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This study was designed to establish the validity and nature of a relationship between the cost of producing prototype units and the cost of producing the units in the investment phase where only costs recurring with the units was considered. A secondary objective of this analysis involved the assessment of the most appropriate cost improvement theory to employ in developing the theoretical first unit cost of an item.

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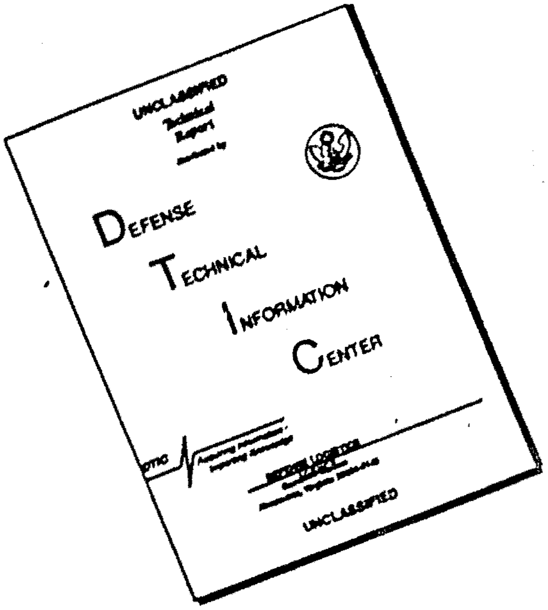

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TM-47

THE RELATIONSHIP BETWEEN DEVELOPMENT AND PRODUCTION HARDWARE COSTS IN MILITARY WEAPONS SYSTEMS

by

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September 1976

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SUMMARY

The study documented in this paper was a three-month effort supported by the Precision Laser Designator Program Office within the USA MICOM. The study was designed to establish the validity and nature of a relationship between the cost of producing prototype units and the cost of producing the units in the investment phase where only costs recurring with the units was considered. A secondary objective of this analysis involved the assessment of the most appropriate cost improvement theory to employ in developing the theoretical first unit cost of an item.

The methodology employed considered several prominent hypotheses on the form and variable content of the relationship between prototype and production units and also involved the identification of two methods for assessing the validity of the two improvement curve theories. Testing of these hypotheses on data for 16 electronics subsystems for various applications produced several good models of the prototype/production relationship.

The figure shows the results for one of the models assuming log linear cumulative average costs which fit the data very well with little scatter evident. This model represents a simple proportional relationship between T_1, the theoretical first production unit cost, and T_{1P}, the theoretical first unit prototype cost, derived based on the improvement rate observed in the production lot data and the average prototype cost. The particular hypothesis shown in this figure, called the sequential hypothesis, assumes that the production quantities add to and continue in sequence the prototype quantity. The alternative to this theory, called the disjoint theory, assumes the improvement curve for production begins again at unit number one. We were unable to determine statistically which of these two theories was better because both produced good results. However, on the basis of the statistical results, we did conclude that prototype cost was a good estimator of production costs.

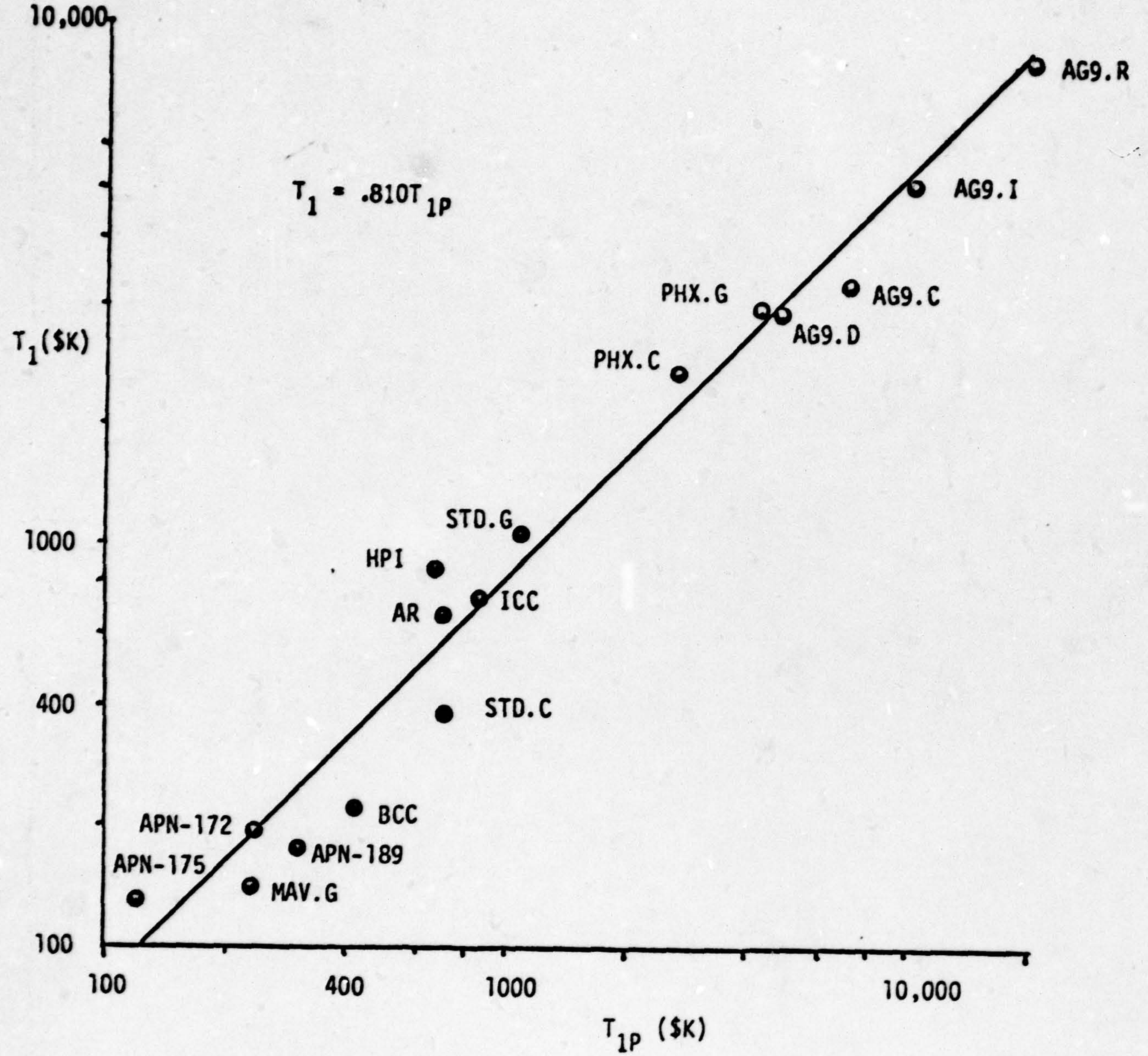


FIGURE 1 SEQUENTIAL MODEL, LOG LINEAR CUM. AVE. THEORY

Similarly, we were unable to establish the superior improvement rate theory. However, it was established that for typical first lot production quantities, log linear fits of production lot data under either theory are virtually identical. In addition, it was shown that there is some impact on the prototype/production cost relationship where one theory is correct and the other is assumed and where prototype quantities are small.

The analysis also revealed (1) no distinguishable difference between the relationship for missiles and other electronics units, and (2) that cost improvement occurs in the production of prototypes which should be taken into account in estimating production costs.

1.0 BACKGROUND AND OBJECTIVE OF STUDY

1.1 BACKGROUND

In general, it has been an established practice in the cost analysis community to apply a cost-to-cost factor to the cost of producing prototype hardware to obtain an estimate of manufacturing this hardware in the production or investment phase. The use of such a methodology in making cost estimates involve (1) an assumption that a relationship between prototype production and investment phase production costs exists, and (2) the development of a factor based usually on limited information. There is good justification for the assumption. Essentially, the same performance characteristics apply for each model and often the models contain many of the same component parts. That this approach is used rather widely, alone, is one justification for its validity. However, to our knowledge, there has been no careful statistical verification of such a relationship.

1.2 STUDY OBJECTIVE

In light of the above facts, Tecolote Research, Inc. (TRI) was asked by the Laser Designator Program Office (USA MICOM) to investigate the quantitative validity of this hypothesis for electronic programs--especially as they pertain to the requirements of the Ground and Airborne Laser Locator Designators (GLLD and ALLD, respectively). Specifically, two principal tasks were addressed in this analysis:

- Examine the validity of a relationship between the cumulative average cost (independent variable, $\overline{C}_p$) of engineering development phase (prototype production) hardware to the first unit cost of producing investment phase hardware (dependent variable, T_1), and present substantiation of the position on validity including any resulting relationship.
- Examine the validity of two cost improvement theories, log-linear cumulative average and log linear unit in determining the theoretical first unit cost of production (T_1) used in the above relationship.

The remaining sections of this document addressing these objectives are organized in the following way: Section 2.0 addresses the methodology employed in the study and develops the rationale for the formulation of the various cost estimating relationships. Section 3.0 outlines the data base used for this study including a discussion on work breakdown structure (WBS) detail, normalizations, and data sources. Section 4.0, Hypothesis Testing, presents the results of the study both from the standpoint of the principal tasks outlined earlier as well as other observations and results that were revealed in the course of the analysis.

2.0 HYPOTHESIS DEVELOPMENT

The goals of the hypothesis development task are twofold: (1) to develop a model which relates a variable depicting the cost of prototype units to a measure of investment phase production costs* (theoretical first unit cost, T_1); and (2) to develop a method for the determination of the most appropriate cost improvement theory. Refinements of the basic factor model in terms of other cost impacts are also a consideration in the development of the hypothesis.

The major issues that need resolution related to these goals are:

(1) Is the production cost improvement curve a continuation of the prototype cost-quantity curve with the exception of a displacement in the curve when production begins.

(2) Does the quantity of prototypes built affect the relationship between production and prototype costs?

(3) Is log-linear cumulative average cost theory more appropriate than log-linear unit cost theory in depicting cost quantity relationships?

Concerning the first of these issues, there are two prominent theories and they differ only with respect to production quantity. One theory which we call "disjoint" (Figure 2.1) consideres the first production unit as unit number one, while the other theory called "sequential" (Figure 2.2) considers the first production unit as unit number (Q_P+1) , where Q_P is the number of prototype units. The result of the sequential assumption is a differing cost impact in production (cost reductions over the early production units is smaller), and T_1 , the theoretical first unit cost, is measured at a different point (the first prototype unit). As is evident T_1/T_{1P} is greater in the sequential theory than in the disjoint theory. Here the improvement

*Appendix B discusses an alternative formulation of the production/prototype cost relationship wherein a theoretical first unit cost and improvement rate are not derived for each program. Instead, the complete cost-quantity population is estimated in a single regression model. This is an unconventional approach which was not pursued at length, but the results are included for the interested analyst.

