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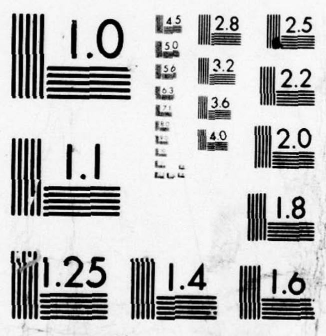

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by

Anthony V. Fiacco
Abolfazl Ghaemi

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The sensitivity is analyzed of the optimal waste treatment cost of a water pollution control model, with respect to the parameters involved in the formulation of the model. This analysis is conducted for the policy mandating a fixed dissolved oxygen requirement, using an upper Hudson River data base. Details concerning the formulation and solution of this problem were reported in another paper. It is shown here that the maximum allowable dissolved oxygen deficit is unequivocally the parameter to which the optimal waste
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1. Introduction

The present paper reports on the sensitivity analysis study of a geometric programming model* of a water pollution control system applied by J. G. Ecker [7] to define and solve three geometric programming problems involving data for the upper Hudson River. We have formulated and solved the convex equivalents of the above problems using the SUMT code [12], and have developed a computer routine to calculate the coefficients involved in the dissolved oxygen constraints in terms of the model parameters. These results are reported in [9]. By sensitivity analysis is meant an analysis of the effect on the optimal objective function value and on an optimal solution point of small perturbations in the model parameters. The importance of such an analysis in real world

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*The general model was apparently originally proposed by Charnes and Gemmell. See Section 5 of this paper.

optimization problems cannot be overstated. It provides the model maker and user with invaluable information regarding the functional relationship between a solution and the design parameters. This has many potential applications. For example, identification of those parameters having the most significant impact on the optimal solution can provide a basis for developing educated guidelines for taking appropriate and efficient action toward effecting parameter changes that will give an optimal marginal improvement of system performance.

Similar studies have been conducted by Armacost and Fiacco on a variety of problems, including a cattle feed problem [2] and a multi-item inventory problem [3]; and by the authors on a nonlinear structural design problem [10].

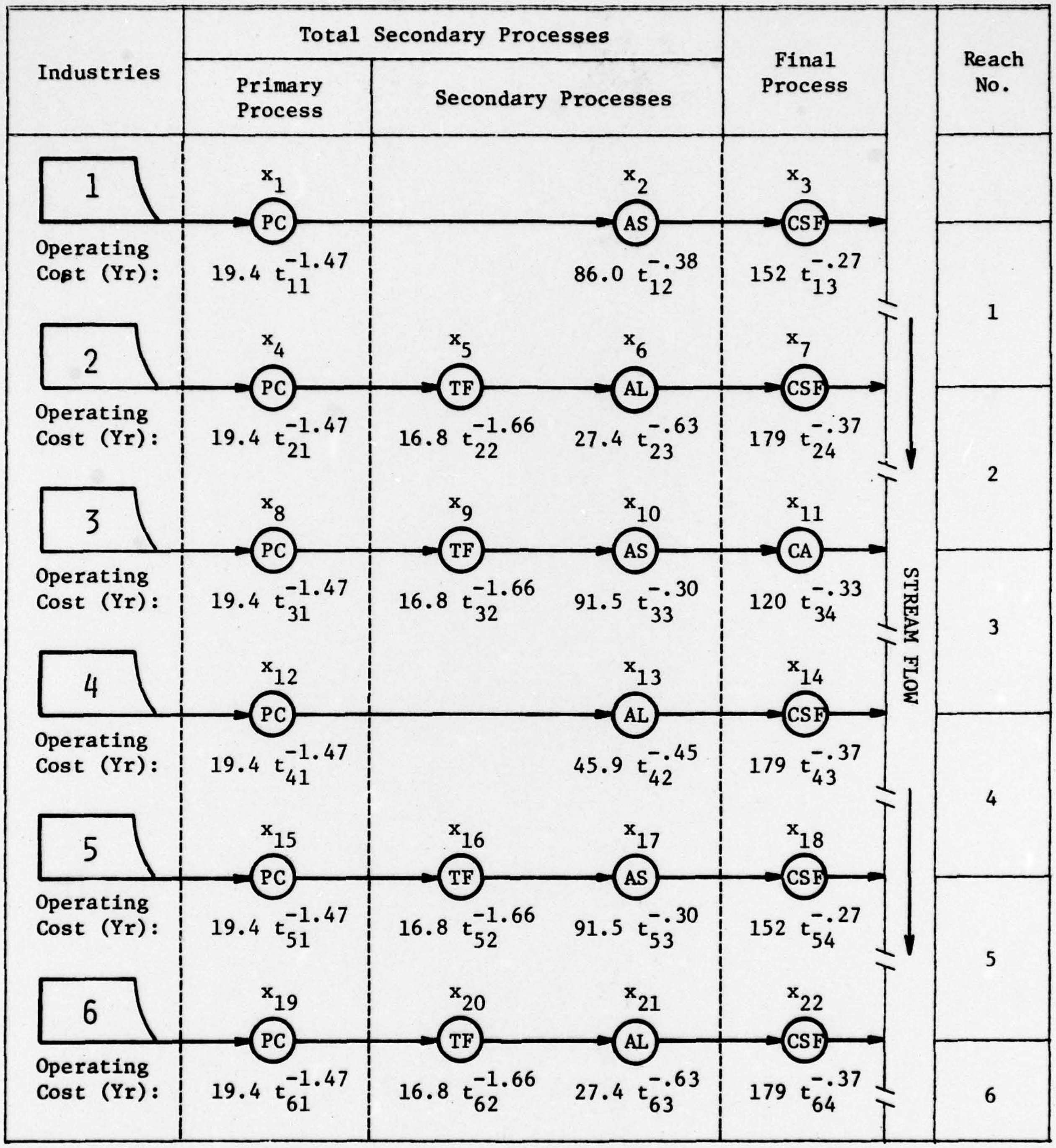
The theoretical basis for the approach taken here to calculate solution sensitivity was originally given in the work of Fiacco and McCormick [11]. Fiacco subsequently generalized this theory and established a theoretical basis for utilizing a penalty function method to estimate the sensitivity information of a local solution and its associated Lagrange multipliers, for a large class of nonlinear programming problems, with respect to general parametric variations of the problem functions [8]. A computational algorithm, "SENSUMT," was devised [4] to implement this method, and subsequently integrated with SUMT [12], evolving through a sequence of refinements. The latest revision by Armacost [1] is filed in The George Washington University Center for Academic and Administrative Computing. SENSUMT provided the main tool for the present study.

