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CIRCUIT ANALYSIS OF A HIGH Q SUPERCONDUCTING HF NOTCH FILTER.(U)

JUL 77 J J HUDAK

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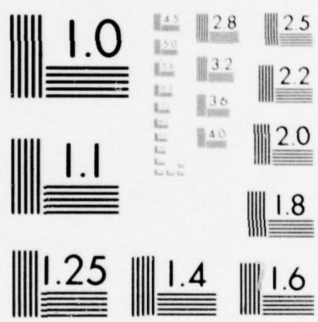

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CIRCUIT ANALYSIS
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I. INTRODUCTION

This technical memorandum presents an analysis of the high frequency (HF) notch filter circuit used by Cutler Hammer Airborne Instruments Lab (AIL). This analysis leads to a set of equations which can be used to determine the values of the circuit components needed to achieve a specific notch depth and bandwidth in the filter's frequency response. These design equations are then interpreted geometrically in terms of the location of the poles and zeros in the S-plane. Several plots of the filter's frequency response as a function of critical circuit parameters are also presented.

II. CIRCUIT THEORY

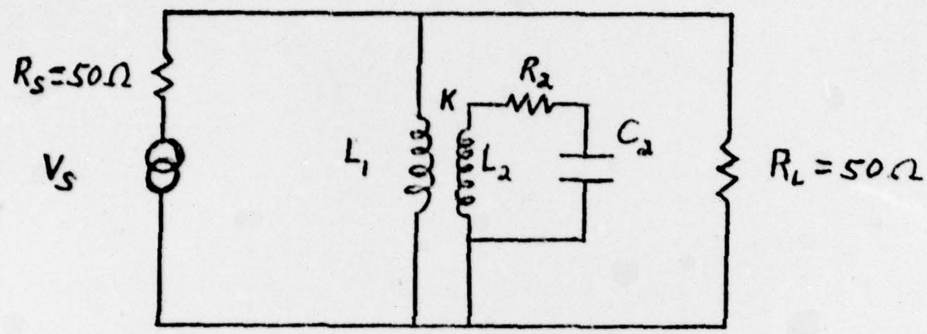
A. General

The present work deals mainly with analyzing the AIL filter and investigating those parameters that are critical to its performance. During the course of the work two other types of notch filter circuits were studied in an attempt to find if other circuit types might offer comparable or improved performance characteristics. These other circuits were a series LC circuit placed in parallel with the load of the receiver and a parallel LC circuit placed in series with the load of the receiver. It was found that these circuit types did not offer the many advantages of the AIL filter circuit.

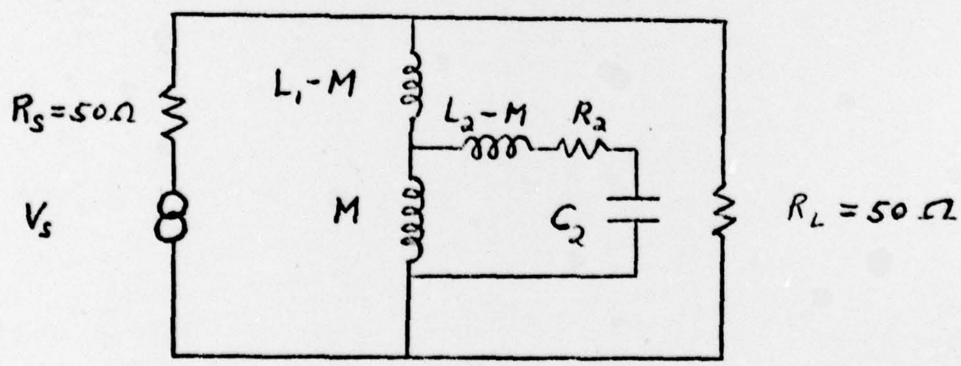
B. Analysis of the AIL Circuit

The circuit used in the AIL notch filter is shown in Figure 1. The filter is an inductively coupled superconducting tank circuit which is placed in parallel with the receiver. The primary and secondary inductances and the capacitor are all superconducting circuit elements. For a certain narrow band of frequencies the impedance of this branch becomes very small with respect to the load thereby reducing the signal strength at the receiver.

The equivalent circuit is also shown in Figure 1. M is the mutual inductance defined by $M = k\sqrt{L_1 L_2}$, where k is the coupling constant between the primary and secondary inductances, L_1 and L_2, respectively. We are interested in calculating the insertion loss caused by the superconducting branch of the circuit. The insertion loss (I.L.) at a given frequency caused by the insertion of the branch is the ratio, expressed in decibels, of the powers delivered to that part of the



AIL Filter Circuit



Equivalent Circuit

Fig. 1

circuit beyond the point of insertion before and after the insertion[1]. The insertion loss includes the effect of mismatch as well as attenuation. However, these effects will not be taken into account in the analysis presented here. They would become important when several filters are used in parallel.

