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MAXIMUM NUMBER OF BEAMS IN MULTIBEAM SONARS. (U)  
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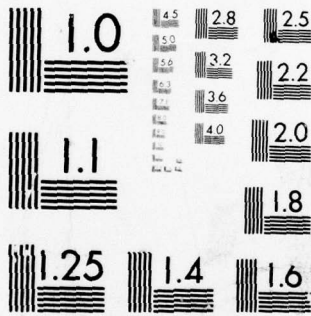
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NAVY UNDERWATER SOUND LABORATORY  
FORT TRUMBULL, NEW LONDON, CONNECTICUT

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6 MAXIMUM NUMBER OF BEAMS  
IN MULTIBEAM SONARS

by

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USL Technical Memorandum No. 2242-99-69

11 16 April 1969

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### Introduction

The Nyquist sampling theorem is used to derive the maximum number of preformed beams required to adequately sample a signal field with a given line array so that the angle of signal arrival can be determined. Consider a single frequency plane wave incident upon a line array. After forming an infinite number of beams (or scanning) a plot of beam output power versus steered angle is obtained. The target bearing is the angle at which this plot is a maximum.

By Nyquist sampling the plot of output power versus bearing, the plot is reconstructed from a finite number of beams. This number of beams is minimum in the sense that fewer beams result in a signal error while more beams are redundant. The Rayleigh resolvability criteria for multiple detections is investigated and compared with the previous result.

### Finite Aperture Continuous Array

For a single frequency uniform plane sound wave incident upon a continuous line array of aperture L (Fig. 1)

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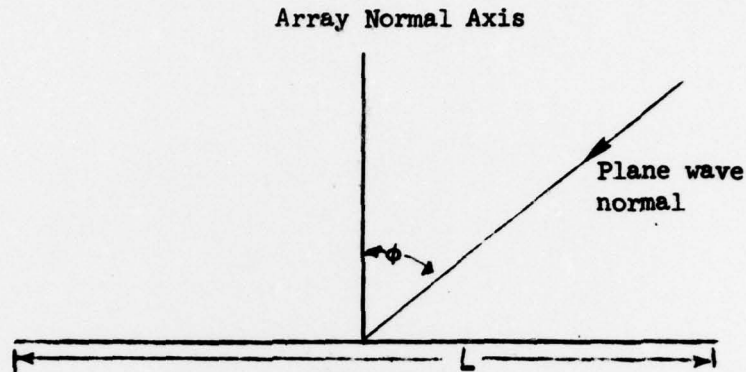


Fig. 1.

the array output is given by:

$$P = A \int_{-L/2}^{L/2} \sin \omega \left( t - \frac{x \sin \phi}{c} \right) dx \quad (1)$$

$$= AL \sin \omega t \left[ \frac{\sin \left( \frac{\omega L}{2c} \sin \phi \right)}{\frac{\omega L}{2c} \sin \phi} \right] \quad (2)$$

$$= AL \sin \omega t [ K(\phi) ]$$

where:

A = constant depending on wavelength of sound,  $\lambda$ , medium of propagation, and array sensitivity;

c = velocity of sound;

$$\omega = \frac{2\pi c}{\lambda}$$

$$K(\phi) = \frac{\sin \left( \frac{\omega L}{2c} \sin \phi \right)}{\frac{\omega L}{2c} \sin \phi}$$

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(3)

and

$\phi$  = arrival angle.

$K(\phi)$  is referred to as the directional characteristic or directivity pattern function and expresses the angular variation of the array output.

To steer the major lobe of the array away from the normal position (by an angle say  $\theta_i$ ) time delay is introduced and the general expression for  $K(\phi)$  becomes:

$$K_i(\phi) = \frac{\sin \left[ \frac{\omega L}{2c} (\sin \phi - \sin \theta_i) \right]}{\frac{\omega L}{2c} (\sin \phi - \sin \theta_i)} \quad (4)$$

where:  $-\frac{\pi}{2} \leq \theta_i \leq \frac{\pi}{2}$

$i$  = the  $i^{\text{th}}$  beam. Note that the waveforms of all the  $K_i(\phi)$  are identical to the one of  $K(\phi)$  but displaced by their respective  $\sin \theta_i$  value.

Following a square law detector-averager, the normalized output signal from the  $i^{\text{th}}$  beam is expressed as:

$$B_i = \left[ \frac{\sin \frac{\omega L}{2c} (\sin \phi - \sin \theta_i)}{\frac{\omega L}{2c} (\sin \phi - \sin \theta_i)} \right]^2 \quad (5)$$

All the  $B_i$  values from expression (5) are interpreted as sampled values of the function:

$$B = \left[ \frac{\sin \frac{\omega L}{2c} \sin \phi}{\frac{\omega L}{2c} \sin \phi} \right]^2 \quad (6)$$

in the range  $-1 \leq \sin \phi \leq 1$ .

To find the minimum number of independent samples required to reconstruct in this range, the Nyquist sampling rate is derived. Its derivation is based on the following argument:

With  $\sin \phi$  taken as the variable, relation (6) is rewritten as:

$$B = B(y) = \left[ \frac{\sin \frac{\omega L}{2c} y}{\frac{\omega L}{2c} y} \right]^2 \quad (7)$$

where:

$$y = \sin \phi$$

The range of interest of  $y$  is  $-1 \leq y \leq 1$ ; it is assumed for convenience that  $B(y)$  is a  $\text{Si}^2$  (1) function with the range  $-\infty \leq y \leq \infty$ . The convenience lies in the fact that the bandwidth of such a function is limited and recognized to be  $\frac{2\omega L}{c}$  (2). The Nyquist sampling interval (3) is then given by:

$$\Delta y = \Delta \sin \phi = \frac{\pi}{2\omega L/c} = \frac{\lambda}{4L} \quad (8)$$

The maximum number,  $M$ , of independent samples of  $B$  in the range  $-1 \leq y = \sin \phi \leq 1$  is:

$$M = \frac{2}{\lambda/4L} = \frac{8L}{\lambda} \quad (9)$$

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(1)  $\text{Si}(x) = \frac{\sin x}{x}$

(2) The bandwidth of  $\text{Si} \frac{a}{2} x$  is  $a$  and the bandwidth of  $\text{Si}^2$  is then  $2a$ .

