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PRIME AND PDQ SORTS - EFFICIENT MINIMAL STORAGE SORTING ALGORIT--ETC(U)
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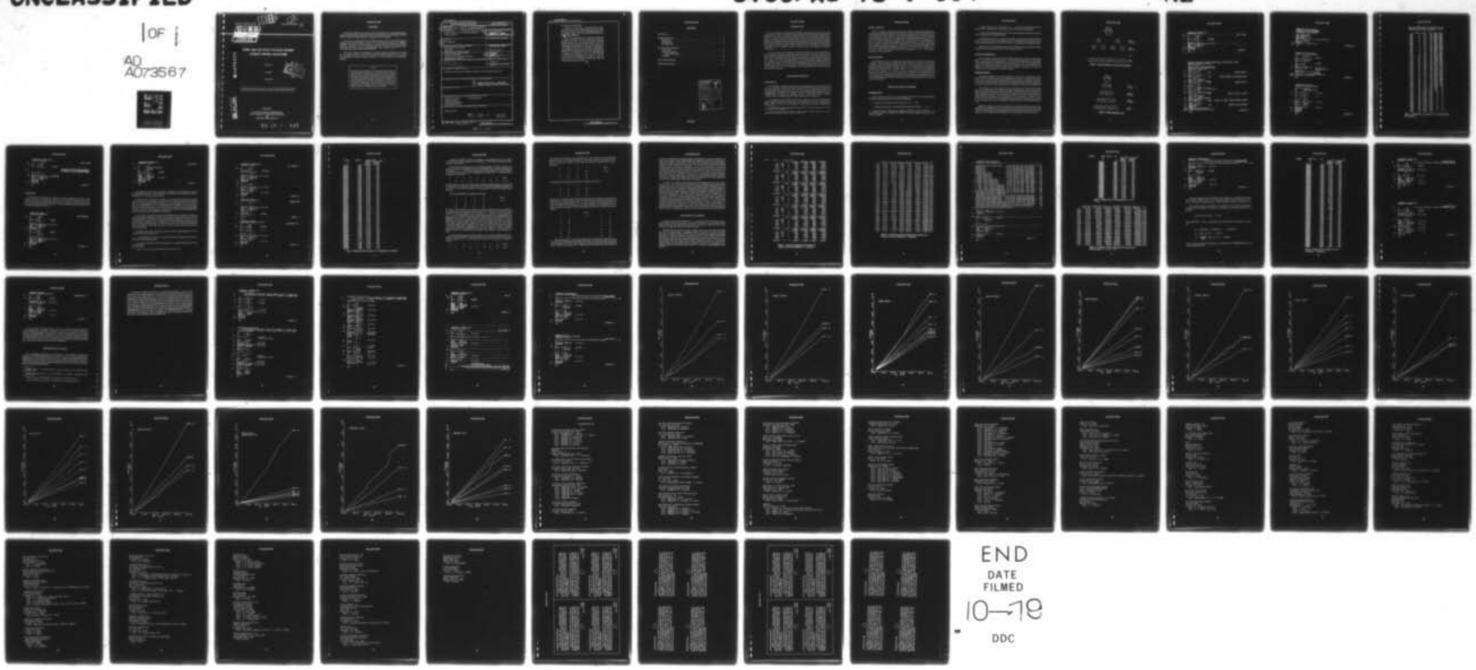

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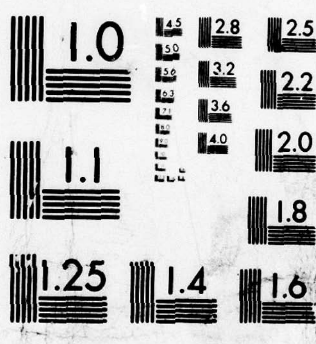

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PRIME AND PDQ SORTS EFFICIENT MINIMAL STORAGE SORTING ALGORITHMS

Final Report

R. Hilbrand

August 1979

Approved for public release; distribution unlimited; statement applied August 1979.

Prepared for

THE JOINT LOGISTICS COMMANDERS
JOINT TECHNICAL COORDINATING GROUP
ON
AIRCRAFT SURVIVABILITY

FOREWORD

This report summarizes the results of research performed by the Aeronautical Systems Division, Wright-Patterson AFB, OH. The work was performed between November 1976 and March 1978, and the Project Engineer for this effort was G. B. Bennett.

The work was sponsored by the JTCG/AS as part of the 3-year TEAS (Test and Evaluation Aircraft Survivability) program. The TEAS program was funded by DDR&E/ODDT&E. The effort was conducted under the direction of the JTCG/AS Vulnerability Assessment Subgroup, as part of JTCG/AS Work Unit VA-6-02F, *Development of Aircraft Preliminary Design Assessment Methodology.*

This report presents and summarizes two sorting algorithms, PRIME Sort and PDQ Sort, developed in the Aeronautical Systems Division Computer Science Center in support of vulnerability assessment computer programs in use by the Deputy for Development Planning.

NOTE

This technical report was prepared by the Vulnerability Assessment Subgroup of the Joint Technical Coordinating Group on Aircraft Survivability in the Joint Logistics Commanders' organization. Because the Services' aircraft survivability development programs are dynamic and changing, this report represents the best data available to the subgroup at this time. It has been coordinated and approved at the JTCG subgroup level. The purpose of the report is to exchange data on all aircraft survivability programs, thereby promoting interservice awareness of the DoD aircraft survivability program under the cognizance of the Joint Logistics Commanders. By careful analysis of the data in this report, personnel with expertise in the aircraft survivability area should be better able to determine technical voids and areas of potential duplication or proliferation.

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Aeronautical Systems Division

PRIME and PDQ Sorts – Efficient Minimal Storage Sorting Algorithms, by R. R. Hilbrand, Wright-Patterson AFB, OH, ASD, for Joint Technical Coordinating Group/Aircraft Survivability, August 1979. 54 pp. (JTCG/AS-78-V-004, publication UNCLASSIFIED.)

One of the problems involved in computer programs for vulnerability assessment is that of rapidly sorting and arranging large sets of data. Two sorting algorithms, designated PRIME and PDQ, have been developed at ASD to more efficiently perform this function in vulnerability programs such as SESTEM and FASTGEN II. The results are compared to those obtained with three other algorithms, SHELLSORT, TREESORT3, and SINGLETON. The newly developed sorts are shown to be significantly faster on the ASD CDC 6600 computer than the existing sorts. When used in an ASD missile endgame model SESTEM, the average run time was reduced by 20 to 25%. Program listings, flow charts, and typical output data are presented.

CONTENTS

INTRODUCTION

One of the steps in most automated vulnerability assessments involves the rapid sorting of large strings of data. In the ASD (Aeronautical Systems Division) missile endgame model SESTEM,[1] for example, the aircraft components struck by the expanding fragment spray-band are calculated and stored, and then must be sorted. The sorted data are then used to compute the components struck in order of time intercepted and the resulting aircraft probability of kill. Similar data string storage and sort problems are involved in other vulnerability assessment computer programs such as FASTGEN II.[2] In SESTEM, about 40% of the computation time is typically involved with the sorting and fragment vulnerability computations.

For these reasons, the development of efficient algorithms to sort these strings of data are of considerable importance. Since large portions of the total computer run times are typically involved in performing this sorting, any improvements in them will show direct payoffs in terms of decreased run times, more rapid turnaround, greater program efficiencies, and decreased costs. The sorting algorithms in use in the vulnerability assessment programs were evaluated, and two new and more efficient programs were written. When the sorting algorithm was inserted in a new version of the SESTEM program, the average time for a run was decreased by 20 to 25%. These programs, and comparisons with some existing programs are presented in this report.

PROGRAM DEVELOPMENT

BACKGROUND

The availability of efficient general purpose sort algorithms and the sharp reduction of cost per computation of present generation computers has reduced interest in research into sort algorithms from earlier levels. Aside from the intrinsic challenge presented by the sorting problem, optimization of existing and developing applications programs written in FORTRAN IV, or similar level languages, indicate a still existing need for compact, efficient, in-line sort algorithms that are readily adaptable to specialized ends.

One approach to meeting this need is to devise a *partial sort*, an efficient algorithm by which a string of numbers is nearly sorted; and complete the sort by a method such as binary insertion, which can take advantage of a high degree of order in a number string.

[1] Aeronautical Systems Division. "SESTEM Missile Endgame Model", by G.B. Bennett, Wright-Patterson AFB, OH, ASD. September 1977. (ASD/XRH memo.)

[2] Aeronautical Systems Division. *FASTGEN II Target Description Computer Program*, by D. Cudney, Wright-Patterson AFB, OH, ASD. March 1978. (Report ASD-TR-77-24.)

PARTIAL SORTING

The basic strategy in producing partially sorted strings is to successively partition a set of numbers by exchange comparisons. Let NUM be a string of numbers of length NO equal to an integral power of two. The set of NO numbers is divided into two subsets of equal size, and the elements of one subset are compared (and exchanged when necessary) to the elements of the other subset in such a way that every element is involved in a comparison once only. As a worst case, after any comparison of sets as described, 25% of the elements less than or equal to the median value will be located below the median. Successive set divisions and comparisons continue the process, removing extreme values from the middle of the array and enabling further distribution to take place, until there are NO subsets of 1 element each. If the NO elements are distinct and represented by 1, 2, . . ., NO, Figure 1 represents the process described.

PDQ ALGORITHM

The successive division of subsets into equal subsets, as illustrated, will cause the 1 element to migrate to the proper postion in the array; but in the worst case, the 2 element will migrate toward the middle position of the array. The 2 element can be forced left by dividing the subsets unequally, as shown in Figure 2. Let the kth interval of comparison be defined as the integer part of $I_k = NO^*F^k$, where $0.<F<1$. A code to achieve the partitioning is given in Listing 1. F = .8 seems to produce a complete sort on a uniform random distribution; however, the author has not been able to characterize the optimal value for F for a particular given distribution. Observation of empirical test results suggests that certain structured distributions are difficult to sort; therefore, the PDQ algorithm may be useful as a test of randomness.

SORT CODE TESTING ALGORITHM

DISTRIBUTIONS

The sort code was tested by a control program (Listings 2A to 2D) that generates strings of a specified length from six different distributions:

1. Sorted arrays: the elements of the sequence 1, 2, 3, . . ., NO.

2. Arrays in reverse order: the elements of the sequence NO, NO-1, . . ., 2, 1.

3. Random arrays: numbers from a uniform distribution over the interval (0. , 1.). The random numbers are multiplied by 100,000 to minimize duplicate numbers when converted to integers.

4. Arrays almost in sort: the sequence, 1, 2, . . ., NO with a specified number of elements, chosen at random, set to values taken from a uniform random distribution.

5. Arrays of equal length sorted blocks: this is the distribution described in (3) sorted in successive segments of a specified length.

6. Constant value arrays: a sequence of numbers of equal value. These distributions were selected with a view to test algorithm behavior on distributions that might be encountered in practical applications, such as (3) and (4), or to demonstrate unusual characteristics.

Elapsed time to sort is measured, and a count of departures from a monotonic sequence is made. If this count is greater than 0, an error message is printed.

TESTING ENVIRONMENT

All test runs were conducted on a CDC CYBER 74, with the same level of optimization (OPT = 2). Observations of repeated runs on the CYBER 74, operating in a time-sharing mode, suggest that time measurements can vary about 20%; however~~, by~~ specifying large arrays to sort (90K), the job will be made to run on the machine in a more dedicated configuration. In this dedicated mode, elapsed times are highly reproducible. Sort times are sensitive to the values taken for F, as shown in Figure 3.

Machine Dependancy

An apparent anomally is that it takes more time to sort a sorted string than to sort a string that has an initial uniform random distribution. This seems to indicate that it takes more time to execute a branch instruction than the three arithmetic replacement statements involved in the number exchange. This is true in the aggregate for the repetitive execution of the code in the DO loop used to compare and exchange the elements of the NUM array.

The CYBER 74 is a stack machine. Up to seven words of packed instructions containing up to 28 instructions can be retained in registers constituting an instruction stack. This device can increase instruction execution speed by reducing memory references; however, a forward branch in the stack "voids the stack", therefore is an expensive operation. For this reason, PDQ will sort faster if the "less than equal" test is replaced by a "less than" test in the exchange algorithm. This increased speed is demonstrated in the decreased times required to sort a constant value array, as compared to the time required to sort an array already in sort.

Markedly different results may be expected on some other computer systems. The expected time relationship may be obtained by replacing the DO loop with an IF loop, where the branch will be in a backward direction in the instruction stack (Listing 3). Unfortunately, code optimization is not as intensive now, and overall sort times are increased.

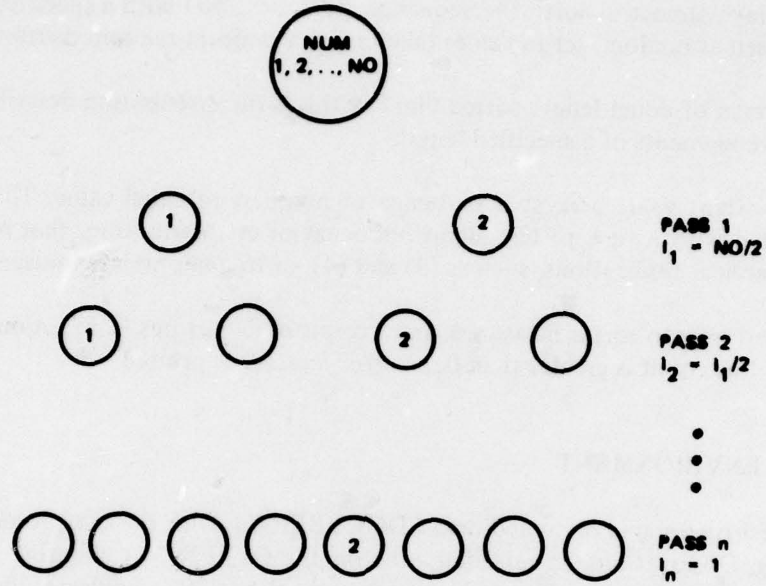


Figure 1. Successive Partition of Sets into Equal Subsets.

