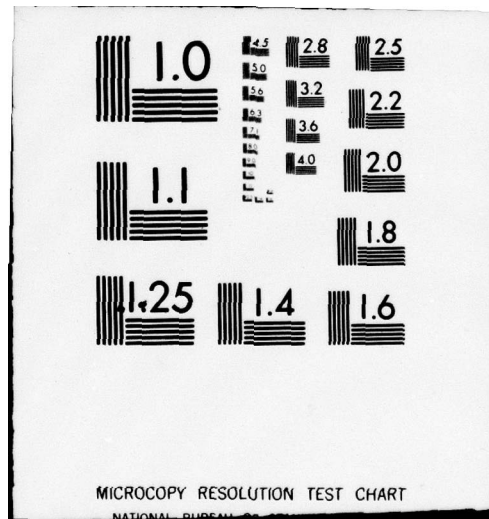
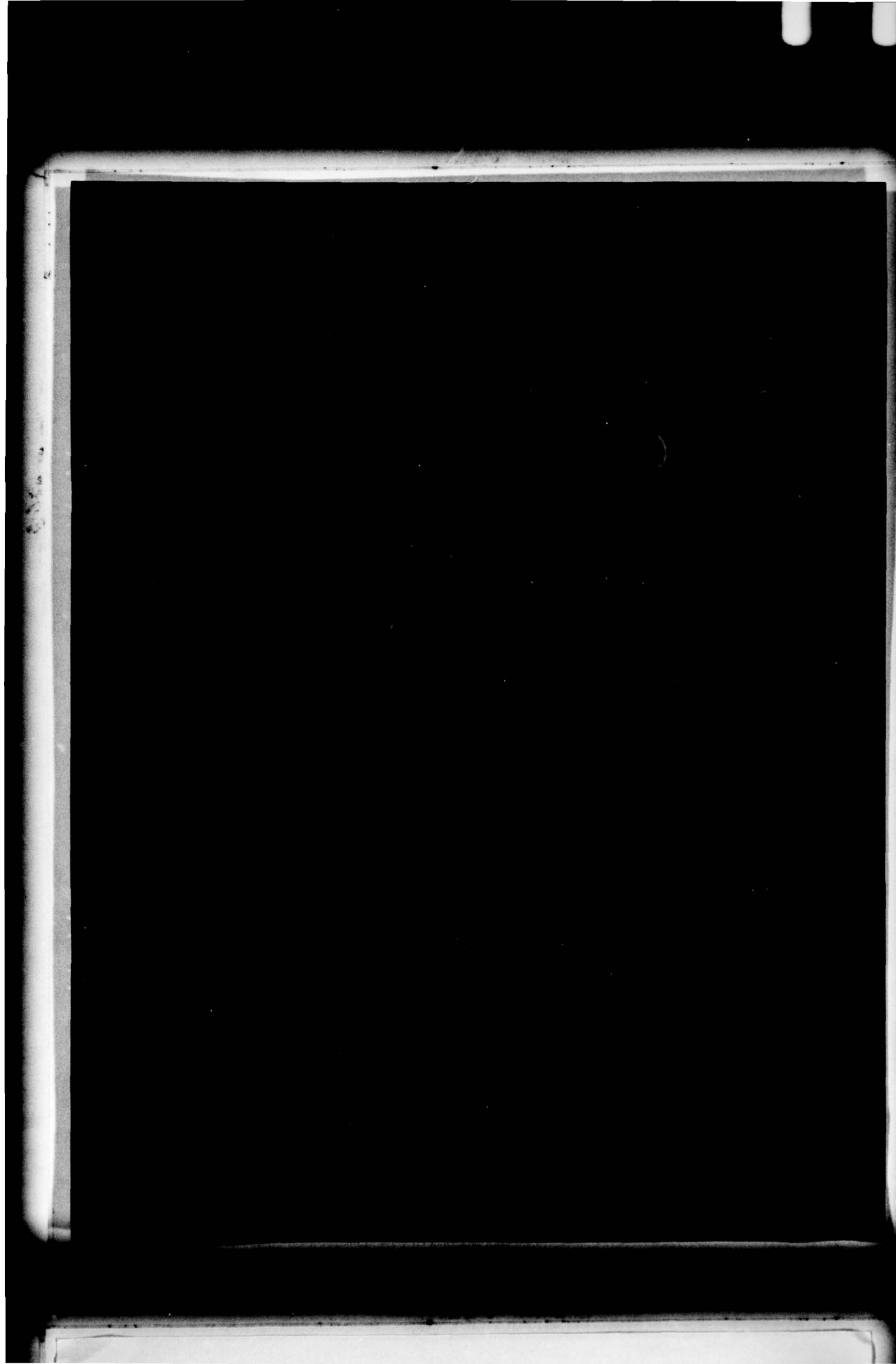