The I.L. in this model is defined as

$$I.L. = 20 \log \frac{V_f}{V_o} ,$$

where V_o and V_f are defined in Figure 2a. We define the ratio V_f/V_o to be the transfer function (T.F.). From Figure 2a we see that T.F. = $Z_f/(Z_f + 25)$. The impedance of the filter, Z_f, must now be calculated using Figure 2b. The impedance is given by $Z_f = V_f/I_f$.

The loop equations are,

$$V_f = j\omega L_1 I_f + j\omega M I_2 \qquad \text{Eq. 1}$$

$$V_2 = j\omega M I_f + (R_2 + j\omega L_2) I_2 \qquad \text{Eq. 2}$$

and

$$V_2 = j \frac{I_2}{\omega C} .$$

Substituting in and solving for I_f in Eq. 2 we can use this result in Eq. 1 to obtain,

$$V_f = j\omega L_1 I_f + \frac{\omega^2 M^2 I_1}{R_2 + j\left(\omega L_2 - \frac{1}{\omega C}\right)} .$$

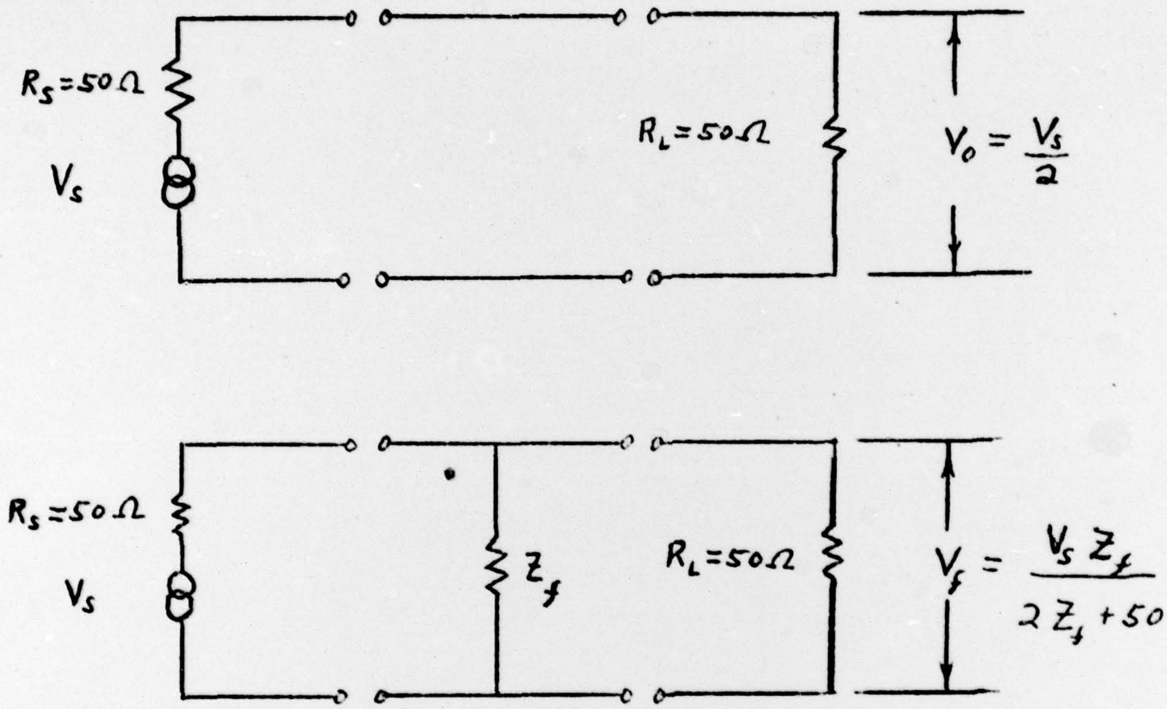
Now

$$Z_f = \frac{V_f}{I_f} = \frac{j\omega L_1 R_2 - \omega^2 L_2 L_1 \left[(1-k^2) - \frac{\omega_0^2}{\omega^2}\right]}{R_2 + j\omega L_2 \left(1 - \frac{\omega_0^2}{\omega^2}\right)} ,$$