(3) M. Schwartz; Information Transmission, Modulation, Noise: M.G.H. 1959.

M represents the maximum number of directivity patterns or simply beams, to be formed in a multibeam sonar and they are formed such that the sine of the angle between adjacent beams is equal to  $\frac{\lambda}{4L}$ .

#### Finite Aperture Element Array

A similar procedure is followed for the case of a half wavelength spaced element array. The only difficulty which arises is that no simple expression exists for the bandwidth; this is circumvented by assuming that the bandwidth extends up to the harmonic at which the Fourier coefficient is 40 db below the coefficient of the fundamental.

The directivity pattern function for an array of N equidistant elements separated by a distance d is given by (4):

$$K(\phi) = \frac{\sin \left( N \frac{\pi d \sin \phi}{\lambda} \right)}{N \sin \left( \frac{\pi d \sin \phi}{\lambda} \right)} \quad (10)$$

With beam steering used, the general expression for  $K(\phi)$  is:

$$K_1(\phi) = \frac{\sin \left[ N \frac{\pi d}{\lambda} (\sin \phi - \sin \theta_1) \right]}{N \sin \left[ \frac{\pi d}{\lambda} (\sin \phi - \sin \theta_1) \right]} \quad (11)$$

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(4) R. L. Pritchard; Directivity of Acoustic Linear Point Arrays, Tech. Memo. No. 21, Acoustic Research Lab., Harvard Univ.

Following a square law detector-averager, the normalized output signal from the  $i^{\text{th}}$  channel is:

$$B_i = \left[ \frac{\sin N \frac{\pi d}{\lambda} (\sin \phi - \sin \theta_i)}{N \sin \frac{\pi d}{\lambda} (\sin \phi - \sin \theta_i)} \right]^2 \quad (12)$$

which may also be interpreted here as sampled values of the function

$$B = \left[ \frac{\sin N \frac{\pi d}{\lambda} \sin \phi}{N \sin \frac{\pi d}{\lambda} \sin \phi} \right]^2 \quad (13)$$

Using the substitution,  $y = \sin \phi$ ,

$$B \equiv B(y) = \left[ \frac{\sin \left( \frac{N \pi d}{\lambda} y \right)}{N \sin \left( \frac{\pi d}{\lambda} y \right)} \right]^2 \quad (14)$$

An argument similar to the preceding case leads to no simplification for finding the bandwidth. The procedure to be followed then consists of expanding  $B(y)$  in a Fourier series and limiting the bandwidth to the  $n^{\text{th}}$  harmonic at which the magnitude of its Fourier coefficient,  $C_n$ , is 40 db below that of the fundamental. The sampling theorem then gives  $2n$  as the number of independent samples. This leads to the conclusion that the maximum number,  $M$ , of beams to be formed in the multibeam sonar is equal to  $2n$  and the formation must be such that the sine of the angle between the two adjacent beams be equal to  $\frac{1}{n}$ .

The following results have been obtained by computer solution using an available Fast Fourier Transform (FFT) subroutine with 512

samples, and with  $d$ , distance between adjacent elements, equal to  $\frac{\lambda}{2}$ .

N	2	12	22	32	42	52	62	72	82	92	102
M	2	22	38	54	68	82	94	108	120	132	142

where M beams are found for N array elements.

#### Rayleigh Resolvability Criterion

This criterion is used as an empirical criterion for resolvability between two adjacent targets. In essence it specifies that for resolvability between two such targets, the axis of a beam pattern  $K_i(\phi)$  must coincide with the first zero value of the adjacent pattern  $K_{i+1}(\phi)$ , counted from the axis of  $K_{i+1}(\phi)$ . Application of this criterion is made for both the continuous and element arrays.

#### Case a: Continuous Array

From relation (3), it can be seen that the first zero of the beam pattern  $K(\phi)$  from its axis, occurs at  $\sin \phi = \frac{2\pi c}{\omega L} = \frac{\lambda}{L}$ .

Therefore, based on Rayleigh criterion, beams have to be formed such that the sine of the angle between them equals  $\lambda/L$ , this compared to the value  $\frac{\lambda}{4L}$  found for the number of independent beams.

#### Case b: N Element Array

From relation (10), it is also seen that the first zero of the beam pattern  $K(\phi)$  in this case occurs as  $\sin \phi = \frac{\lambda}{Nd}$ . The number of beams to be found in the case  $d = \frac{\lambda}{2}$ , is equal to N. This is to be compared with the values of M shown in the preceding table.

Conclusion

The maximum number of beams required for independent signals in multibeam sonars have been derived for both the cases of continuous and N-element arrays. In the latter case some approximation has to be made and no simple analytical expression is found for the maximum number; a computer solution is required.

The number of beams required in multibeam sonars for a resolvability between two adjacent targets based on the Rayleigh criterion, is also derived and a comparison is made.

Acknowledgement

The idea of maximum number of beams and its derivation by means of the sampling theorem originated with Mr. H. S. Newman. The help from and the fruitful discussions with both Mr. H. S. Newman and Dr. E. S. Eby are greatly appreciated. Acknowledgement is also extended to Mr. R. L. Gordon for the use of FFT and plot subroutines he has developed.

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