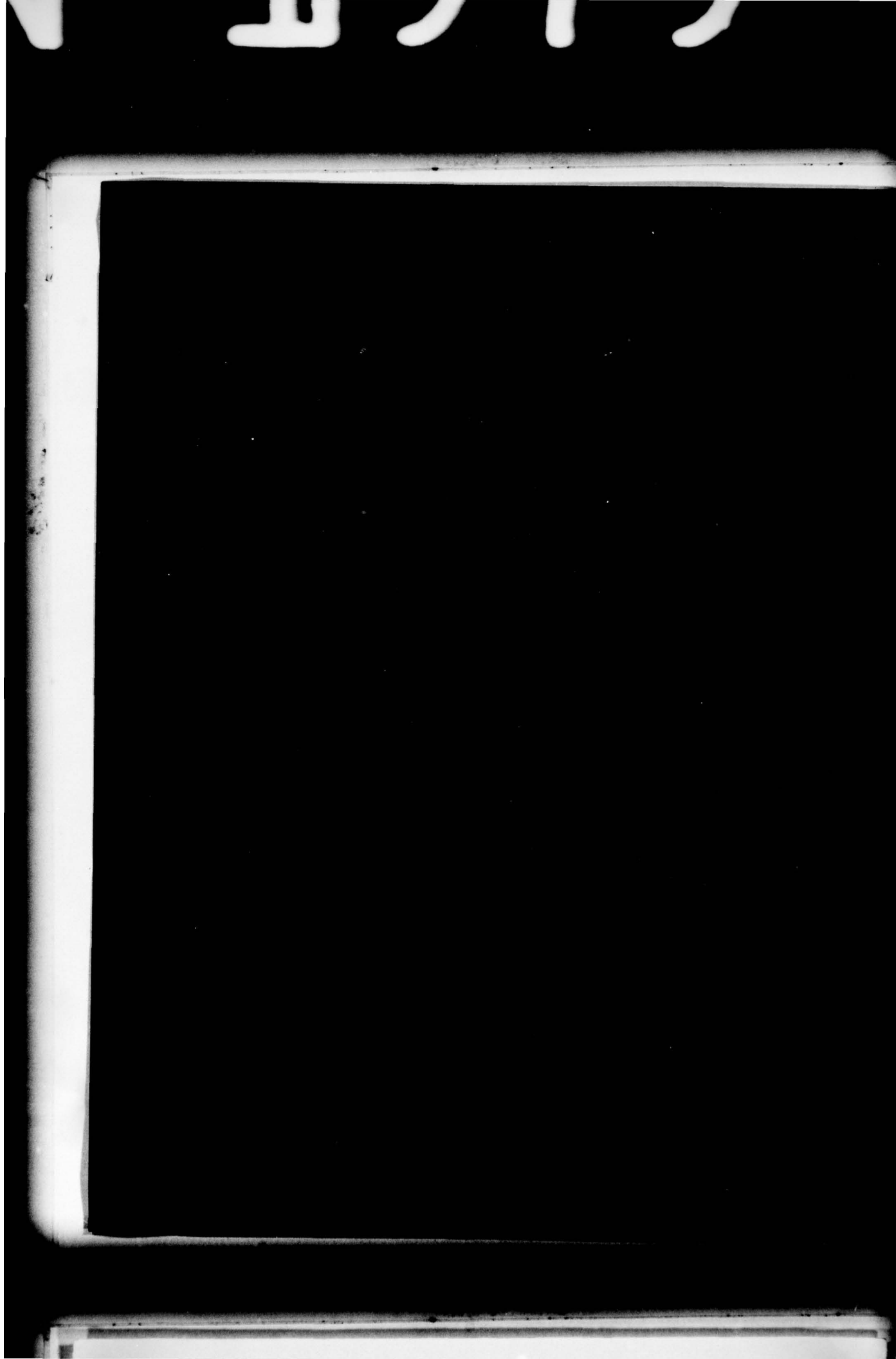
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DIFFERENTIAL ORBIT CORRECTION FOR
NEAR-STATIONARY ARTIFICIAL SATELLITES

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ABSTRACT

This report presents a new concept for the differential correction of orbits. It is developed in detail for near-stationary artificial satellites. For these satellites a few (<10) observations over a short ($5-15^m$, $\lesssim 1\%$ period) time interval allows the reacquisition of the satellite an hour later. Moreover, with only slightly more data points (total number <15) over a slightly longer (total duration $<0.5^h$) time interval it is capable of producing an excellent set of orbital elements. The technique is self-starting and does not use any of the classical initial orbit determination procedures. It can be used by a radar or extended to include angular velocity data. Its success appears to be based on the ability to find a coordinate system in which the object's motion is nearly stationary and the extensive use of analytical (instead of numerical) procedures. Extensions of the theory to include all first-order perturbations and to all orbital types are possible.

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I. INTRODUCTION

Professor Paul Herget, former director of the Minor Planet Center at the University of Cincinnati, has written¹, "It would be a constructive achievement to dispel the myth that 'a preliminary orbit can be computed from three observations'." He was referring to the wide-spread belief that good orbital elements can be computed from a minimum of data. While we agree with his statement, we will present in this report a differential correction procedure, for a particular class of objects, which accurately works when the observation interval is $\lesssim 1\%$ of the period of revolution. The ability to do this is of importance for passive and active artificial satellites, for solar system objects (meteors, comets, and asteroids), and for binary star systems. Moreover, our technique is capable of including all first-order perturbations. It may be possible to extend the concept to all types of orbits, not just near-stationary artificial satellites (the class dealt with here).

How can we do this? We exploit the fact that for any orbital motion there exists a unique coordinate system in which the heretofore moving object is stationary. Consider, for example, an asteroid being observed from the earth. It's apparent motion is complicated by the earth's annual parallax, the observer's diurnal parallax, the earth's and

the observer's motion, the minor planet's motion, and the fact that the force's center is the Sun. To eliminate these effects we first transform to a heliocentric coordinate system, rotate into the asteroid's invariable plane, align with the line of apsides, then rotate the coordinate system with the asteroid's instantaneous angular velocity, and lastly use the asteroid's instantaneous heliocentric distance as the unit of distance. This coordinate system rotates and pulsates with a period equal to the asteroid's orbital period but the asteroid is fixed. Clearly to perform the coordinate transformation, one needs to know the orbit, the earth's orbit, and the observer's location on the earth. However, if one had an approximate set of orbital elements for the asteroid, one could construct a coordinate system in which the asteroid was nearly stationary. If, in addition, one were at the origin of this coordinate system, then the description of the departures from stationarity would not only be small but would also be easily modeled.

