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ASYMPTOTICALLY OPTIMAL DETECTOR OF MEMORY P FOR K-DEPENDENT RAN--ETC(U)

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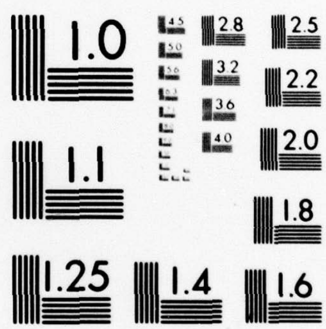

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(6) ASYMPTOTICALLY OPTIMAL DETECTOR OF MEMORY p FOR k-DEPENDENT RANDOM SIGNALS.

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Abstract

This note describes a general method for discriminating between two k-dependent stationary discrete random variables using marginal statistics and their first order correlations.

Asymptotically Optimal Detector of Memory p for k - dependent Random Signals

Lee K. Jones

A stationary discrete time series $\{X_i\}_{i=1}^{\infty}$ is said to be k-dependent if for every integer m, $\{X\}_{i\leq m}$ is independent of $\{X_i\}_{i>k+m}$. Suppose $\{X_i^1\}$ and $\{X_i^2\}$ are two stationary k-dependent random signals occurring with prior probabilities α and 1-α respectively. If n pulses of a random signal are observed (n large compared to k) how do we decide whether we observed $\{X_i^1\}_{i=1}^n$ or $\{X_i^2\}_{i=1}^n$?

A detector L of memory p is given by a function $L(X) = L(x_1, x_2, \ldots, x_n) = \sum_{i=p+1}^{n} g_i(x_i, x_{i-1}, \ldots, x_{i-p})$ and a set A(1) of real numbers such that:

for $L(X) \varepsilon A$ we choose class 1

for $L(X) \varepsilon A^c$ we choose class 2

In view of the stationarity we need only consider (for n large compared to p) detectors for which $g_i = g$ for all i.

Suppose both $\{X_i^1\}$ and $\{X_i^2\}$ are bounded with known (or estimates of) statistical properties (correlations, moments, etc.). In this note we use a straightforward extension of the method of minimal marginal moment variance(2) to determine the g which

(1) For the cases we shall consider, A is the union of one or two intervals.

(2) Introduced in [2] to solve for the case p=0, k=0.

minimizes the probability of error. This solves the problem addressed in [1] ($p=0$, $x_i^1=\theta+n_i$, $x_i^2=n_i$, n_i k-dependent).

Solution: Let 1, h_1, h_2, ... be a complete set of continuous functions on R^{p+1}. For $g_q = \sum_1^q a_j h_j$ we find the coefficients a_j which minimize the probability of error. The constant function 1 need not be present in this expansion since it trivially has the same statistical properties under both hypotheses. As q becomes large the error probability using g_q converges to the error probability for the optimal g.

Consider $L(X) = \sum_{i=p+1}^{n} g_q (x_i, x_{i-1}, \ldots, x_{i-p})$. It is a sum of bounded k+p - dependent random variables. Hence L is asymptotically normal under each hypothesis. The set A will in general be described by 2 thresholds. (If a single threshold detector is desired the following analysis remains the same.)

Let $\mathcal{E}(a_j)$ = probability of error = $\int_{-\infty}^{+\infty} \min \{\alpha p_1, (1-\alpha) p_2\}$ dx where p_i is the probability density of L under hypothesis i. By normality p_1 and p_2 are characterized by their means and variances. We now restrict a_j such that $E_2 L(X) - E_1 L(X) = 1$. $\mathcal{E}$ is then a function of the variances of L under each hypothesis: $\mathcal{E} = \mathcal{E}(v_1(a_j), v_2(a_j))$. Taking the gradient wrt a_j and using λ as a Lagrange multiplier we have:

$$\frac{\partial \mathcal{E}}{\partial v_1} \nabla v_1 + \frac{\partial \mathcal{E}}{\partial v_2} \nabla v_2 - \lambda(\nabla(E_2 L - E_1 L)) = 0 \qquad (1)$$

The partials $\frac{\partial \mathcal{E}}{\partial v_i}$ may be negative (even for a single threshold detector) but are not both zero. It is easy to see that a solution of (1) is a critical point of the objective function $\beta v_1 + (1 - |\beta|)v_2 - \Phi(E_2L-E_1L-1)$ where Φ is a Lagrange multiplier and $-1 \leq \beta \leq +1$. This critical point may be determined for various values of β and the β corresponding to minimum probability of error determined from normal tables.

We now proceed with the calculations. Let

$$h_j^i = h_j(x_i, x_{i-1}, \ldots, x_{i-p})$$

$$\rho_{ijs}^{\ell} = E_i\left[(h_j^{\,1} - E_i\, h_j^{\,1})\ (h_s^{1+\ell} - E_i\, h_s^{1+\ell})\right]$$

By differentiating the above objective function wrt a_j and setting the result equal to zero we obtain:

$$\left[\beta\ A_1 + (1-|\beta|)A_2\right]\ \vec{a} = \frac{\Phi}{2}\ (n-p)\ \vec{m} \tag{2}$$

where $m_j = E_2h_j^1 - E_1h_j^1$

and $(A_i)_{js} = (n-p)\rho_{ijs}^0 + \sum_{\ell=1}^{k+p} (n-p-\ell)\ (\rho_{ijs}^{\ell} + \rho_{isj}^{\ell})$

Solving equation (2) -

$$\vec{a} = \Phi\ (\frac{n-p}{2})\ \left[\beta A_1 + (1-|\beta|)A_2\right]^{-1}\ \vec{m} = \Phi\ \vec{g}_{\beta}$$

Then

$$\Phi = (\sum_{j=1}^{q} g_{\beta j}\ m_j)^{-1} \quad \text{from the condition} \quad E_2L-E_1L=1.$$

v_1 and v_2 may now be calculated and the error (as a function of β) determined. The β corresponding to minimum error is then obtained by a one-parameter minimization. The preceding method will yield an optimal detector whenever the p_i occurring in the expression for $\mathcal{E}(a_i)$ depend only on the means and variances of L. We need only estimate the error as a function of β from the performance of L on sample data.

References

[1] H. V. Poor and J. B. Thomas, "Time Detection of a Constant Signal in m-Dependent Noise," IEEE Trans. Inform. Theory IT-25, Pages 54-61, (1979).

[2] On Optimal Discriminants between two Classes of Random Variables in Terms of the Moments of their Distributions, submitted to SIAM Journal of Appl. Math.

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