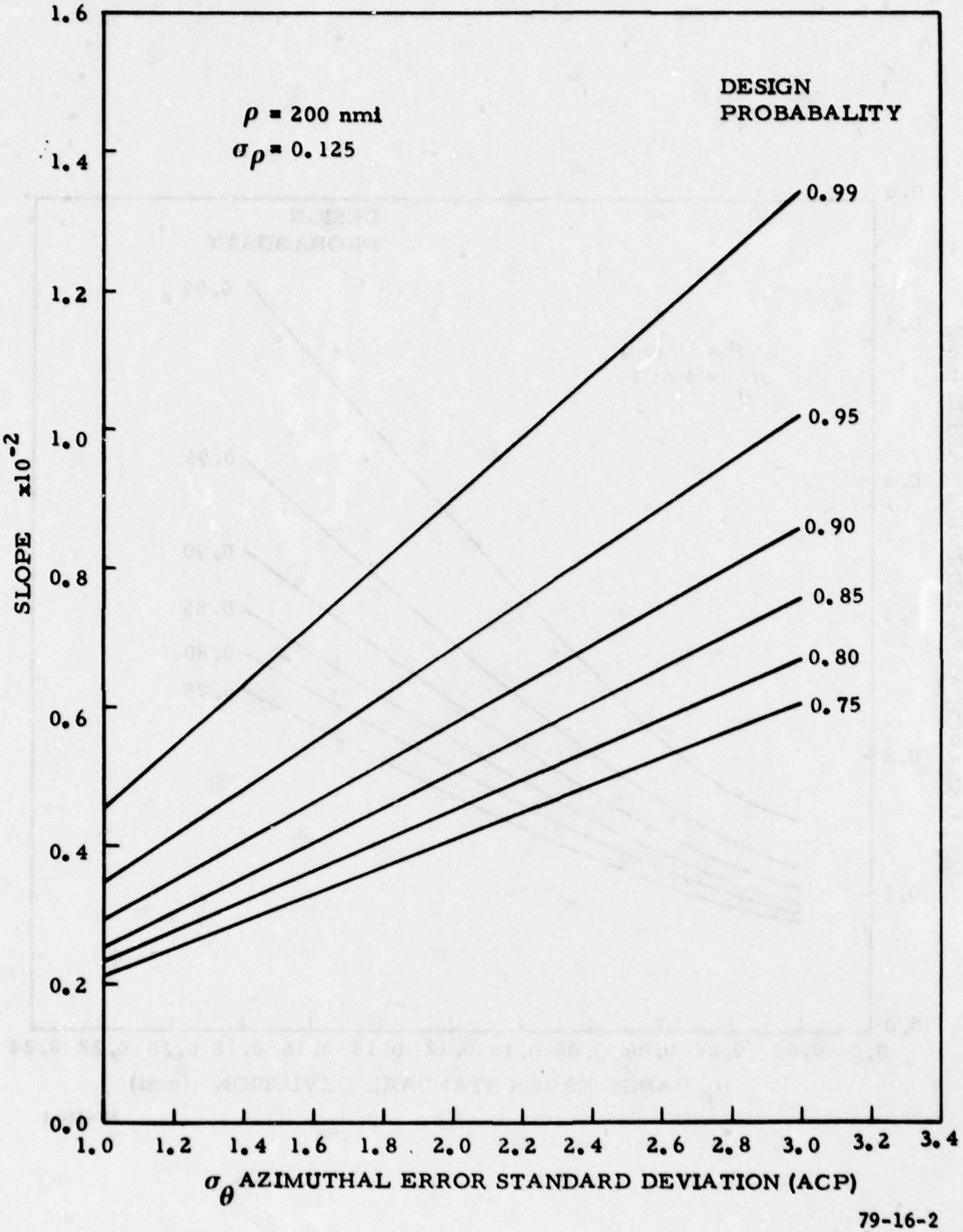
(c) In general, the reduction in the design probability from 0.95 to 0.75 resulted in a reduction in the bias errors of

- several hundreths of a nautical mile in position
- several knots in velocity
- and one to two degrees in heading.



79-16-1

FIGURE 1. MINIMUM SEARCH AREA RADIUS AS A FUNCTION OF RADAR RANGE ERROR STANDARD DEVIATION



79-16-2

FIGURE 2. SLOPE USED FOR SEARCH AREA RADIUS CALCULATIONS AS A FUNCTION OF AZIMUTH ERROR STANDARD DEVIATION

TABLE 7. SIMULATION RESULTS FOR COMPARISON OF SMALL SEARCH AREAS DESIGNED WITH CORRELATION PROBABILITIES OF 0.95 AND 0.75