We are at the origin of any topocentric coordinate system. There also happens to be a populous, growing in number and importance, frequently maneuvered class of satellites which are naturally nearly stationary for all earth-bound observers. The geocentric orbits of these satellites are also simple: low inclination, small eccentricity, and mean

motion $\approx \dot{t}$ ($\equiv 1.0027379093$ rev/day). There are no meaningful perturbations acting on them over a time span of a period ($= 1$ sidereal day) and the analysis will be the simplest, analytically, for this orbital type. There are also many practical problems having to do with searching for such satellites by optical means² which require essentially instantaneous orbital element set construction. Moreover, there are two natural topocentric coordinate systems in which the analysis can be performed and all "small" quantities of the theory have comparable magnitudes. For all of these reasons the exploration of this new concept of differential correction is most efficiently performed for this type of orbit.

We do wish to stress two points: The development presented here is truly a differential correction procedure. It is not an initial orbit procedure nor a dead reckoning technique. The other point is that the optimization problem we have posed (and solved) is not the one that one really wants to solve. The correct optimization problem is the following: Let us define a function $\underline{f}[\underline{a}(t), t]$ which predicts, at time t , the location and velocity of a celestial object from the parameters $\underline{a}$. Let $S(t, T)$ be the propagator for $\underline{f}$ so

$$\underline{f}[\underline{a}(T), T] = S(t, T)\underline{f}[\underline{a}(t), t], \quad S(t, t) = 1.$$

Suppose we also have knowledge, at epoch t , $K(t)$, about the celestial object. If $\underline{a}'(t)$ is an approximation for $\underline{a}(t)$ then we want to minimize, with respect to $\underline{a}'(t)$, at some specific time T ,

$$|\underline{f}[\underline{a}(T), T] - S(t, T)\underline{f}[\underline{a}'(t), t]|$$

subject to our knowledge $K(t)$. It is not clear that the differential correction of orbits is an equivalent formulation of this problem*. It's also not clear how to formulate this problem mathematically.

Let us now turn to some other aspects of the near-stationary artificial satellite problem and its third-order solution for angles-only data in the topocentric equatorial coordinate system (§II). We also include the distance and radial velocity results, to second-order, in any topocentric spherical coordinate system. The Appendix discusses the angles-only problem, to second-order, in the topocentric horizon coordinate system. Section III discusses the results of observational tests conducted at the Experimental Test Site of the Ground-based Electro-Optical Deep Space Surveillance program. This network of five, computer-controlled, observatories is replacing the Baker-Nunn photographic camera system. We

*That is, $\underline{a}(t)$ = orbital element set at epoch t , $\underline{f} = \begin{pmatrix} \underline{r} \\ \underline{\dot{r}} \end{pmatrix}$, and $S(t, T)$ incorporates the physics of the problem.

also include some numerical tests to ascertain the long time ($\sim$ one period) usefulness of the present development. The last section indicates the directions one could take for other types of orbits. This is, generally, only a first-order development.

II. NEAR-STATIONARY ARTIFICIAL SATELLITES

A satellite, whose motion will be modeled by our theory for a maximum time duration $T_{\max} > 0$, is defined to be a near-stationary satellite if

$$i \leq 0.15, e \leq 0.15, |n/\dot{\tau} - 1| \leq 0.15, \text{ and } |n - \dot{\tau}| T_{\max} \leq 0.15. \quad (1)$$

The satellite's inclination is i , its mean motion is n , and its eccentricity is e . The numerical factor of 0.15 is a stationary satellite's equatorial horizontal parallax, π_s . This quantity is beyond our control and has been taken to be the upper limit for a small quantity. The theory developed below is complete through all third-order terms of this magnitude. We shall find, as with many other orbital analysis techniques, that the mean motion is a difficult quantity to accurately compute. However, we shall also find that as $n \rightarrow \dot{\tau}$ the practical limits on i and e rise to $\sim 25^\circ$ and ~ 0.25 respectively. Moreover, the size of the parallax implies that the data need not even be reduced before the numerical computations are performed*.

*We are not advocating incorrect, incomplete, or sloppy data reduction. We merely point out that the largest correction, the astronomical refraction correction, is numerically $\sim \pi_s^4$ at a zenith distance of 60° . Similarly perturbations due to the earth's oblateness or luni-solar forces are inconsequential at this level (e.g., $J_2 \pi_s^2$ is numerically less than π_s^5).

Using standard notation to describe the Keplerian orbit (ω = argument of perigee, Ω = longitude of the ascending node, v = true anomaly, r = geocentric distance) our starting point is

$$h = \tau - \Omega - \tan^{-1}(\cos i \tan u), \quad (2a)$$

$$\delta = \sin^{-1}(\sin i \sin u), \quad (2b)$$

$$r = a(1 - e^2)/(1 + e \cos v), \quad (2c)$$

$$u = v + \omega. \quad (2d)$$

Here $h(H)$ is the geocentric (topocentric) hour angle, $\delta(\Delta)$ is the geocentric (topocentric) declination, $r(R)$ is the geocentric (topocentric) distance, and τ is the mean sidereal time on the observer's celestial meridian. Once the appropriate approximate expressions have been developed for the geocentric variables, we transform to the topocentric coordinate system via

$$\tan(H - h) = p \sinh / (1 - p \cosh), \quad (3a)$$

$$\tan(\delta - \Delta) = q \sin(\gamma - \delta) / [1 - q \cos(\gamma - \delta)], \quad (3b)$$

$$R = r \sin(\delta - \gamma) \csc(\Delta - \gamma), \quad (3c)$$

$$p = (\rho/r) \cos \phi' \sec \delta, \quad (3d)$$

$$q = (\rho/r) \sin \phi' \csc \gamma, \quad (3e)$$

$$\tan \gamma = \tan \phi' \cos[(H - h)/2] \sec[h - (H - h)/2], \quad (3f)$$

where ρ is the observer's geocentric distance and ϕ' is the observer's geocentric latitude.

