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AN INTERMEDIATE-AVERAGED THEORY FOR HIGH ALTITUDE ORBITS.(U)
JUN 79 R SRIDHARAN, W P SENIW

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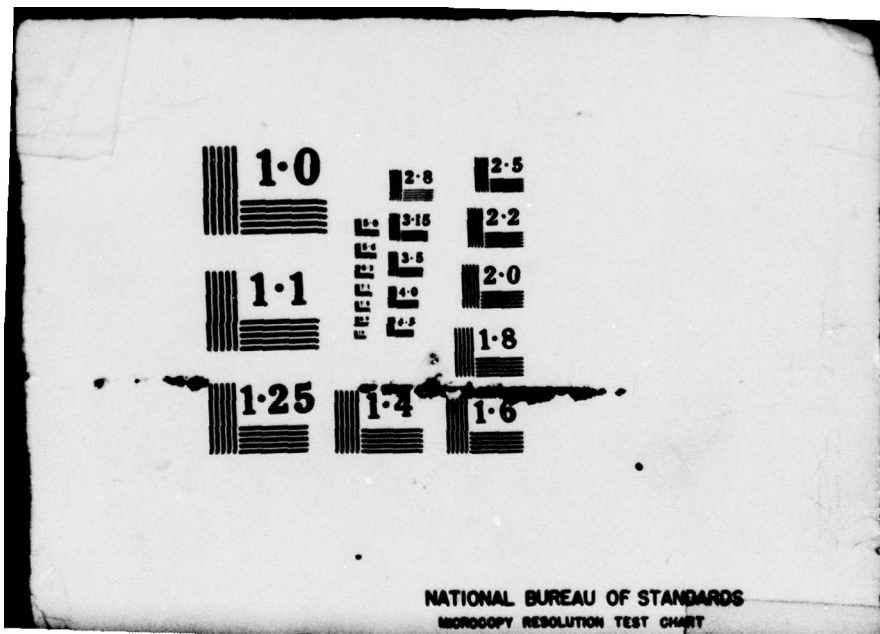
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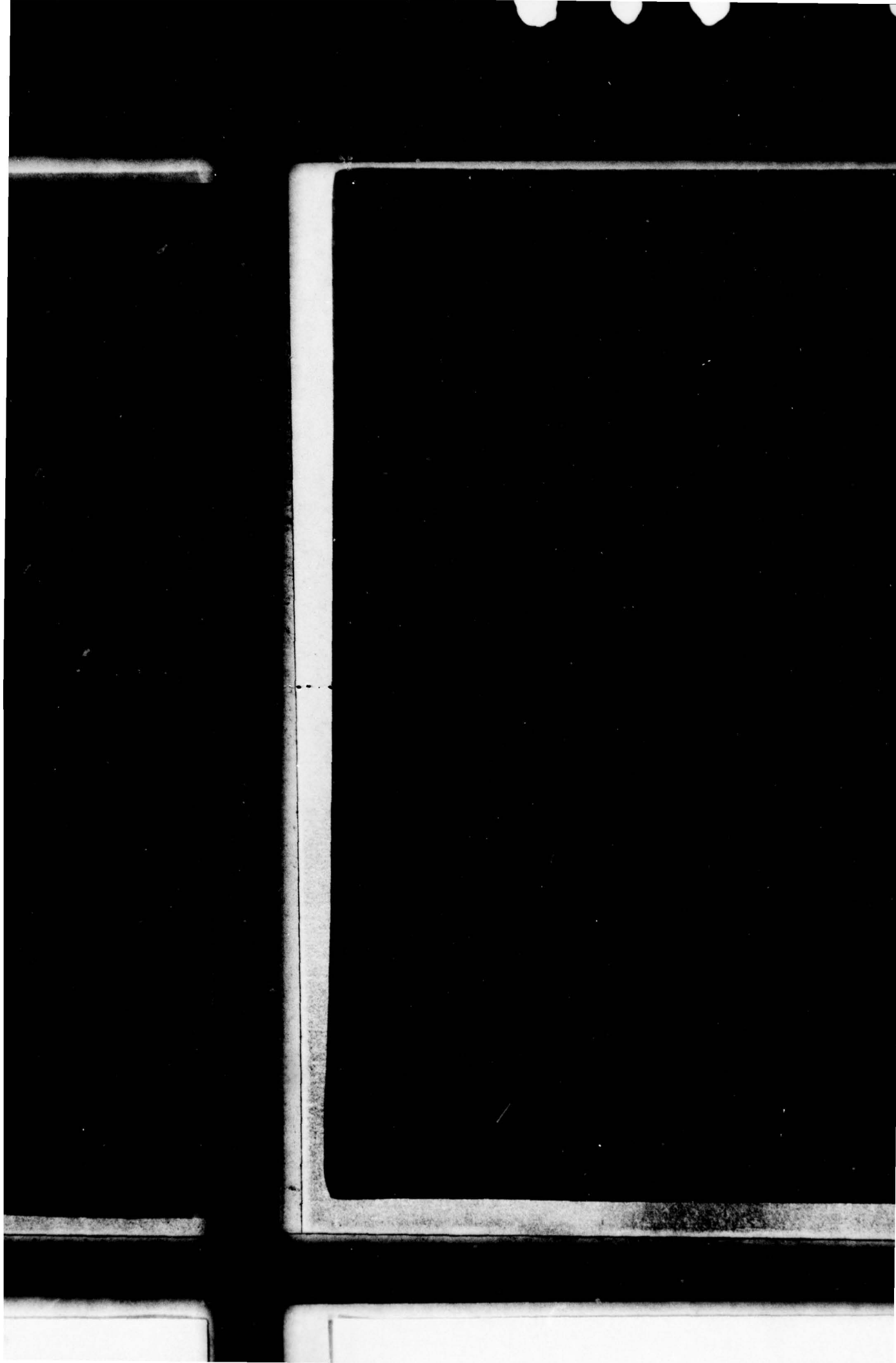
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

