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AERO-SERVO-ELASTIC STABILITY ANALYSIS

by
William P. Rodden, Mildred R. Zeifman
and
John M. Powers, Jr.

Prepared for
DEPARTMENT OF THE NAVY
NAVAL AIR SYSTEMS COMMAND
Washington, D.C. 20361

Under
Contract N00019-76-C-0346
April 1979

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SUMMARY

A general matrix formulation of the aero-servo-elastic stability problem for a closed-loop system is presented. It is based on a classical method for determining servomechanism stability (using linear differential equations) and the British method of flutter analysis which represents the aerodynamic forces as frequency dependent springs and dampers. The combination of the aerodynamic forces with the mechanical springs and dampers in the equations of motion and then with their electro-mechanical equivalents in the servo system leads to a consistent formulation of the linear equations of motion for the closed-loop aero-servo-elastic system. An iterative real eigenvalue solution accounts for the frequency dependence of the aerodynamic forces which is only secondary.

The digital computer program MPASES (modified program for aero-servo-elastic stability) developed to perform the analysis is described. Dynamic storage allocation is utilized throughout MPASES to provide the most efficient use of core storage through the variable dimensioning of all arrays. Program limitations are minimal allowing stability results in one computer run to be obtained for up to ten velocities at each of five altitudes. In addition, as many as twenty-five servo element gain variations are permitted.

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SECTION I

INTRODUCTION

This report documents a modification to Ref. 1 which presented a digital computer program for the analysis of aero-servo-elastic system stability. Reference 1 formulated the problem by combining one of the classical methods for determining servomechanism system stability with the American method of flutter analysis. Since the American method of flutter analysis represents the aerodynamic forces as frequency dependent masses and utilizes the mathematical concept of an artificial structural damping to determine the reduced frequency at flutter, the combined aero-servo-elastic stability analysis does not obtain physically meaningful frequencies and dampings at flight conditions other than neutrally stable ones. On the other hand, the British method of flutter analysis represents the aerodynamic forces more realistically as frequency dependent springs and dampers. This is only an approximation for transient motion since the aerodynamic forces are generally known only for harmonic motion and it is not completely correct to identify the forces in phase with the displacements as aerodynamic springs and the forces in phase with the velocities as aerodynamic dampers. However, comparisons of results from the various methods for flutter analysis have been made by Jocelyn Lawrence and Jackson in Ref. 2, and it has been found that the British method of flutter analysis leads to reasonable predictions of transient aero-elastic behavior when the damping levels are low.

The British method of treating the aerodynamic loads as springs and dampers permits combining the aerodynamic forces with the mechanical springs and dampers in the equations of motion and then with their electro-mechanical equivalents in the servo system. This results in a consistent formulation of the equations of motion for the closed-loop aero-servo-elastic system because the frequency dependence of the aerodynamic forces is only secondary.

Although the frequency dependence is secondary, however, it is important and an iteration is necessary to "line-up" the reduced frequency for which the aerodynamics are determined and the frequency determined by the equations of motion; one iteration, however, is sufficient to achieve sufficient accuracy between the aerodynamic forces and the equations of motion.

The iteration begins at a particular velocity by assuming the reduced frequency $k=0$. (The aerodynamic damping can only be found from a low frequency $k=0.0$ but an interpolation scheme will estimate the damping at $k=0$.) Solving the eigenvalue problem from the equations of motion yields either separate and real roots or complex conjugate pairs. Any real roots found for $k=0$ are correct, e.g., the rigid body roll-damping root or a structural divergence root. However, the complex conjugate roots must be lined up and the lowest frequency roots permit the aerodynamic forces to be revised (by interpolation) to the corresponding reduced frequency. With the revised aerodynamics, the eigenvalue problem is solved again and the lowest oscillatory root is obtained along with an estimate of the next higher frequency. The aerodynamic forces are then revised again to correspond to the estimate of the reduced frequency of the second oscillatory mode and the new eigenvalue problem is solved. This results in the second oscillatory root and an estimate of the third frequency. The process of revising the aerodynamics and solving new eigenvalue problems continues until the desired number of roots have been obtained for the particular velocity. The whole procedure is repeated for the next higher velocity and is continued until the velocity range of interest has been covered. From the solutions for frequency and damping of each mode, root locus charts can be drawn for use in redesign of servo components, e.g., amplifier gains, or mechanical components, e.g., mass balancing.

The modified program for aero-servo-elastic stability (MPASES) requires four revisions to the program PASES of Ref. 1. The first is the inclusion of the aerodynamic forces as spring and damping terms in the equations of motion rather than as complex inertial terms. The second is the iterative eigenvalue solution required to line up the frequencies between the aerodynamic forces and the equations of motion as discussed above. The third problem is the interpolation of the aerodynamic forces necessary in the iteration to minimize aerodynamic computational expense. The last problem is the result of the new formulation dealing only with real matrices; a specialized eigenvalue extraction method that analyzes real matrices which have either real or complex conjugate roots may be utilized for computational efficiency. Each of these modifications is discussed in the following sections.

SECTION II

CLOSED-LOOP AERO-SERVO-ELASTIC STABILITY ANALYSIS

New Equations of Motion

The theoretical derivation of the equations of motion for the PASES computer program is taken from Ref. 1 and is reproduced in this report in the Appendix for ease of reference. It is only necessary here to rederive the aeroelastic equations of motion since the servo equations are not affected by the change in representation of the aerodynamic forces from complex masses to real springs and dampers.

Our new definitions of aerodynamic influence coefficients (AIC's) are taken from the survey of unsteady AIC's in Ref. 3. The unsteady force is given in terms of the deflections and their velocities by

$$\{F\} = (qS/\bar{c}) \left([C_{hs}] \{h\} + [C_{hDh}] \{\dot{h}\bar{c}/V\} \right) \quad (1)$$

The AIC's $[C_{hs}]$ and $[C_{hDh}]$ may be regarded as static and dynamic stability derivatives but are, in general, functions of the reduced frequency

$$k = \omega b_T / V \quad (2)$$

where

$$b_T = \bar{c}/2 \quad (3)$$

Assuming the AIC's to be constants independent of k simplifies the analysis and is adequate for low frequencies, but it is inaccurate at higher frequencies and the reduced frequencies should be lined up with the frequencies obtained from the equations of motion as discussed in the Introduction.

However, the choice of lining up the frequencies or not is left to the user in his choice of frequency dependence of the AICs.

A definition of complex oscillatory AIC's was also given in Ref. 3 as

$$\{F\} = \rho \omega^2 b_r^2 s [C_h] \{h\} \quad (4)$$

for use in the American method of flutter analysis. Equations (1) and (4) must be identical for harmonic motion and we have

$$(qS/\bar{c}) \left([C_{hs}] + i(\omega \bar{c}/V) [C_{hDh}] \right) \{h\} = \rho \omega^2 b_r^2 s \left([C_{hR}] + i[C_{hI}] \right) \{h\} \quad (5)$$

where $[C_{hR}]$ and $[C_{hI}]$ are the real and imaginary parts of $[C_h]$, respectively. Identifying the real and imaginary parts of Eq. (5) leads to

$$[C_{hs}] = 2k^2 (\bar{c}s/S) [C_{hR}] \quad (6)$$

and

$$[C_{hDh}] = k(\bar{c}s/S) [C_{hI}] \quad (7)$$

Equations (6) and (7) determine the aerodynamic stiffness and damping AIC's when the oscillatory AIC's are given in the format of Eq. (4), as is the case with a number of available computer programs, e.g., Refs. 4 and 5.

In terms of the stiffness and damping AIC's, the equations of motion of the aeroelastic system including the rotations of control surfaces appear as (cf. Appendix A, Eq. (2-15))

$$[M]\{\ddot{h}\} + [C]\{\dot{h}\} + [K](\{h\} - [h_\delta]\{\delta\}) = \{F\} \quad (8a)$$

$$= (qS/\bar{c}) \left([C_{hs}]\{h\} + [C_{hDh}]\{\dot{h}\bar{c}/V\} \right) \quad (8b)$$

or

$$[M]\{\ddot{h}\} + [\bar{C}]\{\dot{h}\} + [\bar{K}]\{h\} - [K][h_\delta]\{\delta\} = 0 \quad (9)$$

where

$$[\bar{C}] = [C] - k_{pVS}[C_{hDh}] \quad (10)$$

$$[R] = [K] - (qS/\bar{C})[C_{hs}] \quad (11)$$

The modal solution proceeds as before (see Appendix A). The series for the deflections is

$$\{h\} = [h_F]\{a_F\} + [h_R]\{a_R\} + [h_\delta]\{\delta\} \quad (12)$$

Substituting Eq. (12) into Eq. (9) and premultiplying by $[h_F]^T$ leads to the modal equations for the flexible degrees of freedom.

$$\begin{aligned} & [M_F]\{\ddot{a}_F\} + [M_{FR}]\{\ddot{a}_R\} + [M_{F\delta}]\{\ddot{\delta}\} \\ & + [\bar{C}_F]\{\dot{a}_F\} + [\bar{C}_{FR}]\{\dot{a}_R\} + [\bar{C}_{F\delta}]\{\dot{\delta}\} \\ & + [R_F]\{a_F\} + [R_{FR}]\{a_R\} + [R_{F\delta}]\{\delta\} = 0 \end{aligned} \quad (13)$$

where

$$[M_F] = [h_F]^T [M] [h_F] \quad (14)$$

$$[M_{FR}] = [h_F]^T [M] [h_R] \quad (15)$$

$$[M_{F\delta}] = [h_F]^T [M] [h_\delta] \quad (16)$$

$$[\bar{C}_F] = [h_F]^T [\bar{C}] [h_F] \quad (17)$$

$$[\bar{C}_{FR}] = [h_F]^T [\bar{C}] [h_R] \quad (18a)$$

$$= -k_{pVS}[h_F]^T [C_{hDh}] [h_R] \quad (18b)$$

$$[\bar{C}_{F\delta}] = [h_F]^T [\bar{C}] [h_\delta] \quad (19a)$$

$$= -\omega_0^2 VS [h_F]^T [C_{hDh}] [h_\delta] \quad (19b)$$

$$[\bar{K}_F] = [h_F]^T [K] [h_F] \quad (20)$$

$$[\bar{K}_{FR}] = [h_F]^T [K] [h_R] \quad (21a)$$

$$= -(qS/\bar{c}) [h_F]^T [C_{hs}] [h_R] \quad (21b)$$

$$[\bar{K}_{F\delta}] = [h_F]^T [K] [h_\delta] \quad (22a)$$

$$= -(qS/\bar{c}) [h_F]^T [C_{hs}] [h_\delta] \quad (22b)$$

Equation (13) may be compared to Eq. (2-20) in Appendix A. We have utilized the fact above that the rigid body displacements cause no internal damping or structural forces. We have assumed the vibration modes are either free-free modes for the entire system or restrained modes for individual components, but may not be arbitrarily chosen modes. The limited orthogonality of the free-free or restrained modes (i.e., $[M_F]$ is not a diagonal matrix unless all modes are free-free modes) leads to a diagonal form for the generalized stiffness matrix $[K_F]$. If we denote the diagonal elements of $[M_F]$ by $[M_F]$, then the generalized stiffness matrix is

$$[K_F] = [h_F]^T [K] [h_F] \quad (23a)$$

$$= [\omega_F^2] [M_F] \quad (23b)$$

and

$$[\bar{K}_F] = [K_F] - (qS/\bar{c}) [h_F]^T [C_{hs}] [h_F] \quad (24)$$

The generalized structural damping matrix does not have a diagonal form, as does the generalized stiffness, but is assumed so as an approximation that is justified by low levels of structural damping. The approximate form of the equivalent viscous structural damping is

$$[C_F] = [g_F/\omega_F][K_F] \quad (25a)$$

$$= [g_F][\omega_F][M_F] \quad (25b)$$

so that

$$[\tilde{C}_F] = [C_F] - 4\rho VS[h_F]^T[C_{hDh}][h_F] \quad (26)$$

Next, substituting Eq. (12) into Eq. (9) and premultiplying by $[h_R]^T$ leads to the modal equations for the rigid body degrees of freedom.

$$\begin{aligned} & [M_{RF}]\{\ddot{x}_F\} + [M_R]\{\ddot{x}_R\} + [M_{R\delta}]\{\ddot{\delta}\} \\ & + [\tilde{C}_{RF}]\{\dot{x}_F\} + [\tilde{C}_R]\{\dot{x}_R\} + [\tilde{C}_{R\delta}]\{\dot{\delta}\} \\ & + [\tilde{K}_{RF}]\{x_F\} + [\tilde{K}_R]\{x_R\} + [\tilde{K}_{R\delta}]\{\delta\} = 0 \end{aligned} \quad (27)$$

where

$$[M_{RF}] = [M_{FR}]^T \quad (28)$$

$$[M_R] = [h_R]^T[M][h_R] \quad (29)$$

$$[M_{R\delta}] = [h_R]^T[M][h_\delta] \quad (30)$$

$$[\tilde{C}_{RF}] = [h_R]^T[\tilde{C}][h_F] \quad (31a)$$

$$= - 4\rho VS[h_R]^T[C_{hDh}][h_F] \quad (31b)$$

$$[\tilde{C}_R] = [h_R]^T [\tilde{C}] [h_R] \quad (32a)$$

$$= -\frac{1}{2} \rho V S [h_R]^T [C_{hDh}] [h_R] \quad (32b)$$

$$[\tilde{C}_{R\delta}] = [h_R]^T [\tilde{C}] [h_\delta] \quad (33a)$$

$$= -\frac{1}{2} \rho V S [h_R]^T [C_{hDh}] [h_\delta] \quad (33b)$$

$$[\tilde{K}_{RF}] = [h_R]^T [\tilde{K}] [h_F] \quad (34a)$$

$$= -(qS/\bar{c}) [h_R]^T [C_{hs}] [h_F] \quad (34b)$$

$$[\tilde{K}_R] = [h_R]^T [\tilde{K}] [h_R] \quad (35a)$$

$$= -(qS/\bar{c}) [h_R]^T [C_{hs}] [h_R] \quad (35b)$$

$$[\tilde{K}_{R\delta}] = [h_R]^T [C_{hs}] [h_\delta] \quad (36a)$$

$$= -(qS/\bar{c}) [h_R]^T [C_{hs}] [h_\delta] \quad (36b)$$

and we have again noted that rigid body displacements produce no internal damping or structural forces. Equation (27) should be compared to Eq. (2-29) in Appendix A.

The new matrix partitions in the aero-servo-elastic equations of motion now become

$$[X_{\ddot{x}}] \{\ddot{x}\} = \begin{bmatrix} [M_F] & [M_{FR}] & [M_{F\delta}] & 0 \\ [M_{RF}] & [M_R] & [M_{R\delta}] & 0 \\ 0 & 0 & [CC] & [CS2] \\ [FSA] & [RSA] & [SC] & [SS2] \end{bmatrix} \begin{Bmatrix} \ddot{a}_F \\ \ddot{a}_R \\ \delta \\ \ddot{e}_2 \end{Bmatrix} \quad (37)$$

$$[X_x]\{\dot{x}\} = \begin{bmatrix} [C_F] & [C_{FR}] & [C_{F\delta}] & 0 \\ [C_{RF}] & [C_R] & [C_{R\delta}] & 0 \\ 0 & 0 & [CC] & [CS2] \\ [FSG] & [RSG] & [SC] & [SS2] \end{bmatrix} \begin{Bmatrix} \dot{a}_F \\ \dot{a}_R \\ \dot{\delta} \\ \dot{e}_2 \end{Bmatrix} \quad (38)$$

$$[X_x]\{x\} = \begin{bmatrix} [K_F] & [K_{FR}] & [K_{F\delta}] & 0 \\ [K_{RF}] & [K_R] & [K_{R\delta}] & 0 \\ 0 & 0 & [CC] & [CS2] \\ 0 & 0 & [SC] & [SS2] \end{bmatrix} \begin{Bmatrix} a_F \\ a_R \\ \delta \\ e_2 \end{Bmatrix} \quad (39)$$

These may be compared with Eqs. (2-43), (2-44) and (2-45) in Appendix A.

The coefficient matrices $[X_x]$ and $[X_x]$ each have five new nonzero partitions as expected from moving the aerodynamic terms from the mass matrix to the stiffness and damping matrices.

The Eigenvalue Problem

The new representation of the aerodynamic forces changes the eigenvalue problem only to the extent that the matrices are real now rather than complex. The equation to be solved is still

$$(\gamma[A] + [B])\{V\} = 0 \quad (40)$$

where the amplitudes of motion, $\{V\}$, are defined by

$$\{v\} = \{V\}\exp(\gamma t) \quad (41)$$

and instability occurs when the airspeed and/or control system gains are such that the real part of γ is positive.

Although $[A]$ and $[B]$ are now real matrices, they still may be singular and obtaining the canonical form of the eigenvalue problem by a shift in eigenvalues is still appropriate. We let

$$\gamma = \gamma_0 - 1/\lambda \quad (42)$$

where γ_0 is an arbitrarily chosen real number, and then the new eigenvalue is

$$\lambda = 1/(\gamma_0 - \gamma) \quad (43)$$

and the new eigenvalue problem is

$$\lambda\{V\} = (\gamma_0[A] + [B])^{-1}[A]\{V\} \quad (44)$$

The shift value γ_0 is arbitrary to the extent that it must be chosen so the linear combination $\gamma_0[A] + [B]$ is nonsingular. A value which scales $[A]$ to be the same order of magnitude as $[B]$ and of the same sign is recommended.

The eigenvalues of Eq. (44) are either real or complex conjugates. A subroutine ALLMAT (Ref. 6) for complex matrices was used in Ref. 1.

A more recent development for the real case of Eq. (44) is the subroutine EIGRF given in the International Mathematical and Statistical Library (IMSL, Ref. 7). Subroutine EIGRF calls IMSL routine EBALAF to balance the matrix. Then IMSL routine EHESF reduces the balanced matrix to an upper Hessenberg form and routine EQRH3F computes all of the real and/or complex conjugate pairs of eigenvalues of the Hessenberg matrix.

The IMSL Package is universally used and is usually incorporated into the scientific libraries of major computer systems. When requested, the eigenvectors are found in two iterations by the Inverse Power Method with Shifts (Ref. 8, pp. 323, 626-628) in subroutine EGVCT (Ref. 9). Subroutine

EQNVCT finds the eigenvector $\{u\}$ from a complex matrix $[U]$ and its complex eigenvalue Λ by solving the equation

$$[U - \Lambda I]\{u\} = 0 \quad (45)$$

The subroutine is used in the present development by setting

$$\Lambda = 0 \quad (46)$$

and

$$[U] = \gamma[A] + [B] \quad (47)$$

in Eq. (45). If eigenvectors are requested, all real eigenvalues are used in Eq. (47), but only the complex conjugate eigenvalues with positive imaginary parts (positive frequencies) are used, since the eigenvectors are also complex conjugate pairs.

Writing the complex eigenvalue as

$$\gamma = \mu + i\omega \quad (48)$$

where μ is the decay rate and ω is the damping frequency, we find the cyclic frequency to be

$$f = \omega/2\pi \quad (49)$$

and the fraction of critical damping ζ to be

$$\zeta = -\mu/\sqrt{\mu^2 + \omega^2} \quad (50)$$

For comparison to structural damping levels, twice the damping ratio ζ is a preferable output quantity since

$$\zeta = g/2 \quad (51)$$

for a structurally-damped single degree of freedom oscillator. For a non-oscillatory root a different definition of damping ratio is necessary and we chose the time to half amplitude

$$T_{\frac{1}{2}} = \ln 2 / (-\mu) \quad (52)$$

If the motion is unstable, i.e., $\mu > 0$, Eq. (52) gives the (negative) time to double amplitude. The stability of the oscillatory roots can also be compared using Eq. (52) and this will be an additional output quantity.

Lining Up the Reduced Frequency

The need for lining up the reduced frequency for a specified mode of motion with the frequency determined by the eigenvalue problem for that mode was discussed briefly in the Introduction and at some length in Ref. 2. The necessary equations for the iteration are given in this section.

It is possible to begin the iteration with any value of k . However, a finite value of k may be representative of an oscillatory mode but we are equally interested in static modes. Therefore, we begin with $k=0$ and any real roots will be determined, e.g., the roll-damping root, a static structural divergence root, and any over-damped roots from control system components. The first oscillatory root for a free vehicle may be either the short period mode or the Dutch roll mode and the choice of $k=0$ provides a good estimate of that.

Let the complex conjugate pairs of roots be denoted by

$$\gamma_{rs} = \mu_{rs} \pm i\omega_{rs} \quad (53)$$

where r denotes the oscillatory mode number ordered by frequency,

($\omega_{1s} < \omega_{2s} < \dots$), and s denotes the number of the mode under investigation.

The reduced frequency is

$$k = \omega \bar{c} / 2V \quad (54)$$

and the aerodynamic forces are determined for k_s which should be

$$k_s = \omega_{ss} \bar{c} / 2V \quad (55)$$

However, ω_{ss} is not known at the outset. From the initial solution with $k_0=0$, we estimate the nonzero frequencies ($\omega_{10}, \omega_{20}, \omega_{30}, \dots$). Refining the aerodynamics by interpolation for $k_1 = \omega_{10} \bar{c} / 2V$ and repeating the eigenvalue solution, we find a new set of frequencies ($\omega_{11}, \omega_{21}, \omega_{31}, \dots$). The value of ω_{11} is taken as the correct value for the first mode and its damping α_{11} determines its stability. The eigenvectors for the first root ω_{11} may then be calculated if the mode shapes are desired.

The aerodynamics are next determined by interpolation for $k_2 = \omega_{21} \bar{c} / 2V$ and the next eigenvalue solution yields the frequencies ($\omega_{12}, \omega_{22}, \omega_{32}, \dots$). The frequency ω_{22} is now taken to be correct and its damping α_{22} measures the second oscillatory mode stability. We continue with $k_3 = \omega_{32} \bar{c} / 2V$, finding the aerodynamics for k_3 , and then the eigenvalues; ω_{32} and α_{32} are assumed to be correct. The process continues until all roots of interest, and all eigenvectors, if requested, have been found.

Care must be exercised in tracking the oscillatory roots because, as the reduced frequency is increased the roots which were real at $k=0$ may become complex at higher values of k or roots which were complex at $k=0$ may become real at higher k 's. A suitable algorithm is to choose ω_{ss} as the closest value to $\omega_{s,s-1}$, and then to choose $\omega_{s+1,s}$ as the closest higher value to ω_{ss} .

Linear Spline Interpolation of Aerodynamic Terms

In the modal matrix equations of motion, twelve of the partitions depend on the aerodynamic influence coefficients (AIC's) which are functions of the reduced frequency k . As the reduced frequency changes when the roots are tracked in the frequency lining up process, the AIC's will change and interpolation on k is necessary. It is computationally simpler to interpolate the partitions rather than the AIC's. Two of the partitions depend on structural parameters also but these are independent of k and can be carried along in the interpolation. A linear spline is chosen for the interpolation because it offers cubic accuracy and continuity throughout the regions of interpolation and extrapolation.

A linear spline is a mathematical device for interpolating for a function $y(x)$ for all points x , when y is known for a discrete set of points, $y_i = y(x_i)$. The spline passes through all of the known points. The mathematical spline takes its name from the plastic spline used by draftsmen for drawing curves through specified points. If the plastic spline may be regarded as a uniform beam, the linear spline representing it mathematically is a solution to the uniform beam differential equation. The following derivation for the linear spline is taken from Ref. 10, App. E, by R. L. Harder.

We wish to determine the deflection curve of a continuous beam over multiple supports. Consider the fundamental solution to the deflection equation

$$EI \frac{d^4 y}{dx^4} = w \quad (56a)$$

$$= 0 \quad (56b)$$

that is symmetrical about a support at the origin $x = 0$. The solution is

$$y = A + B|x| + Cx^2 + D|x|^3 \quad (57)$$

Continuity of slope requires $B = 0$, and equilibrium with the support load P requires

$$\lim_{\epsilon \rightarrow 0} \int_{-\epsilon/2}^{\epsilon/2} w dx = \lim_{\epsilon \rightarrow 0} EI \int_{-\epsilon/2}^{\epsilon/2} \frac{d^3 y}{dx^3} dx \quad (58a)$$

$$= \lim_{\epsilon \rightarrow 0} EI \left. \frac{d^3 y}{dx^3} \right|_{-\epsilon/2}^{+\epsilon/2} \quad (58b)$$

$$= P \quad (58c)$$

From Eq. (57) the third derivative is found to be

$$\frac{d^3 y}{dx^3} = 6D \operatorname{sgn} x \quad (59)$$

and Eq. (58b) becomes

$$(12EI) D = P \quad (60)$$

Combining Eqs. (57) and (60) and generalizing for all supports at $x = x_i$ leads to

$$y(x) = \left[A_i + C_i (x - x_i)^2 + Q_i |x - x_i|^3 \right] \quad (61)$$

where $Q_i = P_i/12EI$. At a large distance from the supports, the deflection curve should be linear. For large $x \rightarrow \pm\infty$, Eq. (61) behaves like

$$y(x) = x^3 \operatorname{sgn} x \sum Q_i + x^2 \sum (C_i - 3Q_i x_i \operatorname{sgn} x) + O(x) \quad (62)$$

and the required linear behavior necessitates

$$\sum Q_i = 0 \quad (63)$$

$$\sum C_i = 0 \quad (64)$$

$$\sum Q_i x_i = 0. \quad (65)$$

Equations (63) and (65) are equilibrium equations, and Eq. (64) permits writing

$$\sum (A_i + C_i (x - x_i)^2) = a_0 + a_1 x \quad (66)$$

in Eq. (61), where a_0 and a_1 are new constants, so that Eq. (61) becomes

$$y(x) = a_0 + a_1 x + \sum Q_i |x - x_i|^3 \quad (67)$$

In the case where the spline is symmetrical about the origin, as is the case for the AICs $[C_{hs}(k)]$ and $[C_{hDh}(k)]$ which are symmetrical functions of the reduced frequency k , we may use the method of images and Eq. (67) becomes

$$y(x) = a_0 + \sum Q_i (|x - x_i|^3 + |x + x_i|^3) \quad (68)$$

and only Eq. (63) needs to be satisfied in addition to Eq. (68). The matrix form for Eqs. (63) and (68) in terms of the specified values of $y(x_j)$ is

$$\begin{Bmatrix} 0 \\ y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_N \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & \dots & 1 \\ 1 & K_{11} & K_{12} & K_{13} & \dots & K_{1N} \\ 1 & K_{12} & K_{22} & K_{23} & \dots & K_{2N} \\ 1 & K_{13} & K_{23} & K_{33} & \dots & K_{3N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & K_{1N} & K_{2N} & K_{3N} & \dots & K_{NN} \end{bmatrix} \begin{Bmatrix} a_0 \\ Q_1 \\ Q_2 \\ Q_3 \\ \vdots \\ Q_N \end{Bmatrix} \quad (69)$$

$$\text{where } K_{ji} = |x_j - x_i|^3 + |x_j + x_i|^3 \quad (70)$$

The left hand side of Eq. (69) may be written

$$\begin{Bmatrix} 0 \\ y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_N \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_N \end{Bmatrix} \quad (71)$$

The matrix form for an interpolated value of $y(x_k)$ is

$$y_k = [1 \ K_{k1} \ K_{k2} \ \dots \ K_{kN}] \begin{Bmatrix} a_0 \\ Q_1 \\ Q_2 \\ \vdots \\ Q_N \end{Bmatrix} \quad (72)$$

where

$$K_{kj} = |x_k - x_j|^3 + |x_k + x_j|^3 \quad (73)$$

Combining Eqs. (69), (71) and (72) leads to the desired interpolation coefficients I_{kj} defined by

$$y_k = [I_{kj}]\{y_j\} \quad (74)$$

where $\{y_j\}$ denotes

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}$$

and

$$[I_{kj}] = [1 \ K_{k1} \ K_{k2} \ \dots \ K_{kN}] \begin{bmatrix} 0 & 1 & 1 & 1 & \dots & 1 \\ 1 & K_{11} & K_{12} & K_{13} & \dots & K_{1N} \\ 1 & K_{12} & K_{22} & K_{23} & \dots & K_{2N} \\ 1 & K_{13} & K_{23} & K_{33} & \dots & K_{3N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & K_{1N} & K_{2N} & K_{3N} & \dots & K_{NN} \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \quad (75)$$

Equation (74) provides the linear spline interpolation scheme needed to obtain the AICs in the frequency lining up process. The inverse required in Eq. (75) is ill-conditioned for a large number N of interpolated points, say $N=50$; however, a large number are not needed in the flutter analysis and N will be limited to $N \leq 10$ in the present applications.*

* A lower limit, say $N \leq 5$ is adequate for most applications unless an extremely wide frequency spectrum must be covered.

SECTION III

AERO-SERVO-ELASTIC STABILITY ANALYSIS PROGRAM: MPASES

Program Description

MPASES is a general purpose digital computer program for the analysis of the closed-loop stability problem. This program, which is a modification of PASES (Ref. 1), formulates the problem by combining a classical method for determining servomechanism system stability with the more realistic British method of flutter analysis in lieu of the American method used in PASES. With the input of an arbitrary number of elastic degrees of freedom, aerodynamic influence coefficients and the control system description, the stability of a missile or aircraft configuration as an aero-servo-elastic system can be investigated. When no aerodynamics are involved, the servo-elastic system stability can be determined.

Dynamic storage allocation utilized throughout MPASES provides the most efficient use of core storage through the variable dimensioning of all arrays. Because the program is based on the theory that permits the inclusion of the aerodynamic forces as spring and damper terms in the equations of motion rather than as complex inertial terms, only real matrices are used in the eigenvalue solution. This makes for additional economy in computer usage.

MPASES uses the method of revising the aerodynamics by lining up the frequencies and performing the spline interpolation of the aerodynamic terms necessary in the iterative eigenvalue solution. This process is repeated until the required number of modes and eigenvectors are obtained for a particular velocity. Moreover, if the analysis Mach number (velocity determining factor) differs from the AIC Mach numbers by more than a specified 'deviation' value (input by the submitter), spline interpolation of the

aerodynamic forces is performed also. In this case, the program interpolates for Mach number before the reduced frequency interpolation for each mode is carried out.

The variables in the stability analysis of an aero-servo-elastic system are servo-gains, velocities (through Mach number and speed of sound input) and altitude (air density input). An option is provided to vary the gain of a single servo component or control surface with the density and Mach number held constant. When this option is executed, the coefficients of the output of the desired servo element or control surface are divided by the gain factor, K. This method facilitates programming, since there may be more than one input.

Since dynamic dimensioning is used, the program limitations are minimal. Stability results can be obtained for up to five (5) altitudes and for each density, as many as ten (10) different velocities can be analyzed. In addition, twenty-five (25) gain variations are permitted. The number of AIC matrices that can be input is limited to ten (10) reduced frequencies for each of five (5) Mach numbers. There are no size restrictions for any of the other parameters existing in the program.

The total number of memory units (words or bytes) required to execute MPASES is completely dependent upon input. The size of the program is reflected in the length of the blank common block found in the 'MAIN' section of the program. The length can readily be altered to accommodate different analyses. Likewise, dimensions of the arrays limiting the program as stipulated above can be changed to the desired size.

Subroutine Description

MAIN Main program for MPASES.
 Reads and prints basic input data.
 Allocates dynamic storage for all arrays.
 Calling program for subroutines MASS, SERVØ, FØRMAB, AIC, MPSE
 and MPAS.

AIC Reads and prints complex AIC matrices.
 Separates AIC's into real and imaginary parts.
 Formulates generalized aerodynamic forces and stores on temporary
 File 1.
 Calls subroutine GENMAT.

EGNVCT Finds the eigenvector from a real matrix for which the complex
 eigenvalue is known.
 Uses the inverse power method with shifts.
 The eigenvector obtained is complex with the largest value
 normalized to unity.
 A singular matrix returns an eigenvector of zeros.

EIGRF Computes eigenvalues of a real, triangular matrix.
 Calls routines from the IMSL Package (see Ref. 7).
 Called by subroutine SOLV.

FØRMAB Generates the A and B matrices which constitute the eigenvalue
 problem.
 Stores [A] and [B] on temporary File 2.

GENMAT Formulates the generalized mass and aerodynamic force matrices
 utilizing rigid body, control surface and vibration modes.
 Calls subroutine MATMPL.
 Called by subroutines AIC and MASS.

INTERP Generates the constant portion of the linear spline interpolation coefficients matrix.
 Prints message if spline interpolation matrix is singular.
 Calls subroutine INVERS.

INVERS Finds the inverse of a real, square matrix.
 Called by subroutines INTERP and SØLV.

MASS Reads and prints the weight matrix in pounds. Converts weight to mass units (slugs).
 Reads and prints rigid body control surface modes, free-free vibration modes and restrained (rigid body) modes.
 Stores modal data on temporary File 4.
 Reads and prints damping coefficients and vibration frequencies (Hertz). Converts frequencies to rps units.
 Formulates generalized mass matrix.
 Generates generalized stiffness and damping matrices if flexible modes are input.
 Calls subroutine GENMAT.

MATMPL Multiplies real, two-dimensional matrices.
 If requested, transposes post-multiplier matrix to perform as a pre-multiplier.
 Called by subroutines INTERP, SERVØ, GENMAT and SØLV.

MPAS Obtains stability results from aero-servo-elastic analysis for altitudes and velocities requested.
 Reads Mach numbers at each altitude for which analysis is to be made.
 Calculates velocity (from Mach number and speed of sound) and constants for the aerodynamic forces.
 Performs Mach number interpolation if necessary.
 Reads File 2 - brings [A] and [B] into core.
 Executes gain option if requested; stores A and B matrices reflecting gain factor on temporary File 3.

MPAS (Cont.) Carries out spline interpolation of the aerodynamic forces for reduced frequency for each mode.
 Generates new A and B matrices with changed aerodynamics for the particular mode.
 Calls subroutine SØLV for eigenvalue solution of each mode.
 Prints stability results.
 Prints eigenvectors, if requested, for the analysis.
 Reads File 4.
 Computes and prints structural deflections at system mass points for each mode.
 Calls subroutines INTERP, SPLINE and SØLV.

MPSE Obtains stability results from servo-elastic analysis for number of modes requested.
 Reads File 2 - places [A] and [B] into core.
 Executes gain option if requested.
 Calls subroutine SØLV for eigenvalue solution of required modes.
 Prints stability results.
 Prints eigenvectors for each mode.
 Note that the eigenvectors are always computed and printed for a servo-elastic analysis.
 Reads File 4.
 Computes and prints structural deflections at system mass points for each mode.
 Calls subroutine SØLV.

RESULT Forms the array where stability results for each mode are stored.
 Calls subroutine EGNVCT to compute the eigenvectors (only upon request for aero-servo-elastic analysis, always for servo-elastic analysis).
 Called by subroutine SØLV.

SERVØ Reads coefficients from servo differential equations.
 Prints control system description.
 Computes input to rate gyros and accelerometers from differentiation or interpolation matrices.
 Determines maximum order of servo elements.
 Prints pertinent information concerning all servo elements.

SERVØ Generates [X2], [X1], [X0], [Y1], [Y0], [Z0].
 (Cont.) Calls subroutine MATMPL.

SØLV Prints the A and B matrices.
 Generates the dynamic matrix (eigenvalue problem).
 Calls subroutine EIGRF for eigenvalue solution.
 "Lines-up the frequencies" in aero-servo-elastic analysis.
 Determines real and oscillatory roots for each mode.
 Estimates new reduced frequencies.
 Calls subroutine RESULT to compute stability results and eigen-
 vector for each mode.
 Calls subroutines MATMPL, INVERS, EIGRF (from IMSL Package),
 and RESULT.
 Called by subroutines MPAS and MPSE.

SPLINE Generates the linear spline interpolation matrix.
 Determines the Lagrangian coefficients.
 Used in the frequency lining-up process.
 Called by subroutine MPAS.

Programming Symbols - A partial list of FØRTRAN symbols used in MPASES is presented. Additional symbols are defined in the Input Instructions.

1. Integer Variables

ID	Density index
IE	Mode counter
IG	Gain index
IK	Reduced frequency index
IM	Mach number index
IMØDE	Actual number of modes calculated for stability analysis

I0B	First real root (eigenvalue with positive real part and zero imaginary part)
I0E	Last real root
ISING	Singular matrix control. If ISING = 1, matrix is non-singular; return from subroutine INVERS.
J1	BLANK COMMON length required to call subroutine MASS
J2	BLANK COMMON length required to call subroutine SERVØ
J3	BLANK COMMON length required to call subroutine FØRMAB
J4	BLANK COMMON length required to call subroutine MPSE or subroutine AIC
J5	BLANK COMMON length required to call subroutine MPAS
JTØT	Largest values of J1, J2, J3 and J4 for servo-elastic analysis; or largest value of J1, J2, J3, J4 and J5 for aero-servo-elastic analysis. JTØT is the minimum BLANK COMMON length required to execute MPASES.
KØP	Column assignment in A and/or B matrices for zero and first order servos.
KØ1	Column assignment in A and/or B matrices for second order servo-velocity.
KØ2	Column assignment in A and/or B matrices for second order servo-displacement.
KXM	Control for Mach number interpolation.
K1	Order of servo element for which the gain varies.
K2	Column assignment in the X, Y and/or Z arrays for servo whose gain varies.
L	Counter for coefficients from servo differential equations including inputs to control surfaces, outputs from servos, and inputs to servos.
MDF	Number of mass points; used as a variable dimension involving flexible modes. (MDF = 1 when NFM = 0)
MDR	Number of mass points; used as a variable dimension involving rigid body modes. (MDR = 1 when NRM = 0)
MFM	Number of flexible modes; used as a variable dimension. (MFM = 1 when NFM = 0)

MRM	Number of rigid body modes; used as a variable dimension. (MRM = 1 when NRM = 0)
M1	Number of flexible modes <u>or</u> rigid body modes, whichever is larger.
M2	M1 <u>or</u> the number of control surfaces, whichever is larger.
M3	Number of reduced frequencies <u>or</u> Mach numbers, whichever is larger. (AIC input)
M31	$M3 + 1$.
NFR	Sum of the number of flexible modes and rigid body modes.
NI0	Counter for real roots.
NK1	$NK + 1$
NMODE	Sum of the number of flexible modes, rigid body modes and control surfaces.
NM1	$NM + 1$
NPART	Number of partitions in AIC matrix. (NPART = NA if NA > 1)
NPR	Counter for number of actual modes in the servo-elastic analysis.
NRO	Control for determining the number of real roots.
NSERV	Total number of coefficients from servo differential equations including inputs to control surfaces, outputs from servos, and inputs to servos; used as a variable dimension.
NT	Sum of the number of flexible modes, rigid body modes, control surfaces and <u>all</u> servo elements.
NTC	Sum of the number of flexible modes, rigid body modes, control surfaces and second order servo elements.
NTT	Order of the eigenvalue problem, $2*NTC+NO+N1$.
NVEC	Control for calculation and printing of eigenvectors.
NO	Number of zero order servos
N1	Number of first order servos
N2	Number of second order servos

2. Real Variables

ANORM	Test for normalizing structural deflections.
CANAI	Constant for generalized aerodynamic forces-imaginary part, $-4\rho V S$.
CANAR	Constant for generalized aerodynamic forces-real part, $-4\rho V^2 S/\bar{c}$.
CANI	Constant for imaginary part of AIC matrix, $k(\bar{c}s/S)$.
CANR	Constant for real part of AIC matrix, $2k^2(\bar{c}s/S)$.
DET	Value of determinant returned from subroutine INVERS if matrix is non-singular.
EK	Reduced frequency, $b_r\omega/V$, used in the iterative procedure; determined by the estimated frequency, ω .
EW	Estimated frequency for successive iterations of the eigenvalue solution; determined by the 'lining-up the frequencies' process.
VEL	Velocity for the stability analysis, ft/sec.
VELK	Velocity for the stability analysis, knots.
XNORM	Normalizing factor for structural deflections.

3. Integer Arrays (Variable dimensions indicated in parentheses)

IANA(NTT)	Used in subroutine SOLV as an argument when subroutine RESULT is called. This vector is eventually used in EGNVCT as L1 described below.
INDEX(NTT,3)	Used in subroutine INVERS for working storage.
K0(NSE,2)	Stores information concerning the <u>maximum</u> order of all servo elements and their column assignments in the X, Y and Z arrays.
L1(NTT)	Used in subroutine EGNVCT to restore order of the elements in the eigenvector.
L2(NTT)	Used in subroutine EGNVCT as working storage.
NL(NSERV,2)	Stores information concerning the number of the servo element from which there is input and output.
NOR (NSE,NSERV)	Stores information concerning the order of the servo element coefficients.

4. Real Arrays (Variable dimensions indicated in parentheses)

A(NTT,NTT)	Matrix in the eigenvalue problem formulation.
AFI (NFR,NMODE,NK)	Generalized aerodynamics used for reduced frequency spline interpolation - imaginary part.
AFR (NFR,NMODE,NK)	Generalized aerodynamics used for reduced frequency spline interpolation - real part.
AMI (NFR,NMODE,NM)	Generalized aerodynamics used for Mach number spline interpolation - imaginary part.
AMR (NFR,NMODE,NM)	Generalized aerodynamics used for Mach number spline interpolation - real part.
B(NTT,NTT)	Matrix in the eigenvalue problem formulation.
BINV(NTT,1)	Column of constants used in subroutine INVERS.
BC(NTT,NTT)	$\gamma_0[A] + [B]$, where γ_0 is the eigenvalue shift.
C(NTT,NTT)	Dynamic matrix used for the eigenvalue solution.
CF(MFM)	Structural damping matrix in its equivalent viscous form (diagonal)
CHDH(NDF,NDF)	$k(\bar{c}s/S)[C_{hI}]$, where $[C_{hI}]$ is the imaginary part of the AIC matrix $[C_h]$.
CHS(NDF,NDF)	$2K^2(\bar{c}s/S)[C_{hR}]$, where $[C_{hR}]$ is the real part of the AIC matrix $[C_h]$.
DFM(1,MFM)	$-K[D][h_F]$, where K is the rate gyro or accelerometer gain, D is the differentiation or interpolation vector, and h_F are the flexible modes.
DRM(1,MRM)	$-K[D][h_R]$, where K is the rate gyro or accelerometer gain, D is the differentiation or interpolation vector, and h_R are the rigid body modes.
FRC(M2,M2)	Partition of generalized mass or aerodynamic matrix used in subroutine GENMAT.
GAI(NFR,NMODE)	Matrix of generalized aerodynamics as output from subroutine GENMAT - imaginary part.
GAR(NFR,NMODE)	Matrix of generalized aerodynamics as output from subroutine GENMAT - real part.

GENM (NFR,NMODE)	Generalized mass matrix
P(M31)	Row vector for spline interpolation used in sub- routine SPLINE.
PSK(NK)	Vector of LaGrangian coefficients from reduced frequency spline interpolation
PSM(NM)	Vector of LaGrangian coefficients from Mach num- ber spline interpolation
RI(NTT)	Imaginary part of eigenvalue
RR(NTT)	Real part of eigenvalue
SAFI (NFR,NMODE)	Matrix of generalized aerodynamic forces for esti- mated reduced frequency obtained from spline inter- polation - imaginary part.
SAFR (NFR,NMODE)	Matrix of generalized aerodynamic forces for esti- mated reduced frequency obtained from spline inter- polation - real part.
SI(M31,M3)	Matrix of 'ones' for spline interpolation - used in subroutine INTERP.
SM(NM1,NM)	Constant spline interpolation matrix for Mach num- bers.
SMK(M31,M31)	Matrix of Mach numbers or reduced frequencies in the constant portion of the spline interpolation matrix - used in subroutine INTERP.
SK(MFM)	Generalized stiffness matrix (diagonal).
SPK(NK1,NK)	Constant spline interpolation matrix for reduced frequencies.
STAB(MODE,6)	Stores stability results for each mode.
TM(M1,NDF)	Intermediate array used in subroutine GENMAT.
XMODE (NDF,NMODE)	Matrix of flexible, rigid body and control surface modes.
X0(NT,NT)	Coefficient matrix of second order variables, dis- placement.
X1(NT,NT)	Coefficient matrix of second order variables, velo- city.
X2(NT,NT)	Coefficient matrix of second order variables, ac- celeration.