When developing the formulas we find that the Keplerian orbital element set is not a convenient framework to use. Instead, we shall use

$$N = n - \dot{\tau} \quad (4a)$$

$$\lambda = \tau - \dot{\tau} T - (M_O + \omega + \Omega), \quad (4b)$$

$$E_C = e \cos M_O, \quad E_S = e \sin M_O, \quad (4c)$$

$$I_C = i \cos(\omega + M_O), \quad I_S = i \sin(\omega + M_O), \quad (4d)$$

where M_O is the value of the mean anomaly at the epoch t_O and $T = t - t_O$. We shall further abbreviate some frequently

occurring combinations of the parameters by

$$m = \dot{t}T, \quad (5a)$$

$$E = e \sin(m + M_0), \quad E' = \dot{E}/\dot{\tau} = e \cos(m + M_0), \quad (5b)$$

$$I = i \sin(\omega + m + M_0), \quad I' = \dot{I}/\dot{\tau} = i \cos(\omega + m + M_0). \quad (5c)$$

Note $d\lambda/dt \equiv 0$ so that the new parameters are constants.

The procedure is straightforward: We express the true anomaly as a power series in e and NT , substitute into Eqs. (2), replace trigonometric functions of i by their Maclaurin series, and then expand everything else about $e = 0$, $i = 0$, and $N = 0$. The result is

$$\begin{aligned} h = \lambda - (NT + 2E) - 2NTE' - 5EE'/2 + II'/2 - 5NT(E'^2 - E^2)/2 \\ + (NT + 2E)(I'^2 - I^2)/2 + EN^2T^2 + 13E^3/3 - 3E(E_C^2 + E_S^2), \end{aligned} \quad (6a)$$

$$\begin{aligned} \delta = I + I'(NT + 2E) - I(NT + 2E)^2/2 + 2(NT + 2E)E'I' - 3EE'I'/2 \\ + I^3/6 - I(I_C^2 + I_S^2)/6, \end{aligned} \quad (6b)$$

$$r/a_0 = [1 - 2N/(3\dot{\tau})](1 - E') + (NT + E)E + 5N^2/(9\dot{\tau}^2), \quad (6c)$$

$$\dot{\tau}^2 a_0^3 = GM_{\oplus}. \quad (6d)$$

We now repeat the analysis by using Eqs. (6) in Eqs. (3) and regarding ρ/a_0 as a small parameter. The result is

$$\begin{aligned} (H-h)\sec\phi' / (\rho/a_0) = & [1 + 2N/(3\dot{\tau}) + E' - (NT + 2E)^2/2 \\ & + 2NE'/(3\dot{\tau}) - NTE + E'^2 - E^2 - N^2/(9\dot{\tau}^2) + I^2/2]\sin\lambda \\ & - \{(NT + 2E) + 2NTE' + 5EE'/2 - II'/2 + (NT + 2E)[2N/(3\dot{\tau}) \\ & + E']\}\cos\lambda + (\rho/a_0)[1/2 + 2N/(3\dot{\tau}) + E']\cos\phi'\sin 2\lambda \\ & - (\rho/a_0)(NT + 2E)\cos\phi'\cos 2\lambda + (\rho/a_0)^2\cos^2\phi'\sin 3\lambda/3, \end{aligned} \quad (7a)$$

$$\begin{aligned} (\Delta - \delta) / (\rho/a_0) = & - [1 + 2N/(3\dot{\tau}) + E' + 2NE'/(3\dot{\tau}) - NTE + E'^2 \\ & - E^2 - N^2/(9\dot{\tau}^2) - I^2/2]\sin\phi' + \{I[1 + 2N/(3\dot{\tau}) + E'] \\ & + I'(NT + 2E)\}\cos\phi'\cos\lambda - I(NT + 2E)\cos\phi'\sin\lambda \\ & - (\rho/a_0)\{1 + 2[2N/(3\dot{\tau}) + E']\}\cos\phi'\sin\phi'\cos\lambda \\ & + (\rho/a_0)(I/2)(2\cos^2\lambda - \sin^2\lambda)\cos^2\phi' + (\rho/a_0)[(NT \\ & + 2E)\cos\phi'\sin\lambda - I\sin\phi']\sin\phi' + (\rho/a_0)^2[\sin^2\phi'/3 \\ & + \cos^2\phi'(\sin^2\lambda - 2\cos^2\lambda)/2]\sin\phi', \end{aligned} \quad (7b)$$

$$\begin{aligned}
R/a_0 = 1 - [2N/(3\dot{\tau}) + E'] - (\rho/a_0)\cos\phi'\cos\lambda + 2NE'/(3\dot{\tau}) \\
+ NTE + E'^2 + 5N^2/(3\dot{\tau})^2 + (\rho/a_0)[(NT + 2E)\cos\phi'\sin\lambda - I\sin\phi'] \\
+ [(\rho/a_0)^2/2](\sin^2\phi' + \cos^2\phi'\sin^2\lambda), \quad (8a)
\end{aligned}$$

$$\begin{aligned}
\dot{R}/a_0 = (N + \dot{\tau})E + (NT + 2E)\dot{\tau}E' + (\rho/a_0)[(N + 2\dot{\tau}E')\cos\phi'\sin\lambda \\
- \dot{\tau}I'\sin\phi'] - 2NE/3. \quad (8b)
\end{aligned}$$

We can now formulate our optimization problem. Suppose we have N observations of topocentric hour angle and declination $\{H_n, \mathcal{D}_n, t_n\}$, $n = 1, 2, \dots, N$. We define*

$$S = \sum_{n=1}^N [(H_n - H_n)^2 + (\mathcal{D}_n - \Delta_n)^2], \quad (9)$$

and demand that S be a minimum with respect to $N, \lambda, E_c, E_s, I_c$, and I_s . H_n and Δ_n are expressions (7a, 7b) evaluated at time T_n ,

$$T_n = t_n - \langle t \rangle, \quad \langle t \rangle = \sum_{n=1}^N t_n / N \equiv t_0. \quad (10)$$