where $M^2 = k^2 L_1 L_2$ was used. Using $S = j\omega$ we have

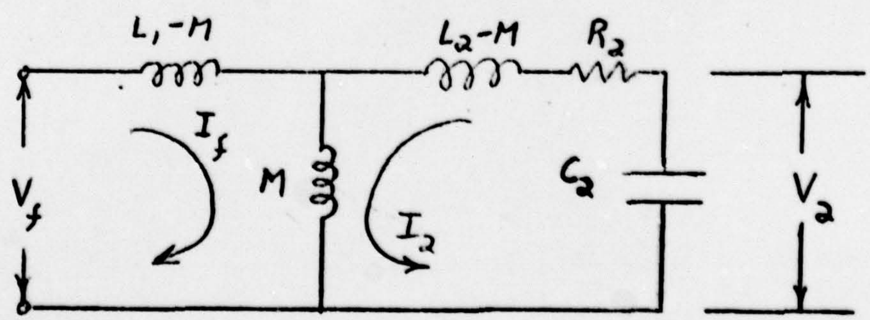
$$Z_f = L_1 S \frac{N(S)}{D(S)} = L_1 S \frac{L_2 (1-k^2) S^2 + R_2 S + L_2 \omega_0^2}{L_2 S^2 + R_2 S + L_2 \omega_0^2} .$$

Fig. 2a



$$\therefore \ I.L. = 20 \log \frac{Z_f}{Z_f + 25}$$

Fig. 2b



The transfer function becomes,

$$T.F. \equiv H(S) = \frac{Z_f}{Z_f + R_c} \quad ; \quad R_c = 25\ \Omega.$$

and

$$H(S) = \frac{NS}{NS + \frac{R_c}{L_1} D} .$$

In the S plane, the zeros and poles of H(S) are the roots of its numerator and denominator, respectively. If the zeros are given by H_o^+ and H_o^- and the poles are given by H_p^+, H_p^- and r, then H(S) can be written,

$$H(S) = |\underset{\sim}{S}| \frac{|\underset{\sim}{S} - \underset{\sim}{H}_o^+|\,|\underset{\sim}{S} - \underset{\sim}{H}_o^-|}{|\underset{\sim}{S} - \underset{\sim}{r}|\,|\underset{\sim}{S} - \underset{\sim}{H}_p^+|\,|\underset{\sim}{S} - \underset{\sim}{H}_p^-|} \quad ; \qquad \text{Eq. 3}$$

where the corresponding vectors in the S plane are used. The zeros of H(S) are quickly found by setting the numerator of H(S) equal to zero and solving for S. The result is

$$H_o^{\pm} = \frac{-\omega_o}{2Q(1-k^2)} \pm j \frac{\omega_o}{(1-k^2)^{1/2}} \left[1 - \frac{1}{4Q^2(1-k^2)}\right]^{1/2} \qquad \text{Eq. 4}$$

where

$$\omega_o = \frac{1}{\sqrt{L_2 C}} \quad \text{and} \quad Q = \frac{\omega_o L_2}{R_2} .$$

Poles of H(S) are found by setting the denominator to zero and solving for S,

$$(1-k^2) S^3 + \left(\frac{R_2}{L_2} + \frac{R_c}{L_1}\right) S^2 + \left(\frac{R_c R_2}{L_1 L_2} + \frac{1}{L_2 C}\right) S + \left(\frac{R_c}{L_1}\right) \frac{1}{L_2 C} = 0$$

Eq. 5

One root may be approximated in the following manner. The values of the coefficients can be found from the approximate values of the variables, $k \sim 0.1$, $R_c = 25\,\Omega$, $L_1 = 10\,\mu H$, $L_2 = 20\,\mu H$, $R_2 = 10\,m\Omega$ and $C = 10\,pf$.

We have,

$$0.99 S^3 + 5\times10^6 S^2 + (10^{10} + 10^{16}) S + \left(\frac{R_c}{L_1}\right) 10^{16} = 0 .$$

Therefore for small S ($S \sim 10^3$) we have an approximate root $S = r \sim -\dfrac{R_c}{L_1}$.

We can now symbolically divide this root out of Eq. 5,

$$\begin{array}{rl}
 & aS^2 + (b+ra)S + [c + r(b+ra)] \\
S - r \;\big) & aS^3 + bS^2 + cS + d \\
 & \underline{aS^3 - raS^2} \\
 & (b+ra)S^2 + cS \\
 & \underline{(b+ra)S^2 - r(b+ra)S} \\
 & [c - r(b+ra)]S + d \\
 & \underline{[c - r(b+ra)]S - r[c + r(b+ra)]} \\
 & d + r[c + r(b+ra)]
\end{array}$$

Eq. 5

Note that the remainder is approximately zero. The remaining quadratic becomes,

$$(1-k^2)S^2 + \left(\frac{R_2}{L_2} + k^2\frac{R_c}{L_1}\right)S + \frac{1}{L_2 C} + \frac{k^2 R_c^2}{L_1^2} = 0 .$$