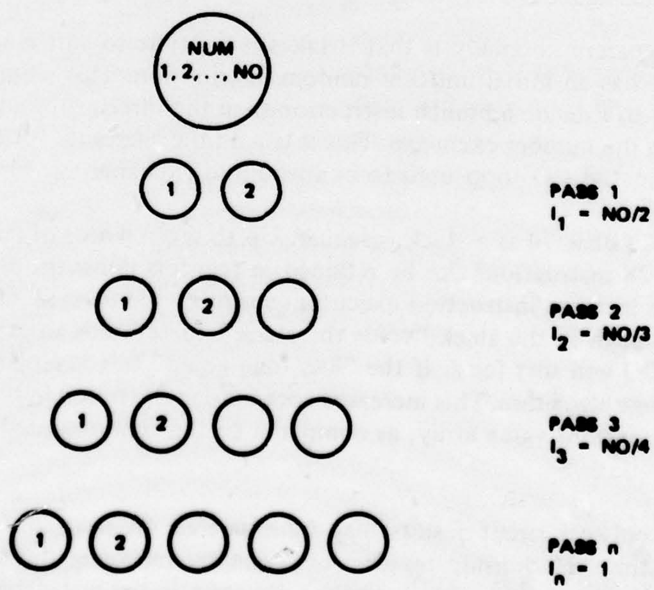


Figure 2. PDQ Partitioning Process.

```
      SUBROUTINE SORT(NUM,NO)
      DIMENSION NUM(NO)
C                                                              PDQ - BASIC
      A        = NO
10    A        = A*.8
      I        = A
      IF(I .LE. 0)            RETURN
      K        = NO - I
      DO 15 J = 1,K
      IF(NUM(J).LE.NUM(I+J)) GO TO 15
      MAX      = NUM(  J)
      NUM(  J)= NUM(I+J)
      NUM(I+J)= MAX
15    CONTINUE
      GO TO 10
C                                                              LISTING  1
      END
```

```
      PROGRAM SORT(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,PUNCH)
      COMMON TIME(7),NO,A(90000)
      DATA ICEED/78/
      READ (5,100) IP1,IP2,IP3,ALF,M
      WRITE(6,115) IP1,IP2,IP3,ALF,M
      DO 80 NO= IP1,IP2,IP3
      DO 10 N = 1,NO
10    A(N)     = N
      CALL TEST(1,          40H                         SORTED ARRAYS)
      DO 20 N = 1,NO
20    A(N)     = NO - N + 1
      CALL TEST(2,          40H          ARRAYS SORTED IN REVERSE ORDER)
      CALL RANSET(ICEED)
      DO 30 N = 1,NO
30    A(N)     = RANF(B)*100000.
      CALL TEST(3,          40H                         RANDOM ARRAYS)
      DO 40 N = 1,NO
40    A(N)     = N
      DO 45 I = 1,M
      N        = RANF(B)*100000.
42    IF(N .LT. NO)           GO TO 45
      N        = N/2
      GO TO 42
45    A(N)     = RANF(B)*100000.
      CALL TEST(4,          40H                  ARRAYS ALMOST IN SORT)
      CALL RANSET(ICEED)
      DO 50 N = 1,NO
50    A(N)     = RANF(B)*100000.
      CALL STRING(M)
      CALL TEST(5,          40H  ARRAYS OF EQUAL LENGTH SORTED BLOCKS)
      DO 60 N = 1,NO
60    A(N)     = 8.
      CALL TEST(6,          40H                  CONSTANT VALUE ARRAYS)
80    CONTINUE
      STOP
100   FORMAT(3I10,A10,4I10 )
115   FORMAT(1H1,3I10,2X,A10,4I10 )
C                                                              LISTING 2A
      END
```

```
      SUBROUTINE TEST(K,ALF)
      COMMON TIME(7),NO,A(90000)
      DIMENSION ALF(5),NUM(90000)
      EQUIVALENCE (A(1),NUM(1))

      CALL TYME (K,K)
      CALL SORT(NUM,NO)
      CALL ETIME(K,K)
      CALL CHECK
      WRITE(6,200) (ALF(I),I=1,4),NO,TIME(K)
      N1        = NO - 8
      CALL OUT(N1,NO)
      RETURN
  200 FORMAT(1H ,4A10,I8,F10.2 )
C
      END
```

LISTING 2B

```
      SUBROUTINE TYME(N1,N2)
      COMMON TIME(7),NO,A(90000)

      TP        = SECOND(B)
                                        RETURN
      ENTRY ETIME
      TC        = SECOND(B)
      TIME(N1)= TC - TP
      TP        = TC
                                        RETURN
      ENTRY CHECK
      IE        = 0
      DO 20 J = 2,NO
   20 IF(A(J-1) .GT. A(J))      IE = IE + 1
      IF(IE .NE. 0)             WRITE(6,100) IE
                                        RETURN
      ENTRY OUT
      WRITE(6,300) (A(N),N=N1,N2)
                                        RETURN
  100 FORMAT(1H , 35X,12HERROR COUNT=,I6)
  300 FORMAT(1H+,65X,9F7.0 )
C
      END
```

LISTING 2C

```
      SUBROUTINE STRING(M)
      COMMON TIME(7),NO,A(90000)

      DO 30 N1= 1,NO,M
      N2          = MINO(N1+M-1,NO)
      I           = N2 - N1 + 1
      B           = I
   10 IF(I .LE. 1)              GO TO 30
      B           = B*.425
      I           = B
      IF(I .LT. 1)              I = 1
      K           = N2 - I

      DO 25 J = N1,K
      L           = J
   15 IF(A(L) .LE. A(I+L))      GO TO 25
      BIG         = A(  L)
      A(  L)      = A(I+L)
      A(I+L)      = BIG
      L           = L - I
      IF(L .GE. N1)             GO TO 15
   25 CONTINUE
      GO TO 10
   30 CONTINUE
      RETURN
C
      END
```

LISTING 2D

TIME IN SECONDS AND ERROR COUNTS FOR TWO SUCCESSIVE PDQ SORTS ON A UNIFORM RANDOM DISTRIBUTION OF 10000 NUMBERS

F	PASSES	TIME1	TIME2	ERR1	ERR2
.200	5	.27	.26	5030	5009
.210	5	.24	.22	5011	5023
.220	6	.28	.27	4995	5014
.230	6	.30	.28	4984	4991
.240	6	.28	.26	5019	4389
.250	6	.29	.27	5016	5009
.260	6	.28	.27	5005	4981
.270	7	.32	.32	4977	4970
.280	7	.33	.31	5005	4956
.290	7	.32	.32	5021	4993
.300	7	.33	.31	5029	4921
.310	7	.32	.31	5016	4989
.320	8	.37	.35	4960	4945
.330	8	.36	.37	5020	4975
.340	8	.37	.34	4948	4995
.350	8	.37	.35	4969	4974
.360	9	.40	.39	4947	4932
.370	9	.40	.39	4941	4967
.380	9	.41	.40	4941	4898
.390	9	.42	.40	4985	5021
.400	10	.45	.44	4989	4897
.410	10	.45	.44	4948	4924
.420	10	.44	.43	4978	4958
.430	10	.45	.42	4996	4954
.440	11	.49	.48	4981	4806
.450	11	.51	.48	4983	4830
.460	11	.49	.46	4924	4838
.470	12	.54	.52	4943	4760
.480	12	.53	.50	4960	4793
.490	12	.54	.51	4912	4773
.500	13	.56	.56	4893	4777
.510	13	.56	.54	4906	4724
.520	14	.61	.59	4892	4438
.530	14	.61	.60	4897	4475
.540	14	.61	.58	4912	4572
.550	15	.64	.63	4685	4123
.560	15	.65	.63	4824	4320
.570	16	.69	.67	4643	3867
.580	16	.69	.66	4790	4336
.590	17	.70	.69	4562	3194
.600	18	.76	.70	4103	2451
.610	18	.73	.75	4269	2545
.620	19	.80	.79	4135	2556
.630	19	.79	.78	4244	3117
.640	20	.81	.80	3200	1212
.650	21	.85	.86	3319	1073
.660	22	.90	.89	1888	96
.670	22	.88	.93	2162	293
.680	23	.91	.95	1273	9
.690	24	.94	1.04	1242	18
.700	25	.99	1.13	401	0
.710	26	1.00	1.15	354	0
.720	28	1.06	1.30	23	0
.730	29	1.10	1.37	20	0
.740	30	1.15	1.48	4	0
.750	32	1.20	1.52	0	0
.760	33	1.24	1.61	0	0
.770	35	1.30	1.71	0	0
.780	37	1.44	1.80	0	0
.790	39	1.49	1.91	0	0
.800	41	1.55	2.00	0	0
.810	43	1.61	2.10	0	0
.820	46	1.76	2.21	0	0
.830	49	1.90	2.43	0	0
.840	52	1.96	2.51	0	0
.850	56	2.13	2.73	0	0
.860	61	2.34	3.01	0	0
.870	66	2.52	3.21	0	0
.880	72	2.77	3.53	0	0
.890	79	3.01	3.75	0	0
.900	87	3.28	4.22	0	0

Figure 3. Elapsed Time as a Function of F for the PDQ Partial Sort (Listing 1).

```
      SUBROUTINE SORT(NUM,NO)
      DIMENSION NUM(NO)
C                                                   PDQ IF LOOP
      A        = NO
 10   A        = A*.8
      I        = A
      IF(I .LE. 0)          RETURN
      K        = NO - I
C                                         LOOP WILL EXECUTE ONCE AS WITH A
C                                         FORTRAN4 DO - LOOP, REGUARDLESS
C.                                        OF INITIAL INDEX PARAMETER VALUES
      J        = 1
      GO TO 20
 15   J        = J + 1
      IF(J .GT. K)          GO TO 25
 20   IF(NUM(J).LE.NUM(I+J)) GO TO 15
      MAX      = NUM(   J)
      NUM(   J)= NUM(I+J)
      NUM(I+J)= MAX
      GO TO 15
 25   CONTINUE
      GO TO 10
C                                                   LISTING  3
      END
```

VARIATIONS

The simplicity of the PDQ code lends it to numerous variations; for instance, PDQ modules can be stacked with different values for F in each module. Changing the direction of the comparisons on alternate passes causes sorting to occur in fewer passes but at the cost of increased complexity (Listing 4). Listing 5 is a format of PDQ using a partitioning scheme involving the logarithm to the base 12.

```
      SUBROUTINE SORT(F)
      COMMON NUM(10000),NO
C                                                   PDQ ALTERNATE
      A        = NO
 10   A        = A*F
      I        = A
      IF(I .LE. 0)          RETURN
      K        = NO - I

      DO 15 J = 1,K
      IF(NUM(J).LE.NUM(I+J)) GO TO 15
      MAX      = NUM(   J)
      NUM(   J)= NUM(I+J)
      NUM(I+J)= MAX
 15   CONTINUE
      A        = A*F
      I        = A
      IF(I .LE. 0)          RETURN
      K        = NO - I
      L        = K + 1

      DO 20 J = 1,K
      L        = L - 1
      IF(NUM(L).LE.NUM(I+L)) GO TO 20
      MAX      = NUM(   L)
      NUM(   L)= NUM(I+L)
      NUM(I+L)= MAX
 20   CONTINUE
      GO TO 10
C                                                   LISTING  4
      END
```

```
      SUBROUTINE SORT(NUM,NO)
      DIMENSION NUM(NO)
C                                                     PDQ LOG12
      A       = NO
      P       = ALOG10(A)/1.07918
 10   P       = P - .1
      A       = 12.**P
      I       = A
      IF(I .LE. 0)            RETURN
      K       = NO - I
      DO 15 J = 1,K
      NUM1    = NUM(  J)
      NUM2    = NUM(I+J)
      IF(NUM1 .LT. NUM2)      GO TO 15
      NUM(  J)= NUM2
      NUM(I+J)= NUM1
 15   CONTINUE
      GO TO 10
C                                                     LISTING 5
      END
```

The original intention of the sort strategy is achieved, with the assurance of a sort by following PDQ with a sort by direct insertion. The insertion code is derived from PDQ by the addition of a few lines of code (Listing 6).

Insertion means the addition of numbers to an existing sorted string (which initially may be of length 1) by inserting the new element into, or at the ends of the sorted string to form a new sorted string of increased length. For any distribution, the sort is completed in one pass. An INSERTION sort is to be distinguished from a BUBBLE sort where successive elements are selected and added to one end of a string, initially of length 0. The BUBBLE sort may require up to NO-1 passes to complete the sort. Tests show the INSERTION sort to be more efficient than the BUBBLE sort (Listing 7).

If the features of the PDQ sort and the INSERTION sort are combined, a particularly efficient algorithm resembling the SHELL sort is obtained (Listing 8). The specification of an optimal sequence of intervals to control partitioning is a difficult task; however, if a geometric sequence is assumed, an F can empirically be found which will yield minimum sort times for a given distribution (Figure 4). If two or more values of F produce minimal elapsed times, the smallest of these values should be selected to reduce sort times on sorted or nearly sorted strings.

The SHELL sort seems to have been intended as a type of merge algorithm.[3] The term "merge" may be used in several ways:

1. The combination of two or more sorted strings into a resultant sorted string irrespective of any particular algorithm.

2. A specific method by which sorted strings can be combined efficiently into resultant sorted strings.

---

[3]Gotlieb, C.C. Sorting on Computers Communications, ACM 6 (May 1963), p. 194.

```
      SUBROUTINE SORT(NUM,NO)
      DIMENSION NUM(NO)
C                                                        PDQ INSERT #1
      A          = NO
10    A          = A*.7
      I          = A
      IF(I .LE. 6)            GO TO 20
      IF(MOD(I,2) .EQ. 0)     I = I + 1
      K          = NO - I

      DO 15 J = 1,K
      IF(NUM(J).LE.NUM(I+J)) GO TO 15
      MAX        = NUM(   J)
      NUM(   J)= NUM(I+J)
      NUM(I+J)= MAX
15    CONTINUE
      GO TO 10

20    K          = NO - 1
      DO 30 J = 1,K
      L          = J
25    IF(NUM(L).LE.NUM(L+1)) GO TO 30
      MAX        = NUM(L   )
      NUM(L   )= NUM(L+1)
      NUM(L+1)= MAX
      L          = L - 1
      IF(L .GT. 0)            GO TO 25
30    CONTINUE
      RETURN
C                                                        LISTING 6
      END
      SUBROUTINE SORT
      COMMON NUM(10000),NO
C                                                        BUBBLE SORT
      K          = NO - 1
5     L          = 1
      DO 10 J = 1,K
      IF(NUM(J).LE.NUM(J+1)) GO TO 10
      MAX        = NUM(J   )
      NUM(J   )= NUM(J+1)
      NUM(J+1)= MAX
      L          = J
10    CONTINUE
      IF(L .EQ. 1)            RETURN
      K          = L - 1
      GO TO 5
C                                                        LISTING 7
      END

      SUBROUTINE SORT(F)
      COMMON NUM(90000),NO,TIME(7)
C                                                        DISTRIBUTION #1
      A          = NO
      I          = NO
10    IF(I .LE. 1)            RETURN
      A          = A*F
      I          = A
      IF(I .LT. 1)            I = 1
      K          = NO - I

      DO 20 J = 1,K
      L          = J
15    IF(NUM(L).LE.NUM(I+L)) GO TO 20
      MAX        = NUM(   L)
      NUM(   L)= NUM(I+L)
      NUM(I+L)= MAX
      L          = L - I
      IF(L .GT. 0)            GO TO 15
20    CONTINUE
      GO TO 10
C                                                        LISTING 8
      END
```

10000 10000 100C0 D#1 L8 C C

NO	F	TIME IN SECONDS TIME1 RANDOM	TIME2 SORTED
10000	.200	2.93	.36
10000	.205	1.33	.33
10000	.210	1.31	.33
10000	.215	1.40	.34
10000	.220	1.30	.33
10000	.225	1.48	.34
10000	.230	1.41	.34
10000	.235	1.54	.34
10000	.240	1.31	.33
10000	.245	1.37	.39
10000	.250	2.81	.40
10000	.255	1.32	.39
10000	.260	1.22	.39
10000	.265	1.30	.39
10000	.270	1.21	.39
10000	.275	1.38	.39
10000	.280	1.17	.39
10000	.285	1.29	.39
10000	.290	1.61	.39
10000	.295	1.28	.40
10000	.300	1.19	.45
10000	.305	1.29	.44
10000	.310	1.41	.45
10000	.315	1.20	.45
10000	.320	1.13	.44
10000	.325	1.18	.45
10000	.330	1.20	.45
10000	.335	1.35	.44
10000	.340	1.41	.44
10000	.345	1.3[illegible]	.52
10000	.350	1.56	.50
10000	.355	1.12	.50
10000	.360	1.16	.49
10000	.365	1.13	.50
10000	.370	1.14	.49
10000	.375	1.13	.49
10000	.380	1.13	.50
10000	.385	1.47	.50
10000	.390	1.12	.56
10000	.395	1.21	.55
10000	.400	1.25	.55
10000	.405	1.15	.55
10000	.410	1.12	.55
10000	.415	1.15	.54
10000	.420	1.10	.54
10000	.425	1.10	.54
10000	.430	1.12	.60
10000	.435	1.13	.60
10000	.440	1.13	.60
10000	.445	1.18	.60
10000	.450	1.14	.61
10000	.455	1.12	.60
10000	.460	1.12	.59
10000	.465	1.15	.66
10000	.470	1.15	.66
10000	.475	1.16	.66
10000	.480	1.14	.65
10000	.485	1.17	.65
10000	.490	1.19	.67
10000	.495	1.25	.70
10000	.500	1.46	.71

Figure 4. Elapsed Time as a Function of F for Distribution 1 (Listing 8).