The problem formulated is a non-linear least squares problem which takes cognizance of the fact that there are only six

*The factor of $\cos^2\Delta_n$ multiplying $(H_n - H_n)^2$ is effectively always unity for near-stationary satellites.

independent parameters. Because of the lack of a theoretical basis for solving such problems, we make no pretense concerning the properties of the values of the estimates obtained for the six parameters. Moreover, we continue our analytical bent and use the method of steepest descent³ to solve the minimization problem. This is theoretically more powerful than the usual iterative linearization technique normally employed to solve non-linear least squares estimation problems. It also converges more quickly and is second-order. It only remains to supply initial guesses for the six parameters. Satisfactory ones are

$$\lambda = \sum_{n=1}^N H_n / N, \text{ all others} = 0. \quad (11)$$

Note in particular that setting E_c , E_s , I_c , and I_s equal to zero does not prejudice the values for ω , Ω , and M_0 . This is important since there is little forcing changes in these angles in the gradient or Hessian matrix of S .

Now let us see what we can anticipate for the numerical results. Since the first-order terms will carry most of the weight, we shall concentrate on them. From Eqs. (6b and 7b) both the inclination and $\omega + M_0$ can be well determined. From Eqs. (6a and 7a) we see that $\omega + \Omega + M_0$ will be well determined but that an eccentricity/mean motion swap is possible. This

follows because as $T \rightarrow 0$ the coefficient of T in h is
 $-(N + 2\epsilon \cos M_0)$ and there will be no way to distinguish
 which part of this quantity is being contributed by $N \neq 0$
 and which part is being contributed by $\epsilon \neq 0^*$. This
 difficulty (especially for N) can be partially ameliorated
 if the units of N are rev/day instead of rad/day**. When
 computations are done using the same data with N in the different
 units there can be a noticeable difference between the results.

*When $T \rightarrow 0$ the constant part of $h \rightarrow \lambda - 2\epsilon \sin M_0$ but λ is of
 the zero'th order while ϵ is of the first-order.⁰ Hence, the
 numerical separation problem is not as severe.

**The difference of 2π accentuates small changes in N .

III. TESTS OF THE THEORY

A. Short Time Observational Tests

It was originally thought that this development would remove the necessity for a traditional differential correction procedure. Such procedures frequently have artificial singularities for $e = 0$ or $i = 0$. Hence, the tests described in §IIIB were performed first. When faced with the reality of a full search² --time-sharing a telescope between a dozen unknown satellites and attempting to initialize data acquisition on four unknown satellites simultaneously--it was pressed into use. Much to our surprise it worked extremely well. In Table 1 is summarized, 1) the total time duration of the first three observation sequences, 2) the number of observations performed in each of these observing sessions, 3) the time interval between the successive observing intervals, and 4) the ratio of the time difference between the start of the last and first observing intervals to the sum of the first and second observing durations. The last quantity is a crude measure of the "gain" (e.g., lever arm) of the technique. Quite clearly a lever arm of 10 can be achieved. In the intervening time other tasks can be performed by the telescope. No satellites were lost due to the use of the technique. In addition, the actual orbital element sets converged quickly towards the true values (within the limitations discussed above and in the next subsection).

TABLE 1

Satellite Number	Observing Time Duration	Number of Observations	Successive Observing Time Interval	Ratio
1	13 ^m .3	8	40 ^m .4	10.8
	2.7	3	126.9	
	2.2	3		
2	5.9	6	31.7	7.7
	3.4	3	38.0	
	2.0	3		
3	4.1	5	22.9	5.4
	1.9	3	8.7	
	2.6	3		
4	7.4	5	24.6	7.2
	0.8	2	39.6	
	17.6	4		
5	4.0	5	23.0	8.7
	1.2	2	20.9	
	0.9	2		
6	7.1	5	163.0	25.6
	2.0	2	72.4	
	1.4	3		
7	4.3	6	18.0	9.5
	1.3	3	33.6	
	1.7	3		
8	5.3	6	12.7	3.3
	1.8	3	9.1	
	1.9	3		
9	3.7	5	21.0	10.8
	2.0	3	39.5	
	2.2	3		
10	4.7	5	54.7	10.4
	8.3	3	78.8	
	8.3	3		

B. Long Time Numerical Tests

To ascertain the limits of the theory a series of long time numerical tests were run on eleven different satellites. For each satellite one positional measurement per half-hour was supplied for an entire night. The data had a numerical accuracy of $\sim \pi_s^5$. The appropriate fits were made and the theory used to predict the position at the beginning of the next evening ($T_{\max} \approx 0.8^d$). This was compared against the actual position at the time. The results are in Table 2 for the a) second-order theory with N in rad/day, b) the third-order theory with N in rad/day, and c) the third-order theory with N in rev/day. Also we have listed the Space Defense Center satellite identification number, the satellites' inclinations, eccentricities, and N values. We have also performed similar numerical experiments wherein the orbital element set and the usual equations of Keplerian motion were used to predict the position. These results do not significantly differ from those obtained with the use of the appropriate order theory series expansion.