The remaining poles are therefore,

$$H_p^{\pm} = \frac{-\omega_o}{2Q(1-k^2)} - \frac{k^2 R_c}{2L_1(1-k^2)} \pm j\frac{\omega_o}{(1-k^2)^{1/2}}\left[1 + \frac{k^2 R_c^2}{\omega_o^2 L_1^2} - \frac{1}{4(1-k^2)^2}\left(\frac{1}{Q} + \frac{k^2 R_c}{\omega_o L_1}\right)^2\right]^{1/2}$$

Eq. 6

The expressions for $H_o^{\pm}$ and $H_p^{\pm}$ may be simplified. By substituting in the approximate values of the variables it is found that the terms in brackets in Eqs. 4 and 6 are essentially unity. Neglecting these small contributions results in the following excellent approximations,

$$H_o^{\pm} \simeq \frac{-\omega_o}{2Q(1-k^2)} \pm j\,\frac{\omega_o}{(1-k^2)^{1/2}} \qquad \text{Eq. 7}$$

and

$$H_p^{\pm} \simeq \frac{-\omega_o}{2Q(1-k^2)} - \frac{k^2 R_c}{2L_1(1-k^2)} \pm j\,\frac{\omega_o}{(1-k^2)^{1/2}} \,. \qquad \text{Eq. 8}$$

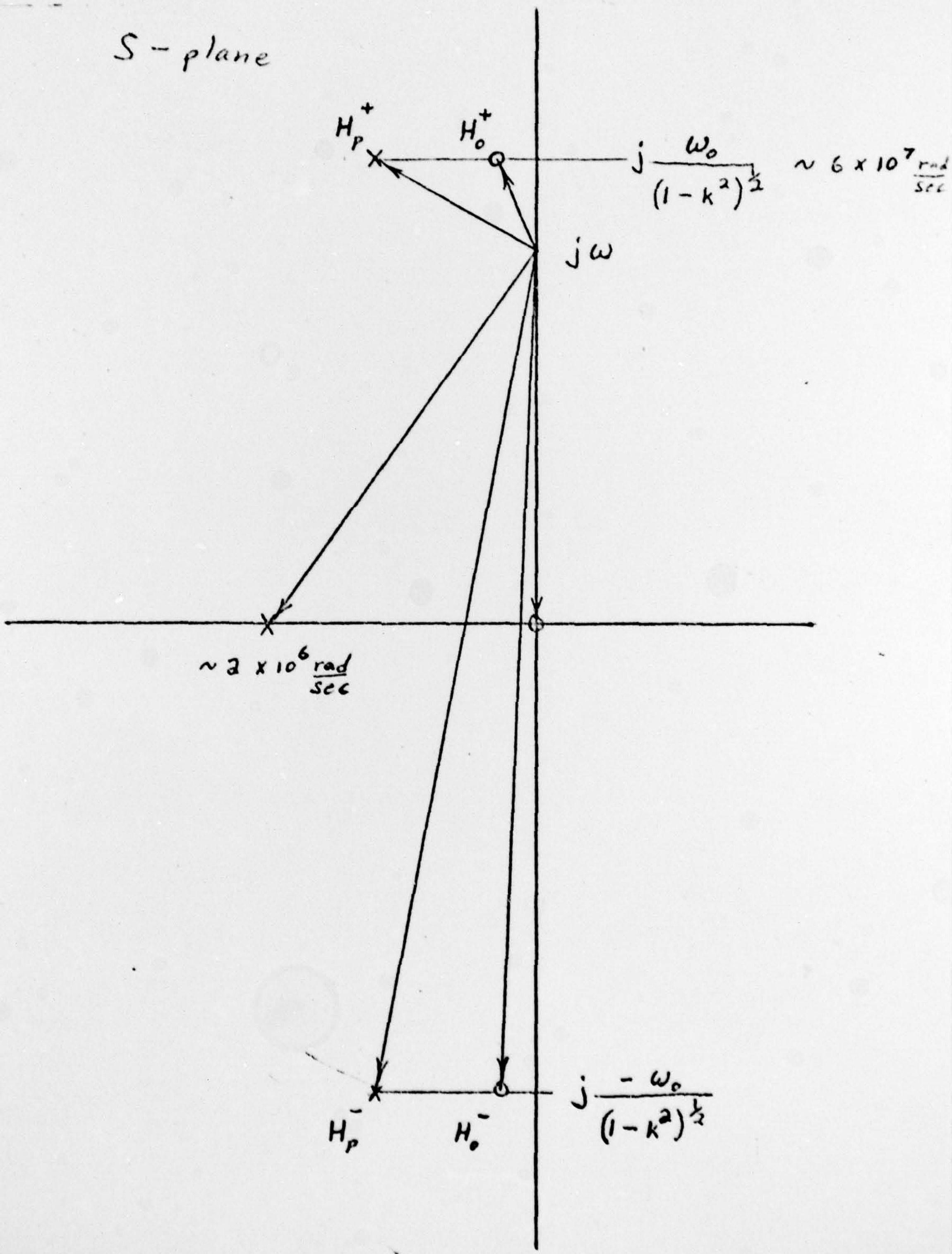
These may now be substituted into Eq. 3. This vector interpretation of the transfer function is shown in Fig. 3. In the frequency band of interest, 2 - 30 MHz, and in the vicinity of the stop band of the notch filter only two terms dominate the value of the transfer function and it may be approximated by

$$H(s) \simeq \frac{|\underset{\sim}{s} - \underset{\sim}{H}_o^{+}|}{|\underset{\sim}{s} - \underset{\sim}{H}_p^{+}|} \,.$$