Y0(NT,NSE)	Coefficient matrix of first order variables, displacement.
Y1(NT,NSE)	Coefficient matrix of first order variables, velocity.
Z0(NT,NSE)	Coefficient matrix of zero order variables, displacement.
Z	Dynamic dimensioning array in BLANK COMMON.
5. <u>Complex Arrays</u> (Variable dimensions indicated in parentheses)	
C2(NTT)	Complex working storage for subroutine EGNVCT.
C3(NTT)	Complex eigenvector for a particular mode, computed by subroutine EGNVCT.
DEFL (NDF)	Structural deflections for the system mass points (determined by input modal data and eigenvectors).
E(NTT)	Complex eigenvalues resulting from the eigenvalue solution in subroutine EIGRF.
U(NTT,NTT)	$\gamma[A] + [B]$, where γ is the complex eigenvalue for a particular mode; used in subroutine EGNVCT to determine eigenvector.
V(1,1)	Pseudo complex eigenvector storage in subroutine EIGRF (not used).
VEC(MODE,NTT)	Stores eigenvectors for each mode (complex).
WK(NTT)	Work area in subroutine EIGRF.

Processing and Programming Considerations

1. Operation

Standard FORTRAN IV processor system. Operable on the CDC computer; model 6600, Cyber 175-Scope 3.4.3 system.

Note: MPASES can be made operable on all computer systems with minor modifications. Probable necessary changes are listed below:

Deletion of PROGRAM statement at beginning of 'MAIN' section of program.

END-OF-FILE (EOF) statement alterations.

Alphanumeric modifications dependent upon the number of characters per word in the operating system used.

Single precision to double precision accuracy. (MPASES, as presented in this report, has single precision accuracy.)

2. Core Storage

Number of memory units (words, bytes, etc.) required to execute is dependent upon input data reflected in the length of BLANK

COMMON. Estimate of BLANK COMMON length can be accomplished as follows:

Servo-Elastic Analysis:

$$\text{LENGTH} = 1 + \text{NTT}(4\text{NTT} + 2\text{MODE} + 9) + \text{NDF}(\text{NMODE} + 2) + 6\text{MODE}$$

OR

$$\text{LENGTH} = 1 + \text{NDF}(\text{NDF} + \text{NMODE} + \text{M1}) + \text{NFR}(\text{NMODE}) + 4\text{NFM} + (\text{M2})^2$$

Use the larger of the above two estimates.

Aero-Servo-Elastic Analysis:

$$\begin{aligned} \text{LENGTH} = & 11 + \text{NTT}(4\text{NTT} + 2\text{MODE} + 9) \\ & + 2(\text{NFR})(\text{NMODE})(\text{NK} + \text{NM} + 1) + 2(\text{M3} + 1)^2 \\ & + \text{NM}(\text{NM} + 2) + \text{NK}(\text{NK} + 2) + \text{NDF}(\text{NMODE} + 2) \\ & + 6\text{MODE} \end{aligned}$$

OR

$$\text{LENGTH} = 1 + \text{NDF}(4\text{NDF} + \text{NMODE} + \text{M1}) + 2(\text{NFR})(\text{NMODE}) + (\text{M2})^2$$

Use the larger of the two. If a number greater than NTT was assigned to the parameter 'MODE,' use MODE=NTT in the above calculations. Note that the first estimate for each analysis is eigenvalue problem size oriented, while the second is dependent upon the number of system mass points.

Refer to this section under Programming Symbols for definitions.

Actual BLANK COMMON length required for the analysis follows the stability results in the program's printed output. This information will enable the user to make more efficient use of core storage in subsequent analyses by recompiling the 'MAIN' section with a more realistic BLANK COMMON length.

Note. MPASES, as presented in this report, contains a BLANK COMMON length of 10,000. This length was more than adequate to perform the analyses in the sample problems used as examples. (see Section 4.0.)

3. Auxiliary Files

Standard input read file (5).

Standard output print file (6).

Four temporary utility files (1, 2, 3, and 4).

Note: To avoid the cumbersome handling of data cards, it is suggested that data sets be generated containing the AIC's, modal data, etc. Consequently, MPASES can be modified to accept this input from auxiliary files in lieu of cards.

Input Instructions

Units - All units are taken in the pound-foot-second system with the exception of density, which must be in slugs/cubic feet. The weight, which is input in pounds, is converted to slugs internally in the program. Likewise, MPASES converts the frequency from Hertz to rps units within the program. The AIC's must be non-dimensional when input with no altitude consideration (i.e., $\rho=1$).

Data Deck Setup

1. Title cards (2).
2. Control card describing data input.
3. Eigenvalue shift, γ_0 .
4. Geometric properties of air vehicle (s , $\bar{c}$, S) and Mach number deviation (DM).
5. Mach numbers and reduced frequencies, i.e., for AIC input.
6. Density and speed of sound for each altitude.
7. Number of velocities to be analyzed for each altitude.
8. Gain factors, if gain option is to be exercised.
9. Weight matrix.
10. Rigid body control surface modes.
11. Structural damping coefficients if NFM > 0.
12. Frequencies of vibration (flexible) modes if NFM > 0.
13. Flexible modes, if any.

14. Restrained (rigid body) modes, if any.
15. Coefficients of differential equations describing the servo elements in the control system.
16. AIC matrices.
17. Mach numbers to determine velocities to be analyzed at each altitude.

Items 4, 5, 6, 7, 16 and 17 are included for aero-servo-elastic analysis only.

Detailed description of input data follows.

Input Data Description

NO.	CARD	FORMAT	COLUMNS	FORTTRAN NAME	DESCRIPTION
1.	TITLE	18A4	1-72		Any alphanumeric statement.
2.	TITLE	18A4	1-72		Any alphanumeric statement.
NOTE: Two cards <u>must</u> be input. May be blank.					
3.	CONTROL	14I5	1-5	MCODE	= 1, diagonal weight matrix input. = 2, coupled weight matrix input.
			6-10	NDF	Number of mass points in the complete system - degrees of freedom (NDF > 0).
			11-15	NFM	Number of flexible modes (NFM > 0). If NRM = 0, NFM > 0.
			16-20	NRM	Number of rigid body modes (NRM ≥ 0). If NFM = 0, NRM > 0.
			21-25	NC	Number of rigid body control surface modes; i.e., number of control surfaces (NC > 0).
			26-30	NSE	Total number of servo elements in the control system, including rate gyros and accelerometers (NSE > 0).

NO.	CARD	FORMAT	COLUMNS	FORTTRAN NAME	DESCRIPTION
			31-35	NA	= 0, no aerodynamics, i.e., servo-elastic analysis only. > 0, AIC input, i.e., aero-servo-elastic analysis. Input as follows: NA = number of partitions contained in each AIC matrix.
NOTE: NA = 1, AICs derived from Doublet-Lattice, Mach Box theories, etc. NA > 1, AICs derived from Strip, Piston Theories, etc. (NA = number of strips). It is assumed that all AICs are obtained from the <u>same</u> method for any <u>one</u> aero-servo-elastic analysis.					
			36-40	NM	Number of Mach numbers for which there is AIC input (NM ≤ 5). If NA = 0, NM = 0.
			41-45	NK	Number of reduced frequencies (k) for each Mach number (NK ≤ 10). If NA = 0, NK = 0.
NOTE: The reduced frequencies <u>must</u> be the same for each Mach number.					
			46-40	ND	Number of altitudes (ND ≤ 5). If NA = 0, ND = 0.
			51-55	MODE	Number of modes requested for the stability analysis. Use a <u>large</u> number if <u>all</u> modes inherent to the analysis are to be considered.
NOTE: Due to the nature of the 'lining up the frequencies' method of analysis, the eigen-value solution <u>may</u> result in <u>less</u> modes than requested.					
			56-60	NG	= 1, <u>no</u> gain variation. > 1, gain option executed. Input as follows: NG = number of gain variations (K). (1 ≤ NG ≤ 25)
			61-65	NGS	= 0, <u>no</u> gain variation. > 0, NGS = number of the particular <u>servo element</u> for which the gain varies. < 0, NGS = number of the <u>control surface</u> for which the gain varies.

NO.	CARD	FORMAT	COLUMNS	FORTTRAN NAME	DESCRIPTION
NOTE: The gain factor (K) can vary for only <u>one</u> servo element <u>or</u> control surface in any <u>one</u> stability analysis.					
66-70	NAB				= 0, A and B matrices not printed. = 1, A and B matrices printed by rows.

NOTE: If NA = 0, [A] and [B] as generated for the analysis are printed.
 If NA > 0, [A] and [B] as generated for k=0 only are printed.

4.	SHIFT	E12.0	1-12	GAMMA	Shift eigenvalue (γ_0). Choice of γ_0 is left to the user.
----	-------	-------	------	-------	---

NOTE: See Section 2 (Theoretical Development) and Section 4 (Sample Problems) for details.

If NA = 0, OMIT the following cards: 5, 6, 7, 8 and 9.

5.	AERO CONSTANTS	4E12.0	1-12	SS	Semi-span (s), feet.
			13-24	CBAR	Mean aerodynamic chord ($\bar{c}$), feet.
			25-36	S	Surface area (S), sq.ft.
			37-48	DM	Mach number deviation. Choice of DM is left to the user.

NOTE: If the difference between the Mach number for which the analysis is to be performed and the Mach number for which there is AIC input is greater than DM, Mach number interpolation of the AICs will be made. If NM = 1, no interpolation.

If NA = 0, OMIT Card 5.

6.	AIC DATA	5E12.0		XM(I)	Mach number; I = 1, NM. If NA = 0, <u>OMIT</u> Card 6.
----	----------	--------	--	-------	---

7.	AIC DATA	6E12.0		XK(I)	Reduced frequency, k; I=1, NK. Continue on next card if necessary.
----	----------	--------	--	-------	--

If NA = 0, OMIT Card 7.

NO.	CARD	FORMAT	COLUMNS	FORTTRAN NAME	DESCRIPTION
8.	AERØ DATA	6E12.0	1-12	DENS(1)	Air density, ρ , associated with the first altitude at which the analysis is to be made; slugs/cu.ft.
			13-24	SØS(1)	Speed of sound associated with the first altitude; ft/sec.
			25-36	DENS(2)	Density for second altitude.
			37-48	SØS(2)	Speed of sound for second altitude.
			.	.	.
			.	.	.
			.	.	.
					DENS(I), SØS(I); I=1, ND. Continue on next card if necessary.
					If NA = 0, <u>OMIT</u> Card 8.
9.	AERØ DATA	S15		MD(I)	Number of velocities (determined by Mach number input - see Card 26) for each altitude; I = 1, ND. (MD ≤ 10)
					If NA = 0, <u>OMIT</u> Card 9.
10.	GAIN	6E12.0		GAIN(I)	Gain factor, K. I = 1, NG. Continue on next card if necessary. (NG ≤ 25)
					If NGS = 0, <u>OMIT</u> Card 10.
11.	WEIGHT	6E12.0		WT(I,I)	If MCØDE = 1, vector of diagonal elements of weight matrix, pounds. I = 1, NDF. Continue on next card if necessary.
				WT(I,J)	If MCØDE = 2, elements of upper triangle of coupled weight matrix, lbs. Input by rows; each row starts on a new card. I = 1, NDF, J = I, NDF. Continue on successive cards to complete each row.

NO.	CARD	FORMAT	COLUMNS	FORTTRAN NAME	DESCRIPTION
12.	MØDE	6E12.0		CMØDE(J,I)	Rigid body control surface modes. Each mode starts on a new card. I = 1, NC, J = 1, NDF. Continue on successive cards to complete each mode.
13.	DAMPING	6E12.0		GF(I)	Structural damping coefficient corresponding to vibration mode (I). I = 1, NFM. Continue on next card if necessary. If NFM = 0, <u>OMIT</u> Card 13.
14.	FREQUENCY	6E12.0		FREQ(I)	Frequency of vibration mode (I), Hz. I = 1, NFM. Continue on next card if necessary. If NFM = 0, <u>OMIT</u> Card 14.
15.	MØDE	6E12.0		FMØDE(J,I)	Free-free vibration (flexible) modes. Each mode starts on a new card. I = 1, NFM, J = 1, NDF. Continue on successive cards to complete each mode. If NFM = 0, <u>OMIT</u> Card 15.
16.	MØDE	6E12.0		RMØDE(J,I)	Restrained (rigid body) modes. Each mode starts on a new card. I = 1, NRM, J = 1, NDF. Continue on successive cards to complete each mode. If NRM = 0, <u>OMIT</u> Card 16.

Input description of the servo differential equation coefficients follows:

IMPORTANT: All servo differential equations must be equated to zero, i.e., all terms should be on one side of the equation, before attempting to input the coefficients.

NOTE: Reference to servo elements and control surfaces is by number; therefore, numbers should be assigned to each servo element (1 to NSE) and control surface (1 to NC). Servo elements also include the rate gyros and accelerometers in the control system.

NO.	CARD	FORMAT	COLUMNS	FORTTRAN NAME	DESCRIPTION
-----	------	--------	---------	------------------	-------------

REPEAT the following cards 17, 18 and 19 for each control surface, i.e., NC times.

17. Output from Control Surface. FORMAT (3E12.0)

Columns	1-12	X0(I)	Zero order coefficient for Control Surface (I).
	13-24	X1(I)	First order coefficient for Control Surface (I).
	25-36	X2(I)	Second order coefficient for Control Surface (I).

Note: I=1, NC.

18. Input Control Card. FORMAT (I5)

Columns	1-5	INC	Number of inputs from servo elements to control surface (I).
---------	-----	-----	--

REPEAT card 19 for input from each servo element to Control Surface (I), i.e., INC times.

19. Input from Servo Element. FORMAT (2I5,2X,3E12.0)

Columns	1-5	K	Servo element from which there is input.
	6-10	NØ	Order of servo element coefficients.
	13-24	C0	Zero order coefficient.
	25-36	C1	First order coefficient.
	37-48	C2	Second order coefficient.

If NØ = 0, zero order; only C0 input.
 = 1, first order; C1 must be input (C0 may be zero).
 = 2, second order; C2 must be input (C0 and C1 may be zero).

REPEAT the above card 19 INC times.

REPEAT the above cards 17, 18 and 19 NC times.

REPEAT the following cards 20, 21 and 22 for each servo element, i.e., NSE times (not necessarily in sequential order).

NO.	CARD	FORMAT	COLUMNS	FORTTRAN NAME	DESCRIPTION
20.	<u>Output from Servo Element.</u> FORMAT (2I5,2X,3E12.0)				
	Columns	1-5	I		Servo element number from which there is output.
		6-10	NØ		Order of servo element coefficients.
		13-24	C0		Zero order coefficient
		25-36	C1		First order coefficient.
		37-48	C2		Second order coefficient.

If NØ = 0, only C0 is input.
 = 1, C1 must be input (C0 may be zero).
 = 2, C2 must be input (C0 and C1 may be zero).

21. Input Control Card. FORMAT (I5)

Columns	1-5	INS	Number of inputs <u>from</u> servo elements (or control surfaces) <u>to</u> servo element (I).
---------	-----	-----	--

If there are no inputs to Servo Element (I), INS = 0.

REPEAT card 22 for input from each servo element or control surface to Servo Element (I), i.e., INS times.

If INS = 0, OMIT Card 22.

22. Input from Servo Element or Control Surface. FORMAT (2I5,2X,3E12.0)

Columns	1-5	K	Servo element number from which there is input.
		-K	Control surface number from which there is input.
	6-10	NØ	If K > 0, NØ = order of servo element coefficients. If K < 0, NØ = order of control surface coefficients.

NØ = -1, input from body angular rate to rate gyro.
 = -2, input from body acceleration to accelerometer.

NOTE: If NØ < 0, K = 0.

13-24	C0	Zero order coefficient.
25-36	C1	First order coefficient.
37-48	C2	Second order coefficient.

NO.	CARD	FORMAT	COLUMNS	FORTTRAN NAME	DESCRIPTION
-----	------	--------	---------	------------------	-------------

If $N_0 = 0$, only C_0 input.
 = 1, C_1 must be input (C_0 may be zero)
 = 2, C_2 must be input (C_0 and C_1 may be zero).
 = -1, only C_0 input, where
 $C_0 = -K_g$ (rate gyro gain)
 = -2, only C_0 input, where
 $C_0 = -K_a$ (accelerometer gain)

NOTE: When $N_0 < 0$, Card 22 must be followed by Card 23.

23. If $N_0 = -1$,
Differentiation Row Vector used to determine angular velocity for
 the rate gyro.
 If $N_0 = -2$,
Interpolation Row Vector used to describe body acceleration for the
 accelerometer.

FORMAT 6E12.0 D(I) Element of differentiation or interpolation row
 matrix. I = 1, NDF.
 Continue on next card if necessary.

NOTE: See Ref. 1 for description.

REPEAT the above Card 22 (and Card 23, if applicable) INS times.

REPEAT the above Cards 20, 21 and 22 NSE times.

Input instructions for the AIC matrices $[C_h]$ follow:

NOTE: It is assumed that the AIC's are dimensionless and do not
 reflect density (i.e., $\rho = 1$).

If $NA = 0$, OMIT Cards 24 and 25.

If $NA = 1$, input only Card 25.

If $NA > 1$, input Cards 24 and 25.

The AICs are input for all the reduced frequencies, i.e., for $XK(I)$,
 where $I = 1, NK$ for each Mach number $XM(J)$, where $J = 1, NM$.

NOTE: The reduced frequencies, k , must be the same for each Mach
 number.

If $NA = 1$, REPEAT Card 25 for each AIC matrix, i.e., $(NK \cdot NM)$ times.

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION
-----	------	--------	---------	-----------------	-------------

If $NA > 1$, REPEAT Cards 24 and 25 NA times for each AIC matrix.
See Card 3. NA = Number of partitions (NPART) in the AIC matrix.
All the input AIC's must have the same number of partitions.

REPEAT until all AIC's are input, i.e., $(NK \cdot NM)$ times.

24. Partition Control Card. FORMAT (2I5)

Columns	1-5	MS	Size of partition.
---------	-----	----	--------------------

NOTE: The sum of the sizes (MS) of all the partitions in each AIC matrix must equal NDF.

6-10 NZERØ = 1, all elements of partitions are equal to zero. Do not input. OMIT Card 25.

= 0, non-zero partition. Input Card 25.

25. AIC Partition or Matrix. FORMAT (6E12.0)

CH(I,J) If $NA = 1$, complete AIC matrix (complex).
I = 1, NDF, J = 1, NDF.

If $NA > 1$, AIC partition (complex).
I = 1, MS, J = 1, MS.

Input by rows. Each row starts on a new card. Continue on successive cards to complete each row.
The imaginary part of each element follows the real part, e.g.:

Columns	1-12	CH(1,1)	Real
	13-24	CH(1,1)	Imaginary
	25-36	CH(1,2)	Real
	37-48	CH(1,2)	Imaginary
	.	.	
	.	.	
	.	.	

OMIT the following Card 26, if $NA = 0$.

REPEAT Card 26 for each altitude, i.e., ND times.

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION
26.		<u>Mach Numbers for Analysis.</u> FORMAT (6E12.0)			
				XMACH(I)	Mach numbers selected for stability analysis at a particular altitude. $I = 1, NMACH$, where $NMACH$ is the number of velocities (determined by the Mach number, $XMACH(I)$ and the speed of sound, $SOS(J)$ for altitude J), i.e., $NMACH = MD(J)$. Refer to Card 9. ($NMACH \leq 10$). Continue on next card if necessary. <u>No</u> eigenvectors are obtained or printed with the stability analysis when $XMACH(I) > 0$.
				-XMACH(I)	Same as above. Complex eigenvectors are calculated and printed for each mode when $XMACH(I) < 0$.
				NOTES: $XMACH(I)$ may be the same or <u>different</u> for each altitude. Do not confuse $XMACH$ with XM which are the Mach numbers for the input AIC's.	

REPEAT above Card 26 ND times.

Program Output Description

Input Data

1. Upper triangle of weight matrix (lbs.).
2. Damping coefficients and frequencies (if free-free vibration modes are present).
3. Mode shapes - flexible, rigid body and control surface.
4. Control system description - coefficients from servo differential equations, rate gyro gains and/or accelerometer gains.
5. Maximum order of each servo element (determined internally in the program).
6. Rows and columns (assigned by the program) in the A and/or B matrices for the coefficients of each servo element.
7. Eigenvector element associated with each servo element (velocity and/or displacement).
8. AIC matrix (by rows) for each reduced frequency and Mach number, if aero-servo-elastic analysis.

A and B Matrices (if requested)

1. For an aero-servo-elastic analysis, only [A] and [B] for $k=0$ are printed.
2. For a servo-elastic analysis, [A] and [B] as generated for the analysis are printed.

Stability Analysis Results

The results are identified and tabulated for each gain factor variation (if any). In addition, when aerodynamics are included, the results are printed for the velocities analyzed at each altitude.

1. For each mode calculated, the following is printed in tabulated form:

Complex eigenvalue, γ - real part, μ , 1/sec.
Complex eigenvalue, γ - imaginary part, ω , rad/sec.
Damped frequency, Hz.
Undamped frequency, Hz.
Reduced frequency, k , used to determine eigenvalue problem for particular mode.
Fraction of critical damping, ζ .
Time to half amplitude, sec.

2. Complex eigenvectors corresponding to each eigenvalue, if requested for the aero-servo-elastic analysis of a particular velocity. The eigenvectors are always printed with the stability results of a servo-elastic analysis.
3. Structural deflections (complex) representing the system mass points for each mode are printed whenever eigenvectors are calculated.

BLANK COMMON Length

The minimum BLANK COMMON length required to execute the program. This length is dependent upon the input data.

SECTION IV SAMPLE PROBLEMS

As examples of the program capabilities we consider a simplified missile and servo configuration first, Case 1, as a rigid body in vacuo, second, Case 2, as a flexible body in vacuo, and third, Case 3, as a flexible body in an incompressible airstream at sea level. The missile is idealized as a uniform beam and control surface as shown in Fig. 1.

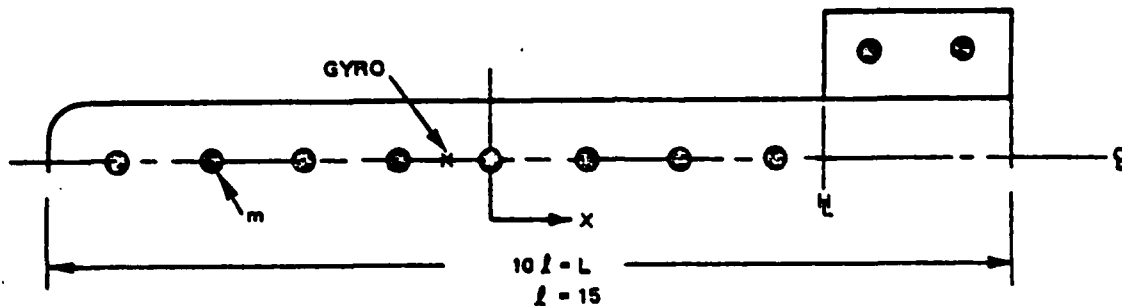


Figure 1. Uniform free-free missile.

The modal characteristics and differentiation matrix for this configuration are given in Ref. 11. For the numerical work we assume the missile to weigh $10m = 1000$ lbs., to be 150 inches long, and to have a fundamental frequency of 45 Hz. From the data in Ref. 11, this results in higher frequencies of $f_2 = 125.4$ Hz and $f_3 = 248.2$ Hz; only three modes will be considered. The structural dampings in the three modes are assumed to be $g_1 = 0.03$, $g_2 = 0.05$, and $g_3 = 0.08$.

A simple servo system is considered whose block diagram is shown in Fig. 2.

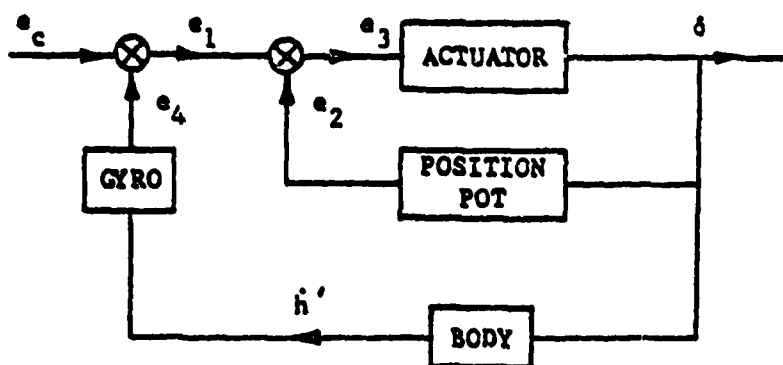


Figure 2. Servo block diagram.

We assume the transfer functions appear as follows.

For the command error,

$$e_1 + e_4 = e_c \quad (76)$$

for the servoposition error,

$$e_1 - e_2 - e_3 = 0 \quad (77)$$

for the actuator,

$$e_3/\delta = K_a s / (T_a s + 1) \quad (78)$$

for the position potentiometer,

$$e_2/\delta = K_p \quad (79)$$

and for the rate gyro,

$$e_4/\dot{h}' = K_g / (s^2/\omega_g^2 + 2\zeta_g s/\omega_g + 1) \quad (80)$$

The differential equations corresponding to Eqs. (78) and (80) are

$$T_a \dot{e}_3 + e_3 - K_a \delta = 0 \quad (81)$$

and

$$(1/\omega_g^2) \ddot{e}_4 + (2\zeta_g/\omega_g) \dot{e}_4 + e_4 - K_g \dot{h}' = 0 \quad (82)$$

The actuator is seen to be a first order servo element and the rate gyro is a second order element. Therefore the second order variables are

$$\{x\} = \begin{Bmatrix} a_F \\ a_R \\ \delta \\ e_4 \end{Bmatrix} \quad (83)$$

the first order variable is

$$\{y\} = \{e_3\} \quad (84)$$

and the zero order variables are

$$\{z\} = \begin{Bmatrix} e_1 \\ e_2 \end{Bmatrix} \quad (85)$$

For the numerical work we assume $K_a = 1/6$ deg. per deg./sec., $T_a = 0.01$ sec., $K_p = 1$ deg./deg., $\omega_g = 376.991$ rad/sec. (60 Hz), $\zeta_g = 0.70$, and $K_g = 0.3$ deg. per deg./sec. The differentiation matrix for the rate gyro from Ref. 11 is

$$[D] = (1/24\ell) \begin{bmatrix} 0 & 0 & +1 & -27 & +27 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (86)$$

where $\ell = 15$ inches.

The case descriptions below present the variables in the eigenvectors and discuss the solutions. The data input code sheets and the program printed output for the three sample problems appear following the case descriptions.

Case 1 Description

The first case is the rigid body in vacuo. There are two rigid body modes of plunging and pitching. The eigenvalue calculation is made choosing

$\gamma_0 = -1.0$. The 11th order eigenvector that appears is in the following order:

$$\{V\}^T = [\dot{a}_{R1} \dot{a}_{R2} \delta \dot{e}_4 a_{R1} a_{R2} \delta e_4 e_3 e_1 e_2] \quad (87)$$

There are four zero eigenvalues from the two rigid body modes and four infinite eigenvalues from the command error, the servoposition error, the position potentiometer, and the actuator. The three non-trivial eigenvalues consist of the real actuator damping and the complex conjugate rate gyro frequency and damping. However, since only the eigenvalues with positive frequencies in the conjugate pairs are printed in the stability results, only nine solutions are presented in the printed output. The structural deflections at the ten system mass points for each mode are also printed.

Case 2 Description

The second case is the flexible body in vacuo. The addition of three flexible modes to Case 1 results in a 17th order eigenvector which has the variables printed out in the following order:

$$\{V\}^T = [\dot{a}_{F1} \dot{a}_{F2} \dot{a}_{F3} \dot{a}_{R1} \dot{a}_{R2} \delta \dot{e}_4 a_{F1} a_{F2} a_{F3} a_{R1} a_{R2} \delta e_4 e_3 e_1 e_2] \quad (88)$$

The eigenvalues are obtained by again choosing $\gamma_0 = -1.0$. Nine non-trivial eigenvalues are obtained in this case. Three are those obtained in Case 1 and the additional three complex conjugate pairs correspond to the three flexible modes. The stability results present the modes with the frequencies in ascending order, the negative values being omitted. Therefore, only thirteen solutions are printed; the structural deflections for the thirteen modes are also printed.

An unsatisfactory design is seen to exist from the negative damping in the 133.0 Hz mode; the missile, will "buzz" in this mode.

Case 3 Description

The third case is the flexible body in an incompressible flow at sea level with a density $\rho = 0.00237692$ slugs/cu. ft. and velocity $V = 500$ fps. It is the same as Case 2 with the addition of aerodynamic loads. The aerodynamic loads are assumed to act only on the control surface and are derived from the incompressible strip theory presented in Ref. 15. The control surface has an exposed span of 20 inches and a semichord of 15 inches. Five reduced frequencies are chosen for the aerodynamic interpolation, $k = 0.05, 0.10, 0.20, 0.50,$ and 1.00 ; the minimum value of $k = 0.05$ is chosen because of the singularity in the aerodynamic damping at zero frequency when it is computed from Theodorsen's function.

Although there are 17 degrees of freedom in the eigenvalue problems solved, only nine modes can be obtained. This results from the frequency "lining-up" process. The reduced frequencies determined by the estimated frequency in each eigenvalue solution are shown along with the stability results for each mode. In addition, the time to half the amplitude as well as the damping ratio is indicated to facilitate stability evaluation.

The solutions for Case 3 are seen not to be significantly different from those of Case 2. However, we note that the short period mode shows up in place of the rigid body pitching mode. The reduced frequency of the short period mode is $k = 0.03346$ and its damping ratio is $\zeta = 0.1401$. The aerodynamic loads do not cause a large disturbance for the (impractical)

configuration chosen as the example. The servoelastic "buzz" observed in Case 2 is still unstable here. More practical configurations should be studied that have more critical hinge line locations and mass balancing on the control surface.

POSTAL CODES

[illegible]

RESTAURANT CATERING FORM

[illegible][illegible]

WILLIAM L. BROWN, JR.

[illegible]

SAMPLE PROBLEM - CASE 1 - PROGRAM MPASIS
MISSILE AND SERVO SYSTEM - RIGID BODY IN VACUO

SERVO ELASTIC STABILITY ANALYSIS

10 DEGREES OF FREEDOM
0 FLEXIBLE MODES
2 RIGID BODY MODES
1 CONTROL SURFACES
0 SERVO ELEMENTS

SHIFT EIGENVALUE(GAMMA) = 1.000E+00

11 MODES REQUESTED

UPPER TRIANGLE OF WEIGHT MATRIX

ROW 1	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 2	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 3	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 4	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 5	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 6	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 7	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 8	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 9	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 10	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

[illegible]

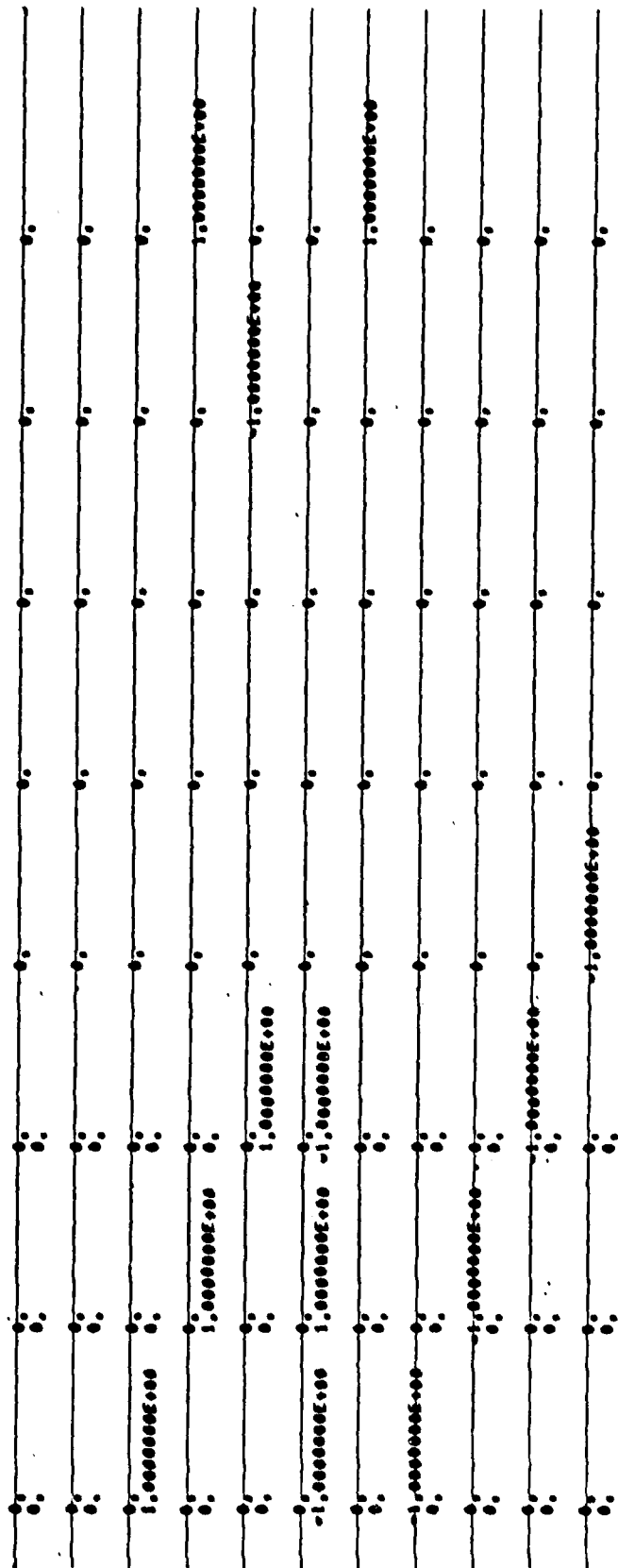
INMNC3V7PDISD A112073A 8 RD/GNV Y NI 8 RD/GNV Y NI 8 RD/GNV Y NI INMNC3V7PDISD
 INMNC3V7PDISD A112073A INMNC3V7PDISD INMNC3V7PDISD INMNC3V7PDISD INMNC3V7PDISD

[illegible]

XIV

Handwritten musical notation on ten staves.

8 MATRIZ



STABILITY ANALYSIS RESULTS

MODE	EIGENVALUE-H MU-RPS	EIGENVALUE-1 OMEGA-RPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	3.783702E-10	0.	0.	6.022070E-11		-1.831890E+09
2	-3.785559E-10	0.	0.	6.024904E-11		1.831830E+09
3	-4.839451E+00	0.	0.	1.782236E-01		1.432203E-01
4	-1.000000E+30	0.	0.	1.591549E+37		6.931672E-39
5	-1.000000E+30	0.	0.	1.591549E+37		6.931672E-39
6	-2.21737E-13	4.417513E-09	7.030691E-10	7.030691E-10	5.147897E-05	3.040494E+12
7	-3.842603E+02	1.234004E+02	1.987161E+01	6.485220E+01	9.528854E-01	1.785172E-03
8	-8.897610E+03	3.677655E+04	5.052852E+07	5.852852E+07	2.419502E-05	7.790262E+05

EIGENVECTORS FOR MODES REQUESTED

EIGENVECTOR FOR MODE 1

3.703702E-10 0.
1.000000E+00 0.
-6.567432E-19 0.

-2.1202197E-12 0.
-7.2102901E-03 0.
0.1024774E-13 0.

-3.940252E-10 0.
-1.041193E-00 0.
7.004290E-11 0.

-0.017359E-12 0.
-0.1024774E-13 0.

EIGENVECTOR FOR MODE 2

-3.705550E-10 0.
1.000000E+00 0.
-6.572310E-19 0.

2.7294997E-12 0.
-7.2102957E-03 0.
-0.1041305E-13 0.

-3.951960E-10 0.
1.041601E-00 0.
-7.647862E-11 0.

-0.024407E-19 0.
0.1041305E-13 0.

EIGENVECTOR FOR MODE 3

1.000000E+00 0.
-2.664374E-01 0.
1.204346E-01 0.

7.003333E-02 0.
-1.4634427E-02 0.
-2.1626751E-02 0.

6.075000E-01 0.
-1.4206130E-01 0.
-1.4206130E-01 0.

-1.0066174E-01 0.
2.1626751E-02 0.

EIGENVECTOR FOR MODE 4

1.000000E+00 0.
-1.000000E-30 0.
-1.140062E-37 0.

7.003333E-02 0.
-7.003333E-40 0.
-1.214012E-37 0.

6.075000E-01 0.
-6.075000E-30 0.
-6.075000E-30 0.

-3.015973E-35 0.
3.015973E-73 0.

EIGENVECTOR FOR MODE 5

1.000000E+00 0.
-1.000000E-30 0.
-1.140062E-37 0.

7.003333E-02 0.
-7.003333E-40 0.
-1.214012E-37 0.

6.075000E-01 0.
-6.075000E-30 0.
-6.075000E-30 0.

-3.015973E-35 0.
3.015973E-73 0.

EIGENVECTOR FOR MODE 6

-2.273740E-13 0.
1.000000E+00 0.
0.951566E-17 0.

1.7005507E-15 -3.105150E-11
-7.210296E-03 -1.303497E-08
-1.539490E-14 9.951650E-12

5.3015620E-16 5.542254E-20
0.201500E-12 -1.215505E-07
-6.277862E-14 0.9245970E-10

1.0927051E-16 1.1346740E-20
1.539490E-14 -9.551650E-12

EIGENVECTOR FOR MODE 7

-6.393050E-02 0.
1.4393951E-04 0.
2.232660E-03 0.

-4.520034E-03 4.205350E-04
1.090404E-05 2.307990E-06
2.3304937E-03 7.444075E-04

-4.3957700E-02 4.0816630E-03
1.0203341E-04 2.3177550E-05
1.0503341E-04 2.3177550E-05

1.000000E+00 0.
-2.3304937E-03 -7.444075E-04

EIGENVECTOR FOR MODE 8

1.000000E+00 0.
-6.570283E-14 0.
-7.654444E-13 0.

7.003333E-02 3.153763E-14
-4.0666030E-15 -1.9261500E-10
-0.1090447E-13 -3.3016927E-08

6.075000E-01 2.7793970E-13
-4.523295E-14 -1.004490E-09
-4.562530E-14 -1.0007110E-09

-1.066042E-10 -8.2012510E-04
-2.2301451E-14 1.0472247E-10

STRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR EACH MODE

DEFLECTIONS FOR MODE 1

1.000000E+00	0.	9.2154423E-01	0.	0.4300044E-01	0.	7.646326E-01	0.
6.661769E-01	0.	6.077211E-01	0.	5.2926537E-01	0.	6.500095E-01	0.
3.723530E-01	0.	2.9309022E-01	0.				

DEFLECTIONS FOR MODE 2

1.000000E+00	0.	9.2154423E-01	0.	0.4300044E-01	0.	7.646327E-01	0.
6.661769E-01	0.	6.0772123E-01	0.	5.2926540E-01	0.	6.500072E-01	0.
3.7235391E-01	0.	2.9309005E-01	0.				

DEFLECTIONS FOR MODE 3

3.1521739E-01	0.	1.9202099E-01	0.	6.0049500E-02	0.	-5.9347026E-02	0.
-1.7753623E-01	0.	-3.0072444E-01	0.	-4.2391304E-01	0.	-5.4710145E-01	0.
-7.246376E-02	0.	1.0000000E+00	0.				

DEFLECTIONS FOR MODE 4

3.1521739E-01	0.	1.9202099E-01	0.	6.0049500E-02	0.	-5.9347026E-02	0.
-1.7753623E-01	0.	-3.0072444E-01	0.	-4.2391304E-01	0.	-5.4710145E-01	0.
-7.246376E-02	0.	1.0000000E+00	0.				

DEFLECTIONS FOR MODE 5

3.1521739E-01	0.	1.9202099E-01	0.	6.0049500E-02	0.	-5.9347026E-02	0.
-1.7753623E-01	0.	-3.0072444E-01	0.	-4.2391304E-01	0.	-5.4710145E-01	0.
-7.246376E-02	0.	1.0000000E+00	0.				

DEFLECTIONS FOR MODE 6

1.000000E+00	0.	9.2154424E-01	-1.0921779E-07	0.4300047E-01	-2.1043557E-07	7.6463271E-01	-3.2765334E-07
6.661769E-01	-4.3607115E-07	6.0772110E-01	-5.6608044E-07	5.2926542E-01	-6.5530672E-07	6.5000966E-01	-7.6452451E-07
3.7235390E-01	-2.1239044E-07	2.9309013E-01	1.0010713E-06				

DEFLECTIONS FOR MODE 7

3.1521739E-01	-1.3754917E-15	1.9202099E-01	-1.1462431E-15	6.0049500E-02	-1.7193046E-16	-5.9347026E-02	3.9307292E-16
-1.7753623E-01	1.1462431E-15	-3.0072444E-01	1.3754917E-15	-4.2391304E-01	2.7509033E-15	-5.4710145E-01	3.2094005E-15
-7.246376E-02	3.2094005E-15	1.0000000E+00	0.				

DEFLECTIONS FOR MODE 8

3.1521739E-01	-1.0830594E-09	1.9202099E-01	-7.4449654E-10	6.0049500E-02	-4.0513345E-10	-5.9347026E-02	-6.9770364E-11
-1.7753623E-01	2.7359272E-10	-3.0072444E-01	6.1295501E-10	-4.2391304E-01	9.5231090E-10	-5.4710145E-01	1.2916020E-09
-7.246376E-02	9.7424236E-10	1.0000000E+00	0.				

MINIMUM BLANK COMMON LENGTH REQUIRED = 946.
BASED ON INPUT DATA AND ANALYSES REQUESTED.

