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A UTILITY REPRESENTATION FOR TEMPORALLY MYOPIC PARTIAL ORDERING--ETC(U)
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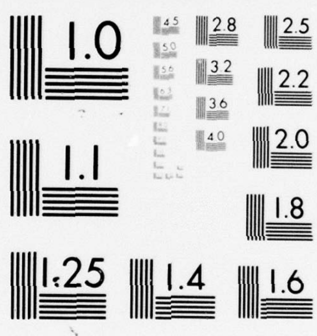

MICROCOPY RESOLUTION TEST CHART

CENTER ON DECISION AND CONFLICT
IN
COMPLEX ORGANIZATIONS

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A UTILITY REPRESENTATION FOR TEMPORALLY MYOPIC PARTIAL ORDERINGS.

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Introduction

Following the framework of Aumann [1], necessary and sufficient conditions for the existence of a bounded utility representation on a partially ordered space of programmes with denumerably infinite dimension is provided.

Emphasis is placed on the application of nonstandard techniques by which the space of programmes is imbedded in a nonstandard space of *Finite dimension. Additionally, the relative topology, γ^*_X, on the space of programmes induced by the product S-topology on the hyper-finite linear space, L*, is sufficient for the existence of a continuous linear functional in the algebraic dual of X,X*.

Our consideration of the partial order framework is motivated by the fact that an indefinite sequence of events in time is in principle unknowable and that therefore any choice made between such elements is ultimately based on some judgment as to consequences beyond some finite time base of reference, bounded, if not by anything else, by the present state of knowledge. That rationality should force preferences to be complete in such a context is less than compelling.

I. Preliminaries

Let X be an infinitely dimensioned real linear space, the elements of which are programmes $x = (x_1, \ldots, x_n, \ldots)$, where each x_i is from a compact subset Ω of R^n_+. Allow an order, $\geq$, to be defined on $X \times X$ such that $\geq$ is reflexive and transitive but not necessarily complete, and such that the following is satisfied for $x, y \in X$:

I.1 $\quad x \geq y \Rightarrow x + z \geq y + z \quad \forall z \in X$

I.2 $\quad x \geq y \wedge \alpha \in (R_+ - \{0\}) \Rightarrow \alpha x \geq \alpha y$

I.3 $\quad x > Kz \quad \forall K \in Z^+ \Rightarrow \sim(z > 0)$

where $x > y$ is to mean $x \geq y \wedge \sim(y \geq x)$ and $x \sim y$ is to mean $x \geq y \wedge y \geq x$.

Assumptions I.1 and I.2 simply state that $\geq$ preserves the linearity of the space X, while I.3 ensures that $\geq$ is not lexicographic.(1) As an easy consequence of the assumptions, one has:

Proposition I.3.1: The ordering $\geq$ satisfies:

(i) $\quad x \geq y \Leftrightarrow (x - y) \geq 0$

$\quad x > y \Leftrightarrow (x - y) > 0$

(ii) $\quad x \geq y$ and $z \geq y$ and $\alpha \in (0,1) \Rightarrow x + (1 - \alpha) z \geq y$

$\quad x > y$ and $z > y$ and $\alpha \in (0,1) \Rightarrow x + (1 - \alpha) z > y$

(iii) $x \geq 0 \Leftrightarrow -x \leq 0 \wedge x > 0 \Rightarrow -x < 0$

For $T_X = \{x : x > 0\}$ and $S_X = \{x : x \geq 0\}$

(iv) $\quad S_X$ and T_X are convex cones for which $0 \notin T_X$, $0 \in S_X$, $T_X \subseteq S_X$.

For subsets $A, B \subseteq X$, let $A + B = \{x + y : x \in A, y \in B\}$ and $-A = \{x : -x \in A\}$

(v) The Kernel of the induced relation, $\sim$, on X is

$$S_X \cap (-S_X) = \{x : x \sim 0\}$$

A utility on X is a real linear functional, u, that represents the order, $\geq$, in the following sense: $u(x) \geq 0$ if $x \in S_X$, $u(x) > 0$ if $x \in T_X$, and $u(x) = 0$ if $x \in (S_X \cap (-S_X))$.