The sensitivity analysis is conducted with respect to all the model parameters for the fixed dissolved oxygen level policy, which yielded the minimum annual pollution control cost for an upper Hudson River data base, among three environmental control policies applied [7], [9]. It is concluded that the optimal waste treatment cost is most sensitive to the maximum allowable oxygen deficit. It is shown that a relaxation of one percent of this requirement, which amounts to about .0218 mg/ℓ, would save approximately $10,270 per year in waste treatment costs. It is also shown that the optimum cost is quite sensitive to several parameters involved in the dissolved oxygen deficit equation ([9], page 12, Equation (1)), and in particular to the fraction of the river bottom covered with sludge, the oxygen uptake rate per unit of river bottom surface area, and the hydraulic radius of the river. The solution is also rather sensitive to certain Biochemical Oxygen Demand (BOD) removal requirements (by primary and secondary treatments), and moderately sensitive to the deoxygenation constant, BOD concentration of the effluent before treatment, the volume of the effluent released into the river, and the river's flow rate. The optimal treatment cost appears relatively insensitive to the remaining model parameters, at least for small data changes.

2. Problem Statements and Solution

We have chosen to conduct the sensitivity analysis for the policy mandating a fixed dissolved oxygen requirement, using an upper Hudson River data base. Although the details concerning the formulation and solution of this problem (Problem P_1) were presented in our previous work [9], for completeness we give the original GP formulation, the formulation of the convex equivalent, and the respective optimal solutions that were obtained. The general problem is to determine treatment levels of various system components that minimize the total annual treatment cost, subject to constraints on maximum allowable oxygen deficits and component treatment levels that may vary between reaches.

A depiction of the system treatment facility configuration is given in Figure 1, and the associated problem formulations follow.



Abbreviations:

- AL Activated Lagoon
- AS Activated Sludge
- CA Carbon Absorption
- CSF Coagulation/Sedimentation/Filtration
- PC Primary Clarifier
- TF Trickling Filter

Transformation:

$$t_{ij} = e^{-x_k}$$

Figure 1.--Configuration of treatment facilities along the upper Hudson River.

Problem P_1: Corresponding to a "fixed dissolved oxygen requirement policy" of 6.2 mg/ℓ along the river.
[Dimensions: 22 variables, 42 constraints]

$$\text{Minimize } F = 19.4\, t_{11}^{-1.47} + 86.0\, t_{12}^{-.38} + 152\, t_{13}^{-.27}$$
$$+ 19.4\, t_{21}^{-1.47} + 16.8\, t_{22}^{-1.66} + 27.4\, t_{23}^{-.63} + 179\, t_{24}^{-.37}$$
$$+ 19.4\, t_{31}^{-1.47} + 16.8\, t_{32}^{-1.66} + 91.5\, t_{33}^{-.30} + 120\, t_{34}^{-.33}$$
$$+ 19.4\, t_{41}^{-1.47} + 45.9\, t_{42}^{-.45} + 179\, t_{43}^{-.37}$$
$$+ 19.4\, t_{51}^{-1.47} + 16.8\, t_{52}^{-1.66} + 91.5\, t_{53}^{-.30} + 152\, t_{54}^{-.27}$$
$$+ 19.4\, t_{61}^{-1.47} + 16.8\, t_{62}^{-1.66} + 27.4\, t_{63}^{-.63} + 179\, t_{64}^{-.37}$$

subject to:

a. Dissolved oxygen deficit constraint, i.e., dissolved oxygen deficit for all reaches $\leqq$ 2.18 mg/ℓ .

(1) $u_{11}t_{11}t_{12}t_{13} \leqq 1$

(2) $u_{21}t_{11}t_{12}t_{13} + u_{22}t_{21}t_{22}t_{23}t_{24} \leqq 1$

(3) $u_{31}t_{11}t_{12}t_{13} + u_{32}t_{21}t_{22}t_{23}t_{24} + u_{33}t_{31}t_{32}t_{33}t_{34} \leqq 1$

(4) $u_{41}t_{11}t_{12}t_{13} + u_{42}t_{21}t_{22}t_{23}t_{24} + u_{43}t_{31}t_{32}t_{33}t_{34} + u_{44}t_{41}t_{42}t_{43} \leqq 1$

(5) $u_{51}t_{11}t_{12}t_{13} + u_{52}t_{21}t_{22}t_{23}t_{24} + u_{53}t_{31}t_{32}t_{33}t_{34} + u_{54}t_{41}t_{42}t_{43} + u_{55}t_{51}t_{52}t_{53}t_{54} \leqq 1$

(6) $u_{61}t_{11}t_{12}t_{13} + u_{62}t_{21}t_{22}t_{23}t_{24} + u_{63}t_{31}t_{32}t_{33}t_{34} + u_{64}t_{41}t_{42}t_{43} + u_{65}t_{51}t_{52}t_{53}t_{54} + u_{66}t_{61}t_{62}t_{63}t_{64} \leqq 1$

b. Constraints on combination of processes.

(7) $.10\ t_{11}^{-1}\ t_{12}^{-1} \leqq 1$ at most 90 percent removal by total secondary treatment on reach 1

(8) $.15\ t_{21}^{-1}\ t_{22}^{-1}\ t_{23}^{-1} \leqq 1$ at most 85 percent removal by total secondary treatment on reach 2

(9) $.15\ t_{31}^{-1}\ t_{32}^{-1}\ t_{33}^{-1} \leqq 1$ at most 85 percent removal by total secondary treatment on reach 3

(10) $.15\ t_{41}^{-1}\ t_{42}^{-1} \leqq 1$ at most 85 percent removal by total secondary treatment on reach 4

(11) $.15\ t_{51}^{-1}\ t_{52}^{-1}\ t_{53}^{-1} \leqq 1$ at most 85 percent removal by total secondary treatment on reach 5

(12) $.15\ t_{61}^{-1}\ t_{62}^{-1}\ t_{63}^{-1} \leqq 1$ at most 85 percent removal by total secondary treatment on reach 6

(13) $1.4286\ t_{61}\ t_{62} \leqq 1$ at least 30 percent removal by first two components in reach 6

c. Operating range constraint for components of treatment facilities.

(14) $1.25\ t_{41} \leqq 1$ at least 20 percent removal by first component in reach 4

d. Redundant constraints (added to prevent numerical overflow).