Therefore,

$$I.L. \simeq 20 \log \frac{|\underset{\sim}{s} - \underset{\sim}{H}_o^{+}|}{|\underset{\sim}{s} - \underset{\sim}{H}_p^{+}|} \,. \qquad \text{Eq. 9}$$

Fig. 3



S - plane
H_p^+
H_o^+
$j \frac{\omega_o}{(1-k^2)^{\frac{1}{2}}}$
$\sim 6 \times 10^7 \frac{rad}{sec}$
$j\omega$
$\sim 2 \times 10^6 \frac{rad}{sec}$
$j \frac{-\omega_o}{(1-k^2)^{\frac{1}{2}}}$
H_p^-
H_o^-

C. THE DESIGN EQUATIONS

Inspection of Fig. 3 suggests that the maximum rejection by the filter occurs for

$$S \simeq j \frac{\omega_0}{(1-k^2)^{1/2}} .$$

The maximum I.L. may therefore be calculated from Eq. 9 with the use of Eqs. 7 and 8. This value is the maximum depth of the notch, D.

$$I.L._{max} \equiv D = -20 \log\left(1 + \frac{k^2 R_c L_2}{R_2 L_1}\right) \qquad \text{Eq. 10}$$

The 3 dB bandwidth of the notch may also be found by using Eqs. 7,8, and 9. The value of I.L. is set to -3 dB and the resulting equation solved for ω.

$$I.L. = -3\,dB \simeq 20 \log \frac{|\underset{\sim}{S} - \underset{\sim}{H}_0^+|}{|\underset{\sim}{S} - \underset{\sim}{H}_p^+|}$$

$$\frac{1}{\sqrt{2}} = \frac{|\underset{\sim}{S} - \underset{\sim}{H}_0^+|}{|\underset{\sim}{S} - \underset{\sim}{H}_p^+|}$$

The result is

$$\omega = \frac{\omega_0}{(1-k^2)^{1/2}} \pm \frac{\Delta\omega}{2} ,$$

where

$$\Delta\omega = \left[\frac{k^4 R_c^2}{L_1^2(1-k^2)^2} + \frac{2k^2 R_c R_2}{L_1 L_2 (1-k^2)^2} - \frac{R_2^2}{L_2^2(1-k^2)^2}\right]^{1/2} . \qquad \text{Eq. 11}$$

$\Delta\omega$ is the 3dB bandwidth.

Eqs. 10 and 11 are related in the following way. From Eq. 10

$$\frac{k^2 R_c L_2}{R_2 L_1} \equiv C = \left(10^{\frac{-D}{20}} - 1\right) \qquad \text{Eq. 12}$$

and so

$$\frac{R_2}{L_2} = \frac{k^2 R_c}{C L_1} .$$

This result can be used in Eq. 11 to obtain,

$$\Delta\omega = \frac{k^2 R_2}{L_1(1-k^2)}\left[1 + \frac{2}{C} - \frac{1}{C^2}\right]^{1/2} . \qquad \text{Eq. 13}$$

Equations 12 and 13 are selfconsistent. They may be used as design equations for selecting the values of the lumped parameters in order to acheive the required values of maximum insertion loss and 3 dB bandwidth when designing a filter.

From experimental measurements of the notch depth, D, and the 3 dB bandwidth, derived values of the coupling constant, k, and the equivalent resistance in the superconducting tank circuit, R_2, may be obtained. The value of C is found from the measured value of D using Eq. 12. Then Eq. 13 may be solved for k. This value of k may be used in Eq. 12 to solve for R_2. Changes to L_1 and L_2 due to the proximity of the superconducting filter body are very small compared to that for copper and may be neglected.