AN INTERMEDIATE-AVERAGED THEORY
FOR HIGH ALTITUDE ORBITS

R. SRIDHARAN

W. P. SENIW

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Abstract

An Analytic theory for the evolution of high altitude satellite orbits is developed in this note. The distinctive feature of the theory lies in the double averaging of the differential equations--once over the period of the orbit, and secondly over the period of the moon. This technique is called "intermediate averaging" to distinguish it from the conventional doubly averaged theories, and to denote the time scales inherent in the averaging technique.

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I. INTRODUCTION

An analytic theory for the evolution of high altitude satellite orbits is developed in this note. The distinctive feature of the theory lies in the double averaging of the differential equations--once over the period of the orbit, and secondly over the period of the moon. This technique is called "intermediate averaging" to distinguish it from the conventional doubly averaged theories, and to denote the time scales inherent in the averaging technique.

II. BACKGROUND

Analytical theories for the evolution of orbits of artificial satellites of the earth have been developed from the early days of satellites. The aim has been to provide computationally efficient formulations that could be used both for orbital analysis and orbit estimation. (A comprehensive bibliography is provided by Dallas: Ref. 1). The alternate technique is the accurate, relatively compact but computationally expensive method of direct numerical integration of the differential equations for orbital evolution.

By far the most common technique that has been applied in the development of analytical theories is that of averaging over the time scales inherent in the differential equations. An adequate exposition of the method of averages is provided by Lorell et al. (Ref. 2). The characteristic features of orbital theories developed using the technique are:

1. They are expressed as analytical formulae for the rates of change of mean elements.

2. The theories are generally either to first or to second order in the expansion of the geopotential (higher order theories do exist and are employed occasionally).
3. The first averaging is over the mean anomaly of the satellite which is the fastest periodic variable in the system, with the necessary short-periodic terms also being formulated.
4. The second averaging is over the period of the argument of perigee which is typically the second fastest variable, at least for low altitude orbits; and the necessary long-periodic terms are also derived.

A complete theory would then consist of the doubly averaged secular rates of change of the mean elements, along with the long-periodic and the short-periodic terms.

III. THE SYSTEM OF EQUATIONS

The well-known Gauss form of the variational equations in the Keplerian elements of the orbit are used in this paper to develop an alternate theory for high altitude satellite orbits. The equations are as follows:

$$\frac{da}{dt} = 2 \frac{a}{\epsilon} \left(\frac{p}{\mu} \right)^{1/2} [e F_1 \sin v + F_2 \frac{p}{r}]$$

$$\frac{de}{dt} = - \frac{r}{ae} \left(\frac{p}{\mu} \right)^{1/2} F_2 + \frac{\epsilon}{2ae} \frac{da}{dt}$$