(15) $e^{-80}\ t_{11}^{-1}\ t_{12}^{-1}\ t_{13}^{-1} \leqq 1$

(16) $e^{-80}\ t_{21}^{-1}\ t_{22}^{-1}\ t_{23}^{-1}\ t_{24}^{-1} \leqq 1$

(17) $e^{-80}\ t_{31}^{-1}\ t_{32}^{-1}\ t_{33}^{-1}\ t_{34}^{-1} \leqq 1$

(18) $e^{-80}\ t_{41}^{-1}\ t_{42}^{-1}\ t_{43}^{-1} \leqq 1$

(19) $e^{-80}\, t_{51}^{-1}\, t_{52}^{-1}\, t_{53}^{-1}\, t_{54}^{-1} \leqq 1$

(20) $e^{-80}\, t_{61}^{-1}\, t_{62}^{-1}\, t_{63}^{-1}\, t_{64}^{-1} \leqq 1$

e. Natural constraints.

(21)–(42) $0 < t_{ij} \leqq 1$, $i=1,6$; $j=1,m_i$

The coefficients u_{ij} in the constraints 1-6 are complicated functions ([9], Appendix 1) of the maximum allowable dissolved oxygen deficit and the physical parameters characterizing the reaches. These coefficients may be evaluated for any problem parameter values via a computer program developed by the authors ([9], Appendix 2). The values of the coefficients obtained using the upper Hudson River data are listed in Appendix 3 along with the respective data base.

Problem C_1: Convex equivalents of Problem P_1.

[Dimensions: 22 variables, 42 constraints]

$$
\begin{aligned}
\text{Minimize} \quad F = \; & 19.4\, e^{1.47x_1} + 86.0\, e^{.38x_2} + 152\, e^{.27x_3} \\
& + 19.4\, e^{1.47x_4} + 16.8\, e^{1.66x_5} + 27.4\, e^{.63x_6} + 179\, e^{.37x_7} \\
& + 19.4\, e^{1.47x_8} + 16.8\, e^{1.66x_9} + 91.5\, e^{.30x_{10}} + 120\, e^{.33x_{11}} \\
& + 19.4\, e^{1.47x_{12}} + 45.9\, e^{.45x_{13}} + 179\, e^{.37x_{14}} \\
& + 19.4\, e^{1.47x_{15}} + 16.8\, e^{1.66x_{16}} + 91.5\, e^{.30x_{17}} + 152\, e^{.27x_{18}} \\
& + 19.4\, e^{1.47x_{19}} + 16.8\, e^{1.66x_{20}} + 27.4\, e^{.63x_{21}} + 179\, e^{.37x_{22}}
\end{aligned}
$$

subject to:

a. Dissolved oxygen deficit constraint, i.e., the dissolved oxygen deficit for all reaches $\leqq$ 2.18 mg/ℓ .

(1) $-u_{11}e^{-(x_1+x_2+x_3)} + 1 \geqq 0$

(2) $-u_{21}e^{-(x_1+x_2+x_3)} - u_{22}e^{-(x_4+x_5+x_6+x_7)} + 1 \geqq 0$

(3) $-u_{31}e^{-(x_1+x_2+x_3)} - u_{32}e^{-(x_4+x_5+x_6+x_7)}$

$-u_{33}e^{-(x_8+x_9+x_{10}+x_{11})} + 1 \geqq 0$

(4) $-u_{41}e^{-(x_1+x_2+x_3)} - u_{42}e^{-(x_4+x_5+x_6+x_7)}$

$-u_{43}e^{-(x_8+x_9+x_{10}+x_{11})} - u_{44}e^{-(x_{12}+x_{13}+x_{14})} + 1 \geqq 0$

(5) $-u_{51}e^{-(x_1+x_2+x_3)} - u_{52}e^{-(x_4+x_5+x_6+x_7)}$

$-u_{53}e^{-(x_8+x_9+x_{10}+x_{11})} - u_{54}e^{-(x_{12}+x_{13}+x_{14})}$

$-u_{55}e^{-(x_{15}+x_{16}+x_{17}+x_{18})} + 1 \geqq 0$

(6) $-u_{61}e^{-(x_1+x_2+x_3)} - u_{62}e^{-(x_4+x_5+x_6+x_7)}$

$-u_{63}e^{-(x_8+x_9+x_{10}+x_{11})} - u_{64}e^{-(x_{12}+x_{13}+x_{14})}$

$-u_{65}e^{-(x_{15}+x_{16}+x_{17}+x_{18})} - u_{66}e^{-(x_{19}+x_{20}+x_{21}+x_{22})} + 1 \geqq 0$

b. Constraints on combination of processes.

(7) $-(x_1+x_2) + \ell n 10 \geqq 0$ — at most 90 percent removal by total secondary treatments in reach 1

(8) $-(x_4+x_5+x_6) + \ell n \frac{1}{.15} \geqq 0$ — at most 85 percent removal by total secondary treatments in reach 2

(9) $-(x_8+x_9+x_{10}) + \ell n \frac{1}{.15} \geq 0$ at most 85 percent removal by total secondary treatments in reach 3

(10) $-(x_{12}+x_{13}) + \ell n \frac{1}{.15} \geq 0$ at most 85 percent removal by total secondary treatments in reach 4

(11) $-(x_{15}+x_{16}+x_{17}) + \ell n \frac{1}{.15} \geq 0$ at most 85 percent removal by total secondary treatments in reach 5

(12) $-(x_{19}+x_{20}+x_{21}) + \ell n \frac{1}{.15} \geq 0$ at most 85 percent removal by total secondary treatments in reach 6

(13) $x_{19} + x_{20} + \ell n\ .7 \geq 0$ at least 30 percent removal by first two components in reach 6

c. Operating range constraint for components of treatment facilities.

(14) $x_{12} + \ell n\ .8 \geq 0$ at least 20 percent removal by first component in reach 4

d. Redundant constraints added to prevent numerical overflow.

(15) $-(x_1+x_2+x_3) + 80 \geq 0$

(16) $-(x_4+x_5+x_6+x_7) + 80 \geq 0$

(17) $-(x_8+x_9+x_{10}+x_{11}) + 80 \geq 0$

(18) $-(x_{12}+x_{13}+x_{14}) + 80 \geq 0$

(19) $-(x_{15}+x_{16}+x_{17}+x_{18}) + 80 \geq 0$

(20) $-(x_{19}+x_{20}+x_{21}+x_{22}) + 80 \geq 0$

e. Natural constraints.