Generally, the SHELL sort does not qualify as a merge algorithm, but it can be made to operate as such under definition (1) by selecting F = .5 and the first interval as the largest power of 2 less than NO.

This choice of F turns out to be one of the worst possible. To see why, it may be constructive to consider this procedure as a type of distribution sort. Consider the sequence of 16 numbers, 1, 2, . . ., 16 where the integers represent the position in an array of numbers to be sorted. Take the first interval of comparison, I_1 equal to 8 and F = .5, then the following illustration can be used:

1	2	3	4	5	6	7	8	Pass 1
9	10	11	12	13	14	15	16	$I_1 = 8$

to suggest that the array to be sorted has been divided into eight subsequences represented by the columns. The numbers in each column are to be sorted in ascending order from top to bottom; i.e., the number in position 1 is to be less than or equal to the number in position 9, etc.

The following illustration represents the next pass:

1	2	3	4	Pass 2
5	6	7	8	$I_2 = 4$
9	10	11	12	
13	14	15	16	

Note that: (1) the subsequences are reduced in number in proportion to F, (2) the length of the subsequences is increased in inverse proportion to F, and (3) the elements of a given column are composed alternately of the n and n + $I_1/2$ columns of the previous pass, n = 1, 2, 3, $I_1/2$. Consequently, there is a high degree of order in any column and an element is likely to be close to its sorted position in a subsequence, and an INSERTION sort or BUBBLE sort applied to a column may be expected to operate faster than on a random distribution of the same length. The worst case in this example occurs when the even and odd columns represent disjoint ranges, and the set of numbers from the even columns contains the smaller numbers. Insofar as the median of each column represents the median of the entire distribution, subsequent passes may be expected to produce increasingly well-ordered subsequences representative of the entire sequence; therefore, needing a minimum number of comparisons and exchanges to sort. However, certain difficulties can arise.

In the following example, the array notation of the previous illustration will be retained, but now the integers represent the actual numbers in the array that are to be sorted. Given the sequence 3, 5, 6, 7, 2, 8, 9, 10, 4, 11, 12, 13, 1, 14, 15, 16, let F = .5 and I_1 = 8, we have:

3	5	6	7	2	8	9	10	Pass 1
4	11	12	13	1	14	15	16	$I_1 = 8$

All columns but the fifth are in sort; therefore, the 1 and 2 elements are exchanged. This could be called an unfavorable exchange, as it improves the order of the array very little; both the 1 and the 2 should be in the first row. After the exchange, pass 2 can be represented:

3	5	6	7	Pass 2
1	8	9	10	$I_2 = 4$
4	11	12	13	
2	14	15	16	

The columns are well-ordered except column 1. After the sort we have:

1	5	6	7
2	8	9	10
3	11	12	13
4	14	15	16

Column 1 has a median atypical of the array, and most of its elements are far from their final position. It will take a large number of small steps to move them into postion in later passes. The elements in column 1 after row 1 may be thought of as blocking elements because they inhibit efficient distribution of the array elements in the early stages of the sort when an element can proceed toward its destination by large steps. To follow the example further:

1	5	Pass 3	1	5
6	7	$I_3 = 2$	2	7
2	8		3	8
9	10		4	10
3	11		6	11
12	13		9	13
4	14		12	14
15	16		15	16

The last pass with $I_4 = 1$ will require a large number of exchanges to complete the sort. The collection of blocking elements can be prevented by a number of methods, such as sorting up a diagonal from left to right, following a column sort. But most of the comparisons will not result in an exchange due to a high degree of order produced by the previous pass, and those exchanges that do result are bought at the expense of an extra pass. Similar objections may be raised to other modifications to the algorithm to eliminate the blocking elements. Examination of the problem reveals that the accumulation of blocking elements from pass to pass is particularly severe if the rows in the array are divided into an integral number of rows in the next pass; therefore, .25 and .5 are bad values for F. This can be verified readily by rewriting the sort algorithm to include counts of primary exchanges and secondary exchanges on a pass-by-pass basis, and printing these values (as well as their cumulative totals), for selected values of F (Figures 5 and 6). It is interesting to note that the minimum sort times do not correspond to the values of F that result in minimum exchanges, because as F becomes larger, there is an increase in overhead associated with the increase in the number of passes. The elapsed times shown are distorted by the code modification required for the accumulation and printing of the statistics.

A study of the exchange counts suggests that the blocking phenomena come into play whenever there is a series of intervals that have factors in common, and that the problem becomes more severe as F decreases; therefore, the intervals should be relatively prime. Consider the first column of the array on the mth pass. The location of a blocking element in the first column can be expressed as 1 + M*Pm. On the next pass, if that same element is to appear in the same column, it must have a location expressible as 1 + N*Pn, where Pn < Pm. Then N*Pn = M*Pm, and N = Pm, M = Pn if Pm, Pn are relatively prime. But these values for M and N are impossible during the earlier passes of the sort for large arrays and the range of F considered here. To test this hypothesis, sequences of relatively prime numbers corresponding to the geometric sequences associated to values for F (F = .2 to F = .46) were used in the sort algorithm (Listing 9) to conduct elapsed time tests (Figure 7). The tests seem to confirm the hypothesis; however, the use of prime sequences is only a partial solution. A blocking element may be replaced in a column with another element that also serves as a blocking element. This situation is likely to occur when sorting arrays of equal length sorted blocks. Counts of exchanges and comparisons for the prime sequences are given in Figure 8.

PRIME SORTING ALGORITHM

The prime sequence corresponding to F = .3 was selected with a view to minimum combined elapsed times for both random and sorted distributions for the algorithm PRIME SORT (Listing 10). This algorithm also incorporates a change that significantly improves the efficiency of exchanges, especially secondary exchanges.

The improvement in execution speed was largely lost when the data was passed through the CALL list. This illustrates a compilation problem that affects the various algorithms given here to differing extents; the PDQ codes were degraded least by passing data through the CALL list. Efficient compilation is important because the PDQ, PRIME, and DISTRIBUTION sort codes given here derive their speed from compact code that requires a minimum of instructions to execute, and from improved partitioning schemes. Full realization of the potential of the code requires effective register assignment to indices, etc., but variables passed through CALL lists inhibit optimal compilation. Generally, an improvement in performance may be expected when data is passed to a subroutine through COMMON storage (Figure 9).

10000 10000 10000 COUNTS 0 0

F	I	PASS	PRIMARY EXCHANGE	SECONDARY EXCHANGE	PRIMARY COMPARE	SECONDARY COMPARE
.200	2000	1	5435	4515	8000	7451
.200	400	2	6531	13457	9600	19496
.200	80	3	7321	39342	9920	46552
.200	16	4	7720	101590	9984	109354
.200	3	5	9036	222497	9997	231548
.200	1	6	5471	9871	9999	14341
TOTALS			41536	390372	57500	428772
F	I	PASS	PRIMARY EXCHANGE	SECONDARY EXCHANGE	PRIMARY COMPARE	SECONDARY COMPARE
.210	2199	1	5305	4187	7901	6871
.210	440	2	6645	13879	9560	19973
.210	92	3	7719	29051	9908	36644
.210	19	4	8125	39808	9981	47903
.210	4	5	7399	33340	9996	46325
.210	1	6	6553	15958	9999	22509
TOTALS			42333	141333	57345	180225
F	I	PASS	PRIMARY EXCHANGE	SECONDARY EXCHANGE	PRIMARY COMPARE	SECONDARY COMPARE
.220	2199	1	5185	3850	7801	5391
.220	483	2	6561	12443	9517	18392
.220	106	3	7556	24802	9894	32235
.220	23	4	7941	36819	9977	44732
.220	5	5	8022	38136	9995	45149
.220	1	6	7108	24748	9999	31853
TOTALS			42373	140798	57183	179752
F	I	PASS	PRIMARY EXCHANGE	SECONDARY EXCHANGE	PRIMARY COMPARE	SECONDARY COMPARE
.230	2299	1	5078	3409	7701	5877
.230	528	2	6415	11366	9472	17118
.230	171	3	7409	23585	9879	30875
.230	27	4	8019	35224	9973	43210
.230	6	5	7831	30339	9994	38164
.230	1	6	7550	41386	9999	48936
TOTALS			42303	145309	57018	184180
F	I	PASS	PRIMARY EXCHANGE	SECONDARY EXCHANGE	PRIMARY COMPARE	SECONDARY COMPARE
.240	2399	1	4927	3079	7601	5335
.240	575	2	6211	10322	9425	15851
.240	138	3	7294	20518	9862	27639
.240	33	4	7672	26512	9967	34256
.240	7	5	7905	32142	9993	40040
.240	1	6	7847	41734	9999	49580
TOTALS			41847	134407	56847	172701
F	I	PASS	PRIMARY EXCHANGE	SECONDARY EXCHANGE	PRIMARY COMPARE	SECONDARY COMPARE
.250	2499	1	4830	2738	7501	4845
.250	624	2	6213	9635	9376	15090
.250	156	3	5364	14599	9844	20770
.250	39	4	6937	35514	9961	43405
.250	9	5	8664	105480	9991	115136
.250	2	6	8045	223317	9998	231359
.250	1	7	3590	1819	9999	5408
TOTALS			44643	393092	66670	436013
F	I	PASS	PRIMARY EXCHANGE	SECONDARY EXCHANGE	PRIMARY COMPARE	SECONDARY COMPARE
.260	2599	1	4687	2519	7401	4504
.260	675	2	5935	8384	9325	13590
.260	175	3	7048	15849	9825	23699
.260	45	4	7655	25151	9955	32750
.260	11	5	7897	23527	9989	37407
.260	3	6	7284	20924	9997	28204
.260	1	7	5334	6263	9999	11595
TOTALS			45840	103617	66491	151739

Figure 5. Counts of Exchanges and Comparisons as a Function of F for the Distribution # 1 Algorithm.

F	PRIMARY EXCHANGE	SECONDARY EXCHANGE	PRIM.+S. EXCHANGE	PRIMARY COMPARE	SECONDARY COMPARE	PRIM.+S. COMPARE	TIME
.200	41534	390372	431906	77500	428772	506272	4.52
.210	42339	141223	183562	57345	180225	237570	1.99
.220	42373	140798	183171	57143	179792	236935	2.00
.230	42302	145309	187611	57018	184180	241198	2.06
.240	41947	134317	176264	56647	172901	229548	1.92
.250	44703	395032	439735	66670	436013	502683	4.40
.260	43940	109517	153457	66491	151739	218230	1.79
.270	45231	137556	182787	66306	149050	215356	1.76
.280	45156	101248	146404	66116	142560	208676	1.71
.290	48849	181379	230228	65922	222388	288310	2.44
.300	47372	86175	133547	75719	129578	205297	1.70
.310	47356	129384	176740	75513	172832	248345	2.05
.320	47518	81923	129441	75300	125242	200542	1.62
.330	47771	91686	139457	75030	135255	210285	1.75
.340	46851	134366	181217	74854	176341	251195	2.08
.350	48763	146663	195426	84620	191968	276588	2.26
.360	42932	77229	120161	84382	121693	206075	1.65
.370	49940	71377	121317	84133	116831	200964	1.61
.380	[illegible]	67314	117812	83077	112068	195145	1.58
.390	51248	55440	106688	93611	102137	195748	1.53
.400	51705	75196	126901	93341	122374	215715	1.72
.410	51545	54414	105959	93056	101342	194398	1.52
.420	51567	53978	105545	92766	101867	194633	1.49
.430	52140	45966	98106	102461	93305	195766	1.50
.440	52700	49156	101856	102151	97004	199155	1.54
.450	52879	47114	99993	101825	95040	196865	1.52
.460	52879	46437	99316	101490	94170	195660	1.49
.470	53917	40506	94423	111140	89195	200335	1.51
.480	53908	41987	95895	110778	90526	201304	1.52
.490	54019	41568	95587	110402	90220	200622	1.56
.500	57933	86443	144376	120909	138791	259700	2.02
.510	55314	38397	93711	119601	88099	207700	1.55
.520	55790	33114	88904	129175	83315	212490	1.52
.530	56279	28921	85200	128733	79624	208357	1.41
.540	55234	30272	85506	128270	79905	208175	1.50
.550	55232	25961	81193	137787	76618	214405	1.56
.560	55672	26324	81996	137284	76395	213679	1.58
.570	57067	26437	83504	146755	77873	224629	1.60
.580	56458	29623	86081	146202	80390	226592	1.64
.590	57741	20546	78287	155622	72690	228312	1.59
.600	56770	20297	77067	155015	71339	226354	1.59
.610	57385	18012	75397	164371	69952	234323	1.64
.620	57610	24339	81949	163696	[illegible]	240104	1.73
.630	56762	23336	80098	172985	74279	247264	1.73
.640	57142	15616	72758	182234	67060	249294	1.70
.650	56616	16366	72982	181442	67088	248530	1.69
.660	56863	13103	69966	190604	64040	254644	1.72
.670	58470	11772	70242	199710	64190	263900	1.79
.680	58212	10576	68788	208765	62663	271428	1.84
.690	58525	10901	69426	207759	62125	269884	1.82
.700	57044	10050	67094	216687	60625	277312	1.90
.710	57277	8336	65613	225535	60110	285645	1.96
.720	57117	7999	65116	234303	58582	292885	1.92
.730	58044	8178	66222	252980	59675	312655	2.05
.740	57850	6939	64789	261558	58280	319838	2.11
.750	57688	5906	63594	270020	56934	326954	2.12
.760	58102	6159	64261	288355	57597	345952	2.19
.770	57237	4958	62195	296546	55446	351992	2.24
.780	57737	4830	62567	314569	55556	370125	2.35
.790	57292	3494	60786	332408	53800	386208	2.47
.800	57691	3137	60828	350026	53734	403760	2.52