	Scans Since Start of Turn	V_x=400		V_y=400		V_x=200		V_y=200	
		P=0.95	P=0.75	P=0.95	P=0.75	P=0.95	P=0.75	P=0.95	P=0.75
Radial Position Error	1	0.2012	0.2038	0.1847	0.1890	0.1809	0.1887	0.1715	0.1776
	2	0.3994	0.3751	0.3863	0.3582	0.3381	0.3023	0.3153	0.2885
	3	0.6165	0.5959	0.6097	0.5827	0.4785	0.4445	0.4734	0.4351
	4	0.8010	0.7868	0.7929	0.7741	0.5410	0.5157	0.5377	0.5063
	5	0.9319	0.9240	0.9318	0.9205	0.5058	0.4868	0.5109	0.4865
	6	0.9357	0.9320	0.9476	0.9415	0.4255	0.4019	0.4436	0.4135
	7	0.8000	0.7987	0.8130	0.8104	0.3414	0.3074	0.3625	0.3188
	8	0.6287	0.6287	0.6298	0.6293	0.2920	0.2506	0.3059	0.2578
Radial Velocity Error	1	135.5	132.5	132.4	130.1	101.4	99.7	99.0	96.9
	2	235.5	230.1	235.5	229.5	178.3	169.9	175.7	168.7
	3	297.8	295.0	298.1	294.0	215.0	210.3	216.3	209.7
	4	331.9	330.9	331.2	329.4	158.0	155.5	158.2	154.2
	5	346.3	346.1	346.2	345.8	105.7	104.2	106.6	104.2
	6	243.5	243.7	242.2	244.3	65.3	62.7	67.9	63.9
	7	154.9	155.2	155.2	155.6	38.8	34.0	42.3	35.3
	8	89.1	89.4	89.0	89.3	27.4	22.5	30.3	24.3
Mean Speed	1	399.2	398.7	400.3	400.1	199.1	198.8	200.5	200.4
	2	393.4	392.6	395.4	394.0	193.0	191.8	196.3	194.3
	3	379.2	379.3	380.6	379.7	176.5	177.2	180.9	178.7
	4	361.9	362.3	361.6	361.6	164.6	165.9	166.5	165.9
	5	345.4	345.5	345.5	345.6	166.0	166.9	167.1	166.8
	6	342.5	342.2	342.4	342.1	174.9	176.8	176.6	176.5
	7	357.7	357.1	356.2	355.7	185.0	188.8	187.4	188.2
	8	378.0	377.4	378.7	378.1	191.2	196.9	196.8	198.9
Mean Heading Error (All Negative)	1	19.41	18.91	19.04	18.69	29.13	28.48	28.63	27.98
	2	34.43	33.64	34.44	33.59	53.69	51.05	52.64	50.67
	3	44.76	44.30	44.83	44.22	69.12	67.22	69.10	67.10
	4	51.27	51.08	51.28	50.97	49.84	48.82	50.17	48.82
	5	54.69	54.66	54.77	54.68	31.35	30.84	32.01	31.15
	6	37.22	37.26	37.36	37.38	18.07	17.29	19.02	17.60
	7	22.65	22.68	22.63	22.68	9.92	8.60	11.06	8.70
	8	12.65	12.68	12.41	12.45	5.76	4.02	6.79	4.01

(d) The largest performance differences between the two design probabilities generally occur in the second or third scan of the turn, which is to be expected since the bias would be developing on these scans and the change in search area size would most likely result in switching the smoothing parameters around this point.

The results obtained in these simulations show that the reduction in the design probability for the small search area from 0.95 to 0.75 does result in measurable reductions in the errors observed during a maneuver; however, these reductions, which are slightly greater than those observed for the noncircular search area, are still relatively insignificant as compared to the total error resulting from the maneuver. Unlike the reduction which was found using the noncircular search area, however, the reductions obtained by changing the search area design probability were obtained with only a change in parameter value and did not involve any additional computation. Therefore, the design probability used for the small search area should be kept rather high to avoid erroneous application of the large search area smoothing constants, but can be reduced from 0.95 to a lower value, probably 0.75 is as low as desirable, which will give a small improvement in tracking performance, but which can be accomplished at no additional computational cost. Since the choice of the design probability used previously (0.95) was rather arbitrary, one might as well choose some other value if there is some benefit, however small, to be gained at no additional cost.

The fact that the performance differences for the change in search area design probability are slightly greater than those for the noncircular search area implies that, if differences of this order of magnitude are considered significant, then the measurement accuracy of the individual site is of greater importance than the shape of the search area. For this reason, the parameters specifying the size of the search areas should be site-dependent parameters with the capability of using a different parameter value for each radar site according to the measurement accuracy performance of the particular site under consideration. For example, the value used for the standard deviation of the azimuthal errors, 3 ACP, is the system standard, yet typical radar sites have a standard deviation on the order of 2 ACP, while values as low as 1.3 ACP have been reported. As a result, the noncircular search area and the search area design probability are seen to be second-order effects compared to the differences resulting from a change in the radar error distribution of the magnitude indicated above. To specify the use of a noncircular search area or to make changes in the search area design probability without making the search area parameters site dependent is totally incongruous with the basic objective of improving the tracking performance.