(21)–(42) $x_i \geq 0$, $i=1,22$.

Optimal Solution of Problem P_1:

OPTIMAL TREATMENT LEVELS

(Using a fixed dissolved oxygen standard of 6.2 mg/ℓ along the entire stream)

t^*_{ij}	j=1	j=2	j=3	j=4
i=1	.7236	.4095	.6617	--
i=2	.7729	.7854	.2471	.4343
i=3	.8640	.8668	.4303	1.0000
i=4	.8000	.3229	1.0000	--
i=5	1.0000	1.0000	1.0000	1.0000
i=6	.7729	.7854	.2471	.9551

•Annual waste treatment cost = $1,837,851

•Binding constraints: (6, 8, 12, and 14)

OPTIMAL PLANT BOD REMOVAL LEVELS AND COSTS

(Using a fixed dissolved oxygen standard of 6.2 mg/ℓ along the entire stream)

Reach No.	Total BOD Removal (%)	Cost/Year ($)	Redundant Components
1	80.39	321,870	None
2	93.48	363,230	None
3	67.77	163,190	CA†
4	74.17	103,260	CSF†
5	0.00	0	All
6	85.67	301,590	None
		1,253,140 (Total)	

†See Figure 1 for definitions.

3. Sensitivity Analysis

A list and description of all model parameters are given in Appendix 1. The analysis was conducted with respect to all the model parameters except for parameter $\overline{\overline{PP}}$, which is not involved in the formulation of Problem P_1. The objective is to assess the relative impact of perturbations of the parameters on the optimal solution vector and cost. Having done this, a number of inferences are made concerning guidelines for improving the treatment operation. These will be directed to the problem at hand, though some general conclusions are rather apparent at the outset.

An obvious use of sensitivity information is extrapolation to estimated solutions with new data. There are many other uses. If a small perturbation is deemed to have significant effect, then, in practice, efforts might be directed toward obtaining a more precise determination of the corresponding parameters involved. Disproportionate effects of small perturbations could signal weaknesses (e.g., instabilities) in the model formulation and a need for alternative formulations, e.g., a rescaling of the involved parameters. Inordinately large effects could cast in serious doubt the validity of the model and a given solution. Even here, sensitivity information could be invaluable in suggesting appropriate modifications of underlying assumptions and model structure. Finally, given a reasonably "stable" model formulation, one can envision further cost-effectiveness studies directed towards efficiently altering certain parameters, e.g., those to which an optimal solution is most sensitive, attempting (at least marginally) to improve the optimal solution at minimum cost.

Because of the usual exponential transformation $t = e^{-x}$ of each variable t of the geometric programming problem, which is required to convert it to an equivalent convex programming problem in x , the sensitivities for the optimal solution vector t^* of the geometric program were obtained by the fact that

$$\frac{\partial t^*(\varepsilon)}{\partial \varepsilon} = -e^{-x^*(\varepsilon)} \frac{\partial x^*(\varepsilon)}{\partial \varepsilon},$$

where $\frac{\partial x^*(\varepsilon)}{\partial \varepsilon}$ could be directly read from the computer output and $-e^{-x^*(\varepsilon)}$ could be calculated. This calculation was not required to determine the sensitivity of the optimal value function of the original geometric program, since the transformation does not effect the values of the problem functions.

Sensitivity information for the optimal value function $f^*(\varepsilon)$ is calculated in Table I. We have tabulated the linear estimates $\partial f^*/\partial \varepsilon_n$ of changes in $f^*(\varepsilon)$ associated with unit changes in a given parameter ε_n, and changes due to a one percentage increase in ε_n, i.e., $.01\varepsilon_n \partial f^*/\partial \varepsilon_n$. For completeness, the sensitivities $\partial t^*_{ij}/\partial \varepsilon_n$ for the solution vector are tabulated in Appendix 2.

As indicated, an extremely important use of such sensitivity information--aside from insight concerning the structure of the model that is acquired--is to provide guidelines for the implementation of efficient modifications to improve the model and, ultimately, the "real world system" performance. In so doing, this information must be analyzed with considerable care and within the context of the application. We note a number of precautions that should be taken in interpreting the results.

The sensitivity measure $\partial f^*/\partial \varepsilon_n$ corresponds to the rate of change of the optimal value function $f^*(\varepsilon)$ with respect to ε_n at the given value of ε_n. Ostensibly it would seem that the larger the absolute value of $\partial f^*/\partial \varepsilon_n$, the "greater the sensitivity" of $f^*(\varepsilon)$ with respect to ε_n. This requires careful interpretation, however. At first the rate of change is instantaneous; yet it can change rapidly near the parameter value of interest, thus resulting in poor estimates of changes in the optimal solution, except for very small changes in the parameter. (This motivates the further development and implementation of

TABLE I

OPTIMAL VALUE FUNCTION SENSITIVITY RESULTS†

Parameter Number (n)	Parameter (ε_n)	Parameter Value	Change Due to One Unit Increase in Parameter Value ($1000's)	Change Due to 1% Increase in Parameter Value ($1000's)
1	V_{R_0}	1162.000	-0.22	-2.556
2	L_{b_0}	1.000	114.82	1.148
3	D_{b_0}	1.000	101.82	1.018
4	K_1	0.126	278.52	0.351
5	K_2	0.126	60.92	0.077
6	K_3	0.126	116.74	0.147
7	K_4	0.126	746.07	0.940
8	K_5	0.126	168.49	0.212
9	K_6	0.126	994.08	1.252
10	r_1	0.080	-157.43	-0.126
11	r_2	0.330	27.94	0.092§
12	r_3	0.224	-39.51	-0.088
13	r_4	0.216	-349.96	-0.756
14	r_5	0.216	-70.53	-0.152
15	r_6	0.250	-202.36	-0.506
16	t_1	4.000	5.62	0.225
17	t_2	0.520	32.49	0.169
18	t_3	0.700	8.37	0.058
19	t_4	2.620	7.02	0.184