In general, the second-order theory yields the same results as the third-order theory. The largest difference is for the International Ultraviolet Explorer satellite (#10637), which clearly shows both the importance of the higher order terms (in this instance) and that the limits

TABLE 2

Satellite Number	i	e	N = n- τ rev/day	Total Positional Error		
				2nd Order rad/day	3rd Order rad/day	3rd Order rev/day
6278	0°00	0.000	-0.0000	0°10	0°10	0°0
83594	1.85	0.001	-0.0000	0.20	0.2	1.1
83598	6.36	0.003	-0.011	4.1	3.9	0.1
1317	11.44	0.000	+0.000	0.1	0.3	0.2
83517	1.56	0.105	0.015	5.5	6.3	14.6
83589	6.60	0.134	0.020	7.6	7.0	10.5
4632	9.41	0.144	0.200	70.8	69.9	35.0
83546	1.77	0.005	0.091	32.8	32.1	17.3
3623	7.86	0.011	-0.105	38.7	37.9	21.2
73505	14.21	0.138	0.014	4.9	3.4	10.2
10637	28.41	0.238	+0.000	4.6	1.0	20.8

for i and e in Eq. (1) can be considerably extended as $N \rightarrow 0$. It is also clear that as N departs from zero the accuracy very rapidly degrades. In fact, almost all of the error is in the hour angle [remember that the declination coordinate has no first-order secular term, cf. Eq. (6b)].

In order to understand the relationship between the two different third-order results, remember that if N is in rev/day we have a multiplier of 2π exaggerating $N \neq 0$ values. When N is large this is good (cf. satellites #4632, #83546, #3623). When N is small but i and e large, this is bad (cf. satellites #73505 and #10637). We conclude that a more traditional differential correction procedure would be much more profitably used than the one presented here over these time spans. Of course, the initial orbital element set for the traditional procedure would be one obtained from our method.

IV. EXTENSIONS

A. Why Does It Work?

To try and answer this question, whose import should be obvious, let us compare a traditional differential correction procedure with the one presented here. Because no one would propose developing a sophisticated, complex algorithm (not to mention the successful coding of it for an electronic digital computer) for use over such short arcs, it would be designed for longer arcs. As the length of the arc increases, the significance of the perturbing effects of the non-sphericity of the earth, the presence of the sun and the moon, of air drag, of solar radiation pressure, etc., all increase. Hence, the physics of a traditional differential correction procedure is much more accurate and complex than we've used. To make this investment profitable, one needs accurate, precisely reduced data. In contradistinction, we should be able to produce the same results with simplified physics and with poor quality, unreduced data. One also formulates the physics in an inertial reference frame (almost) rather than a non-inertial reference frame. Therefore, the coordinate transformation we have handled explicitly and analytically, is handled implicitly and numerically. This complexity forces, as a practical matter, the search for the orbital elements (or geocentric initial conditions) to be an

iterative linear one with the attendant numerical computation of the various partial derivatives one needs. We use a second-order technique and proceed exactly (within the constraints of the order of the theory) because we do it analytically. Another consequence of using a more correct physical model of the situation is the desire to simplify the computations as much as possible. This leads to the use of various analytical devices (e.g., averaging in first or second order). This frequently leads to artificial analytical singularities, typically at zero inclination, zero eccentricity, or the critical inclinations ($5\cos^2 i = 1$). Since our model is pure Keplerian motion, there are no artificial singularities due to the use of analytical devices (and certainly not for $e = 0$, $i = 0$ or at the critical inclinations!). Moreover, one does not expend the effort necessary to design a sophisticated, complex differential correction procedure for one type of orbit. This generality of the traditional methods coupled with the vicissitudes of orbital analysis mean that the art of orbital analysis is frequently as important as the science of orbital analysis for them to work successfully. In our situation human intervention is almost superfluous*. Finally, our analysis is performed in a

*An iterative fitting for the mean motion may help us when n significantly departs from $\dagger$. Solving the problem using both the rad/day and rev/day and then performing a new solution is the maximum art we can envision in our process.

coordinate system that makes the motion nearly stationary. This is not a feature of traditional procedures. These points have been summarized in Table 3.

It would appear, over the arcs with which we are concerned here, that only the first four lines of the Table are in our favor. Of these, it is probably the near-stationary aspect of the motion that is of pre-eminent significance. By accident (for the purpose of this discussion, not for the practical uses of near-stationary artificial satellites) there is a natural, topocentric coordinate system in which some real artificial satellites' motion is nearly stationary. If it is the near-stationarity that really counts, and one can use a minimal amount of data on any orbital type to determine the orbital plane³, then a major change in the short-time differential correction of orbits is at hand.

B. A Non-Near-Stationary Orbit

As we look back at the formulas presented earlier, we see that explicit use of $n \approx \dot{\tau}$ has been made of twice. Once was in computing the satellite's parallax. The other was in the constancy of λ . If we consider any low inclination, small eccentricity orbit, with mean motion n_0 , the near-stationary constraints (1) would be modified to

$$i \leq \pi_0, e \leq \pi_0, |n/n_0 - 1| \leq \pi_0, \text{ and } |n - n_0| T_{\max} \leq \pi_0, \quad (12)$$

TABLE 3

COMPARISON OF TRADITIONAL AND PRESENT DIFFERENTIAL
CORRECTION PROCEDURES

<u>Traditional DC</u>	<u>NSDC</u>
Not near-stationary	Near-stationary
Implicit Geometry, numerical	Explicit Geometry, analytical
Numerical computation of partial derivatives	Analytical computation of partial derivatives
First-order, linear, solution technique	Second-order, non-linear, solution technique
For all types of orbits	For one type of orbit
Complex, accurate physics	Simple, approximate physics
Can involve considerable art	No art
Accurate data	Unreduced data
Artificial singularities	No singularities

where π_0 is the (mean) equatorial horizontal parallax of a satellite with mean motion $n_0 \{ \pi_0 = \sin^{-1} [R_\oplus (n_0^2 / GM_\oplus)^{1/3}] \}$. Then if we redefine

$$N = n - n_0, \quad (13a)$$

$$\lambda = \tau - n_0 T - (M_0 + \omega + \Omega), \quad (13b)$$

$$m = n_0 T, \quad (13c)$$

$$E' = \dot{E} / n_0, \quad (13d)$$

$$I' = \dot{I} / n_0, \quad (13e)$$

all of the analysis of §II and the Appendix will remain valid. The theory is still of order π_0^3 but our earlier comments concerning poor data or data reduction or perturbations may need modification depending on the value of n_0 . Roughly, J_2 will become important when $J_2 \pi_0^2 \sim \pi_0^3$, lunar perturbations will become important when $M_\oplus \pi_p^2 / (M_\oplus \pi_0^2) \sim 1$, and solar perturbations will become important when $M_\odot \pi_\odot^2 / (M_\oplus \pi_0^2) \sim 1$. This extension is not only trivial, it's not very important.