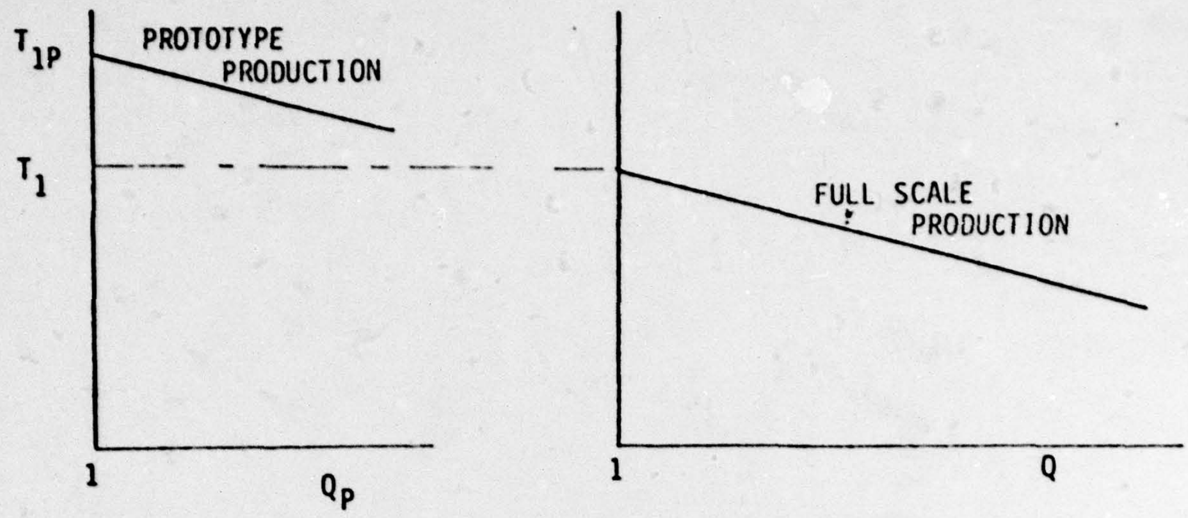


FIGURE 2.1
THE DISJOINT HYPOTHESIS

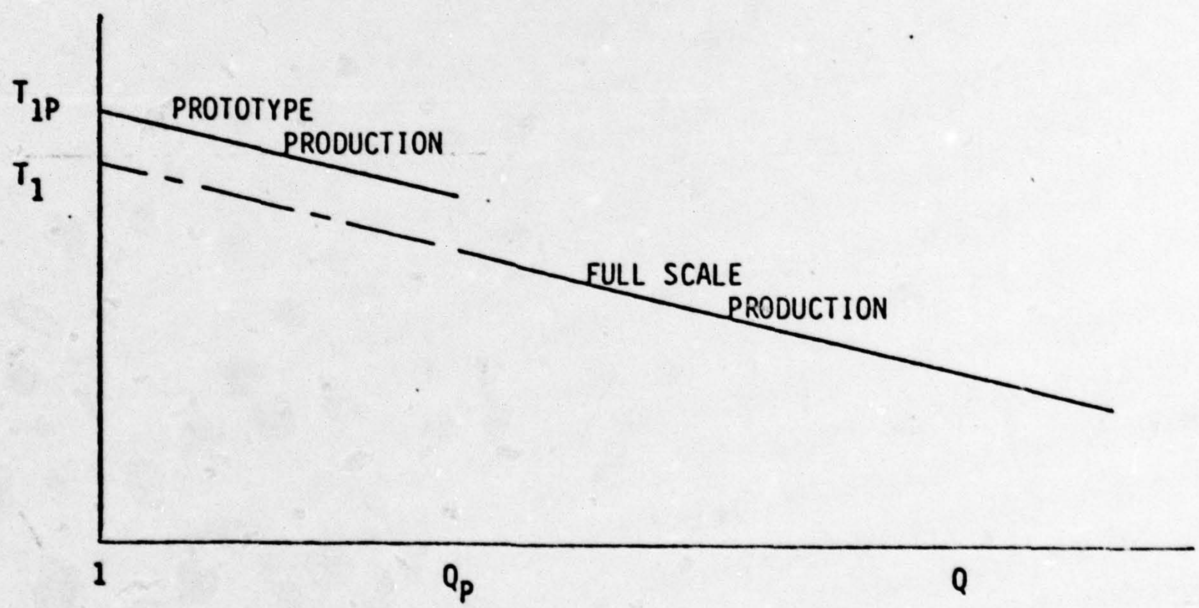


FIGURE 2.2
THE SEQUENTIAL HYPOTHESIS

rate derived from the production lot data is applied to the prototype cost data to determine T_{1P} . T_1 in the sequential case is determined at an earlier quantity (the first prototype unit).* The effect of adding prototype quantities to the production lot data is to steepen the improvement curve fit to the lot data as the lot data at small quantities is shifted farther to the right by the addition of prototype units than that data at larger quantities. The selection of the best hypothesis among these two will depend on which produces the smallest estimating error.

Initially, we intended to examine only the relationship between the average prototype cost $(\bar{C}_P)$ and T_1 ; however, we feel that variations in $\bar{C}_P$ because of differing quantity values is significant and that a better measure of prototype cost is the theoretical first prototype unit cost calculated assuming that production improvement rates are the same as would be observed in the prototype phase. This presumes that any error in such an extrapolation of improvement rates is less than the quantity variation alluded to. We will test this hypothesis by examining T_1 as a function of T_{1P} and C_P separately.

The quantity of prototypes (Q_P) should also be considered as an independent variable for several reasons. The incorporation of Q_P could indicate the use of an improper improvement rate in the prototype phase. If there were no cost improvement in the prototype phase, we would have overstated T_{1P} and Q_P would compensate by having a negative coefficient under each hypothesis. Alternatively, a negative coefficient in the disjoint hypothesis might suggest that there is learning transfer between the prototype and investment phase; i.e., larger prototype quantities lead to a lower T_1 , an implicit feature of the sequential model.** Such a result might verify the sequential hypothesis. A prototype quantity variable should be employed along with $\bar{C}_P$ to assess the significance of cost improvement. Economies of scale in production can be tested by examination of the non-linearity of T_1 with T_{1P} .

---

*T_1 is estimated assuming that the prototype buy is a missing lot and this is then compared with T_{1P} .

**T_1 in the disjoint model is unit number Q_P+1 in the sequential model.

The remaining issue of importance is the appropriate cost improvement theory to apply to determine first unit cost T_1. The obvious criterion here is which theory fits the production lot data best. Unfortunately, the difference between each theory is apparent only at quantities that are small relative to typical production lot quantities. This is true because the cumulative average unit cost curve asymptomatically parallels a log linear unit cost curve so that a unit cost fit is indistinguishable from a cumulative average unit cost curve as is shown in Figure 2.3. The error in assuming log linear unit cost if cumulative average theory is true is shown in Figure 2.4 for an 85% improvement rate. This error is smaller for lower improvement rates. The level of error after a few units is small with respect to typical cost estimating uncertainty.

Because prototype quantities are smaller than their typical production lot buys suggest that prototype data might provide a better indicator of which theory is appropriate. This position may be illuminated by reference to Figure 2.5 where we contrast the results of the factor model fits for large and small quantity prototype programs. The application of an incorrect improvement theory where only a few prototypes are built leads to a bias in the determination of the factor relating production to prototype cost. This figure involves the following assumptions: (1) that the disjoint hypothesis applies; (2) that log linear cumulative average cost theory holds true; (3) that the production and prototype improvement rates are equal; and (4) that T_1 is equal to T_{1P}, i.e., the factor T_1/T_{1P} equals one.

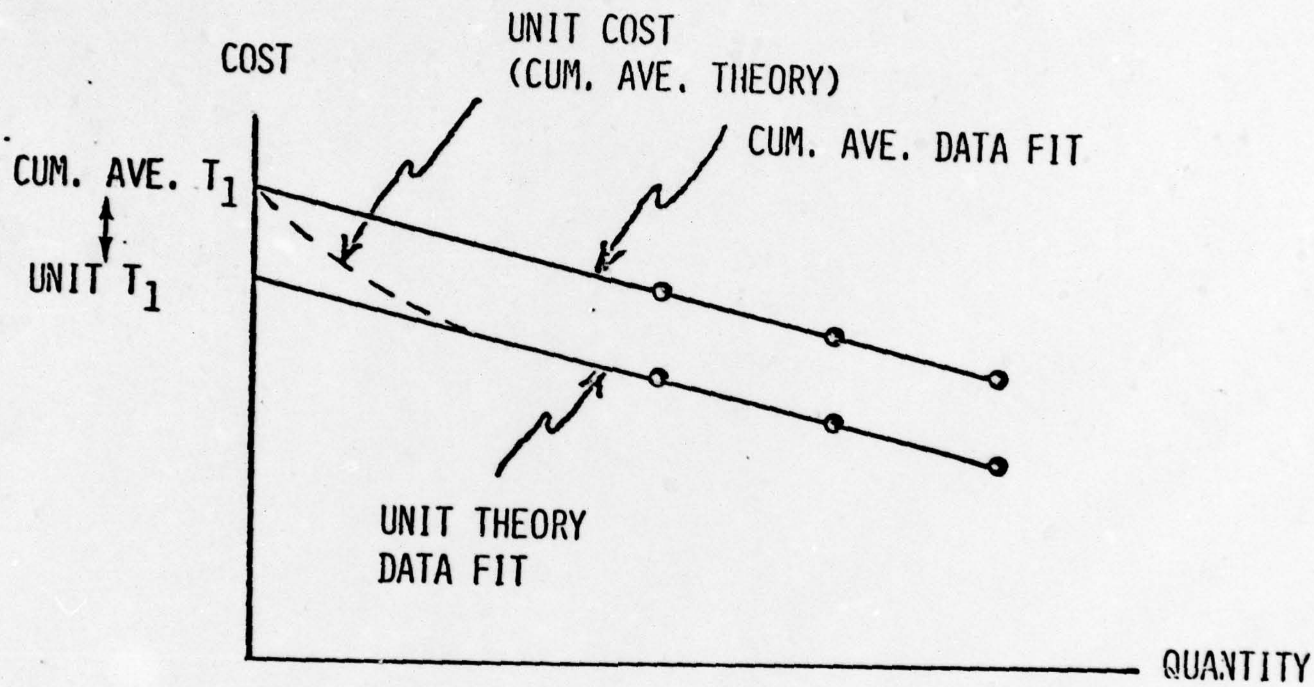
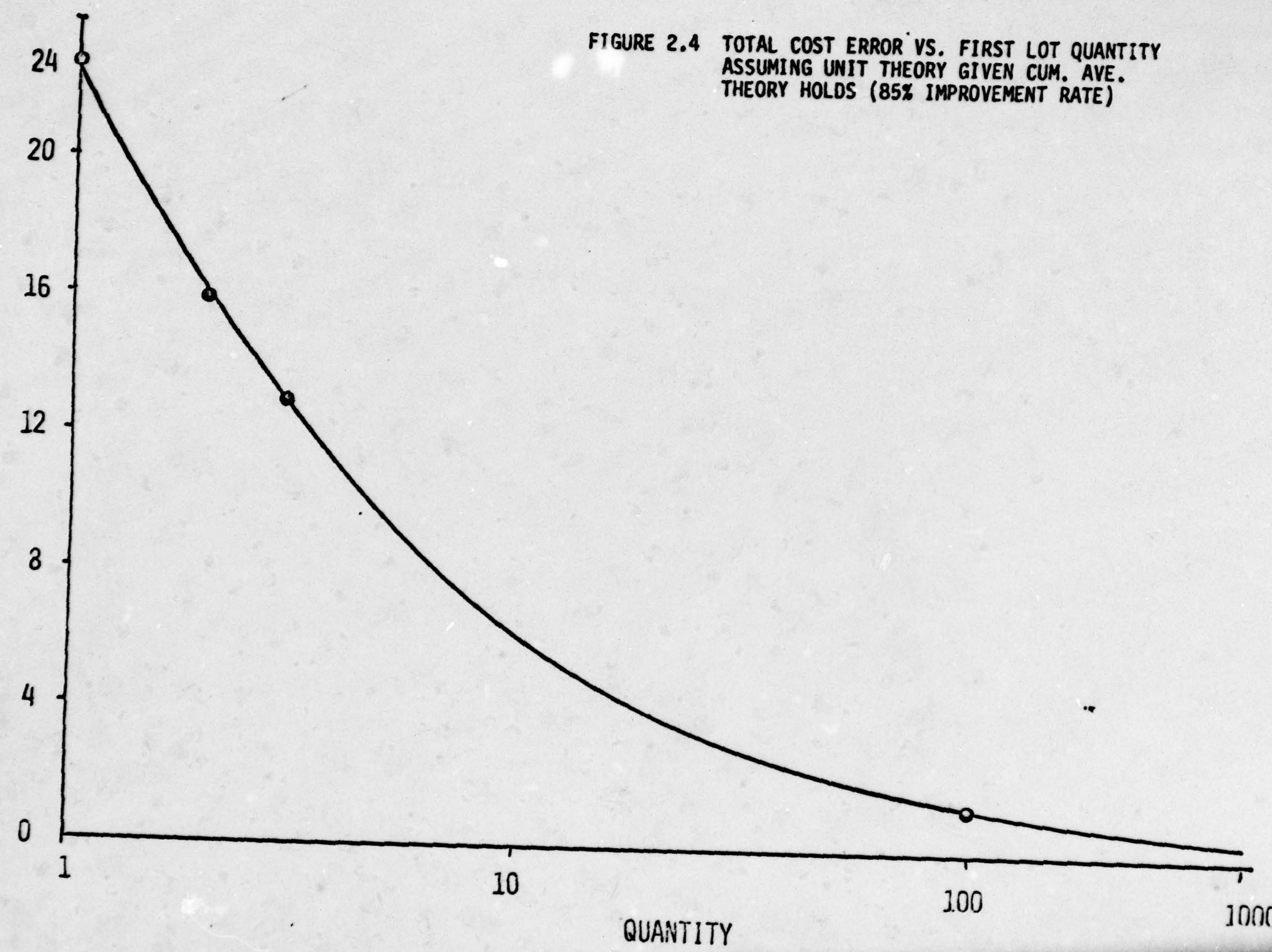