TABLE I--*continued*

Parameter Number (n)	Parameter (ε_n)	Parameter Value	Change Due to One Unit Increase in Parameter Value ($1000's)	Change Due to 1% Increase in Parameter Value ($1000's)
20	t_5	0.370	16.20	0.060
21	t_6	0.950	78.57	0.746
22	F_1	0.700	51.62	0.361
23	F_2	0.700	70.57	0.494
24	F_3	0.700	54.37	0.381
25	F_4	0.700	287.64	2.013
26	F_5	0.700	58.49	0.409
27	F_6	0.700	225.26	1.577
28	Q_1	3.280	11.01	0.361
29	Q_2	3.280	15.06	0.494
30	Q_3	3.280	11.60	0.381
31	Q_4	3.280	61.38	2.013
32	Q_5	3.280	12.48	0.409
33	Q_6	3.280	48.07	1.577
34	K_{s_1}	2.000	18.07	0.361
35	K_{s_2}	2.000	24.70	0.494
36	K_{s_3}	2.000	19.03	0.381
37	K_{s_4}	2.000	100.67	2.013
38	K_{s_5}	2.000	20.47	0.409
39	K_{s_6}	2.000	78.84	1.577

TABLE I--*continued*

Parameter Number (n)	Parameter (ε_n)	Parameter Value	Change Due to One Unit Increase in Parameter Value ($1000's)	Change Due to 1% Increase in Parameter Value ($1000's)
40	R_{w_1}	4.880	-7.40	-0.361
41	R_{w_2}	2.440	-20.25	-0.494
42	R_{w_3}	3.410	-11.16	-0.380
43	R_{w_4}	3.380	-59.57	-2.013
44	R_{w_5}	3.200	-12.79	-0.409
45	R_{w_6}	2.900	-54.37	-1.576
46	V_{E_1}	16.000	2.64	0.422
47	V_{E_2}	32.500	2.57	0.835
48	V_{E_3}	6.400	5.32	0.340
49	V_{E_4}	41.500	0.71	0.295
50	V_{E_5}	48.500	0.12	0.058
51	V_{E_6}	20.000	3.28	0.656
52	E_1	148.000	0.31	0.459
53	E_2	436.000	0.21	0.915
54	E_3	179.000	0.20	0.358
55	E_4	38.000	0.97	0.369
56	E_5	4.830	2.25	0.109
57	E_6	661.000	0.10	0.661

TABLE I--*continued*

Parameter Number (n)	Parameter (ε_n)	Parameter Value	Change Due to One Unit Increase in Parameter Value ($1000's)	Change Due to 1% Increase in Parameter Value ($1000's)
58	S_1	2.180	0.00	0.000
59	S_2	2.180	0.00	0.000
60	S_3	2.180	0.00	0.000
61	S_4	2.180	0.00	0.000
62	S_5	2.180	0.00	0.000
63	S_6	2.180	-471.07	-10.270
64	$\underline{P}_6$	0.300	0.00	0.000
65	$\overline{P}_1$	0.900	0.00	0.000
66	$\overline{P}_{2-6}$	0.850	-494.98	-4.207
67	$\underline{PP}_4$	0.200	3.18	0.006

†See Appendix 1 for parameter definitions.

§This quantity appears inconsistent, but we could not uncover any error in its calculation.

techniques for computing bounds on possible variations as a function of parameter changes, a capability that would be an important addition to sensitivity analysis software.) Second, though dimensionally correct, it is misleading in practice to think of $\partial f^*/\partial\varepsilon_n$ as "the change in $f^*(\varepsilon)$ per unit of change in ε_n." It is misleading because it tacitly suggests that the change can be realized. However, it is obvious that a one unit change for one parameter may be too large to be practically feasible or it may be mathematically impossible, while for another it may be too small to be numerically significant, e.g., less than the typical error involved in the measurement of the given parameter. Examples of these possibilities follow for the present model.

As shown in Table I, the sensitivity of the optimal annual waste treatment cost with respect to the parameter $\overline{P}_{2-6}$, the upper limit of the fraction of BOD removed by the primary and secondary treatment components in reaches 2-6, at the parameter value of 0.85, is -\$494,980. A unit increase in parameter $\overline{P}_{2-6}$ would change its value to 1.85, which is nonsensical in the context of the application, by definition of $\overline{P}_{2-6}$. A mathematically acceptable and physically reasonable perturbation of the parameter might be on the order of .01 to .05, yielding correspondingly "plausible" changes in the annual treatment cost on the order of -\$4,950 to -\$24,750.

On the other hand, a one unit change in the parameter V_{R_0} (river flow rate) at the parameter value of 1162 is so small that it surely falls well within the range of flow rate measurement errors. Thus, a unit change in V_{R_0} is most likely not numerically significant. Allowing a one percent perturbation for this parameter would seem to generate a numerically significant perturbation and would alter the estimate of sensitivity from -\$220 to a more meaningful value of (.01)(1162)(-220), or a decrease in cost of about \$2,556.

Another point that is relevant to this particular model relates to the evaluation of the objective function. Where a decision variable t_{ij} takes on a value of unity, this means that treatment component j in reach i is removing no waste and hence we can (in theory) exclude the corresponding treatment facility from the overall design. This implies that, for all practical purposes, the objective function is actually a piecewise continuous function, although the mathematical formulation of the model does not make this explicit. This introduces considerably more complexity which must be accommodated in attempting to interpret the meaning of a "solution," as well as the practical effect of data changes. For example, as shown in Table I, the sensitivity of the optimal value function with respect to a one unit increase in the parameter S_6, the maximum allowable dissolved oxygen deficit in reach 6, i.e., the natural level of dissolved oxygen (8.38 mg/ℓ) less the minimum allowable dissolved oxygen (6.2 mg/ℓ), is -$471,070. As mentioned previously, this value corresponds to the change in the optimal value function per unit change in the value of the parameter S_6 . (Note that the value of S_6 is 2.18. A one unit increase in this parameter is not unreasonable.) However, if we perturb this parameter by unity, the optimal solution vector t* is also perturbed. The vector of perturbations is shown in Table A.IX of Appendix 2. The components of the solution vector t* , marked with a plus sign (+), extrapolate to approximately unity as a result of this perturbation. This in turn implies that the advanced treatment facilities in reaches 1, 2, and 6, as well as the primary and secondary treatment facilities in reach 3 should be deleted in calculating the resulting cost. (Of course, actual removal or shut-down must be practically justified, though theoretically warranted. It is not within the scope of this study to evaluate the practical feasibility or implications of design modifications.) Assuming a treatment component can be removed or shut down without affecting the performance of the other components, the sensitivity of the optimal value function would change to -$827,930, which is considerably different from the -$471,070 read directly from the computer output. This again emphasizes the importance of careful manipulation and interpretation of sensitivity information and, indeed, of the practical implications and limitations of results obtained with any model.

