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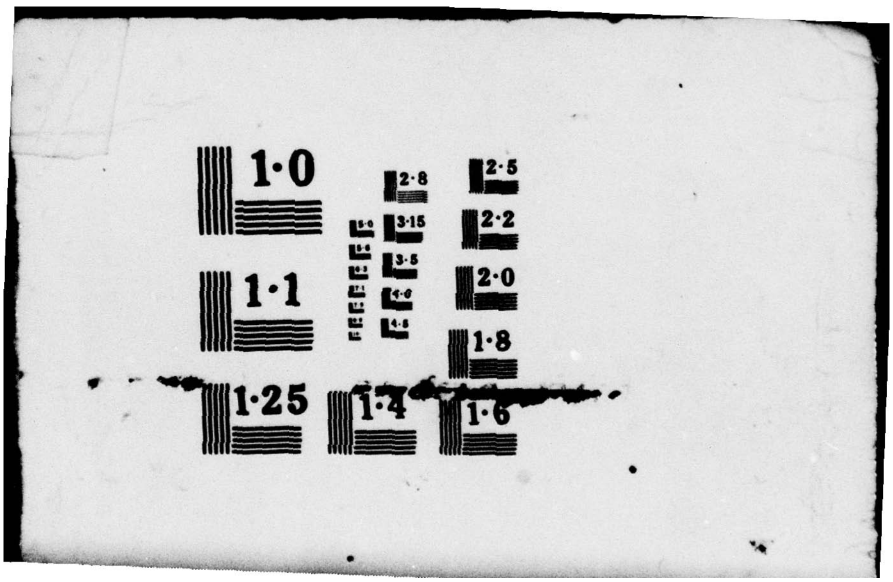
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ON BERRY-ESSEEN RATES FOR STATISTICAL FUNCTIONS, WITH APPLICATION TO L-ESTIMATES

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ABSTRACT

ON BERRY-ESSÉEN RATES FOR STATISTICAL FUNCTIONS, WITH APPLICATION TO L-ESTIMATES

A parameter expressed as a *functional* $T(F)$ of a distribution function (d.f.) F may be estimated by the "statistical function" $T(F_n)$ based on the sample d.f. F_n. Typically, $T(F_n)$ is asymptotically normal. We investigate the rate of this convergence by utilizing the von Mises (1947) representation to express $T(F_n) - T(F)$ as an approximate U-statistic plus R_n, and applying the Berry-Esséen rate $O(n^{-\frac{1}{2}})$ established for U-statistics by Callaert and Janssen (1978). This essentially reduces the problem to a handling of R_n. We carry out this method for linear functions of order statistics ("L-estimates") and obtain results competitive with Bjerve (1977) and Helmers (1977). Also, we briefly indicate the application of the method to M-estimates.

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0. *Summary*. Let $T(\cdot)$ be a real-valued functional defined on distribution functions (d.f.'s). For a sample $X_1, \ldots, X_n$ from a d.f. F, consider estimation of the "parameter" $T(F)$ by the sample analogue estimator $T(F_n)$ based on the usual sample d.f. $F_n(x) = n^{-1}\sum_{i=1}^{n} I(X_i \le x)$, $-\infty < x < \infty$. In many cases, for a suitable constant $\sigma^2(T,F)$, the d.f. of $n^{\frac{1}{2}}[T(F_n) - T(F)]/\sigma(T,F)$ converges weakly to the standard normal d.f. Φ. In Section 1 we formulate a general approach toward investigation of the *rate* of this convergence. Taking two terms of the von Mises (1947) Taylor expansion for statistical functions, we represent $T(F_n) - T(F)$ as an approximate U-statistic plus a remainder term R_n. Under appropriate conditions, we may dispense with R_n, switch to an exact U-statistic, and exploit the Berry-Esséen rate $O(n^{-\frac{1}{2}})$ established for U-statistics by Callaert and Janssen (1978). (On the basis of only one term from von Mises' expansion, the method typically yields a somewhat weaker rate.) In Section 2 the method is applied to linear functions of order statistics ("L-estimates"), as represented by functionals of the form $T(F) = \int_0^1 F^{-1}(u)J(u)du$. The rate $O(n^{-\frac{1}{2}})$ is obtained under conditions on J and F competitive with Bjerve (1977) and Helmers (1977). In Section 3 the application of the approach to M-estimates is sketched.