3.4 TRACKING PERFORMANCE FOR STRAIGHT-LINE TRAJECTORIES.

It has been shown in the previous sections how various factors affect the performance of the tracking algorithm. The objective of the modifications considered previously was to reduce the magnitude of the transient errors in speed and heading, since these errors are an indication of the unresponsiveness of the tracker to maneuvering targets. Another point of interest in the consideration of the tracking algorithm is the performance for straight-line trajectories, since this constitutes the great majority of time a track is under observation. For this reason, it would be undesirable to make use of modifications which improve the tracking performance in maneuvers, which constitute only a small portion of the time a track is under observation, while resulting in a degradation of the tracking performance for straight-line trajectories. While the same performance measures used previously could be used to express the results for straight-line trajectories, it was found more convenient to define a new set of performance measures specifically for straight-line trajectories.

The performance of the tracking algorithm will be expressed in terms of the normalized variance reduction ratios defined for the small search area smoothing constants. Via this technique, it is possible to make a direct comparison between the performance of the tracking algorithm as measured by the simulation results and the theoretical performance of the tracking algorithm which is the optimum which can be achieved for this particular trajectory. Expressing the performance results in this manner provides a direct indication of the magnitude of the performance improvement possible, if any, since any deviation from the predicted performance will be a degradation from the theoretically optimum tracking performance.

Using the same techniques as in a previous report (reference 8), the normalized variance reduction ratios are computed as follows. Assuming the radar errors in range and azimuth are unbiased and independent, the variances in the Cartesian coordinate system are given by

$$\sigma_x^2=(\sin\theta)^2\sigma_\rho^2+(\rho\cos\theta)^2\ \sigma_\theta^2 \tag{11}$$

$$\sigma_y^2=(\cos\theta)^2\sigma_\rho^2+(\rho\sin\theta)^2\ \sigma_\theta^2 \tag{12}$$

where σ_ρ^2 and σ_θ^2 are the variances of the range and azimuth errors, respectively, and ρ and θ are the polar coordinates of the point of interest. For an isotropic alpha-beta tracker, equations (1)-(4), with constant coefficients, it can be shown (e.g., reference 1) that the steady-state variance reductions which result from the use of the tracking algorithm are given by

$$K_p=\frac{2\beta+\alpha\beta+2\alpha^2}{\alpha(4-2\alpha-\beta)} \tag{13}$$

$$K_v=\frac{1}{T^2}\ \frac{2\beta^2}{\alpha(4-2\alpha-\beta)} \tag{14}$$

$$K_s=\frac{2\alpha^2+\beta(2-3\alpha)}{\alpha(4-2\alpha-\beta)} \tag{15}$$

where K_p, K_v, and K_s are the variance reduction ratios, with respect to the measurement errors, in the predicted position, velocity, and smoothed position, respectively. The variance reduction ratios given above are only valid for cases in which the smoothing parameters are constant so that if the smoothing parameters vary, as in the bimodal tracking algorithm, then the interpretation of what is meant by (13) to (15) is open to question.

If the requirements for the validity of the variance reduction ratios are met, then the variance of the errors at the output of the tracking filter will simply be the variance at the input, i.e., (11) or (12), depending on the coordinate of interest, multiplied by the appropriate variance reduction ratio. A measure of the relative performance of a tracking algorithm would be to compare the predicted tracking errors with those actually observed using the ratio of the two quantities so that the closer the ratio of the observed errors to the predicted errors approaches unity the better is the tracking performance as compared to the optimum theoretically achievable performance. The relative tracking performance ratio (referred to previously as the normalized variance reduction ratio) in the case of the smoothed position is given by

$$r_s = \frac{1}{2N_s K_s} \sum_{k=1}^{N_s} \left(\frac{\hat{\sigma}_x^2(k)}{\sigma_x^2(k)} + \frac{\hat{\sigma}_y^2(k)}{\sigma_y^2(k)} \right) \tag{16}$$

where

$$\hat{\sigma}_x^2(k) = \frac{1}{N} \sum_{i=1}^{N} (X_s(k, i) - X_T(k))^2 \tag{17}$$

$$\hat{\sigma}_y^2(k) = \frac{1}{N} \sum_{i=1}^{N} (Y_s(k, i) - Y_T(k))^2 \tag{18}$$

and $\sigma_x^2(k)$ and $\sigma_y^2(k)$ are obtained using the true Cartesian coordinates, $X_T(k)$ and $Y_T(k)$, to calculate the true polar coordinates for use in (11) and (12). Since the statistics used to calculate r_s are functions of the position of the track with respect to the radar, the performance statistic r_s, and similar ones for velocity (r_v) and predicted position (r_p), is an average over the trajectory used in the simulation. By using the three performance measures just defined, the simulation results can be summarized in a very simple manner without requiring the extensive tabulations used previously. In addition to the three performance statistics just discussed, the standard deviation of the heading errors will also be used as a performance measure.