Having the above points in mind, in order to evaluate more realistically and compare directly the changes of the optimal annual waste treatment cost with respect to the corresponding changes in the problem parameters, we base our inferences on the resulting sensitivities given in the last column of Table I that correspond to a one percent increase in the parameter values. (This assumes essentially that such changes are practically feasible, an assumption whose defense is not pursued here.) For this perturbation of the parameters, it is noted that none of the solution vector components for the perturbed problem extrapolate to unity, which implies that the changes in the optimal value function as given in the table can be used directly to extrapolate the perturbed optimal treatment cost.

Furthermore, assuming that the parameters involved in the oxygen deficit constraints (constraints numbered 1-6 in Problems P_1 and C_1) can be varied independently from reach to reach, we can sum the scaled sensitivities in the different reaches to find a linear estimate of the change in $f^*(\varepsilon)$ across all reaches resulting from the simultaneous increase in each reach by one percent of each of the involved parameters. This allows us to calculate the *overall* effect, along the entire length of stream involved, of each model parameter on the optimal annual treatment cost. The results are given in the second column of Table II. The last column gives the ranks of the absolute values of the respective entries in Column 2, ordering these values from the largest to the smallest.

A multitude of inferences can be drawn from the solution vector sensitivity information given in Appendix 2 and the optimal solution value sensitivity given in Tables I and II. The solution vector information can be used to predict optimal system component treatment levels that will be required for any combination of small changes in the parameters, also pinpointing the possible need for adding new components (e.g., if predicted requirements on a given component are deemed excessive) and for removing others (e.g., if predicted requirements are below a given level). Aside from these potentially valuable insights, we shall not further pursue the implications of the solution vector sensitivity information. Several observations based on the optimal value sensitivity information collected in Tables I and II will be offered in the next section.

4. Observations and Conclusions

Based on Tables I and II, the following conclusions are valid.

(1) The order of the parameters, arranged according to the decreasing order of the respective absolute magnitude of the changes in the optimal annual treatment cost resulting from a one percent parameter change, is as follows: S_6 , (F , Q , K_s , R_w) , $\bar{P}_{2-6}$, K , E , V_E , V_{R_0} , r , t , L_{b_0} , D_{b_0} , $\underline{PP}_4$, $(S_1 - S_5 , \underline{P}_6 , \bar{P}_1)$, where the parentheses indicate ties.

TABLE II

CHANGES SUMMED ACROSS REACHES IN THE MINIMUM ANNUAL WASTE TREATMENT COST DUE TO A 1% INCREASE IN THE MODEL PARAMETERS

Parameter	Change in Optimal Value Function Corresponding to a 1% Increase in the Given Parameter Type Across All Reaches	Rank
V_{R_0}	-2.556	7
L_{b_0}	1.148	10
D_{b_0}	1.081	11
K	2.979	4
r	-1.536	8
t	1.442	9
F	5.235	2
Q	5.235	2
K_s	5.235	2
R_w	-5.235	2
V_E	2.606	6
E	2.871	5
S_1 - S_5	0.000	13
S_6	-10.270	1
$\underline{P}_6$	0.000	13
$\bar{P}_1$	0.000	13
$\bar{P}_{2-6}$	-4.207	3
$\underline{PP}_4$	0.006	12

(2) The optimal annual treatment cost decreases with increases in S_6 , R_w , $\bar{P}_{2-6}$, V_{R_0} , and r , is insensitive to changes in $S_1 - S_5$, $\underline{P}_6$ and $\bar{P}_1$, and increases with increases in the remaining parameters.

(3) The magnitude of the significant decreases in the optimal annual treatment cost ranges from \$10,270 per year for a one percent increase in the parameter S_6 to \$1,081 per year for a one percent decrease in the parameter D_{b_0} .

(4) Although the maximum allowable oxygen deficit S_6 clearly dominates the other parameters taken individually, it is interesting to note that the cumulative effects of simultaneously decreasing F , Q , and K_s by one percent of their given values, all such changes associated with removing sludge from the river bottom, would result in an estimated annual treatment cost reduction of \$15,705, a decrease of about 50% below the reduced cost noted for a one percent increase in S_6 . Thus, depending on cost trade-offs, of course, sludge removal would appear to be a good possibility for realizing an optimal marginal improvement.

(5) The preceding point suggests that some parameters may be altered simultaneously by a given change, a fact that should obviously be considered in a cost analysis involved with design modifications. Other parameters, e.g., the flow rate of the stream, may be impossible to control and may change rather abruptly by natural causes. This has numerous implications. For example, an increase of one percent in the parameter V_{R_0} , the flow rate of the river before entering reach 1, will result in an estimated savings of \$2,556 a year in treatment costs. This benefit could be realized at no cost during rainy seasons. It also implies that more waste could be deposited without increasing treatment levels or costs, while meeting the environmental standards, a possibly more interesting alternative to reducing the cost by reducing the requirements on treatment levels for the original level of waste. Thus,

it is seen that numerous alternatives are introduced by having information regarding the effect of parameter changes on a solution. (Although it is obvious that a small increase in V_{R_0} should decrease the treatment cost, other changes in subsets of parameters may be far from obvious and nonintuitive.)

(6) As regards the separate perturbation of each parameter within each reach, it may be noted that the dominant parameter is S_6, followed by F, R_w, Q, and K_s in reaches 4 and 6 and parameter K in reach 6. All have significant impact and reasonably precise determination of these quantities, along with the "initial data" parameters V_{R_0}, L_{b_0}, and D_{b_0}, is essential for an accurate determination of the model functions, and hence for an accurate determination of the optimal value treatment cost. Optimal cost-effective parameter changes would probably involve these parameters. The fact that small changes in $S_1 - S_5$, the maximum allowable oxygen deficits in reaches 1-5, do not affect the optimal value function, while S_6 emphatically does, follows from the data and model structure which result (as shown in Section 4.1 of Reference [9]) in the consequence that the only oxygen deficit constraint that is binding is the one spanning all six reaches [constraint (6) of Problem P_1 or C_1] and involving the oxygen deficit bound S_6. Thus, $S_1 - S_5$ appear only in nonbinding constraints. The same fact accounts for the insensitivity relative to $\underline{P}_6$ and $\bar{P}_1$.