SAMPLE PROBLEM - CASE 2 - PROGRAM MPAGES
MISSILE AND SERVO SYSTEM - FLEXIBLE BODY IN VACUO

SERVO ELASTIC STABILITY ANALYSIS

10 DEGREES OF FREEDOM
3 FLEXIBLE MODES
2 RIGID BODY MODES
1 CONTROL SURFACE
4 SERVO ELEMENTS

SHIFT EIGENVALUE(GAMMA) = 1.000E+00

17 MODES REQUESTED

UPPER TRIANGLE OF WEIGHT MATRIX

ROW 1	1.0000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 2	1.0000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 3	1.0000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 4	1.0000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 5	1.0000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 6	1.0000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 7	1.0000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 8	1.0000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 9	1.0000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
ROW 10	1.0000E+02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

RIGID BODY CONTROL SURFACE MODES

MODE 1

-7.5000E+00 -2.2500E+01

FLEXIBLE MODES

MODE 1 - FREQUENCY = 45.000 CPS
STRUCTURAL DAMPING COEFFICIENT = .030

1.0000E+00 0.11195E-01 -1.2242E-01 -5.32567E-01 -7.5620E-01 -5.32667E-01 -1.2242E-01

MODE 2 - FREQUENCY = 125.000 CPS
STRUCTURAL DAMPING COEFFICIENT = .050

1.0000E+00 -1.50071E-01 -9.10002E-01 -9.0021E-01 -6.19003E-01 9.90215E-01 9.10002E-01

MODE 3 - FREQUENCY = 240.200 CPS
STRUCTURAL DAMPING COEFFICIENT = .080

-0.03195E-01 6.45103E-01 9.9999E-01 1.05906E-01 -9.47093E-01 1.05906E-01 1.0000E+00

RIGID BODY MODES

MODE 1

1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00

MODE 2

-5.2500E+01 -3.7500E+01 -2.2500E+01 -7.5000E+00 7.5000E+01 2.2500E+01 3.7500E+01 5.2500E+01

CONTROL SYSTEM DESCRIPTION

DIFFERENTIAL EQUATION FOR	VARIABLE	COEFFICIENTS		GYRO/ACCEL. GAIN FACTOR
		2ND ORDER	1ST ORDER	
CONTROL SURF. 1	CONTROL SURF. 1	0.	-1.0000E-01	0.
	SERVO ELEMENT 3	-0.	1.0000E-02	1.0000E+00
SERVO ELEMENT 1	SERVO ELEMENT 1	-0.	-0.	1.0000E+00
	SERVO ELEMENT 4	-0.	-0.	1.0000E+00
SERVO ELEMENT 3	SERVO ELEMENT 3	-0.	-0.	-1.0000E+00
	SERVO ELEMENT 1	-0.	-0.	1.0000E+00
	SERVO ELEMENT 2	-0.	-0.	-1.0000E+00
SERVO ELEMENT 4 (RATE GYRO)	SERVO ELEMENT 4	7.0030E-04	3.7150E-03	1.0000E+00
	BODY ANGULAR RATE			-0.3300E+00
SERVO ELEMENT 2	SERVO ELEMENT 2	-0.	-0.	1.0000E+00
	CONTROL SURF. 1	-0.	-0.	-1.0000E+00

INPUT DIFFERENTIATION/INTERPOLATION ROW VECTOR

SERVO ELEMENT 4	0.	0.	1.0000E+00	-2.7000E+01	2.7000E+01	-1.0000E+00	0.	0.
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SERVO ELEMENT	MAXIMUM ORDER	ROW ASSIGNMENT IN A AND/OR B	COLUMN ASSIGNMENT IN A AND/OR B	EIGENVECTOR ELEMENT	
				VELOCITY	DISPLACEMENT

1	0	7	10	16	16
2	0	8	17	17	17
3	1	9	15	15	15
4	2	10	7 AND 14	7	14

[illegible]

[illegible]

STABILITY ANALYSIS RESULTS

MODE	EIGENVALUE-R MU-RPS	EIGENVALUE-I ONE-GA-RPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	-4.4039071E+00	0.	0.	7.707576E+01		1.432220E+01
2	-5.613021E+07	0.	0.	0.934355E+00		1.23470E+00
3	5.613001E+07	0.	0.	0.934641E+00		-1.230720E+00
4	-1.000000E+30	0.	0.	1.501549E+37		6.931472E+39
5	-1.000000E+30	0.	0.	1.501549E+37		6.931472E+39
6	-3.339551E+13	6.53710E+10	1.040412E+10	1.040412E+10	5.1006410E+04	2.075570E+12
7	-5.960559E+13	1.554209E+00	2.473601E+09	2.473601E+09	3.040255E+05	1.161331E+12
8	-2.605417E+02	1.000656E+02	3.005009E+01	5.12151E+01	0.094506E+01	2.660400E+03
9	-1.430040E+00	3.330590E+02	5.300170E+01	5.300080E+01	5.815210E+03	3.578752E+01
10	1.349714E+02	0.366972E+02	1.330053E+02	1.347289E+02	-1.594415E+01	-5.135510E+03
11	-2.037275E+02	1.537013E+01	2.447505E+02	2.443239E+02	1.090276E+01	2.60271E+03

EIGENVECTORS FOR MODES REQUESTED

EIGENVECTION FOR MODE 1

1.2676392E-03	0.	-9.2141874E-05	0.	-1.474987E-05	0.	1.0000000E+00	0.
7.0433134E-02	0.	6.875001E-01	0.	-1.0423449E-01	0.	-2.611002E-04	0.
1.9038472E-05	0.	3.8477044E-06	0.	-2.062561E-01	0.	-1.4635901E-02	0.
-1.4205511E-01	0.	2.1620207E-02	0.	1.2834490E-01	0.	-2.1620207E-02	0.
-1.4205511E-01	0.						

EIGENVECTION FOR MODE 2

1.0000000E+00	0.	-5.6422524E-01	0.	-3.5508449E-01	0.	2.3109830E-01	0.
1.4426596E-02	0.	1.5943574E-01	0.	-6.0421344E-06	0.	-1.7813815E-08	0.
1.0006032E-08	0.	6.3254273E-09	0.	-4.1309950E-09	0.	-2.9262834E-10	0.
-2.0401594E-09	0.	7.2897251E-14	0.	-5.1896096E-04	0.	-5.4638296E-08	0.
-3.0753901E-09	0.						

EIGENVECTION FOR MODE 3

1.0000000E+00	0.	-5.6422462E-01	0.	-3.5508402E-01	0.	2.3109845E-01	0.
1.4426401E-02	0.	1.5943584E-01	0.	4.091868E-06	0.	1.7813245E-08	0.
-1.0006297E-08	0.	-6.3251401E-09	0.	4.1308636E-09	0.	2.9261106E-10	0.
2.4400492E-09	0.	7.2898174E-14	0.	5.1894421E-04	0.	5.4636532E-08	0.
3.0753086E-09	0.						

EIGENVECTION FOR MODE 4

1.0000000E+00	0.	-5.6422493E-01	0.	-3.5508475E-01	0.	2.3109842E-01	0.
1.4426594E-02	0.	1.5943582E-01	0.	4.714480E-13	0.	-8.8817842E-16	0.
-1.1102230E-16	0.	0.	0.	-2.3109842E-39	0.	-1.46426599E-40	0.
-1.5943582E-39	0.	-5.7144800E-51	0.	-2.6577951E-38	9.	4.6773382E-51	0.
2.6577951E-38	0.						

EIGENVECTION FOR MODE 5

1.0000000E+00	0.	-5.6422493E-01	0.	-3.5508475E-01	0.	2.3109842E-01	0.
1.4426594E-02	0.	1.5943582E-01	0.	4.714480E-13	0.	-8.8817842E-16	0.
-1.1102230E-16	0.	0.	0.	-2.3109842E-39	0.	-1.46426599E-40	0.
-1.5943582E-39	0.	-4.7144800E-51	0.	-2.6577951E-38	0.	4.6773382E-51	0.
2.6577951E-38	0.						

EIGENVECTOR FOR MODE 6

9.2509531E-12 -1.0116370E-08 -6.684405E-14 1.3053395E-10 2.550105E-20 2.0145030E-23 -3.3395511E-13 6.5371010E-10
 2.4062440E-15 -4.7101700E-12 2.4665000E-20 2.5201626E-23 2.6512523E-20 2.7000455E-23 1.7560639E-22 1.7950274E-25
 -6.6003200E-26 -6.0050000E-24 -9.3429140E-30 -9.5571854E-33 1.0000000E-00 0. -7.2053001E-03 -4.5070604E-12
 -7.2150547E-16 1.4124802E-12 7.2150612E-16 -1.4124802E-12 -1.5302372E-22 -1.5002042E-25 -7.2150612E-16 1.4124802E-12
 -1.2150547E-16 1.4124802E-12

EIGENVECTOR FOR MODE 7

1.6541127E-11 -6.3072052E-07 -1.1923200E-13 3.1034710E-04 1.4404540E-17 1.1111400E-21 1.1111400E-13 1.5442041E-04
 4.1022134E-15 -1.1114054E-10 1.3942000E-17 1.0710924E-21 1.4404540E-17 1.1111400E-21 1.1111400E-13 1.5442041E-04
 -3.7640147E-23 -2.0910042E-27 -5.2011707E-21 -5.5770195E-31 1.0000000E-00 0. -7.2053001E-03 -1.0450300E-10
 -1.2900500E-15 3.3502191E-11 1.2901450E-15 -3.3502191E-11 -0.7006970E-20 -4.7663296E-25 -1.2901450E-15 3.3502191E-11
 -1.2900500E-15 3.3502191E-11

EIGENVECTOR FOR MODE 8

-1.4195074E-01 1.3521946E-01 5.6706265E-03 -2.4401306E-02 4.9503911E-04 -4.1625411E-03 -6.2647192E-02 1.4573352E-02
 -4.4375745E-03 1.0323412E-03 -4.3070743E-02 1.0019942E-02 1.0000000E-00 0. 6.0379711E-04 -8.1304370E-05
 -5.0722309E-05 5.1052247E-05 -0.0309764E-06 9.5041501E-06 1.0420208E-04 7.7593040E-05 1.3047901E-05 5.4961377E-04
 1.2664200E-04 5.3344769E-05 -2.5100426E-03 -1.0230084E-03 2.3893907E-03 1.7702337E-03 2.5100426E-03 1.0230084E-03
 1.2664200E-04 5.3344769E-05

EIGENVECTOR FOR MODE 9

1.0000000E-00 0. 3.5372026E-02 -2.5524397E-03 4.7371022E-03 -3.1702730E-04 6.4751727E-02 -3.9623410E-03
 4.5070309E-03 -2.0066502E-04 4.4522436E-02 -2.7241093E-03 -7.3801112E-01 -1.6062235E-01 -1.7459707E-05 -3.0023690E-03
 -0.1113490E-06 -1.0315025E-04 -1.0345433E-06 -1.4217230E-05 -1.1026960E-05 -1.9433945E-04 -9.2275096E-07 -1.3767007E-05
 -0.9501326E-06 -1.3362526E-04 -4.0935105E-04 2.2211908E-03 4.7030319E-04 -2.0075736E-03 4.6934705E-04 -2.2211908E-03
 -0.9501326E-06 -1.3362526E-04

EIGENVECTOR FOR MODE 10

-4.0477006E-01 2.3150711E-02 7.3080496E-01 4.5016511E-01 -4.2091772E-02 2.9544420E-02 -0.3070314E-05 9.0360110E-03
 -5.3304703E-03 0.4010324E-04 -5.7638500E-02 6.2127023E-03 1.0000000E-00 0. -4.9200314E-05 4.7640226E-04
 5.0720711E-04 -3.0099240E-04 2.6526404E-03 5.4651470E-05 -5.2517023E-06 9.9468770E-05 -3.7201901E-07 7.0450236E-06
 -3.0100462E-06 0.0307475E-05 1.00334792E-04 -1.1661861E-03 -1.8473707E-04 1.0977906E-03 -1.00334792E-04 1.1661861E-03
 -3.0100462E-06 0.0307475E-05

EIGENVECTOR FOR MODE 11

-3.6203950E-01 2.3971351E-02 2.6052010E-01 -3.6601056E-02 5.4063012E-02 -0.0153300E-01 -0.1490000E-02 4.9114054E-03
 -5.7123753E-03 3.4790465E-04 -5.0026420E-02 3.3767395E-03 1.0000000E-00 0. 5.4450010E-05 2.2660732E-04
 -5.1344755E-05 -1.0000935E-04 -3.1912567E-04 1.7500560E-05 1.930551E-05 5.0944817E-05 0.4510727E-07 3.6080952E-06
 0.2025099E-06 3.5025042E-05 -1.0033277E-04 -6.3169570E-04 1.0013020E-04 5.9666906E-04 1.0033277E-04 6.3169570E-04
 0.2025099E-06 3.5025042E-05

STRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR EACH MODE

DEFLECTIONS FOR MODE 1

3.151301E-01	0.	1.919966E-01	0.	6.886141E-02	0.	-5.428889E-02	0.
-1.774596E-01	0.	-3.886586E-01	0.	-4.238914E-01	0.	-5.471588E-01	0.
-7.253279E-02	0.	1.000000E+00	0.				

DEFLECTIONS FOR MODE 2

-3.127800E-01	0.	4.119731E-01	0.	3.516115E-01	0.	-3.588865E-01	0.
-6.106411E-01	0.	1.958277E-01	0.	1.000000E+00	0.	-3.571826E-01	0.
-8.541344E-01	0.	5.259453E-01	0.				

DEFLECTIONS FOR MODE 3

-3.127641E-01	0.	4.119741E-01	0.	3.516095E-01	0.	-3.588862E-01	0.
-6.106277E-01	0.	1.958289E-01	0.	1.000000E+00	0.	-3.571924E-01	0.
-8.541427E-01	0.	5.259551E-01	0.				

DEFLECTIONS FOR MODE 4

1.000000E+00	0.	3.467432E-01	0.	-2.100268E-01	0.	-5.834169E-01	0.
-7.187416E-01	0.	-6.256210E-01	0.	-3.632801E-01	0.	-7.616667E-03	0.
3.422701E-01	0.	7.777778E-01	0.				

DEFLECTIONS FOR MODE 5

1.000000E+00	0.	3.467432E-01	0.	-2.100268E-01	0.	-5.834169E-01	0.
-7.187416E-01	0.	-6.256210E-01	0.	-3.632801E-01	0.	-7.616667E-03	0.
3.422701E-01	0.	7.777778E-01	0.				

DEFLECTIONS FOR MODE 6

1.000000E+00	0.	4.215836E-01	-3.686243E-11	0.631617E-01	-7.212481E-11	7.647510E-01	-1.001873E-10
0.863474E-01	-1.442497E-10	6.879184E-01	-1.003121E-10	5.295021E-01	-2.163746E-10	4.5188580E-01	-2.524378E-10
3.726694E-01	-2.961856E-10	2.942531E-01	-3.476204E-10				

DEFLECTIONS FOR MODE 7

1.0000000E+00 0. 4.2150369E-01 -0.5739169E-10 0.4316737E-01 -1.7147834E-09 7.6475106E-01 -2.5721751E-09
 6.8633474E-01 -3.4295668E-09 6.0791843E-01 -4.2869585E-09 5.2950211E-01 -5.1443501E-09 4.5108580E-01 -6.0017416E-09
 3.7266948E-01 -7.0418734E-09 2.9425317E-01 -8.2667449E-09

DEFLECTIONS FOR MODE 8

1.0286034E-01 1.8022370E-01 1.2109424E-01 7.1235139E-02 1.2436215E-01 -3.0023243E-02 9.2804327E-02 -1.1449656E-01
 5.8435712E-03 -1.0419500E-01 -1.5145339E-01 -1.6591030E-01 -3.6376126E-01 -7.3970086E-02 -6.0238045E-01 1.3064094E-01
 -2.2940151E-01 1.7049621E-01 1.0000000E+00 0.

DEFLECTIONS FOR MODE 9

1.0000000E+00 0. 3.5225579E-01 4.3585593E-03 -2.1929779E-01 7.2374615E-03 -6.2690844E-01 6.9412312E-03
 -7.9101685E-01 2.3698023E-03 -6.7686704E-01 -6.1501232E-03 -3.0580022E-01 -1.6842565E-02 2.5624155E-01 -2.7475785E-02
 5.3811477E-01 -8.9477464E-03 4.7229743E-01 3.8509085E-02

DEFLECTIONS FOR MODE 10

-7.8829154E-01 3.8698691E-01 9.4574661E-02 -2.0175162E-01 7.1539095E-01 -4.9052319E-01 8.3027476E-01 -3.2848402E-01
 4.2266022E-01 1.3234370E-01 -2.9418074E-01 5.1720405E-01 -8.6911245E-01 4.7410691E-01 -9.1188355E-01 -1.2743405E-01
 -2.0783231E-01 -3.6248849E-01 1.0000000E+00 0.

DEFLECTIONS FOR MODE 11

-7.3720253E-01 8.0522790E-02 6.1118665E-01 2.2588805E-02 9.2680558E-01 -9.1345116E-02 6.9445662E-02 -1.8962641E-01
 -9.2142989E-01 -1.1274572E-01 -8.9602726E-01 1.5609706E-01 1.3352506E-01 3.3447848E-01 1.0000000E+00 0.
 6.3169984E-01 -1.9233059E-01 -8.1800310E-01 -7.6385779E-03

MINIMUM BLANK COMMON LENGTH REQUIRED = 2106.
 BASED ON INPUT DATA AND ANALYSIS REQUESTED.

SAMPLE PROBLEM - CASE 3 - PROGRAM MPABLS

MISSILE AND SERVO SYSTEM - FLEX, BODY IN INCOMPRESS, AINSTREAM AT S, L,

AERO-SERVO-ELASTIC STABILITY ANALYSIS

10 DEGREES OF FREEDOM

1 FLEXIBLE MODES

2 RIGID BODY MODES

1 CONTROL SURFACES

4 SERVO ELEMENTS

SHIFT EIGENVALUE (GAMMA) = 1.000E+00

17 MODES REQUESTED

1 ALTITUDE VARIATIONS

AIC MATRICES FOR 1 MACH NUMBERS AND 5 REDUCED FREQUENCIES

REFERENCE SEMI-CHORD = 1.25000E+00 FT

SEMI-SPAN = 1.66670E+00 FT

SURFACE AREA = 1.0070E+00 SQ FT

MACH NUMBER DEVIATION = 0.0

UPPER TRIANGLE OF WEIGHT MATRIX

ROW 1	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.
ROW 2	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.
ROW 3	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.
ROW 4	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.
ROW 5	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.
ROW 6	1.00000E+02	0.	0.	0.	0.	0.	0.	0.	0.

MODE 7	1.00000E+02	0.	0.	0.
MODE 8	1.00000E+02	0.	0.	0.
MODE 9	1.00000E+02	0.	0.	0.
MODE 10	1.00000E+02	0.	0.	0.

RIGID BODY CONTROL SURFACE MODES

MODE 1	0.	0.	0.	0.	0.	0.
MODE 2	0.	0.	0.	0.	0.	0.

FLEXIBLE MODES

MODE 1 - FREQUENCY = 45.000 CPS						
STRUCTURAL DAMPING COEFFICIENT = .030						
1.00000E+00	4.11195E-01	-1.22424E-01	-5.32547E-01	-7.56204E-01	-5.32467E-01	-1.22424E-01
4.11195E-01	1.00000E+00					
MODE 2 - FREQUENCY = 125.400 CPS						
STRUCTURAL DAMPING COEFFICIENT = .050						
1.00000E+00	-1.50071E-01	-9.10442E-01	-9.90214E-01	-9.19043E-01	9.90215E-01	9.10442E-01
-1.50071E-01	1.00000E+00					
MODE 3 - FREQUENCY = 248.200 CPS						
STRUCTURAL DAMPING COEFFICIENT = .080						
-8.03195E-01	4.45103E-01	9.99999E-01	1.05986E-01	-9.47894E-01	-9.47893E-01	1.05986E-01
4.45103E-01	-8.03195E-01					

RIGID BODY MODES

MODE 1

1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00 1.0000E+00

MODE 2

5.2500E+01 3.7500E+01 2.2500E+01 7.5000E+00 7.5000E+00 3.7500E+01 5.2500E+01

CONTROL SYSTEM DESCRIPTION

DIFFERENTIAL EQUATION FOR	VARIABLE	2ND ORDER	COEFFICIENTS 1ST ORDER	0 ORDER	GYRO/ACCEL, GAIN FACTOR
CONTROL SURF, 1	CONTROL SURF, 1	0.	-1.0670E-01	0.	
	SERVO ELEMENT 3	-0.	1.0000E-02	1.0000E+00	
SERVO ELEMENT 1	SERVO ELEMENT 1	-0.	-0.	1.0000E+00	
	SERVO ELEMENT 4	-0.	-0.	1.0000E+00	
SERVO ELEMENT 3	SERVO ELEMENT 3	-0.	-0.	-1.0000E+00	
	SERVO ELEMENT 1	-0.	-0.	1.0000E+00	
	SERVO ELEMENT 2	-0.	-0.	-1.0000E+00	
SERVO ELEMENT 4 (RATE GYRO)	SERVO ELEMENT 4	7.0030E-06	3.7150E-03	1.0000E+00	
	BCDY ANGULAR RATE				-5.3500E-04
SERVO ELEMENT 2	SERVO ELEMENT 2	-0.	-0.	1.0000E+00	
	CONTROL SURF, 1	-0.	-0.	-1.0000E+00	

GYRO/ACCEL. INPUT DIFFERENTIATION/INTERPOLATION ROW VECTOR

SERVO ELEMENT 4 0. 0. 1.0000E+00 -2.7000E+01 2.7000E+01 -1.0000E+00 0. 0.

SERVO ELEMENT	MAXIMUM ORDER	ROW ASSIGNMENT IN A AND/OR B	COLUMN ASSIGNMENT IN A AND/OR B	EIGENVECTOR ELEMENT	VELOCITY	DISPLACEMENT
1	0	7	14			14
2	0	8	17			17
3	1	9	15			15
4	2	10	7 AND 14	7		14

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

A MATRIX

[illegible]

EIGENVECTORS FOR MODES REQUESTED

EIGENVECTORS FOR MODE 1

1.1342527E-04	0.	1.6629100E-06	0.	3.1421304E-06	0.	1.0000000E-00	0.
2.8747546E-02	0.	5.2630719E-02	0.	-3.3214361E-02	0.	-5.2617907E-05	0.
-7.6083998E-07	0.	-1.4512349E-06	0.	-4.6106336E-01	0.	-2.343432E-02	0.
-2.4308201E-02	0.	1.5368566E-02	0.	0.9677845E-03	0.	-1.5340496E-02	0.
-2.4308201E-02	0.						

EIGENVECTORS FOR MODE 2

1.0000000E-00	0.	-5.6622493E-01	0.	-3.5504475E-01	0.	2.3149842E-01	0.
1.6426594E-02	0.	1.5943582E-01	0.	1.3966315E-11	0.	-6.6613361E-16	0.
-2.5873828E-16	0.	1.1537001E-17	0.	-2.3189842E-39	0.	4.4100090E-16	0.
-1.5943582E-19	0.	-1.3966315E-49	0.	-2.6577951E-38	0.	1.3981900E-49	0.
2.6577951E-38	0.						

EIGENVECTORS FOR MODE 3

1.0000000E-00	0.	-5.6622493E-01	0.	-3.5504475E-01	0.	2.3189842E-01	0.
1.6426594E-02	0.	1.5943582E-01	0.	1.3966315E-11	0.	-6.6613361E-16	0.
-2.5873828E-16	0.	1.1537001E-17	0.	-2.3189842E-39	0.	4.4100090E-16	0.
-1.5943582E-19	0.	-1.3966315E-49	0.	-2.6577951E-38	0.	1.3981900E-49	0.
2.6577951E-38	0.						

EIGENVECTORS FOR MODE 4

1.5666149E-11	1.3234147E-10	5.9191474E-13	-3.1165894E-11	2.1016862E-13	-1.1011013E-11	-8.9302044E-05	1.4056429E-02
3.761667E-08	-2.3886031E-06	-1.0035549E-08	-1.9743494E-10	1.0034940E-08	2.2094750E-10	1.0044404E-08	1.2887895E-10
-2.2174824E-09	-2.621862E-11	-7.8340785E-10	-9.9742650E-12	1.0000000E-00	0.	-1.8938966E-04	-1.5856003E-06
-9.5897130E-09	7.1400767E-07	1.1192645E-04	-7.1397509E-07	-1.6729321E-09	-3.2477281E-11	-1.1182645E-08	7.1397509E-07
-4.5897130E-09	7.1400767E-07						

EIGENVECTORS FOR MODE 5

-3.2828639E-03	-7.5781605E-04	-1.9810102E-06	-8.44071092E-05	-1.5777230E-04	-5.3625593E-05	1.0000000E-00	0.
4.8870363E-02	-6.7296406E-04	-8.8141720E-02	-3.6197312E-02	-1.4651626E-02	1.8993210E-01	-2.1613479E-05	2.4971920E-04
-4.1277164E-06	1.5474474E-05	-2.3861151E-06	1.2186630E-05	-1.0428429E-02	-7.3062033E-02	-5.3798121E-04	-3.4455536E-03
-1.9503010E-03	5.3964496E-03	1.4143461E-02	-8.990547E-04	-1.2187159E-02	-4.4967441E-03	-1.4143461E-02	0.9990547E-04
-1.9563816E-03	5.3964496E-03						

EIGENVECTORS FOR MODE 6

-1.4150340E-01	1.3504809E-01	5.55487194E-03	-2.4348810E-02	4.5303893E-04	-4.1494879E-03	-6.2580589E-02	1.4404512E-02
-4.4308874E-03	1.0274177E-03	-4.368764E-02	1.0026032E-02	1.0000000E-00	0.	6.0230233E-04	-8.171829E-05
-5.8394021E-05	5.1134605E-05	-8.7846195E-06	9.6154713E-06	1.4376681E-04	7.7489504E-05	1.3021690E-05	5.4948765E-06
1.2465557E-04	5.3348874E-05	-2.5164054E-03	-1.8239865E-03	2.3887494E-03	1.7706457E-03	2.5164054E-03	1.8239865E-03
1.2465557E-04	5.3348874E-05						

EIGENVECTORS FOR MODE 7

1.0000000E+00	6.	3.4332129E-02	-2.5507323E-03	4.7302692E-03	-3.4180420E-04	0.4642836E-02	-3.7950704E-03
4.500591E-03	-2.734202E-04	4.500591E-04	-2.734202E-03	-7.3060052E-01	-1.0046620E-01	-1.7721210E-05	-3.0030393E-03
-8.2603564E-06	-1.030555E-04	-1.1105643E-05	-1.4217144E-05	-1.2544121E-05	-1.9405772E-04	-9.0370662E-07	-1.3750775E-05
-9.6401304E-06	-1.3376791E-04	-5.067793E-04	2.2230917E-03	4.7702007E-04	-2.0901230E-03	4.6077993E-04	-2.2230917E-03
-9.6601364E-06	-1.3376791E-04						

EIGENVECTORS FOR MODE 8

-4.0417704E-01	2.3500107E-02	3.3080152E-01	4.5012142E-01	-4.2014120E-02	2.9601831E-02	-8.3644049E-02	9.0833510E-03
-5.9271000E-03	6.4479931E-04	-5.7632777E-02	6.2150324E-03	1.0000000E+00	0.	-4.8542644E-05	4.7595805E-04
5.8740025E-04	-3.0116423E-04	2.6623073E-02	5.4507591E-05	-5.1530110E-06	9.9294719E-05	-3.637593E-07	7.0300077E-04
-3.6019497E-06	6.0411457E-05	1.8028554E-04	-1.1666077E-03	-1.0468359E-04	1.0901962E-03	-1.6028554E-04	1.1666077E-03
-3.6019497E-06	6.0411457E-05						

EIGENVECTORS FOR MODE 9

-3.6313345E-01	2.4065906E-02	2.0076350E-01	-3.6609237E-02	5.5006237E-02	-6.0155314E-01	-0.1544694E-02	4.9430930E-03
-5.7766774E-03	3.4975294E-04	-5.0025304E-02	3.3760470E-03	1.0000000E+00	0.	5.4574415E-05	2.2677417E-04
-5.1436254E-05	-1.0074274E-04	-3.1015700E-04	1.7427940E-05	1.1964375E-05	5.0973943E-05	0.4726351E-07	3.6110780E-06
0.2075271E-06	3.5024236E-05	-1.0042430E-04	-6.3107700E-04	1.0021078E-04	5.5665364E-04	1.0042430E-04	6.3107700E-04
0.2075271E-06	3.5024236E-05						

STRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR EACH MODE

DEFLECTIONS FOR MODE 1

-4.1205554E-01	0.	-2.2402234E-01	0.	-3.5107275E-02	0.	1.5365059E-01	0.
3.4249107E-01	0.	5.3134440E-01	0.	7.2020197E-01	0.	9.0900440E-01	0.
1.0000000E+00	0.	9.9300706E-01	0.				

DEFLECTIONS FOR MODE 2

-6.0970074E-01	0.	-4.0613407E-01	0.	-2.6674599E-01	0.	-7.4940050E-02	0.
1.0007004E-01	0.	2.8657349E-01	0.	4.6250369E-01	0.	6.3977149E-01	0.
0.1916611E-01	0.	1.0000000E+00	0.				

DEFLECTIONS FOR MODE 3

-6.6970074E-01	0.	-4.0613407E-01	0.	-2.6674599E-01	0.	-7.4940050E-02	0.
1.0007004E-01	0.	2.8657349E-01	0.	4.6250369E-01	0.	6.3977149E-01	0.
0.1916611E-01	0.	1.0000000E+00	0.				

DEFLECTIONS FOR MODE 4

1.0000000E+00	0.	9.4748155E-01	-2.3366610E-05	9.9496411E-01	-4.6733220E-05	9.9244666E-01	-7.0099830E-05
9.0992622E-01	-9.3400425E-05	9.4748155E-01	-2.3366610E-05	9.9496411E-01	-4.6733220E-05	9.9244666E-01	-7.0099830E-05
9.7905253E-01	-1.9224042E-04	9.7733624E-01	-2.2622250E-04	9.8400733E-01	-1.4019957E-04	9.8237090E-01	-1.6356600E-04

DEFLECTIONS FOR MODE 5

2506131E-01 3.2076422E-02 -1.1655840E-01 1.7709453E-02 -8.0700742E-03 3.3416042E-03 1.0036432E-01 -1.1032736E-02
 -0.008441E-01 -2.5422970E-02 3.1605401E-01 -3.9834800E-02 4.2488067E-01 -5.4261007E-02 5.3285139E-01 -6.8691747E-02
 7.2454773E-01 -5.0604832E-02 1.0000000E+00 0.

DEFLECTIONS FOR MODE 6

1.0220909E-01 1.7901005E-01 -1.2043049E-01 7.1100922E-02 -1.2379274E-01 -2.9792633E-02 -9.2460324E-02 -1.1412750E-01
 5.8955712E-03 -1.6791834E-01 -1.5044804E-01 -1.6604437E-01 -3.8254593E-01 -7.5100622E-02 -6.8013156E-01 1.2769904E-01
 -2.2750410E-01 1.6780364E-01 1.0000000E+00 0.

DEFLECTIONS FOR MODE 7

1.0000000E+00 0. 3.5331050E-01 4.3347907E-03 -2.1910501E-01 7.2190099E-03 -6.2679355E-01 0.9609362E-03
 -7.9100224E-01 2.4748087E-03 -6.1702902E-01 -5.9416503E-03 -3.6618271E-01 -1.6480374E-02 2.5562791E-01 -2.6070150E-02
 5.3702780E-01 -7.8946074E-03 4.7049592E-01 4.0175078E-02

DEFLECTIONS FOR MODE 8

-7.8843667E-01 3.9029301E-01 -9.4809727E-02 -2.0222521E-01 7.1485620E-01 -4.9362029E-01 -8.3971770E-01 -3.3202221E-01
 4.2359733E-01 1.3072244E-01 -2.9247444E-01 5.1869580E-01 -8.6070930E-01 4.7411844E-01 -9.1400002E-01 -1.2319309E-01
 -2.1023545E-01 -3.5993270E-01 1.0000000E+00 0.

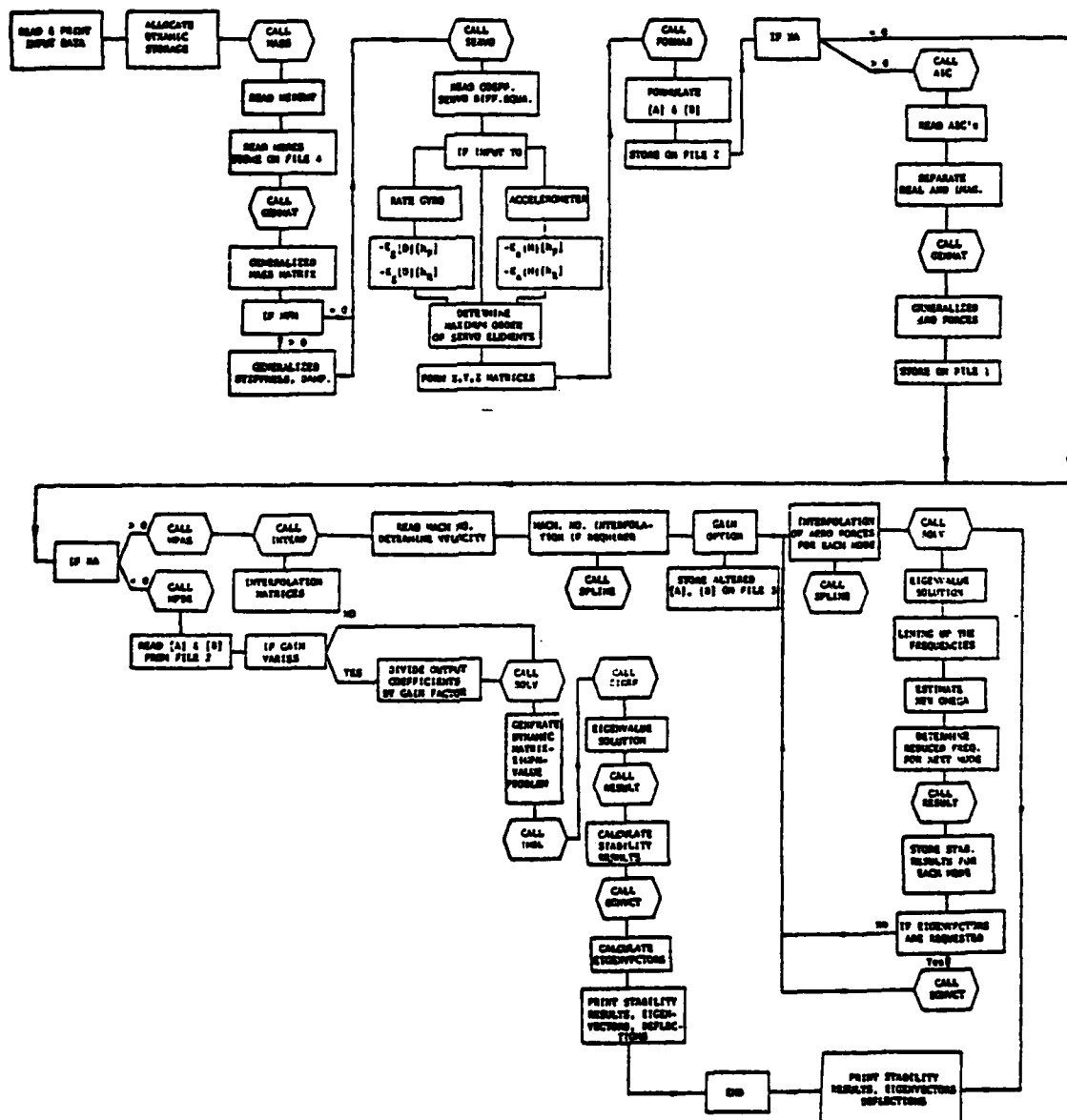
DEFLECTIONS FOR MODE 9

-7.3720154E-01 0.0604275E-02 0.1124510E-01 2.2545943E-02 9.2681927E-01 -9.1539531E-02 6.9330120E-02 -1.0983151E-01
 -9.2158037E-01 -1.1270127E-01 -8.9605228E-01 1.5637495E-01 1.3364941E-01 3.3486304E-01 1.0000000E+00 0.
 6.3141208E-01 -1.9320035E-01 -8.1860077E-01 -9.6625770E-03

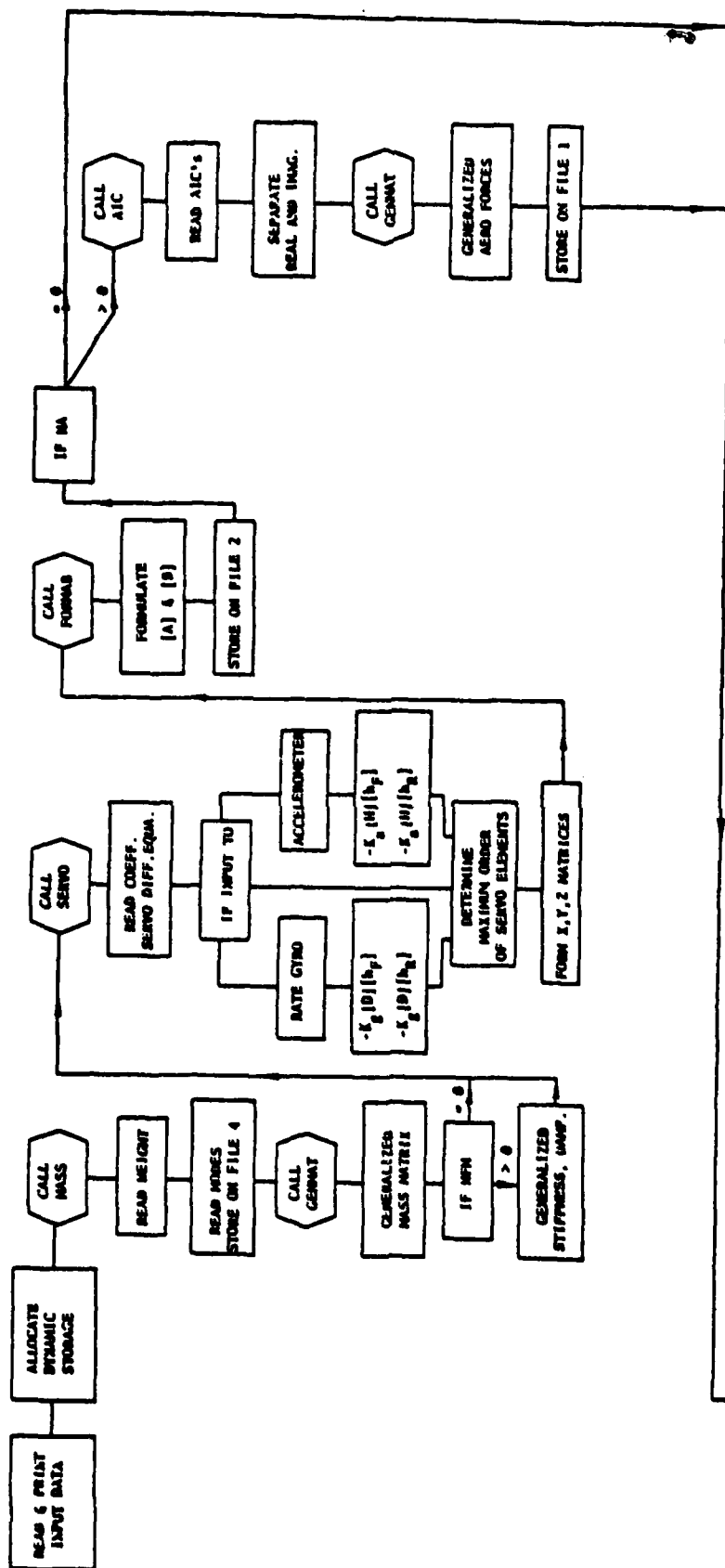
MINIMUM BLANK COMMON LENGTH REQUIRED = 2640.
 BASED ON INPUT DATA AND ANALYSES REQUESTED.

SECTION V

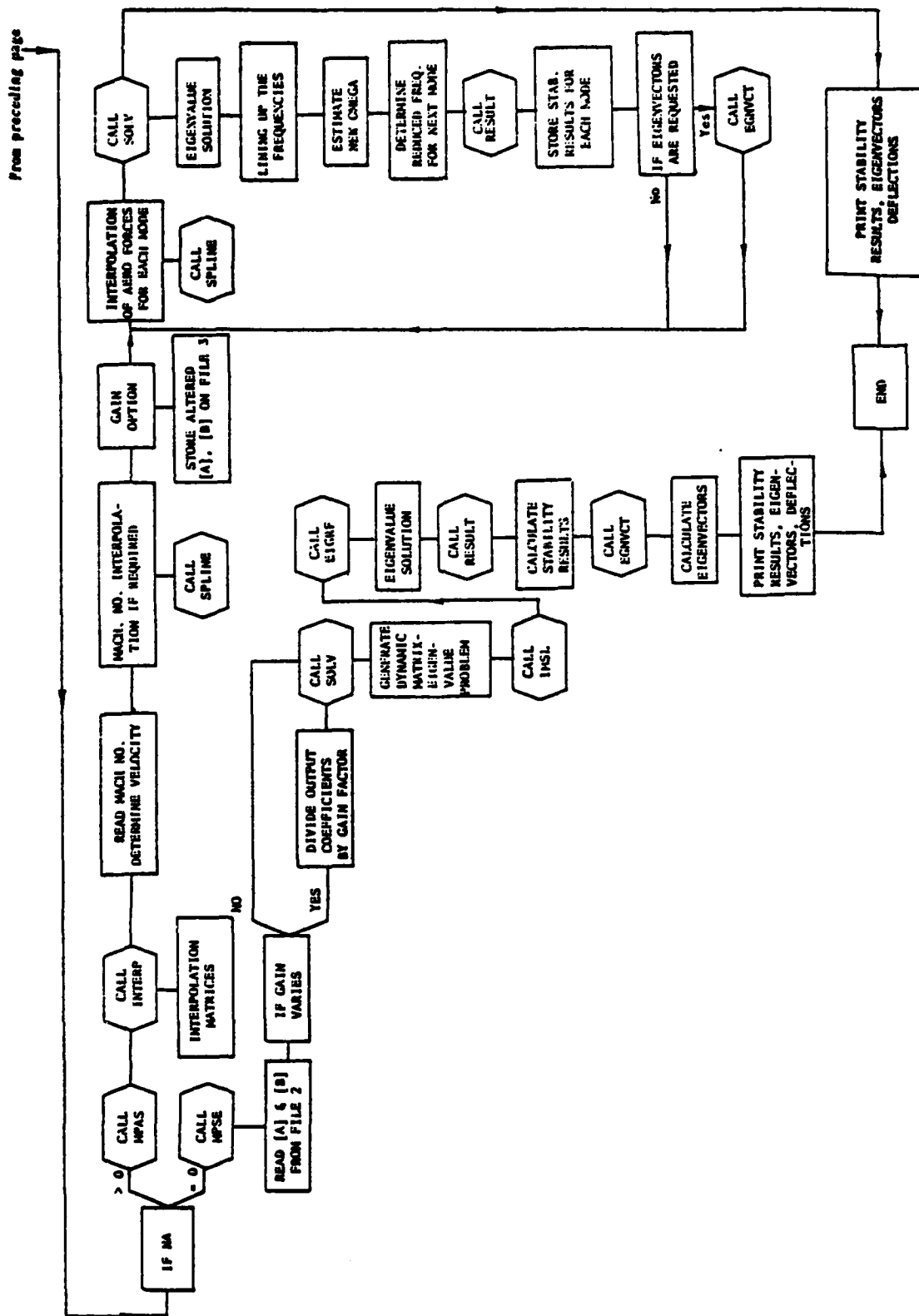
PROGRAM FLOW CHART



Note: See next two pages for larger scale.



See next page.