FIGURE 2.3

LOG LINEAR UNIT THEORY FIT

OF LOG LINEAR CUMULATIVE AVERAGE UNIT DATA

FIGURE 2.4 TOTAL COST ERROR VS. FIRST LOT QUANTITY ASSUMING UNIT THEORY GIVEN CUM. AVE. THEORY HOLDS (85% IMPROVEMENT RATE)



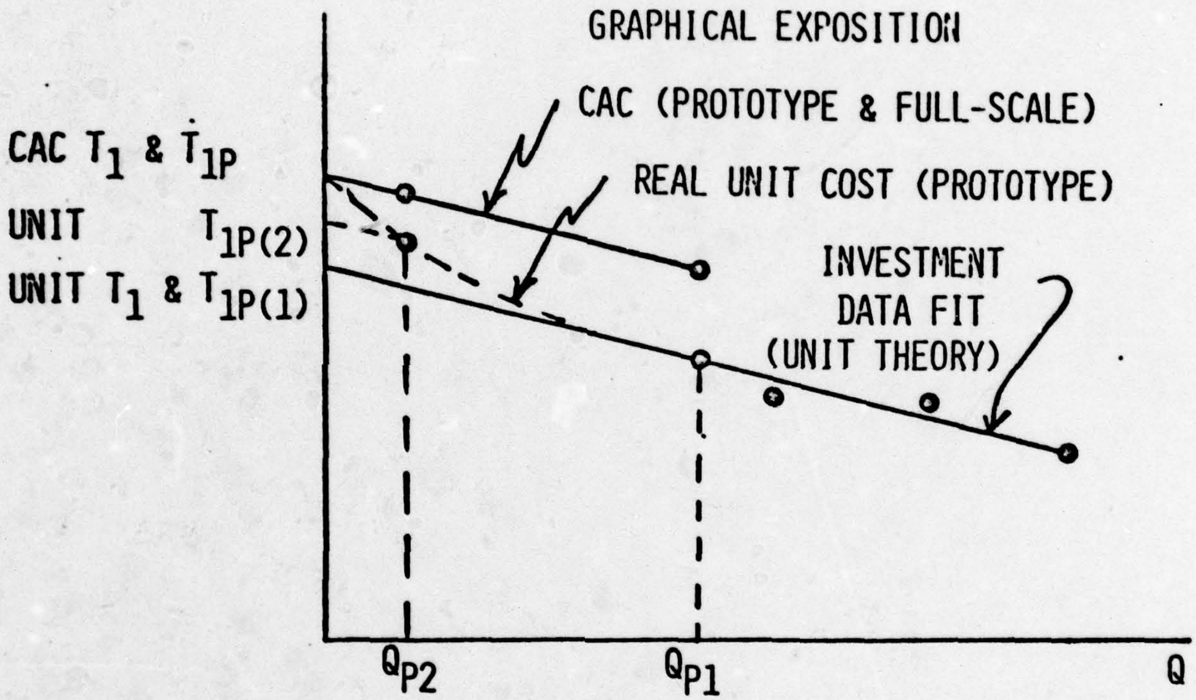


FIGURE 2.5

THE EFFECT ON T_1/T_{1P} OF THE APPLICATION
OF THE IMPROPER IMPROVEMENT THEORY
FOR TWO DIFFERENT PROTOTYPE QUANTITIES

If log-linear unit theory is applied to the production data, the resulting fit, given that first lot buy is not small, will be essentially the same as the real unit cost curve under cumulative average theory at these quantities. Similarly, the application of unit theory to the prototype data will produce an approximation of the real unit cost curve where prototypes are not small as indicated by Q_{P1} .* Should the prototype quantity be small, however, (Q_{P2}) , then the real unit cost would lie above the investment data fit and the calculated first unit prototype cost $(T_{1P(2)})$ would be greater than T_1 , producing a factor that is less than one.** This indicates that the application of the incorrect improvement theory biases the real factor and the degree of bias depends on the quantity of prototypes built. This, in turn, suggests that the incorrest theory will produce poorer fit statistics where a cross section of programs are examined and one theory tends to apply. Poorer factor model fits in one theory given small prototype quantities may then present a basis for selecting the most appropriate theory.

---

*Because the factor is equal to one, the production and prototype improvement curves under unit theory should approximately overlap given a large prototype quantity.

**Where log linear unit theory is true and cumulative average theory is applied, there will be a similar bias in the opposite direction.

3.0 DATA REDUCTION AND NORMALIZATION

The analysis of the relationship between prototype and production costs presents fairly stringent requirements on the data to be employed and substantially limits the number of usable data points. Of course, both prototype and investment phase data is required and this data must exclude all of the non-recurring items such as design engineering in the prototype phase. Only electronics items were considered. Specifically included in the individual item costs are the recurring costs for the items identified in Form DD-1558-1; i.e., recurring engineering tooling, quality control, manufacturing material, and purchased equipment. Excluded are such system level items as program management, data and system engineering. The data set was also reduced by the exclusion of programs where prototype phase data of appropriate detail was available, but production data was available only in the form of an estimate. Probably the most restrictive constraint was the need for recurring costs in the prototype phase.

We expected that the relationship between prototype and investment units would vary with the type of equipment. To verify this, we attempted to incorporate a variety of electronics systems in the data set. Our best source of data initially was in the tactical missile area, so we focussed our effort on adding avionics and ground equipment to this base. We also disaggregated the cost data as much as was possible in the equipment (WBS) dimension in order to broaden the sample and to provide visibility for assessing factor variations with equipment type. Usually, such a breakdown is limited to Level 3 (the subsystem level) of a WBS.

The cost data employed in this report are summarized in Tables 3.1 through 3.3. All of the data shown in these tables have been normalized for cost, quantity, and fiscal escalation using a TRI computer routine called CØSTNØR. All dollar figures are presented in constant fiscal year 1974 dollars and were escalated to this by using the OSD (Comptroller) procurement escalation index shown in Table 3.4. In all cases, the data presented include general and administrative expense, but do not include contractor fee.

TABLE 3.1 DATA SET FOR THE SEQUENTIAL HYPOTHESIS
ALL COSTS IN THOUSANDS OF FY 1974 DOLLARS
LOG LINEAR CUMULATIVE AVERAGE COST IMPROVEMENT CURVE THEORY ASSUMED

	PROTOTYPE			FULL SCALE PRODUCTION		RATIO
	TOTAL QUANTITY Q_p	AVERAGE UNIT COST $\bar{C}_p$	FIRST UNIT COST T_{1P}	IMPROVEMENT CURVE SLOPE $S\%=100 \times 2^b$	FIRST UNIT COST T_1	OF FIRST UNIT COSTS T_1/T_{1P}
MISSILES						
GUIDANCE						
PHOENIX	73	461	4129	70.2	3902	.95
MAVERICK	25	132	263	86.2	142	.54
STANDARD	34	233	1073	73.6	1096	1.02
CONTROL						
PHOENIX	72	230	2610	67.4	2602	1.00
STANDARD	36	202	698	78.7	382	.55
ELECTRONICS						
AWG-9						
RADAR	25	2660*	20305	70.6	16259	.80
INFRARED	25	259*	1053	78.6	810	.77
COMPUTER	25	1274*	6243	75.6	4571	.73
DISPLAY	25	599*	4609	70.6	3941	.86
IMPROVED HAWK						
ACQ. RADAR	3	452	672	77.9	661	.98
COMPUTER	3	686	844	87.7	720	.85
ILL. RADAR	3	357	653	68.3	862	1.32
POWER SUPPLY	3	349	414	89.8	225	.54
DOPPLER RADARS						
APN-189	8	220	302	90.0	175	.58
APN-172	15	157	237	90.0	191	.81
APN-175	10	82	117	90.0	130	1.11

* $\bar{C}_p$ for last 3 prototype units

TABLE 3.2 DATA SET FOR THE DISJOINT HYPOTHESIS
ALL COSTS IN THOUSANDS OF FY 1974 DOLLARS
LOG LINEAR CUMULATIVE AVERAGE COST IMPROVEMENT CURVE THEORY ASSUMED

	PROTOTYPE			FULL SCALE PRODUCTION		RATIO
	TOTAL QUANTITY Q_P	AVERAGE UNIT COST $\bar{C}_P$	FIRST UNIT COST T_{1P}	IMPROVEMENT CURVE SLOPE $S\%=100\times2^b$	FIRST UNIT COST T_1	OF FIRST UNIT COSTS T_1/T_{1P}
MISSILES						
GUIDANCE						
PHOENIX	73	461	1566	82.1	626	.40
MAVERICK	25	132	244	87.7	111	.46
STANDARD	34	233	1058	74.3	953	.90
CONTROL						
PHOENIX	72	230	885	80.4	324	.37
STANDARD	36	202	666	79.4	337	.51
ELECTRONICS						
AWG-9						
RADAR	25	2660*	5614	85.2	2563	.46
INFRARED	25	259*	378	92.1	209	.55
COMPUTER	25	1274*	2379	87.4	1114	.47
DISPLAY	25	599*	1240	85.5	600	.48
IMPROVED HAWK						
ACQ. RADAR	3	452	595	84.1	375	.63
COMPUTER	3	686	800	90.8	558	.70
ILL. RADAR	3	357	558	75.5	357	.64
POWER SUPPLY	3	349	398	92.0	188	.47
DOPPLER RADARS						
APN-189	8	220	302	90.0	166	.55
APN-172	15	157	237	90.0	172	.73
APN-175	10	82	117	90.0	128	1.10

*$\bar{C}_p$ for last 3 prototype units

TABLE 3.3 DATA SET FOR THE DISJOINT HYPOTHESIS
ALL COSTS IN THOUSANDS OF FY 1974 DOLLARS
LOG LINEAR UNIT COST IMPROVEMENT CURVE THEORY ASSUMED

	PROTOTYPE			FULL SCALE PRODUCTION		RATIO OF FIRST UNIT COSTS
	TOTAL QUANTITY Q_P	AVERAGE UNIT COST $\bar{C}_P$	FIRST UNIT COST T_{1P}	IMPROVEMENT CURVE SLOPE $S\%=100x2^b$	FIRST UNIT COST T_1	T_1/T_{1P}
MISSILES						
GUIDANCE						
PHOENIX	73	461	1172	81.7	469	.40
MAVERICK	25	132	204	87.5	91	.45
STANDARD	34	233	660	74.3	545	.83
CONTROL						
PHOENIX	72	230	643	79.8	235	.37
STANDARD	36	202	468	79.4	225	.48
ELECTRONICS						
AWG-9						
RADAR	25	2660*	4646	83.9	2193	.47
INFRARED	25	259*	348	91.3	195	.56
COMPUTER	25	1274*	2031	86.4	974	.48
DISPLAY	25	599*	1037	84.1	518	.50
IMPROVED HAWK						
ACQ. RADAR (AR)	3	452	531	82.2	314	.59
COMPUTER (ICC)	3	686	750	89.9	505	.67
ILL. RADAR (HPI)	3	357	459	73.1	258	.56
POWER SUPPLY (BCC)	3	349	376	91.4	171	.45
DOPPLER RADARS						
APN-189	8	220	268	90.0	122	.46
APN-172	15	157	207	90.0	128	.62
APN-175	10	82	103	90.0	92	.89

*$\bar{C}_P$ for last 3 prototype units

TABLE 3.4

OSD PROCUREMENT ESCALATION INDICES - AUGUST 1975

FY	74	75	76	7T	77
63	1.493	1.808	1.952	2.023	2.100
64	1.484	1.797	1.940	2.011	2.087
65	1.447	1.752	1.892	1.961	2.035
66	1.418	1.717	1.854	1.921	1.994
67	1.387	1.679	1.813	1.879	1.951
68	1.351	1.636	1.766	1.831	1.900
69	1.309	1.585	1.711	1.774	1.841
70	1.247	1.510	1.630	1.690	1.754
71	1.178	1.426	1.540	1.596	1.657
72	1.135	1.374	1.484	1.538	1.596
73	1.100	1.332	1.438	1.491	1.547
74	1.000	1.211	1.307	1.355	1.406
75	0.826	1.000	1.080	1.119	1.162
76	0.765	0.926	1.000	1.037	1.076
7T	0.738	0.893	0.965	1.000	1.038
77	0.711	0.861	0.929	0.963	1.000
78	0.678	0.821	0.886	0.919	0.954
79	0.651	0.788	0.851	0.882	0.916
80	0.626	0.758	0.818	0.848	0.880
81	0.602	0.729	0.787	0.816	0.847
82	0.579	0.701	0.757	0.785	0.814
83	0.557	0.674	0.728	0.755	0.783
84	0.536	0.649	0.701	0.726	0.754

SOURCE: OSD (COMPTROLLER) 22 August 1975

3.1 PRODUCTION LOT DATA ANALYSIS

The availability of a TRI developed computer program called CØSTNØR greatly expedited much of the tedious (and, therefore, error-prone) and time-consuming cost data normalization effort undertaken in estimating production first unit costs for both quantity theories. As used in this study, the program was utilized for determining values of the cost improvement curve parameters when lot cost and lot quantity data were given for two (or more) production lots. The data need not be sequential and need not include the first production lot. This feature allowed the determination of T_1 under the discontinuous hypothesis by considering the prototype lot as a missing lot of quantity, Q_P .