$$\frac{d\Omega}{dt} = \frac{r}{(\sin I) \sqrt{\mu p}} F_3 \sin (\omega + v)$$

$$\frac{dI}{dt} = \frac{r}{\sqrt{\mu p}} F_3 \cos (\omega + v)$$

$$\frac{d\omega}{dt} = \frac{1}{e} \left(\frac{p}{\mu}\right)^{\frac{1}{2}} [-F_1 \cos v + F_2 \sin v(1 + \frac{r}{p})] - (\cos I) \frac{d\Omega}{dt}$$

and

$$\frac{dM}{dt} - n = - \frac{2r}{\sqrt{\mu a}} F_1 - \epsilon^{\frac{1}{2}} [(\cos I) \frac{d\Omega}{dt} + \frac{d\omega}{dt}]$$

where

a = the semi-major axis

e = the eccentricity

I = the inclination of the plane of the orbit to the equator

Ω = the right ascension of the ascending node of the orbit

ω = the argument of perigee of the orbit

M = the mean anomaly of the orbit

$\epsilon = 1 - e^2$

$p = ae$

μ = the gravitational constant of the earth

r = the geocentric radial distance to the satellite

v = the true anomaly of the satellite

n = the mean motion

F_1, F_2, F_3 = the components of the perturbing force along the instantaneous radial, transverse and normal directions to the satellite orbit.

Certain features of the above equations should be noticed. Firstly, the Gauss form has been altered slightly in the equations for e , ω and M in order to take advantage of commonality of terms. Secondly, the differential equations are linear in the perturbing forces. Thirdly, singularities at $e = 0$ or $I = 0$ exist; these will be commented on later.

IV. THE PERTURBING FORCES

The perturbations considered here result from the zonal harmonic terms in the geopotential and the point mass gravitational effects of the sun and the moon. The perturbing force due to the zonal harmonics can be written as

$$\vec{F}_n = \frac{\mu J_n}{r^2} \left(\frac{R_e}{r} \right)^n [(n+1) P_n \vec{u}_r - P'_n \sin I \cos u \vec{u}_t - P'_n \cos I \vec{u}_n]$$

where n = the degree of the zonal harmonic

J_n = the zonal harmonic coefficient

R_e = the equatorial radius of the earth

P_n = the n -th order Legendre polynomial in the argument $(\sin \delta)$

δ = the geocentric latitude of the satellite

$$P'_n = \frac{d}{d(\sin \delta)} P_n$$

$\vec{u}_r, \vec{u}_t, \vec{u}_n$ = the instantaneous unit vectors along the radial, transverse, and normal directions in orbit.

The perturbing force due to a third body (the moon or the sun) can be written as

$$\vec{F}_q = \frac{\mu_k}{r_k^2} \left(\frac{r}{r_k} \right)^q [-P'_q \vec{u}_r + P'_{q+1} \vec{u}_k]$$

where q = the order of the term in the Legendre polynomial expansion of the perturbing force

μ_k = the gravitational constant of the k -th body

r_k = the geocentric distance to the k -th body

P_q = the q -th order Legendre polynomial in the argument $\zeta = (\vec{u}_r \cdot \vec{u}_k)$

$\vec{u}_k$ = the instantaneous geocentric unit vector to the third body.

Geopotential zonal harmonic terms up to order 4 only will be included in the theory. Only the first term in the expansion of the third body perturbation will be considered. Neglected terms are at least an order of magnitude smaller, for satellites in half-synchronous orbits, and at least a factor of 4 smaller for synchronous satellites. Atmospheric drag is neglected in this theory.

V. THE SINGLY AVERAGED EQUATIONS

The first step in developing an analytical theory is to average the differential equations over the fastest variable on the right hand side of the equations, which is the mean anomaly of the satellite. A considerable body of results already exists for this step and we will adopt the results of Liu (Ref. 3) for the singly averaged equations due to the geopotential. Liu tabulates both the averaged rates and the short periodic terms, the latter being computed only to order J_2 .

The third-body perturbations must also be averaged over the period of the satellite. The relevant results for the averaged effects are taken from earlier work (Ref. 4) of an author of this note. The short-periodic terms due to the third body were, however, not derived in Ref. 4, and hence were developed for this note. They are tabulated in Appendix 1 along with the singly averaged rates of the elements due to the third body.

VI. INTERMEDIATE AVERAGING

The singly averaged equations due to the geopotential show a periodic dependence on the argument of perigee. However, the rate of change of the

argument of perigee in high altitude orbits is less than 1 deg/day. Thus, the period is of the order of a year or larger.