Theorem I.4: A utility on X exists only if in any topology which is linear on X such that all linear functionals are continuous, one has

$$(-T_X) \cap (c\ell S_X) = \phi$$

Proof: (Kannai [2])

Suppose then for some $x, x \in (-T_X) \cap (c\ell S_X)$. From (iii) in Proposition I.3.1 it follows that $u(x) < 0$. However, in every neighborhood of $x, B(X)$, there is some $y \in B(x)$ such that $y \in S$, since $x \in c\ell(S_X)$. Hence $u(y) \geq 0$ and by the continuity of $u(\cdot)$, $u(x) \geq 0$, which contradicts $u(x) < 0$.

Q.E.D.

It should be remarked that Theorem I.4 requires that conditions I.3 be necessary for a utility in R^n with the natural topology, and Aumann has shown that in R^n, I.3 implies $(-T_X) \cap (c\ell S_X) = \phi$, which, by the way, is easily seen to imply $(-T_X) \cap (c\ell T_X) = \phi$, which, in turn, indicates that the cone T_X is improper[†] (cf. Aumann, [1], p. 456).

We shall be interested in obtaining a utility representation for the ordered structure $(X, \geq)$ where the partial order, $\geq$, exhibits a form of temporal perspective which we characterize as follows:

(Π) Let $(X, \geq)$ be such that $\forall x \in X\ \exists y \in (X|_{X^K})$ such that for some $\tilde{z}(K) \in X$, $(y + \tilde{z}(K)) \sim x$, where $X|_{X^K}$ is the space of programmes truncated to K periods, and $\tilde{z}(K)$ is a positive constant on X for $\tilde{z}_i(K)$ $i > K$ and $\tilde{z}(K) = 0$, for $\tilde{z}_i(K)$ $i \leq K$.

It should be said that (Π) is a weak form of temporal myopia. We term it weak since it is implied by stronger forms. For example, the concept of monotone myopia found in Brown and Lewis [3] in the case where $\geq$ is complete and continuous on a subspace of ℓ_∞, in the appropriate topology, implies (Π). The item of distinction is that the topology we employ presently may not be strong enough to yield continuity in the Brown and Lewis context. This is an issue for future research to be pursued in a sequel paper. The interpretation given to (Π) is that given an arbitrary programme, of infinite length, there is a programme with constant tail, for sufficiently large K, to which the former programme is indifferent. That a preference can exhibit such a property is highly plausible in the context of indefinite time horizons since the framework includes comparisons to be made between programme lengths of 10^{90} time units, and beyond.

[†]A proper cone is one for which $(c) \cap (-c) = \{0\}$.

Intuitively, there should be some number, say, 1037, for which the consequences of a programme, x(t), for $t > 10^{1037}$ just simply don't matter enough to warrant more than indifference between those consequences and some constant tail. Alternatively, one could argue philosophically on the grounds that an infinite sequence of events is in principle unknowable and that at best one can only extrapolate onward from some finite time frame of reference. In that vein, temporal myopia could be seen as the result of limited capacities of extrapolation in the sense that the hypothesis that constant consequences will obtain is as good as any other attempt to specify them in further detail.

II. The Nonstandard Framework

Let M be a super structure sufficiently rich to model the linear space described earlier as X. Let M* be an enlargement of M in the sense of [9], that provides a non-standard model of M. We shall make use of the specific requirement that M* be polysaturated as in Reference 4, Definition 7.4.6, p. 182 (see Appendix).

A basis for X is the collection $\mathbb{B} = \{H(n)_j\}_{j=1}^{\infty}$ where each element is a basis for R^n. The dimensionality of $\mathbb{B}$ is then denumerable. Then because M* is polysaturated, it is permissible to imbed X in a hyperlinear space L* (Theorem 7.6.1 of Reference 4). A basis for L* is then $\mathbb{B}^* = \{H^*(n)_j\}_{j=1}^{\omega}$ for $\omega \in N^* - N$. Then L* is an internal hyper-finite linear space of dimension $n\omega$, an element of which is a *finite sequence of vectors $x^* = (x_1^*, \ldots, x_\omega^*)$, where each $x_j^* \in \Omega^* \subseteq R_+^{*n}$ (reference Note 10.1.12, p. 271, of Reference 4), and each Ω^* is near-standard from the compactness of Ω in R_+^n.