The CØSTNØR routine employs an iterative procedure to establish the theoretical first unit cost and improvement rate under either log linear cumulative average or log linear unit theory. It operates on lot average cost data so that no smoothing of data occurs, thereby retaining all the information content of the original data. This also permits the determination of cumulative average curves where some lot data is missing. Appendix A describes this computer program in greater detail.

3.2 PROGRAM DATA DESCRIPTIONS

The following subsection briefly addresses each of the data elements used in this analysis.

3.2.1 Phoenix Missile - Guidance and Control

The Phoenix missile cost data were obtained directly from the accounting records of the Hughes Aircraft Co. production facility in Tucson, Arizona, and the Missile Division at Canoga Park. The data for production during fiscal years '73, '74, and '75 are actual recurring costs incurred based on personnel time card entries and material costs.* The FY'76 data are actuals plus purchase order commitments as of 6 June 1975. Quantity information for FY'76 is the number of "equivalent units". The accounting records are maintained separately for each fiscal year '73 through '74.

The production cost accounting records are very detailed, providing fabrication, assembly, inspection and test, and material cost information for each WBS element. These detailed accounting records were aggregated and summarized by WBS elements. Fabrication and Assembly activities were judged to be most nearly comparable with CIR/CCDR descriptions of manufacturing. Similarly, Inspection and Test were judged to be comparable to quality assurance. It was not possible, however, to relate the engineering costs to WBS elements, nor was it possible to clearly identify the recurring tooling costs. CIR's for the Phoenix were examined to verify these observations. All of the 1558-1 CIR forms available at this time (through 12/74) confirm that recurring tooling costs are not identifiable. It was explained that recurring tooling costs are accrued as incurred into the activity accounts described above (e.g., fabrication, assembly, etc.). Factory sustaining and support costs could not be related to WBS elements and were, therefore, aggregated and allocated to the hardware elements.

*The fiscal years indicated here represent the approximate data at the midpoint of the actual delivery schedule.

The RDT&E phase cost data was obtained from the accounting records for three separate contracts: (1) the initial RDT&E program, (2) the Test Prototype Missile program, and (3) the Separation and Test Missile program. Information was available to the subsystem level on manufacturing and test costs. Production (factory) support costs were not available to this level, however, and were allocated to the subsystems on the basis of particular subsystems cost. The average prototype cost $(\overline{C}_p)$ in Table 3.1 - 3.3 is the average for all three contracts.

3.2.2 Maverick Missile - Guidance and Control

Maverick missile production cost data were obtained from the System Program Office (SPO) at Wright-Patterson Air Force Base, Dayton, Ohio. The data represent the results of a detailed analysis of contractor records conducted by the SPO over a period of several weeks at the Hughes Aircraft Co. production facility in Tucson, Arizona. Adjustments were made for material burden, material allowances, abnormal economic escalation, and contract incentives. The allocations and adjustments were performed by SPO personnel with assistance and general guidance from HAC.

There were data from four full scale production contracts (Options A, B, C, D) and one pilot production contract used to fit the cost improvement curve for the Maverick Missile.

The prototype data were obtained from the Hughes Aircraft Co., Canoga Park facility. These data did not provide the level of detail available for the production history, hence, production cost data were aggregated to a level comparable with the prototype information. Prototype fabrication occurred in Canoga Park, California, (an R&D facility) and full scale production in the Tucson, Arizona, production plant. The prototypes were built as part of the single DDT&E program which also involved the fabrication of the pilot production missiles or Tucson "G" missiles built at the Hughes Tucson facility. The detailed accounting records for this program allowed distinguishing between the costs for each, which cost performance and CIR reports did not permit. The prototype cost detail, however, did not allow the separation of guidance and control sections except for the hydraulic actuation system which was not of concern in this study. Prototype and pilot production model quantities were obtained from Maverick SPO.

3.2.3 Standard Missile - Guidance and Control

Standard missile production cost data were obtained from the General Dynamics Corporation production facility in Pomona, California. The data were extracted from ledgers in their industrial accounting office and represent actual costs incurred. The contractor maintains a contract WBS to which he assigns accounting work order (AWO) numbers. While the WBS structure appeared to be quite detailed, it provided very limited hardware visibility below the missile section level; e.g., guidance section.*

Prototype cost data (Contract No. NOW65-0119i) was also extracted from General Dynamics accounting data. This contract also funded the production of the 100 pilot line missiles. The accounting record detail allowed the separation of costs for guidance, control, and ordnance sections.** The average costs for the standard prototype sections may be slightly overstated since the prototype fabrication assembly and checkout task encompassed some completion of the design task of the prior contract.

3.2.4 AWG-9 Fire Control Radar

The cost data for the AWG-9 system was obtained from CIR reports for both RDT&E and investment phase equipment. Production lot data consisted of production buys for FY'71 through FY'75. The CIR's provided this information at the subsystem level, i.e., radar, computer, etc. Indicated CIR quantities contained equivalent spares where the costs for spares were separately identified necessitating quantity adjustments.

*In the course of the data analysis, it was learned that a pilot production program for 100 missiles had preceded the first full scale production contract. While cost data was not available for the pilot production effort, the improvement curve analysis allows for these 100 units by treating them as missing lot data.

**Ordnance section cost data was not analyzed in this study because of the many participants (contractor and government) in both RDT&E and investment phase and the corresponding problem of complete cost data.

The prototype cost information was obtained from Contract No. N00019-70-C-0207 wherein three prototype models (XN-3) and four pilot models (YN-3) were produced. These three prototypes were the last of a series of 25 prototype models built in the AWG-9 RDT&E phase. Recurring costs for prototypes of earlier contracts could not be established by the contractor. It was necessary to estimate recurring engineering for the pilot production units in order to establish prototype recurring costs in the -0207 contract. This cost was estimated as equal to the average recurring engineering fraction observed in the following four production lots where little variance in this fraction was noted.

3.2.5 Improved HAWK - Ground Fire Control

Cost and quantity data for the Improved Hawk (I/H) acquisition radar (AR), data processor (ICC), high-powered illuminator (HPI), and battery control center (BCC) were obtained from CEIS files at USA MICOM. The information for both prototype and full-scale production was retrieved directly (with appropriate fiscal escalation) from government forms DD-1737 and DD-1177, the reporting forms required to be completed by the prime contractor (Raytheon, Andover Plant) for the single prototype production contract (three units) and the three full-scale production contracts. Complete and reliable data was not available on the fourth and last full-scale production buy.

In all cases, cost data represents recurring costs for engineering, tooling, quality control, and manufacturing and material. A G&A rate of 11.3% was shown to be representative by USA MICOM personnel and was applied to the direct charges.

3.2.6 Doppler Navigation Systems

Costs for these avionics units were provided by the Canadian Marconi Corporation based on their accounting records and corresponding to the cost elements of Army Regulation 37-18. No production lot data was available; the contractor indicated that a 90% learning rate was consistent with their experience. These units were all developed in the mid to late 1960's with the APN-175 representing the oldest technology.

3.2.7 Program Deletions

It had originally been the intent to incorporate Redeye, TOW, Shillelagh, Improved Hawk, and Sparrow missiles to the data base in an effort to enrich the sample. Early in the data compilation effort, however, it was learned that RDT&E information for the Redeye, TOW, and Shillelagh systems was sparse in both cost and quantity detail. For example, a visit to the TOW Project Office indicated that although very aggregate or "bottom line" RDT&E cost data were available, it consisted of cost elements from the inception of the RDT&E phase through its completion. At best, it would have been very difficult to break out or even estimate prototype production costs. With respect to the Redeye and Shillelagh RDT&E costs, information was virtually unavailable.

The Improved Hawk and Sparrow missiles were excluded from the data base for other reasons. In the case of the Improved Hawk, it could not accurately be determined what elements made up the guidance and control subsystems for the second, third, and fourth full-scale production contracts. The reason for this is that in the middle of the second production buy, Raytheon changed accounting systems. This necessitated an amendment to their governmental cost reporting requirements. In the course of this action, the WBS structure was altered, making homogeneous comparisons between contracts infeasible within the time frame of the study. Further, DD-1737 quantity information for the last three production buys is no longer required by the government on a routine basis. As such, the information would have to be acquired directly from Raytheon (Andover).

It was felt that the Sparrow missile system should not be included in the study's data base due principally to the large degree of uncertainty in the costs for the full-scale production phase. Apparently, once full-scale production began and after a number of units had been manufactured, reliability and performance requirements (MILSPEC) were made more stringent. This caused the prices for the latter articles to be somewhat higher in cost than the earlier models. The problem was determining the magnitude of this MILSPEC change on costs, since to incorporate the cost data without an accounting of the cost increase would lead to biased and erroneous results when fitting the cost improvement relationship. This, in turn, would incorrectly cause full-scale production costs to appear greater than prototype costs.

4.0 HYPOTHESIS TESTING

The three dimensional figure of Figure 4.1 reveals the breadth of the analysis undertaken in this study. The major issue to be addressed by the analysis are depicted in the three dimensions of the figure. The checkmarks in the individual blocks indicate the areas addressed in the analysis. We did not apply log linear unit theory in the testing of the sequential hypothesis. This area was deleted because we did not expect to see any significant differences between the sequential and disjoint results employing unit theory. Table 4.1 lists the algebraic forms of the models that were statistically tested. The logarithmic form was employed because log normal disturbances are considered most appropriate.* T_{1P} was incorporated in the dependent variable in order to constrain the exponent on T_{1P} to a value of one, i.e., a proportional relationship between T_1 and T_{1P} . Also note that Models 5 through 8 involve the substitution of $\overline{C}_P$ for T_{1P} in Models 1 through 4, respectively.

The results of the hypothesis testing corresponding to the blocks of Figure 4.1 are presented in Tables 4.2 through 4.4 . The tables contain the values of the coefficients for the various independent variables and fit statistics for the various models. Each of the issues addressed by this study will be considered in the light of the statistical results of these tables.

The most important conclusion that can be made based on the results shown in the tables is that the estimated first unit prototype cost (T_{1P}) is a good estimator of the first unit production cost. Both quantity hypotheses involving only T_{1P} produce R^2 in the .90 area with standard errors under 0.3.** The goodness of the fit is better observed graphically as shown in Figure 4.2 through 4.4 for the various formulations of the proportional model.

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*This specification implies a constant percentage error across the range of T_{1P} values. It also prohibits negative values of the ratio.

**The values of R^2 were calculated on variations in $\ell n T_1$, not $\ell n T_1/T_{1P}$. The latter form of independent variable was necessary to constrain the relationship to be a proportional one. The variation in T_1 is the proper criterion, i.e., the matter of concern.