The singly averaged equations due to the third body exhibit dependence on the position of the third body, and hence on its period. And, for the moon, the period is 29 days. Hence, the fastest variable left in the singly averaged equations is the mean anomaly of the moon. Mathematically,

$$\left\langle \frac{dz}{dt} \right\rangle = f_G(z, J_n) + f_T(z, M_T)$$

where $\langle \rangle$ = singly averaged rates

f_G = terms due to the geopotential

f_T = terms due to the third body

M_T = the mean anomaly of the third body

z = any element of the satellite orbit in the set $\underline{z}$.

The next step in averaging is

$$\langle \left\langle \frac{dz}{dt} \right\rangle \rangle = f_G(z, J_n) + \frac{1}{\tau_M} \int_0^{\tau_M} f_T(z, M_T) dt$$

where τ_M is the moon's period. Neither the geopotential terms nor the third body terms due to the sun are affected by this step of averaging. But the mean anomaly of the moon is averaged out of the equations. The rates averaged over the intermediate period for the moon are tabulated in Appendix 2.

The theory would not be complete without the intermediate periodic terms, whose basic period is that of the moon. These terms have been derived and are also given in Appendix 2.

VII. PROBLEM OF SINGULARITIES

The original differential equations are singular at the values $e = 0$ and $I = 0$. These singularities do carry over into the theory developed here.

As the eccentricity goes to zero, the perigee becomes ill-defined. Hence, the short-periodic corrections in the mean anomaly and in the argument of perigee become large. However, these two corrections are of nearly equal magnitude and opposite in sign. Thus, the position of the satellite in the orbit as defined by $\omega + M$ does not exhibit any singularity. Unless the eccentricity becomes so small that the precision of computation affects results (an unlikely case), the theory remains valid.

As the inclination goes to zero, the line of nodes, and hence the value of Ω , become ill-defined. The theory does exhibit this singularity in the secular terms. And again, the change in the value of Ω is nearly equal and opposite in sign to that of ω , the argument of perigee. Thus the position of perigee can be defined using the value of $(\Omega + \omega)$. Another solution is also possible: the reference plane can be changed from the equator to, say, the ecliptic, or preferably, a plane containing the earth's polar axis. Very little needs to be changed in the theory to accommodate a plane change.

Finally, the classic problem of singularity at critical inclination does not exist in this theory as no averaging is carried out over the period of the argument of perigee.

VIII. CONCLUSION

A theory for high altitude orbits has been developed using the concept of intermediate averaging. The final theory consists of the following additive, separable parts:

- 1) The averaged rates of change of the orbital elements due to the geopotential (intermediate averaging leaves the singly averaged expressions unchanged);

- 2) The short periodics due to the geopotential;
- 3) The intermediate averaged rates due to the moon;
- 4) The intermediate periodic terms due to the moon;
- 5) The averaged rates due to the sun (intermediate averaging leaves the singly averaged expressions unchanged);
- 6) The short periodic terms due to the sun and the moon.

The theory is being implemented in an analytic orbit determination program called ANODE (Ref. 5) for the Millstone Hill Radar.

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3. J. J. F. Liu, "A Second-Order Theory of an Artificial Satellite Under the Influence of the Oblateness of the Earth," presented at the AIAA Twelfth Aero. Sci. Meeting, 1974 (Paper No. A74-18799).
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APPENDIX 1

SINGLY AVERAGED RATES AND SHORT PERIODICS DUE TO THIRD BODY

The singly averaged rates of change of the orbital elements of a satellite due to the point mass perturbations of a third body are recorded in this part of the appendix. The theory is restricted to the first term in the Legendre polynomial expansion of the perturbing force due to the third body.