It was stated previously that the variance reduction ratios, (13)-(15), are only valid for constant smoothing parameters, so there is some question as to what parameter values to use for the smoothing constants. Since a comparison based on the variance reduction ratios will only be valid in a steady-state situation, this implies a constant-velocity, straight-line trajectory. All of the theoretical reduction ratios will be computed using the small search area smoothing parameters, since these are the parameters which should be used if it is known, a priori, that a straight-line trajectory is under observation. Any large search area correlations which occur will be the result of the statistical nature of the radar errors rather than the start of a target maneuver so that the heavier smoothing (larger α and β values) used for large search area correlations will be detrimental to the performance statistics, which is the proper interpretation of a large search area return (and the use of the large search area smoothing constants) in this case.

The simulation results for the straight-line tracks are given in table 8 in terms of the performance measures just discussed. Results are presented for three distinct methods of handling large search area returns. The results for the column denoted "No Delay" are for the case where all large search area returns are immediately smoothed with the large search area smoothing parameters, while the results denoted "Delay" are for the case where the first large search area return is smoothed with the small search area smoothing parameters (the one-scan delay concept introduced in Section 3.2). In the case of the results denoted "SSA Values," the large search area smoothing parameters are not used at all, and each return is smoothed with the small search area smoothing parameters. Since a circular search area is being used, it was not thought necessary to vary both speed and heading, so only one heading was used in these results.

One of the justifying factors in the use of a one-scan delay before application of the large search area smoothing parameters was the desire to avoid unwanted perturbations of the tracking algorithm output because of occasional large search area returns observed for straight-line tracks. The results given in table 8 confirm that these occasional large search area returns (and their associated smoothing parameters) cause considerable degradation in the performance of the tracking algorithm as compared to the theoretical performance. In the case of the velocity data, the standard bimodal tracking algorithm (without delay) produces errors which are 4.4 times larger than the theoretical performance. The introduction of a one-scan delay, however, results in a considerable reduction in both the position and velocity errors, with position errors being a factor of 1.6 smaller and velocity errors a factor of 3.4 smaller, bringing these errors down to approximately those obtainable theoretically. When the large search area parameters are not used at all, the results improve even further and are very close to one, as expected, since this is the constant-coefficient case assumed in the derivation of the variance reduction ratios, (13)-(15), and confirms the validity of the simulation model. The smoothing equations in this case are linear, since the nonlinear effect of the bimodal operation of the tracking algorithm has been eliminated and the fact that the normalized variance reduction ratios are so close to one indicates that the theoretical basis for the performance predictions is very good. A significant reduction (about 46 percent) in the magnitude of the standard deviation of the heading error was also noted when the one-scan delay was introduced.

Another important performance measure of interest is the number of large search area returns actually smoothed with the large search area smoothing parameters as compared to the total number of large search area returns. The large reduction (of about 24 percent) observed in the total number of large search area returns in the case where delay was used shows that about one in four of the large search area returns is actually caused by the erroneous application of

TABLE 8. SIMULATION RESULTS FOR CONSTANT-VELOCITY STRAIGHT-LINE TRAJECTORIES

Velocity:	V_x=400			V_x=200		
LSA Usage:	No Delay	Delay	SSA Values	No Delay	Delay	SSA Values
r_p	1.638	1.070	1.005	1.622	1.055	0.991
r_v	4.429	1.292	1.003	4.383	1.268	0.986
r_s	1.418	1.046	1.000	1.415	1.039	0.993
Standard Deviation in Heading (Deg.)	1.91	1.02	0.90	4.98	2.63	2.41
LSA Parameters Used	2029	199	0	1778	105	0
Total LSA	2029	1541	1482	1778	1355	1307
TOTAL Returns	25000	25000	25000	25000	25000	25000

the large search area smoothing parameters in a situation in which the use of these parameters is inappropriate, namely straight-line tracks. When the large search area smoothing parameters are not used, the total number of large search area returns is slightly lower than when one-scan delay is used, and the number of returns agrees very well with the expected number of large search area returns based on the design probability for the small search area, 0.95, implying 5 percent (or 1,250) of the returns are in the large search area. The simulation results in this case are considered to be in very good agreement with theory (1,482 and 1,307 compared to 1,250) when all the approximations and assumptions used in the probablistic design of the small search area are considered.

The obvious conclusion which must be reached with regard to the one-scan delay is that due to the highly significant improvement in the tracking performance for straight-line tracks, which are the majority of tracks under observation, and the relatively insignificant change in the transient performance, the use of a one-scan delay should be incorporated in the tracking algorithm in connection with a dynamic search area. Previous studies (references 3 and 9) have also considered the one-scan delay to be useful.