Let Z*(K) be an internal supplementary space co-finite (not co*finite) to L*. Then $L^* \oplus Z^*(K) = G^*(K)$ where G*(K) has dimension $K \in N$ and Z*(K) is the collection of internal final segments of elements of L* with negative sign and identically zero for the first K places. The reference on supplementary space as used here is that of Grothendiek [5], Ch. I.11.(2)

Let $\geq^*$ be a partial order on L* satisfying the following:

II.1* $\forall x^*, y^* \in L^* \quad x^* + z^* \geq^* y^* + z^* \quad \forall z^* \in L^*$

II.2* $\forall x^*, y^* \in L^* \quad \alpha \in (R_+ - \{0\})\,(x^* \geq^* y^* \Rightarrow \alpha x^* \geq^* \alpha y^*)$

II.3* $\forall x^* \in L^* \quad \forall K^* \in N^* \; \left(x^* >^* K^* z^* \Rightarrow \sim(z^* >^* 0)\right) \; \forall z^* \in L^*$

where $x^* >^* y^*$ means $x^* \geq^* y^* \ldots \sim(y^* \geq^* x^*)$ and $x^* \sim^* y^*$ means $x^* \geq^* y^* \ldots y^* \geq^* x^*$.

The use of standard positive scalars in Property II.2* yields that the cones, $S^*_X = \{x^* : x^* \geq^* 0\}$ and $T^*_X = \{x^* : x^* >^* 0\}$, are S-convex in L^* and that $0 \notin \text{S-int}(T^*_X)$ while $0 \in S^*_X$ and $T^*_X \subset S^*_X$.(3) An analogous version of Proposition I.3.1 is true of $\geq^*$, which we will refer to as Proposition II.3.1*.

(Π^*) Let $(L^*, \geq^*)$ be such that $\forall x^* \in L^* \; \exists Z^*(K)$ for some $K \in N$ such that $x^* \sim^* (y^* - \dot{\bar{z}})$, where $\bar{z}$ is a constant element of $Z^*(K)$ and $y^* \in G^*(K) = L^* \oplus Z^*(K)$.

Π^* is the formal restatement of Π in the context of L^*, the interpretation of which is that $\geq^*$ is standardly temporally myopic.

Following Kannai [2], we note that the space X can be topologized by the product topology $\gamma = \prod_{j=1}^{\infty} (\gamma_j^n)$, where γ_j^n is the natural product topology on R^n. A typical neighborhood of the origin in γ is a sequence, $\{e_j\}_{j=1}^{\infty}$, such that $|x(j)| < e_j$ for $x \in X$ and each $e_j > 0$. It can also be verified that members of the algebraic dual of X,X* are continuous linear functionals on X in γ. The use of algebraic dual is the same as found in Aumann [1], p. 458.

Consider now the product S-topology, as defined by Robinson [9], on L*, which we denote as γ^*. Then $\gamma^* = \prod_{j=1}^{\omega} (\gamma_j^*)$, where γ_j^* is the product S-topology on $(R^*)^n$, for n fixed and standard for all j. Since $R \subsetneq R^*$ in M* and n is fixed, from the fact that $X \subsetneq L^*$, one obtains in straightforward manner:

Lemma II.4: $\gamma_X^* = \gamma$

Proof: The open sets of γ_X^* are precisely $\{X \cap O_{\gamma^*} : O_{\gamma^*} \in \gamma^*\}$, where the O_{γ^*} are characterized by standard neighborhoods in the S-topology.

Q.E.D.

Definition II.5: Let $\Phi : N^* \to R^*$ be a sequence on the extended natural numbers N* with values in R*. A point $\tilde{x}^*$ is said to be an F-limit of $\Phi(n)$ if $(\forall \delta \varepsilon R\ \delta > 0)(\exists n \varepsilon N)(\forall m \geq n) \Rightarrow |\Phi(m) - \tilde{x}^*| < \delta$. Alternatively, we say that $\Phi(n)$ F-converges to $\tilde{x}^*$.

A set defined in M* is said to be S-closed if it is closed under F-limits. The S-closure of a set we denote by $S\text{-}c\ell(\cdot)$.

Lemma II.6: Let $(L^*, \gtrsim^*)$ be endowed with the product S-topology, γ^*. Then $(-T_X^*) \cap (S\text{-}c\ell T_X^*) = \phi$.

Proof: Let $\tilde{x}^* \varepsilon (-T^*_X)$. Then $-\tilde{x}^* \varepsilon T^*_X$, which means $-\tilde{x}^* >^* 0$. By virtue of the fact that T^*_X is an S-convex cone in L^*, if $\tilde{x}^* \varepsilon S\text{-}c\ell(T^*_X)$, then for some $y^* \varepsilon T^*_X$, $\underset{K \to \infty}{F\text{-}\lim} \left(\frac{y^*}{K} + \tilde{x}^*\right) = \tilde{x}^*$. But this contradicts II.3* since in that case, because $0 \notin S\text{-int}(T^*_X)$, $\left(\frac{y^*}{K} + x^*\right) >^* 0$ for all $K \varepsilon N$.