Figure 6. Cumulative Counts of Exchanges and Comparisons as a Function of F for the Distribution #1 Algorithm.

```
      SUBROUTINE SORT(NUM,NO,N)
      DIMENSION NUM(NO),INK(16,27)

      DATA((INK(I,J),I=1,16),J=1,19)   /1,5,23,127,631,3121,15629,78121,   F= .20
     *390625,7*9,                       1,5,23,107,509,2447,11657,55529,      .21
     *264390,7*9,                       1,5,23, 97,431,1933, 8821,40093,      .22
     *182229,7*9,                       1,4,19, 93,359,1553, 6761,29363,      .23
     *127696,7*9,                       1,4,17, 71,307,1259, 5237,21803,      .24
     * 90847,378529,6*9,                1,4,17, 61,257,1021, 4093,16381,      .25
     * 65537,262144,6*9,                1,4,13, 59,223, 839, 3229,12451,      .26
     * 47881,184179,6*9,                1,4,13, 53,191, 701, 2579, 9551,      .27
     * 35407,131137,6*9,                1,4,13, 47,153, 577, 2081, 7411,      .28
     * 25479, 94331,337613,5*9,         1,3,13, 41,139, 487, 1693, 5801,      .29
     * 19991, 68927,237695,5*9,         1,3,11, 37,127, 409, 1373, 4567,      .30
     * 15241, 50921,169351,5*9,         1,3,11, 31,107, 349, 1129, 3637,      .31
     * 11731, 37831,122007,5*9,         1,3,11, 31, 97, 293,  929, 2909,      .32
     *  9091, 28429, 89817,277556,4*9,  1,3,11, 29, 83, 257,  773, 2347,      .33
     *  7109, 21557, 65293,197835,4*9,  1,3, 7, 23, 73, 223,  647, 1901,      .34
     *  5591, 16477, 48437,142473,4*9,  1,3, 7, 23, 67, 191,  547, 1553,      .35
     *  4441, 12589, 36251,103574,4*9,  1,3, 7, 19, 59, 167,  457, 1277,      .36
     *3547,9851,27361,75979,211043,3*9, 1,3, 7, 19, 53, 139,  389, 1051,      .37
     *2851,7699,20789,56207,151908,3*9, 1,3, 7, 17, 47, 127,  331,  877,      .38
     *2297,6053,15923,41911,110395,3*9/
      DATA((INK(I,J),I=1,16),J=20,27)/1,3,7,17,43,113,283,727,1867,4793,      .39
     *12281,31489,80761,207090,2*9,   1,3,7,17,37, 97,241,613,1523,3821,      .40
     * 9539,23833,59611,149012,2*9,   1,2,7,13,37, 83,211,509,1249,3049,      .41
     *7451,18169,44293,108096, 2*9,   1,2,7,13,31, 79,181,433,1033,2459,      .42
     *5857,13933,33191,79031,188152,9,1,2,5,13,29, 67,157,367, 857,1993,      .43
     *4621,10753,25031,58199,135345,9,1,2,5,13,29, 61,137,313, 709,1619,      .44
     *3677,8353,19001,43159,98101,222951,1,2,5,11,23,53,113,269,593,1321      .45
     *,2939,6529,14503,32233,71633,159152,1,2,5,11,23,47,107,229,499,         .46
     *1087,2357,5119,11149,24223,52639,114454/

      DO 5 IN = 1,16
      IF(NO .LE. INK(IN,N))   GO TO 10
    5 CONTINUE

   10 IN       = IN - 1
      IF(IN .LE. 0)           RETURN
      I        = INK(IN,N)
      K        = NO - I

      DO 20 J  = 1,K
      L        = J
   15 IF(NUM(L).LE.NUM(I+L))  GO TO 20
      MAX      = NUM(  L)
      NUM(  L) = NUM(I+L)
      NUM(I+L) = MAX
      L        = L - I
      IF(L .GT. 0)            GO TO 15
   20 CONTINUE
      GO TO 10
C                                                          LISTING 9
      END
```

10000 1000 PRIME L9 0 0

NO	F	TIME IN SECONDS TIME1 RANDOM	TIME2 SORTED
10000	.200	1.41	.38
10000	.210	1.37	.39
10000	.220	1.41	.40
10000	.230	1.33	.42
10000	.240	1.36	.43
10000	.250	1.29	.44
10000	.260	1.39	.45
10000	.270	1.25	.46
10000	.280	1.23	.48
10000	.290	1.24	.49
10000	.300	1.23	.50
10000	.310	1.24	.51
10000	.320	1.22	.53
10000	.330	1.24	.54
10000	.340	1.23	.56
10000	.350	1.21	.57
10000	.360	1.20	.58
10000	.370	1.18	.60
10000	.380	1.20	.62
10000	.390	1.20	.64
10000	.400	1.23	.64
10000	.410	1.21	.67
10000	.420	1.24	.69
10000	.430	1.23	.70
10000	.440	1.26	.73
10000	.450	1.24	.74
10000	.460	1.28	.76

Figure 7. Elapsed Time for Prime Sequences, F = .2 to F = .46.

F	PRIMARY EXCHANGE	SECONDARY EXCHANGE	PRIM.+S. EXCHANGE	PRIMARY COMPARE	SECONDARY COMPARE	PRIM.+S. COMPARE	TIME
.200	41462	147881	189343	56092	185639	241731	2.06
.210	42016	139874	181890	56908	178340	235248	1.99
.220	42572	143802	186374	58689	183067	241756	2.02
.230	43221	125396	168617	61220	164868	226088	1.88
.240	44256	127638	171894	63104	167525	230629	1.91
.250	44676	112908	157584	64546	153442	217988	1.79
.260	45485	128351	173836	65632	169789	235421	1.95
.270	45620	101598	147218	66907	143503	210410	1.72
.280	45771	95038	140809	69703	137030	206733	1.69
.290	46559	93253	139812	71822	135500	207322	1.71
.300	47248	88133	135381	73472	130919	204391	1.66
.310	47959	87356	135315	74732	131027	205759	1.66
.320	48197	82176	130373	76635	126432	203067	1.63
.330	49022	80827	129849	79387	125723	205110	1.68
.340	49553	77781	127334	81531	122710	204241	1.62
.350	49793	70060	119853	83167	115293	198460	1.56
.360	50400	67542	117942	84612	113562	198174	1.57
.370	49707	59500	109207	87788	104974	192762	1.51
.380	50391	59552	109943	90240	105303	195543	1.51
.390	51120	55681	106801	92146	101906	194052	1.51
.400	51650	59045	110695	94101	106178	200279	1.55
.410	51510	51370	102880	97388	98356	195744	1.50
.420	52509	52483	104992	99904	100123	200027	1.53
.430	53395	48715	102110	101888	97187	199075	1.52
.440	53763	49395	103158	105081	98489	203570	1.57
.450	53747	43001	96748	108141	91929	200070	1.49
.460	55065	44032	99097	110513	93768	204281	1.53

Figure 8. Cumulative Counts of Exchanges and Comparisons for Prime Sequences, F = .2 to F = .46.

```
      SUBROUTINE SORT(NUM,NO)
      DIMENSION NUM(NO),INK(11)
C                        IT IS MORE EFFICIENT TO PASS DATA THROUGH COMMON
C                                                              PRIME SORT
      DATA INK /1,3,11,37,127,409,1373,4567,15241,50821,169351/       F = .30

      DO 5 IN = 1,11
      IF(NO .LE. INK(IN))     GO TO 10
 5    CONTINUE

 10   IN        = IN - 1
      IF(IN   .LE. 0)         RETURN
      I         = INK(IN)
      K         = NO - I

      DO 20 J = 1,K
      L         = J
      NUM2      = NUM(I+L)
 15   NUM1      = NUM(   L)
      IF(NUM1 .LE. NUM2)      GO TO 20
      NUM(   L)= NUM2
      NUM(I+L)= NUM1
      L         = L - I
      IF(L .GT. 0)            GO TO 15
 20   CONTINUE
      GO TO 10
C                                                              LISTING 10
      END
```

Assembly language coding can minimize these problems, and candidates for that purpose are given in Listings 11 and 12. An algorithm suitable for in-line code is given in Listing 13. The effect of a prime sequence is approximated by making all intervals odd, etc.

Expected sort behavior is indicated by the expression for the number of comparisons required by the PDQ sort. The intervals of comparison are given approximately by the sequence:

$$F*NO,\ F^2*NO,\ F^3*NO,\ \ldots,\ F^n*NO$$

such that $F^n*NO = 1$. Then $n = -(\text{Log NO})/(\text{Log F})$. The number of comparisons for n passes are:

$$K = (NO-F*NO) + (NO-F^2*NO) + \ldots + (NO-F^n*NO)$$

$$K = n*NO - (F + F^2 + \ldots + F^n)*NO$$

$$K = -\frac{\text{Log NO}}{\text{Log F}}*NO - (F + F^2 + \ldots + F^n)*NO$$

This is also the expression for the primary comparisons of the DISTRIBUTION sort, but with a smaller value for F.

10000 1000 D#2 L13 0 0

NO	F	TIME IN SECONDS TIME1 RANDOM	TIME2 SORTED
10000	.200	1.24	.33
10000	.205	1.22	.33
10000	.210	1.16	.33
10000	.215	1.25	.33
10000	.220	1.20	.33
10000	.225	1.37	.34
10000	.230	1.20	.33
10000	.235	1.21	.33
10000	.240	1.39	.33
10000	.245	1.07	.39
10000	.250	1.11	.39
10000	.255	1.05	.39
10000	.260	1.06	.39
10000	.265	1.07	.39
10000	.270	1.21	.38
10000	.275	1.16	.39
10000	.280	1.06	.38
10000	.285	1.05	.39
10000	.290	1.08	.38
10000	.295	1.06	.38
10000	.300	1.02	.44
10000	.305	1.10	.44
10000	.310	1.00	.44
10000	.315	.97	.44
10000	.320	1.08	.44
10000	.325	1.10	.44
10000	.330	1.08	.44
10000	.335	1.14	.43
10000	.340	1.04	.43
10000	.345	.98	.49
10000	.350	.95	.49
10000	.355	1.00	.49
10000	.360	1.00	.49
10000	.365	.95	.49
10000	.370	.97	.49
10000	.375	.97	.49
10000	.380	.96	.48
10000	.385	.95	.49
10000	.390	.98	.54
10000	.395	1.02	.55
10000	.400	.97	.54
10000	.405	.94	.54
10000	.410	.94	.54
10000	.415	.97	.54
10000	.420	.96	.54
10000	.425	.94	.54
10000	.430	.97	.60
10000	.435	.95	.59
10000	.440	.95	.59
10000	.445	.94	.59
10000	.450	.94	.59
10000	.455	.96	.59
10000	.460	1.00	.59
10000	.465	.95	.65
10000	.470	.98	.64
10000	.475	.96	.64
10000	.480	.95	.64
10000	.485	.94	.64
10000	.490	.96	.64
10000	.495	.99	.70
10000	.500	1.13	.70

Figure 9. Elapsed Time as a Function of F for Distribution 2 (Listing 13).

```
      SUBROUTINE SORT(NUM,NO)
      DIMENSION NUM(NO)
C                                                    DISTRIBUTION #3
C                        IT IS MORE EFFICIENT TO PASS DATA THROUGH COMMON
      A        = NO
      I        = NO
   10 IF(I .LE. 1)          RETURN
      A        = A*.381
      I        = A
      IF(MOD(I,2) .EQ. 0)   I = I + 1
      IF(MOD(I,9) .EQ. 0)   I = I + 2
      K        = NO - I

      J        = 0
   15 J        = J + 1
      IF(J .GT. K)          GO TO 10
      L        = J
      NUM2     = NUM(I+L)
   20 NUM1     = NUM(  L)
      IF(NUM1 .LE. NUM2)    GO TO 15
      NUM(  L)= NUM2
      NUM(I+L)= NUM1
      IF(L .LE. I)          GO TC 15
      L        = L - I
      GO TO 20
C                                                         LISTING 11
      END
```

```
      SUBROUTINE SORT(NUM,NO)
      DIMENSION NUM(NO)

C                                                    DISTRIBUTION #4
C                        IT IS MORE EFFICIENT TO PASS DATA THROUGH COMMON
      A        = NO
      I        = NO
   10 IF(I .LE. 1)          RETURN
      A        = A*.381
      I        = A
      IF(MOD(I,2) .EQ. 0)   I = I + 1
      IF(MOD(I,9) .EQ. 0)   I = I + 2
      K        = NO - I

      J        = 0
   15 J        = J + 1
      IF(J .GT. K)          GO TO 10
      NUM1     = NUM(   J)
      NUM2     = NUM(I+J)
      IF(NUM1 .LE. NUM2)    GO TO 15
      L        = J
   20 NUM(I+L)= NUM1
      IF(L .LE. I)          GO TO 25
      L        = L - I
      NUM1     = NUM(  L)
      IF(NUM1 .GT. NUM2)    GO TO 20
      L        = L + I
   25 NUM(  L)= NUM2
      GO TO 15
C                                                         LISTING 12
      END
```

```
      SUBROUTINE SORT(F)
      COMMON NUM(90000),NO,TIME(7)                    DISTRIBUTION #2
C
      A          = NO
      I          = NO
 10   IF(I .LE. 1)                  RETURN
      A          = A*F
      I          = A
      IF(MOD(I,2) .EQ. 0)           I = I + 1
      IF(MOD(I,9) .EQ. 0)           I = I + 2
      K          = NO - I

      DO 20 J = 1,K
      L          = J
      NUM2       = NUM(I+L)
 15   NUM1       = NUM(  L)
      IF(NUM1 .LE. NUM2)            GO TO 20
      NUM(  L)=  NUM2
      NUM(I+L)=  NUM1
      L          = L - I
      IF(L .GT. 0)                  GO TO 15
 20   CONTINUE
      GO TO 10
C                                                     LISTING 13
      END
```

For each primary comparison, there is a chance of a primary exchange which will be followed generally by a secondary comparison, etc. Figure 6 indicates that secondary exchanges will be substantial for useful values of F; therefore, the expression for the comparisons of the DISTRIBUTION or PRIME sort will be that for PDQ sort, plus a series of terms involving probabilities which represent the various orders of exchanges (primary, secondary, etc.). Sort times for large NO may be expected to be proportional to Log NO for the PDQ sort, and to increase faster than a Log NO rate for PRIME sort.

PERFORMANCE EVALUATION

For a comparative evaluation of sort performance, some establised sort algorithms published in the COMMUNICATIONS of the ACM[4-7] were used. These algorithms were adapted for the sake of uniform notation and style. In addition, the subroutine SIFTUP of TREESORT3 was coded in-line to reduce the substantial overhead involved in the frequent calls to this procedure. The sort designated SINGLETON is one of the faster, more stable members of the QUICKSORT family.

---

[4] Singleton, Richard C. An efficient algorithm for sorting with minimal storage. Communications ACM 12 (March 1969), p. 185.

[5] Loeser, Rudolf. Some performance tests of "QUICKSORT" and decendants. Communications ACM 17 (March 1974), p. 143.

[6] Boothroyd, J. Algorithm 201, SHELLSORT. Communications ACM 6, 8 (August 1963), p. 445.

[7] Floyd, R. W. Algorithm 245, TREESORT3. Communications ACM 7, 12 (December 1964), p. 701.