SECTION VI

PROGRAM LISTING

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      PROGRAM MPASP (INPUT, OUTPUT, TAPE1, TAPE2, TAPE3, TAPE4, TAPE5=INPUT, MAIN0010
      1 TAPE5=OUTPUT) MAIN0020
C----- MPASES = MODIFIED MPASES, ARNU-SERVO-ELASTIC STABILITY ANALYSIS MAIN0030
C----- DYNAMIC STIMULUS ALLOCATION USED MAIN0040
C----- INSL EIGENVALUE PACKAGE USED MAIN0050
C----- COMMON/CONTROL/ MCONR, NCONI, NZ, FL, NA, NU, NG, NBS, A1, R2, NPM, NNM, NVEC, MAIN0060
      1 NAM MAIN0070
      COMMON/INPUT/NU(5), AM(5), AR(10), UENS(5), SUS(5), GAIN(25) MAIN0080
      COMMON/ARNU/SS, GAMMA, S, OM, OM MAIN0090
      COMMON Z(10000) MAIN0100
      DIMENSION TITLE(10) MAIN0110
      EQUIVALENCE (MU(1), TITLE(1)) MAIN0120
C----- MAIN0130
      CALL INSL MAIN0140
      READ (INPUT) AND PRINT MAIN0150
      1 HEAD(5,50) TITLE MAIN0160
      IF (FUP(5)) 2, 3 MAIN0170
      2 STOP MAIN0180
      3 CONTINUE MAIN0190
      HEAD(10) 1 MAIN0200
      HEAD(10) 2 MAIN0210
      WRITE(0,50) TITLE MAIN0220
      HEAD(5,50) MU(1), NPM, NNM, NC, NSE, NA, NM, NK, NU, MUUE, NG, NUS, NAM MAIN0230
      HEAD(5,50) GAMMA MAIN0240
      NPM, NNM, NNM MAIN0250
      NPM, NNM, NNM MAIN0260
      NPM, NNM, NNM MAIN0270
      NPM, NNM, NNM MAIN0280
      IF (NA, 50, 0) GO TO 5 MAIN0290
      HEAD(5,50) SS, GAMMA, S, OM MAIN0300
      OM, GAMMA, S, OM MAIN0310
      HEAD(5,50) (AM(1), (R1, NM) MAIN0320
      HEAD(5,50) (AR(1), (R1, NM) MAIN0330
      HEAD(5,50) (UENS(1), SUS(1), (R1, NU) MAIN0340
      HEAD(5,50) (MU(1), (R1, NU) MAIN0350
      WRITE(0,50) MAIN0360
      GO TO 10 MAIN0370
      5 WRITE(0,50) MAIN0380
      10 WRITE(0,50) MU, NPM, NNM, NC, NSE, GAMMA, MUUE MAIN0390
      IF (NA, 50, 0) GO TO 15 MAIN0400
      WRITE(0,50) NU, NPM, NNM, NM, SS, S, OM MAIN0410
      15 IF (NUS, 50, 0) GO TO 25 MAIN0420
      HEAD(5,50) (GAIN(1), (R1, NG) MAIN0430
      IF (NUS, 50, 0) GO TO 20 MAIN0440
      WRITE(0,50) NU, NUS MAIN0450
      GO TO 25 MAIN0460
      20 INQUIRE(ABS(NUS)) MAIN0470
      WRITE(0,50) NU, INUS MAIN0480
C----- DYNAMIC DIMENSIONING MAIN0490
      25 DIMENSION MAIN0500
      NPM, NNM MAIN0510
      IF (NPM, 50, 0) NPM=1 MAIN0520
      NPM, NNM MAIN0530
      IF (NPM, 50, 0) NPM=1 MAIN0540
      NPM, NNM MAIN0550

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C

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SUBROUTINE MASS(FMODE,RMODE,CMODE,GENM,SK,CF,GF,FREQ,WT,TM,FRC,
1  NPM,NRM,NC,NRF,NRR,NRF,NRMIDE,-1,-2)
FORM GENERALIZED MASS MATRIX
COMMON/CONTROL/MCODE,N0,N1,N2,NTC,NA,NO,NG,NBS,N1,N2,NFM,NRM,NVEC,
1  NAB
0(MPMSTNM FMODE(MOF,NFM),RMODE(MOR,NRM),CMODE(MOF,NC),
1  GENM(NRF,NRMIDE),SK(NFM),CF(NFM),GF(NFM),FREQ(NFM),WT(MOF,MOF),
2  TM(N1,MOF),FRC(N2,N2)
C
READ HEIGHT MATRIX
RETN=0
IF(MCODE,EQ,2) GO TO 35
DO 30 I=1,MOF
DO 30 J=1,MOF
30  WT(I,J)=0
READ(5,463) (WT(I,I),I=1,MOF)
GO TO 45
35  DO 40 I=1,MOF
READ(5,501) (WT(I,J),J=1,MOF)
IF(I,FQ,MOF) GO TO 40
I=I+1
DO 30 J=1,MOF
30  WT(J,I)=WT(I,J)
40  CONTINUE
45  WRITE(6,604)
DO 50 I=1,MOF
50  WRITE(6,609) I,(WT(I,J),J=1,MOF)
DO 55 I=1,MOF
DO 55 J=1,MOF
55  WT(I,J)=WT(I,J)/32,174
C
READ MODES AND PRINT
WRITE(6,610)
DO 60 I=1,NC
READ(4,501) (CMODE(J,I),J=1,MOF)
60  WRITE(6,611) I,(CMODE(J,I),J=1,MOF)
IF(NPM,NE,0) GO TO 70
READ(5,501) (GF(I),I=1,NFM)
READ(5,503) (FREQ(I),I=1,NFM)
WRITE(6,612)
DO 65 I=1,NFM
65  READ(4,503) (FMODE(J,I),J=1,MOF)
WRITE(6,613) I,FREQ(I),GF(I),FMODE(J,I),J=1,MOF)
65  FREQ(I)*FREQ(I)=4,241853
WRITE(6,614)((FMODE(I,J),I=1,MOF),J=1,NFM)
70  IF(NRM,EQ,0) GO TO 80
WRITE(6,614)
DO 75 I=1,NRM
75  READ(4,503) (RMODE(J,I),J=1,MOF)
WRITE(6,611) I,(RMODE(J,I),J=1,MOF)
WRITE(6,614)((RMODE(I,J),I=1,MOF),J=1,NRM)
80  CALL GENMAT(WT,RETN,TM,FRC,FMODE,RMODE,CMODE,MOF,MOR,MOF,NC,NPM,
1  N1,N2)
WRITE(6,614)((CMODE(I,J),I=1,MOF),J=1,NC)
C
FORM GENERALIZED STIFFNESS AND DAMPING MATRICES
IF(NPM,EQ,0) RETURN
DO 85 I=1,NPM
85  SK(I)=FREQ(I)*FREQ(I)*GENM(I,I)
85  CF(I)=GF(I)*FREQ(I)*GENM(I,I)
RETURN
903  P11PMAT(NG12,0)
904  P11PMAT(1M0 2X,31MUPPER TRIANGLE OF HEIGHT MATRIX //)
905  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
906  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
907  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
908  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
909  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
910  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
911  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
912  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
913  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
914  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
915  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
916  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
917  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
918  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
919  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
920  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
921  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
922  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
923  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
924  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
925  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
926  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
927  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
928  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
929  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
930  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
931  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
932  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
933  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
934  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
935  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
936  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
937  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
938  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
939  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
940  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
941  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
942  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
943  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
944  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
945  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
946  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
947  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
948  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
949  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
950  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
951  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
952  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
953  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
954  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
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956  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
957  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
958  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
959  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
960  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
961  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
962  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
963  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
964  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
965  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
966  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
967  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
968  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
969  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
970  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
971  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
972  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
973  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
974  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
975  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
976  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
977  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
978  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
979  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
980  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
981  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
982  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
983  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
984  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
985  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
986  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
987  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
988  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
989  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
990  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
991  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
992  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
993  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
994  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
995  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
996  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
997  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
998  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
999  P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
1000 P11PMAT(1M0 2X,31MUPPER 13 // (3X,1PBE15,5))
END

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SUBROUTINE SERVO(FMODE,RMODE,X0,X1,X2,Y0,Y1,Z0,K0,NOR,NL,C0,C1,      SERVO010
1 C2,O,OPM,OPM,MFM,MHM,NSE,NOP,NSESV,NT,NOP,NOR,NC)      SERVO020
SERVO = SERVO DIFFERENTIAL EQUATIONS      SERVO030
COMMON/CONTROL/MODE,N0,N1,N2,NTC,NA,N0,NG,NGS,K1,K2,MFM,MHM,MVEC, SERVO040
1 NAB      SERVO050
OIMNS:(IN FMODE(MOP,MFM),RMODE(MOR,MHM),X0(NT,NT),X1(NT,NT),      SERVO060
1 X2(NT,NT),Y0(NT,NSE),Y1(NT,NSE),Z0(NT,NSE),O(1,NOP),      SERVO070
2 OPM(1,MFM),OPM(1,MHM),K0(NSE,2),NOR(NSE,NSESV),NL(NSESV,2),      SERVO080
3 C0(NSESV),C1(NSESV),C2(NSESV)      SERVO090
C      SERVO100
MFM,MHM,MHM      SERVO110
MOP,MOP,MOP+MC      SERVO120
DO 7 J=1,NT      SERVO130
DO 5 J=1,NT      SERVO140
X0(I,J)=0.0      SERVO150
X1(I,J)=0.0      SERVO160
5 X2(I,J)=0.0      SERVO170
DO 6 JJ=1,NSE      SERVO180
Y0(I,JJ)=0.0      SERVO190
Y1(I,JJ)=0.0      SERVO200
6 Z0(I,JJ)=0.0      SERVO210
7 CONTINUE      SERVO220
C      SERVO230
C READ COEFFICIENTS FROM SERVO DIFFERENTIAL EQUATIONS      SERVO240
C PRINT CONTROL SYSTEM DESCRIPTION      SERVO250
C WRITE(6,600)      SERVO260
L=0      SERVO270
IN=0      SERVO275
GU 25 I=1,NC      SERVO280
IC=FR=1      SERVO290
READ(5,503) X0(IC,IC),X1(IC,IC),X2(IC,IC)      SERVO300
WRITE(6,601) I,I,X2(IC,IC),X1(IC,IC),X0(IC,IC)      SERVO310
READ(5,502) INC      SERVO320
DO 20 J=1,INC      SERVO330
L=L+1      SERVO340
READ(5,504) K,N0,C0(L),C1(L),C2(L)      SERVO350
WRITE(6,602) K,C2(L),C1(L),C0(L)      SERVO360
DO 15 KK=1,NSE      SERVO370
IF(KK,FO,K) GO TO 10      SERVO380
NOR(KK,L)=0      SERVO390
GO TO 15      SERVO400
10 NOR(KK,L)=N0      SERVO410
15 CONTINUE      SERVO420
NL(L,1)=I      SERVO430
NL(L,2)=K      SERVO440
20 CONTINUE      SERVO450
25 CONTINUE      SERVO460
NINC=L      SERVO470
DO 115 J=1,NSE      SERVO480
L=L+1      SERVO490
READ(5,504) K,N0,C0(L),C1(L),C2(L)      SERVO500
WRITE(6,603) I,I,C2(L),C1(L),C0(L)      SERVO510
NL(L,1)=I      SERVO520
NL(L,2)=I      SERVO530
DO 35 KK=1,NSE      SERVO540
IF(KK,EO,I) GO TO 30      SERVO550
NOR(KK,L)=0      SERVO560
GO TO 35      SERVO570
30 NOR(KK,L)=N0      SERVO580
35 CONTINUE      SERVO590
READ(5,502) INS      SERVO600
IF(INS,FO,0) GO TO 115      SERVO610
DO 110 M=1,INS      SERVO620
L=L+1      SERVO630
READ(5,504) K,N0,C0(L),C1(L),C2(L)      SERVO640
IF(K,L,0) GO TO 100      SERVO650
IF(N0,L,0) GO TO 50      SERVO660
WRITE(6,602) K,C2(L),C1(L),C0(L)      SERVO670
DO 45 KK=1,NSE      SERVO680
IF(KK,EO,K) GO TO 40      SERVO690
NOR(KK,L)=0      SERVO700
GO TO 45      SERVO710
40 NOR(KK,L)=N0      SERVO720
45 CONTINUE      SERVO730
NL(L,1)=I      SERVO740
NL(L,2)=K      SERVO750
GO TO 110      SERVO760
C INPUT TO RATE GYRO OR ACCELEROMETER      SERVO770

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90	HL(L,1)=0	SENV0780
	HL(L,2)=0	SENV0790
	IF(MODE,(-1)) GO TO 95	SENV0800
	WRITE(6,604) CO(L)	SENV0810
	GO TO 88	SENV0820
95	WRITE(6,605) CO(L)	SENV0830
88	DO 89 K=1,NMF	SENV0840
85	NOR(M,L)=0	SENV0850
	INOR=1	SENV0860
	READ(5,903) (O(1,JJ),JJ=1,NOF)	SENV0870
	WRITE(1) I,(O(1,JJ),JJ=1,NOF)	SENV0880
	DO 78 JJ=1,NOF	SENV0890
78	O(1,JJ)=CO(L)+O(1,JJ)	SENV0900
	IF(MMF,FO,0) GO TO 85	SENV0910
	CALL MATMPL(D,MNOE,OFM,1,NOF,1,1,NMF,NOF,1)	SENV0920
	DO 88 JJ=1,NMF	SENV0930
	IF(MN,EO,(-1)) GO TO 75	SENV0940
	X2(IH,JJ)=X2(IH,JJ)+OFM(1,JJ)	SENV0950
	GO TO 88	SENV0960
75	X1(IH,JJ)=X1(IH,JJ)+OFM(1,JJ)	SENV0970
88	CONTINUE	SENV0980
85	IF(MMF,C9,0) GO TO 110	SENV0990
	CALL MATMPL(D,MNOE,OFM,1,NOF,1,1,NMF,NOF,1)	SENV1000
	DO 95 JJ=1,NMF	SENV1010
	IC=MF+JJ	SENV1020
	IF(MN,EO,(-1)) GO TO 98	SENV1030
	X2(IH,IC)=X2(IH,IC)+OFM(1,JJ)	SENV1040
	GO TO 95	SENV1050
98	X1(IH,IC)=X1(IH,IC)+OFM(1,JJ)	SENV1060
95	CONTINUE	SENV1070
	GO TO 110	SENV1080
C	INPUT FROM CONTROL SURFACE	SENV1090
100	HL(L,1)=0	SENV1100
	HL(L,2)=0	SENV1110
	IAH=IABS(N)	SENV1120
	WRITE(6,606) IAH,C2(L),C1(L),CO(L)	SENV1130
	DO 104 K=1,NMF	SENV1140
103	NOR(M,L)=0	SENV1150
	INOR=1	SENV1160
	IC=MF+IAH(N)	SENV1170
	X2(IH,IC)=CO(L)	SENV1180
	X1(IH,IC)=C1(L)	SENV1190
	X2(IH,IC)=C2(L)	SENV1200
110	CONTINUE	SENV1210
115	CONTINUE	SENV1220
	IF(MN,FO,0) GO TO 118	SENV1230
	ENOFIL=1	SENV1240
	RETN=1	SENV1250
	WRITE(6,610)	SENV1260
116	READ(1) I,(O(1,JJ),JJ=1,NOF)	SENV1270
	IF(POP(1)) 117,111	SENV1280
111	CONTINUE	SENV1290
	WRITE(6,611) I,(O(1,JJ),JJ=1,NOF)	SENV1300
	GO TO 116	SENV1310
117	RETN=1	SENV1320
118	CONTINUE	SENV1330
C	DETERMINE MAXIMUM ORDER OF SERVO ELEMENTS	SENV1340
	DO 140 K=1,NMF	SENV1350
	DO 128 LL=1,L	SENV1360
	IF(MNR(M,LL),EO,2) GO TO 135	SENV1370
128	CONTINUE	SENV1380
	DO 125 LL=1,L	SENV1390
	IF(MNR(M,LL),EO,1) GO TO 134	SENV1400
125	CONTINUE	SENV1410
	NO(M,1)=0	SENV1420
	NO(M,2)=0	SENV1430
	NO(M,3)=0	SENV1440
	GO TO 140	SENV1450
134	NO(M,1)=1	SENV1460
	NO(M,2)=1	SENV1470
	NO(M,3)=1	SENV1480
	GO TO 140	SENV1490
135	NO(M,1)=2	SENV1500
	NO(M,2)=2	SENV1510
	NO(M,3)=2	SENV1520
140	CONTINUE	SENV1530
C	FORM X, Y, Z MATRICES	SENV1540
	DO 170 I=1,L	SENV1550

IF (NL(I,1),EQ,0) GO TO 170	SENV1440
IF (I,GT,MINC) GO TO 145	SENV1450
II=NMFO=NL(I,1)	SENV1460
GO TO 150	SENV1470
145 II=NMFO=NL(I,1)	SENV1480
150 NMNL(I,2)	SENV1490
MMNN(M,1)	SENV1500
IF (M=1) 145,160,155	SENV1510
155 JJ=NMFO=NM(M,2)	SENV1520
XZ(I,J)=CZ(I)	SENV1530
YI(I,J)=CI(I)	SENV1540
YB(I,J)=CB(I)	SENV1550
GO TO 170	SENV1560
160 JJ=NM(M,2)	SENV1570
VI(I,J)=CI(I)	SENV1580
VB(I,J)=CB(I)	SENV1590
GO TO 170	SENV1600
165 JJ=NM(M,2)	SENV1610
ZC(I,J)=CO(I)	SENV1620
170 CONTINUE	SENV1630
NTCONMODE=42	SENV1640
PRINT SERV ELEMENT INFORMATION	SENV1650
WRITE(6,607)	SENV1660
DO 200 I=1,NSE	SENV1670
II=NMFO=I	SENV1680
IF (KO(I,1)=1) 175,180,185	SENV1690
175 KOP=2+NTCONMODE=KO(I,2)	SENV1700
GO TO 190	SENV1710
180 KOP=2+NTCONMODE=KO(I,2)	SENV1720
GO TO 190	SENV1730
185 KO1=NMFO=NM(I,2)	SENV1740
KO2=NTCONMODE	SENV1750
GO TO 195	SENV1760
190 WRITE(6,608) I,KO(I,1),II,KOP,KOP	SENV1770
GO TO 200	SENV1780
195 WRITE(6,609) I,KO(I,1),II,KO1,KO2,KO1,KO2	SENV1790
200 CONTINUE	SENV1800
SAVE FOR GAIN OPTION	SENV1810
IF (NGS,EQ,0) RETURN	SENV1820
NI=NM(NGS,1)	SENV1830
N2=NM(NGS,2)	SENV1840
RETURN	SENV1850
502 FORMAT(10I5)	SENV1860
503 FORMAT(4E12,0)	SENV1870
504 FORMAT(2I5,2X,3E12,0)	SENV1880
600 FORMAT(1M1 50X,2NMCONTROL SYSTEM DESCRIPTION///1X,	SENV1890
125NDIFFERENTIAL EQUATION FOR,22X,8NVAR,30X,12MCOEFFICIENTS,	SENV1900
220X,11MBYRD/ACCEL./	SENV1910
176X,4M2ND ORDER,4X,4M1ST ORDER,4X,7MO ORDER,9X,11MGA IN FACTOR //	SENV1920
601 FORMAT(1M0 2X,13MCONTROL SURF, 13,27X,13MCONTROL SURF, 13,10X,	SENV1930
11PE14,4,1P2E13,4)	SENV1940
602 FORMAT(1M 45X,13MSERVO ELEMENT 13,10X,1PE14,4,1P2E13,4)	SENV1950
603 FORMAT(1M0 2X,13MSERVO ELEMENT 13,27X,13MSERVO ELEMENT 13,10X,	SENV1960
11PE14,4,1P2E13,4)	SENV1970
604 FORMAT(1M 2X,13M(ACCELEROMETER),20X,4MBODY/46X,13MACCELERATION,	SENV1980
154X,1PE14,4)	SENV1990
605 FORMAT(1M 2X,13M(RATE GYRO),32X,4MBODY/46X,13MANGULAR RATE,	SENV2000
154X,1PE14,4)	SENV2010
606 FORMAT(1M 45X,13MCONTROL SURF, 13,10X,1PE14,4,1P2E13,4)	SENV2020
607 FORMAT(/// 4X,5MSERVO,10X,7MMAXIMUM,5X,10MROM ASSIGNMENT,3X,	SENV2030
1 17MROM ASSIGNMENT,8X,10MEIGENVECTOR ELEMENT/1X,7MELEMENT,	SENV2040
2 10X,5MORFO,7X,13MIN A AND/OR B,7X,13MIN A AND/OR B/73X,	SENV2050
3 8MVELOCITY,1X,12MISPLACEMENT /)	SENV2060
608 FORMAT(1M 4X,12,15X,11,14X,12,22X,12,26X,12)	SENV2070
609 FORMAT(1M 4X,12,15X,11,14X,12,15X,12,4M AND,13,13X,12,11X,12)	SENV2080
610 FORMAT(/// 4X,11MBYRD/ACCEL.,15X,4MINPUT DIFFERENTIATION/INTERPOL	SENV2090
1ATION ROM VECTOR //)	SENV2062
611 FORMAT(1M0,2X,13MSERVO ELEMENT,12,22X,1P0E13,4/(21X,1P0E13,4)	SENV2083
END	SENV2090

	SUBROUTINE FORMAN(GENM,SK,CF,X0,X1,X2,Y0,Y1,Z0,A,B,NPR,NMODE,NFM, AM	0010
	1 NT,NSE,NTT)	AB 0020
C	FORM A AND B MATRICES	AB 0030
	COMMON/CONTROL/NCODE,N0,N1,N2,NTC,NA,ND,NG,NGS,X1,X2,NFM,NMM,NVEC,AB	0040
	1 NAB	AB 0050
	DIMENSION GENM(NPR,NMODE),SK(NFM),CF(NFM),X0(NT,NT),X1(NT,NT),	AB 0060
	1 X2(NT,NT),Y0(NT,NSE),Y1(NT,NSE),Z0(NT,NSE),A(NT,NT),	AB 0070
	2 B(NT,NT)	AB 0080
C		AB 0090
	DO 110 I=1,NTT	AB 0100
	DO 110 J=1,NTT	AB 0110
	A(I,J)=0.0	AB 0120
110	B(I,J)=0.0	AB 0130
	DO 115 I=1,NT	AB 0140
	DO 115 J=1,NTC	AB 0150
	A(I,J)=X2(I,J)	AB 0160
	JJ=NTC+J	AB 0170
	A(I,JJ)=Y1(I,J)	AB 0180
115	B(I,JJ)=X0(I,J)	AB 0190
	IF(N1,EQ,0) GO TO 125	AB 0200
	DO 120 J=1,N1	AB 0210
	JJ=NTC+J	AB 0220
	A(I,JJ)=Y1(I,J)	AB 0230
120	B(I,JJ)=Y0(I,J)	AB 0240
125	IF(N0,EQ,0) GO TO 135	AB 0250
	DO 130 J=1,N0	AB 0260
	JJ=NTC+N1+J	AB 0270
130	B(I,JJ)=Z0(I,J)	AB 0280
135	CONTINUE	AB 0290
	DO 140 I=1,NTC	AB 0300
	II=NT+I	AB 0310
	JJ=NTC+I	AB 0320
	A(II,JJ)=1.0	AB 0330
140	B(II,JJ)=1.0	AB 0340
	IF(NFM,EQ,0) GO TO 145	AB 0350
	DO 145 I=1,NFM	AB 0360
	JJ=NTC+I	AB 0370
	A(I,JJ)=CF(I)	AB 0380
145	B(I,JJ)=SK(I)	AB 0390
148	DO 150 I=1,NFM	AB 0400
	DO 150 J=1,NMODE	AB 0410
150	A(I,J)=GENM(I,J)	AB 0420
	WRITE(2) ((A(I,J),I=1,NTT),J=1,NTT),((B(I,J),I=1,NTT),J=1,NTT)	AB 0430
	RETURN	AB 0440
	END	AB 0450

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SUMMONTIME MMSE (A.M. STAN.VEC.C.MC.E.M.M.H.I.A.N.A.B.INV.INDEA.GAIN. MMSE 0010
1 NTF.MOUE.NMUNE.NPM.GAMMA.NUP.UEFL.AMUE.WA.V) MMSE 0020
C SEMYU.ELASTIC.ANALYSIS MMSE 0030
COMMON/CONTROL/ACUOE.MUOH.ONZ.NTC.NA.ND.NG.NGS.A.KZ.NPM.NMM.NVEC. MMSE 0040
1 NAR MMSE 0050
DIMENSION A(NTF,NTF),M(NTF,NTF),STAN(MOUE,7),VEC(MOUE,NTF), MMSE 0060
1 C(NTF,NTF),HC(NTF,NTF),E(NTF),MM(NTF),M(NTF),IANA(NTF), MMSE 0070
2 MINV(NTF,1),INDEA(NTF,1),GAIN(1),UEFL(INUP),XMOUE(INDF,NMUNE) MMSE 0080
3 MM(NTF),V(1,1) MMSE 0090
COMPLEX VEC.E,UEFL,ANUMM.V MMSE 0100
C MMSE 0110
C GAIN FACTOR OPTION MMSE 0120
NVEC=1 MMSE 0130
DO 1M5 I=1,NM5 MMSE 0140
NEXTNU 2 MMSE 0150
HEAD(2) ((A(I,J),I=1,NTF),J=1,NTF),((B(I,J),I=1,NTF),J=1,NTF) MMSE 0160
IF(NM5) 120,125,101 MMSE 0170
101 L=1 MMSE 0180
I1=NMUNE+NM5 MMSE 0190
IF(L=1) 114,110,105 MMSE 0200
105 J1=NMUNE+NM5 MMSE 0210
J=NTC+JJ MMSE 0220
IF(GAIN(I6).EQ.0.) GO TO 108 MMSE 0230
A(I1,J)=A(I1,J)/GAIN(I6) MMSE 0240
A(I1,J)=A(I1,J)/GAIN(I6) MMSE 0250
B(I1,J)=B(I1,J)/GAIN(I6) MMSE 0260
GO TO 125 MMSE 0270
108 A1=A(I1,JJ) MMSE 0280
A2=A(I1,J) MMSE 0290
B1=B(I1,J) MMSE 0300
DO 109 LL=1,NTF MMSE 0310
A(I1,LL)=0. MMSE 0320
109 B(I1,LL)=0. MMSE 0330
A(I1,JJ)=A1 MMSE 0340
A(I1,J)=A2 MMSE 0350
B(I1,J)=B1 MMSE 0360
GO TO 125 MMSE 0370
110 J1=NK2 MMSE 0380
J=2*NTC+JJ MMSE 0390
IF(GAIN(I6).EQ.0.) GO TO 113 MMSE 0400
A(I1,J)=A(I1,J)/GAIN(I6) MMSE 0410
B(I1,J)=B(I1,J)/GAIN(I6) MMSE 0420
GO TO 125 MMSE 0430
113 A1=A(I1,J) MMSE 0440
B1=B(I1,J) MMSE 0450
DO 114 LL=1,NTF MMSE 0460
A(I1,LL)=0. MMSE 0470
114 B(I1,LL)=0. MMSE 0480
A(I1,J)=A1 MMSE 0490
B(I1,J)=B1 MMSE 0500
GO TO 125 MMSE 0510
115 J1=NK2 MMSE 0520
J=2*NTC+NT1+JJ MMSE 0530
IF(GAIN(I6).EQ.0.) GO TO 118 MMSE 0540
B(I1,J)=B(I1,J)/GAIN(I6) MMSE 0550
GO TO 125 MMSE 0560
118 B1=B(I1,J) MMSE 0570
DO 119 LL=1,NTF MMSE 0580
B(I1,LL)=0. MMSE 0590
119 B(I1,J)=B1 MMSE 0600
GO TO 125 MMSE 0610
120 I1=NPM+IAN5(NM5) MMSE 0620
J1=NTC+NPM+IAN5(NM5) MMSE 0630
IF(GAIN(I6).EQ.0.) GO TO 121 MMSE 0640
A(I1,I1)=A(I1,I1)/GAIN(I6) MMSE 0650
A(I1,J1)=A(I1,J1)/GAIN(I6) MMSE 0660
B(I1,J1)=B(I1,J1)/GAIN(I6) MMSE 0670
GO TO 125 MMSE 0680
121 A1=A(I1,I1) MMSE 0690
A2=A(I1,J1) MMSE 0700
B1=B(I1,J1) MMSE 0710
DO 122 LL=1,NTF MMSE 0720
A(I1,LL)=0. MMSE 0730
122 B(I1,LL)=0. MMSE 0740
A(I1,I1)=A1 MMSE 0750
A(I1,J1)=A2 MMSE 0760
B(I1,J1)=B1 MMSE 0770

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129 ENDO.
END.
1E00.
C EIGENVALUE SOLUTION
CALL SOLVER(001E,GAMMA,A,B,C,D,CONV,INT,E,LANA,MINV,INDEX,STAB,
1 VEC,NT,MODE,MODE,00,V)
C PRINT STABILITY RESULTS
WRITE(UNIT,*)
IF(MODE) 100,105,104
100 WRITE(UNIT,*) GAIN(10),MODE
GO TO 105
105 WRITE(UNIT,*) GAIN(10),MODE
DO 175 101,1000
IF(NTAB(1,2),PU,0.) GO TO 170
WRITE(UNIT,*) 1.(STAB(1,1),J,101,1.(STAB(1,1),J,100,7)
GO TO 175
170 WRITE(UNIT,*) 1.(STAB(1,1),J,101,1.(STAB(1,1),J,100,7)
175 CONTINUE
C PRINT EIGENVECTORS
WRITE(UNIT,*)
DO 176 101,1000
176 WRITE(UNIT,*) 1.(VECT(1,1),J,101,NT)
C PRINT STRUCTURAL DEFLECTIONS
NF=100
IF(NF,0.0) GO TO 177
NFAD(1)(AMODE(1,1),101,NUF),J=1,NF
177 NF=NF+1
IF(NF,0.0) GO TO 176
NFAD(1)(AMODE(1,1),101,NUF),J=NF,NF
176 NF=NF+1
HEAD(1)(AMODE(1,1),101,NUF),J=NF,NF,AMODE)
WRITE(UNIT,*)
DO 100 1001,1000
DO 101 101,1000
DEFL(1)=0.0
DO 100 1001,1000
CONTINUE
100 DEFL(1)=DEFL(1)+AMODE(1,1)*VECT(100,1)
101 CONTINUE
TST=0.0
DO 102 101,1000
ANORM=CABS(DEFL(1))
IF(TST,0.1,ANORM) GO TO 102
L=1
TST=ANORM
102 CONTINUE
ANORM=DEFL(1)
ANORM=CABS(ANORM)
IF(ANORM,0.0) GO TO 103
DO 103 101,1000
103 DEFL(1)=DEFL(1)/ANORM
103 WRITE(UNIT,*) 100,DEFL(1),101,NUF
104 CONTINUE
105 CONTINUE
RETURN
015 FORMAT(1M,1J,12E10,EIGENVECTORS FOR MODES REQUESTED //)
016 FORMAT(1M,1J,12E10,STABILITY ANALYSIS RESULTS //)
017 FORMAT(1M,1J,12E10,GAIN FACTOR = 1PE11.4,1A,17HPUN DEGRU ELEMENT (3)
018 FORMAT(1M,1J,12E10,GAIN FACTOR = 1PE11.4,1A,17HPUN CONTINUAL SURFACE)
019 FORMAT(1M,1J,12E10,MODE=2A,12E10,EIGENVALUE=0.0A,17HPUN EIGENVALUE=1.0A,
1 AMPLITUDE=12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
2 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
3 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
020 FORMAT(1M,1J,12E10,MODE=2A,12E10,EIGENVALUE=0.0A,17HPUN EIGENVALUE=1.0A,
1 AMPLITUDE=12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
2 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
3 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
021 FORMAT(1M,1J,12E10,MODE=2A,12E10,EIGENVALUE=0.0A,17HPUN EIGENVALUE=1.0A,
1 AMPLITUDE=12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
2 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
3 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
022 FORMAT(1M,1J,12E10,MODE=2A,12E10,EIGENVALUE=0.0A,17HPUN EIGENVALUE=1.0A,
1 AMPLITUDE=12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
2 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
3 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
023 FORMAT(1M,1J,12E10,MODE=2A,12E10,EIGENVALUE=0.0A,17HPUN EIGENVALUE=1.0A,
1 AMPLITUDE=12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
2 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
3 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
024 FORMAT(1M,1J,12E10,MODE=2A,12E10,EIGENVALUE=0.0A,17HPUN EIGENVALUE=1.0A,
1 AMPLITUDE=12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
2 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
3 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
025 FORMAT(1M,1J,12E10,MODE=2A,12E10,EIGENVALUE=0.0A,17HPUN EIGENVALUE=1.0A,
1 AMPLITUDE=12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
2 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
3 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
026 FORMAT(1M,1J,12E10,MODE=2A,12E10,EIGENVALUE=0.0A,17HPUN EIGENVALUE=1.0A,
1 AMPLITUDE=12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
2 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
3 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
027 FORMAT(1M,1J,12E10,MODE=2A,12E10,EIGENVALUE=0.0A,17HPUN EIGENVALUE=1.0A,
1 AMPLITUDE=12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
2 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
3 1J,12E10,AMPLITUDE=0.0A,17HPUN AMPLITUDE=1.0A,17HPUN
END

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103

105

SUBROUTINE GENMAT(AM,GEN,TH,PRC,FMODE,RMODE,CMODE,NOF,NOR,NOP,NC,	GENM0010
1 NPM,M1,M2)	GENM0020
C GENMAT - FORM GENERALIZED MATRICES	GENM0030
COMMON/CINTROU/CMODE,N0,M1,M2,NYC,N4,N0,N6,N03,M1,M2,NPM,NRM,NVEC,	GENM0040
1 NAM	GENM0050
DIMENSION AM(NOF,1),GEN(NPR,1),TH(M1,1),PRC(M2,1),FMODE(NOP,1),	GENM0060
1 RMODE(NOR,1),CMODE(NOP,1)	GENM0070
C	GENM0080
IF(NPM.EQ.0) GO TO 3	GENM0090
CALL MATMPL(FMODE,AM,TH,NOP,NOP,M1,NPM,NOP,NOP,2)	GENM0100
CALL MATMPL(TH,FMODE,PRC,M1,NOP,M2,NPM,NPM,NOP,1)	GENM0110
DO 1 J=1,NPM	GENM0120
DO 1 J=1,NPM	GENM0130
1 GEN(I,J)=PRC(I,J)	GENM0140
CALL MATMPL(TH,CMODE,PRC,M1,NOP,M2,NPM,NC,NOP,1)	GENM0150
DO 2 J=1,NPM	GENM0160
DO 2 J=1,NC	GENM0170
JJ=J*NC+J	GENM0180
2 GEN(I,JJ)=PRC(I,J)	GENM0190
3 IF(NRM.EQ.0) RETURN	GENM0200
IF(NPM.EQ.0) GO TO 5	GENM0210
CALL MATMPL(TH,RMODE,PRC,M1,NOR,M2,NPM,NRM,NOP,1)	GENM0220
DO 4 J=1,NPM	GENM0230
DO 4 J=1,NRM	GENM0240
JJ=J*NRM+J	GENM0250
4 GEN(I,JJ)=PRC(I,J)	GENM0260
5 CALL MATMPL(RMODE,AM,TH,NOR,NOP,M1,NRM,NOP,NOP,2)	GENM0270
CALL MATMPL(TH,RMODE,PRC,M1,NOR,M2,NRM,NRM,NOP,1)	GENM0280
DO 6 J=1,NRM	GENM0290
II=J*NRM+J	GENM0300
DO 6 J=1,NRM	GENM0310
JJ=J*NRM+J	GENM0320
6 GEN(II,JJ)=PRC(I,J)	GENM0330
CALL MATMPL(TH,CMODE,PRC,M1,NOP,M2,NRM,NC,NOP,1)	GENM0340
DO 7 J=1,NRM	GENM0350
II=J*NRM+J	GENM0360
DO 7 J=1,NC	GENM0370
JJ=J*NRM+J	GENM0380
7 GEN(II,JJ)=PRC(I,J)	GENM0390
IF(NPM.EQ.0) RETURN	GENM0400
CALL MATMPL(TH,FMODE,PRC,M1,NOP,M2,NRM,NPM,NOP,1)	GENM0410
DO 8 J=1,NRM	GENM0420
II=J*NRM+J	GENM0430
DO 8 J=1,NPM	GENM0440
8 GEN(II,J)=PRC(I,J)	GENM0450
RETURN	GENM0460
END	GENM0470

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SUBROUTINE HPSOL(M,N,NH,NV,A,B,C,D,CJ,L1,L2,STAN,VEC,MUOR,EN) NSL10010
RESULT = COMPUTES STABILITY RESULTS AND EIGENVECTORS FOR MODE NSL10020
DIMENSION A(N+1),B(N+1),C(N+1),D(N+1),CJ(N+1),L1(N+1) NSL10030
L2(N+1),STAN(MODE+1),VEC(MODE+1) NSL10040
COMPLEX GAMMA,B,C,D,CJ,VEC,EN NSL10050
C NSL10060
C STABILITY RESULTS NSL10070
STAN(M,1)=0 NSL10080
STAN(M,2)=0 NSL10090
STAN(M,3)=0 NSL10100
STAN(M,4)=0 NSL10110
STAN(M,5)=0 NSL10120
STAN(M,6)=0 NSL10130
STAN(M,7)=0 NSL10140
STAN(M,8)=0 NSL10150
EIGENVECTORS NSL10160
IF(MV,EN,0) RETURN NSL10170
GAMMA=CMPLX(MH,MN) NSL10180
DO 10 I=1,N NSL10190
DO 10 J=1,N NSL10200
10 U(I,J)=A(N+1,I)*B(N+1,J) NSL10210
EIGENVECTORS NSL10220
CALL EIGENVECTORS(MH,MN,CJ,L1,L2,N) NSL10230
DO 10 I=1,N NSL10240
10 VEC(M,1)=CJ(I) NSL10250
RETURN NSL10260
END NSL10270

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SUBROUTINE INTERP(XMK,S,SI,SMK,BINV,INDEX,N,N1) INTR0010
INTERP = GENERATES CONSTANT PORTION OF LINEAR SPLINE INTERPOLATION INTR0020
DIMENSION XMK(1),S(N1,1),SI(N1,1),SMK(N1,1),BINV(N1,1),INDEX(N1,1) INTR0030
C INTR0040
DO 30 I=1,N1 INTR0050
DO 30 J=1,N1 INTR0060
IF(I,NE,1) GO TO 10 INTR0070
SMK(I,J)=1 INTR0080
IF(J,EQ,1) SMK(I,J)=0 INTR0090
GO TO 20 INTR0100
10 SMK(I,J)=ABS(XMK(I-1)-XMK(J-1))*3+ABS(XMK(I)-XMK(J-1))*3 INTR0110
20 SMK(I,J)=SMK(I,J) INTR0120
30 CONTINUE INTR0130
DO 40 I=1,N1 INTR0140
DO 40 J=1,N INTR0150
SI(I,J)=0 INTR0160
IF(I-J,EQ,1) SI(I,J)=1 INTR0170
40 CONTINUE INTR0180
CALL INVERS(N1,SMK,N1,BINV,0,0,1,SING,INDEX) INTR0190
IF(SING,EQ,1) GO TO 45 INTR0200
WRITE(0,00) INTR0210
STOP INTR0220
45 CALL MATMPL(SMK,SI,S,N1,N1,N1,N1,N1,1) INTR0230
60 FIRMAT(INO,2X,40NSPLINE INTERPOLATION MATRIX IS SINGULAR,) INTR0240
RETURN INTR0250
END INTR0260

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SUBROUTINE SPLINE(CMK,XMK,S,PS,P,N,N1) SPLN0010
SPLINE = GENERATES SPLINE INTERPOLATION MATRIX SPLN0020
DIMENSION CMK(1),S(N1,1),PS(1),P(1) SPLN0030
C SPLN0040
P(1)=1 SPLN0050
DO 10 I=1,N SPLN0060
P(I)=1 SPLN0070
P(I)=ABS(CMK-XMK(I))*3+ABS(CMK,XMK(I))*3 SPLN0080
10 CONTINUE SPLN0090
C COEFFICIENTS SPLN0100
DO 20 I=1,N SPLN0110
PS(I)=0 SPLN0120
DO 15 J=1,N SPLN0130
15 PS(I)=PS(I)+P(J)*S(J,I) SPLN0140
20 CONTINUE SPLN0150
RETURN SPLN0160
END SPLN0170

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SUBROUTINE INVERS(NDIM,A,N,R,M,DETERM,ISING,INDEX)	INVS001
C	INVS002
C ***** INVERSE OR LINEAR EQUATIONS SOLVER *****	INVS003
C	INVS004
C	INVS005
C NDIM IS THE ACTUAL SIZE OF A IN CALLING PROGRAM,	INVS006
C EG. A(NDIM,NDIM)	INVS007
C A IS SQUARE MATRIX TO BE INVERTED,	INVS008
C N IS SIZE OF UPPER LEFT PORTION BEING INVERTED. MINIMUM	INVS009
C R IS COLUMN OF CONSTANTS (OPTIONAL INPUT), SUPPLY SPACE B(NDIM,1)	INVS010
C M IS THE NUMBER OF COLUMNS OF CONSTANTS	INVS011
C DETERM RETURNS THE VALUE OF DETERMINANT IF NON-SINGULAR	INVS012
C ISING RETURNS, 2 IF MATRIX A(N,N) IS SINGULAR	INVS013
C , 1 IF MATRIX A(N,N) IS NON-SINGULAR	INVS014
C INVERSE RETURNS IN A	INVS015
C SOLUTION VECTORS RETURN IN B	INVS016
C INDEX IS WORKING STORAGE (N,3)	INVS017
C	INVS018
C DIMENSION A(NDIM,1), B(NDIM,1), INDEX(N,3)	INVS019
C EQUIVALENCE (IROW,JROW), (ICOL,JCOL), (AMAX, Y, SWAP)	INVS020
C	INVS021
C INITIALIZE	INVS022
C	INVS023
C DETERM = 1.0E0	INVS024
C DO 10 J=1,N	INVS025
10 INDEX(J,3) = 0	INVS026
DO 130 I=1,N	INVS027
C	INVS028
C SEARCH FOR PIVOT	INVS029
C	INVS030
C AMAX = 0.0E0	INVS031
DO 40 J=1,N	INVS032
IF (INDEX(J,3) .EQ. 1) GO TO 40	INVS033
DO 30 K=1,N	INVS034
IF (INDEX(K,3) = 1) 20,30,100	INVS035
20 IF (ABS(A(J,K)) .LE. AMAX) GO TO 30	INVS036
IROW = J	INVS037
ICOL = K	INVS038
AMAX = ABS(A(J,K))	INVS039
30 CONTINUE	INVS040
40 CONTINUE	INVS041
IF (AMAX.EQ.0.0) GO TO 100	INVS042
INDEX(ICOL,3) = INDEX(ICOL,3) + 1	INVS043
INDEX(I,1) = IROW	INVS044
INDEX(I,2) = ICOL	INVS045
C	INVS046
C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL	INVS047
C	INVS048
C IF (IROW .EQ. ICOL) GO TO 70	INVS049
DETERM = -DETERM	INVS050
DO 50 L=1,N	INVS051
SWAP = A(IROW,L)	INVS052
A(IROW,L) = A(ICOL,L)	INVS053
50 A(ICOL,L) = SWAP	INVS054
IF (M .LE. 0) GO TO 70	INVS055
DO 60 L=1,M	INVS056
SWAP = B(IROW,L)	INVS057
B(IROW,L) = B(ICOL,L)	INVS058
60 B(ICOL,L) = SWAP	INVS059
C	INVS060
C DIVIDE PIVOT ROW BY PIVOT ELEMENT	INVS061
C	INVS062
70 PIVOT = A(ICOL,ICOL)	INVS063
DETERM = DETERM * PIVOT	INVS064
A(ICOL,ICOL) = 1.0E0	INVS065
DO 80 L=1,N	INVS066
80 A(ICOL,L) = A(ICOL,L) / PIVOT	INVS067
IF (M .LE. 0) GO TO 100	INVS068
DO 90 L=1,M	INVS069
90 B(ICOL,L) = B(ICOL,L) / PIVOT	INVS070
C	INVS071
C REDUCE NON PIVOT ROWS	INVS072
C	INVS073
100 DO 130 L=1,N	INVS074
IF (L .EQ. ICOL) GO TO 130	INVS075
P = A(L,ICOL)	INVS076
A(L,ICOL) = 0.0E0	INVS077
DO 110 I=1,N	INVS078

110 A(I,L) = A(I,L) + A(JCOLUM,L) * Y	INVMS070
IF(M.LE. 0) GO TO 130	INVMS080
DO 120 L=1,M	INVMS081
120 B(I,L) = B(I,L) + B(JCOLUM,L) * Y	INVMS082
130 CONTINUE	INVMS083
C	INVMS084
C INTERCHANGE COLUMNS	INVMS085
C	INVMS086
DO 150 I=1,M	INVMS087
L = N + 1	INVMS088
IF(INDEX(L,1).EQ. INDEX(L,2)) GO TO 150	INVMS089
JNO = INDEX(L,1)	INVMS090
JCOLUM = INDEX(L,2)	INVMS091
DO 180 K=1,M	INVMS092
SNAP = A(K,JNO)	INVMS093
A(K,JNO) = A(K,JCOLUM)	INVMS094
A(K,JCOLUM) = SNAP	INVMS095
140 CONTINUE	INVMS096
150 CONTINUE	INVMS097
DO 170 K=1,M	INVMS098
IF(INDEX(K,3).EQ. 1) GO TO 160	INVMS099
ISING = 2	INVMS100
GO TO 180	INVMS101
160 CONTINUE	INVMS102
170 CONTINUE	INVMS103
ISING = 1	INVMS104
180 RETURN	INVMS105
190 ISING = 2	INVMS106
RETURN	INVMS107
END	INVMS108