1. *A general approach.* Utilizing the notion of Taylor expansion for statistical functions introduced by von Mises (1947), and taking two terms, we represent $T(F_n) - T(F)$ as V_n^* + remainder, where

$$V_n^* = \frac{d}{d\lambda} T((1-\lambda)F+\lambda F_n)\big|_{\lambda=0+} + \frac{1}{2}\frac{d^2}{d\lambda^2} T((1-\lambda)F+\lambda F_n)\big|_{\lambda=0+} .$$

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Typically, the terms in V_n^* may be represented in the forms $\int \alpha(x)dF_n(x)$ and $\frac{1}{2}\int\int \beta(x,y)dF_n(x)dF_n(y)$, respectively. In this case V_n^* has the form (1.1) below, with $h(x,y) = \frac{1}{2}[\alpha(x) + \alpha(y) + \beta(x,y)]$. The following theorem, stated for this setting of a statistical function $T(F_n)$ estimating $T(F)$, is in fact valid in connection with *any* statistic $T_n = T_n(X_1, \ldots, X_n)$ estimating a parameter T_0, but depends upon finding a suitable function $h(x,y)$.

THEOREM 1.1. *Suppose that* $T(F_n) - T(F)$ *may be represented as* $V_n + R_n$, *where* V_n *is given by*

$$V_n = n^{-2} \sum_{i=1}^{n} \sum_{j=1}^{n} h(X_i, X_j) , \tag{1.1}$$

with $h(x,y)$ *symmetric in its arguments and satisfying*

$$E_F\{h(X_1,X_2)\} = 0 , \tag{1.2a}$$

$$E_F|h(X_1,X_2)|^3 < \infty , \tag{1.2b}$$

$$E_F|h(X_1,X_1)|^{3/2} < \infty , \tag{1.2c}$$

and where, for some $A > 0$,

$$P(|R_n| > An^{-1}) = O(n^{-\frac{1}{2}}),\ n\to\infty . \tag{1.3}$$

Put $\sigma^2(T,F) = 4\mathrm{Var}_F\{g(X_1)\}$, *where* $g(x) = E_F\{h(x,X_1)\}$. *Then*

$$\sup_t |P(n^{\frac{1}{2}}[T(F_n)-T(F)]/\sigma(T,F) \le t) - \Phi(t)| = O(n^{-\frac{1}{2}}),\ n\to\infty . \tag{1.4}$$

We obtain Theorem 1.1 by successive application of the following well-known and easily proved device.

LEMMA 1.1 *Let the sequence of rv's* $\{\xi_n\}$ *satisfy*

(*) $$\sup_t |P(\xi_n \le t - \Phi(t)| = O(n^{-\frac{1}{2}}) .$$

Then, for any sequence of rv's $\{\Delta_n\}$ *and constant* B ,

(**) $$\sup_t |P(\xi_n + \Delta_n \le t) - \Phi(t)| = O(n^{-\frac{1}{2}}) + P(|\Delta_n| > Bn^{-\frac{1}{2}}) .$$

For the first application, put $\xi_n = n^{\frac{1}{2}} V_n / \sigma(T,F)$ and $\Delta_n = n^{\frac{1}{2}} R_n / \sigma(T,F)$. Then, by (1.3), it suffices for (1.4) to establish (*). Now, for the *U-statistic* of Hoeffding (1948) associated with the kernel h, namely $U_n = \binom{n}{2}^{-1} \Sigma_{1 \le i < j \le n} h(X_i, X_j)$, Callaert and Janssen (1978) establish (*) for $\xi_n = n^{\frac{1}{2}} U_n / \sigma(T,F)$, under the conditions (1.2a,b). Therefore, to complete the proof of Theorem 1.1, it suffices to show that $P(n^{\frac{1}{2}} |U_n - V_n| > Bn^{-\frac{1}{2}}) = O(n^{-\frac{1}{2}})$ *for some constant* B. This is established in the following result.