$$\left\langle \frac{da}{dt} \right\rangle = 0$$

$$\left\langle \frac{dI}{dt} \right\rangle = \frac{3}{2} \frac{A}{\epsilon} \frac{1}{2} [(5 - 4\epsilon)\xi_1\xi_3 \cos \omega - \epsilon\xi_2\xi_3 \sin \omega]$$

$$\left\langle \frac{d\Omega}{dt} \right\rangle = \frac{3}{2} \frac{A}{\epsilon} \frac{1}{\sin I} [(5 - 4\epsilon)\xi_1\xi_3 \sin \omega + \epsilon\xi_2\xi_3 \cos \omega]$$

$$\left\langle \frac{de}{dt} \right\rangle = -\frac{15}{2} A\epsilon \frac{1}{2} \xi_1\xi_2$$

$$\left\langle \frac{d\omega}{dt} \right\rangle = \frac{3}{2} A\epsilon \frac{1}{2} (4\xi_1^2 - \xi_2^2 - 1) - \cos I \left\langle \frac{d\Omega}{dt} \right\rangle$$

$$\begin{aligned} \left\langle \frac{dM}{dt} - n \right\rangle &= \frac{A}{4} [8 + 12e + 15e^2 - 3(1 + 12e + 22e^2)\xi_1^2 - 21\epsilon\xi_2^2] \\ &\quad - \epsilon \frac{1}{2} (\cos I \left\langle \frac{d\Omega}{dt} \right\rangle + \left\langle \frac{d\omega}{dt} \right\rangle) \end{aligned}$$

where

$$A = n \left(\frac{\mu_k}{\mu} \right) \left(\frac{a}{r_k} \right)^3$$

n = the mean motion of satellite

μ_k, μ = gravitational constants of k -th body and the earth

a = semi-major axis of satellite orbit

r_k = geocentric distance to third body

I = inclination of the satellite orbit to equator

e = eccentricity of satellite orbit

$$\epsilon = 1 - e^2$$

ω = argument of perigee of satellite orbit

ξ_1, ξ_2, ξ_3 = projections of the geocentric unit vector to the third body
on the satellite orbital axes $\vec{u}_n$, unit normal, $\vec{u}_p$, to
perigee, and $\vec{u}_Q$, normal to $\vec{u}_p$ in the orbit plane
M = mean anomaly of satellite.

The short-periodic perturbations due to a third body are tabulated below.

$$\begin{aligned}\Delta I_p = & 3 \frac{A}{n} \frac{1}{\epsilon^{\frac{1}{2}}} [\xi_1 \xi_3 \cos \omega (-\frac{11}{4} e \sin E + \frac{\sin 2E}{4} + \frac{e^2}{2} \sin 2E - e^3 \sin E \\ & - \frac{e}{12} \sin 3E) + \epsilon^{\frac{1}{2}} \xi_1 \xi_3 \sin \omega (-\frac{5e}{4} \cos E + \frac{1}{4} \cos 2E + \frac{e^2}{4} \cos 2E \\ & - \frac{e}{12} \cos 3E) + \epsilon^{\frac{1}{2}} \xi_2 \xi_3 \cos \omega (\frac{5e}{4} \cos E - \frac{1}{4} \cos 2E - \frac{e^2}{4} \cos 2E \\ & + \frac{e}{12} \cos 3E) - \epsilon \xi_2 \xi_3 \sin \omega (-\frac{e}{4} \sin E - \frac{1}{4} \sin 2E + \frac{e}{12} \sin 3E)] \\ \Delta \Omega_p = & 3 \frac{A}{n} \frac{1}{\epsilon^{\frac{1}{2}} \sin I} [\epsilon^{\frac{1}{2}} \xi_1 \xi_3 \cos \omega (\frac{5e}{4} \cos E - \frac{1}{4} \cos 2E - \frac{e^2}{4} \cos 2E \\ & + \frac{e}{12} \cos 3E) + \xi_1 \xi_3 \sin \omega (-\frac{11}{4} e \sin E - e^3 \sin E + \frac{1}{4} \sin 2E \\ & + \frac{e^2}{2} \sin 2E - \frac{e}{12} \sin 3E) + \epsilon^{\frac{1}{2}} \xi_2 \xi_3 \sin \omega (\frac{5e}{4} \cos E - \frac{1}{4} \cos 2E \\ & - \frac{e^2}{4} \cos 2E + \frac{e}{12} \cos 3E) + \epsilon \xi_2 \xi_3 \cos \omega (-\frac{e}{4} \sin E - \frac{1}{4} \sin 2E \\ & + \frac{e}{12} \sin 3E)]\end{aligned}$$