3.5 INTERACTION OF VARIOUS MODIFICATIONS.

In the previous sections, only one modification of the basic bimodal tracking algorithm was considered at a time, yet there could be either beneficial or adverse interactions between various modifications. It would not be feasible to consider all possible combinations of modifications and scenarios, since the number of simulations required would grow exponentially. However, a limited number of simulations were performed in which interactions were considered, but if nothing significant were found, these results were not presented. For example, simulations of a noncircular search area with delay were performed, but the results closely parallel those obtained using a circular search area with delay and so were not reported.

Certain combinations of the tracking algorithm modifications may prove to be beneficial in that the adverse performance in one may be offset by a performance improvement in the other, with the resulting combination yielding an acceptable level of total performance. One example of two complementary modiications is the combination of the one-scan delay and a reduction in the search area design probability. It was shown previously that the use of a one-scan delay before use of the large search area smoothing constants results in a considerable improvement in the straight-line tracking performance of the bimodal tracking algorithm; however, it was also shown that the introduction of a one-scan delay results in an increase in the peak transient errors for maneuvering targets. A reduction in the search area design probability was shown to result in a decrease in the peak transient error, so a natural combination would be the use of one-scan delay with a reduced search area design probability. It would be hoped that a synergistic effect would be observed in which the reduced design probability compensated for the increase in the peak transient produced by the one-scan delay, and yet the increase in the number of large search area returns due to the reduced design probability will result in some loss in performance for straight-line trajectories. As a result, a parametric study of tracking performance was made to determine the magnitude of the changes which result from these two modifications and to allow a tradeoff between the two performance measures of interest which will result in an acceptable design compromise.

The simulation results to evaluate the combination of a one-scan delay with a reduced search area design probability are given in figure 3 and table 9. The scenario is the same as that used previously with V_x=200 knots, and some of the results in table 9 were obtained from tables 4 and 5 and are repeated here to facilitate comparison with the new results. The results presented in figure 3 show the straight-line performance of the tracking algorithm as a function of the search area design probability for the cases with and without delay, while the data tabulated in table 9 give the transient performance for several search area probabilities. In both cases, the results for a fixed search area (without delay) have also been included to provide a comparison with the present bimodal tracking algorithm as it is used in practice. It should be noted that as the search area design probability approaches one, the effect of the one-scan delay becomes negligible, and the results for the two cases approach the same limiting values as shown in figure 3. The simulation results for the case where a one-scan delay is used show that the performance measures decrease in an approximately linear relationship (on semilogarithmic coordinates) as the design probability increases. Comparing these results to the fixed search area results, it is seen that, for design probabilities in excess of about 0.8, all performance measures for the dynamic search area with one-scan delay are better than those for the fixed search area for this particular scenario. When the one-scan delay is not used, the performance measures are significantly less sensitive to the design probability, with significant changes occurring only when the design probability is in excess of 0.9, and in order to improve the tracking performance over that of the fixed search area, a design probability greater than 0.98 must be used. In general, the use of a dynamic search area with one-scan delay can result in significant reductions in the velocity and heading errors for straight-line trajectories as compared to the performance for a fixed search area or for a dynamic search area with no delay.

The motivation for reducing the search area design probability in the case where the one-scan delay is used is to improve the transient performance, and the simulation results in table 9 show the trends expected; namely, that as the design probability is reduced, the transient errors are also reduced. Comparing the peak mean heading error and the peak radial velocity error, in the case of a fixed search area, with those of the dynamic search area with one-scan delay, it is seen that the fixed search area results fall between the dynamic search area results for design probabilities of 0.85 and 0.95 and should be approximately equal at a design probability of 0.9. In summary, the transient errors for the dynamic search area with delay are less than those for the fixed search area when the design probability is less than 0.9, while the straight-line tracking performance of the dynamic search area is better than that of the fixed search area without delay for design probabilities greater