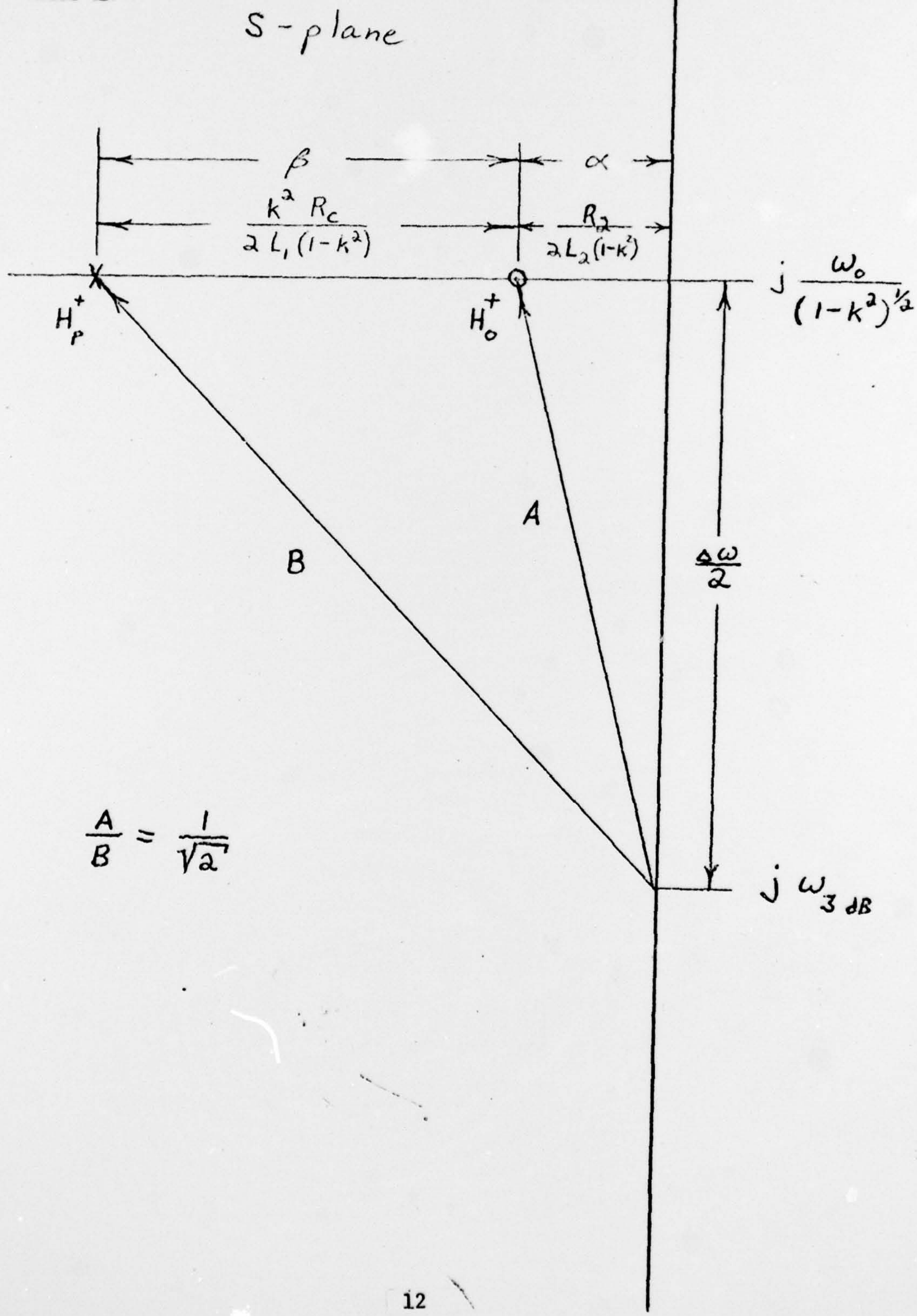
D. Geometric Interpretation of the Design Equations

Further insight into the relation of the values of the lumped parameters to the filter response can be obtain by considering the geometric relations shown in Fig. 4. From Eq. 9 the maximum depth of the notch is seen to be simply related to the distances α and β,

$$D = 20 \log\left(\frac{\alpha}{\alpha+\beta}\right) . \qquad \text{Eq. 14}$$

Reducing α will increase the notch depth. Therefore, reducing R_2 or increasing L_2 will increase the notch depth. Increasing β increases the notch depth while increasing the bandwidth. As seen from Eq. 11 and Fig. 4, the

Fig. 4


S-plane
β
α
$\frac{k^2 R_c}{2L_1(1-k^2)}$
$\frac{R_2}{2L_2(1-k^2)}$
$j\frac{\omega_0}{(1-k^2)^{1/2}}$
H_p^+
H_o^+
A
B
$\frac{\Delta\omega}{2}$
$j\,\omega_{3\,dB}$
$\frac{A}{B} = \frac{1}{\sqrt{2}}$

bandwidth can be written

$$\Delta\omega = 2\left[\beta^2 + 2\alpha\beta - \alpha^2\right]^{1/2} \quad . \qquad \text{Eq. 15}$$

Therefore, increasing the coupling constant k or reducing L_1 increases the bandwidth. Most importantly, the bandwidth and notch depth can be controlled by varying the ratios k^2/L_1 and R_2/L_2, respectively.