LEMMA 1.2. *Suppose that* $h(x,y)$ *is symmetric in its arguments and satisfies* $Eh^2(X_1,X_2) < \infty$ *and* $E|h(X_1,X_1)|^{3/2} < \infty$. *Then, for* $B > 2|E\{h(X_1,X_2) - h(X_1,X_1)\}|$,

$$P(|U_n - V_n| > Bn^{-1}) = o(n^{-\frac{1}{2}}) , n \to \infty .$$

PROOF. Check that $U_n - V_n = n^{-1}(U_n - W_n)$, where $W_n = n^{-1} \sum_{i=1}^n h(X_i, X_i)$. Then, for $B > 2|E\{h(X_1,X_2) - h(X_1,X_1)\}|$, we have

$$P(|U_n - V_n| > Bn^{-1}) = P(|U_n - W_n| > B)$$
$$\le P(|U_n - W_n - E\{U_n\} + E\{W_n\}| > \tfrac{1}{2}B)$$
$$\le P(|U_n - E\{U_n\}| > B/4) + P(|W_n - E\{W_n\}| > B/4) .$$

The first term on the right is $O(n^{-1})$ by Chebyshev's inequality and the well-known rate $\mathrm{Var}\{U_n\} = O(n^{-1})$. For the second term, we use Theorem 4 of Baum and Katz (1965), which implies: *for* $\{Y_i\}$ *i.i.d. with* $E\{Y_1\} = 0$ *and* $E|Y_1|^r < \infty$, *where* $r \geq 1$, $P(|Y| > \epsilon) = o(n^{1-r})$, *for all* $\epsilon > 0$. We thus apply this result with $r = 3/2$. □

REMARK. Note (using (**)) that even if the optimum rate $O(n^{-\frac{1}{2}})$ is not known for R_n in (1.3), we still may obtain a (weaker) Berry-Esséen rate by replacing $O(n^{-\frac{1}{2}})$ in (1.4) by $O(P(|R_n| > An^{-1}))$. □

2. *L-estimates.* Consider the functional $T(F) = \int_0^1 F^{-1}(u)J(u)du$ and the corresponding L-estimate $T(F_n)$. Implementing the approach of Section 1, we find (under regularity conditions on J)

$$V_n^* = -\int_{-\infty}^{\infty}[F_n(x) - F(x)]J\circ F(x)dx - \tfrac{1}{2}\int_{-\infty}^{\infty}[F_n(x) - F(x)]^2 J'\circ F(x)dx$$

and thus $V_n^* = V_n$ of form (1.1) with $h(x,y) = \frac{1}{2}[\alpha(x) + \alpha(y) + \beta(x,y)]$, where

$$\text{(2.1a)} \qquad \alpha(x) = -\int_{-\infty}^{\infty}[I(x \leq t) - F(t)]J\circ F(t)dt$$

and

$$\text{(2.1b)} \qquad \beta(x,y) = -\int_{-\infty}^{\infty}[I(x \leq t) - F(t)][I(y \leq t) - F(t)]J'\circ F(t)dt .$$

Writing $K(u) = \int_0^u J(u)du$, we obtain by integration by parts that $T(G) - T(F) = -\int_{-\infty}^{\infty}[K\circ G(x) - K\circ F(x)]dx$. Therefore, for $R_n = T(F_n) - T(F) - V_n$ we obtain

(2.2) $$R_n = -\int_{-\infty}^{\infty} W_{F_n,F}(x)dx ,$$

where

$$W_{G,F} = K\circ G - K\circ F - (J\circ F)(G-F) - \tfrac{1}{2}(J'\circ F)(G-F)^2 .$$