Q.E.D.

Definition II.7: The partial order $\geq^*$ on L^* is pure in the sense of Aumann [1] if $x^* \sim^* y^*$ is true if and only if $x^* = y^*$. Alternatively, we may characterize $\geq^*$ as being pure on L^* if $\left(S_X \cap (-S_X)\right) \subseteq \text{Kernel}\left(L^* - (L^*)\right)$, where $\text{Kernel}\left(L^* - (L^*)\right) = \left\{(x^*,y^*) \varepsilon (L^* \times L^*) : (x^* - y^*) = 0\right\}$.

The existence proof that follows is based on a construction found in Kannai's generalization of Aumann's Theorem A [1,2]. A point of distinction is that the assumption that T^*_X is improper on L^* is not required in the present framework, however, as that condition is a natural consequence of Lemma II.6.

▶Theorem II.8: There exists a utility representing $\geq$ on X.

Proof: We begin by considering the ordered structure $(\tilde{L}^*, \geq^*)$, where $\tilde{L}^*$ is the quotient space $L^*(\text{Mod } \tilde{Z}^*)$,(4) for $\tilde{Z}^* = \bigcup_{K \varepsilon N} Z^*(K)$. By Property (Π^*) one has $(L^*, \geq^*) \subseteq (\tilde{L}^*, \geq^*)$. W.O.L.G. one can assume that $\geq^*$ is pure on $\tilde{L}^*$ since if not, we can work with $(\tilde{L}^*/[E], \geq^*)$, where $[E] = \{x^* \varepsilon \tilde{L}^* : x^* \sim^* 0\}$.

Let $y^* \varepsilon T^*_X$ in $\tilde{L}^*$, and consider the Q-convex closure of $(V(y^*) \cup T^*_X)$, which we denote as Q-con$(V(y^*) \cup T^*_X)$ and such that Q-con $(V(y^*) \cup T^*_X)$ contains all *Finite linear combinations† of points in $(V(y^*) \cup T^*_X)$ and $V(y^*)$ is an open S-neighborhood of y^*.

Lemma II.9: For some $V(y^*)$, if $y^* \varepsilon T^*_X$ then $0 \notin$ Q-con$(V(y^*) \cup T^*_X)$.

Proof: (Klee [10]) If 0ε Q-con$(V(y^*) \cup T^*_X))$, then for $y^*_1 \varepsilon V(y^*)$, $y^*_2 \varepsilon T^*_X$, and some standard $\alpha \varepsilon (0,1)$, $\alpha y^*_1 + (1-\alpha)y^*_2 = 0$ for all $V(y^*)$. Then $-\alpha y^*_1 = (1-\alpha)y^*_2$ and $-y^*_1 = \left(\frac{1-\alpha}{\alpha}\right)y^*_2$. But because T^*_X is an S-convex cone, $\left(\frac{1-\alpha}{\alpha}\right)y^*_2 \varepsilon T^*_X$ and therefore $-y^*_1 \varepsilon T^*_X$. This contradicts Lemma II.6.

Q.E.D.

Then by the fact that $\gtrsim^*$ is pure on $\tilde{L}^*$, by the lemma, for a given $y^* \varepsilon T^*_X$, for some $V(y^*)$, Q-con$(V(y^*) \cup T^*_X)$ is non-void and plainly distinct from 0, by Proposition II.3.1*.

By an analog of a standard separation result, as found in Brown and Pearcy [3], Proposition 14.14 for example, since Q-con$(V(y^*) \cup T^*_X)$ is internal, there exists an S-continuous linear functional on the algebraic dual of $\tilde{L}^*$, $\tilde{u}^*_{y^*}$, that supports Q-con$(V(y^*) \cup T^*_X)$. Then for $x^* \varepsilon (V(y^*) \cup T^*_X)$

†That is, combinations of the form $\sum_{j \varepsilon F^*} \alpha^*_j x^*_j$ for $\alpha^*_j \varepsilon [0,1]^*$ $\sum_{j \varepsilon F^*} \alpha^*_j = 1$, $x^*_j \varepsilon (V(y^*) \cup T^*_X)$ and F^* internally *Finite.