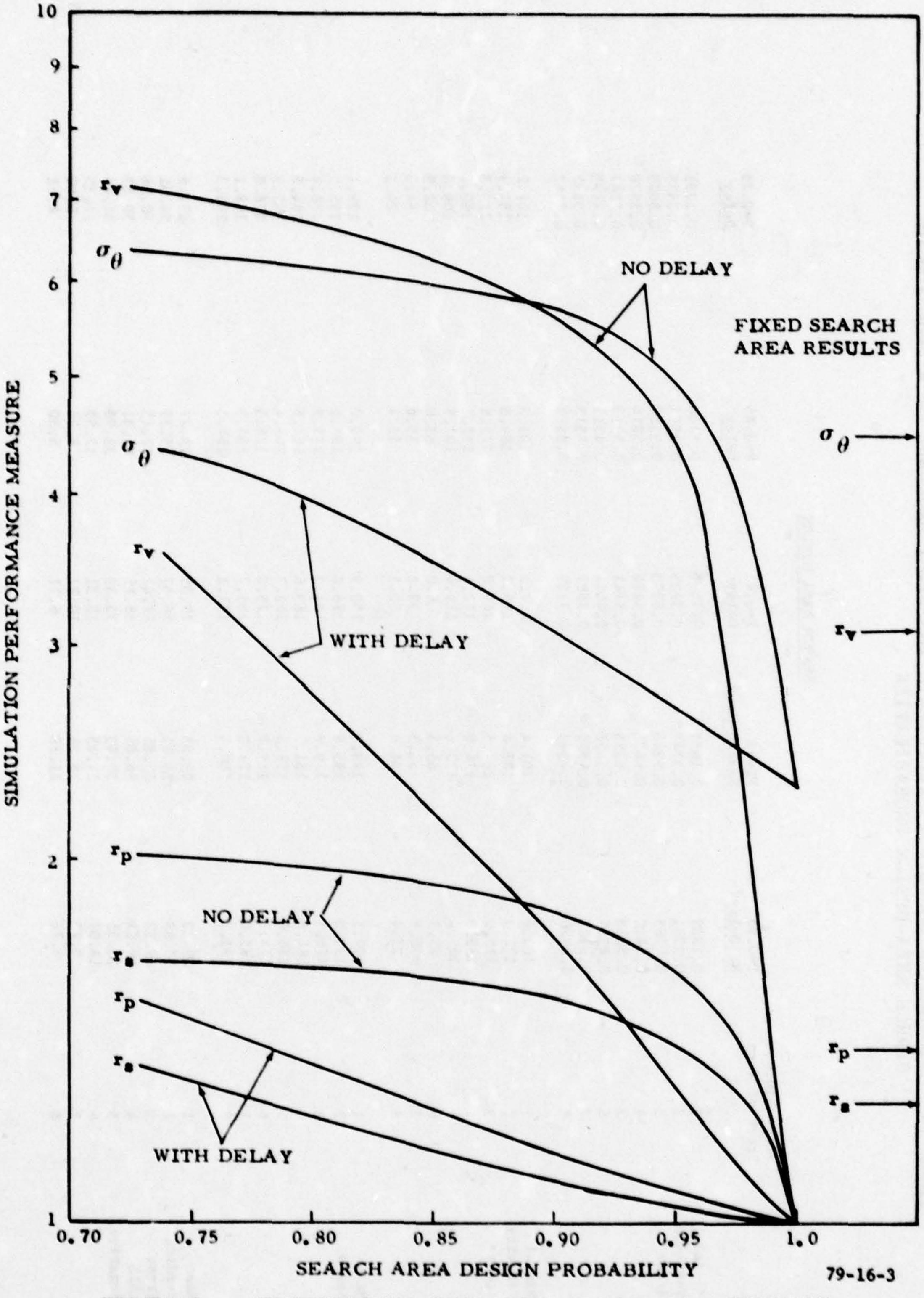


FIGURE 3. STRAIGHT-LINE SIMULATION RESULTS AS A FUNCTION OF SEARCH AREA DESIGN PROBABILITY

TABLE 9. SIMULATION RESULTS TO ILLUSTRATE TRANSIENT RESPONSE AS A FUNCTION OF SEARCH AREA DESIGN PROBABILITIES

		SEARCH AREA DESIGN				
	Scans Since Start of Turn	P=0.95 No Delay	Fixed Search Area	P=0.95 Delay	P=0.85 Delay	P=0.75 Delay
Radial Position Error	1	0.1809	0.1655	0.1676	0.1700	0.1791
	2	0.3381	0.3747	0.3735	0.3591	0.3479
	3	0.4785	0.5848	0.6093	0.5328	0.5026
	4	0.5410	0.6476	0.6420	0.5814	0.5578
	5	0.5058	0.6255	0.5741	0.5313	0.5147
	6	0.4255	0.5658	0.4642	0.4313	0.4193
	7	0.3414	0.4882	0.3687	0.3313	0.3195
	8	0.2920	0.4369	0.3189	0.2807	0.2642
Radial Velocity Error	1	101.4	101.4	101.3	101.3	101.2
	2	178.3	188.6	188.2	184.8	181.9
	3	215.0	234.5	240.8	225.6	219.8
	4	158.0	174.0	172.1	163.3	160.2
	5	105.7	122.6	111.6	107.5	106.0
	6	65.3	85.3	66.0	63.6	62.9
	7	38.8	58.5	39.6	35.6	34.5
	8	27.4	44.4	30.1	26.4	24.1
Mean Speed	1	199.1	199.3	198.9	199.0	199.1
	2	193.0	195.8	196.2	195.6	195.5
	3	176.5	179.8	184.3	179.9	178.4
	4	164.6	161.7	163.6	164.5	164.8
	5	166.0	159.5	163.3	165.4	165.9
	6	174.9	165.7	175.0	176.5	176.8
	7	185.0	174.2	185.7	188.3	189.1
	8	191.2	180.4	191.1	194.6	196.3
Mean Heading Error (All Negative)	1	29.13	29.35	29.35	29.29	29.16
	2	53.69	56.81	56.67	55.59	54.61
	3	69.12	75.85	77.41	72.48	70.56
	4	49.84	56.05	55.15	51.81	50.63
	5	31.35	37.43	33.39	31.96	31.45
	6	18.07	24.45	18.31	17.59	17.35
	7	9.92	15.75	10.27	9.14	8.80
	8	5.76	11.38	6.71	5.30	4.51