Having found h and R_n heuristically, we now rigorously establish two Berry-Esséen theorems for L-estimates by introducing sets of conditions on J and F under which h and R_n are well-defined and satisfy the requirements of Theorem 1.1. The relevant asymptotic variance parameter is

$$\sigma^2(J,F) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J\circ F(x)J\circ F(y)[F(\min(x,y)) - F(x)F(y)]dxdy .$$

THEOREM 2.1. *Suppose that J vanishes outside an interval $[a,b]$, $0 < a < b < 1$, and that J' exists and satisfies a Lipschitz condition of order $\delta > 0$ on an open interval containing $[a,b]$. Assume $\sigma^2(J,F) > 0$. Then*

(2.3) $$\sup_t |P(n^{\frac{1}{2}}[T(F_n) - T(F)]/\sigma(J,F) \leq t) - \Phi(t)| = O(n^{-\frac{1}{2}}),\ n\to\infty .$$

THEOREM 2.2. *Suppose that J' exists and satisfies a Lipschitz condition of order $\delta > 1/3$ on $(0,1)$. Assume that $E_F|X_1|^3 < \infty$ and that $0 < \sigma^2(J,F) < \infty$. Then (2.3) holds.*

REMARKS. (i) Theorem 2.1 requires that J be trimmed away from 0 and 1, but imposes no conditions on F other than $\sigma^2(J,F) > 0$. Bjerve (1977), also for the case of J trimmed, obtains (2.3) requiring only that J be Lipschitz of order 1 on an open interval containing $[a,b]$, but furthermore requiring that F^{-1} possess a second derivative also satisfying such a Lipschitz condition.

(ii) Theorem 2.2 allows J to have support $(0,1)$ but imposes a standard moment condition on F. Helmers (1977) proves a similar theorem. He allows J' not to exist at finitely many points but requires J' to be Lipschitz of order $> \frac{1}{2}$ on the open intervals where it exists and F^{-1} to be Lipschitz of order $> \frac{1}{2}$ on neighborhoods of the points where J' does not exist. (The additional restriction $\int |J'| dF^{-1} < \infty$ required in Helmers (1977a) is eliminated in Helmers (1977b).)

(iii) Theorems 2.1 and 2.2 remain valid with $T(F_n)$ replaced in (2.3) by the closely related statistic $T_n = n^{-1}\sum_{i=1}^n J(i/(n+1))X_{ni}$, where $X_{n1} \le \dots \le X_{nn}$ denote the ordered sample values. To see this, write $T(F_n) = \sum_{i=1}^n [K(i/n) - K((i-1)/n)]X_{ni}$ and apply standard arguments. For example, in the case of Theorem 2.2, we use boundedness of J' to write $|T(F_n) - T_n| \le Mn^{-2}\sum_{i=1}^n |X_i|$ for a constant M. Put $\Delta_n = n^{\frac{1}{2}}[T(F_n) - T_n]$ and obtain $P(|\Delta_n| > 2ME|X_1|n^{-\frac{1}{2}}) \le P(n^{-1}\sum_{i=1}^n |X_i| > 2E|X_1|) \le P(|n^{-1}\sum_{i=1}^n |X_i| - E|X_1|| > E|X_1|) = O(n^{-1})$ by Chebyshev's inequality, since $E X_1^2 < \infty$. Then apply Lemma 1.1. □

PROOF OF THEOREM 2.1. First we utilize the assumption that J is trimmed. Let $0 < \epsilon < \min\{a, 1-b\}$. Then there exist A, B such that $-\infty < A < F^{-1}(a-\epsilon) < F^{-1}(b+\epsilon) < B < \infty$. Hence, defining $||q||_\infty = \sup_x |q(x)|$, we have that $||G-F||_\infty < \epsilon$ implies $W_{G,F}(x) = 0$ for $x \notin [A,B]$. Therefore, defining

$$R_n(A,B) = -\int_A^B W_{F_n,F}(x)dx ,$$

we have

$$P(|R_n| > Cn^{-1}) \le P(|R_n(A,B)| > Cn^{-1}) + P(||F_n - F||_\infty \ge \epsilon) .$$