$u^*_{y^*}(x^*) \geq 0$ and since $y^* \in$ S-int$(V(y^*) \cup T^*_X)$ $\tilde{u}^*_{y^*}(y^*) > 0$ and by S-continuity of $\tilde{u}^*_{y^*}$, $\tilde{u}^*_{y^*}(\bar{y}^*) > 0$ for any $\bar{y}^* \in V(y^*)$ with $\tilde{u}^*_{y^*}$ taking values in R^*.

Zakon [6] has shown (Theorem 4.1) that the S-topology is Second Countable[(5)] and therefore $\tilde{L}^*$ is separable in the product S-topology, γ^*. It follows that there is an F-sequence $\{y^*_j\}_{j \in N}$ such that each $y^*_j \in T^*_X$ and $T^*_X \subset \bigcup_{j \in N} V(y^*_j)$, where $V(y^*_j)$ is an S-neighborhood of y^*_j. We can then form a family $\{S_j\}_{j \in N}$, such that $S_j = V(y^*_j) \cup S_{j-1}$, where $S_{j-1} = \bigcup_{j=1}^{j-1} V(y^*_j)$, which has the finite intersection property. Therefore, making use of the polysaturated character of M^* once more ([4], p. 182), $T^*_X \subset \bigcup_{j=1}^{\omega} V(y^*_j)$ for $\omega \in N^* - N$.

For each $j = 1, \ldots, \omega$, consider the restriction of $\tilde{u}^*_{y^*j} | (R^*_+)^{jn}$. Since Ω^* is near standard, $\tilde{u}^*_{y^*j}$ is bounded there and its norm in the relative S-topology we denote as $\|\tilde{u}^*_{y^*j}\|_{jn}$ and remark that $(\|\tilde{u}^*_{y^*j}\|_{jn}) \in M^+_0$ for each standard $n \in N$, where $M^+_0 = \{r \in R^*_+ : |r| < n \; n \in N\}$.

Consider the following construction on $\tilde{L}^*$:

$$\text{Ш}(x^*) = \sum_{j=1}^{\omega} \frac{\tilde{u}^*_{y^*j}(x^*)}{2^j \cdot \|\tilde{u}^*_{y^*j}\|_{jn} + 1}$$

where $\text{Ш}(\cdot) : \tilde{L}^* \to R^*$.

▶Lemma II.10: For all $x^* \in \tilde{L}^*$, $\text{Ш}(x^*) \in M^+_0$.

Proof: By property (Π^*) any $x^* \in \tilde{L}^*$ is an element of $({}^*R_+^{nK})$ for K standard so that for some fixed standard K, $x^* \in ({}^*R_+^{nj})$ for $j \geq K$ and for $m \geq K$,

$$\sum_{j=1}^{m} \left| \frac{\tilde{u}^*_{y^*j}(x^*)}{2^j \|\tilde{u}^*_{y^*j}\|_{jn} + 1} \right| \leq \sum_{j=1}^{K-1} \frac{|\tilde{u}^*_{y^*j}(x^*)|}{2^j \|\tilde{u}^*_{j^*j}\|_{jn} + 1} + \sum_{j=K}^{m} \frac{\|\tilde{u}^*_{y^*j}\|_{jn} \|x^*\|_{jn}}{2^j \|\tilde{u}^*_{y^*j}\|_{jn} + 1}$$

$$\leq \sum_{j=1}^{K-1} \frac{|\tilde{u}^*_{y^*j}(x^*)|}{2^j \|\tilde{u}_{y^*j}\|_{jn} + 1} + \sum_{j=K}^{m} \frac{\|x^*\|}{2^j}$$

where $\|x^*\|_j = \|x^*\| = \|x^*\|_{nK}$ for all $j \geq K$.

In the last expression, since K is standard, the values in the term on the left are constant on R^*_+ and plainly in M_0^+, the values in the term on the right are infinitesimal for all $j \in N^* - N$ and by Robinson's [9] Theorem 3.5.13, the F-limit of the partial sums $(S_n - S_{n-1})$ is zero and therefore $\sum_{j=K}^{m} \frac{\|x^*\|}{2^j} \in M_0^+$ for arbitrary subscripts. Since the sum of two elements in M_0^+ is also in M_0^+, we are done.

Q.E.D.