$$\begin{aligned}\Delta e_p = & \frac{\epsilon}{2ae} (\Delta a_p) - 3\epsilon^{\frac{1}{2}} \frac{A}{n} \frac{1}{e} [\epsilon^{\frac{1}{2}} (\xi_2^2 - \xi_1^2) (\frac{5}{4} e \cos E - \frac{1}{4} \cos 2E \\ & - \frac{e^2}{4} \cos 2E + \frac{e}{12} \cos 3E) + \xi_1 \xi_2 (\frac{1}{2} \sin 2E - 2e \sin E - \frac{e^2}{4} \sin 2E) \\ & - e \xi_1 \xi_2 (\frac{1}{2} \sin E + \frac{5e^2}{4} \sin E + \frac{1}{6} \sin 3E - \frac{e^2}{12} \sin 3E - \frac{e}{2} \sin 2E)]\end{aligned}$$

$$\begin{aligned}\Delta\omega_p = & -\cos I(\Delta\Omega_p) + \frac{A}{n} \frac{\epsilon^{\frac{1}{2}}}{e} [\delta \sin E - \frac{e}{4} \sin 2E] (1 - 3\xi_1^2) \\ & - 3\epsilon^{\frac{1}{2}} \xi_1 \xi_2 (-\cos E + \frac{e}{4} \cos 2E) + 3\frac{A}{n} \frac{1}{e} [\xi_1 \xi_2 (\frac{1}{4} \sin 2E - 2e \sin E) \\ & + \frac{\epsilon^{\frac{1}{2}}}{2} (\xi_2^2 - \xi_1^2) (\frac{1}{2} \sin E + \frac{e}{2} \sin 2E - \frac{1}{6} \sin 3E) + \frac{\xi_1 \xi_2}{4} \epsilon \sin 2E]\end{aligned}$$

$$\begin{aligned}\Delta M_p = & -\epsilon^{\frac{1}{2}} [\cos I(\Delta\Omega_p) + (\Delta\omega_p)] - 2\frac{A}{n} [3e(\sin E + \frac{e^2}{4} \sin E - \frac{e}{4} \sin 2E \\ & + \frac{e^2}{36} \sin 3E) + 3\xi_1^2 (-\frac{11}{4} e \sin E - e^3 \sin E + \frac{1}{4} \sin 2E \\ & + \frac{e^2}{2} \sin 2E - \frac{e}{12} \sin 3E) + \frac{3}{2} \xi_2^2 \epsilon (-\frac{e}{2} \sin E - \frac{1}{2} \sin 2E \\ & + \frac{e}{6} \sin 3E) + 6\xi_1 \xi_2 \epsilon^{\frac{1}{2}} (\frac{5e}{4} \cos E - \frac{1}{4} \cos 2E - \frac{e^2}{4} \cos 2E \\ & + \frac{e}{12} \cos 3E)]\end{aligned}$$

and

$$\begin{aligned}\Delta a_p = & \frac{2a}{\epsilon^{\frac{1}{2}}} \frac{A}{n} [\epsilon \epsilon^{\frac{1}{2}} (3\xi_2^2 - 1) (-\cos E + \frac{e}{4} \cos 2E) + 3e\xi_1 \xi_2 (\delta \sin E \\ & - \frac{e}{4} \sin 2E) + 3\epsilon^{\frac{1}{2}} (\xi_2^2 - \xi_1^2) (e \cos E - \frac{\cos 2E}{4}) + 3\xi_1 \xi_2 (\frac{1}{4} \sin 2E \\ & - 2e \sin E) + \frac{3}{4} \epsilon \xi_1 \xi_2 \sin 2E]\end{aligned}$$

where E is the eccentric anomaly of the satellite, and $\delta = 1 + e^2$.

APPENDIX 2

INTERMEDIATE AVERAGING OF THE PERTURBATIONS DUE TO THE MOON

This part of the appendix records the intermediate averaged perturbations of a satellite orbit due to the point mass effect of the moon.

$$\begin{aligned} \langle \langle \frac{dI}{dt} \rangle \rangle &= -\frac{3}{4} \frac{n}{\epsilon} \frac{B}{\sin I} \sin J_k \cos J_k [(5 - 4\epsilon) \cos \omega \sin AP_k \\ &\quad - \epsilon \sin \omega \cos AP_k] \end{aligned}$$

$$\begin{aligned} \langle \langle \frac{d\Omega}{dt} \rangle \rangle &= -\frac{3}{4} \frac{n}{\epsilon} \frac{B}{\sin I} \sin J_k \cos J_k [(5 - 4\epsilon) \sin \omega \sin AP_k \\ &\quad + \epsilon \cos \omega \cos AP_k] \end{aligned}$$