An attempt to evaluate the efficiency of a procedure by frequency counts of critical parameters is not entirely satisfactory, even for differing versions of that procedure on the same machine. The particular form of a procedure is very important. Machine independent comparisons between differing procedures are even more difficult. Some of the results were omitted from the graphical sort performance data when interference between curves obscured the comparisons. Each curve has a label that refers to the listing of the code used in generating the data. (e.g., PDQ I L18 means PDQ INSERT #2, Listing 18.) The test results indicate that the PRIME and DISTRIBUTION sorts compare favorably overall to the QUICKSORT; and in the case of random distributions, the results increasingly favor the PDQ, PRIME, and DISTRIBUTION sorts as the array size decreases. The DISTRIBUTION sort is a compact and efficient sort suitable for in-line code applications because it is generally understandable; therefore, it may be modified easily for particular uses.

```
      SUBROUTINE SORT(NUM,NO)
      DIMENSION NUM(NO)
C         ADAPTATION OF SHELLSORT (ACM ALGORITHM 201 BY J. BOOTHROYD)
C      COMMUNICATIONS OF THE ACM - VOLUME 17 / NUMBER 3 / MARCH, 1974
      I         = 1
 5    I         = I + I
      IF(I .LE. NO)          GO TO 5
      I         = I - 1
 10   I         = I/2
      IF(I .LE. 0)           RETURN
      K         = NO - I
      DO 20 J = 1,K
      L         = J + I
 15   L         = L - I
      IF(L .LE. 0)           GO TO 20
      M         = L + I
      IF(NUM(L) .LE. NUM(M)) GO TO 20
      MAX       = NUM(L)
      NUM(L)    = NUM(M)
      NUM(M)    = MAX
      GO TO 15
 20   CONTINUE
      GO TO 10
C                                                          LISTING 14
      END
```

```
      SUBROUTINE SORT(NUM,NO)
C      COMMUNICATIONS OF THE ACM - VOLUME 17 / NUMBER 3 / MARCH, 1974
C         ADAPTATION OF TREESORT3 (ACM ALGORITHM 245 BY R. W. FLOYD)
      DIMENSION NUM(NO)
      N1        = NO
      L         = NO/2 + 1
 10   L         = L - 1
      IF(L .LE. 1)           GO TO 35
C                            SUBROUTINE SIFTUP
      I         = L
      NNNN      = NUM(I)
 20   J         = I + I
      IF(J .GT. N1)          GO TO 30
      IF(J .EQ. N1)          GO TO 25
      IF(NUM(J).LT.NUM(J+1)) J = J + 1
 25   IF(NUM(J).LE.NNNN    ) GO TO 30
      NUM(I)    = NUM(J)
      I         = J
      GO TO 20
 30   NUM(I)    = NNNN
      GO TO 10
 35   L         = N1 + 1
 40   L         = L - 1
      IF(L .LE. 1)           RETURN
C                            SUBROUTINE SIFTUP
      I         = 1
      NNNN      = NUM(1)
 45   J         = I + I
      IF(J .GT. L)           GO TO 60
      IF(J .EQ. L)           GO TO 55
      IF(NUM(J).LT.NUM(J+1)) J = J + 1
 55   IF(NUM(J).LE.NNNN    ) GO TO 60
      NUM(I)    = NUM(J)
      I         = J
      GO TO 45
 60   NUM(I)    = NNNN
      NNNN      = NUM(1)
      NUM(1)    = NUM(L)
      NUM(L)    = NNNN
      GO TO 40
C                                                          LISTING 15
      END
```

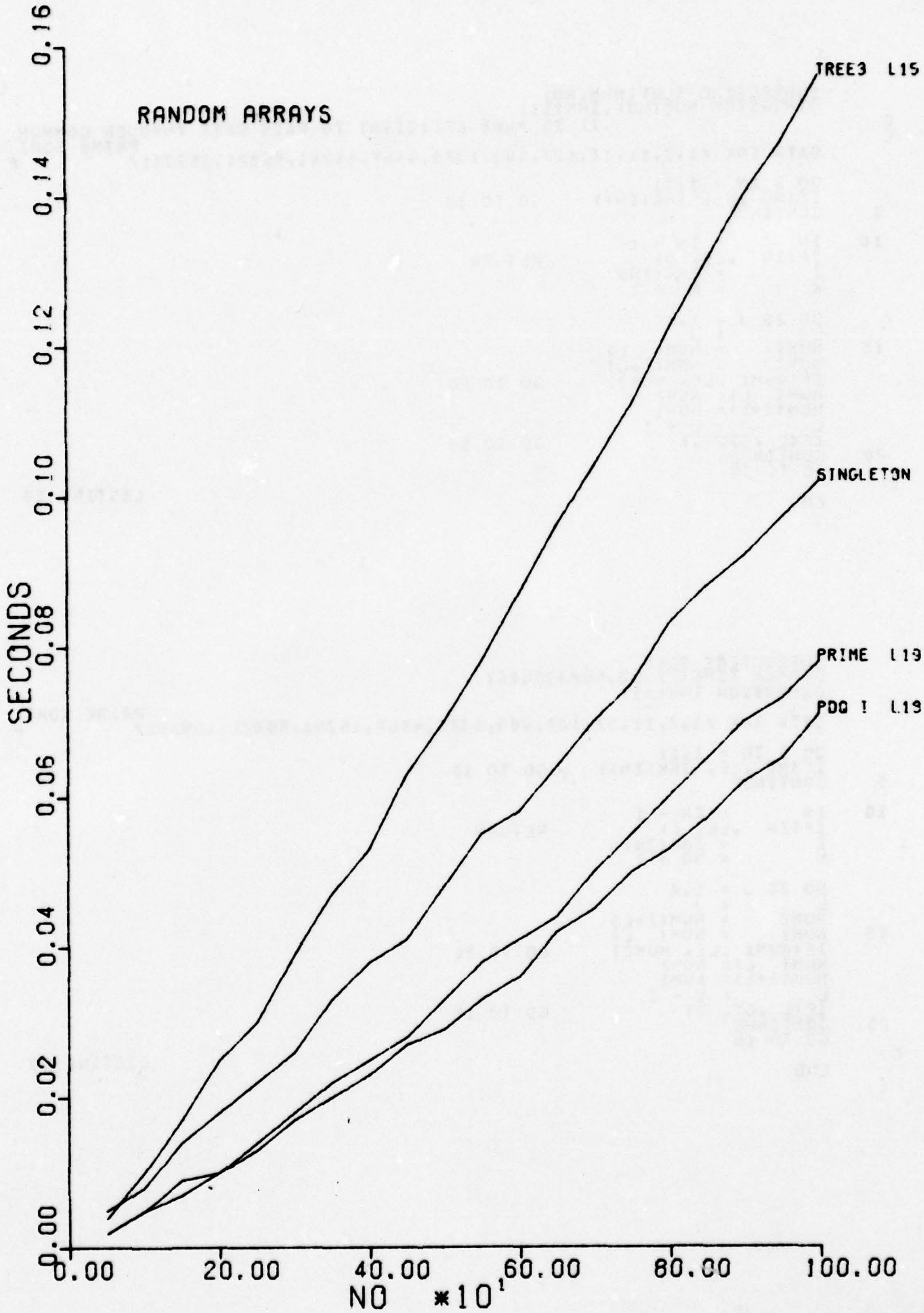
```
      SUBROUTINE SORT(NUM,NO)
C                               (ACM ALGORITHM 347) BY RICHARD C. SINGLETON
C     . COMMUNICATIONS OF THE ACM - VOLUME 12 / NUMBER 3 / MARCH, 1969
      DIMENSION IL(17),IU(17),NUM(NO)
      M         = 1
      I         = 1
      J         = NO
    5 IF(I .GE. J)              GO TO 70
   10 K         = I
      IJ        = (J+I)/2
      NT        = NUM(IJ)
      IF(NUM(I) .LE. NT)        GO TO 20
      NUM(IJ) = NUM(I )
      NUM(I ) = NT
      NT        = NUM(IJ)
   20 L         = J
      IF(NUM(J) .GE. NT)        GO TO 40
      NUM(IJ) = NUM(J )
      NUM(J ) = NT
      NT        = NUM(IJ)
      IF(NUM(I) .LE. NT)        GO TO 40
      NUM(IJ) = NUM(I )
      NUM(I ) = NT
      NT        = NUM(IJ)
                                GO TO 40
   30 NUM(L ) = NUM(K )
      NUM(K ) = NTT
   40 L         = L - 1
      IF(NUM(L) .GT. NT)        GO TO 40
      NTT       = NUM(L )
   50 K         = K + 1
      IF(NUM(K) .LT. NT)        GO TO 50
      IF(K .LE. L)              GO TO 30
      IF(L-I  .LE. J-K)         GO TO 60
      IL(M)     = I
      IU(M)     = L
      I         = K
      M         = M + 1
                                GO TO 80
   60 IL(M)     = K
      IU(M)     = J
      J         = L
      M         = M + 1
                                GO TO 80
   70 M         = M - 1
      IF(M .EQ. 0)              RETURN
      I         = IL(M)
      J         = IU(M)
   80 IF(J-I .GE. 1)            GO TO 10
      IF(I    .EQ. 1)           GO TO 5
      I         = I - 1
   90 I         = I + 1
      IF(I .EQ. J)              GO TO 70
      NT        = NUM(I+1)
      IF(NUM(I) .LE. NT)        GO TO 90
      K         = I
  100 NUM(K+1)= NUM(K )
      K         = K - 1
      IF(NT .LT. NUM(K))        GO TO 100
      NUM(K+1)= NT
                                GO TO 90
C                                                         LISTING 16
      END
```

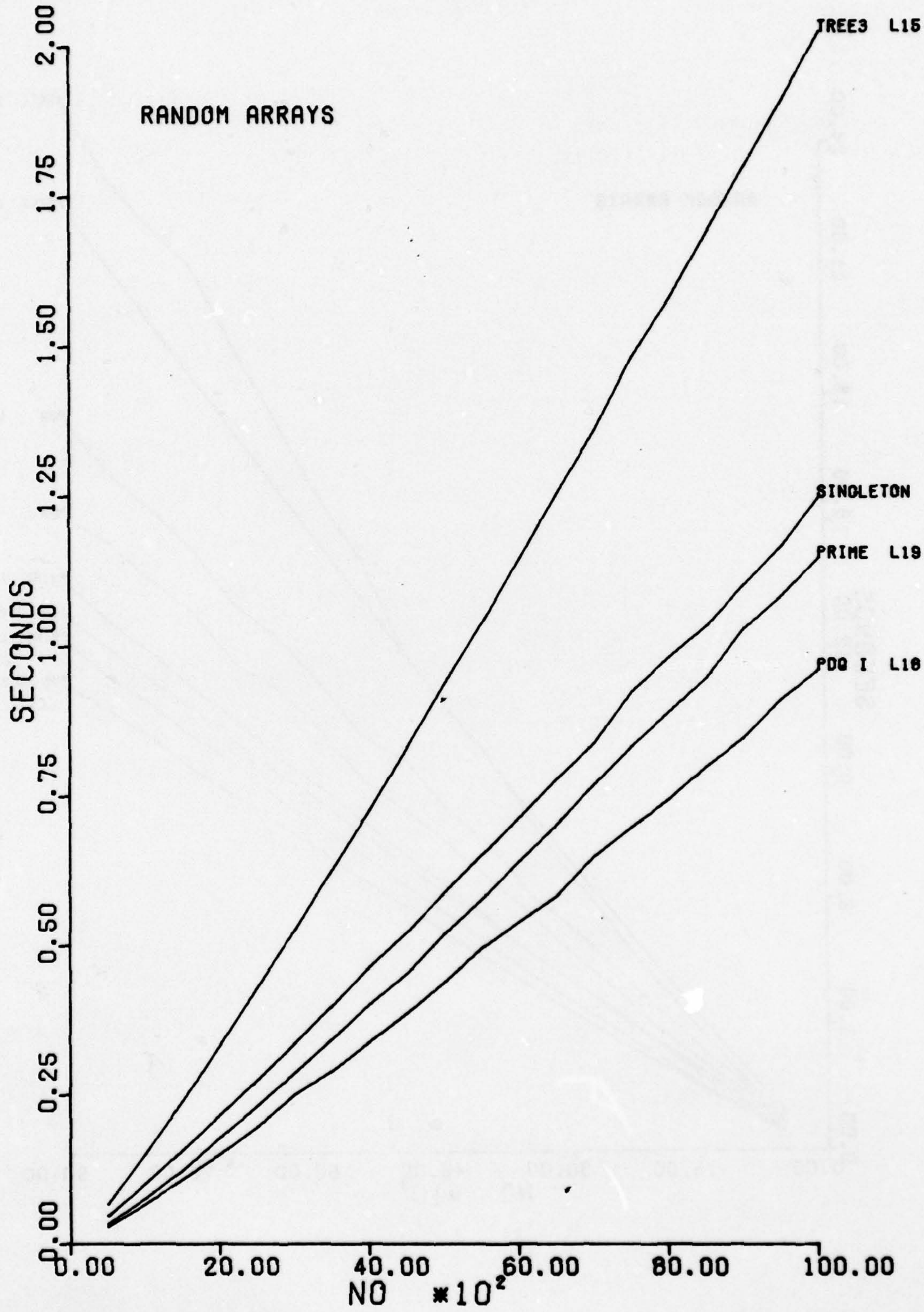
```
      SUBROUTINE SORT(NUM,NO)
      DIMENSION NUM(NO)
C                                                              PDQ #2
      A         = NO
 10   A         = A*.8
      I         = A
      IF(I  .LE. 0)          RETURN
      K         = NO - I
      DO 15 J = 1,K
      NUM1      = NUM(  J)
      NUM2      = NUM(I+J)
      IF(NUM1 .LT. NUM2)     GO TO 15
      NUM(  J)= NUM2
      NUM(I+J)= NUM1
 15   CONTINUE
      GO TO 10
C                                                         LISTING 17
      END
```

