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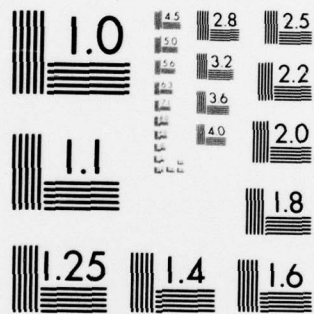
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)                                                                                                                                                                                                                                               |                                   |                                                             |  |
| <p>Work began on this contract in the summer of 1975. Through the succeeding years, work has evolved in several directions. It is convenient to group the work into four related categories: 1. Nonparametric density estimation; 2. Probabilistic automata theory, 3. Isotonic inference and splines and 4. Miscellaneous.</p> |                                   |                                                             |  |

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AFOSR-TR- 78 - 1419 Inference under Order+Smoothness  
Restrictions with Applications in Time Series  
Analysis.

FINAL REPORT ON

AFOSR-75-2840

EDWARD J. WEGMAN

Work began on this contract in the summer of 1975. Through the succeeding years, work has evolved in several directions. It is convenient to group the work into four related categories: 1. Nonparametric density estimation, 2. Probabilistic automata theory, 3. Isotonic inference and splines and 4. Miscellaneous. We discuss each category in turn.

#### NONPARAMETRIC DENSITY ESTIMATION

We have had an ongoing interest in density estimation dating to a time prior to the Air Force contract. Several of our early papers were on maximum likelihood density estimation under order restrictions. It became clear that a restriction of unimodality or monotonicity was not always desirable. Wegman (1975) addressed the problem of maximum likelihood density estimation when such restrictions were unsuitable. This work involved some fairly delicate convergence results and has, as far as we know, the weakest assumptions for obtaining results on strong consistency in the literature.

Davies and Wegman (1975) inaugurates the concept of sequential nonparametric density estimation. We introduce in this paper several stopping rules and discuss asymptotic distributions as well as the closedness moments and distribution of the stopping variable. We also investigate the ASN for various error accuracies and provide some practical applications.

Wegman and Davies (1979) develops the idea of a recursive density estimator for use with sequential procedures. Using the almost sure invariance principal, we develop exact rates of strong consistency (laws of the iterated logarithm). This is the only discussion so far as we know of laws of the iterated logarithm for any nonparametric density estimator.

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## PROBABILISTIC AUTOMATA THEORY

Work in probabilistic automata theory originally stemmed from an interest in black box models. Gould and Wegman (1975), Gould (1975a) and Gould (1975b) form a rather complete theory of automata in environments. Gould and Wegman (1975) introduced this concept and demonstrated that automata in environments were strictly more powerful than the prior automata model (without environments). This paper also demonstrated that several other models meant to generalize ordinary probabilistic automata were special cases of automata in environments.

Gould (1975a) introduced the notion of a metric on the environment and with this he was able to make rigorous the concept of stability of an automaton. Gould (1975b) introduced the notion of the environment as a probability space, thus allowing a theory of random environments. This paper has considerable import on the theory of artificial intelligence since, roughly speaking, it proves that a hard-wired automaton (computer) can simulate in the mean a probabilistic automaton (e.g., such as a human brain).

Unfortunately, because of the level of the probability treatment, particularly in Gould (1975b) which is beyond that usually found among automata theorists, these papers have had a rough go in the refereed journals. (They are still under consideration as of this writing.) However, in 1976, we were invited to present a paper, Gould and Wegman (1976), at the International Conference on Information Theory. This paper contains a summary of all 3 papers. In 1977, we were invited, again, to the International Conference on Mathematical Modeling where we presented Wegman (1977), a shortened version of Gould and Wegman (1975). We expect ultimately to publish refereed versions of this work.

## ISOTONIC INFERENCE AND SPLINES

Interest in black box models and in density estimation, both focussed our interest in time series analysis, particularly in estimation of functions related to filters. Robertson and Wegman (1978) was motivated by the desire to test for the existence of trend, although it evolved into a much more general work. It includes the only systematic extension of these testing procedures to the non-Gaussian case. Wegman (1977) extended the notion of periodic spline to the multivariate context and showed several applications to filter function estimation.

In a series of papers, Wright and Wegman (1977), Wright (1977a) and Wright (1977b), the role of splines in statistics are explored and a theory of isotonic splines is developed. This first paper recognized the parallel between isotonic methods and spline theory and provides a function estimator when the function is known to be both smooth and isotonic. Applications of this theory is to density estimation, filter function estimation and trend, among others. Work and interest in this area is not complete.

#### MISCELLANEOUS

Two papers fall into the miscellaneous category, which arose from unique opportunities to interact with colleagues. Nour and Wegman (1978) develops an important stochastic fertility model which represents a genuine breakthrough. Previous unrealistic models had been based on renewal theory which implied an infinite reproductive period. Wegman and Carroll (1977) took exception to the well publicized Princeton Robustness Study and showed, contrary to the conclusions of the PRS, that adaptive estimators can be as good as the so-called M-estimators. Professor Tukey has recently reincarnated the adaptive-M-estimates proposed in Wegman and Carroll (1977) in a Tukeyesque guise at the recent ARO Robustness Workshop.

In summary, the AFOSR support we feel has allowed us to carry out some interesting and important research. It has been a genuine pleasure in all respects to deal with AFOSR and particularly with Dr. I. N. Shimi.

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