SUBROUTINE MATMPL(A,B,C,MA,MB,MC,M,N,L,ICH)	MATM0010
CHAMPL MATRIX MULTIPLICATION (REAL AND T-C=0-DIMENSIONAL)	MATM0020
C	MATM0030
C ROW DIMENSION OF MATRIX A IN CALLING PROGRAM = MA	MATM0040
C ROW DIMENSION OF MATRIX B IN CALLING PROGRAM = MB	MATM0050
C ROW DIMENSION OF MATRIX C IN CALLING PROGRAM = MC	MATM0060
C M = NO. OF ROWS IN PRODUCT MATRIX C	MATM0070
C N = NO. OF COLUMNS IN C	MATM0080
C L = COMMON DIMENSION OF A AND B	MATM0090
C 1. A * B = C	MATM0100
C 2. A(TRANSPOSE) * B = C	MATM0110
C 3. A * B(TRANSPOSE) = C	MATM0120
C	MATM0130
C DIMENSION A(MA,1),B(MB,1),C(MC,1)	MATM0140
C	MATM0150
GO TO (100,200,300),ICH	MATM0160
100 DO 175 I=1,M	MATM0170
DO 150 J=1,N	MATM0180
C(I,J)=0.	MATM0190
DO 125 K=1,L	MATM0200
C(I,J)=C(I,J)+A(I,K)*B(K,J)	MATM0210
125 CONTINUE	MATM0220
150 CONTINUE	MATM0230
175 CONTINUE	MATM0240
GO TO 400	MATM0250
200 DO 275 I=1,M	MATM0260
DO 250 J=1,N	MATM0270
C(I,J)=0.	MATM0280
DO 225 K=1,L	MATM0290
C(I,J)=C(I,J)+A(K,I)*B(K,J)	MATM0300
225 CONTINUE	MATM0310
250 CONTINUE	MATM0320
275 CONTINUE	MATM0330
GO TO 400	MATM0340
300 DO 375 I=1,M	MATM0350
DO 350 J=1,N	MATM0360
C(I,J)=0.	MATM0370
DO 325 K=1,L	MATM0380
C(I,J)=C(I,J)+A(I,K)*B(J,K)	MATM0390
325 CONTINUE	MATM0400
350 CONTINUE	MATM0410
375 CONTINUE	MATM0420
400 RETURN	MATM0430
END	MATM0440

C	SUBROUTINE EGVCT (C1, C2, EIGEN, C3, N1, N2, N)	EUNVCO01
C	SUBROUTINE TO OBTAIN EIGENVECTOR FROM REAL NON-SYMMETRIC	EUNVCO02
C	MATRICES FOR WHICH THE EIGENVALUE IS KNOWN. THE METHOD	EUNVCO03
C	USED IS THE DIRECT METHOD OUTLINED IN ERM-F- BY DR.	EUNVCO04
C	S. M. LUNNINGHAM	EUNVCO05
C	COMPLEX C1(N,N), C2(N), C3(N),	EUNVCO06
C	EIGEN, D1, D2, D3, D4, D5, D6, D8	EUNVCO07
C	INTEGER N1(N), N2(N)	EUNVCO08
C		EUNVCO09
C	I13 = N	EUNVCO10
C	I12 = N - 1	EUNVCO11
C	X1 = 0.0	EUNVCO12
C	DO 20 J=1,N	EUNVCO13
C	N1(J) = J	EUNVCO14
C	N2(J) = J	EUNVCO15
C	C1(J,J) = C1(J,J) - EIGEN	EUNVCO16
C	DO 10 I=1,N	EUNVCO17
C	X2 = CABS(C1(I,J))	EUNVCO18
C	IF (X1-X2) 5,10,10	EUNVCO19
C	5 X1 = X2	EUNVCO20
C	I1 = I	EUNVCO21
C	J1 = J	EUNVCO22
C	10 CONTINUE	EUNVCO23
C	20 CONTINUE	EUNVCO24
C	DO 150 K=2,N	EUNVCO25
C	IF (CABS(C1(I1,J1))) 50,30,50	EUNVCO26
C	30 X5 = X6 = 1	EUNVCO27
C	SINGULAR MATRIX RETURN ZERO	EUNVCO28
C	DO 36 I=1,N	EUNVCO29
C	C3(I) = 0.0	EUNVCO30
C	GO TO 1800	EUNVCO31
C		EUNVCO32
C	50 D1 = (1.0,0.0)/C1(I1,J1)	EUNVCO33
C	D2 = C1(I1,I13)	EUNVCO34
C	D3 = C1(I13,J1)	EUNVCO35
C	D4 = C1(I13,I13)	EUNVCO36
C	DO 80 I=1,I12	EUNVCO37
C	C3(I) = C1(I,J1)	EUNVCO38
C	C1(I,J1) = C1(I,I13)	EUNVCO39
C	C1(I,I13) = -C3(I)*D1	EUNVCO40
C	D5 = -C1(I1,I13)*D1	EUNVCO41
C	C1(I1,I1) = C1(I13,I1)	EUNVCO42
C	C1(I13,I1) = D5	EUNVCO43
C	80 CONTINUE	EUNVCO44
C	C3(I1) = D3	EUNVCO45
C	C1(I1,J1) = D4	EUNVCO46
C	C1(I13,J1) = -D2*D1	EUNVCO47
C	C1(I1,I13) = -D3*D1	EUNVCO48
C	C1(I13,I13) = D1	EUNVCO49
C	IF (I13, EQ, N) GO TO 80	EUNVCO50
C	I14 = I13 + 1	EUNVCO51
C	DO 70 I=1,I14,N	EUNVCO52
C	D6 = C1(I1,I1)	EUNVCO53
C	C1(I1,I1) = C1(I13,I1)	EUNVCO54
C	C1(I13,I1) = D6	EUNVCO55
C	C3(I) = C1(I,J1)	EUNVCO56
C	C1(I,J1) = C1(I,I13)	EUNVCO57
C	70 C1(I,I13) = C3(I)	EUNVCO58
C	80 I=N1(J1)	EUNVCO59
C	N1(J1) = N1(I13)	EUNVCO60
C	N1(I13) = I	EUNVCO61
C	I = N2(I1)	EUNVCO62
C	N2(I1) = N2(I13)	EUNVCO63
C	N2(I13) = I	EUNVCO64
C	X1 = 0.0	EUNVCO65
C	DO 100 J=1,I12	EUNVCO66
C	D7 = C1(I13,J)	EUNVCO67
C	DO 130 I=1,I12	EUNVCO68
C	C1(I,J) = C1(I,J) + C3(I)*D6	EUNVCO69
C	X2 = CABS(C1(I,J))	EUNVCO70
C	IF (X1-X2) 120,130,130	EUNVCO71
C		EUNVCO72
C		EUNVCO73

120 X1 = X2	EGNVC014
11 = 1	EGNVC015
J1 = J	EGNVC016
130 CONTINUE	EGNVC017
140 CONTINUE	EGNVC018
113 = 113 - 1	EGNVC019
112 = 112 - 1	EGNVC020
150 CONTINUE	EGNVC021
C	EGNVC022
160 C3(2) = C1(2,1)	EGNVC023
C3(1) = (1.0,0.0)	EGNVC024
DO 180 J=3,N	EGNVC025
C3(J) = (0.0,0.0)	EGNVC026
J1 = J-1	EGNVC027
DO 170 I=1,J1	EGNVC028
C3(J) = C3(J) + C3(I)*C1(J,I)	EGNVC029
170 CONTINUE	EGNVC030
180 CONTINUE	EGNVC031
IF(CABS(C1(1,1)) .LT. 1.0E-20) GO TO 202	EGNVC032
DO 201 K=1,2	EGNVC033
C	EGNVC034
DO 184 J=1,N	EGNVC035
11 = N2(J)	EGNVC036
DO 182 I=1,N	EGNVC037
IF(11 .EQ. N1(I)) GO TO 184	EGNVC038
182 CONTINUE	EGNVC039
184 C2(J) = C3(I)	EGNVC040
C	EGNVC041
DO 190 J=2,N	EGNVC042
11 = N - J + 1	EGNVC043
J1 = 11 + 1	EGNVC044
DO 185 I=1,11	EGNVC045
C2(I) = C2(I) + C1(I,J1)*C2(J1)	EGNVC046
185 CONTINUE	EGNVC047
190 CONTINUE	EGNVC048
O1 = C1(1,1)/C2(1)	EGNVC049
C3(1) = (1.0,0.0)	EGNVC050
DO 200 J=2,N	EGNVC051
11 = J - 1	EGNVC052
C3(J) = C2(J) + C1(J,J1)*O1	EGNVC053
DO 195 I=1,11	EGNVC054
C3(J) = C3(J) + C1(J,I)*C3(I)	EGNVC055
195 CONTINUE	EGNVC056
200 CONTINUE	EGNVC057
201 CONTINUE	EGNVC058
C	EGNVC059
C3(1) NOW CONTAINS THE EIGENVECTOR WHICH MUST BE RE-ARRANGED	EGNVC060
C ACCORDING TO THE ORDER DICTATED BY N1(I) BACK TO THE ORIGINAL	EGNVC061
C ORDER.	EGNVC062
C	EGNVC063
202 DO 230 I=1,N	EGNVC064
11 = N1(I)	EGNVC065
N1(I) = 1	EGNVC066
205 IF(11-1) 210,230,210	EGNVC067
210 O1 = C3(11)	EGNVC068
C3(11) = C3(I)	EGNVC069
C3(I) = O1	EGNVC070
K = N1(11)	EGNVC071
N1(11) = 11	EGNVC072
11 = K	EGNVC073
GO TO 205	EGNVC074
230 CONTINUE	EGNVC075
260 N1(1) = 2	EGNVC076
C	EGNVC077
1000 RETURN	EGNVC078
END	EGNVC079

APPENDIX A

THEORETICAL DERIVATION FROM REFERENCE 1

2.0 THE CLOSED-LOOP AERO-SERVO-ELASTIC STABILITY PROBLEM

2.1 NOMENCLATURE

A	Element of coefficient matrix of $\{\dot{v}\}$ in Equation (2-4)
AIC	Aerodynamic influence coefficient
a	Amplitude of generalized displacement coordinate
B	Element of coefficient matrix of $\{v\}$ in Equation (2-4)
b_r	Reference dimension
C	Element of discrete damping matrix
c	Element of generalized damping matrix
D	Element of differentiation matrix
e	Servo signal
F	Element of forcing function matrix
g_F	Modal structural damping coefficient
H	Element of interpolation matrix
h	Deflection of aeroelastic system
h_δ	Deflection due to unit shaft rotation
I	Element of unit matrix
i	Imaginary unit
K	Servo system gain constant; element of stiffness matrix
k	Element of generalized stiffness matrix
k	Reduced frequency (Strouhal number)
M	Element of discrete mass matrix
m	Element of generalized mass matrix
Q	Element of generalized force matrix
Q_a	Element of AIC matrix

$\text{Re}()$	Denotes real part of ()
s	Laplace transform parameter
T	Servo system time constant
u	$u = \dot{x}$
V	Amplitude of v ; velocity of flight
v	$\{v\} = [u \ x \ y \ z]^T$
X	Element of coefficient matrix of second order variables
x	Second order variable; forward Cartesian coordinate
Y	Element of coefficient matrix of first order variables
y	First order variable
Z	Element of coefficient matrix of zero order variables
z	Zero order variable
γ	Eigenvalue, i. e., coefficient in Equation (2-5) to describe the time dependence of transient motion. Note $\text{Re}(\gamma) = 0$ indicates neutral stability.
γ_0	Shift value of γ in eigenvalue problem
δ	Shaft rotation of control surface
ζ	Ratio of viscous damping coefficient to the critical viscous damping coefficient
λ	Eigenvalue of Equation (2-8)
λ_0	Shift value of λ in eigenvalue problem
ω	Angular frequency
$(\bar{\ })$	Denotes complex amplitude

Subscripts

a	Actuator; aerodynamic
F	Flexible body motion
g	Gyro
h	Hinge line

p	Potentiometer
R	Rigid body motion
x	Corresponds to magnitude of second order variable
$\dot{x}$	Corresponds to velocity of second order variable
$\ddot{x}$	Corresponds to acceleration of second order variable
y	Corresponds to magnitude of first order variable
$\dot{y}$	Corresponds to velocity of first order variable
z	Corresponds to magnitude of zero order variable
δ	Control surface rotation

Matrix Notation

[]	Square or rectangular
[] ^T	Transpose
[] ⁻¹	Inverse
{ }	Column
[]	Row
[]	Diagonal

2.2 THEORETICAL DERIVATION

All components of an aero-servo-elastic system can be regarded as being composed of elements whose characteristics are described by second order (or lower) differential equations. Let $\{x\}$ denote the set of second order variables, $\{y\}$ the set of first order variables, and $\{z\}$ the set of zero order variables. The aero-servo-elastic equations of motion can be written in the general form

$$\begin{aligned} & [X_{\ddot{x}}]\{\ddot{x}\} + [X_{\dot{x}}]\{\dot{x}\} + [X_x]\{x\} \\ & + [Y_{\dot{y}}]\{\dot{y}\} + [Y_y]\{y\} + [Z_z]\{z\} = \{F\} \end{aligned} \quad (2-1)$$

where $\{F\}$ denotes a forcing function. The mathematical formulation of the stability problem requires first order differential equations. We introduce the variable

$$\{u\} = \{\dot{x}\} \quad (2-2)$$

and combine Equations (2-1) and (2-2) into the matrix form

$$\begin{aligned} & \begin{bmatrix} X_{\ddot{x}} & X_{\dot{x}} & Y_{\dot{y}} & 0 \\ 0 & I & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix} \\ & + \begin{bmatrix} 0 & X_x & Y_y & Z_z \\ -I & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u \\ x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix} \end{aligned} \quad (2-3)$$

which we may abbreviate as

$$[A] \{\dot{v}\} + [B] \{v\} = \{Q\} \quad (2-4)$$

By setting the forcing function to zero, and letting

$$\{v\} = \{V\} \exp(\gamma t) \quad (2-5)$$

we obtain the eigenvalue formulation of the stability problem

$$(\gamma[A] + [B]) \{V\} = 0 \quad (2-6)$$

Instability occurs when the velocity and/or system gains are such that the real part of γ is positive.

Since $[A]$ is singular and $[B]$ may be singular, we let

$$\gamma = \gamma_0 - 1/(\lambda - \lambda_0) \quad (2-7)$$

where γ_0 and λ_0 are arbitrarily chosen complex numbers. Then Equation (2-6) can be rewritten in the canonical form of the eigenvalue problem as

$$\lambda \{V\} = (\gamma_0[A] + [B])^{-1} \left[[A] + \lambda_0 (\gamma_0[A] + [B]) \right] \{V\} \quad (2-8)$$

where the new eigenvalue is

$$\lambda = \lambda_0 - 1/(\gamma - \gamma_0) \quad (2-9)$$

The eigenvalue extraction leads to convergence to the eigenvalues λ corresponding to the value of γ closest to γ_0 . The non-Hermitian matrices yield roots that appear separately, as close pairs, or as complex conjugates.

The frequency and damping for each mode are found from the real and imaginary parts of each eigenvalue. Letting α denote the decay rate coefficient, β the damped frequency, ζ the viscous damping coefficient, and ω the undamped frequency of each mode, we may write

$$\gamma = \alpha + i\beta \quad (2-10a)$$

$$= \omega \left(\zeta + i\sqrt{1 - \zeta^2} \right) \quad (2-10b)$$

from which we find

$$\omega = \sqrt{\alpha^2 + \beta^2} \quad (2-11)$$

and

$$\zeta = \alpha/\omega \quad (2-12)$$

The equations of motion of the aeroelastic system including the shaft rotation of the control surface (flipper) appear as

$$[M]\{\ddot{h}\} + [C]\{\dot{h}\} + [K](\{h\} - [h_\delta]\{\delta\}) = \{F\} \quad (2-13)$$

where $\{h\}$ denotes the displacement of the control point masses in the lumped-parameter aeroelastic system, $[M]$ is the mass matrix, $[C]$ is the viscous damping matrix, $[K]$ is the stiffness matrix, $[h_\delta]$ is the flipper displacement matrix (the elements of $[h_\delta]$ are zero for points off the flipper and equal $(x_h - x_i)$ for the i^{th} control point where x_h is the hinge line coordinate), and $\{F\}$ is the external force matrix. For the purposes of closed-loop stability analysis, the external force of interest is the aerodynamic force induced by the motion. A survey of unsteady aerodynamic influence coefficients (AICs) has been given in Reference 3. For present purposes it is sufficient to write the aerodynamic force as

$$\{F\} = -[\bar{Q}_a]\{\ddot{h}\} \quad (2-14)$$

where $[\bar{Q}_a]$ is a complex matrix of oscillatory AICs valid only for harmonic motion. The AICs $[\bar{Q}_a]$ are dependent on the planform, the altitude, the flight Mach number, and the reduced frequency (Strouhal number) of the motion $k = \omega b_r/V$ where ω is the frequency, b_r is a reference dimension, and V is the velocity of flight. This limitation to harmonic motion reflects the state-of-the-art of unsteady aerodynamic theory, viz., considerably more solutions have been found for harmonic motion than for arbitrary transient motion. For this reason flutter analysis has traditionally required a trial-and-error solution to find the velocity and frequency for neutrally stable oscillations. To the same extent the aero-servo-elastic stability analysis must be carried out by trial and error. Equations (2-13) and (2-14) may be combined to appear as

$$[\bar{M}]\{\ddot{h}\} + [C]\{\dot{h}\} + [K](\{h\} - [h_\delta]\{\delta\}) = 0 \quad (2-15)$$

where the mass matrix now includes the aerodynamics.

$$[\bar{M}] = [M] + [\bar{Q}_a] \quad (2-16)$$

The inputs to the hydraulic actuators come from rate gyro and accelerometer feedback loops. The angular velocity of a rate gyro may be found by numerical differentiation of the displacement velocities.

$$\dot{h}' = [D] \{\dot{h}\} \quad (2-17)$$

where $[D]$ is a differentiation row matrix. Methods of numerical differentiation have been discussed thoroughly, e. g., by Milne⁴. If the differentiation is carried out locally, i. e., "in-the-small", then the elements of $[D]$ will only be nonzero for control points surrounding the gyro location, e. g., parabolic differentiation would involve only the three control points closest to the gyro. The acceleration at an accelerometer is found by numerical interpolation of the displacement accelerations.

$$\ddot{h} = [H] \{\ddot{h}\} \quad (2-18)$$

where $[H]$ is an interpolation row matrix. Methods of numerical interpolation have also been discussed by Milne⁴. If the interpolation is also carried out locally, the elements of $[H]$ will also only be nonzero for control points surrounding the accelerometer location.

Since the aeroelastic system has a large number of degrees of freedom, it is desirable to reduce them by a series or modal method. We write the deflections in terms of a series of vibration modes, rigid-body modes, and the shaft rotation(s) of the flipper(s).

$$\{h\} = [h_F] \{a_F\} + [h_R] \{a_R\} + [h_\delta] \{\delta\} \quad (2-19)$$

where $[h_F]$ is a matrix of restrained and/or free-free vibration modes, $\{a_F\}$ is the corresponding set of generalized coordinates of the vibration modes, $[h_R]$ is the matrix of rigid body deflection modes, and $\{a_R\}$ is the set of amplitudes of the rigid body motions. Substituting Equation (2-19) into Equation (2-15) leads to a truncation error in the solution since the series, Equation (2-19), is finite. The error matrix may be interpreted physically as a distributed force on the system. If we impose the condition that this error does no work in any of the flexible or rigid modes (N. B., this is the method of Galerkin) then the generalized equations of motion are found by premultiplying Equation (2-15) by $[h_F]^T$ and setting

to zero, and also by premultiplying by $[h_R]^T$ and setting to zero. Substituting Equation (2-19) into Equation (2-15) and premultiplying by $[h_F]^T$ leads to

$$\begin{aligned} & [\bar{m}_F] \{\ddot{a}_F\} + [\bar{m}_{FR}] \{\ddot{a}_R\} + [\bar{m}_{F\delta}] \{\ddot{\delta}\} \\ & + [c_F] \{\dot{a}_F\} + [k_F] \{a_F\} = 0 \end{aligned} \quad (2-20)$$

where we have noted that rigid body displacements produce no internal damping or structural forces, and

$$[\bar{m}_F] = [h_F]^T [\bar{M}] [h_F] \quad (2-21)$$

$$[\bar{m}_{FR}] = [h_F]^T [\bar{M}] [h_R] \quad (2-22)$$

$$[\bar{m}_{F\delta}] = [h_F]^T [\bar{M}] [h_\delta] \quad (2-23)$$

$$[c_F] = [h_F]^T [C] [h_F] \quad (2-24)$$

and

$$[k_F] = [h_F]^T [K] [h_F] \quad (2-25)$$

We also have noted that the generalized stiffness matrix has a diagonal form by virtue of the orthogonality of vibration modes, whether they be restrained modes or free-free modes,* and can be written in terms of the generalized mass matrix and frequency matrix as

$$[k_F] = [\omega_F^2] [m_F] \quad (2-26a)$$

$$= [\omega_F^2 m_F] \quad (2-26b)$$

where

$$[m_F] = [h_F]^T [M] [h_F], \text{ diagonal elements} \quad (2-27)$$

* This is also the case for arbitrarily chosen modes, e. g., modes given by polynomial expressions, if their associated frequencies can be defined. However, inclusion of arbitrary modes leads to a convergence requirement of a large number of terms in Equation (2-19) and therefore will not be permitted in this formulation.

and ω_F is the frequency of the vibration mode $\{h_F\}$. The generalized damping matrix does not have a diagonal form, as does the generalized stiffness, but may be assumed so as an approximate means of including structural damping. If we represent the structural damping in its equivalent viscous form we may write

$$[c_F] = [k_F g_F / \omega_F] \quad (2-28a)$$

$$= [g_F \omega_F m_F] \quad (2-28b)$$

where g_F is the structural damping coefficient (N. B., $g_F = 2\zeta_F$ where ζ is the ratio of viscous damping coefficient to the critical viscous damping coefficient) corresponding to the vibration mode $\{h_F\}$.

Substituting Equation (2-19) into Equation (2-15) and premultiplying by $[h_R]^T$ leads to

$$[\bar{m}_{RF}]\{\ddot{a}_F\} + [\bar{m}_R]\{\ddot{a}_R\} + [\bar{m}_{R\delta}]\{\ddot{\delta}\} = 0 \quad (2-29)$$

where we again have noted that rigid body displacements produce no internal damping or structural forces, and

$$[\bar{m}_{RF}] = [h_R]^T [\bar{M}] [h_F] \quad (2-30)$$

$$[\bar{m}_R] = [h_R]^T [\bar{M}] [h_R] \quad (2-31)$$

and
$$[\bar{m}_{R\delta}] = [h_R]^T [\bar{M}] [h_\delta] \quad (2-32)$$

If the generalized aerodynamic forces for specific modes are considered, we may rewrite Equations (2-21)-(23) and (30)-(32) as

$$[\bar{m}_F] = [m_F] + [\bar{Q}_F] \quad (2-33)$$

$$[\bar{m}_{FR}] = [m_{FR}] + [\bar{Q}_{FR}] \quad (2-34)$$

$$[\bar{m}_{F\delta}] = [m_{F\delta}] + [\bar{Q}_{F\delta}] \quad (2-35)$$

$$[\bar{m}_R] = [m_R] + [\bar{Q}_R] \quad (2-36)$$

$$[\bar{m}_{RF}] = [m_{FR}]^T + [\bar{Q}_{RF}] \quad (2-37)$$

$$[\bar{m}_{R\delta}] = [m_{R\delta}] + [\bar{Q}_{R\delta}] \quad (2-38)$$

where

$$[m_{FR}] = [h_F]^T [M] [h_R] \quad (2-39)$$

$$[m_{F\delta}] = [h_F]^T [M] [h_\delta] \quad (2-40)$$

$$[m_R] = [h_R]^T [M] [h_R] \quad (2-41)$$

and $[m_{R\delta}] = [h_R]^T [M] [h_\delta] \quad (2-42)$

Note that $[m_{FR}]$ vanishes if free-free modes are used throughout. Also, $[m_R]$ is the rigid body mass matrix consisting of total mass, static unbalances about the coordinate origin (not necessarily the centroid), and moments and products of inertia about the coordinate origin. The generalized aerodynamic forces are found (e. g., from Volumes I or II of this report) for the appropriate vibration, rigid body, and flipper modes, including their coupling.

The matrix partitions in Equation (2-3) may now be defined as follows:

$$[X_{\ddot{x}}] \{\ddot{x}\} = \begin{matrix} & \begin{matrix} m & n & c & s \end{matrix} \\ \begin{matrix} m \\ n \\ c \\ ns \end{matrix} & \begin{bmatrix} [\bar{m}_F] & [\bar{m}_{FR}] & [\bar{m}_{F\delta}] & 0 \\ [\bar{m}_{RF}] & [\bar{m}_R] & [\bar{m}_{R\delta}] & 0 \\ 0 & 0 & [CC] & [CS2] \\ [FSA] & [RSA] & [SC] & [SS2] \end{bmatrix} \end{matrix} \begin{Bmatrix} \ddot{a}_F \\ \ddot{a}_R \\ \ddot{\delta} \\ \ddot{e}_2 \end{Bmatrix} \quad (2-43)$$

$$[X_{\dot{x}}] \{\dot{x}\} = \begin{matrix} & \begin{matrix} m & n & c & s \end{matrix} \\ \begin{matrix} m \\ n \\ c \\ ns \end{matrix} & \begin{bmatrix} [c_F] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & [CC] & [CS2] \\ [FSG] & [RSG] & [SC] & [SS2] \end{bmatrix} \end{matrix} \left\{ \begin{matrix} \dot{a}_F \\ \dot{a}_R \\ \dot{\delta} \\ \dot{e2} \end{matrix} \right\} \quad (2-44)$$

$$[X_x] \{x\} = \begin{matrix} & \begin{matrix} m & n & c & s \end{matrix} \\ \begin{matrix} m \\ n \\ c \\ ns \end{matrix} & \begin{bmatrix} [k_F] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & [CC] & [CS2] \\ 0 & 0 & [SC] & [SS2] \end{bmatrix} \end{matrix} \left\{ \begin{matrix} a_F \\ a_R \\ \delta \\ e2 \end{matrix} \right\} \quad (2-45)$$

where e2 refers to the set of second order servo elements and

- CC = Control surface coefficients.
- CS2 = Coefficients for second order servo element from which there is input to a control surface.
- FSA = Elements of the matrix $-K_a[H][h_F]$ for the accelerometer into which there is input from the body acceleration.
- FSG = Elements of the matrix $-K_g[D][h_F]$ for the rate gyro into which there is input from the body angular rate.
- RSA = Elements of the matrix $-K_a[H][h_R]$ for the accelerometer into which there is input from the body acceleration.
- RSG = Elements of the matrix $-K_g[D][h_R]$ for the rate gyro into which there is input from the body angular rate.
- SC = Coefficients for control surface from which there is input to a servo element.
- SS2 = Coefficients for second order servo element from which there is input to another servo element.

The number of rows and columns in each partition is indicated by

- m = Number of flexible modes.
- n = Number of rigid body modes.
- c = Number of control surfaces.
- ns = Total number of servo elements (zero, first and second order), $ns = s + q + r$.
- s = Number of second order servo elements.
- q = Number of first order servo elements.
- r = Number of zero order servo elements.

$$[Y_{\dot{y}}] \{\dot{y}\} = \begin{matrix} & q \\ m & \begin{bmatrix} 0 \\ 0 \\ [CS1] \\ [SS1] \end{bmatrix} \end{matrix} \{e1\} \quad (2-46)$$

$$[Y_y] \{y\} = \begin{matrix} & q \\ m & \begin{bmatrix} 0 \\ 0 \\ [CS1] \\ [SS1] \end{bmatrix} \end{matrix} \{e1\} \quad (2-47)$$

where $e1$ refers to the set of first order servo elements and

- $CS1$ = Coefficients for first order servo element from which there is input to a control surface.
- $SS1$ = Coefficients for first order servo element from which there is input to another servo element.

$$[Z_z] \{z\} = \begin{matrix} & r \\ m & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ n & \\ c & [CS0] \\ ns & [SS0] \end{matrix} \{e0\} \quad (2-48)$$

where $e0$ refers to the set of zero order servo elements and

CS0 = Coefficients for zero order servo element from which there is input to a control surface.

SS0 = Coefficients for zero order servo element from which there is input to another servo element.

A study of the format of Equations (2-43)-(48) shows that each of the six matrices is composed of partitions that may be described as aeroelastic, servoeelastic, or simply servo terms. PASES, a computer program designed to calculate closed-loop aero-servo-elastic stability, forms the various matrices from the servo partitions which are input directly and the aeroelastic partitions which are generated from basic input (e. g. , modes, frequencies, mass, and aerodynamic data).

APPENDIX B

AN APPLICATION OF PROGRAM MPASES

TO A

TYPICAL AIR-TO-AIR MISSILE

Introduction

The computer program MPASES developed in this report used the same example problem to demonstrate its usage as had been used in the original version of the program. That example of an air-to-air missile was not a very practical one to the extent that it had no wing, the weight of the aft end of the fuselage was assumed to oscillate with the control surface (flipper), and there was only a single (rate-gyro) feedback loop. In addition, the example used an incorrect transfer function for the actuator [Eq. (78) should have read $\delta/e_3 = K_a/s(T_a s + 1)$].

The new example studied here is still somewhat idealized but is more practical to the extent that a wing, with its attendant aerodynamic loads, is added, and a light-weight dynamically balanced flipper with bending and torsion degrees of freedom is considered separately from the body. A more realistic servo system is also considered with three feedback loops, a rate feedback loop (with a rate-gyro as before), and attitude and acceleration feedback loops.

In addition to being a more practical problem which demonstrates a more general use of Program MPASES, the sample also illustrates certain peripheral aspects of the problem. One is the calculation of a coupled mass matrix for the flipper. Another is the calculation of the wing and

flipper aerodynamic influence coefficients from Piston Theory for a more practical range of supersonic Mach numbers and their assembly to obtain the proper partitioned format for the complete vehicle.

The servoelastic stability problem is considered first, and then the aerodynamic influence coefficients are added next to consider the aero-servo-elastic stability analysis.

Typical Air-to-Air Missile Example

The typical air-to-air missile is idealized as shown in Fig. 3. The fuselage is idealized as a uniform beam with ten equally spaced masses. The wing is assumed to be rectangular and massless, rigid in the spanwise direction and to bend with the fuselage in the chordwise direction. The flipper is also rectangular and is assumed to be rigid in both spanwise and chordwise directions but to have root flexibility in bending and torsion. The flipper mass distribution is simulated by eight masses connected by rigid massless bars to the four degrees of freedom of the flipper. The coupled mass matrix for the flipper is obtained from the least squares solution in Appendix B of Ref. 12. The various input matrices to MPASES are derived below.

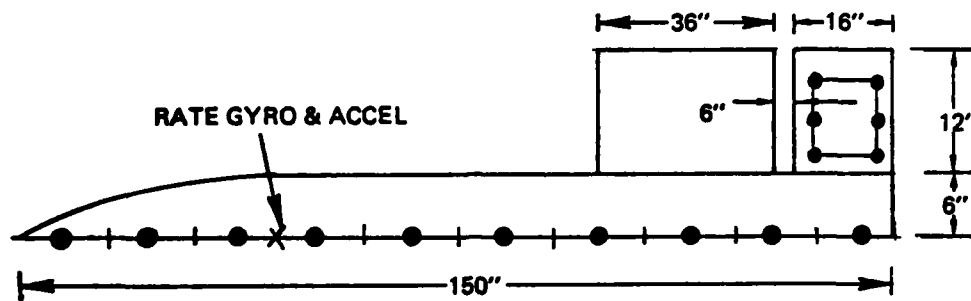


Fig. 3 - Idealization of Typical Air-to-Air Missile

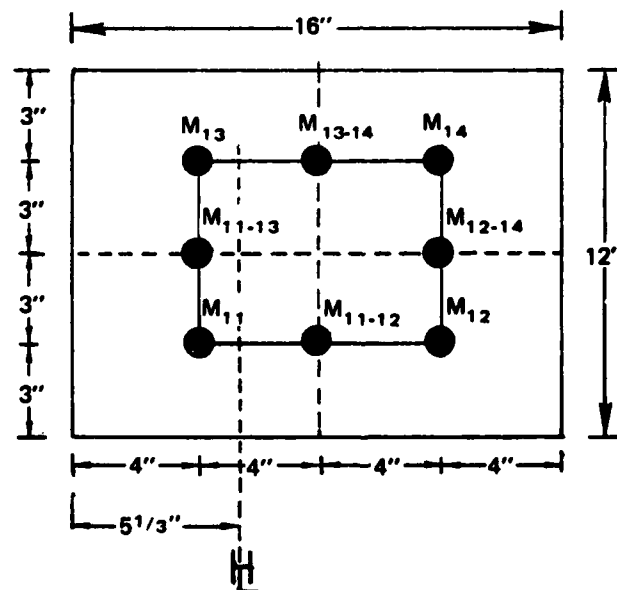


Fig. 4 - Idealization of Flipper Mass Distribution

The Mass Matrix

The missile fuselage is assumed to weigh 1000.0 lbs. and each flipper weighs 5.0 lbs. The ten equal fuselage weights on the half-fuselage are 50.0 lbs. apiece. The flipper idealization is shown in Fig. 4. The flipper center of gravity is assumed to be 0.50 in. forward of the hinge line and 3.86 in. outboard of the fuselage side; the hinge line is located 5.333 in. aft of the leading edge. The pitching moment of inertia about the hinge line is 50.0 lb-in² and the rolling moment of inertia about the fuselage side is 137 lb-in². The flipper is balanced so it has no product of inertia with respect to the hinge line and fuselage side.

The input to Program MASS consists of the inertial data and the coordinates of the eight mass points. The annotated input deck and program output are shown below in Tables 1 and 2, respectively.

The coupled mass matrix for the four flipper degrees of freedom is:

$$[M] = \begin{bmatrix} M_{11} + (\frac{1}{2})(M_{11-12} + M_{11-13}) & (\frac{1}{2})M_{11-12} & (\frac{1}{2})M_{11-13} & 0 \\ (\frac{1}{2})M_{11-12} & M_{12} + (\frac{1}{2})(M_{11-12} + M_{12-14}) & 0 & (\frac{1}{2})M_{12-14} \\ (\frac{1}{2})M_{11-13} & 0 & M_{13} + (\frac{1}{2})(M_{11-13} + M_{13-14}) & (\frac{1}{2})M_{13-14} \\ 0 & (\frac{1}{2})M_{12-14} & (\frac{1}{2})M_{13-14} & M_{14} + (\frac{1}{2})(M_{12-14} + M_{13-14}) \end{bmatrix}$$

where, from the program output:

$$M_{11} = 4.828472$$

$$M_{11-12} = 0.004167$$

$$M_{12} = 1.695139$$

$$M_{11-13} = -1.584722$$

$$M_{13} = 1.825694$$

$$M_{12-14} = -2.904167$$

$$M_{14} = 2.320139$$

$$M_{13-14} = -1.184722$$

Table 1 - Input Cards for Program MASS

```

C      TITLE AND SUBTITLE CARDS
      LEAST SQUARES MASS MATRIX
      MASS ELEMENTS FOR TYPICAL AIR-TO-AIR MISSILE
C      NUMBER OF STRIPS
      1
C      NUMBER OF MASSES
      6
C      INERTIAL PROPERTIES
      5.0      2.5      19.3      50.0      137.0      0.0
C      MASS POINT COORDINATES
      1.33333333 3.0      -2.66666667 3.0      -6.66666667 3.0
      1.33333333 6.0      1.33333333 9.0      -6.66666667 6.0
      -6.66666667 9.0      -2.66666667 9.0
  
```

Table 2 - Output from Program MASS

LEAST SQUARES MASS MATRIX
MASS ELEMENTS FOR TYPICAL AIR-TO-AIR MISSILE

MASS MATRIX INPUT DATA
1 STRIPS

NUMBER OF MASSES BY STRIP

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

STRIP 1

TOTAL MASS M = .50000000E+01
X UNBALANCE MA BAK = .25000000E+01
Y UNBALANCE MY BAK = .14300000E+02
X MOMENT OF INERTIA I SUB Y = .50000000E+02
Y MOMENT OF INERTIA I SUB X = .13700000E+03
PRODUCT OF INERTIA I SUB XY = 0.

X(1) = .13333333E+01	Y(1) = .30000000E+01
X(2) = -.20000000E+01	Y(2) = .30000000E+01
X(3) = -.66666667E+01	Y(3) = .30000000E+01
X(4) = .13333333E+01	Y(4) = .60000000E+01
X(5) = .13333333E+01	Y(5) = .40000000E+01
X(6) = -.66666667E+01	Y(6) = .60000000E+01
X(7) = -.66666667E+01	Y(7) = .40000000E+01
X(8) = -.20000000E+01	Y(8) = .40000000E+01

M = 6 MM = 2

(1) BASIC MASS AND REDUNDANT MASS GEOMETRY MATRICES

.500000E+01	.100000E+01	.100000E+01	.100000E+01	.100000E+01	.100000E+01	.100000E+01
.250000E+01	.133333E+01	-.200000E+01	-.666667E+01	.133333E+01	.133333E+01	-.666667E+01
.193000E+02	.300000E+01	.300000E+01	.300000E+01	.600000E+01	.400000E+01	.600000E+01
.500000E+02	.177778E+01	.711111E+01	.444444E+02	.177778E+01	.177778E+01	.444444E+02
.137000E+03	.400000E+01	.400000E+01	.400000E+01	.300000E+02	.610000E+02	.300000E+02
0.	.400000E+01	-.800000E+01	-.200000E+02	.600000E+01	.120000E+02	-.400000E+02
	.100000E+01	.100000E+01				
	-.666667E+01	-.200000E+01				
	.400000E+01	.400000E+01				
	.444444E+02	.711111E+01				
	.610000E+02	.610000E+02				
	-.600000E+02	-.240000E+02				

MASS MATRIX

STRIP 1 STRIP

.4828722E+01
.4100000E+02
.1445130E+01
-.1504722E+01
.1025090E+01
-.2904100E+01

.2320130E+01
-.1104722E+01

As discussed in Appendix B of Ref. 12, the negative values have no particular significance. The numerical value of the coupled mass matrix becomes

$$[M] = \begin{bmatrix} 4.43333 & 0.0010417 & -0.396181 & 0.0 \\ & 0.970139 & 0.0 & -0.726042 \\ & & 1.133333 & -0.296181 \\ \text{symmetrical} & & & 1.297917 \end{bmatrix}$$

Modal Matrices and Frequencies

The modal matrices and frequencies for the fuselage are the same as those used in the MPASES example problems with the addition of the deflections of the four flipper mass points. These are found from numerical interpolation and differentiation among the last four fuselage control point deflections, i.e., mass points 7, 8, 9, and 10, to find the deflection and slope at the flipper hinge line from which the deflections on the flipper follow. The Lagrangian interpolation formula is

$$\begin{aligned}h(x) = & h_7 \frac{(x-x_8)(x-x_9)(x-x_{10})}{(x_7-x_8)(x_7-x_9)(x_7-x_{10})} \\& + h_8 \frac{(x-x_7)(x-x_9)(x-x_{10})}{(x_8-x_7)(x_8-x_9)(x_8-x_{10})} \\& + h_9 \frac{(x-x_7)(x-x_8)(x-x_{10})}{(x_9-x_7)(x_9-x_8)(x_9-x_{10})} \\& + h_{10} \frac{(x-x_7)(x-x_8)(x-x_9)}{(x_{10}-x_7)(x_{10}-x_8)(x_{10}-x_9)}\end{aligned}$$

The hinge line is at $x = 139.333$ and the remaining coordinates are $x_7 = 97.5$, $x_8 = 112.5$, $x_9 = 127.5$, and $x_{10} = 142.5$, so the interpolation formula becomes

$$h_{HL} = 0.049654 h_7 - 0.232235 h_8 + 0.526618 h_9 + 0.655963 h_{10}$$

By differentiating the interpolation formula, we obtain the fuselage slope at the hinge line, and the result is

$$h'_{HL} = -0.009633745 h_7 + 0.048160494 h_8 \\ -0.134086420 h_9 + 0.095559671 h_{10}$$

From the hinge line deflection and slope the flipper deflections are found from

$$h_{11} = h_{13} = h_{HL} - 1.333333 h'_{HL}$$

$$h_{12} = h_{14} = h_{HL} + 6.666667 h'_{HL}$$

This generates the four flipper deflections required to complete the description of the three fuselage modes.

The fourth and fifth vibration modes are bending and torsion of the flipper uncoupled with the fuselage. In bending, if $h_{13} = h_{14} = 1.0$, then $h_{11} = h_{12} = 0.333333$ since the flipper is assumed rigid, and in torsion, if $h_{12} = h_{14} = 1.0$, then $h_{11} = h_{13} = -0.2$, as can be seen from the geometry in Fig. 4.

The control surface deflection mode is similar to the flipper torsion mode except that it corresponds to a unit control surface rotation (one radian, positive trailing edge up), so that $h_{11} = h_{13} = 1.333333$ and $h_{12} = h_{14} = -6.666667$.

The previously assumed frequencies for the fuselage were 45.0, 125.4, and 248.2 Hz. The flipper frequencies are assumed to be 100.0 Hz in bending and 115.0 Hz in torsion.

The damping coefficients in the three fuselage modes are assumed to be $g = 0.01$, 0.02 , and 0.03 , respectively, and $g = 0.03$ is also assumed for both flipper modes.

The rigid body modes require the addition of the four flipper mass points: four unit plunging displacements, and four x-coordinates for pitching about the fuselage nose, $x_{11} = x_{13} = 138.0$ and $x_{12} = x_{14} = 146.0$.

Servo System

The servo system block diagram is shown in Fig. 5. The subscripts on the outputs are the same as the servo element numbers. The system consists of three loops with rate, attitude, and acceleration feedbacks. The transfer functions and their corresponding differential equations and numerical values are discussed in the following sections for each component in the three loops.

Control Surface Actuator - The actuator servo is assumed to move the control surface shaft with an angular rate proportional to the difference between the desired and actual angular displacements. Its transfer function is

$$Y_{FS} = \frac{\delta}{e_5} = \frac{1}{s/K_v + 1}$$

and the corresponding differential equation is

$$(1/K_v)\dot{\delta} + \delta - e_5 = 0$$

The numerical value $1/K_v = 0.016$ sec. is used in the analysis.