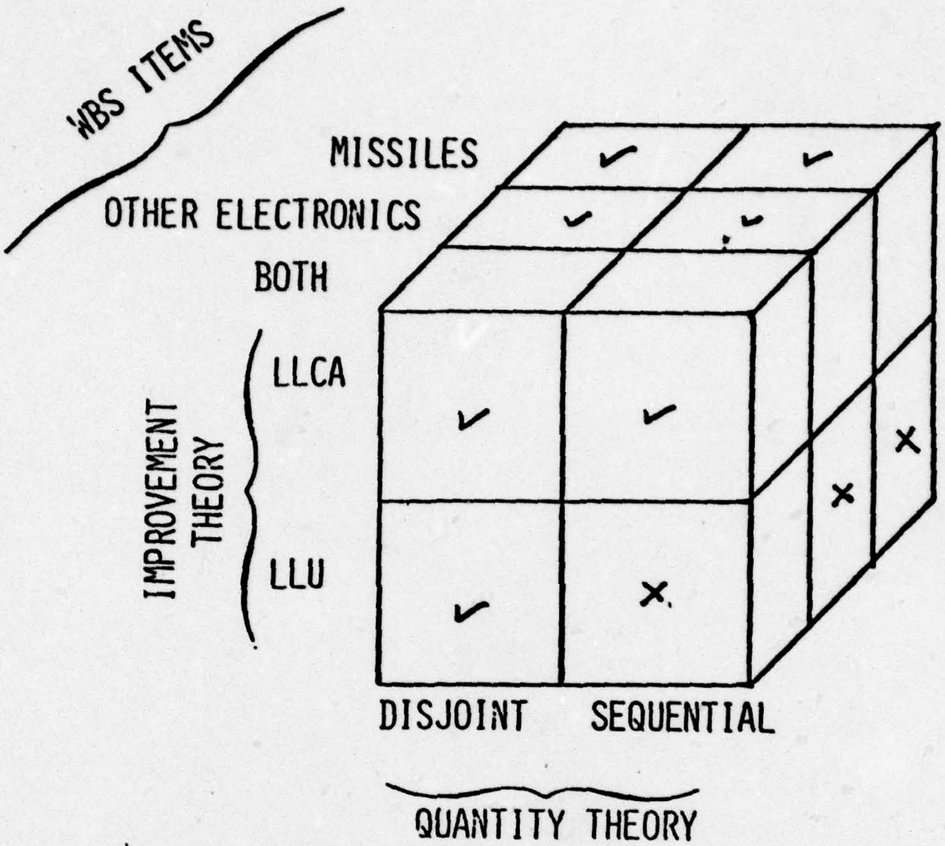

FIGURE 4.1 SPECTRUM OF TESTED MODELS

Model No. 1 $\ln T_1/T_{1P} = \ln A$

2 $\ln T_1/T_{1P} = \ln A + \alpha \ln T_1P$

3 $\ln T_1/T_1P = \ln A + \beta \ln Q_P$

4 $\ln T_1/T_1P = \ln A = a \ln T_{1P} + \beta \ln Q_P$

5 - 8 SUBSTITUTE $\overline{C}_P$ FOR T_{1P} IN EACH OF ABOVE MODELS ($\overline{C}_P$ NOT ANALYZED IN DISCONTINUOUS CASE)

TABLE 4.1 ALGEBRAIC FORMS OF TESTED MODELS

TABLE 4.2

HYPOTHESIS TESTING RESULTS
CUM. AVE. THEORY
SEQUENTIAL HYPOTHESIS

	MODEL NO.	lnA	lnT_{1P}	lnQ_P	σ	R^{2*}
ALL	1	-.211			.272	.965
	2	-.577	.026X		.279	.965
	3	-.166		-.016X	.281	.965
	4	-.676	.042X	-.040X	.286	.966
MISSILES	1	-.252			.329	.942
	2	-3.654	.244Y		.225	.980
	3	-2.007		.464X	.279	.969
	4	-5.253	-.561 X	-.750X	.233	.985
OTHER ELECTRONICS	1	-.192			.258	.972
	2	.012	-.015X		.270	.973
	3	-.904		-.044X	.268	.973
	4	-.089	-.0004X	-.044X	.284	.973

*CALCULATED WITH lnT_1 AS DEPENDENT VARIABLE: not lnT_1/T_{1P1}

SIGNIFICANCE LEVELS

X = >10%

Y = > 5%

Z = >2.5%

TABLE 4.3

HYPOTHESIS TESTING RESULTS
CUM. AVE. COST THEORY
DISJOINT HYPOTHESIS

	MODEL NO.	lnA	lnT_{1P}	lnQ_P	$ln\bar{C}_P$	σ	R^{2*}
ALL	1	-.575				.294	.884
	2	1.403	-.148Z			.266	.911
	3	-.281		-.109Y		.277	.904
	4	1.206	-.118Y	-.073X		.264	.919
	5	..130				.453	.620
	6	1.187			-.083X	.464	.628
	7	-.508		.236		.382	.747
	8	.320		..233	-.065X	.392	.752
OTHER ELECTRONICS	1	-.519				.262	.919
	2	1.769	-.171			.193	.961
	3	-.353		-.075X		.265	.926
	4	1.752	-.169	-.009X		.204	.961
	5	-.0295				.292	.842
	6	.252			-.022X	.307	.843
	7	-.432		.182Z		.246	.899
	8	.210		.188Y	-.051	.256	.903
MISSILES	1	-.698				.354	.811
	2	-1.319	.046X			.407	.812
	3	.803		-.397X		.344	.866
	4	-4.794	.599Z	-1.061Z		.170	.978

*Calculated with lnT_1 as dependent variables; not lnT_1/T_{1P}

SIGNIFICANCE LEVELS

X = >10%

Y = > 5%

Z = >2.5%

TABLE 4.4

HYPOTHESIS TESTING RESULTS
LOG-LINEAR UNIT THEORY
DISJOINT HYPOTHESIS

	MODEL NO.	lnA	lnT_{1P}	lnQ_P	$ln\bar{C}_P$	σ
ALL	1	-.658				.287
	2	..598	-.095X			.284
	3	-.484		-.064X		.288
	4	.521	-.079X	-.047X		.289
	5	-.115				.360
	6	-.0083			-.0084X	.373
	7	-.650		.198		.295
	8	-.745		.198	.007X	.306
OTHER ELECTRONICS	1	-.625				.282
	2	.826	-.110X			.271
	3	-.552		-.033X		.295
	4	.840	-.112X	.008X		.288
	5	-.224				.271
	6	-.723			.039X	.284
	7	-.657		.196		.205
	8	-.766		.195Z	.009X	.218