III. VARIATION OF FREQUENCY RESPONSE WITH CIRCUIT PARAMETERS

The variation of the filter's notch shape with different values of the circuit parameters is shown graphically in this section. The circuit analysis leading to Eqs. 14 and 15 show that the notch shape is dominated by the relative locations of the pole, H_p^+, and zero, H_o^+, in the S-plane. In turn, Figure 4 shows that for the range of values of the circuit parameters considered here, the relative locations of H_p^+ and H_o^+ are dominated by the values of the ratios k^2/L_1 and R_2/L_2. Figures 5 and 6 show how the shape of the notch changes for different values of these ratios. Figure 5 shows notch shapes for a fixed value of k^2/L_1 while the value of R_2/L_2 changes. The value of R_2/L_2 controls the amount of attenuation achieved by the filter. Note that except for the largest value of R_2/L_2, the 3 dB bandwidth remains unchanged. Also, for a sufficiently low value of R_2, lower values of this variable will not improve the filtering action since attenuation increases only in a bandwidth much narrower than the bandwidth of most signals.

Figure 6 shows notch shapes for a fixed value of R_2/L_2 while k^2/L_1 changes. It is readily seen that the value of k^2/L_1 controls the width of the notch as well as effecting the amount of attenuation. Note that the largest coupling constant results in an unacceptably large 3 dB bandwidth. Narrow bandwidth can be achieved at the sacrifice of attenuation.