Servo Element No. 1 - The rate gyro transfer function is

$$e_1/\dot{\theta} = \frac{K_g}{s^2/\omega_g^2 + (2\zeta_g/\omega_g)s + 1}$$

and the differential equation is

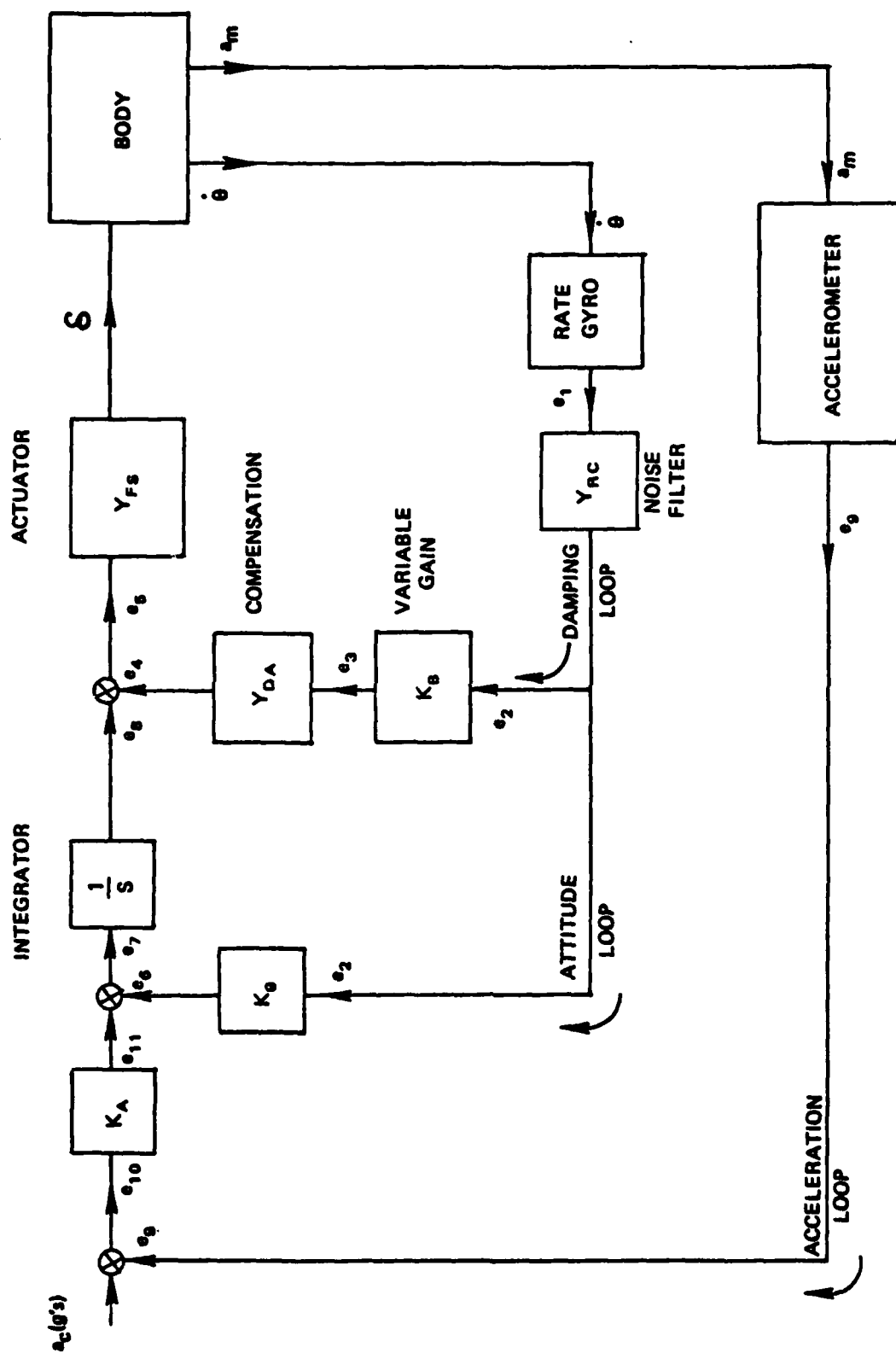


Fig. 5 - Servo System Block Diagram

$$(1/\omega_g^2)\ddot{e}_1 + (2\zeta_g/\omega_g)\dot{e}_1 + e_1 - K_g\dot{\theta} = 0$$

The numerical values are taken as $1/\omega_g^2 = 0.659 \times 10^{-5} \text{ sec}^2$,

$2\zeta_g/\omega_g = 0.359 \times 10^{-2} \text{ sec}$, and $K_g = 1.0 \text{ deg/sec per deg/sec}$.

The numerical differentiation matrix to obtain the rate $\dot{\theta}$ is based on a cubic spline fit among the deflections of mass points 2, 3, 4, and 5. The differentiation matrix is (see Ref. 11)

$$[D] = (1/24\ell)[0 \quad +1 \quad -27 \quad +27 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

where $\ell = 15.0 \text{ in.}$, the distance between fuselage mass points.

The coefficient $1/24\ell = 0.002777778$ is input as the rate gyro gain, and the integer elements of the differentiation matrix are input directly.

Servo Element No. 2 - The noise filter transfer function and differential equation are

$$Y_{RC} = e_2/e_1 = \frac{1}{\tau s + 1}$$

and

$$\tau\dot{e}_2 + e_2 - e_1 = 0$$

where $\tau = 0.001 \text{ sec}$.

Servo Element No. 3 - For the variable gain amplifier we use

$$e_3/e_2 = K_B$$

or

$$e_3 - K_B e_2 = 0$$

where $K_B = 0.1 \text{ deg/deg/sec}$ for a Mach number of 3.0 at sea level. Although the aero-servo-elastic stability analysis covers a range of supersonic Mach numbers, a constant value of K_B is used.

Servo Element No. 4 - The compensating damping amplifier equations are

$$Y_{DA} = e_4/e_3 = \frac{T_d s + 1}{s^2/\omega_d^2 + (2\zeta_d/\omega_d)s + 1}$$

and

$$(1/\omega_d^2)\ddot{e}_4 + (2\zeta_d/\omega_d)\dot{e}_4 + e_4 - T_d\dot{e}_3 - e_3 = 0$$

where $1/\omega_d^2 = 0.253 \times 10^{-5} \text{ sec}^2$, $2\zeta_d/\omega_d = 0.223 \times 10^{-2} \text{ sec}$, and $T_d = 0.016 \text{ sec}$.

Servo Element No. 5 - The equation for the rate loop feedback junction is

$$e_5 = e_8 - e_4$$

or

$$e_5 + e_4 - e_8 = 0$$

Servo Element No. 6 - The attitude amplifier gain is

$$e_6/e_2 = K_\theta$$

or

$$e_6 - K_\theta e_2 = 0$$

where $K_9 = 2.0 \text{ deg/sec per deg/sec}$.

Servo Element No. 7 - The equation for the attitude loop feedback junction is

$$e_7 = e_{11} - e_6$$

or

$$e_7 + e_6 - e_{11} = 0$$

Servo Element No. 8 - The electronic integrator in the attitude loop has the equation

$$e_8/e_7 = 1/s$$

or

$$\dot{e}_8 - e_7 = 0$$

Servo Element No. 9 - The accelerometer has the transfer function

$$e_9/a_m = \frac{1}{s^2/\omega_a^2 + (2\zeta_a/\omega_a)s + 1}$$

and equation of motion

$$(1/\omega_a^2)\ddot{e}_9 + (2\zeta_a/\omega_a)\dot{e}_9 + e_9 - a_m = 0$$

where $1/\omega_a^2 = 1.013 \times 10^{-5} \text{ sec}^2$ and $2\zeta_a/\omega_a = 0.6366 \times 10^{-2} \text{ sec}$.

The numerical interpolation matrix to obtain the acceleration a_m is also based on a cubic spline fit among the deflections of mass points 2, 3, 4, and 5. The interpolation matrix is (see Ref. 11)

$$[H] = (1/16)[0 \quad -1 \quad +9 \quad +9 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

The coefficient $1/16 = 0.0625$ is input as the accelerometer gain, and the integer elements of the interpolation matrix are input directly.

Servo Element No. 10 - The acceleration loop feedback equation is

$$e_{10} = a_c - e_9$$

which reduces to

$$e_{10} + e_9 = 0$$

for the stability problem.

Servo Element No. 11 - The acceleration loop gain is

$$e_{11}/e_{10} = K_a$$

or

$$e_{11} - K_a e_{10} = 0$$

where $K_a = 4.0 \text{ deg/sec per } g = 0.002170 \text{ sec/ft.}$

Servoelastic Stability Analysis

The input data cards are prepared according to the MPASES input instructions in Sect. III. The data are found in the previous sections of this appendix with the exception of the fuselage vibration modes which are found in the original MPASES example problem; the vibration modes for the present example are found by combining the previous fuselage modes with the additional flipper mass point deflections derived above in this report. The shift eigenvalue $\gamma_0 = +100.0$ is chosen to scale [A] to be of the same order of magnitude as [B] and with the same sign. The input data deck, with comments added, is reproduced in Table 3.

The printed output is reproduced in Table 4 except that only the modes and deflections corresponding to eigenvalues of practical interest are shown. (The modes and deflections corresponding to zero or infinite eigenvalues are not shown.)

The stability analysis results come from the solution of a 30th order eigenvalue problem. The order of the eigenvalue problem is found from

$$N = 2(m+n+c) + ns + s$$

where N = order of the eigenvalue problem

m = number of flexible modes

n = number of rigid body modes

c = number of control surfaces
 ns = number of servo elements
 s = number of second order servo elements.

In this example, $m = 5$, $n = 2$, $c = 1$, $ns = 11$, and $s = 3$, so that $N = 30$. The stability analysis results are printed for 22 modes since 8 of the eigenvalues are complex conjugate pairs. Only modes 4-7, and 16-22 are of practical interest. The seven oscillatory and four non-oscillatory solutions are all seen to be stable. The motions involved in the modes are summarized as follows.

<u>Mode No.</u>	<u>Dominant Motion</u>
4	Actuator
5 & 6	Accelerometer (critically damped)
7	Noise filter
16	Rate gyro
17	First body bending
18	Compensating damping amplifier
19	Flipper bending
20	Flipper torsion
21	Second body bending
22	Third body bending

Table 3 - Input Cards for Program MPASES for
Servoelastic Stability Analysis

```

C      TITLE CARDS
TYPICAL AIM-FU-AIM MISSILE
SERVOELASTIC ANALYSIS
C      CONTROL CARD
      2      14      5      2      1      11      0      0      0      0      50      1      0      1
C      SHIFT EIGENVALUE
100.0
C      MASS MATRIX
C      1ST ROW
50.0      0.0      0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0      0.0
C      2ND ROW
50.0      0.00      0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0      0.0
C      3RD ROW
50.0      0.0      0.00      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0      0.0
C      4TH ROW
50.0      0.0      0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0      0.0
C      5TH ROW
50.0      0.0      0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0      0.0
C      6TH ROW
50.0      0.0      0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0      0.0
C      7TH ROW
50.0      0.0      0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0      0.0
C      8TH ROW
50.0      0.0      0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0      0.0
C      9TH ROW
50.0      0.0      0.0      0.0      0.0      0.0
C      10TH ROW
50.0      0.0      0.0      0.0      0.0
C      11TH ROW
4.4333333  0.0010417 -0.3961806  0.0
C      12TH ROW
0.9701389  0.0      -0.7260417
C      13TH ROW
1.1333332 -0.2961806
C      14TH ROW
1.2979166
C      CONTROL SURFACE DEFLECTION MODE
0.0      0.0      0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      1.333333333 -6.666666667
1.333333333 -6.666666667
C      DAMPING COEFFICIENTS, G
0.01      0.02      0.03      0.03      0.03
C      MODAL VIBRATION FREQUENCIES
45.0      125.4      248.2      100.0      115.0
C      VIBRATION MODE SHAPES
C      1ST MODE
1.0      0.411195 -0.122424 -0.532567 -0.756204 -0.756204
-0.532467 -0.122424 0.411195 1.0      0.821620 1.138881
0.821620 1.138881
C      2ND MODE
1.0      -0.150871 -0.910842 -0.990216 -0.419043 0.419043

```

0.990215	0.910842	0.150871	-1.0	-0.630256	-1.281954
-0.630256	-1.281954				
C 3RD MODE					
-0.803195	0.645103	1.0	0.105986	-0.947894	-0.947893
0.105986	1.0	0.645103	-0.803195	-0.259298	-1.188204
-0.259298	-1.188204				
C 4TH MODE					
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	.3333333333	.3333333333
1.0	1.0				
C 5TH MODE					
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	-0.2	1.0
-0.2	1.0				
C RIGID BODY MODES					
C PLUNGE MODE					
1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0				
C PITCH MODE					
7.5	22.5	37.5	52.5	67.5	82.5
97.5	112.5	127.5	142.5	158.0	146.0
138.0	146.0				
C CONTROL SURFACE OUTPUT					
1.0	0.016	0.0			
C NUMBER OF INPUTS TO CONTROL SURFACE					
1					
C CONTROL SURFACE INPUTS FROM SERVOS					
5	0	-1.0			
C OUTPUT FROM SERVO ELEMENT					
1	2	1.0	0.359	-2 0.659	-5
C NUMBER OF INPUTS TO SERVO ELEMENT					
1					
C INPUT FROM BODY PITCH RATE					
0	-1	-2.7777778	-3		
C DIFFERENTIATION MATRIX FOR RATE GYRO					
0.0	1.0	-27.0	27.0	-1.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0				
C OUTPUT FROM SERVO ELEMENT					
2	1	1.0	0.001		
C NUMBER OF INPUTS TO SERVO ELEMENT					
1					
C INPUTS TO SERVO ELEMENT					
1	0	-1.0			
C OUTPUT FROM SERVO ELEMENT					
3	0	1.0			
C NUMBER OF INPUTS TO SERVO ELEMENT					
1					
C INPUTS TO SERVO ELEMENT					
2	0	-0.1			
C OUTPUT FROM SERVO ELEMENT					
4	2	1.0	0.223	-2 0.253	-5
C NUMBER OF INPUTS TO SERVO ELEMENT					
1					
C INPUTS TO SERVO ELEMENT					
3	1	-1.0	-0.016		
C OUTPUT FROM SERVO ELEMENT					
5	0	1.0			

C NUMBER OF INPUTS TO SERVO ELEMENT
2

C INPUTS TO SERVO ELEMENT
4 0 1.0
8 0 -1.0

C OUTPUT FROM SERVO ELEMENT
6 0 1.0

C NUMBER OF INPUTS TO SERVO ELEMENT
1

C INPUTS TO SERVO ELEMENT
2 0 -2.0

C OUTPUT FROM SERVO ELEMENT
7 0 1.0

C NUMBER OF INPUTS TO SERVO ELEMENT
2

C INPUTS TO SERVO ELEMENT
6 0 1.0
11 0 -1.0

C OUTPUT FROM SERVO ELEMENT
8 1 0.0 1.0

C NUMBER OF INPUTS TO SERVO ELEMENT
1

C INPUTS TO SERVO ELEMENT
7 0 -1.0

C OUTPUT FROM SERVO ELEMENT
4 2 1.0 0.0266 -2 1.013 -5

C NUMBER OF INPUTS TO SERVO ELEMENT
1

C INPUT FROM BODY ACCELERATION
0 -2 -0.0625

C INTERPOLATION MATRIX FOR ACCELEROMETER
0.0 -1.0 4.0 4.0 -1.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0

C OUTPUT FROM SERVO ELEMENT
10 0 1.0

C NUMBER OF INPUTS TO SERVO ELEMENT
1

C INPUTS TO SERVO ELEMENT
9 0 1.0

C OUTPUT FROM SERVO ELEMENT
11 0 1.0

C NUMBER OF INPUTS TO SERVO ELEMENT
1

C OUTPUT FROM SERVO ELEMENT
10 0 -0.002170

Table 4 - Output from Program MPASES for Servoelastic Stability Analysis

TYPICAL AIR-TO-AIR MISSILE SERVOELASTIC ANALYSIS

SEMO ELASTIC STABILITY ANALYSIS

14 DEGREES OF FREEDOM
5 FLEXIBLE MODES
2 RIGID BODY MODES
1 CONTROL SURFACES
11 SERVO ELEMENTS

SHIFT EIGENVALUE (GAMMA) = 1.000E+02

as soon as possible

UPPER TRIANGLE OF WEIGHT MATRIX

[illegible]

ROW 7	5.00000E+01	0.	0.	0.	0.	0.	0.	0.
ROW 8	5.00000E+01	0.	0.	0.	0.	0.	0.	0.
ROW 9	5.00000E+01	0.	0.	0.	0.	0.	0.	0.
ROW 10	5.00000E+01	0.	0.	0.	0.	0.	0.	0.
ROW 11	4.43333E+00	1.00170E-03	-3.96101E-01	0.				
ROW 12	9.70119E-01	0.	-7.24022E-01					
ROW 13	1.13333E+00	-2.96101E-01						
ROW 14	1.20792E+00							

RIGID BODY CONTROL SURFACE MODES

MODE 1	0.	0.	0.	0.	0.	0.	0.	0.
0.	1.33333E+00	0.	0.66667E+00	1.33333E+00	0.	0.66667E+00	1.33333E+00	0.

FLEXIBLE MODES

MODE 1 - FREQUENCY = 45.000 CPS STRUCTURAL DAMPING COEFFICIENT = .010	1.00000E+00	0.11195E-01	-1.22024E-01	-5.32537E-01	-7.56200E-01	-7.56200E-01	-1.13000E+00	-5.32007E-01	-1.22024E-01
0.11195E-01	1.00000E+00	0.21020E-01	1.13000E+00	0.21020E-01	0.21020E-01	0.21020E-01	1.13000E+00	0.21020E-01	1.13000E+00
MODE 2 - FREQUENCY = 125.400 CPS STRUCTURAL DAMPING COEFFICIENT = .020	1.00000E+00	-1.50071E-01	-9.10002E-01	-9.00210E-01	-4.19003E-01	-4.19003E-01	-9.00210E-01	9.00210E-01	9.10002E-01
1.50071E-01	-1.00000E+00	-6.30256E-01	-1.20195E+00	-1.20195E+00	-6.30256E-01	-6.30256E-01	-1.20195E+00	-6.30256E-01	-1.20195E+00
MODE 3 - FREQUENCY = 200.200 CPS STRUCTURAL DAMPING COEFFICIENT = .030	-0.03195E-01	0.45103E-01	1.00000E+00	1.05900E-01	-0.47003E-01	-0.47003E-01	1.05900E-01	1.05900E-01	1.00000E+00
0.03195E-01	-0.03195E-01	-2.59200E-01	-1.10020E+00	-1.10020E+00	-2.59200E-01	-2.59200E-01	-1.10020E+00	-2.59200E-01	-1.10020E+00

[illegible]

00-300000-1
10-300000-2
00-300000-1
10-300000-2

00-300000-1
10-300000-2
00-300000-1
10-300000-2

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CONTROL SYSTEM DESCRIPTION

DIFFERENTIAL EQUATION FOR	VARIABLE	2ND ORDER	COEFFICIENTS 1ST ORDER	0 ORDER	GYRO/ACCEL. GAIN FACTOR
CONTROL SURF. 1	CONTROL SURF. 1	0.	1.000E-02	1.000E+00	
	SERVO ELEMENT 5	0.	0.	-1.000E+00	
SERVO ELEMENT 1 (RATE GYRO)	SERVO ELEMENT 1	0.5000E-06	1.5000E-03	1.000E+00	
	BODY ANGULAR RATE				-2.777E+03
SERVO ELEMENT 2	SERVO ELEMENT 2	0.	1.000E-03	1.000E+00	
	SERVO ELEMENT 1	0.	0.	-1.000E+00	
SERVO ELEMENT 3	SERVO ELEMENT 3	0.	0.	1.000E+00	
	SERVO ELEMENT 2	0.	0.	-1.000E-01	
SERVO ELEMENT 4	SERVO ELEMENT 4	2.5300E-06	2.2300E-03	1.000E+00	
	SERVO ELEMENT 3	0.	-1.6000E-02	-1.000E+00	
SERVO ELEMENT 5	SERVO ELEMENT 5	0.	0.	1.000E+00	
	SERVO ELEMENT 4	0.	0.	1.000E+00	
	SERVO ELEMENT 6	0.	0.	-1.000E+00	
SERVO ELEMENT 6	SERVO ELEMENT 6	0.	0.	1.000E+00	
	SERVO ELEMENT 2	0.	0.	-2.000E+00	
SERVO ELEMENT 7	SERVO ELEMENT 7	0.	0.	1.000E+00	
	SERVO ELEMENT 6	0.	0.	1.000E+00	
	SERVO ELEMENT 11	0.	0.	-1.000E+00	
SERVO ELEMENT 8	SERVO ELEMENT 8	0.	1.000E+00	0.	
	SERVO ELEMENT 7	0.	0.	-1.000E+00	
SERVO ELEMENT 9 (ACCELEROMETER)	SERVO ELEMENT 9	1.0130E-05	0.3600E-03	1.000E+00	
	BODY ACCELERATION				-0.2500E-02
SERVO ELEMENT 10	SERVO ELEMENT 10	0.	0.	1.000E+00	
	SERVO ELEMENT 9	0.	0.	1.000E+00	
SERVO ELEMENT 11	SERVO ELEMENT 11	0.	0.	1.000E+00	
	SERVO ELEMENT 10	0.	0.	-1.000E-02	

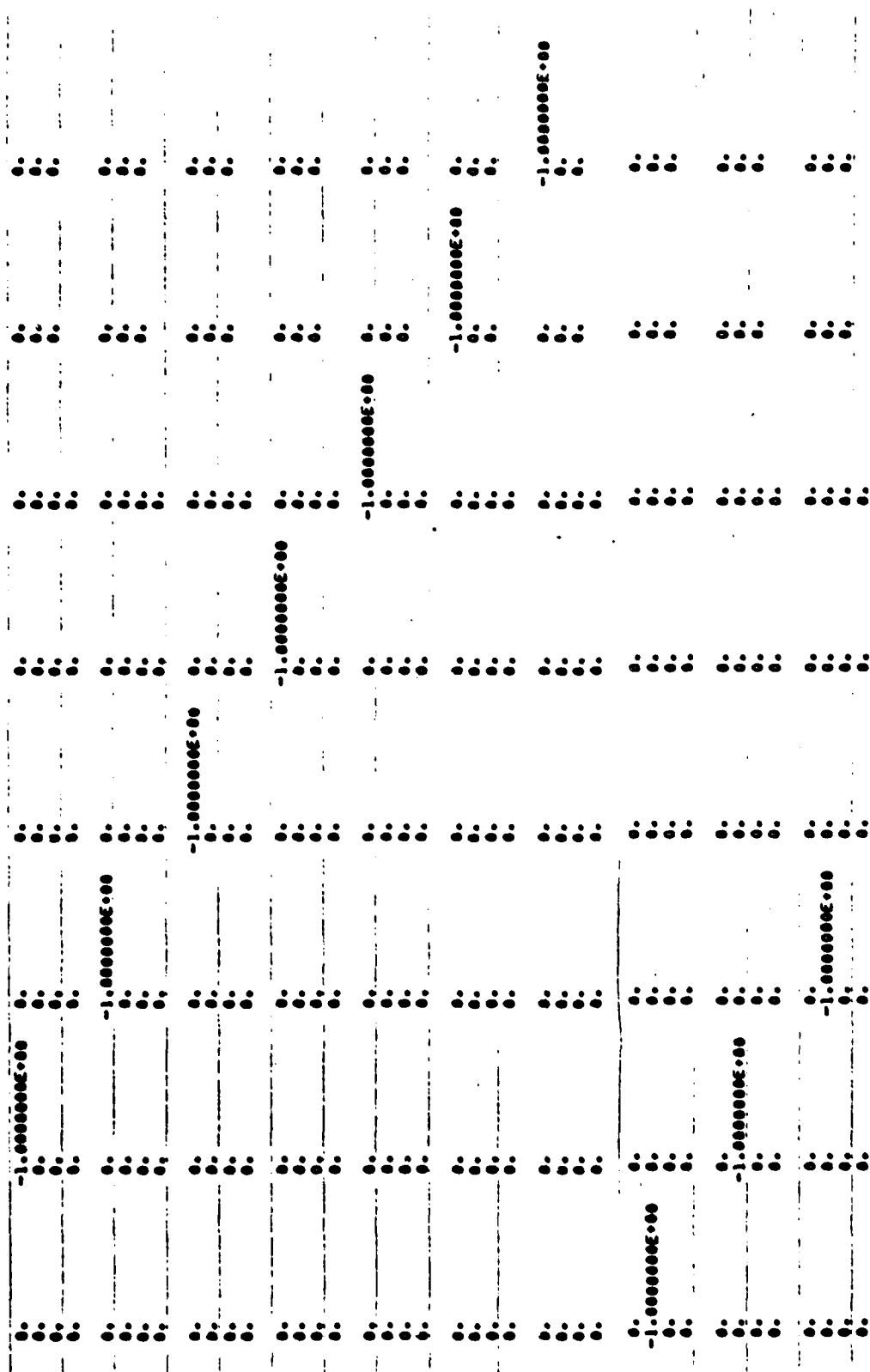
GYRO/ACCEL.		INPUT DIFFERENTIATION/INTERPOLATION ROW VECTOR									
SERVO ELEMENT	1	0.	1,000E+00	-2,700E+01	2,700E+01	-1,000E+00	0.	0.	0.	0.	0.
SERVO ELEMENT	9	0.	-1,000E+00	9,000E+00	9,000E+00	-1,000E+00	0.	0.	0.	0.	0.

SERVO ELEMENT	MAXIMUM ORDER	ROW ASSIGNMENT IN A AND/OR B	COLUMN ASSIGNMENT IN A AND/OR B	EIGENVECTOR ELEMENT	VELOCITY	DISPLACEMENT
1	2	9	9 AND 29	9	20	20
2	1	10	21		21	21
3	1	11	24		24	24
4	2	12	10 AND 21	10	21	21
5	0	13	26		26	26
6	0	14	27		27	27
7	0	15	28		28	28
8	1	16	25		25	25
9	2	17	11 AND 22	11	22	22
10	0	18	29		29	29
11	0	19	30		30	30

[illegible]

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STABILITY ANALYSIS RESULTS

MODE	EIGENVALUE-M MU-MPS	EIGENVALUE-1 ONEGA-MPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	-2.50110E-11	0.	0.	3.988641E-12		2.771358E-10
2	-3.00133E-11	0.	0.	4.776769E-12		2.309465E-10
3	-1.32600E-06	0.	0.	2.161663E-07		5.248572E-05
4	-6.249950E-01	0.	0.	9.447104E-00		1.109044E-02
5	-3.09344E-02	0.	0.	4.924886E-01		2.240407E-01
6	-3.189769E-02	0.	0.	5.076075E-01		2.173832E-01
7	-9.985427E-02	0.	0.	1.569230E-02		6.941588E-04
8	-3.565272E-15	0.	0.	5.614305E-14		-1.944184E-16
9	-9.001199E-15	0.	0.	1.633540E-15		7.695480E-17
10	-1.000000E-34	0.	0.	1.591549E-37		6.931472E-34
11	-1.000000E-34	0.	0.	1.591549E-37		6.931472E-34
12	-1.000000E-34	0.	0.	1.591549E-37		6.931472E-34
13	-1.000000E-34	0.	0.	1.591549E-37		6.931472E-34
14	-1.000000E-34	0.	0.	1.591549E-37		6.931472E-34
15	6.002100E-07	2.957891E-06	4.706357E-07	4.822312E-07	-2.179007E-01	-1.049678E-06
16	-2.699049E-02	2.011408E-02	4.475291E-01	6.203306E-01	6.924887E-01	2.568116E-03
17	-1.451224E-00	2.424129E-02	5.494736E-01	6.494736E-01	5.138594E-03	4.776293E-01
18	-4.435253E-02	4.440194E-02	7.066788E-01	9.468389E-01	7.067131E-01	1.562413E-03
19	-9.571353E-00	6.344437E-02	1.010305E-02	1.010500E-02	1.507500E-02	7.241693E-02
20	-1.101639E-01	7.250824E-02	1.154113E-02	1.152548E-02	1.521252E-02	6.291960E-02
21	-8.106528E-00	7.931509E-02	1.262339E-02	1.262403E-02	1.022264E-02	6.548416E-02
22	-2.369782E-01	1.562317E-03	2.460505E-02	2.466791E-02	1.510666E-02	2.924941E-02

EIGENVECTORS FOR MODES REQUESTED

EIGENVECTOR FOR MODE 4

4.0544204E-09	0.	2.5798854E-08	0.	3.0744610E-06	0.	-1.8234730E-05	0.
-5.9591332E-03	0.	-4.8874007E-04	0.	1.3738340E-05	0.	-1.2011885E-01	0.
-1.0554439E-03	0.	-1.0249120E-04	0.	1.0000000E-08	0.	-6.4419247E-04	0.
-6.9278846E-08	0.	-4.9151711E-04	0.	2.9175802E-07	0.	9.5346847E-05	0.
7.1799449E-09	0.	-2.1981505E-01	0.	1.9219110E-03	0.	1.6727877E-05	0.
1.6463085E-11	0.	-1.0000124E-02	0.	1.1643059E-05	0.	1.7843060E-07	0.
1.5653501E-08	0.	1.5437697E-04	0.	3.5086119E-05	0.	-9.6543980E-07	0.
1.6080129E-02	0.	3.4720274E-05	0.				

EIGENVECTOR FOR MODE 5

6.2800004E-10	0.	1.4506925E-09	0.	9.0817447E-10	0.	-4.3603665E-09	0.
-1.0303166E-06	0.	-5.6324449E-04	0.	1.7269464E-10	0.	-1.7614219E-06	0.
-5.3625051E-08	0.	5.5582152E-08	0.	1.0000000E-00	0.	-2.0298662E-12	0.
-4.0162529E-12	0.	-2.9354399E-12	0.	1.4053083E-11	0.	5.4419101E-09	0.
-1.0205375E-11	0.	-5.5016711E-13	0.	5.043114E-09	0.	1.7333019E-10	0.
-1.1752471E-10	0.	-3.4322245E-03	0.	2.5098014E-10	0.	2.5098006E-11	0.
-2.2668465E-08	0.	-2.2464440E-06	0.	5.6196029E-10	0.	7.014252E-06	0.
3.2322245E-03	0.	7.4134272E-06	0.				

LINEARVECTION FOR MODE 6

3.505052E-10	0.	1.4520134E-09	0.	0.4025124E-10	0.	-6.2194927E-09	0.
-1.0074411E-06	0.	-5.2090057E-09	0.	1.5913087E-10	0.	-1.6447416E-06	0.
-4.0104091E-06	0.	5.3144424E-08	0.	1.0000000E-00	0.	-1.0763200E-12	0.
-4.5549187E-12	0.	-2.7910405E-12	0.	1.4220471E-11	0.	5.0463604E-09	0.
-1.6332061E-11	0.	-4.9090271E-11	0.	5.1503026E-09	0.	1.5099553E-10	0.
-1.6660901E-10	0.	-3.1350230E-03	0.	2.2171067E-10	0.	2.2171055E-11	0.
-2.1340172E-04	0.	-2.1159563E-04	0.	6.6443735E-10	0.	6.8025566E-06	0.
3.1350230E-03	0.	6.8030000E-06	0.				

LINEARVECTION FOR MODE 7

-6.4120904E-06	0.	-3.5506504E-05	0.	-3.0342119E-05	0.	5.5904401E-05	0.
1.0224244E-01	0.	2.4007244E-05	0.	-9.002512E-07	0.	2.3040007E-02	0.
-4.343735E-04	0.	3.4440004E-01	0.	1.0000000E-00	0.	6.421400E-09	0.
3.5630442E-04	0.	3.0445230E-04	0.	-5.5905990E-08	0.	-1.0230204E-04	0.
-2.0930903E-04	0.	4.0024012E-10	0.	-2.4074434E-05	0.	4.3500747E-07	0.
-3.4440214E-04	0.	-1.0014594E-03	0.	2.9050101E-04	0.	2.9050101E-05	0.
-3.5509057E-07	0.	3.4557044E-04	0.	5.9700363E-06	0.	5.9403046E-06	0.
1.0014594E-03	0.	2.1731070E-06	0.				

LINEARVECTION FOR MODE 16

-2.452047E-04	0.	0.7219157E-07	-3.0132701E-06	5.4030712E-06	-1.1094501E-06	3.2724100E-06	1.3567979E-05	-1.5032604E-05
1.7623434E-03	0.	0.4940444E-03	1.2375321E-06	3.7476692E-06	-3.7800330E-07	-1.1028471E-07	3.5177637E-03	6.0422571E-05
-1.724476E-03	-1.0172707E-02	1.1903670E-02	-1.5601100E-02	-1.5601100E-02	1.0000000E-00	0.	5.2461000E-09	2.972500E-09
1.5350492E-04	-2.0230705E-09	0.0209361E-09	-3.7019089E-09	-5.1930490E-08	1.5943420E-09	1.5943420E-09	-1.5040130E-05	8.3081444E-06
-1.5049944E-04	-2.950403E-04	0.5030442E-10	4.0020414E-10	6.1260034E-06	-6.6301077E-06	-6.6301077E-06	-5.9049604E-06	3.1330101E-05
-5.0200474E-05	5.7159390E-04	-1.7706011E-03	-1.8509512E-03	7.4440474E-06	4.0329921E-05	4.0329921E-05	7.4440474E-07	9.0329921E-06
-1.2220409E-07	1.5659315E-07	5.0154215E-05	-5.5593065E-06	1.4069095E-05	8.0059042E-05	8.0059042E-05	-1.1034340E-05	-7.0043270E-05
1.7706011E-03	1.0509512E-03	3.0553540E-05	4.0105041E-06					

LINEARVECTION FOR MODE 17

1.1042003E-04	0.	0.4709434E-05	2.3057443E-08	-0.2192000E-08	1.5000164E-08	4.7551507E-09	1.4622272E-06	1.7374346E-05
-3.2711545E-05	-0.2070275E-05	-0.2070275E-05	2.2000450E-07	1.1302303E-06	-5.9920454E-09	-2.5921340E-06	2.7144974E-05	3.5556156E-05
1.4910910E-04	-0.300312E-04	1.3732065E-04	1.4955070E-04	-1.4955070E-04	1.0000000E-00	0.	2.2995449E-07	-2.6746610E-04
-2.0041044E-10	-0.2037192E-11	3.0711004E-11	-5.0331551E-11	-5.0331551E-11	6.105641E-08	-6.9094444E-09	-1.5122202E-07	1.1000422E-07
3.9970527E-09	-0.2316222E-10	-9.1613090E-11	2.1091261E-11	2.1091261E-11	1.2549715E-07	-9.0702524E-06	-1.5447107E-06	-5.2001475E-07
-5.3203174E-07	-0.6352050E-07	-1.0345122E-05	-3.5400253E-03	-3.5400253E-03	-1.2549715E-07	-7.576269E-04	-1.573301E-07	-7.576269E-04
2.7004464E-04	-1.1424007E-04	5.5971074E-07	4.7210391E-07	4.7210391E-07	-3.1407922E-06	-1.5150570E-07	3.1002750E-06	7.0351567E-06
1.0195122E-05	3.5400253E-03	3.4903410E-08	7.0033909E-06					

LINEARVECTION FOR MODE 18

-2.1133004E-06	3.0107345E-07	-0.4102225E-06	7.0093014E-06	-0.4102225E-06	-2.0097591E-06	6.2740724E-06	2.590410E-05	-9.4403624E-06
1.9211271E-03	-1.2950362E-02	9.7210534E-06	1.4177027E-06	1.4177027E-06	-2.550002E-07	-4.5040044E-06	2.0100340E-03	-1.510002E-03
-2.541303E-04	1.0104340E-04	6.3075442E-03	-2.4215130E-02	-2.4215130E-02	1.0000000E-00	0.	2.1076632E-09	1.9751105E-03
1.042734E-06	-0.0410074E-10	9.4313000E-09	-7.715110E-09	-7.715110E-09	-3.9044903E-08	-1.0030532E-06	-1.0702075E-05	1.2417350E-05
-9.302612E-09	-1.2550252E-08	2.0204113E-10	3.0392070E-10	3.0392070E-10	-3.9044903E-08	-1.0030532E-06	3.9110542E-07	1.6197044E-07
-0.0003210E-05	2.572047E-05	-1.1200764E-03	1.1273300E-03	1.1273300E-03	5.7139404E-07	-1.040002E-07	5.7139404E-04	-1.040002E-07
1.0040944E-09	-0.5924001E-09	4.0004000E-05	-2.5747434E-05	-2.5747434E-05	1.1427997E-06	-3.2909004E-07	1.3007050E-06	2.7700042E-04
1.1200764E-03	1.1273300E-03	2.4435055E-06	2.4463074E-06					

EIGENVECTORS FOR MODE 19

-0.224747E-04	0.059742E-04	-5.213032E-04	0.080117E-04	-2.951180E-07	9.223607E-07	5.015212E-04	-7.791004E-04
-5.007017E-05	2.351697E-05	3.752607E-04	-5.163015E-04	-0.107101E-04	1.120504E-07	2.900645E-04	-1.072655E-04
-0.207790E-07	-0.315041E-05	-2.130510E-05	-2.545205E-05	1.000000E-04	0.	1.370504E-04	9.603070E-04
1.093252E-04	0.141156E-04	0.121545E-04	0.547221E-04	-1.240392E-04	-0.050829E-07	3.033333E-04	0.553003E-04
-0.220921E-04	-5.700016E-04	1.703949E-04	1.259507E-04	-3.020130E-04	-0.005012E-04	-0.360202E-04	2.614492E-04
-3.957023E-04	3.420242E-04	-2.374329E-04	-1.570831E-04	-5.009413E-04	4.020145E-04	-5.009413E-04	4.020145E-04
2.250991E-04	-3.452351E-04	4.402723E-04	-3.462700E-04	-1.173002E-07	0.052240E-04	1.009112E-07	3.330064E-04
2.374324E-05	1.574033E-03	5.152245E-04	3.417387E-04				

EIGENVECTORS FOR MODE 20

5.919320E-07	-0.043004E-07	-0.331101E-04	9.026047E-04	-9.707204E-07	1.107965E-04	-5.371051E-05	5.942424E-05
1.443401E-03	-1.052709E-03	-1.009420E-04	1.319073E-04	3.111719E-04	-3.792447E-04	1.419041E-04	-1.490301E-04
-2.290324E-04	-3.700500E-05	-1.041072E-05	-1.227404E-05	1.000000E-04	0.	-9.572707E-10	-0.029242E-10
1.263051E-04	1.131350E-04	1.032166E-04	1.320830E-04	0.317702E-04	7.292291E-04	-2.509502E-04	-2.623143E-04
1.045260E-04	1.476409E-04	-5.301747E-11	-4.210032E-11	-2.090093E-04	-1.924934E-04	-5.107507E-04	3.952322E-04
-1.650026E-04	2.500720E-04	-2.100095E-05	-1.300130E-05	-3.424307E-04	2.700303E-04	-3.224307E-04	2.700303E-04
4.050430E-04	-2.137620E-04	2.001070E-04	-2.500305E-04	-6.440735E-04	5.520690E-04	1.100724E-07	2.940490E-04
2.100495E-05	1.300730E-03	4.550509E-04	2.990203E-04				

EIGENVECTORS FOR MODE 21

0.130100E-04	-0.366072E-04	-0.207406E-04	7.751020E-04	5.577574E-04	-5.120449E-04	-2.102050E-05	1.930051E-05
3.401513E-04	-1.570604E-04	-5.403052E-04	4.107351E-04	1.105191E-04	-9.137912E-10	4.444753E-07	-3.002304E-07
-3.495720E-07	-1.073400E-05	-5.244202E-04	-2.502021E-04	1.000000E-04	0.	-0.130302E-11	-1.017030E-10
0.077204E-04	1.024007E-04	-6.527291E-12	-6.965432E-12	2.405054E-04	-5.720671E-04	-2.020350E-04	-4.207474E-04
5.340513E-11	6.757500E-11	-1.107257E-12	-1.002308E-12	-4.920427E-10	-5.553555E-10	-1.352744E-04	5.790310E-10
-3.100455E-04	6.044557E-04	-1.200497E-05	-1.200062E-05	-0.341010E-04	7.000112E-04	-0.034101E-10	7.000112E-04
3.400477E-04	-9.050505E-11	6.010903E-04	-6.735166E-04	-1.000020E-04	1.001027E-04	4.403510E-04	2.721621E-04
1.200497E-05	1.200062E-05	2.140004E-04	2.135037E-04				

EIGENVECTORS FOR MODE 22

4.435135E-04	-1.917203E-04	-2.752210E-04	1.440124E-04	1.504703E-05	-6.322400E-04	9.262096E-04	-3.790101E-04
1.027510E-05	1.550504E-04	-3.367474E-04	1.304002E-04	7.472396E-10	-2.934095E-10	0.033150E-07	7.000000E-07
5.001241E-04	9.904430E-05	1.009040E-05	-1.110001E-05	1.000000E-04	0.	-1.269960E-11	-2.019555E-11
9.012600E-12	1.740095E-11	-9.191900E-04	-9.507010E-04	-2.520410E-04	-5.040205E-04	0.362002E-10	-1.042404E-04
-7.292152E-04	-1.190503E-04	-1.450025E-13	-6.753307E-13	4.070039E-10	-3.107613E-10	0.371000E-04	-4.000304E-04
9.270710E-10	-0.011504E-12	-9.700500E-04	-0.399270E-04	1.020000E-04	-3.005194E-04	1.020000E-04	-3.005194E-04
9.700002E-04	0.399270E-04	0.220124E-04	1.197002E-04	3.441201E-04	-0.130302E-04	-1.134054E-04	1.449471E-04
		2.100347E-04	1.300043E-04				

STRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR EACH MODE

DEFLECTIONS FOR MODE 4									
-2.251011E-04	0.	-1.110014E-04	0.	0.	0.	0.024704E-05	0.	3.1295730E-04	0.
5.9051430E-04	0.	0.551262E-04	0.	0.	0.	1.123132E-03	0.	1.306414E-03	0.
1.6416101E-03	0.	1.004775E-03	0.	0.	0.	-1.970037E-01	0.	1.000000E+00	0.
51.9761901E-01	0.	9.999874E+01	0.	0.	0.				
DEFLECTIONS FOR MODE 5									
-2.976421E-04	0.	-1.1500711E-04	0.	0.	0.	2.9602350E-05	0.	1.7004490E-04	0.
4.0541510E-04	0.	7.915322E-04	0.	0.	0.	1.248145E-03	0.	1.004261E-03	0.
1.251512E-03	0.	1.123438E-04	0.	0.	0.	-1.980093E-01	0.	1.000000E+00	0.
-1.003023E-01	0.	9.997697E-01	0.	0.	0.				
DEFLECTIONS FOR MODE 6									
-2.942120E-04	0.	-1.120591E-04	0.	0.	0.	2.7452550E-05	0.	1.015706E-04	0.
3.553067E-04	0.	7.004109E-04	0.	0.	0.	1.250077E-03	0.	1.017326E-03	0.
1.727502E-03	0.	1.133311E-03	0.	0.	0.	-1.980040E-01	0.	1.000000E+00	0.
-1.905915E-01	0.	9.996933E-01	0.	0.	0.				
DEFLECTIONS FOR MODE 7									
-2.000144E-04	0.	-2.152200E-04	0.	0.	0.	1.424012E-04	0.	-3.122007E-04	0.
-4.402050E-04	0.	3.940594E-04	0.	0.	0.	1.0339510E-03	0.	2.720232E-03	0.
2.201042E-03	0.	7.909003E-04	0.	0.	0.	-1.990169E-01	0.	1.000000E+00	0.
-1.997425E-01	0.	9.992765E-01	0.	0.	0.				
DEFLECTIONS FOR MODE 10									
-2.943225E-04	0.	-1.9406770E-04	0.	0.	0.	2.655747E-05	0.	2.445770E-04	0.
5.127649E-04	0.	0.104390E-04	0.	0.	0.	1.120554E-03	0.	1.420734E-03	0.
1.003010E-03	0.	1.927730E-03	0.	0.	0.	-1.970930E-01	0.	1.000000E+00	0.
-1.981360E-01	0.	9.997505E-01	0.	0.	0.				
DEFLECTIONS FOR MODE 17									
-1.773901E-01	-1.437520E-01	-7.469141E-02	-5.957000E-02	-2.000722E-02	1.003505E-02	9.910024E-05	2.445770E-04	2.700492E-04	0.
1.415795E-01	1.049353E-01	1.350024E-01	1.097309E-01	9.745596E-02	7.043025E-02	-2.577413E-04	1.420734E-03	3.904157E-04	0.
-6.524703E-02	-5.433309E-02	-1.070025E-01	-1.375247E-01	-3.931951E-01	-1.540449E-01	5.502354E-04	1.000000E+00	0.	0.
-4.249100E-01	-1.005500E-01	5.007044E-01	-2.550000E-02						
DEFLECTIONS FOR MODE 18									
-4.909105E-05	-2.333000E-04	-1.433745E-04	-1.006104E-04	-1.091650E-04	8.944490E-05	-3.105324E-05	4.051704E-04	0.	0.
3.752520E-04	0.021077E-04	0.594821E-04	1.097310E-04	1.300070E-03	-5.395033E-04	1.000000E+00	0.	0.	0.
1.753200E-03	-3.055025E-04	1.007245E-03	7.402742E-04	-1.985175E-01	0.004074E-04	1.000000E+00	0.	0.	0.
-1.994550E-01	1.419901E-03	9.990017E-01	5.344010E-04						

DEFLECTIONS FOR MODE 19

-1.412365E-02	-1.3302031E-04	-1.5508720E-04	0.1161011E-05	1.0270958E-02	1.6019720E-04	1.3046230E-02	1.3012657E-04
4.5971927E-03	-6.1000091E-06	-1.3346434E-04	-1.0777134E-04	-1.0510084E-02	-2.3482843E-04	-1.7002901E-02	-1.6415076E-04
-1.0050357E-02	4.5369714E-05	-1.5013148E-02	3.2551607E-04	3.2243175E-01	5.5021949E-03	2.4259795E-01	-4.2915770E-02
1.0000000E+00	0.	9.1010620E-01	-5.0477970E-02				

DEFLECTIONS FOR MODE 20

-9.0537474E-03	-3.3444954E-04	2.6076173E-04	4.5790541E-05	4.0061474E-03	3.0353762E-04	5.2205017E-03	3.4005716E-04
3.1717221E-03	1.5244651E-04	-8.6300697E-04	-1.3011047E-04	-4.1073731E-03	-3.4544534E-04	-3.4030037E-03	-3.2093294E-04
0.0141505E-04	-3.9703520E-05	0.3197004E-01	3.9311014E-04	-2.0501447E-01	-4.2104259E-04	1.0000000E+00	0.
-2.2017004E-01	-1.7135433E-03	9.7744043E-01	-1.2417107E-03				

DEFLECTIONS FOR MODE 21

5.1026252E-01	-2.0721171E-02	-7.0749741E-02	4.0140036E-03	-4.7111040E-01	2.159346E-02	-5.1101417E-01	2.64440275E-02
-2.1400062E-01	1.1216770E-02	2.1049635E-01	-1.1153420E-02	5.1196494E-01	-2.440573E-02	4.6755472E-01	-2.4452331E-02
7.0371374E-02	-9.3725170E-03	-5.2422075E-01	2.6042455E-02	1.0691844E-01	1.7308161E-02	-2.1405701E-01	-2.3392117E-02
1.0000000E+00	0.	0.7022455E-01	-0.0700270E-02				

DEFLECTIONS FOR MODE 22

-0.0303467E-01	3.0475201E-04	0.5971232E-01	-2.2422690E-04	9.4979098E-01	-4.3503344E-04	1.0052764E-01	-1.7100164E-03
-6.4700670E-01	2.5770346E-04	-4.4745950E-01	4.3594548E-04	1.0446078E-01	2.6104057E-04	1.0000000E+00	0.
0.4429304E-01	-4.0200970E-05	-7.9250944E-01	1.3672414E-04	-2.1083505E-01	-1.9708036E-02	-1.5067430E-01	1.0953577E-01
1.4755394E-01	-1.0359474E-02	2.2771304E-01	1.1244533E-01				

MINIMUM BLANK COLUMN LENGTH REQUIRED = 6053.
BASED ON INPUT DATA AND ANALYSES REQUESTED.