SIGNIFICANCE LEVELS

X = >10%

Y = > 5%

Z = >2.5%

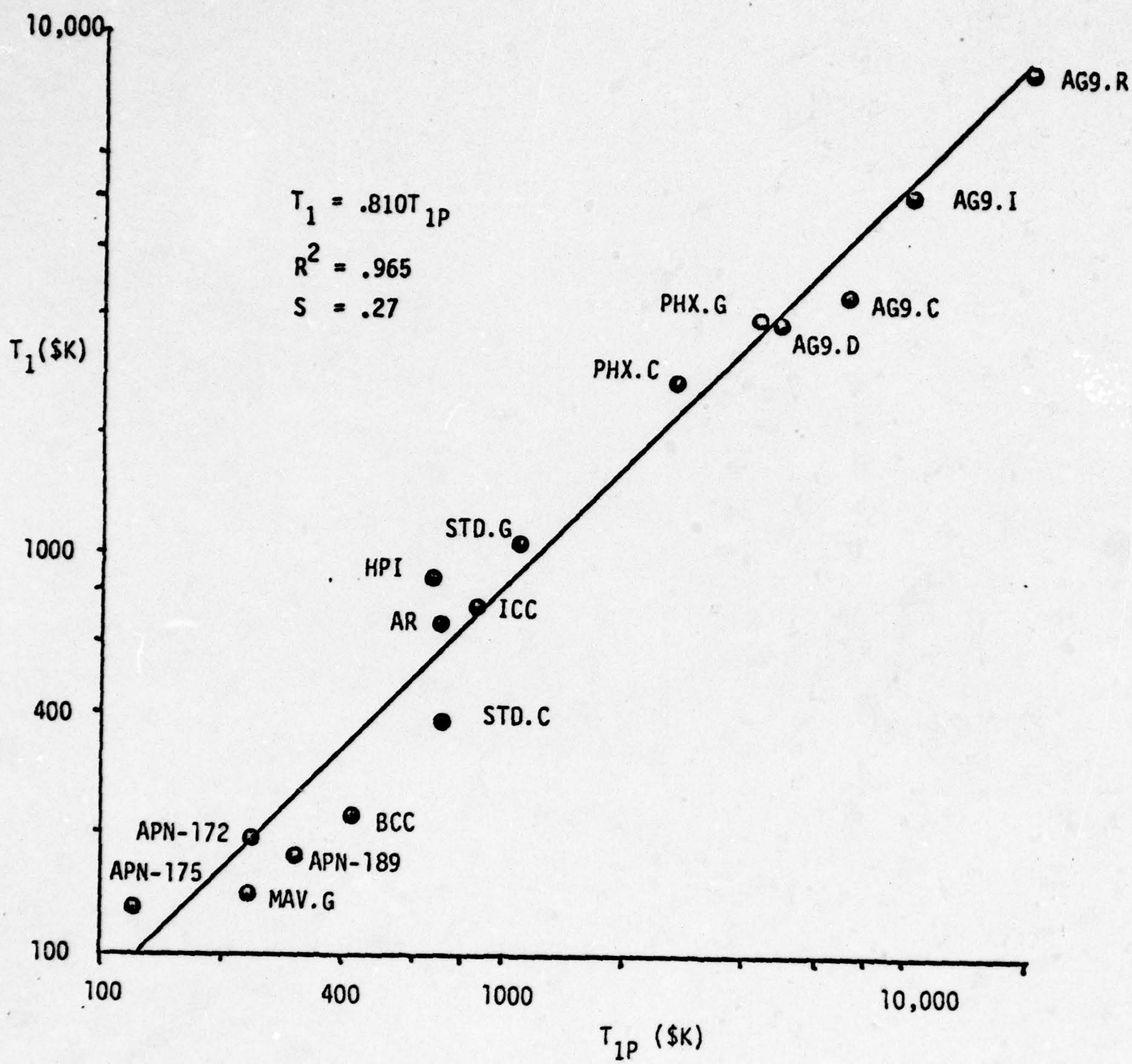

FIGURE 4.2 SEQUENTIAL MODEL, LOG LINEAR CUM. AVE. THEORY

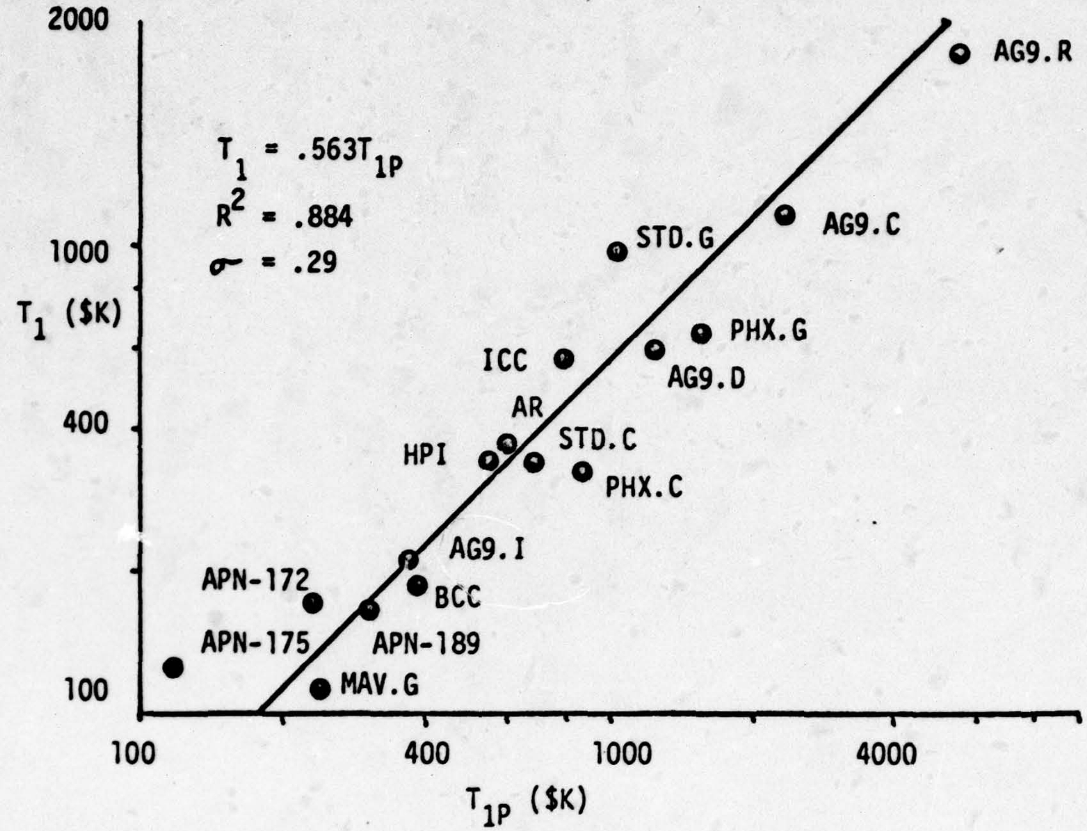

FIGURE 4.3 DISJOINT MODEL LOG LINEAR CUM. AVE. THEORY

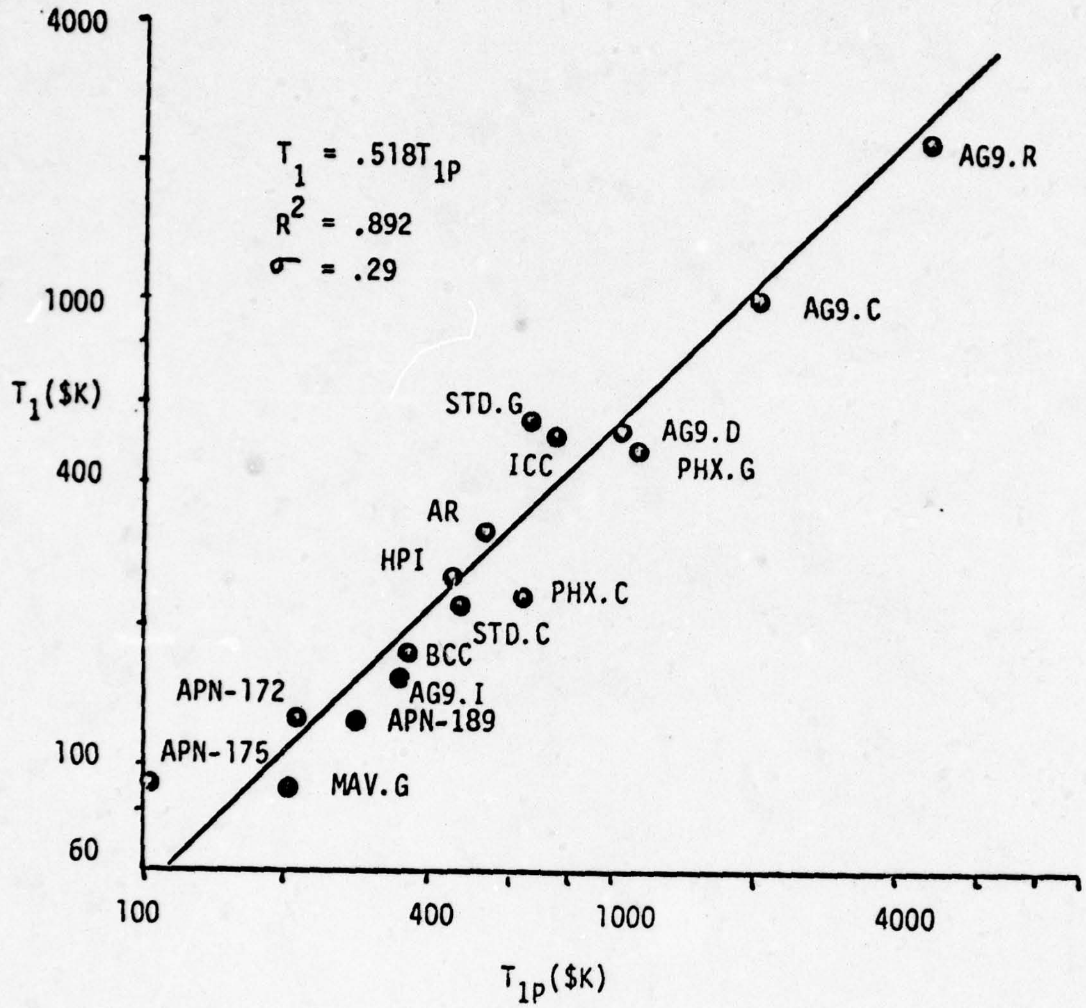

FIGURE 4.4 DISJOINT MODEL, LOG LINEAR UNIT THEORY

We do not note significant differences in any of the hypotheses across the two broad hardware categories. Although the missile data produces poorer fits in all the formulations, they do produce factors (ℓnA) that are very similar to those estimated for the other electronics group. The estimated value of the factor (Model No. 1) for other electronics is well within the probability distribution produced by the missile data. Because we cannot distinguish with a high degree of confidence the difference between the factors in the proportional model under either quantity, hypothesis leads us to conclude that separate models should not be employed for different hardware and that the aggregate factors should be applied.

Comparing the results of the sequential and disjoint hypothesis reveals little difference in the fit characteristics of either theory. The sequential theory produces a better R^2 in the factor only model (Model No. 1); however, the standard errors are virtually the same (.294 vs. .272).* A notable characteristic of the disjoint theory is the significance of the other two variables included in Model No. 2 and 3. A possible interpretation of the coefficient on prototype quantity which produces a smaller T_1 for larger prototype quantities is that the sequential theory is more appropriate. In the sequential theory, the cost of the first production unit (Unit Number Q_P+1) decreases as Q_P increases because of the negative slope of the cost quantity relationship.

The meaning of the significance of the T_{1P} term of Model No. 2 under the disjoint theory is unclear. The coefficient produces a smaller factor for larger cost subsystems. Its magnitude, however, is not large enough to make its impact clearly visible in Figure 4.3. This phenomenon is not observed in the sequential theory except in the missile group where it is opposite in sign. The difficulty of interpreting its meaning and the inconsistency between quantity theory impacts leads us to exclude the variable from selected models.

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*We are able to conclude that the factor A is not equal to one which implies no break or discontinuity in the cost-quantity relationship under the sequential theory. The probability that A is less than one is .995.

The statistical evidence seems to support the sequential theory, although only to a slight degree. A major concern of ours in the application of this theory is the corresponding improvement rate increase observed in the production data. The average production improvement rate for the disjoint theory and the total sample was 85.2%. When the prototype quantity is treated as a missing lot in the sequential theory, the improvement rate steepens to an average of 78.7%. This is a rate which is much lower than what is typically accepted by cost analysts. This steepening results from the fact that addition of Q_p to production lot quantities moves small quantity points farther to the right than large quantities in log space on cost-quantity graph.*

Our conclusion based on the above observations on the quantity theories is that we cannot reject either hypothesis on statistical grounds in that each produces a very good factor-only model. Our recommendation is to apply either one with the appropriate factor and with the understanding that the sequential model implies a higher improvement rate.

Investigation of the fit statistics resulting from using different cost improvement theories indicate little difference for either theory in the disjoint formulation. There is as suggested in Section 2.0 a difference between the factors produced (.563 vs. .518) and the unit theory factor is smaller as hypothesized. We cannot, however, establish whether this is the result of positive bias in the cumulative average theory or negative bias in the unit theory. We, therefore, cannot conclude that one theory is better than the other.** Our recommendation is to apply either with the corresponding factor developed in the analysis.

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*The improvement rate of 90% estimated for the doppler navigation systems was employed in both quantity theory determinations.

**It is possible that one theory applies in some cases and not in others.

The remaining issue is whether T_{1P} obtained by applying observed production improvement rates to prototype average unit costs is a better estimator than the average prototype cost, $\overline{C}_P$. This is clearly the case where only the $\overline{C}_P$ term is employed. Tables 4.3 and 4.4 reveal much higher standard errors for this case (.45 vs. .29 in cumulative average theory). The incorporation of a quantity term along with $\overline{C}_P$, however, improves the fits substantially--almost to the point that the fits are indistinguishable. The exponent on the prototype quantity term corresponds almost exactly to the average improvement rate observed in the production data. This exponent exhibits a high t-value and supports the hypothesis that there is cost improvement in the production of prototypes and that the rate is not significantly different from the production rate. Note also in Table 4.4 the essentially equivalent ℓnA terms of Models 1 and 7 (-.650 vs. .658) when a Q_P variable is added. Given an average improvement rate, these two models produce the equivalent result. Because the cumulative average theory results are better if T_{1P} is calculated using production improvement rate, we suggest that the T_{1P} model be employed where an improvement different than the average is postulated. Otherwise, either Model 1 or Model 7 give essentially the same result.

Our recommendations on model usage are summarized in Table 4.5 for each quantity and improvement theory.

TABLE 4.5

RECOMMENDATIONS ON MODEL USAGE

IMPROVEMENT THEORY		SEQUENTIAL*	DISJOINT
	LOG-LINEAR CUM. AVG.	$T_1 = .810T_{1P}$	$T_1 = .563T_{1P}$ or $T_1 = .602\overline{C}_P\ Q_P^{.236}$
	LOG-LINEAR UNIT THEORY	SAME AS ABOVE	$T_1 = .518T_{1P}$ or $T_1 = .522\overline{C}_P\ Q_P^{.198}$

T_{1P} = $\overline{C}_P Q_P^{b_i}$ where b_i = Programs Production Improvement Rate

T_1 = Theoretical First Unit Cost

$\overline{C}_P$ = Average Prototype Cost

Q_P = Prototype Quantity

*The application of sequential theory implies higher improvement rates than in disjoint theory.

APPENDIX A

ITERATIVE PROCEDURES
FOR FITTING COST IMPROVEMENT CURVES

ITERATIVE PROCEDURES FOR FITTING COST IMPROVEMENT CURVES*

1. INTRODUCTION AND BACKGROUND

(U) Compilation and analysis of cost data consumes a substantial portion of the cost analyst's resources in most study efforts. While there are no short cuts for locating and acquiring useful, detailed cost data, it would be beneficial if maximum use can be made of data once it is in hand and if the routine and repetitious procedures of normalization for fiscal escalation and cost improvement effects were reduced to a mechanical process performed by a computer program. The impetus for this paper was exactly that situation--a study which required the processing of substantial quantities of historical cost information for production hardware.

(U) Maximum use of data has, on occasion, been frustrated by the fragmentary nature of the information available for some production programs. Cost and quantity data may be available for the second, sixth, seventh, and ninth production lots, for example; but the development of log-linear cumulative average cost improvement curves directly from this partial information is not possible. Consequently, considerable effort (and money) may be spent in backfilling or trying to "dig-out" the complete program history, often without success. In the past, the only alternative was to modify one's approach and adopt log-linear individual unit cost improvement curve theory and apply established iterative methods to quantitatively define the cost improvement curve. It is not clear which of these theories, if either, more nearly approximates the real world of the U.S. defense industry.

*This Appendix is extracted from a paper of the same title, Tecolote Research, Inc., TM-28, by A. J. Kluge, dated July 1975.

(U) The purpose in this paper is not to validate one or the other of these theories. Rather, it is to show that the same iterative procedure may be applied to either theory and to describe a computer program for implementing the methods. Results of these procedures will also be presented and compared. Perhaps the greatest benefit of this computer program will be the ability to accept and process directly information reported in the Cost Information Reports (CIR), the DD-1177 Forms, Contractor Cost Data Reports (CCDR) and other official cost reporting systems.

2. MATHEMATICAL FORMULATION

(U) The problem is one of determining values for the parameters of an equation which describes the cost improvement curve, given lot cost (either total cost or lot average unit cost) and lot production unit numbers for two or more production lots. If all of the lots are sequential, and the first lot begins with (production) unit number 1, the problem is simply one of accumulating consecutive lot total costs, accumulating consecutive lot total production quantities, dividing one by the other and calculating the cumulative average unit cost curve. If, however, the cost information for the first lot is not available, this direct approach is not possible. In general, if information for any lot is missing, the information for subsequent lots cannot be used in this direct method.

2.1 DIRECT APPROACH

(U) What is termed the direct approach in this paper is the standard method of developing cumulative average data from lot or contract information.

(U) Given the total cost C_T for several consecutive lots (or contracts), the cumulative average cost $\bar{C}$ versus cumulative quantity is obtained as follows;

$$\bar{C}_1 = \frac{C_{T1}}{Q_1}$$

$$\bar{C}_2 = \frac{C_{T1}+C_{T2}}{Q_1+Q_2}$$

$$\bar{C}_3 = \frac{C_{T1}+C_{T2}+C_{T3}}{Q_1+Q_2+Q_3}$$

$$\vdots$$

$$\bar{C}_i = \frac{C_{T1}+C_{T2}+\ldots+C_{Ti}}{Q_1+Q_2+\ldots+Q_i}$$

(U) The cumulative average costs are thus determined directly and the resulting cost versus quantity data may be fit with the log-linear improvement curve equation. Regression methods may be used in this fitting process.