(7) The fact that the absolute values of the sensitivities for F_i, Q_i, K_s, and R_{w_i} in the last column of Table I are identical for each i may be explained as follows. These parameters enter only in the dissolved oxygen level constraints and have effect on f^* at optimality only through g_6, constraint (6), entering that constraint in the form $(Q_iF_iK_{s_i}/R_{w_i})F(t,\varepsilon)$, where $F(t,\varepsilon)$ is a function of the problem variables t_{ij} and the other problem parameters. From the theory, it

follows that $\partial f^*/\partial\beta_i = u_6^* \cdot \partial g_6/\partial\beta_i$, where u_6^* is the optimal Lagrange multiplier associated with g_6 , and β_i denotes any of the indicated parameters. Denoting $Q_i F_i K_{s_i}/R_{w_i}$ by α_i , it follows that our sensiivity measure $(\partial f^*/\partial\beta_i)(.01\beta_1) = .01u_6^*\alpha_i$ if $\beta_i = Q_i$, F_i , or K_{s_i} , and $-.01u_6^*\alpha_i$ if $\beta_i = R_{w_i}$, which is consistent with the results given in Table I.

Obviously any attempt to alter or to measure more accurately the parameters would generally involve some cost. Such changes or efforts are economically attractive only if it is determined that the cost incurred will be less than the expected gain. Such an assessment would require results such as those provided by the present study. Sensitivity information, coupled with judicious interpretation by users conversant with the application of the model, could provide invaluable insights and guidelines for determining the most cost-effective changes in environmental control policies and system design parameters, providing a more complete basis for determining the economic implications of making these changes. Indeed, such information would appear to be crucial not only for assessing the impact of changes *without re-solving* the problem with new data and new parameter values, but even for interpreting the significance of a "solution."

5. Postscript: Comment on the Development of the Model

While this paper was in its final stages of preparation, we learned [6], in response to our previous paper [9], that A. Charnes and R. E. Gemmell originally formulated a model of similar character [5]. Their model was subsequently included in the 1973 Istanbul NATO conference proceedings. The definition of the model variables, form of the cost function, and structure of the dissolved oxygen deficit constraints utilized by Ecker conform to those introduced in these referenced works. The earlier models apparently assume that the coefficients involved in the oxygen deficit equation relationships are values specified at the outset. One of the important innovations introduced by Ecker is the functional dependence of these coefficients (u_{ij} in Problem P_1) on S_i, the maximum allowable oxygen deficit in reach i, and on the numerous parameters (given in Appendix 1) that are involved in defining the dissolved oxygen deficit equation that is crucial in characterizing the state of the stream. These relationships were developed in some detail in the Ecker paper [7] and in our earlier paper [9].

APPENDIX 1

LISTING AND DESCRIPTION OF THE PARAMETERS INVOLVED IN THE FORMULATION OF THE MODEL

Parameters representing physical measurements involved in the dissolved oxygen deficit equation of Problem P_1:

V_{R_0} : flow rate of the river before entering reach 1, in 10^6 gallons/day

L_{b_0} : initial BOD level of the stream before entering reach 1, in mg/ℓ

D_{b_0} : initial oxygen deficit of the stream before entering reach 1, in mg/ℓ

K : deoxygenation constant, in day^{-1}

r : reaeration constant, in day^{-1}

t : flow time along the reach, in days

F : fraction of river bottom covered with sludge

Q : a coefficient in the sludge term, determined empirically in day^{-1}

K_s : oxygen uptake rate per unit area of stream bottom surface, in $gm/m^2/day$

R_w : hydraulic radius of the river cross section, in meters

V_E : volume of the effluent released into the river, in 10^6 gallons/day

E : BOD concentration of effluent, before treatment, in mg/ℓ

Aside from the above parameters that are involved in the oxygen sag equation of each reach, the following parameters will also enter into the formulation of the model at various stages.

Parameters representing quantities determined by management decisions or feasibility considerations:

S : maximum allowable oxygen deficit, in mg/ℓ

$\underline{P}$: minimum required or feasible BOD removal by a specified sequence of treatment components in a given reach; a fraction

$\bar{P}$: maximum required or feasible BOD removal by a specified sequence of treatment components in a given reach; a fraction

$\underline{PP}$: minimum required or feasible BOD removal by a specified single treatment component in a given reach; a fraction

$\overline{PP}$: maximum required or feasible BOD removal by a specified single treatment component in a given reach; a fraction.

In Table I, parameters $\bar{P}_1$, $\bar{P}_{2-6}$, and $\underline{P}_6$ and $\underline{PP}_4$ have the following meanings:

$\underline{P}_6$: minimum required BOD removal by the first two treatment components in reach 6

$\bar{P}_1$: maximum required BOD removal by the total secondary treatment components in reach 1

$\bar{P}_{2-6}$: maximum required BOD removal by the total secondary treatment components in reaches 2, 3, 4, 5, and 6. (We have taken $\bar{P}_{2-6} = \bar{P}_2 = \bar{P}_3 = \ldots = \bar{P}_6$ in this analysis, although our general model allows for independent variation of $\bar{P}_2 - \bar{P}_6$.)

$\underline{PP}_4$: minimum required BOD removal by the first treatment component in reach 4.

APPENDIX 2†

SENSITIVITIES OF THE PROBLEM VARIABLES t^*_{ij}, THE FRACTION OF WASTE REMOVED BY TREATMENT COMPONENT j IN REACH i AT THE OPTIMAL SOLUTION POINT OF PROBLEM P_1, WITH RESPECT TO THE PROBLEM PARAMETERS ε_n

TABLE A.I

Variable	Parameter	
	$\varepsilon_2 = L_{b_0}$ = 1 mg/ℓ	$\varepsilon_3 = D_{b_0}$ = 1 mg/ℓ
t_{13}	-.1649	-.1462
t_{24}	-.1744	-.1546
t_{33}	-.1387	-.1230
t_{42}	-.1231	-.1092
t_{64}	-.3833	-.3399

†Sensitivities with absolute value less than 0.1, i.e., $|\partial t_{ij}/\partial \varepsilon_n| < 0.1$, though calculated, are not tabulated here. This accounts for blank entries and derivatives not appearing in the tables that follow.