Now apply the Lipschitz condition on J', whereby $|J'(u_1) - J'(u_2)| \leq D|u_1 - u_2|^{\delta}$, to obtain

$$|R_n(A,B)| \leq \tfrac{1}{2}(B-A)D\,||F_n-F||_{\infty}^{2+\delta},$$

where $\delta > 0$. Thus, with $C_1 = 2C/(B-A)D$,

$$(2.4)\quad P(|R_n| > Cn^{-1}) \leq P(n||F_n-F||_{\infty}^{2+\delta} > C_1) + P(||F_n-F||_{\infty} \geq \epsilon)\ .$$

We now apply the inequality $P(n^{\frac{1}{2}}||F_n-F||_{\infty} \geq d) \leq D_o\exp(-2d^2)$, $d > 0$, where D_o is a constant, due to Dvoretzky, Kiefer and Wolfowitz (1956). It is readily seen that the terms on the right in (2.4) are each $O(n^{-\frac{1}{2}})$, so that R_n satisfies (1.3).

The required properties of h are obtained easily. Restricting the range of integration in (2.1) to the interval $[A,B]$, finiteness of moments follows. Further, interchanging expectation and integration by Fubini's theorem, we find $E\{\alpha(X_1)\} = 0$, $E\{\beta(x,X_1)\} = 0$ and thus also $E\{\beta(X_1,X_2)\} = 0$ and $g(x) = E\{h(x,X_1)\} = \tfrac{1}{2}\alpha(x)$. Thus $\sigma^2(T,F)$ of Theorem 1.1 is given by $E\alpha^2(X_1) = \sigma^2(J,F)$. □

In proving Theorem 2.2, we will need a property of $||F_n-F||_{L_2}$, where $||\cdot||_{L_2}$ denotes the L_2-norm, $||q||_{L_2} = [\int q^2(x)dx]^{\frac{1}{2}}$.

LEMMA 2.1. *Let* k *be a positive integer and assume* $E_F|X_1|^k < \infty$. *Then*

$$(2.5)\qquad E\{||F_n-F||_{L_2}^{2k}\} = O(n^{-k})\ ,\ n\to\infty\ .$$

PROOF. Put $Y_i(t) = I(X_i \le t) - F(t)$, $1 \le i \le n$. Then

$$\text{(2.6)}\quad E\{||F_n - F||_{L_2}^{2k}\} = n^{-2k} \sum_{i_1=1}^{n} \sum_{j_1=1}^{n} \cdots \sum_{i_k=1}^{n} \sum_{j_k=1}^{n} E\{\prod_{\ell=1}^{k} \int Y_{i_\ell}(t) Y_{j_\ell}(t) dt\} .$$

Since $E|X_1| < \infty$, integration by parts yields the inequality

$$\text{(2.7)}\quad \int |I(X_1 \le t) - F(t)| dt \le |X_1| + E|X_1| .$$

Consequently, the expectation term on the right in (2.6) is finite since $E|X_1|^k < \infty$ and, by Fubini's theorem,

$$\text{(2.8)}\quad E\{\prod_{\ell=1}^{k} \int Y_{i_\ell}(t) Y_{j_\ell}(t) dt\} = \int \cdots \int E\{\prod_{\ell=1}^{k} Y_{i_\ell}(t_\ell) Y_{j_\ell}(t_\ell)\} dt_1 \ldots dt_k .$$

By independence of $Y_1(s_1), \ldots, Y_n(s_n)$ for any $s_1, \ldots, s_n$, we have

$$E\{Y_{i_1}(t_1) Y_{j_1}(t_1) \ldots Y_{i_k}(t_k) Y_{j_k}(t_k)\} = 0$$

except possibly in the case that each index in the list $i_1, j_1, \ldots, i_k, j_k$ appears at least twice. In this case the number of distinct elements in the set $\{i_1, j_1, \ldots, i_k, j_k\}$ is $\le k$. It follows that the number of ways to choose $i_1, j_1, \ldots, i_k, j_k$ such that the expectation in (2.8) is nonzero is $O(n^k)$. Thus the number of nonzero terms in the summation in (2.6) is $O(n^k)$. □