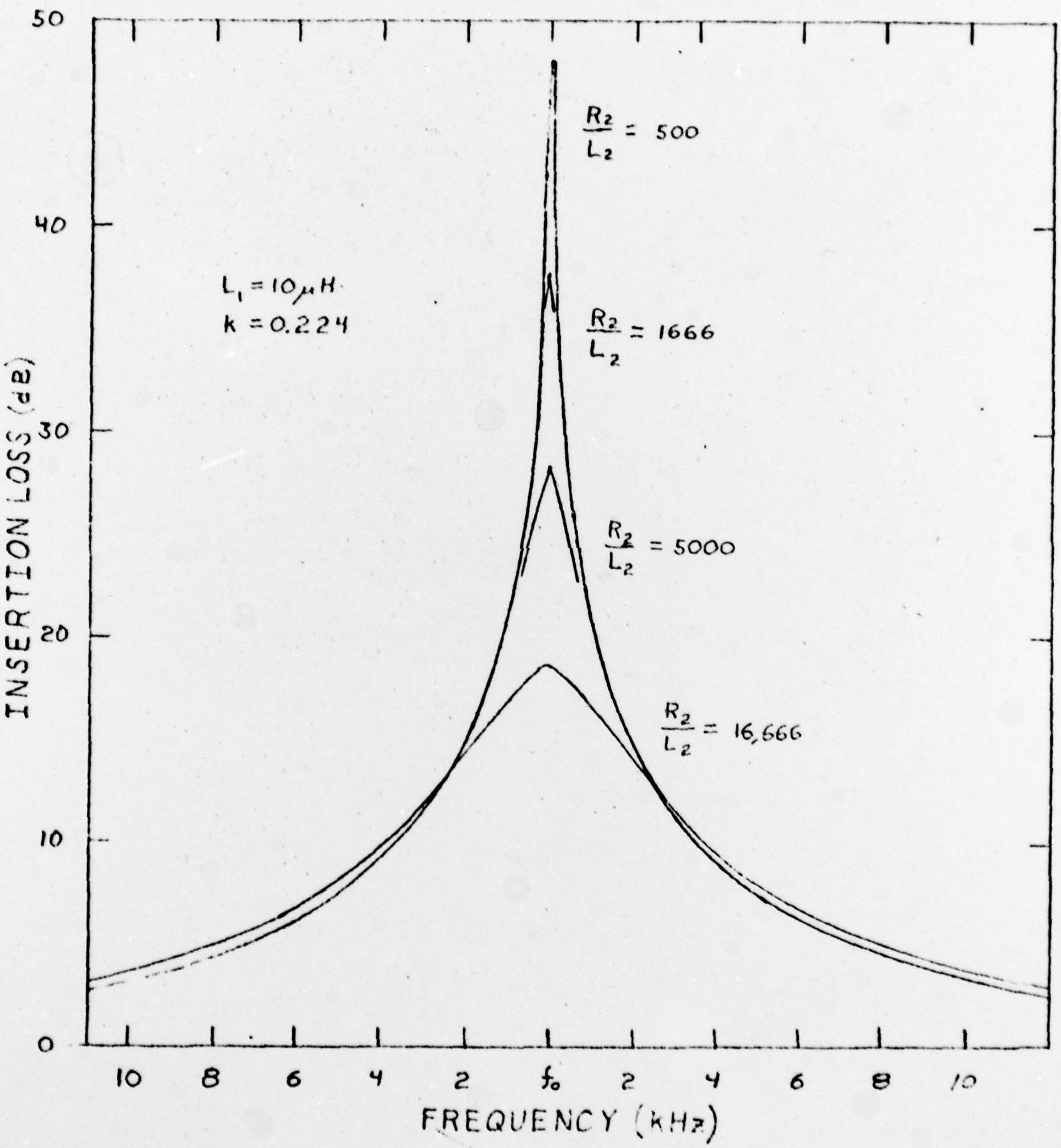


Figure 5. Notch Shape as a Function of R_2/L_2.

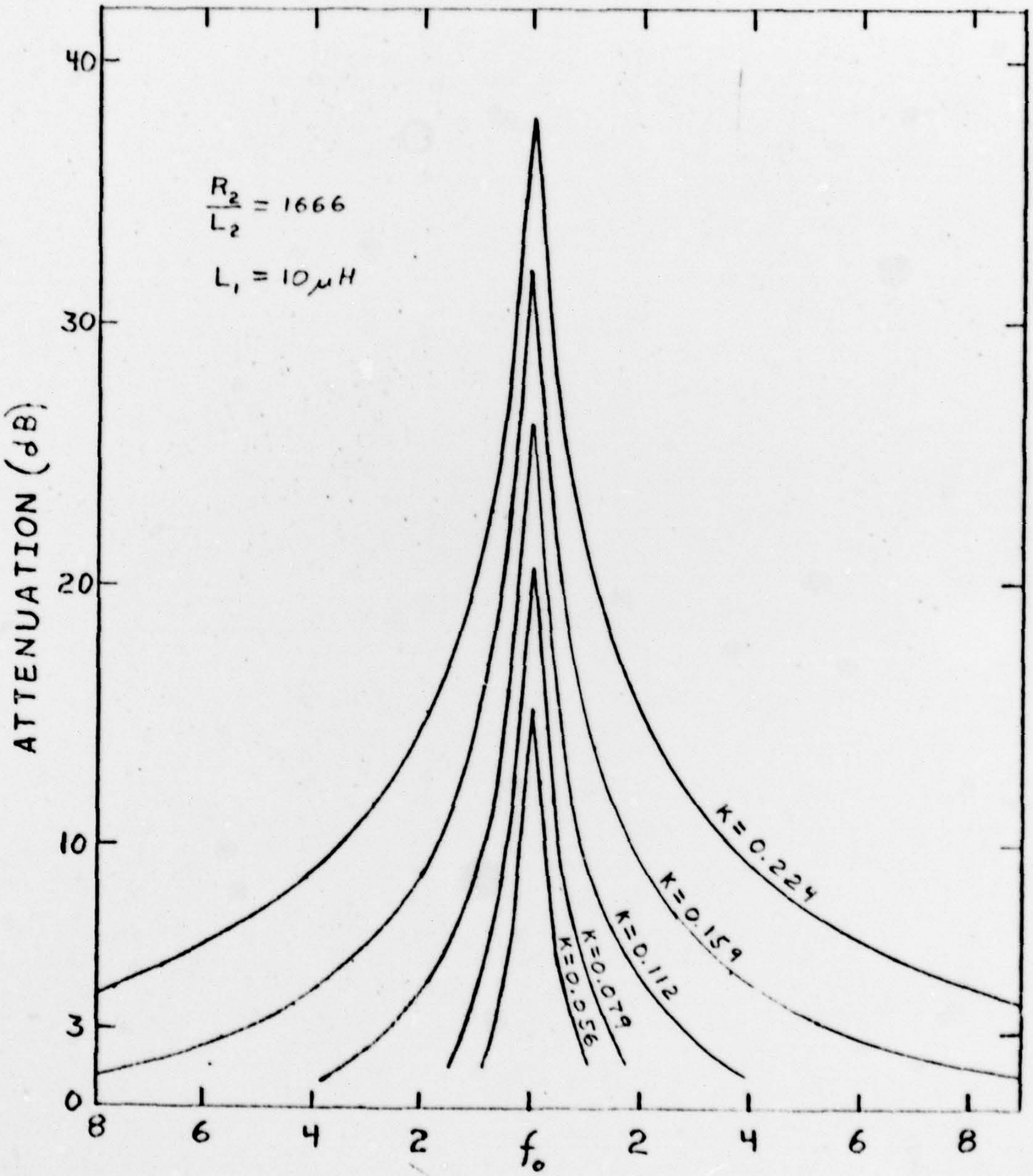


Figure 6. Notch Shape as a Function of k^2/L_2.