Aerodynamic Influence Coefficients

The aerodynamic influence coefficients (AIC's) at supersonic speeds are found from Piston Theory and are generated by Program PISTON (Ref. 4). Three Mach numbers are considered, 2.0, 3.0, and 4.0, and four reduced velocities are considered, 1000.0, 20.0, 5.0, and 2.0 based on the wing semichord of 1.5 ft. The AIC's for both the wing and the tail (flipper) are found for the same reduced velocities by using the wing semichord as the reference for both surfaces.

The wing planform is shown in Fig. 3 and its motion is determined by the deflections of mass points Nos. 7, 8, and 9 which are located at the fractional wing chord positions 0.152778, 0.569444, and 0.986111, respectively. One strip is sufficient to represent the wing since it has been assumed rigid in the spanwise direction. The basic Piston Theory is used without thickness (a small thickness ratio $\tau = 0.001$ is input). The input data cards are listed below with comments and follow the input format described in the Program PISTON User's Manual (Ref. 4). The input data are also echoed in the program output which follows the input data card listing below. At the end of the printed output the punched card output is listed. The punched AIC's are input appropriately according to the proper partitioning format into Program MPASES.

The tail (flipper) planform is also shown in Fig. 4, and its motion is determined by the deflections of its four mass points, Nos. 11, 12, 13, and 14 which are located at the tail quarter- or

three-quarter chord positions. The tail has been assumed rigid in both spanwise and chordwise directions but to bend and twist about the actuator at the root. Two equal width strips are used to account for the bending motion. Except for the chordwise and spanwise locations of the aerodynamic control points, the tail is treated the same as the wing in terms of input to Program PISTON. The input data cards are listed below those for the wing in the following pages and the printed output of AIC's for the tail follows that for the wing. Again, at the end of the printed output the punched AIC's are listed.

Table 5 - Input Cards for Program PISTON

C TITLE AND SUBTITLE CARDS						
PISTON THEORY AERODYNAMICS FOR WING						
NEW AND IMPROVED MAC MISSILE						
C CONTROL CARDS						
0	1	2	1	1		
1	3	0	4	4	4	
C PLANFORM GEOMETRY						
0.0	1.5	1.0	3.0	3.0		
C STRIP GEOMETRY						
1.0	1.5	0.152777778	0.569444444	0.986111111	0.4	
C THICKNESS DATA						
0.001	0.0	0.0				
C SERIES OF MACH NUMBERS						
2.0	3.0	4.0				
C TRIM ANGLE OF ATTACK FOR EACH MACH NUMBER						
0.0						
0.0						
0.0						
C SERIES OF REDUCED VELOCITIES FOR EACH MACH NUMBER						
1000.0	20.0	5.0	2.0			
1000.0	20.0	5.0	2.0			
1000.0	20.0	5.0	2.0			
C TITLE AND SUBTITLE CARDS						
PISTON THEORY AERODYNAMICS FOR TAIL						
NEW MAC MISSILE						
C CONTROL CARDS						
0	0	2	1	1		
2	3	0	4	4	4	
C PLANFORM GEOMETRY						
0.0	1.5	1.0	3.0	3.0		
C STRIP GEOMETRY						
0.5	0.66666667	0.25	0.50	0.75	0.4	
0.5	0.66666667	0.25	0.50	0.75	0.4	
C THICKNESS DATA						
0.001	0.0	0.0				
C SERIES OF MACH NUMBERS						
2.0	3.0	4.0				
C TRIM ANGLE OF ATTACK FOR EACH MACH NUMBER						
0.0						
0.0						
0.0						
C SERIES OF REDUCED VELOCITIES FOR EACH MACH NUMBER						
1000.0	20.0	5.0	2.0			
1000.0	20.0	5.0	2.0			
1000.0	20.0	5.0	2.0			

PISTON THEORY AERODYNAMICS FOR WING
NEW AND IMPROVED MAC MISSILE

Table 6 - Output from Program PISTON

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER
THICKNESS INTEGRALS CALCULATED FOR AIRFOIL 2

INPUT DATA

1 STRIPS
3 MACH NUMBERS
12 REDUCED FREQUENCIES (TOTAL)
SECANT LAMBDA = 0.
REFERENCE SEMI-CHORD = .150000E+01
SEMI-SPAN = .100000E+01
SURFACE AREA = .300000E+01
C BAR = .300000E+01

STRIP NO.	DELTA V	0	21	22	23	ZMAX
1	.100000E+01	.150000E+01	.152778E+00	.569886E+00	.986111E+00	.400000E+00

STRIP NO.	TAU	TAU(M)	TAU(T)
1	.100000E+02	0.	0.

MACH NUMBER = 2.000000
1/K(R) = .100000E+04
1/K(R) = .200000E+02
1/K(R) = .500000E+01
1/K(R) = .200000E+01

STRIP NO. ALPHA ZERO (DEGREES)

1 0.00

MACH NUMBER = 3.000000
1/K(R) = .100000E+04
1/K(R) = .200000E+02
1/K(R) = .500000E+01
1/K(R) = .200000E+01

STRIP NO. ALPHA ZERO (DEGREES)

1 0.00

MACH NUMBER = 4.000000
1/K(R) = .100000E+04
1/K(R) = .200000E+02
1/K(R) = .500000E+01
1/K(R) = .200000E+01

STRIP NO. ALPHA ZERO (DEGREES)

1 0.00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 2.000000

1/K(R) = .100000E+04

1 STRIPS

CM(1) SIZE = 3 BY 3

.13126448E+07	-.74933925E+03	-.18126021E+07	.66239441E+02	.49995734E+06	.58819818E+01
.88239893E+06	.66239441E+02	.37445451E+06	-.95926251E+03	-.77684946E+06	-.89723991E+02
-.11042446E+06	.58819818E+01	.62892598E+06	-.89723991E+02	-.51850062E+06	-.25619977E+03

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 2.000000

1/K(R) = .200000E+02

1 STRIPS

CM(1) SIZE = 3 BY 3

.52505796E+03	-.14986785E+02	-.72504084E+03	.13247888E+01	.19998294E+03	.11763964E+00
.16095798E+03	.13247888E+01	.14978180E+03	-.19185250E+02	-.31073979E+03	-.17944798E+01
-.44169782E+02	.11763964E+00	.25157003E+03	-.17944798E+01	-.20740025E+03	-.51239955E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 2.000000

1/K(R) = .500000E+01

1 STRIPS

CM(1) SIZE = 3 BY 3

.32816119E+02	-.37464963E+01	-.44315052E+02	.33119721E+00	.12498933E+02	.29409909E+01
.18059874E+02	.33119721E+00	.03613628E+01	-.47963125E+01	-.19421237E+02	-.44861996E+00
-.27606114E+01	.29409909E+01	.14723127E+02	-.44861996E+00	-.12962516E+02	-.12809989E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 2.000000

1/K(R) = .200000E+01

1 STRIPS

CM(1) SIZE = 3 BY 3

.52505796E+01	-.14986785E+01	-.72504084E+01	.13247888E+00	.19998294E+01	.11763964E+01
.16095798E+01	.13247888E+00	.14978180E+01	-.19185250E+01	-.31073979E+01	-.17944798E+00
-.44169782E+00	.11763964E+01	.25157003E+01	-.17944798E+00	-.20740025E+01	-.51239955E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .100000E+00

1 STRIPS

CM(1) SIZE = 3 BY 3

.07679779E+00	-.50055253E+03	-.1210031E+07	.44303712E+02	.33380535E+00	.30543149E+01
.26787508E+00	.44303712E+02	.24971050E+00	-.63926042E+03	-.51759158E+00	-.59655906E+02
-.73387325E+05	.30543149E+01	.41040755E+00	-.59655906E+02	-.34502022E+00	-.17052603E+03

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .200000E+02

1 STRIPS

CM(1) SIZE = 3 BY 3

.35071912E+03	-.10011051E+02	-.48427320E+03	.68607420E+00	.13355010E+03	.77000299E-01
.10715003E+03	.68607420E+00	.99806599E+02	-.12789360E+02	-.20703003E+03	-.11931101E+01
-.20354930E+02	.77000299E-01	.16730302E+03	-.11931101E+01	-.13000009E+03	-.34105325E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .500000E+01

1 STRIPS

CM(1) SIZE = 3 BY 3

.21919905E+02	-.25027627E+01	-.30267070E+02	.22151050E+00	.03471337E+01	.10271575E-01
.00900770E+01	.22151050E+00	.62429120E+01	-.31961421E+01	-.12939780E+02	-.29827953E+00
-.10300031E+01	.10271575E-01	.10460109E+02	-.29827953E+00	-.00255050E+01	-.05203313E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .200000E+01

1 STRIPS

CM(1) SIZE = 3 BY 3

.35071912E+01	-.10011051E+01	-.48427320E+01	.68607420E-01	.13355010E+01	.77000299E-02
.10715003E+01	.68607420E-01	.99806599E+00	-.12789360E+01	-.20703003E+01	-.11931101E+00
-.20354930E+00	.77000299E-02	.16730302E+01	-.11931101E+00	-.13000009E+01	-.34105325E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .100000E+04

1 STRIPS

CM(1) SIZE = 3 BY 3

.05007675E+06	-.37614320E+03	-.96972714E+00	.33314115E+02	.25005610E+06	.20504950E+01
.20061447E+06	.33314115E+02	.18734085E+06	-.47925302E+03	-.38796332E+06	-.04629343E+02
-.54868713E+05	.20504950E+01	.31314920E+06	-.44629343E+02	-.25028657E+06	-.12768765E+03

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .200000E+02

1 STRIPS

CM(1) SIZE = 3 BY 3

.26355670E+03	-.75228640E+01	-.36389086E+03	.66628229E+00	.10034016E+03	.57009916E-01
.80245787E+02	.66628229E+00	.72939541E+02	-.95850603E+01	-.15518533E+03	-.89258687E+00
-.21947485E+02	.57009916E-01	.12525971E+03	-.89258687E+00	-.10331223E+03	-.25537530E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .500000E+01

1 STRIPS

CM(1) SIZE = 3 BY 3

.16471919E+02	-.16807160E+01	-.22743179E+02	.16657057E+00	.62712598E+01	.14252879E-01
.50153617E+01	.16657057E+00	.46837213E+01	-.23962651E+01	-.96990630E+01	-.22514672E+00
-.13717178E+01	.14252879E-01	.78287320E+01	-.22314672E+00	-.64570141E+01	-.63643826E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .200000E+01

1 STRIPS

CM(1) SIZE = 3 BY 3

.26355670E+01	-.75228640E+00	-.36389086E+01	.66628229E-01	.10034016E+01	.57009916E-02
.80245787E+00	.66628229E-01	.72939541E+00	-.95850603E+00	-.15518533E+01	-.89258687E-01
-.21947485E+00	.57009916E-02	.12525971E+01	-.89258687E-01	-.10331223E+01	-.25537530E+00

1.00000E+03 PTH 1001
 3 1 1 1 PTH 1002
 3 PTH 1 11
 1.31264E+06-7.49339E+02-1.81260E+06 6.62394E+01 4.99957E+05 5.88198E+00PTN 1 12
 4.02395E+05 6.62394E+01 3.74455E+05-9.59263E+02-7.74849E+05-8.47240E+01PTN 1 13
 -1.10424E+05 5.88198E+00 6.28925E+05-8.97240E+01-5.18501E+05-2.56200E+02PTN 1 14
 2.00000E+01 PTH 2001
 3 1 1 1 PTH 2002
 3 PTH 2 11
 5.25058E+02-1.49868E+01-7.25041E+02 1.32479E+00 1.99983E+02 1.17640E-01PTN 2 12
 1.60958E+02 1.32479E+00 1.49782E+02-1.91853E+01-3.10740E+02-1.79448E+00PTN 2 13
 -4.41698E+01 1.17640E-01 2.51570E+02-1.79448E+00-2.07400E+02-5.12400E+00PTN 2 14
 5.00000E+00 PTH 3001
 3 1 1 1 PTH 3002
 3 PTH 3 11
 3.20141E+01-3.74676E+00-4.53151E+01 3.31197E-01 1.24949E+01 2.94099E-02PTN 3 12
 -1.00599E+01 3.31197E-01 9.36136E+00-4.79631E+00-1.94217E+01-4.48620E-01PTN 3 13
 -2.76061E+00 2.94099E-02 1.57231E+01-4.48620E-01-1.29625E+01-1.28100E+00PTN 3 14
 2.00000E+00 PTH 4001
 3 1 1 1 PTH 4002
 3 PTH 4 11
 5.25058E+00-1.49868E+00-7.25041E+00 1.32479E-01 1.99983E+00 1.17640E-02PTN 4 12
 -1.60958E+00 1.32479E-01 1.49782E+00-1.91853E+00-3.10740E+00-1.79448E-01PTN 4 13
 -4.41698E-01 1.17640E-02 2.51570E+00-1.79448E-01-2.07400E+00-5.12400E-01PTN 4 14
 1.00000E+03 PTH 1001
 3 1 1 1 PTH 1002
 3 PTH 1 11
 8.74798E+05-5.00553E+02-1.21068E+06 4.43037E+01 3.33885E+05 3.85431E+00PTN 1 12
 -2.87875E+05 4.43037E+01 2.49716E+05-4.39268E+02-5.17592E+05-5.96559E+01PTN 1 13
 -7.33873E+04 3.85431E+00 4.18408E+05-5.96559E+01-3.45020E+05-1.70527E+02PTN 1 14
 2.00000E+01 PTH 2001
 3 1 1 1 PTH 2002
 3 PTH 2 11
 3.50719E+02-1.00111E+01-4.84273E+02 8.86074E-01 1.33554E+02 7.70863E-02PTN 2 12
 -1.07150E+02 8.86074E-01 9.94866E+01-1.27854E+01-2.07037E+02-1.19312E+00PTN 2 13
 -2.93549E+01 7.70863E-02 1.67363E+02-1.19312E+00-1.38008E+02-3.41053E+00PTN 2 14
 5.00000E+00 PTH 3001
 3 1 1 1 PTH 3002
 3 PTH 3 11
 2.19199E+01-2.50276E+00-3.02671E+01 2.21519E-01 8.34713E+00 1.92710E-02PTN 3 12
 -8.69688E+00 2.21519E-01 6.24291E+00-3.19634E+00-1.29394E+01-2.98260E-01PTN 3 13
 -1.83468E+00 1.92710E-02 1.04602E+01-2.98260E-01-8.62551E+00-4.52633E-01PTN 3 14
 2.00000E+00 PTH 4001
 3 1 1 1 PTH 4002
 3 PTH 4 11
 3.50719E+00-1.00111E+00-4.84273E+00 8.86074E-02 1.33554E+00 7.70863E-03PTN 4 12
 -1.07150E+00 8.86074E-02 9.94866E-01-1.27854E+00-2.07037E+00-1.19312E-01PTN 4 13
 -2.93549E-01 7.70863E-03 1.67363E+00-1.19312E-01-1.38008E+00-3.41053E-01PTN 4 14
 1.00000E+03 PTH 1001
 3 1 1 1 PTH 1002
 3 PTH 1 11
 6.58877E+05-3.76143E+02-9.09727E+05 3.33141E+01 2.50850E+05 2.85050E+00PTN 1 12
 -2.00614E+05 3.33141E+01 1.87349E+05-4.79253E+02-3.87963E+05-4.46293E+01PTN 1 13
 -5.48687E+04 2.85050E+00 3.13149E+05-4.46293E+01-2.58201E+05-1.27688E+02PTN 1 14
 2.00000E+01 PTH 2001
 3 1 1 1 PTH 2002
 3 PTH 2 11
 2.03551E+02-7.52286E+00-3.63891E+02 6.66282E-01 1.00340E+02 5.70099E-02PTN 2 12
 -8.02458E+01 6.66282E-01 7.49395E+01-9.58506E+00-1.55185E+02-8.92587E-01PTN 2 13
 -2.19475E+01 5.70099E-02 1.25260E+02-8.92587E-01-1.03312E+02-2.55375E+00PTN 2 14
 5.00000E+00 PTH 3001
 3 1 1 1 PTH 3002
 3 PTH 3 11
 1.64719E+01-1.88072E+00-2.27432E+01 1.76571E-01 6.27126E+00 1.42525E-02PTN 3 12
 -3.01536E+00 1.76571E-01 4.68372E+00-2.39627E+00-9.69906E+00-2.23147E-01PTN 3 13
 -1.37172E+00 1.42525E-02 7.62873E+00-2.23147E-01-6.45701E+00-4.38438E-01PTN 3 14
 2.00000E+00 PTH 4001
 3 1 1 1 PTH 4002
 3 PTH 4 11
 2.03551E+00-7.52286E-01-3.63891E+00 6.66282E-02 1.00340E+00 5.70099E-03PTN 4 12
 -8.02458E-01 6.66282E-02 7.49395E-01-9.58506E-01-1.55185E+00-8.92587E-02PTN 4 13
 -2.19475E-01 5.70099E-03 1.25260E+00-8.92587E-02-1.03312E+00-2.55375E-01PTN 4 14

STRIP THEORY AERODYNAMICS FOR TAIL

also not reliable

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

THICKNESS INTEGRALS CALCULATED FOR AIRFOIL 2

INPUT DATA

2 STRIPS
3 MACH NUMBERS
12 REDUCED FREQUENCIES (TOTAL)
SECANT LENGTH = 0.
REFERENCE SLIM-CHORD = .150000E+01
SEMI-SPAN = .100000E+01
SURFACE AREA = .300000E+01
C BAR = .300000E+01

STRIP NO.	DELTA Y	B	21	22	23	ZMAX
1	.500000E+00	.666667E+00	.250000E+00	.500000E+00	.750000E+00	.400000E+00
2	.500000E+00	.666667E+00	.250000E+00	.500000E+00	.750000E+00	.400000E+00

STRIP NO.	TAU	TAU(H)	TAU(T)
1	.100000E-02	0.	0.
2	.100000E-02	0.	0.

MACH NUMBER = 2.000000
1/K(R) = .100000E+04
1/K(R) = .200000E+02
1/K(R) = .500000E+01
1/K(R) = .200000E+01

STRIP NO. ALPHA ZERO (DEGREES)

1	0.00
2	0.00

MACH NUMBER = 3.000000
1/K(R) = .100000E+04
1/K(R) = .200000E+02
1/K(R) = .500000E+01
1/K(R) = .200000E+01

STRIP NO. ALPHA ZERO (DEGREES)

1	0.00
2	0.00

MACH NUMBER = 4.000000
1/K(R) = .100000E+04
1/K(R) = .200000E+02
1/K(R) = .500000E+01
1/K(R) = .200000E+01

STRIP NO. ALPHA ZERO (DEGREES)

1	0.00
2	0.00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 2.000000

1/K(R) = .100000E+04

2 STRIPS

CM(1) SIZE = 2 BY 2

.50100233E+00 -.26004306E+03 -.50100233E+04 .37108691E+02
 .49840100E+00 .37108691E+02 -.49840100E+04 -.25862025E+03

CM(2) SIZE = 2 BY 2

.50100233E+00 -.26004306E+03 -.50100233E+04 .37108691E+02
 .49840100E+00 .37108691E+02 -.49840100E+04 -.25862025E+03

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 2.000000

1/K(R) = .200000E+02

2 STRIPS

CM(1) SIZE = 2 BY 2

.20004093E+03 -.52008612E+01 -.20004093E+03 .74217381E+00
 .19936040E+03 .74217381E+00 -.19936040E+03 -.51724049E+01

CM(2) SIZE = 2 BY 2

.20004093E+03 -.52008612E+01 -.20004093E+03 .74217381E+00
 .19936040E+03 .74217381E+00 -.19936040E+03 -.51724049E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 2.000000

1/K(R) = .500000E+01

2 STRIPS

CM(1) SIZE = 2 BY 2

.12540058E+02 -.13002193E+01 -.12540058E+02 .18554345E+00
 .12460025E+02 .18554345E+00 -.12460025E+02 -.12931012E+01

CM(2) SIZE = 2 BY 2

.12540058E+02 -.13002193E+01 -.12540058E+02 .18554345E+00
 .12460025E+02 .18554345E+00 -.12460025E+02 -.12931012E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 2.000000

1/K(R) = .200000E+01

2 STRIPS

CM(1) SIZE = 2 BY 2

.20004093E+01 -.52008612E+00 -.20004093E+01 .74217381E+01
 .19936040E+01 .74217381E+01 -.19936040E+01 -.51724049E+00

CM(2) SIZE = 2 BY 2

.20004093E+01 -.52008612E+00 -.20004093E+01 .74217381E+01
 .19936040E+01 .74217381E+01 -.19936040E+01 -.51724049E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .100000E+04

2 STRIPS

CNC (1) SIZE = 2 BY 2

.33493683E+06 -.17362410E+03 -.33493683E+06 .24763263E+02
.33173483E+06 .24763263E+02 -.33173483E+06 -.17220099E+03

CNC (2) SIZE = 2 BY 2

.33493683E+06 -.17362410E+03 -.33493683E+06 .24763263E+02
.33173483E+06 .24763263E+02 -.33173483E+06 -.17220099E+03

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .200000E+02

2 STRIPS

CNC (1) SIZE = 2 BY 2

.13397473E+03 -.34724820E+01 -.13397473E+03 .49526565E+00
.13269393E+03 .49526565E+00 -.13269393E+03 -.34440197E+01

CNC (2) SIZE = 2 BY 2

.13397473E+03 -.34724820E+01 -.13397473E+03 .49526565E+00
.13269393E+03 .49526565E+00 -.13269393E+03 -.34440197E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .500000E+01

2 STRIPS

CNC (1) SIZE = 2 BY 2

.83734200E+01 -.86812049E+00 -.83734200E+01 .12381641E+00
.82933700E+01 .12381641E+00 -.82933700E+01 -.86100494E+00

CNC (2) SIZE = 2 BY 2

.83734200E+01 -.86812049E+00 -.83734200E+01 .12381641E+00
.82933700E+01 .12381641E+00 -.82933700E+01 -.86100494E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .200000E+01

2 STRIPS

CNC (1) SIZE = 2 BY 2

.13397473E+01 -.34724820E+00 -.13397473E+01 .49526565E+01
.13269393E+01 .49526565E+00 -.13269393E+01 -.34440197E+00

CNC (2) SIZE = 2 BY 2

.13397473E+01 -.34724820E+00 -.13397473E+01 .49526565E+01
.13269393E+01 .49526565E+00 -.13269393E+01 -.34440197E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .100000E+04

2 STRIPS

CH(1) SIZE = 2 BY 2

.25160467E+06 -.13041501E+03 -.25160467E+06 .18590714E+02
 .24840200E+06 .18590714E+02 -.24840200E+06 -.12899160E+03

CH(2) SIZE = 2 BY 2

.25160467E+06 -.13041501E+03 -.25160467E+06 .18590714E+02
 .24840200E+06 .18590714E+02 -.24840200E+06 -.12899160E+03

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .200000E+02

2 STRIPS

CH(1) SIZE = 2 BY 2

.10064187E+03 -.26083002E+01 -.10064187E+03 .37181428E+00
 .99360800E+02 .37181428E+00 -.99360800E+02 -.25798321E+01

CH(2) SIZE = 2 BY 2

.10064187E+03 -.26083002E+01 -.10064187E+03 .37181428E+00
 .99360800E+02 .37181428E+00 -.99360800E+02 -.25798321E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .500000E+01

2 STRIPS

CH(1) SIZE = 2 BY 2

.62901167E+01 -.65207506E+00 -.62901167E+01 .92953571E+01
 .62100500E+01 .92953571E+01 -.62100500E+01 -.64495802E+00

CH(2) SIZE = 2 BY 2

.62901167E+01 -.65207506E+00 -.62901167E+01 .92953571E+01
 .62100500E+01 .92953571E+01 -.62100500E+01 -.64495802E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .200000E+01

2 STRIPS

CH(1) SIZE = 2 BY 2

.10064187E+01 -.26083002E+00 -.10064187E+01 .37181428E+01
 .99360800E+00 .37181428E+01 -.99360800E+00 -.25798321E+00

CH(2) SIZE = 2 BY 2

.10064187E+01 -.26083002E+00 -.10064187E+01 .37181428E+01
 .99360800E+00 .37181428E+01 -.99360800E+00 -.25798321E+00

1.00000E+03	PTN 1001
0 2 1 1	PTN 1002
2	PTN 1 11
3.01002E+05-2.00003E+02-9.01002E+05 3.71007E+01	PTN 1 12
0.00001E+05 3.71007E+01-0.00001E+05-2.50020E+02	PTN 1 12
2	PTN 1 21
3.01002E+05-2.00003E+02-9.01002E+05 3.71007E+01	PTN 1 22
0.00001E+05 3.71007E+01-0.00001E+05-2.50020E+02	PTN 1 22
2.00000E+01	PTN 2001
0 2 1 1	PTN 2002
2	PTN 2 11
2.00001E+02-5.20000E+00-2.00001E+02 7.02170E-01	PTN 2 12
1.00000E+02 7.02170E-01-1.00000E+02-9.17240E+00	PTN 2 12
2	PTN 2 21
2.00001E+02-5.20000E+00-2.00001E+02 7.02170E-01	PTN 2 22
1.00000E+02 7.02170E-01-1.00000E+02-9.17240E+00	PTN 2 22
0.00000E+00	PTN 3001
0 2 1 1	PTN 3002
2	PTN 3 11
1.25001E+01-1.30022E+00-1.25001E+01 1.05543E-01	PTN 3 12
1.25000E+01 1.05543E-01-1.25000E+01-1.29310E+00	PTN 3 12
2	PTN 3 21
1.25001E+01-1.30022E+00-1.25001E+01 1.05543E-01	PTN 3 22
1.25000E+01 1.05543E-01-1.25000E+01-1.29310E+00	PTN 3 22
2.00000E+00	PTN 4001
0 2 1 1	PTN 4002
2	PTN 4 11
2.00001E+00-5.20000E-01-2.00001E+00 7.02170E-02	PTN 4 12
1.00000E+00 7.02170E-02-1.00000E+00-9.17240E-01	PTN 4 12
2	PTN 4 21
2.00001E+00-5.20000E-01-2.00001E+00 7.02170E-02	PTN 4 22
1.00000E+00 7.02170E-02-1.00000E+00-9.17240E-01	PTN 4 22
1.00000E+03	PTN 1001
0 2 1 1	PTN 1002
2	PTN 1 11
3.30937E+05-1.73020E+02-3.30937E+05 2.07033E+01	PTN 1 12
3.31735E+05 2.07033E+01-3.31735E+05-1.72201E+02	PTN 1 12
2	PTN 1 21
3.30937E+05-1.73020E+02-3.30937E+05 2.07033E+01	PTN 1 22
3.31735E+05 2.07033E+01-3.31735E+05-1.72201E+02	PTN 1 22
2.00000E+01	PTN 2001
0 2 1 1	PTN 2002
2	PTN 2 11
1.33975E+02-3.07240E+00-1.33975E+02 0.95266E-01	PTN 2 12
1.32690E+02 0.95266E-01-1.32690E+02-3.00002E+00	PTN 2 12
2	PTN 2 21
1.33975E+02-3.07240E+00-1.33975E+02 0.95266E-01	PTN 2 22
1.32690E+02 0.95266E-01-1.32690E+02-3.00002E+00	PTN 2 22
0.00000E+00	PTN 3001
0 2 1 1	PTN 3002
2	PTN 3 11
0.37302E+00-0.00120E-01-0.37302E+00 1.23010E-01	PTN 3 12
0.29337E+00 1.23010E-01-0.29337E+00-0.01000E-01	PTN 3 12
2	PTN 3 21
0.37302E+00-0.00120E-01-0.37302E+00 1.23010E-01	PTN 3 22
0.29337E+00 1.23010E-01-0.29337E+00-0.01000E-01	PTN 3 22
2.00000E+00	PTN 4001
0 2 1 1	PTN 4002
2	PTN 4 11
1.33975E+00-3.07240E-01-1.33975E+00 0.95266E-02	PTN 4 12
1.32690E+00 0.95266E-02-1.32690E+00-3.00002E-01	PTN 4 12
2	PTN 4 21
1.33975E+00-3.07240E-01-1.33975E+00 0.95266E-02	PTN 4 22
1.32690E+00 0.95266E-02-1.32690E+00-3.00002E-01	PTN 4 22
1.00000E+03	PTN 1001
0 2 1 1	PTN 1002
2	PTN 1 11
2.51005E+05-1.30015E+02-2.51005E+05 1.05907E+01	PTN 1 12
2.00002E+05 1.05907E+01-2.00002E+05-1.20992E+02	PTN 1 12
2	PTN 1 21
2.51005E+05-1.30015E+02-2.51005E+05 1.05907E+01	PTN 1 22
2.00002E+05 1.05907E+01-2.00002E+05-1.20992E+02	PTN 1 22
2.00000E+01	PTN 2001
0 2 1 1	PTN 2002
2	PTN 2 11
1.00002E+02-2.00000E+00-1.00002E+02 3.71010E-01	PTN 2 12
0.93000E+01 3.71010E-01-0.93000E+01-2.57903E+00	PTN 2 12
2	PTN 2 21
1.00002E+02-2.00000E+00-1.00002E+02 3.71010E-01	PTN 2 22
0.93000E+01 3.71010E-01-0.93000E+01-2.57903E+00	PTN 2 22
0.00000E+00	PTN 3001
0 2 1 1	PTN 3002
2	PTN 3 11
0.29012E+00-0.52075E-01-0.29012E+00 0.20930E-02	PTN 3 12
0.21005E+00 0.20930E-02-0.21005E+00-0.00950E-01	PTN 3 12
2	PTN 3 21
0.29012E+00-0.52075E-01-0.29012E+00 0.20930E-02	PTN 3 22
0.21005E+00 0.20930E-02-0.21005E+00-0.00950E-01	PTN 3 22
2.00000E+00	PTN 4001
0 2 1 1	PTN 4002
2	PTN 4 11
1.00002E+00-2.00000E-01-1.00002E+00 3.71010E-02	PTN 4 12
0.93000E+01 3.71010E-02-0.93000E+01-2.57903E-01	PTN 4 12
2	PTN 4 21
1.00002E+00-2.00000E-01-1.00002E+00 3.71010E-02	PTN 4 22
0.93000E+01 3.71010E-02-0.93000E+01-2.57903E-01	PTN 4 22

Aero-Servo-Elastic Stability Analysis

The input data for the aero-servo-elastic stability analysis are obtained by adding the aerodynamic influence coefficients (AIC's) to the servoeelastic stability analysis data deck and by adding the necessary aerodynamic control cards and related data. The complete input data deck is listed in Table 7. The AIC's were based on the three Mach numbers $M = 2.0$, 3.0 , and 4.0 . The stability analysis is carried out at sea level for which the density is 0.002378 slugs/cu. ft., and the speed of sound is 1100.0 fps, and at the three Mach numbers 2.5 , 3.0 , and 3.5 ; the eigenvectors are requested for $M = 3.0$.

The results of the analyses at the three Mach numbers are shown in Table 8. Table 8 shows an abbreviated output: it shows the first page which echoes the aerodynamic control data, a typical printout of the AIC's for $M = 2.0$ and $k = 0.001$, the three sets of stability results, and the eigenvectors for $M = 3.0$. There are some differences from the servoeelastic results in Table 4. Both the servoeelastic analysis and the aero-servo-elastic analysis have 22 roots. However, the 15th root, which was a complex conjugate "zero" in the servoeelastic analysis, now has become the short period root in the presence of the airstream. In addition, an unstable root, No. 3, has also appeared. A perusal of the 3rd eigenvector and structural deflections indicates it to be some kind of divergence in altitude. This instability is a subject for further research.

The motions involved in the eight oscillatory and five non-oscillatory solutions are summarized below.

<u>Mode No.</u>	<u>Dominant Motion</u>
3	Altitude divergence
4	Actuator
5 & 6	Accelerometer
7	Noise filter
15	Short period
16	First body bending
17	Rate gyro
18	Compensating damping amplifier
19	Flipper bending
20	Second body bending
21	Flipper torsion
22	Third body bending

Table 7 - Input Cards for Program MPASES for
Aero-Servo-Elastic Stability Analysis

```

C   TITLE CARDS
TYPICAL AIR-TO-AIR MISSILE
AERO-SERVO-ELASTIC ANALYSIS USING PISTON THEORY
C   CONTROL CARD
  2   1*   5   2   1   11   5   3   *   1   30   1   0   1
C   SHIFT EIGENVALUE
100.0
C   AERO CONSTANTS
1.0   3.0   3.0   0.05
C   AIC DATA
C   MACH NUMBERS
2.0   3.0   4.0
C   REDUCED FREQUENCIES
0.001   0.05   0.20   0.50
C   AERO DATA
0.002378   1100.0
C   NUMBER OF VELOCITIES
  3
C   MASS MATRIX
C   1ST ROW
50.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
C   2ND ROW
50.0   0.00   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
C   3RD ROW
50.0   0.0   0.00   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
C   4TH ROW
50.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
C   5TH ROW
50.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
C   6TH ROW
50.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
C   7TH ROW
50.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
C   8TH ROW
50.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
C   9TH ROW
50.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
C   10TH ROW
50.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0
C   11TH ROW
4.4333333   0.0010417   -0.3961806   0.0
C   12TH ROW
0.9701389   0.0   -0.7260417
C   13TH ROW
1.1333332   -0.2961806
C   14TH ROW
1.2979166
C   CONTROL SURFACE DEFLECTION MODE
0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   1.3333333333333333   -6.6666666667
1.3333333333333333   -6.6666666667

```

DAMPING COEFFICIENTS, G						
C	0.01	0.02	0.03	0.03	0.03	
C	MODAL VIBRATION FREQUENCIES					
	45.0	125.4	248.2	100.0	115.0	
C	VIBRATION MODE SHAPES					
C	1ST MODE					
	1.0	0.411195	-0.122424	-0.532567	-0.756204	-0.756204
	-0.532467	-0.122424	0.411195	1.0	0.821620	1.138881
	0.821620	1.138881				
C	2ND MODE					
	1.0	-0.150871	-0.910842	-0.990216	-0.419043	0.419043
	0.990215	0.910842	0.150871	-1.0	-0.630256	-1.281954
	-0.630256	-1.281954				
C	3RD MODE					
	-0.803195	0.645103	1.0	0.105986	-0.947894	-0.947893
	0.105986	1.0	0.645103	-0.803195	-0.259298	-1.188204
	-0.259298	-1.188204				
C	4TH MODE					
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	.3333333333	.3333333333
	1.0	1.0				
C	5TH MODE					
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	-0.2	1.0
	-0.2	1.0				
C	RIGID BODY MODES					
C	PLUNGE MODE					
	1.0	1.0	1.0	1.0	1.0	1.0
	1.0	1.0	1.0	1.0	1.0	1.0
	1.0	1.0				
C	PITCH MODE					
	7.5	22.5	37.5	52.5	67.5	82.5
	97.5	112.5	127.5	142.5	158.0	146.0
	138.0	146.0				
C	CONTROL SURFACE OUTPUT					
	1.0	0.016	0.0			
C	NUMBER OF INPUTS TO CONTROL SURFACE					
	1					
C	CONTROL SURFACE INPUTS FROM SERVOS					
	5	0	-1.0			
C	OUTPUT FROM SERVO ELEMENT					
	1	2	1.0	0.359	-2 0.659	-5
C	NUMBER OF INPUTS TO SERVO ELEMENT					
	1					
C	INPUT FROM BODY PITCH RATE					
	0	-1	-2.77777778	-3		
C	DIFFERENTIATION MATRIX FOR RATE GYRO					
	0.0	1.0	-27.0	27.0	-1.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0				
C	OUTPUT FROM SERVO ELEMENT					
	2	1	1.0	0.001		
C	NUMBER OF INPUTS TO SERVO ELEMENT					
	1					
C	INPUTS TO SERVO ELEMENT					
	1	0	-1.0			
C	OUTPUT FROM SERVO ELEMENT					
	3	0	1.0			
C	NUMBER OF INPUTS TO SERVO ELEMENT					
	1					
C	INPUTS TO SERVO ELEMENT					
	2	0	-0.1			
C	OUTPUT FROM SERVO ELEMENT					
	4	2	1.0	0.223	-2 0.253	-5
C	NUMBER OF INPUTS TO SERVO ELEMENT					
	1					

C INPUTS TO SERVO ELEMENT
 3 1 -1.0 -0.016
 C OUTPUT FROM SERVO ELEMENT
 5 0 1.0
 C NUMBER OF INPUTS TO SERVO ELEMENT
 2
 C INPUTS TO SERVO ELEMENT
 4 0 1.0
 8 0 -1.0
 C OUTPUT FROM SERVO ELEMENT
 6 0 1.0
 C NUMBER OF INPUTS TO SERVO ELEMENT
 1
 C INPUTS TO SERVO ELEMENT
 2 0 -2.0
 C OUTPUT FROM SERVO ELEMENT
 7 0 1.0
 C NUMBER OF INPUTS TO SERVO ELEMENT
 2
 C INPUTS TO SERVO ELEMENT
 6 0 1.0
 11 0 -1.0
 C OUTPUT FROM SERVO ELEMENT
 8 1 0.0 1.0
 C NUMBER OF INPUTS TO SERVO ELEMENT
 1
 C INPUTS TO SERVO ELEMENT
 7 0 -1.0
 C OUTPUT FROM SERVO ELEMENT
 9 2 1.0 0.6366 -2 1.013 -5
 C NUMBER OF INPUTS TO SERVO ELEMENT
 1
 C INPUT FROM BODY ACCELERATION
 0 -2 -0.0625
 C INTERPOLATION MATRIX FOR ACCELEROMETER
 0.0 -1.0 9.0 9.0 -1.0 0.0
 0.0 0.0 0.0 0.0 0.0 0.0
 0.0 0.0
 C OUTPUT FROM SERVO ELEMENT
 10 0 1.0
 C NUMBER OF INPUTS TO SERVO ELEMENT
 1
 C INPUTS TO SERVO ELEMENT
 9 0 1.0
 C OUTPUT FROM SERVO ELEMENT
 11 0 1.0
 C NUMBER OF INPUTS TO SERVO ELEMENT
 1
 C OUTPUT FROM SERVO ELEMENT
 10 0 -0.002170
 C AICS
 C 1ST K, 1ST M
 C AIC PARTITION CONTROL
 6 1
 C AIC PARTITION CONTROL
 3 0
 C AIC PARTITION
 1.51264E+06-7.49339E+02-1.81260E+08 6.62394E+01 4.49457E+05 5.88198E+00PTN 1 12
 4.02395E+05 6.62394E+01 3.74455E+05-9.59263E+02-7.76849E+05-8.97240E+01PTN 1 13
 -1.10424E+05 5.88198E+00 6.28925E+05-8.97240E+01-5.18501E+05-2.56200E+02PTN 1 14
 C AIC PARTITION CONTROL
 1 1
 C AIC PARTITION CONTROL
 2 0
 C AIC PARTITION
 5.01602E+05-2.60043E+02-5.01602E+05 3.71087E+01