$$\langle \langle \frac{de}{dt} \rangle \rangle = \frac{15}{8} n B \epsilon \sin^2 J_k \sin 2AP_k$$

$$\langle \langle \frac{d\omega}{dt} \rangle \rangle = \frac{3}{4} n B \epsilon [\sin^2 J_k \cos 2AP_k - 2] - \cos I \langle \langle \frac{d\Omega}{dt} \rangle \rangle$$

$$\begin{aligned} \langle \langle \frac{dM}{dt} - n \rangle \rangle &= -\epsilon \frac{B}{\sin I} [\cos I \langle \langle \frac{d\Omega}{dt} \rangle \rangle + \langle \langle \frac{d\omega}{dt} \rangle \rangle] + \frac{n}{8} B [2(8 + 12e + 15e^2) \\ &\quad - 21\epsilon (\sin^2 AP_k + \cos^2 J_k \cos^2 AP_k) \\ &\quad - 3(1 + 12e + 22e^2) (\cos^2 AP_k + \cos^2 J_k \sin^2 AP_k)] \end{aligned}$$

and

$$\langle \langle \frac{da}{dt} \rangle \rangle = 0$$

Intermediate averaging also yields periodic terms with the moon's period. These are recorded below.

$$\Delta a_{I,P} = 0$$

$$\Delta I_{I,P} = \frac{3}{2} \frac{n}{n_k} \frac{B}{\epsilon} \{ (5 - 4\epsilon) \cos \omega \times (1) - \epsilon \sin \omega \times (2) \}$$

$$\begin{aligned} \text{where } (1) = & \left\{ \frac{1}{4} \cos AP_k \cos (2\omega_k + 2\nu_k) + \frac{1}{4} \sin J_k \cos J_k \sin AP_k \sin (2\omega_k + 2\nu_k) \right. \\ & + \frac{e_k}{12} \cos AP_k \cos (2\omega_k + 3\nu_k) + \frac{e_k}{4} \cos AP_k \cos (2\omega_k + \nu_k) \\ & - \frac{e_k}{2} \sin AP_k \sin J_k \cos J_k \sin \nu_k + \frac{e_k}{12} \sin AP_k \sin J_k \cos J_k \\ & \left. \sin (2\omega_k + 3\nu_k) + \frac{e_k}{4} \sin AP_k \sin J_k \cos J_k \sin (2\omega_k + \nu_k) \right\} \end{aligned}$$

$$\begin{aligned} (2) = & \left\{ -\frac{1}{4} \sin AP_k \cos (2\omega_k + 2\nu_k) + \frac{1}{4} \cos AP_k \sin J_k \cos J_k \sin (2\omega_k + 2\nu_k) \right. \\ & - \frac{e_k}{12} \sin AP_k \cos (2\omega_k + 3\nu_k) - \frac{e_k}{4} \sin AP_k \cos (2\omega_k + \nu_k) \\ & - \frac{e_k}{2} \cos AP_k \sin J_k \cos J_k \sin \nu_k + \frac{e_k}{12} \cos AP_k \sin J_k \cos J_k \\ & \left. \sin (2\omega_k + 3\nu_k) + \frac{e_k}{4} \cos AP_k \sin J_k \cos J_k \sin (2\omega_k + \nu_k) \right\} \end{aligned}$$

$$\sin I \Delta \Omega_{I,P} = \frac{3}{2} \frac{n}{n_k} \frac{B}{\epsilon} \{ (5 - 4\epsilon) \sin \omega \times (1) + \epsilon \cos \omega \times (2) \}$$

where (1) and (2) are as above in $\Delta I_{I,P}$.

$$\begin{aligned} \Delta e_{I,P} = & -\frac{15}{2} \frac{n}{n_k} B \epsilon^{\frac{1}{2}} \left\{ \frac{1}{4} \sin 2AP_k \left\{ -e_k \sin \nu_k - \frac{e_k}{2} \sin (2\omega_k + \nu_k) \right. \right. \\ & \left. \left. - \frac{1}{2} \sin 2(\omega_k + \nu_k) - \frac{e_k}{6} \sin (2\omega_k + 3\nu_k) \right\} + \frac{1}{4} \sin 2AP_k \cos^2 J_k \right. \\ & \left. \left\{ e_k \sin \nu_k - \frac{e_k}{2} \sin (2\omega_k + \nu_k) - \frac{1}{2} \sin (2\omega_k + 2\nu_k) \right. \right. \\ & \left. \left. - \frac{e_k}{6} \sin (2\omega_k + 3\nu_k) \right\} + \frac{1}{4} \cos 2AP_k \cos J_k \left\{ -\frac{e_k}{2} \cos (2\omega_k + \nu_k) \right. \right. \end{aligned}$$