than 0.8, so that a compromise value of the design probability for the dynamic search area with delay might be 0.85. For a design probability of 0.85, the peak radial velocity error is 8.9 knots less than that for the fixed search area, the peak heading error is 3.37 degrees less, and the radial position errors are significantly less (e.g., on scan eight, 0.4369 to 0.2807, or a reduction of 36 percent), while the straight-line tracking performance measures, r_v, r_p, r_s, and σ_θ, are reduced by 29.1, 11.0, 7.7, and 22.4 percent, respectively. The reduction of the design probability to 0.85 has significantly reduced the increases in the transient errors which occurred when the one-scan delay was introduced at a design probability of 0.95.

The results obtained in this section show that the dynamic search area with a one-scan delay can result in a substantial reduction in the tracking errors for straight-line tracks, as compared with the results for fixed search areas, while at the same time reducing the peak transient errors for turning tracks, although the reduction in this case is, admittedly, insignificant. The value of the normalized variance reduction ratio for velocity using fixed search areas was found to be 3.1 or, in other words, the performance of the tracking algorithm using the fixed search areas was 3.1 times worse than that theoretically obtainable using an alpha-beta tracking algorithm. The magnitude of the difference between the actual performance and the theoretical performance indicates that there is considerable room for improvement in the performance of practical tracking algorithms.

The importance of the results presented in this section is that it has been shown possible to obtain an improvement in tracking performance for both straight-line and turning trajectories. Thus, the dynamic search area with a one-scan delay has been shown to be the means by which this performance improvement can be realized. It should be noted that at no time during this study was any consideration given to changing the smoothing parameters, so the possibility exists that even more significant performance improvements can be obtained by allowing the use of different values for the smoothing constants.

4. SUMMARY AND CONCLUSIONS

In the design of practical tracking algorithms, it is frequently necessary to use several sets of smoothing constants to obtain acceptable performance for both straight-line and maneuvering targets. The simplest example of such a tracker is the bimodal algorithm studied in this report in which two sets of smoothing parameters are used with the selection of the smoothing parameters dependent on the magnitude of the deviation between the predicted position of the track and the measured position of the target datum. If this deviation is small, then the smoothing parameters appropriate to a straight-line trajectory are used, and this region is referred to as the small search area. Since the tracking filter assumes a straight-line constant velocity trajectory to calculate the predicted position, when a target maneuvers, a bias will develop because of the violation of this assumption, and this bias will cause the difference between the measured and predicted positions to become sufficiently large that the data point will fall in the large search area, which is the region in which the smoothing parameters appropriate to maneuvering targets are used. As a result, the size and shape of the small search area controls the responsiveness of the tracking algorithm to maneuvers, since it controls the point at which the transition from one set of parameters to the other is made. By making the search area size proportional to the radar errors, it is possible to design a dynamic search area which is adaptive to the radar errors observed at the point of interest and will thus ensure that the same level of tracking performance is maintained throughout the coverage area of the radar.

Since the switching of smoothing parameters is a nonlinear operation, it is not possible to evaluate the effects of changes in the search area via the standard linear theory which is applicable to a constant coefficient alpha-beta tracking filter. As a consequence, the evaluation of the nonlinear interaction between the smoothing parameters and the search area must be done on the basis of the actual algorithm and search area of interest. The work reported in this study involves changes in the structure of the tracking filter to search area interaction and is not concerned with the tracking parameters per se, since optimization of the tracking filter structure must precede any attempt to optimize the smoothing parameter values.

There are two important considerations in the evaluation of the interaction between the search area and the tracking filter. The first is that the erroneous application of the large search area smoothing constants on straight-line trajectories causes an undesirable perturbation of the tracking filter output, which results in tracking performance significantly worse than that theoretically possible. Therefore, it is desirable to make the small search area as large as possible in order to suppress large search area returns which may be erroneously taken as the start of a maneuver. On the other hand, the second consideration is that, in order to minimize the transient or bias errors which develop during a maneuver, it would be desirable to make the small search area as small as possible in order to detect the onset of a maneuver as soon as possible so that the bias can be minimized by application of the smoothing constants appropriate to a maneuver. Variations in the size and shape of the small search area will, therefore, cause a tradeoff between the tracking performance for straight-line and maneuvering targets. In order to maintain the same level of performance throughout the coverage area of the radar, they must be dynamic as opposed to fixed so as to vary with the magnitude of the radar errors.