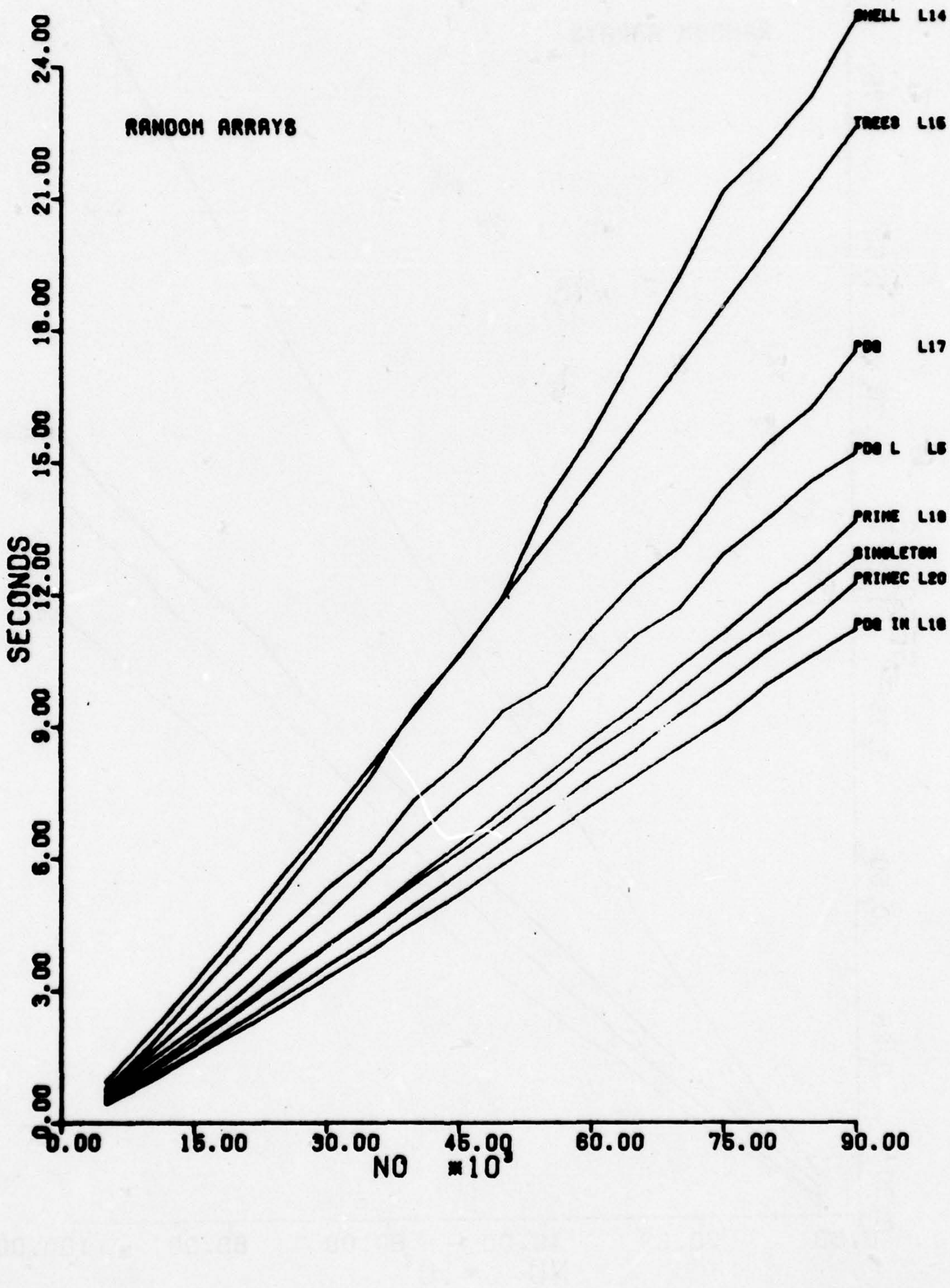
```
      SUBROUTINE SORT(NUM,NO)
      DIMENSION NUM(NO)
C                                                     PDQ INSERT #2
      A         = NO
      I         = NO
 10   IF(I .LE. 0)           GO TO 30
      A         = A*.715
      I         = A
      IF( I.GT.9 .AND. MOD(I,2).EQ.0)  I = I + 1
      K         = NO - I
      DO 15 J = 1,K
      NUM1      = NUM(  J)
      NUM2      = NUM(I+J)
      IF(NUM1 .LT. NUM2)     GO TO 15
      NUM(  J)= NUM2
      NUM(I+J)= NUM1
 15   CONTINUE
      GO TO 10
 30   K         = NO - 1
      DO 40 J = 1,K
      L         = J
      NUM2      = NUM(L+1)
 35   NUM1      = NUM(L  )
      IF(NUM1 .LE. NUM2)     GO TO 40
      NUM(L  )= NUM2
      NUM(L+1)= NUM1
      L         = L - 1
      IF(L .GT. 0)           GO TO 35
 40   CONTINUE
      RETURN
C                            ENHANCEMENTS:
C                            ALTERNATE COMPARISONS
C             IF THE ARRAY NUM IS IMMEDIATELY PRECEEDED BY THE SMALLEST
C                   NUMBER EXPRESSABLE ON THE MACHINE, THAN THE STATEMENT
C                       IF(L.GT.0) GO TO 35 CAN BE REPLACED BY:  GO TO 35
C                                                         LISTING 18
      END
```

```
      SUBROUTINE SORT(NUM,NO)
      DIMENSION NUM(NO),INK(11)
C                          IT IS MORE EFFICIENT TO PASS DATA THROUGH COMMON
C                                                                PRIME SORT
      DATA INK /1,3,11,37,127,409,1373,4567,15241,50821,169351/        F = .30
      DO 5 IN = 1,11
      IF(NO .LE. INK(IN))      GO TO 10
 5    CONTINUE
 10   IN        = IN - 1
      IF(IN  .LE. 0)           RETURN
      I         = INK(IN)
      K         = NO - I
      DO 20 J = 1,K
      L         = J
 15   NUM1      = NUM(  L)
      NUM2      = NUM(I+L)
      IF(NUM1 .LE. NUM2)       GO TO 20
      NUM(  L)= NUM2
      NUM(I+L)= NUM1
      L         = L - I
      IF(L .GT. 0)             GO TO 15
 20   CONTINUE
      GO TO 10
C                                                                LISTING 19
      END
```

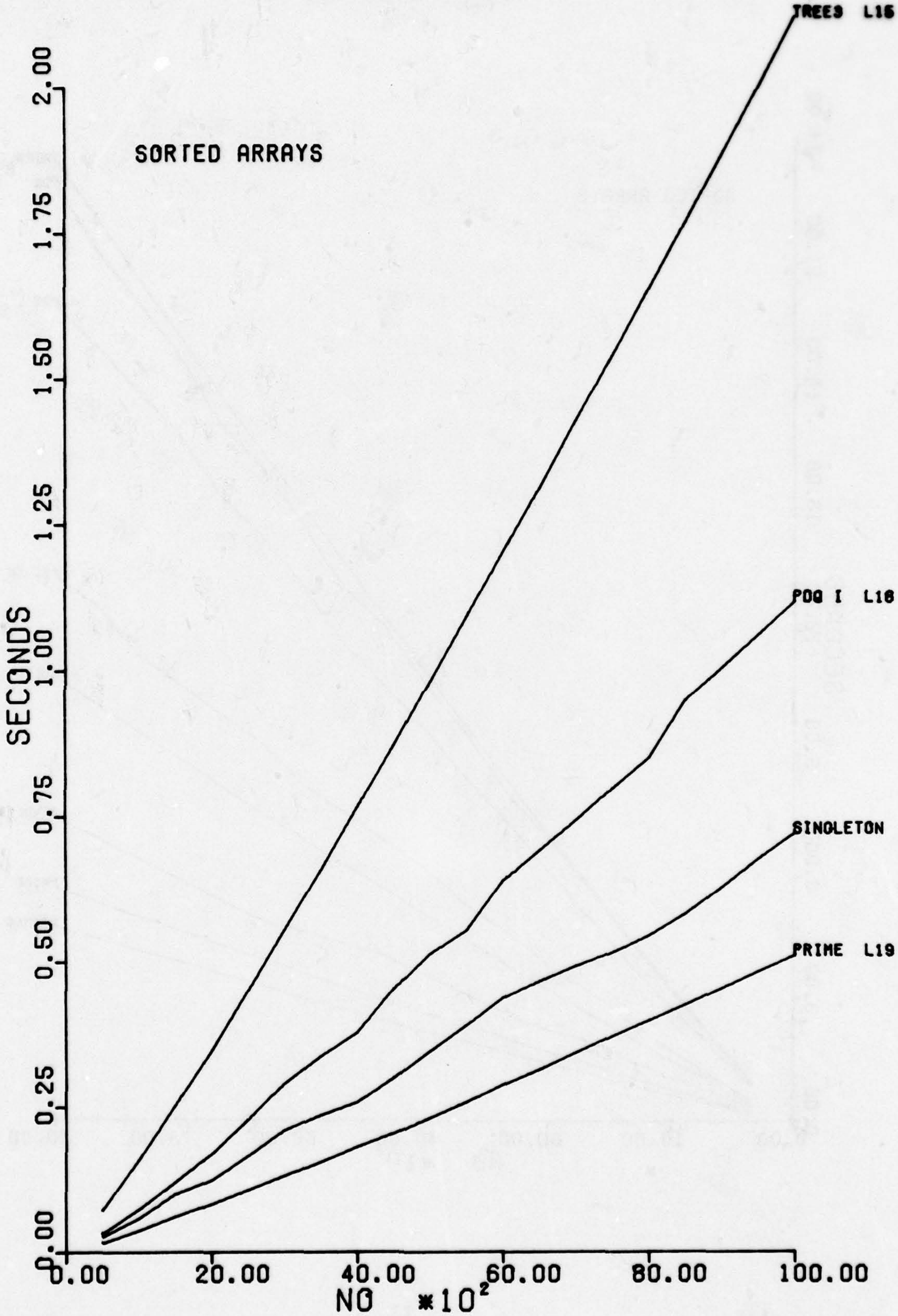
```
      SUBROUTINE SORT
      COMMON TIME(7),NO,NUM(90000)
      DIMENSION INK(11)
C                                                                PRIME SORT
      DATA INK /1,3,11,37,127,409,1373,4567,15241,50821,169351/        F = .30
      DO 5 IN = 1,11
      IF(NO .LE. INK(IN))      GO TO 10
 5    CONTINUE
 10   IN        = IN - 1
      IF(IN  .LE. 0)           RETURN
      I         = INK(IN)
      K         = NO - I
      DO 20 J = 1,K
      L         = J
      NUM2      = NUM(I+L)
 15   NUM1      = NUM(  L)
      IF(NUM1 .LE. NUM2)       GO TO 20
      NUM(  L)= NUM2
      NUM(I+L)= NUM1
      L         = L - I
      IF(L .GT. 0)             GO TO 15
 20   CONTINUE
      GO TO 10
C                                                                LISTING 20
      END
```

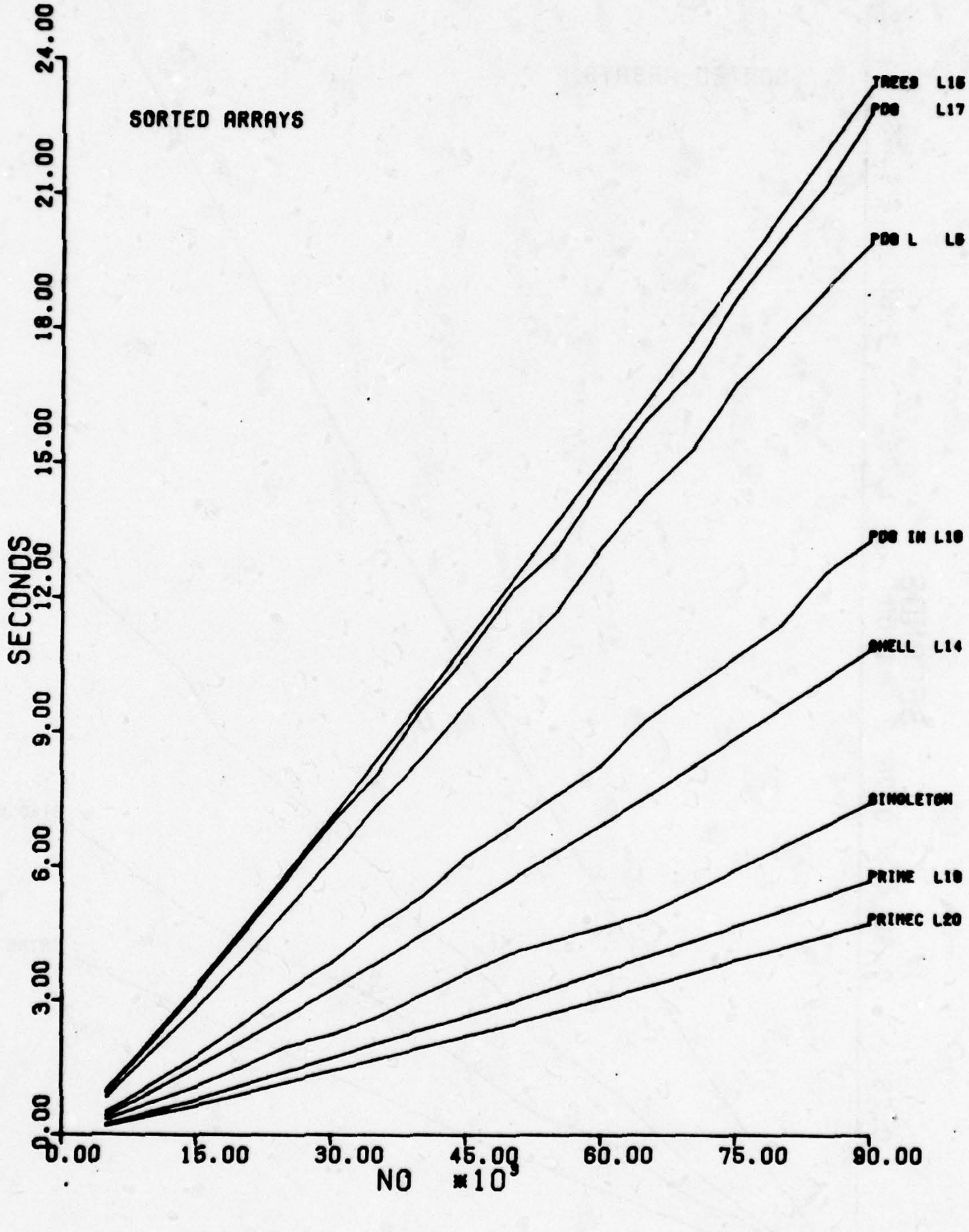

RANDOM ARRAYS
TREE3 L15
SINGLETON
PRIME L19
PDQ I L19
SECONDS
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.00
20.00
40.00
60.00
80.00
100.00
NO *10[1]