(U) If the datum C_{T1} were not available, it would not be possible to evaluate $\bar{C}_1$ or any of the other $\bar{C}_i$'s. An iterative procedure using lot average unit costs may be used instead. This procedure is described in Sections 2.2 and 2.3.

2.2 LOG-LINEAR CUMULATIVE AVERAGE COST THEORY

(U) To make use of lot average unit cost data in constructing log-linear cumulative average cost curves, begin with the following definitions:

$$\bar{C} = C_o Q^b \qquad (1)$$

where $\bar{C}$ is the cumulative average cost from unit one through any quantity, Q,

C_o is the unit cost of the first production article, Q is the cumulative quantity,

b is the improvement curve exponent given by

$$b = \log_2 S = \frac{\log_N S}{\log_N 2},$$

S is the improvement curve slope expressed as a fraction (e.g. 90% = 0.90),

Then the lot average unit cost, $\overline{C}_\ell$, is given by:

$$\overline{C}_\ell = \frac{C_o Q_L^{1+b} - C_o (Q_F - 1)^{1+b}}{Q_L - (Q_F - 1)}$$

or

$$\overline{C}_\ell = C_o \frac{\left[Q_L^{1+b} - (Q_F - 1)^{1+b} \right]}{Q_L - (Q_F - 1)} \qquad (2)$$

where Q_L is the unit number of the last unit in the lot, and

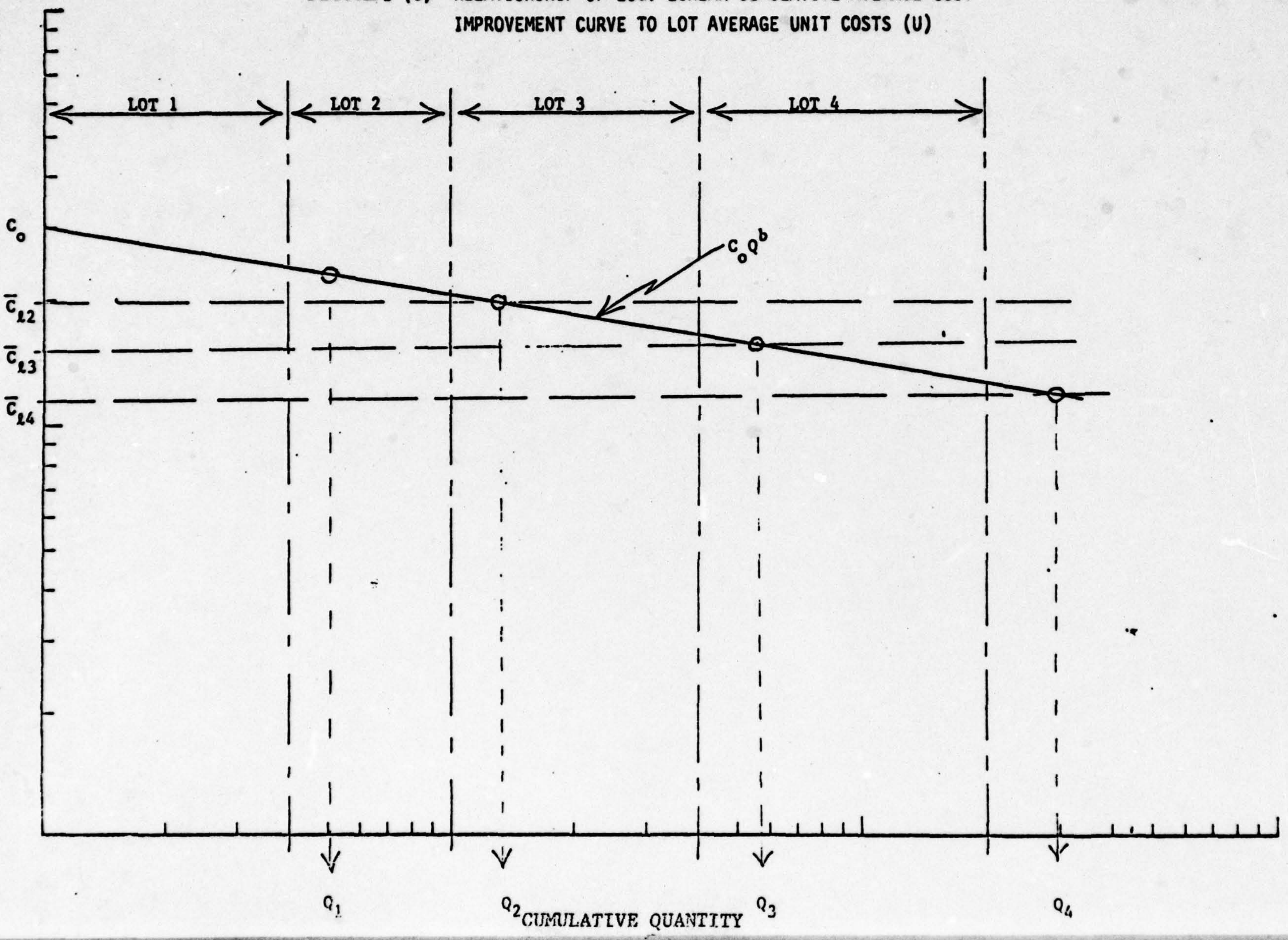
Q_F is the unit number of the first unit in the lot.

(U) The unit numbers and total lot costs ($\overline{C}_\ell$ x lot quantity) are usually available for two or more lots. Our goal is to find C_o and b. The approach is to find quantity plot points for the given $\overline{C}_\ell$ values, such that the data points will fall on the log-linear cumulative average cost improvement curve. Referring to Figure (1), we are given $\overline{C}_\ell(2)$, $\overline{C}_\ell(3)$, $\overline{C}_\ell(4)$, and wish to find the quantity plot points Q(2), Q(3), and Q(4), where the $\overline{C}_\ell$ values equal the values of the log-linear cumulative average cost curve. (For purposes of illustration, it is assumed that Lot 1 cost data is not available.)

(U) The problem is then one of finding values for C_o and b which satisfy the requirement:

$$\overline{C} = \overline{C}_\ell$$

FIGURE 1 (U) RELATIONSHIP OF LOG.-LINEAR CUMULATIVE AVERAGE COST IMPROVEMENT CURVE TO LOT AVERAGE UNIT COSTS (U)



or $$C_oQ^b = \frac{C_o\left[Q_L^{1+b} - (Q_F-1)^{1+b}\right]}{Q_L - (Q_F-1)}$$

or to find Q where: $$Q = \left[\frac{Q_L^{1+b} - (Q_F-1)^{1+b}}{Q_L - (Q_F-1)}\right]^{1/b} \quad (3)$$

(U) Since b is unknown, equation (3) does not lend itself to direct solution, nor will it yield to solution by simultaneous equations. But, an iterative procedure is possible as follows.

(1) Estimate initial quantity plot points for each lot; e.g., the arithmetic mid-point of each lot;

(2) Use these initial estimates to perform a curve fit of $\overline{C}_\ell$ versus Q to obtain an equation of the form $C = C_oQ^b$.

(3) Use the value of b resulting from the curve fitting procedure to refine the initial estimates of Q by substituting b in equation (3).

(4) The refined estimates of Q are used to refit the curve $C = C_oQ^b$.

(5) Repeat Steps 3 and 4 above until the quantity values converge.

(U) The resulting equation provides the sought for parameters C_o and b for the log-linear cumulative average cost improvement curve.

2.3 LOG-LINEAR INDIVIDUAL UNIT COST THEORY

(U) If log-linear unit cost theory is adopted, the individual unit cost, C, is given by the equation

$$C = C_oQ^b \quad (4)$$

where C_o, Q, and b are as previously defined. Then the cumulative average cost of M units is

$$\overline{C} = \frac{\sum_{i=1}^{M} C_o Q_i^{\,b}}{M} \tag{5}$$

Because this equation is cumbersome to evaluate, it is normally approximated by its asymptote, that is;

$$\overline{C} \simeq \frac{C_o Q^b}{1+b} \tag{6}$$

It is then possible to say that;

$$\overline{C}_\ell \simeq \frac{C_o}{1+b}\left[\frac{Q_L^{\,1+b} - (Q_F-1)^{1+b}}{Q_L - (Q_F-1)}\right]$$

Employing the same rationale developed earlier, plot points are found by equating C to $\overline{C}_\ell$.

$$C_o Q^b = \frac{C_o}{1+b}\left[\frac{Q_L^{\,1+b} - (Q_F-1)^{1+b}}{Q_L - (Q_F-1)}\right]$$

or

$$Q = \left\{\frac{1}{1+b}\left[\frac{Q_L^{\,1+b} - (Q_F-1)^{1+b}}{Q_L - (Q_F-1)}\right]\right\}^{1/b} \tag{7}$$

(U) Note that this expression differs from equation (3) by the 1/(1+b) term only. It is, therefore, possible to generalize and rewrite equations (3) and (7) as follows:

$$Q = \left\{\frac{1}{1+Kb}\left[\frac{Q_L^{\,1+b} - (Q_F-1)^{1+b)}}{Q_L - (Q_F-1)}\right]\right\}^{1/b} \tag{8}$$

where Q is the plot point for the log-linear cumulative average cost curve

when K = 0,

or Q = the plot point for the log-linear unit cost curve

when K = 1.

3. TYPICAL RESULTS

(U) Samples of the output of a computer program, using the three curve fitting methods explained above, are shown on the following pages. Figure 2 shows the results of a direct curve fit. The heading identifies the data used and the date on which the program was run. The next heading identifies the curve fit as one based on log-linear cumulative average cost theory. The presence of only one entry under the column headed "Iteration No." indicates that the direct curve fit method was employed since an iterative curve fit always results in two or more lines of output.

(U) Referring to the equation cost = C_oQ^b, where Q is quantity, the column headed "first unit cost" gives the estimate C_o of the cost of the first unit. The column headed "exponent" gives the exponent b and "slope" gives $S = 2^b$. "INDEX OF DETERMINATION" is a statistical measure of how well the learning curve fits the data. The closer the index of determination is to 1.0, the better the curve fit. The columns headed EPS1, EPS2 do not apply to the direct curve fit.

(U) Figure 3 is a typical page of output cost data for the direct curve fit. For convenience, the data source and date is again identified by the first printed line. The first line of data gives actual cumulative average costs escalated to the desired fiscal year as indicated by the heading, "CUM. AVG. COST IN ...". The lines printed under "QUANTITY PLOT POINTS AND ESTIMATED COSTS IN FY75 DOLLARS" gives cumulative quantities and directly below indicates the estimated cost at that quantity as determined by the curve cost = C_oQ^b. This page of output also records the number of data points input to the program, since there is one column of figures for each data point. The entries allow comparison between actual costs and estimated costs on a normalized basis.

(U) Figure 4 of the sample output is similar to Sample 1. The second heading shows that a cumulative average cost improvement curve is being determined, while the presence of more than one iteration indicates that the iterative method is being used. The line labeled "Iteration No. 1" is the first approximation to the learning curve, line 2 is the second, etc. While the index of determination remains a measure of the goodness of the fit, it has a somewhat different interpretation when one of the iterative processes is employed. For this reason, caution should be exercised if any comparison is made between the indices of determination given under direct and iterative curve fits.

(U) The next page affords a comparison of the input data with the estimated costs based on the curve fit. Under the heading, "CUM. AVG. COSTS..." are the costs based on the input data. These costs, at the quantities indicated for each iteration, can be compared with the costs estimated by the curve fit at the same quantity. This comparison is listed for each iteration.

(U) When the iterative process is employed, the columns labeled EPS1 and EPS2 measure the convergence of the method. EPS1 is the fractional change in the calculated first unit cost between successive iterations. EPS2 is the similar change in the exponents. When EPS1 <u>and</u> EPS2 both become small, the iteration process stops.

(U) Figures 6 and 7 of the sample output are the same as Figure 4 and 5, except that log linear unit cost theory is being used to develop unit cost curves instead of cumulative average cost curves. Column headings on Figure 6 have the same meanings as those on Figure 4.

(U) Figure 7 again gives costs based on the input data for comparison with costs estimated by the learning curve at the quantities indicated for each iteration.