TABLE A.II

Variable	Parameter					
	$\varepsilon_4 = K_1 =$ .126 day^{-1}	$\varepsilon_5 = K_2 =$ .126 day^{-1}	$\varepsilon_6 = K_3 =$ .126 day^{-1}	$\varepsilon_7 = K_4 =$ .126 day^{-1}	$\varepsilon_8 = K_5 =$ .126 day^{-1}	$\varepsilon_9 = K_6 =$ .126 day^{-1}
t_{11}				-.1901		-.1569
t_{12}	-.1762			-.4161		-.3434
t_{13}	-.4006		-.1674	-.9430	-.2158	-.7809
t_{24}	-.4233	-.1225	-.2285	-1.3720	-.3054	-1.0615
t_{31}	-.1362			-.4666	-.1035	-.3573
t_{32}	-.1211			-.4146		-.3174
t_{33}	-.3326		-.1879	-1.1389	-.2525	-.8720
t_{42}	-.3007			-1.1312	-.2493	-.8518
t_{64}	-.9306	-.1557	-.2409	-1.0991	-.2749	-.7223

TABLE A.III

Variable	Parameter					
	$\varepsilon_{10} = r_1 =$ .080 day^{-1}	$\varepsilon_{11} = r_2 =$ .330 day^{-1}	$\varepsilon_{12} = r_3 =$ .224 day^{-1}	$\varepsilon_{13} = r_4 =$ .216 day^{-1}	$\varepsilon_{14} = r_5 =$ .216 day^{-1}	$\varepsilon_{15} = r_6 =$ .250 day^{-1}
t_{11}				.1512		
t_{12}	.1918			.3310		.1437
t_{13}	.4362		.1317	.7529	.1314	.3269
t_{24}	.1877			.6125	.1289	.3384
t_{31}				.1829		.1079
t_{32}				.1624		
t_{33}	.1474			.4463		.2633
t_{42}	.1333			.3461		
t_{64}	.4125	-.1296		.4735		.4587

TABLE A.IV

Variable	Parameter	
	$\varepsilon_{17} = t_2$ = .52 day	$\varepsilon_{21} = t_6$ = .95 day
t_{64}	-.1199	-.7700

TABLE A.VI

Variable	Parameter	
	$\varepsilon_{31} = Q_4$ = 3.28 day^{-1}	$\varepsilon_{33} = Q_6$ = 3.28 day^{-1}
t_{64}	.1998	.1823

TABLE A.VII

Variable	Parameter	
	$\varepsilon_{37} = K_{s_4}$ = 2 $gm/m^2/day$	$\varepsilon_{39} = K_{s_6}$ = 2 $gm/m^2/day$
t_{13}	-.1424	-.1115
t_{24}	-.1536	-.1203
t_{33}	-.1206	
t_{64}	-.3377	-.2645

TABLE A.V

Variable	Parameter					
	$\varepsilon_{22} = F_1 =$ .7 fraction	$\varepsilon_{23} = F_2 =$ .7 fraction	$\varepsilon_{24} = F_3 =$ .7 fraction	$\varepsilon_{25} = F_4 =$ .7 fraction	$\varepsilon_{26} = F_5 =$ .7 fraction	$\varepsilon_{27} = F_6 =$ .7 fraction
t_{12}				-.1789		-.1401
t_{13}				-.4069		-.3186
t_{24}		-.1077		-.4389		-.3437
t_{31}				-.1413		-.1106
t_{32}				-.1255		
t_{33}				-.3448		-.2700
t_{42}				-.3108		-.2442
t_{64}	-.1731	-.2368	-.1823	-.9648	-.1962	-.7557

TABLE A.VIII

Variable	Parameter	
	$\varepsilon_{43} = R_{w_4} =$ 3.38 meters	$\varepsilon_{45} = R_{w_6} =$ 2.90 meters
t_{64}	.1998	.1823

TABLE A.IX†

Variable	Parameter
	$\varepsilon_{63} = S_6 =$ 2.18 mg/ℓ
t_{11}	.1339
t_{12}	.2930
$\overset{+}{t}_{13}$	.6663
$\overset{+}{t}_{24}$	.7188
$\overset{+}{t}_{31}$	.2314
$\overset{+}{t}_{32}$	.2055
t_{33}	.5646
t_{42}	.5107
$\overset{+}{t}_{64}$	1.5802

†Components of t_{ij} marked with a plus sign (+) extrapolate to unity as a result of a one unit increase in the dissolved oxygen deficit S_6 in reach 6.

TABLE A.X

Variable	Parameter	
	$\varepsilon_{66} = \bar{P}_{2-6} =$ .85 (fraction)	$\varepsilon_{67} = \underline{PP}_{4} =$ .2 (fraction)
t_{12}	.1612	
t_{13}	.3666	
t_{21}	-1.2210	
t_{22}	-1.0990	
t_{23}	-.9111	
t_{24}	2.5797	
t_{31}	.1264	
t_{32}	.1122	
t_{33}	.3085	
t_{41}		-.9994
t_{42}	.2739	.3040
t_{61}	-1.2212	
t_{62}	-1.0988	
t_{63}	-.9112	
t_{64}	5.6704	

APPENDIX 3

INPUT DATA AND CALCULATED u_{ij}

FOR THE PROBLEM P_1

LIST OF INPUT DATA

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	REACH-1	REACH-2	REACH-3	REACH-4	REACH-5	REACH-6
t	4,0000	0,5200	0,7000	2,6200	0,3700	0,9500
r	0,0800	0,3300	0,2240	0,2160	0,2160	0,2500
K	0,1260	0,1260	0,1260	0,1260	0,1260	0,1260
V_E	16,0000	32,5000	6,4000	41,5000	48,5000	20,0000
E	148,0000	436,0000	179,0000	38,0000	4,8300	661,0000
F	0,7000	0,7000	0,7000	0,7000	0,7000	0,7000
R_w	4,8800	2,4400	3,4100	3,3800	3,2000	2,9000
Q	3,2800	3,2800	3,2800	3,2800	3,2800	3,2800
S	2,1800	2,1800	2,1800	2,1800	2,1800	2,1800
K_s	2,0000	2,0000	2,0000	2,0000	2,0000	2,0000
V_{R_0}	1162					
L_{b_0}	1.0					
D_{b_0}	1.0					

CALCULATED U(I,J)

****

REACH-1	REACH-2	REACH-3	REACH-4	REACH-5	REACH-6
0,7756	0,0000	0,0000	0,0000	0,0000	0,0000
0,8544	0,9172	0,0000	0,0000	0,0000	0,0000
0,8804	2,0082	0,1029	0,0000	0,0000	0,0000
0,8540	4,3310	0,3305	0,4018	0,0000	0,0000
0,8433	4,4959	0,3476	0,4337	0,0120	0,0000
0,8162	4,8288	0,3827	0,5003	0,0379	1,6400

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