PROOF OF THEOREM 2.2. Applying the Lip condition on J', we obtain

$$|R_n| \le \tfrac{1}{2} D \int |F_n(x) - F(x)|^{2+\delta} dx \le \tfrac{1}{2} D ||F_n - F||_\infty^\delta \cdot ||F_n - F||_{L_2}^2 .$$

Thus, for any $A > 0$,

$$P(|R_n| > An^{-1}) \le P(n||F_n-F||_\infty^\delta ||F_n-F||_{L_2}^2 > 2A/D)$$

$$\le P(n^{1/6}||F_n-F||_\infty^\delta > 2A/D) + P(n^{5/6}||F_n-F||_{L_2}^2 > 1) .$$

For $\delta > 1/3$, the first term above is $O(n^{-\frac{1}{2}})$ by the DKW inequality used in the proof of Theorem 2.1. The second term above is $O(n^{-\frac{1}{2}})$ by Lemma 2.1, applied for $k = 3$. Therefore, R_n satisfies (1.3).

The required properties of h are established in similar fashion as in the proof of Theorem 2.1. In this connection, the inequality (2.7) is useful in proving finiteness of moments. □

REMARK. It is evident from the preceding proof that under a higher moment assumption $E|X_1|^\nu < \infty$, where ν is an integer > 3, the Lip condition on J' may be relaxed to order $\delta > 1/\nu$. □

3. *M-estimates*. Let ψ be a function such that the parameter of interest $T(F)$ may be defined as a solution of the equation $\lambda_F(t) = 0$, where $\lambda_F(t) = \int\psi(x-t)dF(x)$. Thus $T(\cdot)$ represents an "M-functional with respect to ψ" if $\lambda_F(T(F)) = 0$, all F . In the following let us put T_n for $T(F_n)$ and t_o for $T(F)$.

The approach of Section 1 leads to

$$\alpha(x) = -\frac{\psi(x-t_o)}{\lambda_F'(t_o)} ,$$

and

$$\beta(x,y) = \frac{2\alpha(x)}{\lambda_F'(t_o)} [\psi'(y-t_o) + \lambda_F'(t_o) - \tfrac{1}{2}\lambda_F''(t_o)\alpha(y)] .$$

Defining $h(t) = [\lambda_F(t) - \lambda_F(t_o)]/(t-t_o)$ or $\lambda_F'(t_o)$, according as $t \neq t_o$ or $t = t_o$, we obtain for $R_n = T_n - t_o - V_n$

$$R_n = -\frac{\int[\psi(x-T_n)-\psi(x-t_o)]d[F_n(x)-F(x)]}{h(T_n)} + \left[\frac{1}{\lambda_F'(t_o)} - \frac{1}{h(T_n)}\right] \int \psi(x-t_o)d[F_n(x)-F(x)]$$

$$+ \frac{\int\psi(x-t_o)d[F_n(x)-F(x)] \int\psi'(x-t_o)d[F_n(x)-F(x)]}{[\lambda_F'(t_o)]^2}$$

$$+ \frac{\lambda_F''(t_o)\{\int\psi(x-t_o)d[F_n(x)-F(x)]\}^2}{2[\lambda_F'(t_o)]^3} .$$

A brute force treatment of R_n leads to

$$|R_n| \leq C_1 \, ||F_n - F||_\infty^3$$

under moderate restrictions on ψ, ψ', ψ'', and on $\lambda_F'(t)$, $\lambda_F''(t)$ and $\lambda_F'''(t)$ for t in a neighborhood of t_o. An application of the DKW inequality cited in Section 2 then yields (1.3) for R_n. Condition (1.2) is easily verified. For other discussion of the Berry-Esséen rate for M-estimates, see Bickel (1974).

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