LOG. LINEAR CUM. AVG. COST THEORY
CUM. AVG. COST IMPROVEMENT CURVE FIT

ITERATION NO.	FIRST UNIT COST	IMPROVEMENT CURVE EXPONENT	IMPROVEMENT CURVE SLOPE	INDEX OF DETERMINATION	EPS1	EPS2
1	28729	-.739	.599	.9984	*000	*000

FIGURE 2

Unclassified

CASE NO. 2.0 XYZ MISSILE SAMPLE PART 04/17/75

```
ITERATION      .......CUM. AVG. COST IN FY 75 DOLLARS AT INDICATED QUANTITY FOR LOTS  1-10..........
   NO.     $   329  $   209  $   112  $    66  $    47  $    32  $    27  $    23  $    20  $    20

               .............QUANTITY PLOT POINTS AND ESTIMATED COSTS IN FY 75 DOLLARS..............

    1          386      807     1974     3979     5855     9380    12316    15302    18104    18749
           $   351  $   204  $   105  $    63  $    47  $    33  $    27  $    23  $    20  $    20
```

FIGURE 3

Unclassified

CASE NO. 3.0 XYZ MISSILE SAMPLE PART 04/17/75 1 0 1

LOG. LINEAR CUM. AVG. COST THEORY
CUM. AVG. COST IMPROVEMENT CURVE FIT

ITERATION NO.	FIRST UNIT COST	IMPROVEMENT CURVE EXPONENT	IMPROVEMENT CURVE SLOPE	INDEX OF DETERMINATION	EPS1	EPS2
1	14273	-.794	.577	.7452	*000	*000
2	17439	-.701	.629	.7653	.182	.134
3	18912	-.696	.618	.7643	.078	.007
4	18975	-.695	.618	.7642	.003	.001
5	18980	-.695	.618	.7642	.000	.000

FIGURE 4

CASE NO. 3.0 XYZ MISSILE SAMPLE PART 04/17/75

ITERATION NO.										
	.......CUM. AVG. COST IN FY 75 DOLLARS AT INDICATED QUANTITY FOR LOTS 1-10.........									
	$ 329	$ 100	$ 44	$ 22	$ 5	$ 7	$ 10	$ 7	$ 4	$ 29
	............QUANTITY PLOT POINTS AND ESTIMATED COSTS IN FY 75 DOLLARS..............									
1	193	597	1391	2977	4917	7618	10848	13809	16703	18427
	$ 218	$ 89	$ 45	$ 25	$ 17	$ 12	$ 9	$ 7	$ 6	$ 6
2	386	4200	9620	21037	35616	54884	79013	100782	122075	134943
	$ 324	$ 65	$ 38	$ 22	$ 16	$ 12	$ 9	$ 8	$ 7	$ 6
3	386	3214	7369	16094	27211	41944	60349	76968	93222	103038
	$ 303	$ 70	$ 39	$ 23	$ 16	$ 12	$ 9	$ 8	$ 7	$ 6
4	386	3178	7286	15913	26904	41471	59666	76097	92167	101871
	$ 302	$ 70	$ 39	$ 23	$ 16	$ 12	$ 9	$ 8	$ 7	$ 6
5	386	3175	7279	15897	26876	41428	59605	76018	92072	101766
	$ 302	$ 70	$ 39	$ 23	$ 16	$ 12	$ 9	$ 8	$ 7	$ 6

FIGURE 5

Unclassified

CASE NO. 4.0 XYZ MISSILE SAMPLE PART 04/17/75 1 1

LOG. LINEAR UNIT COST THEORY
UNIT COST IMPROVEMENT CURVE FIT

ITERATION NO.	FIRST UNIT COST	IMPROVEMENT CURVE EXPONENT	IMPROVEMENT CURVE SLOPE	INDEX OF DETERMINATION	EPS1	EPS2
1	14273	-.794	.577	.7452	*000	*000
2	6029	-.723	.616	.7638	*367	.098
3	6488	-.712	.612	.7631	.071	.016
4	6563	-.710	.612	.7630	.012	.003
5	6576	-.710	.612	.7629	.002	.000
6	6578	-.710	.612	.7629	.000	.000

FIGURE 6

Unclassified

CASE NO. 4.0 XYZ MISSILE SAMPLE PART 04/17/75

ITERATION NO.										
	..........UNIT COST IN FY 75 DOLLARS AT INDICATED QUANTITY FOR LOTS 1-10..........									
	$ 329	$ 100	$ 44	$ 22	$ 5	$ 7	$ 10	$ 7	$ 4	$ 29
	............QUANTITY PLOT POINTS AND ESTIMATED COSTS IN FY 75 DOLLARS..............									
1	193	597	1391	2977	4917	7618	10848	13809	16703	18427
	$ 218	$ 89	$ 45	$ 25	$ 17	$ 12	$ 9	$ 7	$ 6	$ 6
2	74	574	1314	2872	4863	7494	10788	13761	16668	18425
	$ 298	$ 71	$ 40	$ 23	$ 16	$ 12	$ 9	$ 8	$ 7	$ 6
3	80	574	1317	2876	4865	7499	10791	13762	16669	18425
	$ 291	$ 72	$ 40	$ 23	$ 16	$ 12	$ 9	$ 8	$ 7	$ 6
4	81	575	1317	2877	4865	7499	10791	13763	16669	18425
	$ 290	$ 72	$ 40	$ 23	$ 16	$ 12	$ 9	$ 8	$ 7	$ 6
5	82	575	1317	2877	4866	7500	10791	13763	16669	18425
	$ 289	$ 72	$ 40	$ 23	$ 16	$ 12	$ 9	$ 8	$ 7	$ 6
6	82	575	1317	2877	4866	7500	10791	13763	16669	18425
	$ 289	$ 72	$ 40	$ 23	$ 16	$ 12	$ 9	$ 8	$ 7	$ 6

FIGURE 7

Unclassified

APPENDIX B

THE DEVELOPMENT OF A FULL SAMPLE RELATIONSHIP

1.0 INTRODUCTION

In the main body of this paper, models were produced which estimated the production first unit cost, T_1 , as a function of prototype first unit cost, T_{1P} . The derivation of the models was a two step process. For each program in the data base, T_1 was estimated by the regression of costs at various quantities with their associated quantities. The models were then derived by fitting T_1 to a function of T_{1P} . The application of these models involves a similar two step process. T_1 is estimated as a function of T_{1P} and then an extrapolation is made along a cost improvement curve to obtain a cost estimate at the desired quantity.

This appendix discusses an alternate approach which eliminates the multi-step process by incorporating all of the production cost-quantity data points in the model. The full sample is then used to estimate the cost-quantity relationship so that cost at quantity is given directly as a function of prototype cost and production quantity.

2.0 MODEL DESCRIPTION

The simplest model formulation which accounts for both the cost-quantity relationship and the prototype-full scale production relationship is given by

$$\text{COST} = A\, T_{1P} Q^b$$

In this formula, COST is the cumulative average (or unit) cost of full scale production at quantity Q . A describes the drop in cost relative to the prototype first unit cost T_{1P} . The exponent b is the implied cost improvement curve slope. Notice that this model constrains the cost-quantity relationship to a single cost improvement slope for all programs and, therefore, constrains the analyst using this model to a single slope. If it is believed that different programs will exhibit different cost improvement rates, then the model suffers from specification error.

Since T_{1P} is calculated by $T_{1P} = \overline{C}_P / Q_P^b$, and since we are assuming a single slope for all programs, the above model can be written:

$$\text{COST} = A \frac{\overline{C}_P}{Q_P{}^b} Q^b$$

$$\text{or COST} = A\, \overline{C}_P \left(\frac{Q}{Q_P}\right)^b \quad \text{where}$$

the parameters to be found by regression are A and b .

In order to investigate the effects of $\overline{C}_P$ and Q_P on the basic relationship postulated above, additional $\overline{C}_P$ and Q_P terms can be added to the model. When this is done, the four basic models resemble the four basic model forms given in the main body of this paper. These models are given below in Table 1.

TABLE 1

MODEL FORMS

MODEL NO.	FORM
1	$COST = A\,\overline{C}_P \left(\frac{Q}{Q_P}\right)^{b}$
2	$COST = A\,\overline{C}_P \left(\frac{Q}{Q_P}\right)^{b} \overline{C}_P^{\alpha}$
3	$COST = A\,\overline{C}_P \left(\frac{Q}{Q_P}\right)^{b} Q_P^{\beta}$
4	$COST = A\,\overline{C}_P \left(\frac{Q}{Q_P}\right)^{b} \overline{C}_P^{\alpha} Q_P^{\beta}$

3.0 REGRESSION ANALYSIS

The regression analysis was performed on the log transforms of the models given in Table 1. With the inclusion of all the appropriate cost and quantity information, the data base was the same as that used in the main study. The same equipment stratification of the data base was made and both major hypotheses were investigated. The investigation was limited to cumulative average cost improvement curve theory only.

The results of the regression analysis are given in Tables 2 and 3. In Tables 2 and 3, the column labeled Model indicates the model being tested and the sample used for the test. The column labeled $\ln A$ gives intercept term for the model; while the columns labeled $\bar{C}_p$ and Q_p give the exponents α and β of the independent variables $\bar{C}_p$ and Q_p. The column labeled S% gives the implied cost improvement curve slope in percent. S is obtained as 100×2^b where b is the exponent of Q/Q_p in the various models. The last column gives the standard error of the estimate for each of the models.

TABLE 2

DISJOINT HYPOTHESIS

MODEL	ℓnA	$\overline{C}_p$	Q_p	S% $S=100\times2^b$	σ
TOTAL					
N=63					
1	-.642			82.4	.379
2	-.536	-.008X		82.4	.382
3	-.048		-.206	82.9	.317
4	.226	.020X	-.207	82.6	.320
MISSILES					
N=27					
1	-1.056			85.1	.300
2	-4.448	-.433		82.9	.270
3	1.236		-.531	80.9	.264
4	3.084	-.202X	-.366	81.2	.266
ELECTRONICS					
N=36					
1	-.671			87.6	.318
2	1.478	-.159		87.0	.289
3	-.451		-.076	86.6	.314
4	1.433	-.153	-.014X	86.8	.293

MODEL: $COST = A\,\overline{C}_p \left(\frac{Q}{Q_p}\right)^b \quad \overline{C}_p^{\alpha}\, Q_p^{\beta}$

X indicates the parameter is not significant at the 10% level.

TABLE 3

SEQUENTIAL HYPOTHESIS

MODEL	ℓnA	$\overline{C}_P$	Q_P	S%	σ
TOTAL					
N=63					
1	-.394			79.1	.377
2	1.716	-.149		76.7	.355
3	.141		-.203	80.1	.319
4	2.116	-.140	-.198	77.8	.295
MISSILES					
N=27					
1	-.755			82.2	.324
2	4.701	-.425		79.6	.299
3	1.303		-.457	78.0	.302
4	3.836	-.274	-.236	78.3	.301
ELECTRONICS					
N=36					
1	-.375			80.1	.398
2	3.127	-.253		78.9	.307
3	.446		-.242	75.7	.328
4	2.610	-.133	-.183	76.8	.292

MODEL: $COST = A\,\overline{C}_P \left(\frac{Q}{Q_P}\right)^b \quad \overline{C}_P^{\alpha}\, Q_P^{\beta}$

4.0 CONCLUSIONS

The conclusion to be drawn from Tables 2 and 3 essentially parallel the conclusions given in the main body of this paper. In summary, both the disjoint and sequential hypotheses reproduce their respective data bases equally well and, hence, it is impossible to establish the best hypothesis on the basis of Tables 2 and 3. The small sample size in conjunction with the similar fits obtained in the missile and electronics samples makes it inadvisable to conclude that missiles behave inherently different from the other electronics in our data base. Although the addition of the variables $\overline{C}_P$ and Q_P generally improve the fits, the improvement is not large enough to conclude with confidence that the prototype/full scale relation is dependent on either $\overline{C}_P$ or Q_P . Finally, notice that, as expected, the sequential hypothesis implies a steeper cost improvement curve slope in conjunction with a smaller drop from prototype production costs.

Based on the above observations, the two best models are given below.

DISJOINT HYPOTHESIS:

$$\text{CUMULATIVE AVERAGE COST} = .53\ \overline{C}_P\left(\frac{Q}{Q_P}\right)^{-.28}$$

SEQUENTIAL HYPOTHESIS:

$$\text{CUMULATIVE AVERAGE COST} = .67\ \overline{C}_P\left(\frac{Q}{Q_P}\right)^{-.34}$$