PTN 1 12

4,98401E+05 3,71087E+01-4,98401E+05-2,58620E+02	PTN 1 12
C AIC PARTITION CONTROL	
2 0	
C AIC PARTITION	
5,01602E+05-2,60043E+02-5,01602E+05 3,71087E+01	PTN 1 22
4,98401E+05 3,71087E+01-4,98401E+05-2,58620E+02	PTN 1 22
C 2ND K, 1ST M	
C AIC PARTITION CONTROL	
6 1	
C AIC PARTITION CONTROL	
3 0	
C AIC PARTITION	
5,25058E+02-1,49868E+01-7,25041E+02 1,32479E+00 1,99983E+02 1,17640E-01	PTN 2 12
1,60958E+02 1,32479E+00 1,49782E+02-1,91853E+01-3,10740E+02-1,79448E+00	PTN 2 13
-4,41698E+01 1,17640E-01 2,51570E+02-1,79448E+00-2,07400E+02-5,12400E+00	PTN 2 14
C AIC PARTITION CONTROL	
1 1	
C AIC PARTITION CONTROL	
2 0	
C AIC PARTITION	
2,00641E+02-5,20086E+00-2,00641E+02 7,42174E-01	PTN 2 12
1,99360E+02 7,42174E-01-1,99360E+02-5,17240E+00	PTN 2 12
C AIC PARTITION CONTROL	
2 0	
C AIC PARTITION	
2,00641E+02-5,20086E+00-2,00641E+02 7,42174E-01	PTN 2 22
1,99360E+02 7,42174E-01-1,99360E+02-5,17240E+00	PTN 2 22
C 3RD K, 1ST M	
C AIC PARTITION CONTROL	
6 1	
C AIC PARTITION CONTROL	
3 0	
C AIC PARTITION	
3,28161E+01-3,74670E+00-4,53151E+01 3,31197E-01 1,24989E+01 2,94099E-02	PTN 3 12
1,00599E+01 3,31197E-01 9,36136E+00-4,79631E+00-1,94212E+01-4,48620E-01	PTN 3 13
-2,76061E+00 2,94099E-02 1,57231E+01-4,48620E-01-1,29625E+01-1,28100E+00	PTN 3 14
C AIC PARTITION CONTROL	
1 1	
C AIC PARTITION CONTROL	
2 0	
C AIC PARTITION	
1,25401E+01-1,30022E+00-1,25401E+01 1,85543E-01	PTN 3 12
1,24600E+01 1,85543E-01-1,24600E+01-1,29310E+00	PTN 3 12
C AIC PARTITION CONTROL	
2 0	
C AIC PARTITION	
1,25401E+01-1,30022E+00-1,25401E+01 1,85543E-01	PTN 3 22
1,24600E+01 1,85543E-01-1,24600E+01-1,29310E+00	PTN 3 22
C 4TH K, 1ST M	
C AIC PARTITION CONTROL	
6 1	
C AIC PARTITION CONTROL	
3 0	
C AIC PARTITION	
5,25058E+00-1,49868E+00-7,25041E+00 1,32479E-01 1,99983E+00 1,17640E-02	PTN 4 12
1,60958E+00 1,32479E-01 1,49782E+00-1,91853E+00-3,10740E+00-1,79448E-01	PTN 4 13
-4,41698E-01 1,17640E-02 2,51570E+00-1,79448E-01-2,07400E+00-5,12400E-01	PTN 4 14
C AIC PARTITION CONTROL	
1 1	
C AIC PARTITION CONTROL	
2 0	
C AIC PARTITION	
2,00641E+00-5,20086E-01-2,00641E+00 7,42174E-02	PTN 4 12
1,99360E+00 7,42174E-02-1,99360E+00-5,17240E-01	PTN 4 12
C AIC PARTITION CONTROL	
2 0	

C	AIC PARTITION	
	2.00641E+00-5.20086E-01-2.00641E+00 7.42174E-02	PTN 4 22
	1.99360E+00 7.42174E-02-1.99360E+00-5.17240E-01	PTN 4 22
C	1ST K, 2ND M	
C	AIC PARTITION CONTROL	
	6 1	
C	AIC PARTITION CONTROL	
	3 0	
C	AIC PARTITION	
	8.76798E+05-5.00553E+02-1.21068E+06 4.43037E+01 5.33885E+05 3.85431E+00PTN 1 12	
	2.67875E+05 4.43037E+01 2.49716E+05-6.39268E+02-5.17592E+05-5.96559E+01PTN 1 13	
	-7.33873E+04 3.85431E+00 4.18408E+05-5.96559E+01-3.45020E+05-1.70527E+02PTN 1 14	
C	AIC PARTITION CONTROL	
	1 1	
C	AIC PARTITION CONTROL	
	2 0	
C	AIC PARTITION	
	3.34937E+05-1.73624E+02-3.34937E+05 2.47633E+01	PTN 1 12
	3.31735E+05 2.47633E+01-3.31735E+05-1.72201E+02	PTN 1 12
C	AIC PARTITION CONTROL	
	2 0	
C	AIC PARTITION	
	3.34937E+05-1.73624E+02-3.34937E+05 2.47633E+01	PTN 1 22
	3.31735E+05 2.47633E+01-3.31735E+05-1.72201E+02	PTN 1 22
C	2ND K, 2ND M	
C	AIC PARTITION CONTROL	
	6 1	
C	AIC PARTITION CONTROL	
	3 0	
C	AIC PARTITION	
	3.50719E+02-1.00111E+01-4.84273E+02 8.86074E-01 1.33554E+02 7.70863E-02PTN 2 12	
	1.07150E+02 8.86074E-01 9.98866E+01-1.27854E+01-2.07037E+02-1.19312E+00PTN 2 13	
	-2.93549E+01 7.70863E-02 1.67363E+02-1.19312E+00-1.38008E+02-3.41053E+00PTN 2 14	
C	AIC PARTITION CONTROL	
	1 1	
C	AIC PARTITION CONTROL	
	2 0	
C	AIC PARTITION	
	1.33975E+02-3.47248E+00-1.33975E+02 4.95266E-01	PTN 2 12
	1.32694E+02 4.95266E-01-1.32694E+02-3.44402E+00	PTN 2 12
C	AIC PARTITION CONTROL	
	2 0	
C	AIC PARTITION	
	1.33975E+02-3.47248E+00-1.33975E+02 4.95266E-01	PTN 2 22
	1.32694E+02 4.95266E-01-1.32694E+02-3.44402E+00	PTN 2 22
C	3RD K, 2ND M	
C	AIC PARTITION CONTROL	
	6 1	
C	AIC PARTITION CONTROL	
	3 0	
C	AIC PARTITION	
	2.19199E+01-2.50276E+00-3.02671E+01 2.21519E-01 8.34713E+00 1.92716E-02PTN 3 12	
	6.69688E+00 2.21519E-01 6.24291E+00-3.19634E+00-1.29398E+01-2.98280E-01PTN 3 13	
	-1.83468E+00 1.92716E-02 1.04602E+01-2.98280E-01-8.62551E+00-8.52633E-01PTN 3 14	
C	AIC PARTITION CONTROL	
	1 1	
C	AIC PARTITION CONTROL	
	2 0	
C	AIC PARTITION	
	8.37342E+00-8.68120E-01-8.37342E+00 1.23816E-01	PTN 3 12
	-8.29337E+00 1.23816E-01-8.29337E+00-8.61005E-01	PTN 3 12
C	AIC PARTITION CONTROL	
	2 0	
C	AIC PARTITION	
	8.37342E+00-8.68120E-01-8.37342E+00 1.23816E-01	PTN 3 22
	8.29337E+00 1.23816E-01-8.29337E+00-8.61005E-01	PTN 3 22

C 4TH K, 2ND M
C AIC PARTITION CONTROL
6 1
C AIC PARTITION CONTROL
3 0
C AIC PARTITION
3,50719E+00-1,00111E+00-4,84273E+00 8,86074E-02 1,33554E+00 7,70863E-03PTN 4 12
1,07150E+00 8,86074E-02 9,98866E-01-1,27854E+00-2,07037E+00-1,19312E-01PTN 4 13
-2,93549E-01 7,70863E-03 1,67363E+00-1,19312E-01-1,38008E+00-3,41053E-01PTN 4 14
C AIC PARTITION CONTROL
1 1
C AIC PARTITION CONTROL
2 0
C AIC PARTITION
1,33975E+00-3,47248E-01-1,33975E+00 4,95266E-02 PTN 4 12
1,32694E+00 4,95266E-02-1,32694E+00-3,44402E-01 PTN 4 12
C AIC PARTITION CONTROL
2 0
C AIC PARTITION
1,33975E+00-3,47248E-01-1,33975E+00 4,95266E-02 PTN 4 22
1,32694E+00 4,95266E-02-1,32694E+00-3,44402E-01 PTN 4 22
C 1ST K, 3RD M
C AIC PARTITION CONTROL
6 1
C AIC PARTITION CONTROL
3 0
C AIC PARTITION
6,58877E+05-3,76143E+02-9,09727E+05 3,33141E+01 2,50850E+05 2,85050E+00PTN 1 12
2,00614E+05 3,33141E+01 1,67349E+05-4,79253E+02-3,87963E+05-4,46293E+01PTN 1 13
-5,48687E+04 2,85050E+00 3,13149E+05-4,46293E+01-2,58281E+05-1,27688E+02PTN 1 14
C AIC PARTITION CONTROL
1 1
C AIC PARTITION CONTROL
2 0
C AIC PARTITION
2,51605E+05-1,30415E+02-2,51605E+05 1,85907E+01 PTN 1 12
2,48402E+05 1,85907E+01-2,48402E+05-1,28992E+02 PTN 1 12
C AIC PARTITION CONTROL
2 0
C AIC PARTITION
2,51605E+05-1,30415E+02-2,51605E+05 1,85907E+01 PTN 1 22
2,48402E+05 1,85907E+01-2,48402E+05-1,28992E+02 PTN 1 22
C 2ND K, 3RD M
C AIC PARTITION CONTROL
6 1
C AIC PARTITION CONTROL
3 0
C AIC PARTITION
2,63551E+02-7,52286E+00-3,63891E+02 6,66282E-01 1,00340E+02 5,70099E-02PTN 2 12
8,02458E+01 6,66282E-01 7,49395E+01-9,58506E+00-1,55185E+02-8,92587E-01PTN 2 13
-2,19475E+01 5,70099E-02 1,25260E+02-8,92587E-01-1,03312E+02-2,55375E+00PTN 2 14
C AIC PARTITION CONTROL
1 1
C AIC PARTITION CONTROL
2 0
C AIC PARTITION
1,00642E+02-2,60830E+00-1,00642E+02 3,71814E-01 PTN 2 12
9,93608E+01 3,71814E-01-9,93608E+01-2,57983E+00 PTN 2 12
C AIC PARTITION CONTROL
2 0
C AIC PARTITION
1,00642E+02-2,60830E+00-1,00642E+02 3,71814E-01 PTN 2 22
9,93608E+01 3,71814E-01-9,93608E+01-2,57983E+00 PTN 2 22
C 3RD K, 3RD M
C AIC PARTITION CONTROL
6 1

C AIC PARTITION CONTROL

C AIC PARTITION

1,64719E+01-1,88072E+00-2,27432E+01 1,66571E-01 6,27126E+00 1,42525E-02PTN 3 12

5,01536E+00 1,66571E-01 4,68372E+00-2,39627E+00-9,69908E+00-2,23147E-01PTN 3 13

-1,37172E+00 1,42525E-02 7,82873E+00-2,23147E-01-6,45701E+00-6,38438E-01PTN 3 14

C AIC PARTITION CONTROL

C AIC PARTITION CONTROL

C AIC PARTITION

6,29012E+00-6,52075E-01-6,29012E+00 9,29536E-02 PTN 3 12

6,21005E+00 9,29536E-02-6,21005E+00-6,44958E-01 PTN 3 12

C AIC PARTITION CONTROL

C AIC PARTITION

6,29012E+00-6,52075E-01-6,29012E+00 9,29536E-02 PTN 3 22

6,21005E+00 9,29536E-02-6,21005E+00-6,44958E-01 PTN 3 22

C 4TH K, 3RD M

C AIC PARTITION CONTROL

C AIC PARTITION CONTROL

C AIC PARTITION

2,63551E+00-7,52286E-01-3,63891E+00 6,66282E-02 1,00340E+00 5,70099E-03PTN 4 12

8,02458E-01 6,66282E-02 7,49395E-01-9,58506E-01-1,55185E+00-8,92587E-02PTN 4 13

-2,19475E-01 5,70099E-03 1,25260E+00-8,92587E-02-1,03312E+00-2,55375E-01PTN 4 14

C AIC PARTITION CONTROL

C AIC PARTITION CONTROL

C AIC PARTITION

1,00642E+00-2,60830E-01-1,00642E+00 3,71814E-02 PTN 4 12

9,93608E-01 3,71814E-02-9,93608E-01-2,57983E-01 PTN 4 12

C AIC PARTITION CONTROL

C AIC PARTITION

1,00642E+00-2,60830E-01-1,00642E+00 3,71814E-02 PTN 4 22

9,93608E-01 3,71814E-02-9,93608E-01-2,57983E-01 PTN 4 22

C MACH NUMBERS FOR VELOCITIES AND EIGENVECTOR REQUEST

2.5 -3.0 3.5



ANALYSIS OF THE DATA USING THE THEORY

10 DEGREES OF FREEDOM
9 FLEXIBLE MODES
2 RIGID BODY MODES
1 CONTROL SURFACES
11 SERVO ELEMENTS

SHIFT E (GENVALUT (GAMA) = -1,000E+02

03182003Z 6300N 05

! ALTITUDE VARIATIONS !

AIC MATRICES FOR 3 MACH NUMBERS AND 4 REDUCED FREQUENCIES

REFERENCE SERIAL-00000 = 1,500000.00 FY

SEMI-SPAM 1.5000E+00 PY
SHOGACC 1.0000E+00 PY

[illegible]

CO-300000'S - MOLEKULO VZROTHI NIVN

UPPER TRIANGLE OF WEIGHT MATRIX

[illegible]

[illegible]

STABILITY ANALYSIS RESULTS

SPEED OF SOUND = 1.1000E+03 FT/SEC

DENSITY = 2.3700E-03 SLUGS/CU FT

MACH NUMBER = 3.0000E+00

VELOCITY = 3.3000E+03 FT/SEC, 1.0530E+03 M/SEC

MODE	EIGENVALUE - MU-MPS	EIGENVALUE - OMEGA-MPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	REDUCED FREQUENCY	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	-1.3325E-07	0.	0.	2.12239E-04	0.		5.19719E+06
2	-2.0334E-04	0.	0.	4.50944E-05	0.		2.44637E+03
3	6.27331E+00	0.	0.	9.90420E-01	0.		-1.10491E-01
4	-0.49574E-01	0.	0.	1.75217E+01	0.		0.150537E-03
5	-2.9034E-02	0.	0.	4.62002E+01	0.		2.34740E-03
6	-3.3301E-02	0.	0.	5.31447E+01	0.		2.07574E-03
7	-9.90534E-02	0.	0.	1.20421E+02	0.		6.94104E-04
8	-2.24447E-01	0.	0.	3.57103E+02	0.		3.04055E-04
9	-1.01904E-01	0.	0.	2.25067E+02	0.		-4.00461E-04
10	-1.00000E-03	0.	0.	1.25415E+02	0.		6.93147E-03
11	-1.00000E-03	0.	0.	1.25415E+02	0.		6.93147E-03
12	-1.00000E-03	0.	0.	1.25415E+02	0.		6.93147E-03
13	-1.00000E-03	0.	0.	1.25415E+02	0.		6.93147E-03
14	-1.00000E-03	0.	0.	1.25415E+02	0.		6.93147E-03
15	-1.00917E-01	4.23511E+01	6.74030E+00	7.10372E+00	1.92504E-02	3.15710E-01	4.91000E-02
16	-1.30351E-01	2.45700E+02	3.92667E+01	3.93264E+01	1.12160E-01	5.50921E-02	5.00950E-02
17	-2.43305E-02	3.27000E+02	5.21047E+01	6.49037E+01	1.49039E-01	5.45097E-01	2.04010E-03
18	-0.420524E-02	4.30540E+02	6.05315E+01	9.830720E+01	1.45725E-01	7.16450E-01	1.565107E-03
19	-0.005927E-01	9.25004E+02	9.90090E+01	9.90609E+01	2.84004E-01	3.20334E-02	3.45449E-02
20	-0.03353E+00	7.93190E+02	1.20240E+02	1.20240E+02	3.00541E-01	1.11360E-02	7.04671E-02
21	-9.704257E-01	9.93776E+02	1.50104E+02	1.509167E+02	4.51710E-01	9.71000E-02	7.16271E-03
22	-2.52191E+01	1.50233E+03	2.30052E+02	2.500020E+02	7.10151E-01	1.55000E-02	2.001970E-02

EIGENVECTORS FOR MODES REQUESTED

EIGENVECTORS FOR MODES REQUESTED

MODE	EIGENVECTORS FOR MODES REQUESTED
1	-4.030032E-05 0. 7.003405E-06 0. 1.554632E-06 0. -5.97020E-04 0.
2	-4.030032E-05 0. 2.000700E-02 0. -5.64707E-05 0. -1.03375E-04 0.
3	-1.393550E-04 0. -3.059772E-02 0. 1.000000E+00 0. -7.400520E-06 0.
4	1.220777E-06 0. 2.478100E-01 0. -5.527540E-05 0. -6.4252570E-05 0.
5	4.617512E-03 0. -9.001740E-02 0. -2.423140E-05 0. -5.410152E-05 0.
6	-0.330707E-06 0. 1.254054E-01 0. -5.370424E-05 0. -5.370424E-06 0.
7	-3.799930E-05 0. -3.2105530E-05 0. -1.0752350E-06 0. -2.303014E-06 0.
8	-1.574054E-01 0. -3.050092E-04 0. -1.0752350E-06 0. -2.303014E-06 0.

EIGENVECTON FOR MODE 4

-0.859423E-05	0.	7.159270E-04	0.	1.443170E-04	0.	-1.234873E-03	0.
-0.002804E-04	0.	5.687712E-04	0.	-1.894839E-05	0.	-2.848310E-04	0.
1.650611E-03	0.	-4.978470E-05	0.	1.000000E-00	0.	4.063106E-07	0.
-0.424443E-04	0.	-1.040495E-04	0.	1.453401E-05	0.	1.021044E-05	0.
-0.459191E-04	0.	1.207715E-07	0.	3.352544E-04	0.	-1.236584E-05	0.
-0.859423E-05	0.	-1.177020E-04	0.	-1.351479E-06	0.	-1.351479E-06	0.
-0.107707E-07	0.	-1.694797E-04	0.	-2.702959E-05	0.	-5.257110E-05	0.
1.177020E-02	0.	2.554151E-05	0.				

EIGENVECTON FOR MODE 5

-2.312510E-07	0.	3.007611E-04	0.	0.034750E-04	0.	-7.030310E-06	0.
-7.352501E-06	0.	3.119423E-07	0.	-0.400470E-09	0.	-2.174009E-04	0.
4.912310E-07	0.	-4.901419E-07	0.	1.000000E-00	0.	7.944979E-10	0.
-1.311455E-10	0.	-3.043644E-11	0.	2.047490E-04	0.	2.532421E-04	0.
-1.074250E-04	0.	2.622977E-11	0.	7.487941E-09	0.	-1.492630E-09	0.
1.530040E-09	0.	-3.444299E-03	0.	-2.305122E-04	0.	-2.305122E-04	0.
-2.575571E-08	0.	-2.124621E-04	0.	-4.770244E-09	0.	7.470900E-06	0.
3.444299E-03	0.	7.487941E-09	0.				

EIGENVECTON FOR MODE 6

-1.297440E-07	0.	2.564770E-04	0.	5.907804E-09	0.	-5.073330E-06	0.
-5.242745E-06	0.	1.354270E-07	0.	-3.162265E-09	0.	-1.515420E-04	0.
0.743418E-08	0.	-0.100210E-04	0.	1.000000E-00	0.	3.085501E-10	0.
-2.261500E-11	0.	-1.769200E-11	0.	1.519333E-04	0.	1.570000E-08	0.
-0.594740E-10	0.	9.552007E-14	0.	4.342983E-09	0.	-2.014482E-10	0.
2.449760E-10	0.	-2.449765E-03	0.	-3.031003E-10	0.	-3.031003E-11	0.
-2.994345E-08	0.	-1.970432E-04	0.	-0.003766E-10	0.	6.499202E-06	0.
2.994345E-08	0.	6.448596E-04	0.				

EIGENVECTON FOR MODE 7

-2.700725E-04	0.	-2.254711E-04	0.	-1.463279E-04	0.	4.460050E-02	0.
3.250793E-01	0.	-1.306265E-04	0.	2.173330E-04	0.	6.000652E-02	0.
-1.260400E-03	0.	1.000000E-00	0.	9.501700E-01	0.	-2.710099E-07	0.
-2.250021E-07	0.	1.465427E-07	0.	-4.467391E-05	0.	-3.263574E-04	0.
1.304300E-07	0.	-2.176529E-04	0.	-0.044047E-05	0.	1.2702621E-04	0.
-1.001400E-03	0.	-9.595098E-04	0.	0.0052781E-04	0.	0.0052781E-05	0.
1.732510E-06	0.	1.003201E-03	0.	1.733055E-03	0.	-1.730973E-03	0.
9.595098E-04	0.	2.002209E-04	0.				

EIGENVECTON FOR MODE 15

0.392460E-06	6.005197E-05	2.909413E-06	-9.050493E-06	6.113502E-07	-7.945590E-07	3.641906E-04	1.224309E-03
2.002805E-04	0.494402E-04	2.200305E-03	1.189704E-03	-5.065703E-05	-2.232161E-05	2.736497E-05	2.434774E-04
-1.502742E-03	-2.530009E-03	2.924947E-04	-1.344004E-04	1.000000E-00	0.	1.3073204E-06	-6.597764E-07
-1.002760E-07	-3.450472E-04	-2.125000E-06	-7.302000E-04	2.445210E-05	-1.040280E-05	1.022733E-05	-1.154535E-05
9.071025E-06	-5.534200E-05	-5.961455E-04	1.4040530E-06	4.962404E-06	-2.303970E-06	-6.510504E-05	-1.560314E-05
-4.936979E-04	-5.263743E-04	-7.073492E-03	-2.125052E-02	-6.667704E-05	-1.304304E-05	-0.667704E-06	-1.304304E-06
4.033045E-07	-3.072079E-06	5.202430E-06	1.591004E-06	-1.333550E-04	-2.600617E-05	1.4070517E-04	7.2217120E-05
7.073492E-03	2.125052E-02	1.5344470E-05	4.013094E-05				

EIGENVECTORS FOR MODE 16

2.7501134E-05	7.0499744E-05	-1.2254122E-06	-1.7524450E-06	-2.1401571E-07	-3.0015515E-07	2.2932317E-04	2.9490470E-04
1.1053604E-04	1.5369144E-04	1.2593308E-05	1.4615016E-05	-2.6312650E-07	-2.7069547E-07	3.0244494E-05	5.2362531E-05
1.4001616E-04	1.8218441E-04	1.6218441E-04	-1.8347944E-04	1.0000000E-00	0.	3.0250437E-07	-1.2045779E-07
-9.4695307E-09	5.3494804E-09	-1.1949203E-09	9.3701927E-10	1.1394099E-06	-9.9434914E-07	5.5554325E-07	-9.7920201E-07
5.5206054E-08	-5.8199756E-08	-1.1355279E-09	1.2112754E-09	2.0265452E-07	-1.6650374E-07	-2.5371061E-06	-4.4505501E-07
-7.8264470E-07	-6.9456990E-07	-2.2660109E-04	-9.8406676E-03	-2.5275001E-06	1.0100305E-07	-2.5275001E-07	1.0100305E-06
3.2799933E-08	-2.317333E-08	8.1237043E-01	9.7925250E-01	-5.0550001E-06	3.0400730E-07	5.5467245E-04	8.5062914E-04
2.2600100E-04	4.0406676E-03	4.9172437E-07	8.7086447E-06				

EIGENVECTORS FOR MODE 17

1.1266374E-05	1.0456035E-05	-3.4000454E-06	6.0225076E-07	-5.4665014E-07	3.2426510E-07	6.7340054E-04	-2.3093227E-04
2.1021571E-04	-3.7263349E-04	-1.1995510E-06	-1.3086244E-05	2.3001543E-06	2.9590213E-07	0.7204202E-05	-1.0570200E-04
-5.0429369E-04	-1.5732312E-04	-3.0890347E-04	-7.6104019E-04	1.0000000E-00	0.	1.5910045E-06	-4.6175375E-06
0.3131720E-08	5.7040703E-08	1.4555460E-09	6.0109240E-10	-1.5274294E-06	-9.7340490E-07	-1.0390450E-06	1.3034546E-07
-2.5509700E-08	2.2429756E-08	5.4622454E-10	-9.7724420E-10	-3.3526500E-07	-1.1736570E-06	-9.2661040E-07	1.2212105E-06
-1.0507664E-06	1.7037056E-06	-1.5095429E-03	-1.9404443E-03	1.0025355E-06	1.1531327E-06	1.0035355E-07	1.1531327E-07
2.3300500E-09	-4.9070704E-09	1.0013140E-06	-1.7000134E-06	2.1270711E-06	2.3002653E-06	1.0402239E-06	1.9410273E-06
1.5595924E-03	1.9644944E-03	3.1072950E-06	9.2672527E-06				

EIGENVECTORS FOR MODE 18

6.2300740E-06	1.7253106E-05	-6.0071026E-06	-1.3512493E-06	-1.2015459E-06	7.4922210E-07	1.5100775E-03	9.3300234E-05
3.0131072E-04	-1.0340021E-03	-9.4241274E-07	-1.4621500E-05	9.9530044E-09	2.9276255E-07	2.3150034E-04	-3.4764192E-04
-1.1233797E-04	6.1139045E-05	-1.3012014E-03	-3.9864594E-03	1.0000000E-00	0.	1.2239010E-06	-2.7050026E-06
5.5494450E-09	8.5303739E-09	2.3311035E-09	-5.7070013E-10	-1.0500030E-06	-1.0236376E-06	-2.2067794E-06	1.5067717E-06
-1.6200040E-06	1.6000144E-06	3.1565050E-10	-3.5104927E-10	-7.1304042E-07	1.9505227E-07	1.7939404E-07	5.5014900E-06
-2.9000471E-06	6.6901019E-06	-1.1607237E-03	-1.1255942E-03	2.7253066E-07	-1.1040411E-07	2.7253066E-06	-1.1040411E-06
7.2234190E-10	-5.3265906E-09	2.9916094E-06	-6.1042403E-06	5.4506042E-07	-2.2080002E-07	1.9737095E-06	2.6699411E-06
1.1007237E-03	1.1205492E-03	2.5107704E-06	2.4450002E-06				

EIGENVECTORS FOR MODE 19

-5.0101379E-04	9.5670054E-06	-5.1925173E-06	6.0195639E-06	-2.9049591E-07	4.0021244E-07	5.2300054E-04	-0.1707257E-04
-4.0204104E-05	-2.0171232E-05	3.5500727E-06	-5.0191001E-06	-7.7004004E-06	1.2103707E-07	1.2019024E-06	-2.3020714E-06
-9.0007204E-06	-5.0769295E-05	-2.5002500E-05	-2.7134754E-05	1.0000000E-00	0.	1.5500637E-06	8.7043466E-09
1.0031409E-06	7.5750977E-09	7.0219597E-10	4.3972104E-10	-1.3322300E-06	-7.9422135E-07	-0.2904700E-06	6.5740049E-06
-9.1505191E-09	-5.3790716E-09	1.4043407E-10	1.1757451E-10	-3.0345151E-09	-0.9420904E-09	-9.0372717E-06	1.0304004E-06
-4.2019322E-08	4.1255747E-08	-5.1157040E-05	-1.5961444E-03	-6.0055492E-06	4.9362311E-06	-6.0055492E-09	4.9362311E-09
7.3509912E-09	-5.4300554E-10	4.7399923E-06	-9.1030032E-06	-1.2139090E-07	9.0746215E-06	2.3200174E-07	3.3049007E-06
5.1157004E-05	1.5961444E-03	1.1101070E-07	3.0030333E-06				

EIGENVECTORS FOR MODE 20

1.1204204E-07	-6.0900057E-08	-0.2135241E-06	7.7062704E-06	5.0004001E-09	4.5450103E-10	-2.4049001E-05	2.1210400E-05
-9.1099057E-07	5.1002054E-04	-7.1461444E-06	5.4015172E-06	1.5057519E-09	-1.3033089E-09	4.0070940E-07	-3.9140000E-07
-5.0041392E-07	-1.0762281E-05	-5.2702473E-06	-2.5099072E-06	1.0000000E-00	0.	-0.502217E-11	-1.4027024E-10
9.0025093E-09	1.0244903E-08	4.9150470E-13	-7.3104931E-12	2.7074914E-08	3.0010400E-06	5.4550033E-09	9.4357250E-10
0.9000332E-11	0.9191033E-11	-1.0402730E-12	-1.0190413E-12	-9.9961005E-10	-5.4740907E-09	-1.3531347E-08	7.0531211E-10
-3.0066652E-09	0.7140720E-09	-1.4030600E-05	-1.2000752E-03	-7.9370092E-09	7.1450530E-09	-7.9370092E-10	7.1450530E-10
3.4295000E-09	-9.6013914E-11	6.5192153E-09	-6.0100009E-09	-1.5074174E-06	1.4290100E-06	4.0330047E-06	2.7211555E-06
1.9030449E-05	1.2405745E-03	3.08463049E-09	2.7354006E-09				

DISPLACEMENT FOR MODE 21

3.076011E-06	-2.042100E-06	-7.005790E-06	6.000019E-06	3.337400E-06	-1.500500E-07	-2.023000E-03	4.045010E-04
-2.002000E-03	2.500000E-04	-1.043000E-04	1.040102E-06	3.500007E-06	-3.007000E-06	-2.007000E-07	1.070227E-06
1.539413E-06	2.000070E-04	1.111017E-07	7.500242E-06	1.000000E-06	0.000000E-06	-3.000000E-06	2.513100E-04
1.000200E-06	1.111017E-04	-0.000000E-06	-3.110150E-06	7.000002E-07	2.300000E-06	-3.000000E-06	2.700020E-06
1.033000E-06	1.777030E-04	-3.073700E-11	-3.157000E-11	1.010000E-06	1.010000E-10	3.020000E-06	1.100010E-06
5.014702E-06	1.777030E-04	-9.733722E-07	-9.007070E-11	2.070000E-06	-1.000000E-06	2.070000E-06	1.100010E-06
2.100130E-06	-3.700070E-10	-3.000000E-06	1.701010E-06	4.000000E-06	-2.001000E-06	1.011521E-07	2.100070E-06
9.733372E-05	9.007070E-04	2.112101E-07	2.100000E-06				

DISPLACEMENT FOR MODE 22

-2.970970E-06	-1.310531E-06	-2.110003E-04	1.000000E-06	1.500705E-05	-0.315032E-04	7.000500E-06	-3.102703E-04
1.000200E-06	2.220000E-06	-3.270000E-06	0.001010E-06	0.700000E-10	-1.020000E-10	0.001000E-07	7.000500E-07
5.002200E-11	9.971010E-05	1.001000E-06	-1.112207E-07	1.000000E-06	0.000000E-06	-0.707000E-12	-1.000700E-11
5.000000E-12	1.100000E-12	-0.100000E-06	-7.570000E-09	-2.107000E-06	-0.150000E-04	1.201000E-06	-1.057120E-06
-7.300000E-06	2.000021E-11	-1.000000E-13	-5.500000E-13	0.073270E-10	-3.170110E-10	0.370120E-06	-0.002013E-06
9.270000E-10	-1.100000E-06	-0.9100130E-06	-6.300000E-06	1.021000E-06	-3.007072E-06	1.021000E-06	-3.007072E-06
9.9100130E-06	6.300010E-04	2.150021E-06	1.300015E-06	3.203105E-06	-0.150000E-06	-1.000000E-06	1.000000E-06

STRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR EACH MODE

DEFLECTIONS FOR MODE 3

1.000000E-06	0.0	9.710070E-01	0.0	9.010713E-01	0.0	9.120400E-01	0.0
0.030000E-01	0.0	0.5010030E-01	0.0	0.023077E-01	0.0	3.930700E-01	0.0
7.000000E-01	0.0	7.000000E-01	0.0	7.000000E-01	0.0	7.000000E-01	0.0

DEFLECTIONS FOR MODE 4

-1.000000E-01	0.0	-1.113000E-01	0.0	-5.707307E-02	0.0	-1.002530E-01	0.0
5.770702E-02	0.0	1.021000E-01	0.0	1.020117E-01	0.0	2.702000E-01	0.0
3.5012471E-01	0.0	0.013100E-01	0.0	0.057033E-01	0.0	2.103773E-01	0.0

DEFLECTIONS FOR MODE 5

-0.000702E-03	0.0	-7.000000E-03	0.0	-7.000770E-03	0.0	-5.707307E-03	0.0
-2.000000E-03	0.0	0.000000E-03	0.0	1.571570E-02	0.0	3.007070E-02	0.0
5.001015E-02	0.0	0.025135E-02	0.0	0.000307E-01	0.0	3.007070E-02	0.0

DEFLECTIONS FOR MODE 6

-2.000000E-03	0.0	-0.000000E-03	0.0	-5.707307E-03	0.0	-5.707307E-03	0.0
-2.000000E-03	0.0	0.000000E-03	0.0	0.000000E-03	0.0	0.000000E-03	0.0
0.0100270E-02	0.0	7.107013E-02	0.0	0.000000E-01	0.0	-0.130000E-01	0.0

DEFLECTIONS FOR MODE 7

-5.071000E-04	0.0	3.020150E-04	0.0	2.000000E-04	0.0	-3.700000E-04	0.0
-3.000000E-04	0.0	1.100000E-03	0.0	2.071000E-03	0.0	2.670000E-03	0.0
-1.100000E-03	0.0	-7.000000E-03	0.0	-3.700000E-03	0.0	1.000000E-03	0.0

DEFLECTIONS FOR MODE 15

-3.1132647E-01 -5.1734340E-02 -1.6532340E-01 -5.3160250E-02 -1.9621942E-02 -4.8962950E-02 1.2539447E-01 -4.5677950E-02
 2.6937093E-01 -9.3440231E-02 5.1216739E-01 -4.3023187E-02 5.5307407E-01 -4.4700013E-02 0.9401124E-01 -0.0598784E-02
 8.3533606E-01 -5.4322160E-02 9.7571937E-01 -6.1003579E-02 0.8400514E-01 -1.3304455E-01 1.0000000E+00 0.
 0.0939736E-01 -2.3567274E-01 9.2131217E-01 -1.0262824E-01

DEFLECTIONS FOR MODE 16

2.0940042E-01 3.4913904E-02 9.4407305E-02 1.5254147E-02 -0.0043179E-03 -6.0040343E-03 -4.1731797E-02 -2.3344977E-02
 -1.4501023E-01 -3.1372645E-02 -1.642373E-01 -2.9100401E-02 -1.4120704E-01 -1.670923E-02 -0.3725040E-02 4.0365027E-03
 -0.1490067E-04 3.0449221E-02 9.5247005E-02 5.9369025E-02 4.5120504E-01 3.2730714E-02 -1.6151555E-01 5.7325700E-02
 1.0900000E+00 0. 3.8727050E-01 2.4579077E-02

DEFLECTIONS FOR MODE 17

6.3943004E-03 9.0710005E-03 4.9036948E-03 1.2012310E-03 3.0700446E-03 -5.6336665E-03 3.6900314E-05 -1.0174154E-02
 -0.4425267E-03 -1.0602007E-02 -9.4273042E-03 -5.8304267E-03 -1.2630100E-02 4.6701495E-03 -1.1741021E-02 1.9907106E-02
 -0.1748240E-03 3.4462225E-02 2.6430000E-03 5.0315170E-02 4.0913001E-01 3.2570014E-03 -2.7110221E-01 2.082442E-01
 1.0000000E+00 0. 3.1976000E-01 2.9527201E-01

DEFLECTIONS FOR MODE 18

9.3000341E-05 2.1464529E-03 1.0014025E-03 4.5720362E-04 1.1773941E-03 -1.5100040E-03 1.3420050E-03 -3.3753729E-03
 -9.0000000E-04 -4.0267095E-03 -3.4146362E-03 -2.2403360E-03 -5.4491502E-03 2.3390211E-03 -4.5409442E-03 9.3205953E-03
 1.5924330E-05 1.7633070E-02 6.4971505E-03 2.0041254E-02 4.2000092E-01 -5.5050306E-02 -3.8343536E-01 5.9422020E-01
 1.0000000E+00 0. 1.0910711E-01 6.5007451E-01

DEFLECTIONS FOR MODE 19

-1.4151377E-02 -4.2270339E-04 -3.0640071E-04 2.1703162E-04 1.0103600E-02 5.7973705E-04 1.3037397E-02 4.0206719E-04
 4.7905432E-03 -9.0003650E-06 2.0036047E-04 -5.8021704E-04 -1.0265063E-02 -0.4063903E-04 -1.1712004E-02 -5.6473934E-04
 -1.0762145E-02 2.6674274E-04 -1.6992755E-02 1.3402423E-03 3.2470742E-01 4.0051021E-03 2.4640624E-01 -6.3020254E-02
 1.0000000E+00 0. 9.1227002E-01 7.2025437E-02

DEFLECTIONS FOR MODE 20

4.5543903E-01 2.4006300E-02 -6.4447191E-02 -4.0222795E-03 -4.1562572E-01 -2.2302300E-02 -4.5009464E-01 -2.3662014E-02
 -1.0900071E-01 -9.1314103E-03 1.4271415E-01 1.1512050E-02 4.5149714E-01 2.5040422E-02 4.1172001E-01 2.2239323E-02
 9.0157270E-02 2.6322590E-03 -5.7430303E-01 -2.0533334E-02 7.1170501E-02 3.6790702E-02 1.3592004E-01 -7.5040154E-02
 9.3524400E-01 1.1200094E-01 1.0000000E+00 0.

DEFLECTIONS FOR MODE 21

1.1260192E-03 -0.3420910E-04 -7.4027592E-04 1.5922100E-04 -1.9101101E-03 0.3197012E-04 -1.2663330E-03 0.0490111E-04
 4.0519612E-04 3.1417415E-04 1.0749450E-04 -5.1006352E-04 1.0627550E-03 -1.0261520E-03 5.320319E-04 -7.9102634E-04
 -1.0201090E-03 4.0024900E-04 -2.5011600E-03 2.0090120E-03 4.7366300E-02 -1.2962172E-02 0.8737995E-01 1.6742704E-02
 3.5590644E-01 -3.1704076E-02 1.0000000E+00 0.

DEFLECTIONS FOR MODE 22

-0.0470021E-01 1.9312053E-03 -0.4402475E-01 -1.2503407E-03 9.4000223E-01 -2.3306325E-03 1.0505900E-01 -7.9123314E-04
 -0.4055934E-01 1.5751010E-03 -9.4407702E-01 2.4500020E-03 1.0720114E-01 1.3400419E-03 1.0000000E+00 0.
 0.0020040E-01 3.1001970E-05 -7.9242021E-01 1.4004500E-03 -2.0794400E-01 -1.2001092E-02 -1.5400633E-01 1.6678005E-01
 0.0462824E-01 2.1400754E-02 2.3726247E-01 2.0024704E-01

STABILITY ANALYSIS RESULTS

SPEED OF SOUND = 1.1000E+03 FT/SEC

VELOCITY = 2.3700E-03 SLUGS/CU FT

MACH NUMBER = 3.5000E+00

VELOCITY = 3.8500E+03 FT/SEC, 2.2700E+03 KNOTS

MODE	EIGENVALUE-H MU-HPS	EIGENVALUE-I OMEGA-HPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	REDUCED FREQUENCY	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	1.005747E-07	0.	0.	1.000000E+00	0.	0.	6.61188E+06
2	-2.810000E-04	0.	0.	4.472259E-05	0.	0.	2.466713E+03
3	6.923473E+00	0.	0.	1.101937E+00	0.	0.	-1.001126E-01
4	-4.715072E+01	0.	0.	1.307047E+01	0.	0.	7.953430E-01
5	-2.841991E+02	0.	0.	4.602746E+01	0.	0.	2.346707E-03
6	-3.346440E+02	0.	0.	5.326024E+01	0.	0.	2.071297E-03
7	-9.985279E+02	0.	0.	1.589207E+02	0.	0.	6.941691E-04
8	1.204451E+16	0.	0.	2.076894E+15	0.	0.	-5.311072E-17
9	-4.364506E+16	0.	0.	6.978279E+15	0.	0.	1.500874E-17
10	-1.000000E+30	0.	0.	1.591549E+27	0.	0.	6.931472E-39
11	-1.000000E+30	0.	0.	1.591549E+27	0.	0.	6.931472E-39
12	-1.000000E+30	0.	0.	1.591549E+27	0.	0.	6.931472E-39
13	-1.000000E+30	0.	0.	1.591549E+27	0.	0.	6.931472E-39
14	-1.000000E+30	0.	0.	1.591549E+27	0.	0.	6.931472E-39
15	-1.518183E+01	4.477590E+01	7.126320E+00	7.524810E+00	1.745519E-02	3.211061E-01	4.565630E-02
16	-1.372595E+01	2.430803E+02	3.878419E+01	3.884507E+01	9.494340E-02	5.623690E-02	5.049871E-02
17	-2.413766E+02	3.315634E+02	5.277006E+01	6.527241E+01	1.291800E-01	5.805532E-01	2.871642E-03
18	-4.433366E+02	4.294579E+02	6.835035E+01	9.823631E+01	1.673213E-01	7.102000E-01	1.563479E-03
19	-1.904040E+01	6.258509E+02	9.960720E+01	9.965338E+01	2.430300E-01	3.001872E-02	3.639256E-02
20	-8.461159E+00	7.911390E+02	1.262320E+02	1.262304E+02	3.090152E-01	1.117162E-02	7.622274E-02
21	-9.682047E+01	1.029206E+04	1.430032E+02	1.645266E+02	4.008094E-01	9.366712E-02	7.158506E-03
22	-2.621034E+01	1.562336E+03	2.486535E+02	2.486834E+02	8.087025E-01	1.549694E-02	2.862549E-02

MINIMUM BLANK COMMON LENGTH REQUIRED = 1040.
BASED ON INPUT DATA AND ANALYSES REQUESTED.

SYMBOLS

In this modification to the original program PASES, the general notation has been retained. The nomenclature used in the basic development (Ref. 1) has been reproduced in Appendix A. The new symbols introduced in the present modification are defined here. The reader is referred to Appendix A for the definitions of symbols not appearing below:

A	Coefficient in spline fit, Eq. (57); also see Appendix A
a_0, a_1	Coefficients in spline fit, Eqs. (67) and (68)
B	Coefficient in spline fit, Eq. (57); also see Appendix A
b_r	Reference semichord
C	Element of discrete damping matrix; coefficient in spline fit, Eq. (57)
C	Element of generalized damping matrix
C_h	Element of oscillatory AIC matrix
C_{hDh}	Element of damping AIC matrix
C_{hI}	Imaginary part of C_h
C_{hR}	Real part of C_h
C_{hs}	Element of static AIC matrix
$\bar{c}$	Reference chord
D	Coefficient in spline fit, Eqs. (57) and (60)
E	Modulus of elasticity
f	Cyclic frequency
g	Aeroelastic damping coefficient
I	Element of unit matrix; spline bending moment of inertia
I_{kj}	Element of spline interpolation matrix, Eqs. (74) and (75)
K_{ji}, K_{kj}	Spline parameters, Eqs. (70) and (73)
K	Element of generalized stiffness matrix

N	Element of generalized mass matrix
P	Spline reaction force
Q	Spline loading parameter, $Q=P/12EI$
q	Dynamic pressure
S	Reference area
s	Reference span; Laplace transform variable
$T_{1/2}$	Time to half amplitude (negative value denotes time to double amplitude)
U	Element of matrix in canonical form for eigenvector analysis, Eqs. (45) and (47)
w	Spline loading, Eq. (56a)
x	Coordinate along spline; also see Appendix A
y	Spline deflection; also see Appendix A
Λ	Eigenvalue ($=0$) in eigenvector analysis, Eqs. (45) and (46)
μ	Decay rate coefficient
ρ	Atmospheric density
$(\bar{})$	Denotes complex amplitude; denotes inclusion of aerodynamic damping and stiffness in $\bar{C}$ and $\bar{K}$, respectively.

Subscripts:

r	Oscillatory mode number in frequency lining-up process, Eq. (53)
s	Number of mode being lined-up, Eqs. (53) and (55)

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