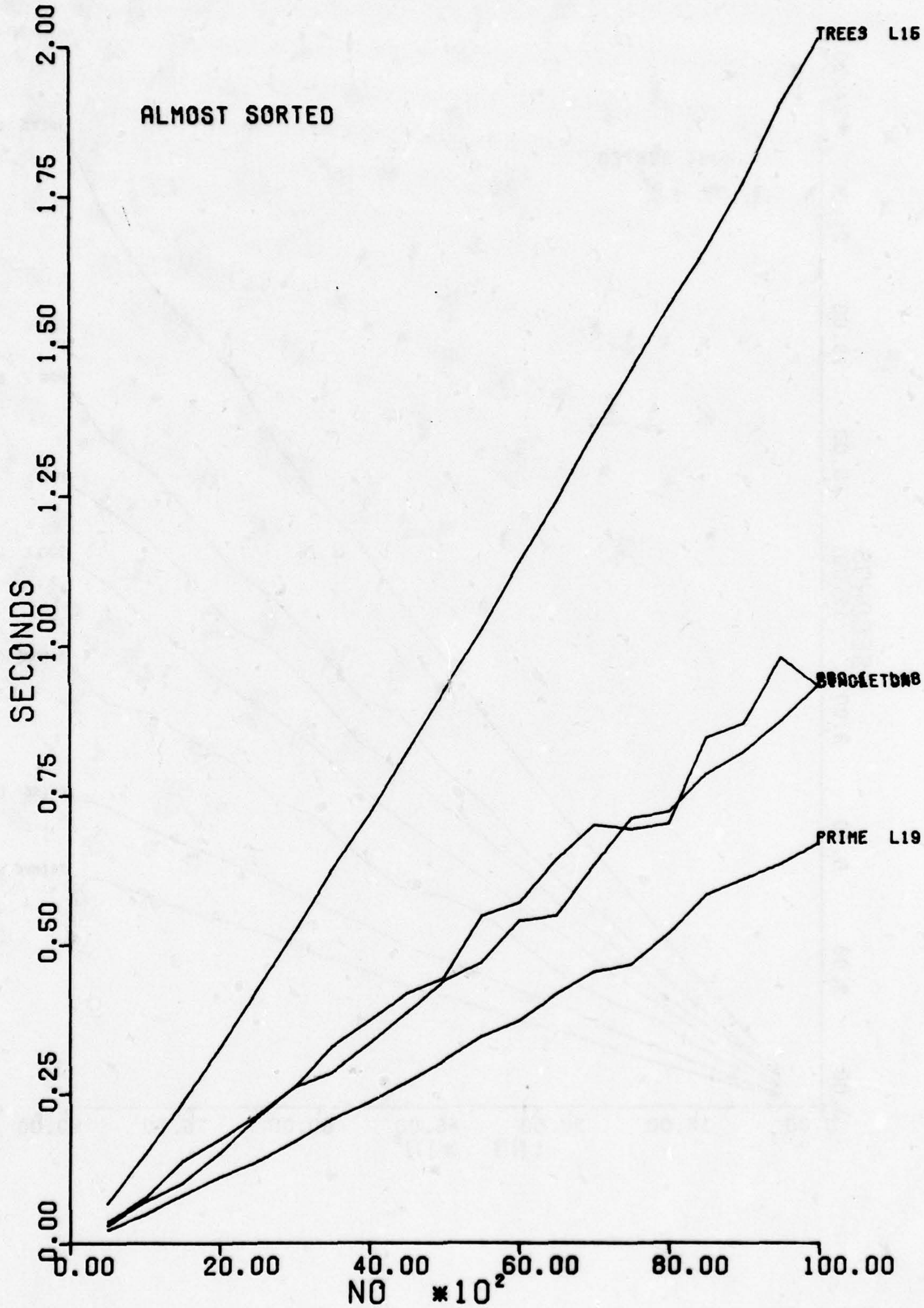

RANDOM ARRAYS
TREE3 L15
SINGLETON
PRIME L19
PDQ I L18
SECONDS
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
0.00
20.00
40.00
60.00
80.00
100.00
NO *10²

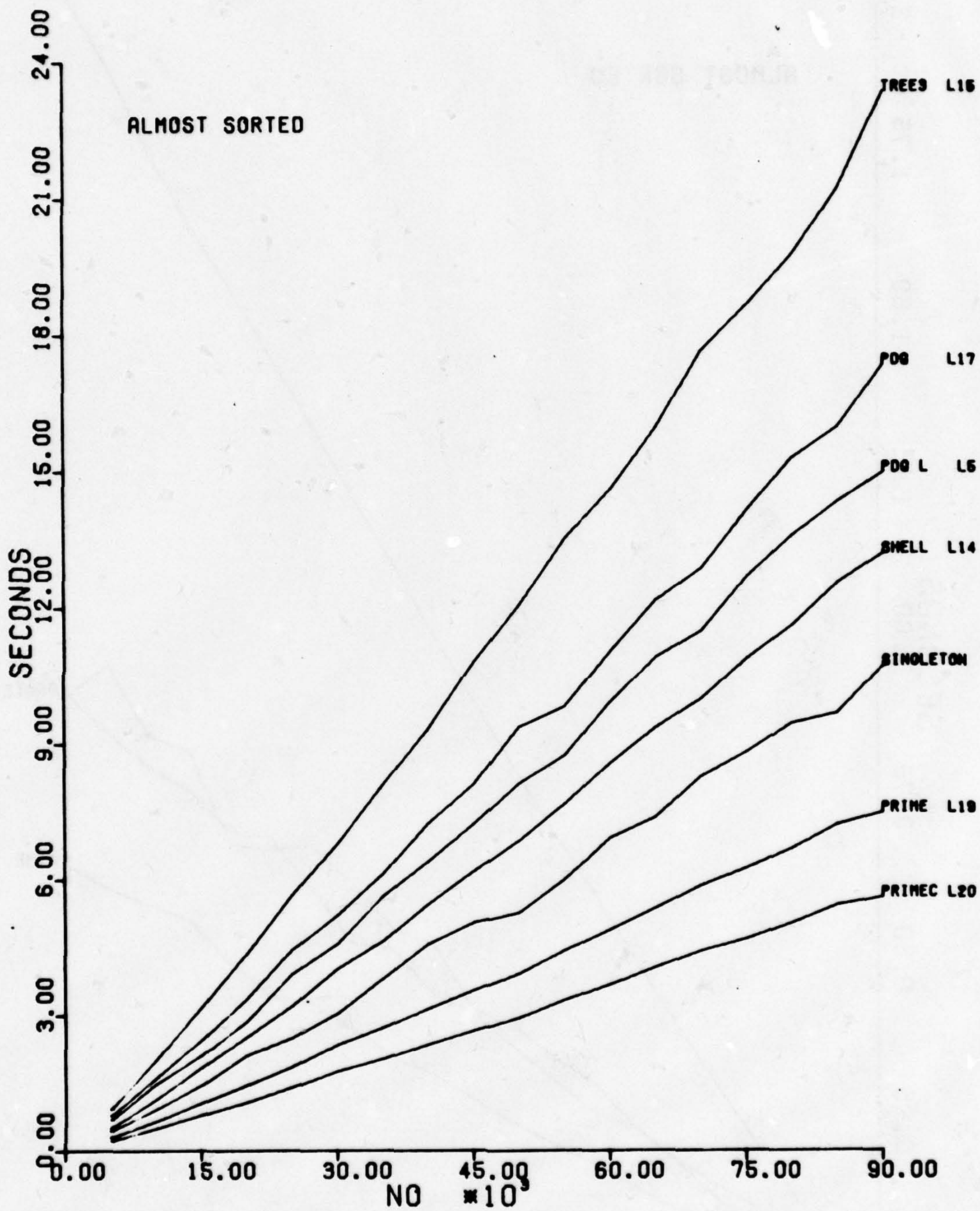


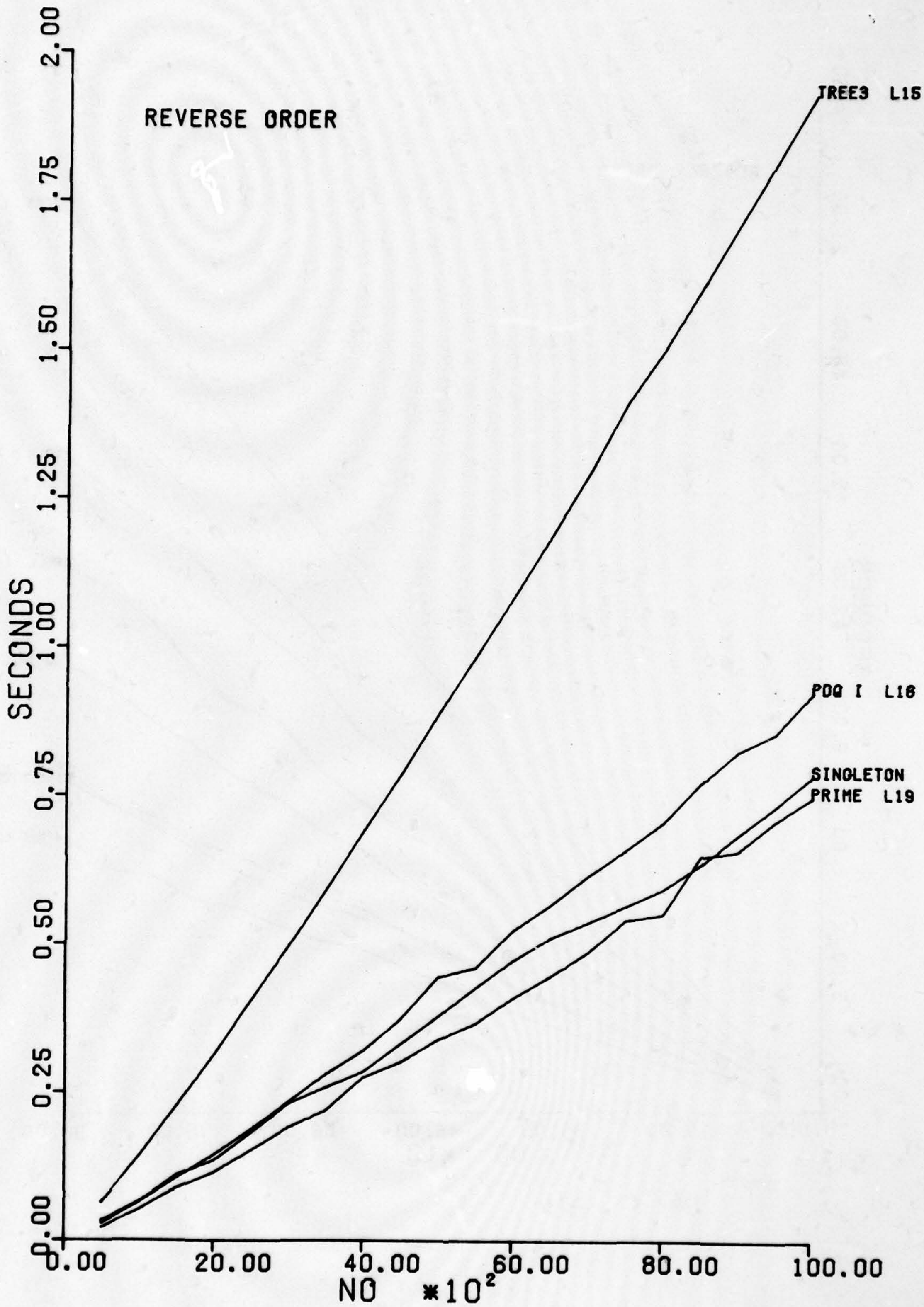
RANDOM ARRAYS
SHELL L14
TREES L16
PDQ L17
PDQ L L5
PRIME L19
SINGLETON
PRIMEC L20
PDQ IN L18
SECONDS
0.00
3.00
6.00
9.00
12.00
15.00
18.00
21.00
24.00
0.00
15.00
30.00
45.00
60.00
75.00
90.00
NO *10

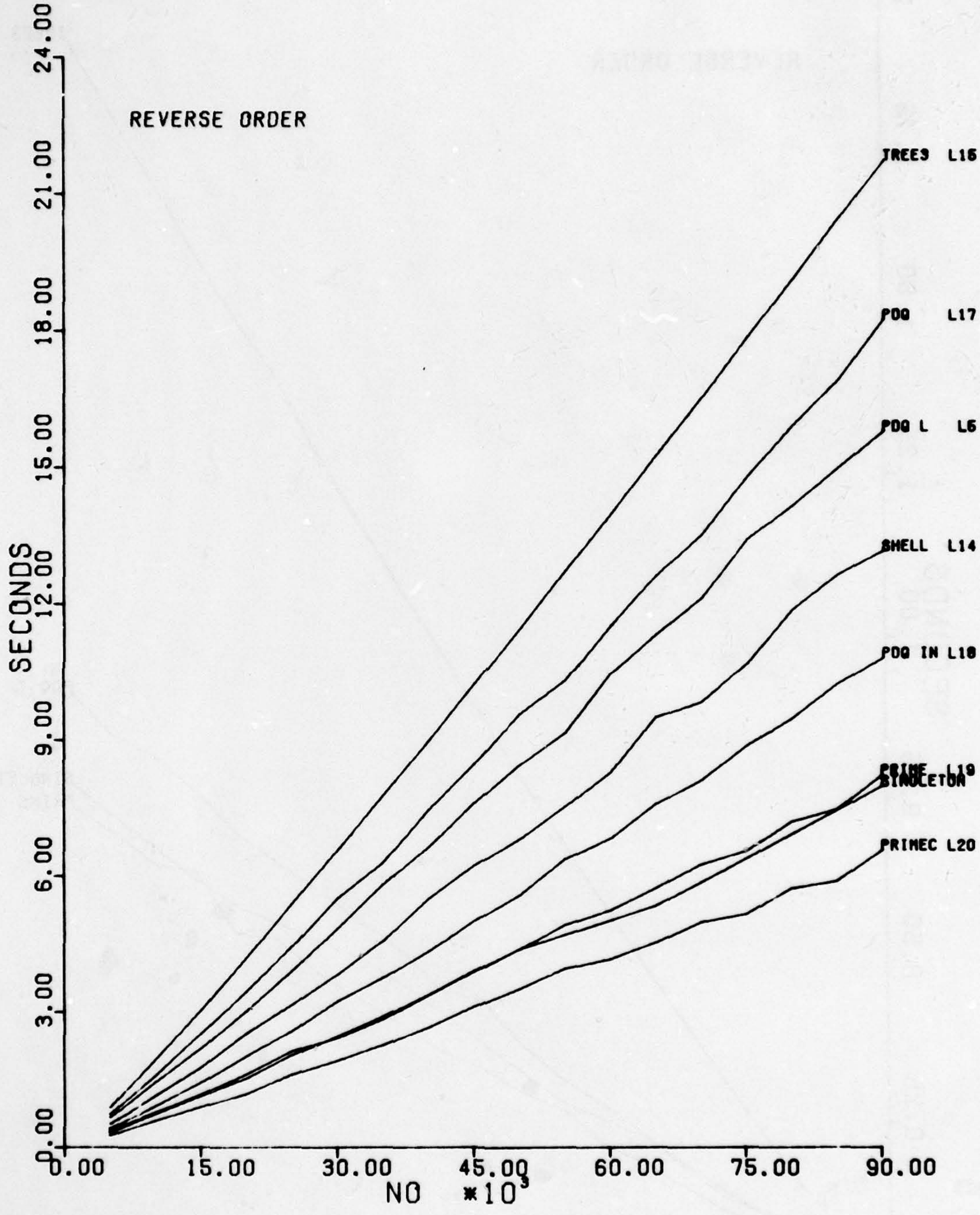

SORTED ARRAYS
TREES L15
PDQ I L16
SINGLETON
PRIME L19
SECONDS
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.00
20.00
40.00
60.00
80.00
100.00
NO *10²


SORTED ARRAYS
SECONDS
0.00
3.00
6.00
9.00
12.00
15.00
18.00
21.00
24.00
0.00
15.00
30.00
45.00
60.00
75.00
90.00
NO *10
TREES L15
PDG L17
PDG L L5
PDG IN L19
SHELL L14
SINGLETON
PRIME L19
PRIMEC L20


TREES L16
ALMOST SORTED
PRIME L19
SECONDS
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
0.00
20.00
40.00
60.00
80.00
100.00
NO *10²


ALMOST SORTED
SECONDS
0.00
3.00
6.00
9.00
12.00
15.00
18.00
21.00
24.00
0.00
15.00
30.00
45.00
60.00
75.00
90.00
NO *10³
TREES L16
PDG L17
PDG L L5
SHELL L14
SINGLETON
PRIME L19
PRIMEC L20