Consider, instead the general problem of making any artificial satellite nearly stationary. The heliocentric

parallax of the earth complicates all other astronomical problems and doesn't add anything. As the problem appears intractable anyhow we'll ignore it for now. We must, at least, have estimates for i , Ω , n , and e . It seems clear that the true longitude [$u = v + \omega$, cf. Eq. (2d)] is the variable to use. From

$$\cos u = \cos \delta \cos(\alpha - \Omega), \quad (14a)$$

$$\sin u = \cos \delta \cos i \sin(\alpha - \Omega) + \sin i \sin \delta, \quad (14b)$$

We can write

$$\begin{aligned} u(i, \Omega, \alpha, \delta) &= u[i, \Omega, \alpha(r, A, \Delta), \delta(r, A, \Delta)] \\ &= u\{i, \Omega, \alpha[r(n, e), A, \Delta], \delta[r(n, e), A, \Delta]\}. \end{aligned} \quad (15)$$

Let i_0 , Ω_0 , n_0 , and e_0 be our initial values for i , Ω , n , and e . Then to first order in $\Delta i = i - i_0$, etc.

$$\begin{aligned} u \approx u_0 &+ \left. \frac{\partial u}{\partial i} \right|_0 \Delta i + \left. \frac{\partial u}{\partial \Omega} \right|_0 \Delta \Omega + \left(\frac{\partial u}{\partial \alpha} \frac{\partial \alpha}{\partial r} + \frac{\partial u}{\partial \delta} \frac{\partial \delta}{\partial r} \right) \left. \frac{\partial r}{\partial n} \right|_0 \Delta n \\ &+ \left(\frac{\partial u}{\partial \alpha} \frac{\partial \alpha}{\partial r} + \frac{\partial u}{\partial \delta} \frac{\partial \delta}{\partial r} \right) \left. \frac{\partial r}{\partial e} \right|_0 \Delta e. \end{aligned} \quad (16)$$

Here u_0 is not really $u(i_0, \Omega_0, n_0, e_0)$ since the dependence of the true anomaly on the mean motion and the eccentricity has

not been taken into account. Thus, with $v_0 = v(n_0, e_0)$,

$$u \approx v_0 + \omega + \left. \frac{\partial u}{\partial i} \right|_0 \Delta i + \left. \frac{\partial u}{\partial \Omega} \right|_0 \Delta \Omega + \left[\frac{\partial v}{\partial n} + \left(\frac{\partial u}{\partial \alpha} \frac{\partial \alpha}{\partial r} + \frac{\partial u}{\partial \delta} \frac{\partial \delta}{\partial r} \right) \frac{\partial r}{\partial n} \right] \Big|_0 \Delta n \\ + \left[\frac{\partial v}{\partial e} + \left(\frac{\partial u}{\partial \alpha} \frac{\partial \alpha}{\partial r} + \frac{\partial u}{\partial \delta} \frac{\partial \delta}{\partial r} \right) \frac{\partial r}{\partial e} \right] \Big|_0 \Delta e. \quad (17)$$

Before, in §II, the three angles of the problem, ω , Ω , and M_0 [= the mean anomaly at the epoch time, not our initial guess for $M(T)$], naturally appeared in analytically convenient forms. While ω and Ω (by default) still do, M_0 does not*. Our numerical experience with the near-stationary satellite problem augers very poorly for the determination of M_0 . Hence, either the formulation presented here needs modification or a more clever choice of variables is required.

*That is, we've actually written out Eq. (17) using the general first-order expressions

$$\Delta r/a_0 = [-2(1 - e \cos E)/(3n) + eT \sin v/(1 - e^2)^{1/2}] \Delta n \\ - \cos v \Delta e + O \Delta i + O \Delta \Omega + O \Delta \omega + [e \sin v/(1 - e^2)^{1/2}] \Delta M_0, \\ \cos \delta_0 \Delta \delta = [T(1 - e^2)^{1/2} \sin i \cos u/(1 - e \cos E)^2] \Delta n \\ + [\sin i \cos u \sin v(2 + e \cos v)/(1 - e^2)] \Delta e + \cos i \sin u \Delta i \\ + O \Delta \Omega + \sin i \cos u \Delta \omega + [(1 - e^2)^{1/2} \sin i \cos u/(1 - e \cos E)^2] \Delta M_0, \\ \cos^2 \delta_0 \Delta \alpha = [T(1 - e^2)^{1/2} \cos i/(1 - e \cos E)^2] \Delta n \\ + [\cos i \sin v(2 + e \cos v)/(1 - e^2)] \Delta e - \sin i \sin u \cos u \Delta i \\ + \cos^2 \delta_0 \Delta \Omega + \cos i \Delta \omega + [(1 - e^2)^{1/2} \cos i/(1 - e \cos E)^2] \Delta M_0.$$

One might inquire as to the advisability of again dealing directly with topocentric coordinates. Since the topocentric coordinate system is not now the unique one referred to in the Introduction, such an analysis would not likely be of much benefit.

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APPENDIX: The Horizon System

Since the satellite is nearly stationary, and the topocentric horizon system is centered on the observer, this appears to be an excellent coordinate system in which to work. The azimuth parallax correction is also extremely simple. The drawback, in our view, is the requirement of undoing the direction cosines.