In Section 3.5, an example was given in which a tradeoff was made between straight-line performance and maneuver performance which achieved an overall level of performance which was better in both cases than that achieved by the tracking algorithm presently in use in which fixed search areas are used. The means by which this was achieved was to use a dynamic search area generally smaller than that presently used (for good maneuver detection) but to incorporate a one-scan delay before the application of the large search area smoothing parameters (to obtain good straight-line performance). The result of this combination was a tracking algorithm with improved performance for both straight-line and maneuvering targets. In practice, if very few large search area returns are noted on straight-line tracks relatively close to the radar, then the small search area is too large and will give poor maneuver detection.

The one-scan delay in association with a dynamic search area is one combination of tracking algorithm modifications which was particularly useful in terms of improved tracking performance. One modification which was not found to be particularly useful in terms of improved tracking performance was the noncircular search area. Intuitively, a noncircular search area should give better maneuver detection because the shape more closely matches that of radar error distribution, and this is, in fact, correct, but the magnitude of the tracking improvements which resulted from the use of a noncircular search area was found to be negligible. Typically, the mean speed error was on the order of 1.5 knots smaller for the noncircular search area, while the mean heading error was on the order of 1 degree smaller, but the magnitude of these improvements is insignificant when the computational burden required to implement the noncircular search area is considered. If the computational resources required to implement a noncircular search area are of no consequence; however, then this technique might be considered acceptable.

On the other hand it was found that even larger improvements could, in fact, be obtained when the size of the circular search area was reduced, which can be accomplished by a simple parameter change with no additional computational burden being imposed. One conclusion which is quite obvious from these results is that, in order to obtain a uniform level of tracking performance throughout a multiradar tracking environment in which the accuracies of the radars are not uniform, it is necessary that the parameters defining the search areas should be site-dependent so that the parameter values can be chosen to match the error performance of the particular site from which a radar datum is received. Such a modification would ensure that the maneuver detection capability of the dynamic search area is used to the greatest extent possible.

The fact that the straight-line tracking performance of the fixed search area tracking algorithm was found to be over three times worse than that theoretically achievable by an alpha-beta tracker indicates that there is considerable room for improvement in both the straight-line performance and the maneuvering performance of the tracking algorithm. Since it was relatively easy to find a modification which would improve the tracking performance in both cases, it is apparent that there are ways by which the present tracking algorithm can be improved, and this is especially true when it is noted that the smoothing parameter values were not changed in order to achieve any of the results presented in this study. Using the additional degrees-of-freedom available when changes in the parameter values are permitted, it should be possible to obtain an even greater improvement in the performance of the tracking algorithm, and this improvement will most likely be found in terms of a reduction of the transient error during maneuvers. The results presented here are a first step in an attempt to optimize the performance of the bimodal tracking algorithm used in the en route air traffic control environment. Additional analytical work is needed to obtain the full performance potential of this algorithm. A more complete evaluation in an environment more representative of actual operational conditions would then be warranted.

5. REFERENCES

1. Lefferts, R. E., Calculation of the Correlation Region Size for Use with Alpha-Beta Tracking Filters, National Aviation Facilities Experimental Center, Atlantic City, N. J., FAA-NA-79-15, April 1979.

2. Lautenschlager, R. W. and Miskill, D. K., Preliminary Functional Specification: En Route Multiple Radar Data Processing (MRDP) for the DABS/ATCRBS Surveillance Environment, MTR-7299, The MITRE Corp., NAFEC, Atlantic City, N. J., March 1977.

3. Mullin, F. R., The Use of Dynamic Search Areas in the National Airspace System En Route Computer Program, Federal Aviation Administration Report, FAA-RD-75-77, NTIS No. AD-A012-496, May 1975.

4. Sinha, L. P., Turn Detection Characteristics of NAS Bimodal and IPC Tracking Algorithms , WP-7711, Series 11, The MITRE Corp., NAFEC, Atlantic City, N. J., March 8, 1974.

5. Lefferts, R. E., The Effect of One Scan Delay on Track Loss, WP-7711, Series 3, The MITRE Corp., NAFEC, Atlantic City, N. J., September 29, 1972.

6. Sokkappa, B. G. and Sinha, L.P., Primary Tracking Performance Results with Supplementary Data 'OFF', 4 nmi Large Search Area and One-Scan Delay, MTR-4465, Series 3, The MITRE Corp., NAFEC, Atlantic City, N. J., September 28, 1972.

7. Happel, F. I., Performance Results of Model 3dl.2 Surveillance Simulation, MTR-4465, Series 6, The MITRE Corp., NAFEC, Atlantic City, N. J., December 3, 1973.

8. Lefferts, R. E., En Route Tracking of DABS Radar Data--Preliminary Study Results, WP-12571, The MITRE Corp., Atlantic City, N. J., September 1977.

9. Greenwood, D. R., Tracking Analysis, MTR-4080, The MITRE Corp., McLean, Va., August 11, 1969.