REVERSE ORDER
TREE3 L15
PDQ I L16
SINGLETON
PRIME L19
SECONDS
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
0.00
20.00
40.00
60.00
80.00
100.00
NO *10²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PRIMEC L20
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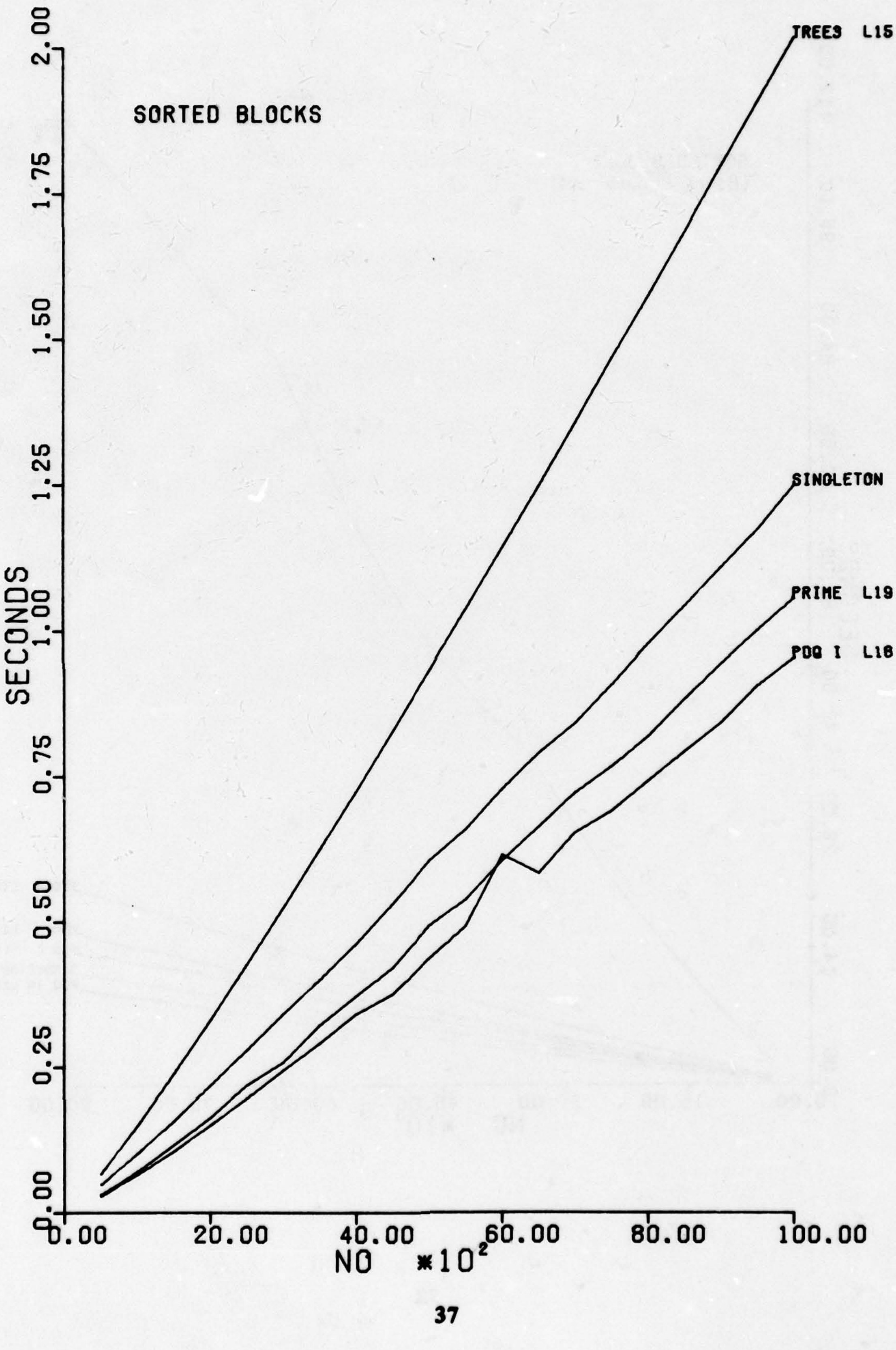
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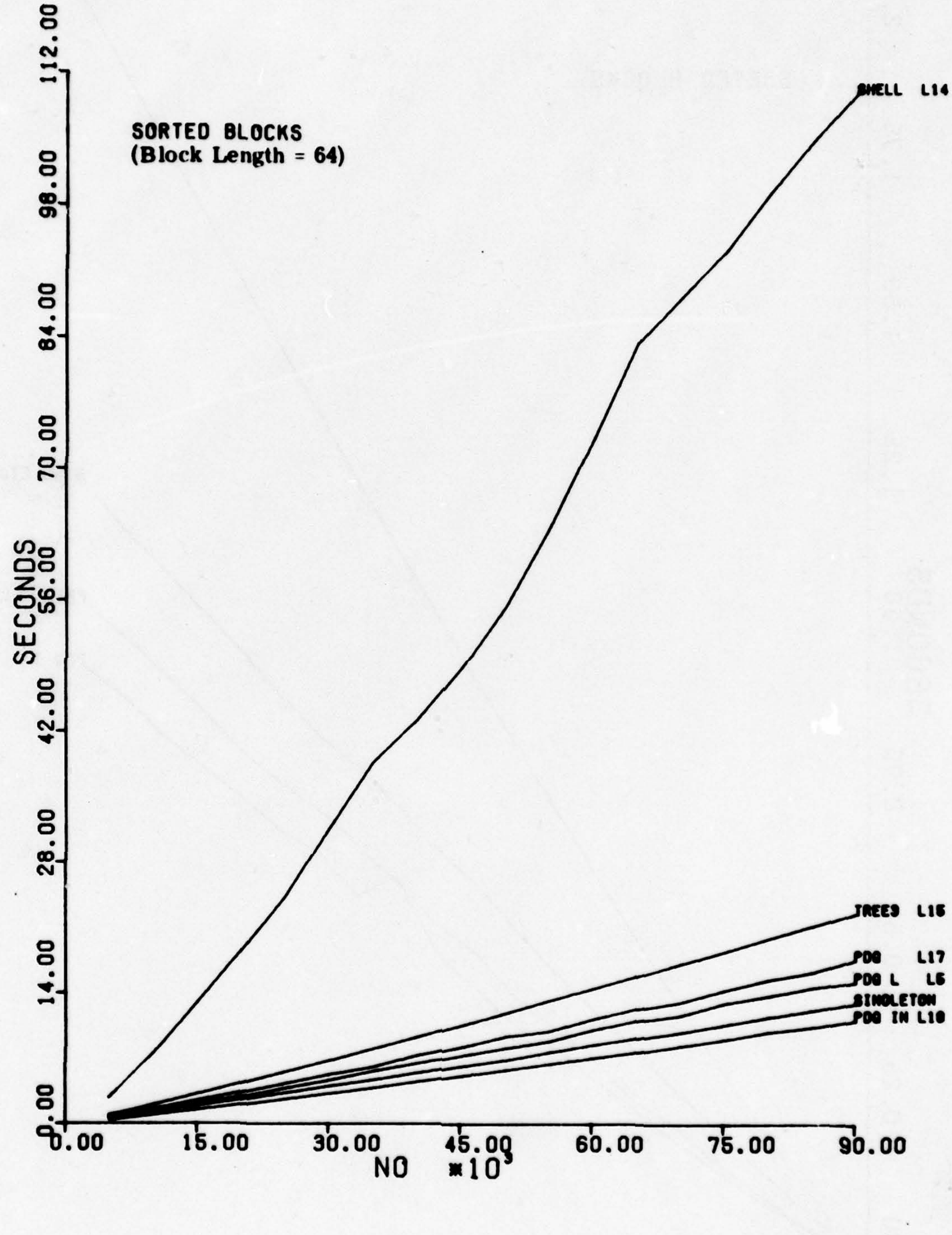
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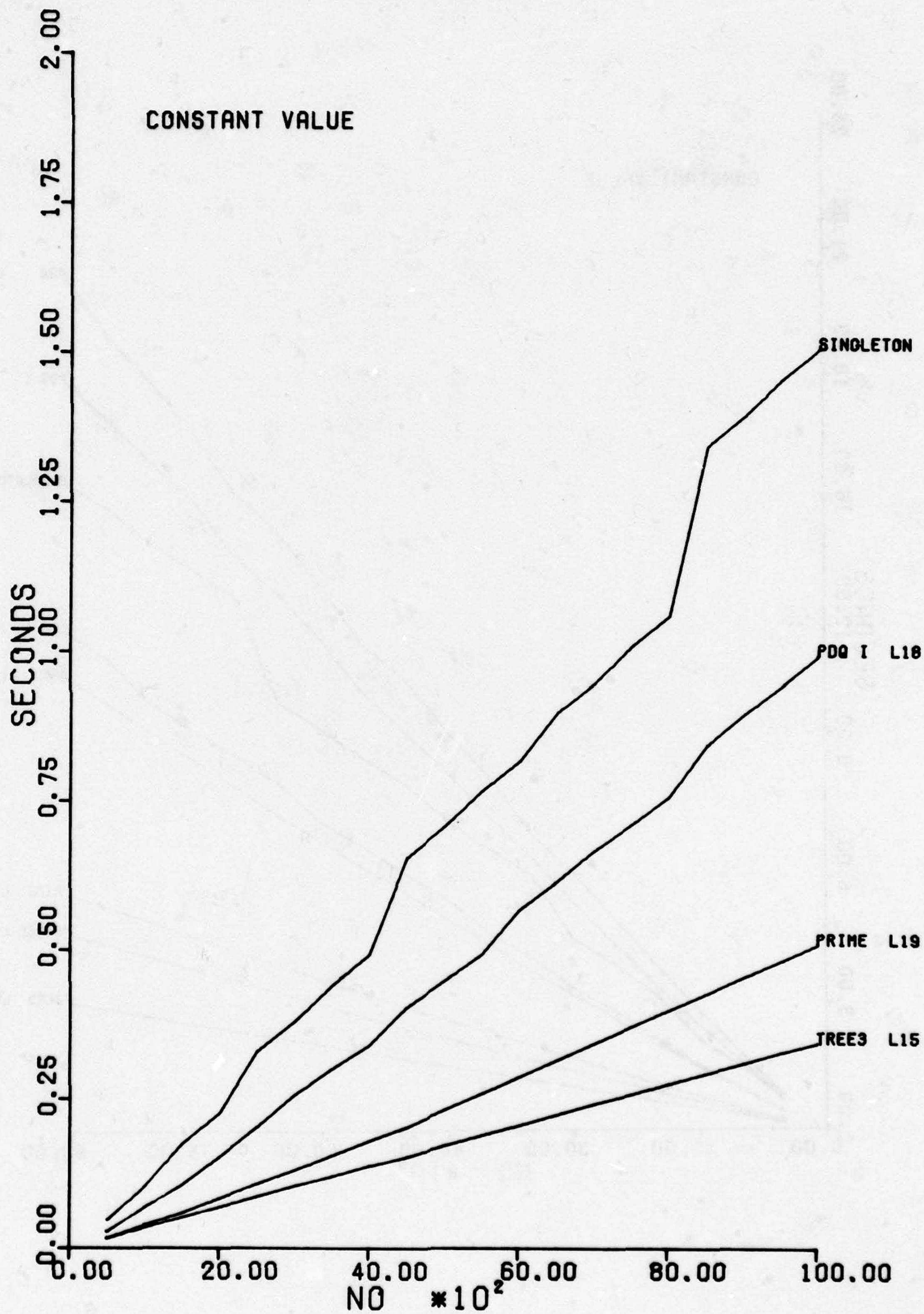
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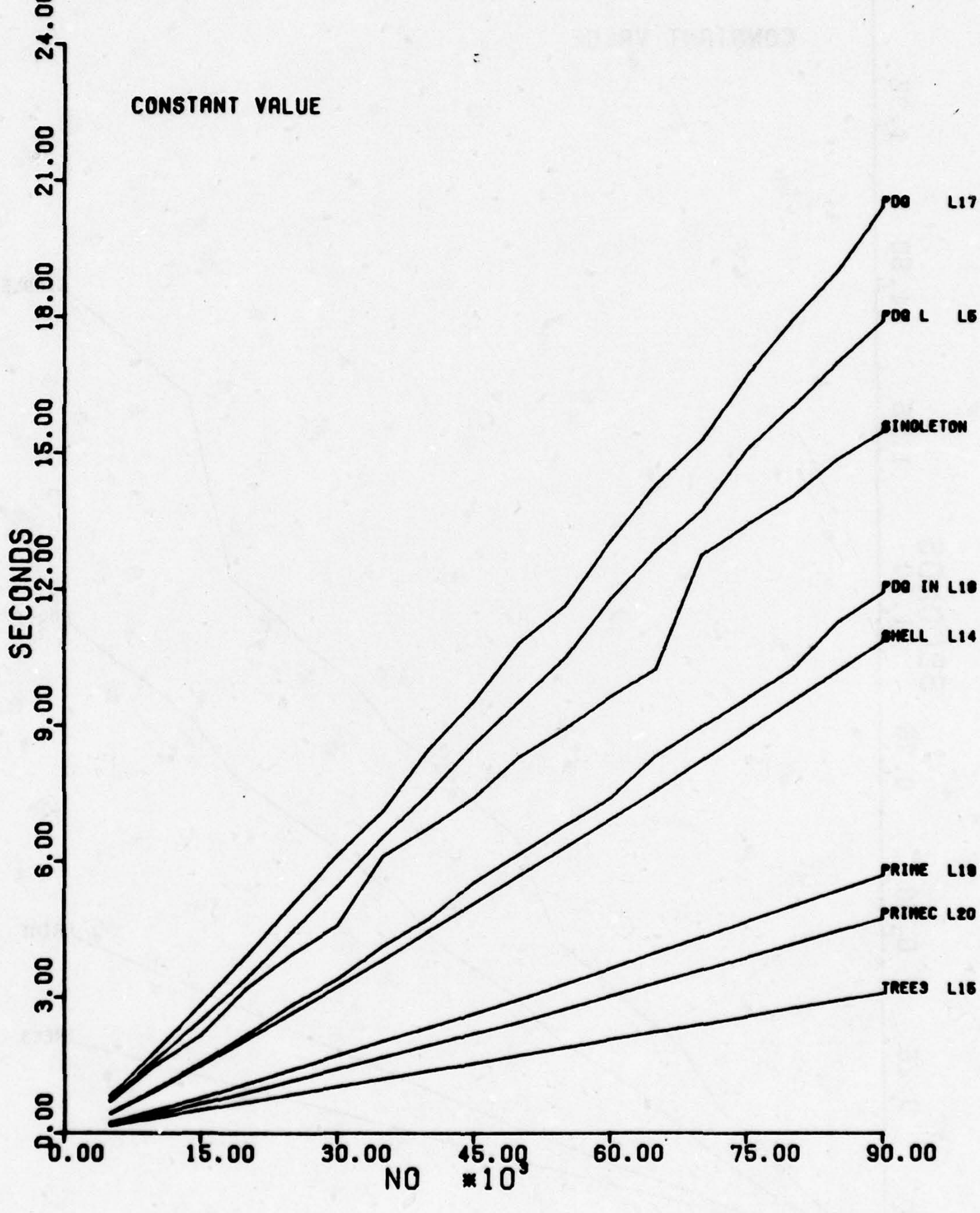
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One of the problems involved in computer programs for vulnerability assessment is that of rapidly sorting and arranging large sets of data. Two sorting algorithms, designated PRIME and PDQ, have been developed at ASD to more efficiently perform this function in vulnerability programs such

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