Since $\mathbb{U}(x^*) \in M_0^+$ for any $x^* \in \tilde{L}^*$, we may take its standard part,[6] and since $X \subset L^*$, the restriction of $\succsim^*$ on X satisfies (Π). Let $\mathbb{U}_X$ be $\mathrm{Im}\left[st(\mathbb{U}(\cdot))\right]$.[7] It is a straightforward matter of verification that $\mathbb{U}_X$ is continuous on X in γ by Lemma II.4. Furthermore, since $\mathrm{Im}\left[st(\mathbb{U}(x^*))\right]$ vanishes on subscripts $j \in N^* - N$, for $x \in X$,

$$\mathbb{U}_X(x) = st(\mathbb{U}(x)) = \sum_{j \in N} \left[\frac{st\, \tilde{u}^*_{j^*j}(x)}{2^j st(\|u^*_{y^*j}\|_{jn} + 1)} \right]$$

the latter series being convergent, since $st(\mathbb{U}(x)) < \infty$. Since $T_X \subset T^*_X$, $\mathbb{U}_X(x) \geqq 0$ and for some $j = 1, \ldots, \omega$, $x \in V(y^*_j)$, since $\{V(y^*_j)\}_{j=1}^{\omega}$ covers T^*_X and a fortiori T_X. Then for some y^*_j, $st(\tilde{u}^*_{y^*j}(x)) > 0$, and thus $\mathbb{U}_X(\cdot)$ represents $\succeq$ on X.

Q.E.D.

Notes

(1) This is proven by Aumann [1], p. 453.

(2) A vector space X is said to be the direct sum of the vector spaces M_j, $j = 1, \ldots, n$, for finite $n \in N$, which one symbolizes as:

$$X = M_1 \oplus M_2 \oplus \ldots \oplus M_n$$

if the spaces M_j are subspaces of X and such that every $x \in X$ can be uniquely represented as:

$$x = m_1 + m_2 + \ldots m_n$$

where $m_j \in M_j$ for $j = 1, \ldots, n$.

Grothendiek [5], p. 34, terms two vector spaces F and G to be supplementary with respect to the vector space X if the mapping

$$(x,y) \longmapsto x + y$$

of $F \times G$ into X is an isomorphism. In particular, this requires that $F \cap G = \{0\}$ and that $F \oplus G = X$, as defined above.

The co-dimension of a space G to F where F and G are supplementary, and G is a subspace of F is equal to $\dim(F) - \dim(G)$.

(3) The S-interior of a set contains open S-neighborhoods of its members.

(4) That is, a member of $\tilde{L}^*$ is a member of some $G^*(K)$ for K standard finite dimension.

(5) The demonstration involves the observation that the neighborhood base for the S-topology on R* is identical to the neighborhood base of R in the natural topology.

(6) The order preserving homomorphism from M_0 to R with Kernel M_1 is denoted as st(·) and is termed the standard part. If $x^* \varepsilon M_0$, then st(x*) is the unique real number in R such that $(st(x^*) - x^*) \varepsilon M_1$.

(7) By Im[st(Ш(x*))] we mean the projection of the standard part of Ш(x*) back onto $\tilde{L}^*$.

Appendix

Polysaturated Enlargements

An internal relation, ψ, is concurrent on a subset $A \subseteq \ell D(\psi)$ if whenever a finite set (standard), $\{a_1, \ldots, a_n\} \subseteq A$, of elements of A is is given, then there exists some $y \in rD(\psi)$ such that $(a_j, y) \in \psi$ for all $j = 1, \ldots, n$.

One says that ψ is satisfied for A if there is some $z \in rD(\psi)$ such that for any $a \in A$, $(a,z) \in \psi$.

Let κ denote an infinite (standard) cardinal number. An enlargement M^* is said to be κ-saturated is whenever an internal binary relation ψ is concurrent on a set A, where A is such that $\text{card}(A) < \kappa$, then ψ is satisfied for A in the sense given above. An enlargement M^* that models a set X is polysaturated if it is κ-saturated where $\text{card}(X) \leq \kappa$.

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13. ABSTRACT

A utility representation for partial orders defined on a denumerably infinite programme space is obtained for the case where the partial orders satisfy a weak form of temporal myopia. Nonstandard techniques are employed, by which the space of programmes is imbedded in a hyper-linear space of *Finite dimension. The relative topology on the space of programmes induced by the product S-topology on the hyper-linear space is sufficient for the existence of a continouos linear functional in the algebraic dual of the programme space. The general technique is attributable in origin to Aumann [1].

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