In addition to geocentric hour angle and declination (h, δ) we need geocentric zenith distance, z , and azimuth A . A is measured from the south positive westward. Finally, if ϕ is the observer's astronomical latitude,

$$\sin z \sin A = \cos \delta \sinh, \quad (\text{A1a})$$

$$\sin z \cos A = \sin \phi \cos \delta \cosh - \cos \phi \sin \delta, \quad (\text{A1b})$$

$$\cos z = \cos \phi \cos \delta \cosh + \sin \phi \sin \delta. \quad (\text{A1c})$$

From Eqs. (2)

$$\sin z \sin A = \cos \delta \sin(\tau - \Omega) - \sin \delta \cos(\tau - \Omega), \quad (\text{A2a})$$

$$\begin{aligned} \sin z \cos A = & \sin \phi [\cos \delta \cos(\tau - \Omega) + \sin \delta \sin(\tau - \Omega)] \\ & - \cos \phi \sin \delta, \end{aligned} \quad (\text{A2b})$$

$$\begin{aligned} \cos z = & \cos \phi [\cos \delta \cos(\tau - \Omega) + \sin \delta \sin(\tau - \Omega)] \\ & + \sin \phi \sin \delta. \end{aligned} \quad (\text{A2c})$$

We proceed as outlined in §II and find

$$\begin{aligned}
\sin z \sin A &= \sin \lambda - (NT + 2E) \cos \lambda - (N^2 T^2 / 2) \sin \lambda \\
&- 2NT(E \sin \lambda + E' \cos \lambda) - 2E^2 \sin \lambda - (5EE' / 2) \cos \lambda \\
&+ [(I_C^2 + I_S^2)^{1/2} I / 2] \cos(\tau - \Omega), \quad (A3a)
\end{aligned}$$

$$\begin{aligned}
\sin z \cos A &= \sin \phi [\cos \lambda + (NT + 2E) \sin \lambda] - I \cos \phi - (N^2 T^2 / 2) \sin \phi \cos \lambda \\
&+ 2NT \sin \phi (E' \sin \lambda - E \cos \lambda) - INT \cos \phi - 2EI' \cos \phi \\
&- \sin \phi \{ 2E^2 \cos \lambda - (5EE' / 2) \sin \lambda + [I(I_C^2 + I_S^2)^{1/2}] \sin(\tau - \Omega) \}, \quad (A3b)
\end{aligned}$$

$$\begin{aligned}
\cos z &= \cos \phi [\cos \lambda + (NT + 2E) \sin \lambda] + I \sin \phi - (N^2 T^2 / 2) \cos \phi \cos \lambda \\
&+ 2NT \cos \phi (E' \sin \lambda - E \cos \lambda) + I' NT \sin \phi + 2IE \sin \phi \\
&- \cos \phi \{ 2E^2 \cos \lambda - (5EE' / 2) \sin \lambda + [I(I_C^2 + I_S^2)^{1/2} / 2] \sin(\tau - \Omega) \}. \quad (A3c)
\end{aligned}$$

If Z , A are topocentric zenith distance and azimuth, then

$$\tan(A - A) = P \sin A / (1 - P \cos A), \quad (A4a)$$

$$\tan(Z - z) = Q \sin(z - \Gamma) / [1 - Q \cos(z - \Gamma)], \quad (A4b)$$

$$P = (\rho/r) \sin(\phi - \phi') \csc z, \quad (A4c)$$

$$\begin{aligned}
Q &= (\rho/r) \cos(\phi - \phi') \sec \Gamma, \\
\tan \Gamma &= \tan(\phi - \phi') \cos[(A + A)/2] \sec[(A - A)/2], \quad (A4d)
\end{aligned}$$

But

$$\phi - \phi' = \eta \sin 2\phi' + (\eta^2 / 2) \sin 4\phi' + \dots \quad (A5a)$$

where

$$\eta = f + f^2 / 2 + \dots \quad (A5b)$$

and f is the flattening of the earth, $1/297.25$. Therefore, through second-order in π_s^2 ,

$$A = A, \quad (A6a)$$

$$Z = z + (\rho/a_0) \sin z [1 + 2N/(3\tau) + E + (\rho/a_0) \cos z]. \quad (A6b)$$

Hence,

$$\begin{aligned} \sin Z \sin A = & [1 - (\rho/a_0)^2 \sin^2 z] \sin z \sin A + \\ & (\rho/a_0) \sin z \sin A \cos z [1 + 2N/(3\tau) + E + (\rho/a_0) \cos z], \end{aligned} \quad (A7a)$$

$$\begin{aligned} \sin Z \cos A = & [1 - (\rho/a_0)^2 \sin^2 z] \sin z \cos A + \\ & (\rho/a_0) \sin z \cos A \cos z [1 + 2N/(3\tau) + E + (\rho/a_0) \cos z], \end{aligned} \quad (A7b)$$

$$\cos Z = \cos z - (\rho/a_0) \sin^2 z [1 + 2N/(3\tau) + E + 2(\rho/a_0) \cos z]. \quad (A7c)$$

Not only must the direction cosines be eliminated to obtain explicit expressions for Z and A , but the problem is implicit. Hence, we have opted for the (relative) simplicity of the equatorial coordinate system.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report presents a new concept for the differential correction of orbits. It is developed in detail for near-stationary artificial satellites. For these satellites a few (<10) observations over a short (5-15 ^m , $\leq 1\%$ period) time interval allows the reacquisition of the satellite an hour later. Moreover, with only slightly more data points (total number <15) over a slightly longer (total duration <0.5) time interval it is capable of producing an excellent set of orbital elements. The technique is self-starting and does not use any of the classical initial orbit determination procedures. It can be used by a radar or extended to include angular velocity data. Its success appears to be based on the ability to find a coordinate system in which the object's motion is nearly stationary and the extensive use of analytical (instead of numerical) procedures. Extensions of the theory to include all first-order perturbations and to all orbital types are possible.		