$$- \frac{1}{2} \cos (2\omega_k + 2\nu_k) - \frac{e_k}{6} \cos (2\omega_k + 3\nu_k) \}]$$

$$\Delta\omega_{I,P} = \frac{3}{2} \frac{n}{n_k} B \epsilon^4 [4 \times (3) - (4) - (5)] - \cos I \Delta\Omega_{I,P}$$

where

$$(3) = \{ \frac{1}{4} \cos^2 AP_k \sin (2\omega_k + 2\nu_k) - \frac{1}{4} \sin^2 AP_k \cos^2 J_k$$

$$\sin (2\omega_k + 2\nu_k) - \frac{1}{8} \sin 2AP_k \cos J_k \cos (2\omega_k + 2\nu_k)$$

$$+ \frac{e_k}{2} \cos^2 AP_k \sin \nu_k + \frac{e_k}{12} \cos^2 AP_k \sin (2\omega_k + 3\nu_k)$$

$$+ \frac{e_k}{4} \cos^2 AP_k \sin (2\omega_k + \nu_k) + \frac{e_k}{2} \sin^2 AP_k \cos^2 J_k \sin \nu_k$$

$$- \frac{e_k}{12} \sin^2 AP_k \cos^2 J_k \sin (2\omega_k + 3\nu_k)$$

$$- \frac{e_k}{4} \sin^2 AP_k \cos^2 J_k \sin (2\omega_k + \nu_k) - \frac{e_k}{24} \sin 2AP_k$$

$$\cos J_k \cos (2\omega_k + 3\nu_k) - \frac{e_k}{8} \sin 2AP_k \cos J_k \cos (2\omega_k + \nu_k) \}$$

$$(4) = \{ \frac{1}{4} \sin^2 AP_k \sin (2\omega_k + 2\nu_k) - \frac{1}{4} \cos^2 AP_k \cos^2 J_k$$

$$\sin (2\omega_k + 2\nu_k) + \frac{1}{8} \sin 2AP_k \cos J_k \cos (2\omega_k + 2\nu_k)$$

$$+ \frac{e_k}{2} \sin^2 AP_k \sin \nu_k + \frac{e_k}{12} \sin^2 AP_k \sin (2\omega_k + 3\nu_k)$$

$$+ \frac{e_k}{4} \sin^2 AP_k \sin (2\omega_k + \nu_k) + \frac{e_k}{2} \cos^2 AP_k \cos^2 J_k \sin \nu_k$$

$$- \frac{e_k}{12} \cos^2 AP_k \cos^2 J_k \sin (2\omega_k + 3\nu_k) - \frac{e_k}{4} \cos^2 AP_k \cos^2 J_k$$

$$\sin (2\omega_k + v_k) + \frac{e_k}{24} \sin 2AP_k \cos J_k \cos (2\omega_k + 3v_k)$$

$$+ \frac{e_k}{8} \sin 2AP_k \cos J_k \cos (2\omega_k + v_k)$$

$$\textcircled{5} = e_k \sin v_k$$

$$\Delta M_{I,P} = \frac{1}{4} \left(\frac{n}{n_k} \right) B [-3(1 + 12e + 22e^2) \times \textcircled{3} - 21e \times \textcircled{4} + (8 + 12e + 15e^2)$$

$$\times \textcircled{5}] - \epsilon^2 [\cos I(\Delta \Omega_{I,P}) + (\Delta \omega_{I,P})]$$

and

$$\Delta a_{I,P} = 0.$$

The only symbols used here but not defined in Appendix 1 are:

n_k = mean motion of moon around earth

$$B = (\mu_k/\mu) (a/p_k)^3 \epsilon_k^{3/2}$$

p_k = parameter of the orbit of the moon

J_k = inclination of the satellite orbit with respect to the moon's orbit

AP_k = argument of perigee of the satellite orbit with respect to the moon's orbit

ω_k = argument of perigee of the moon measured with respect to the satellite orbit

v_k = true anomaly of the moon in its orbit

e_k = eccentricity of the orbit of the moon

$$\epsilon_k = 1 - e_k^2.$$

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