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THE ANALYSIS OF SOUND PROPAGATION IN JET ENGINE DUCTS USING THE FINITE DIFFERENCE METHOD

*ANALYSIS AND OPTIMIZATION BRANCH
STRUCTURES & DYNAMICS DIVISION*

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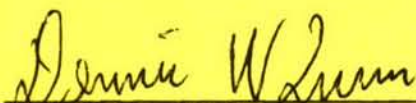
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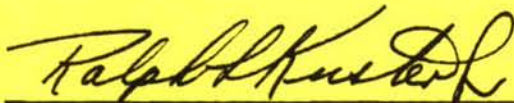


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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In this report, the author derives the partial differential equations which describe sound propagation in jet engine ducts and then presents a finite difference approach for solving these equations. Also included is a computer program listing, sample input and sample output. The program can handle uniform rectangular and cylindrical ducts with or without uniform flow. In addition, if a mapping function which maps a nonuniform duct to a uniform duct is specified, the program can determine sound fields in nonuniform ducts in the absence of flow.		

FOREWORD

This report describes work performed in the Air Force Flight Dynamics Laboratory under Project 2304N112, Theoretical Duct Acoustics. This is an interim report on work carried out between November 1975 and November 1978.

The author thanks Ms. Jean Schwab for her preliminary editing, proofreading and setting up and running sample cases. The author thanks also Ms. Lynn Curtis for her typing in the preparation of this report.

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SECTION I

INTRODUCTION

In this report we consider the problem of computing sound propagation in ducts. These ducts are models of the interior of jet engines. Since the largest source of jet engine noise is fan noise, there has been an increase in interest in duct acoustics in recent years. We will consider ducts of uniform cross section with or without mean flow and ducts with variable cross section not containing flow. The ducts are either two dimensional or axially symmetric three dimensional. To compute the sound field within one of these ducts, the momentum and continuity equations are linearized and combined to yield a linear, second order partial differential equation in the acoustic pressure. The boundary conditions consist of a specified pressure distribution at the duct entrance, a vanishing normal derivative at the centerline, and a specified acoustic impedance at the exit and at the outer wall of the duct which corresponds to an acoustic lining. These equations are then solved using the finite difference method. The large system of algebraic equations which result from using finite differences is solved using an out-of-core block tri-diagonal system solver on the CDC Cyber 74 computer. The acoustic velocities are then determined by integrating the linearized momentum equations and then used to compute the acoustic power at the entrance and exit of the duct. These are then used to determine the attenuation resulting from a particular wall lining.

For a nonuniform duct the additional step of mapping the nonuniform geometry to a uniform duct is required before the finite difference method is applied to obtain the solution. We use a conformal map so

that right angles in the original coordinate system become right angles in the mapped coordinate system. The mapped equations are then solved using finite differences as indicated.

In Section II, we will describe in more detail the equations and boundary conditions which need to be solved and give the expressions used to compute the acoustic power. In Section III, we discuss the conformal mapping procedure and derive the mapped equations and boundary conditions. In Section IV, we derive the finite difference equations for both the original and mapped equations, and indicate the method used to solve the resultant system of algebraic equations. In Section V, we describe the computer program DUCT including minor program modifications for extra capability, and in Section VI we give samples with input and output.

SECTION II

ANALYTIC MODEL

Differential Equations

If viscous and heat transfer terms are neglected, the equations of motion for an ideal gas are

$$\text{Momentum} \quad \rho' D(V')/Dt = -\nabla p' \quad (1a)$$

$$\text{Continuity} \quad D(\rho')/Dt + \rho'(\nabla \cdot V') = 0 \quad (1b)$$

$$\text{Energy and state} \quad p' = \text{const } \rho'^{\alpha} \quad (1c)$$

where ρ' is the density, V' is the velocity vector, and p' is the pressure. If it is assumed that the pressure perturbations are small compared to the average pressure and that the steady velocities are negligible, Eq. (1) becomes

$$\partial \hat{p} / \partial t = -\rho_o c^2 \nabla \cdot \hat{v} \quad (2a)$$

$$\rho_o \partial \hat{v} / \partial t = -\nabla \hat{p} \quad (2b)$$

When combined, Eqs. (2) reduce to the wave equation

$$\nabla^2 \hat{p} = (1/c^2) \partial^2 \hat{p} / \partial t^2 \quad (3)$$

and, with time dependence of the form $e^{i\omega t}$ [that is, $\hat{p}(X,Y,Z,t) = e^{i\omega t} p(X,Y,Z)$], Eq. (3) becomes the Helmholtz equation for the pressure p

$$\nabla^2 p + (\omega^2/c^2) p = 0 \quad (4)$$

Analyzing only one frequency for fan noise is reasonable since noise at different frequencies can be superimposed. In nondimensional coordinates,^{1,2,3,4} Eq. (4) becomes

$$\nabla^2 p + (2\pi\eta)^2 p = 0 \quad (5)$$

with $\eta = R\omega/(2\pi c)$ for a cylindrical duct (R being the duct radius), $\eta = H\omega/(2\pi c)$ for a rectangular duct (H being the duct height). In two dimensions, for rectangular ducts, Eq. (5) is

$$\partial^2 p / \partial x^2 + \partial^2 p / \partial y^2 + (2\pi\eta)^2 p = 0 \quad (6)$$

whereas, in three dimensions with axial symmetry, Eq. (5) becomes

$$\partial^2 p / \partial x^2 + \partial^2 p / \partial r^2 + (1/r) \partial p / \partial r + (2\pi\eta)^2 p = 0 \quad (7)$$

where x is the axial direction and r is the radial direction. In the mean flow case, the steady velocity in the axial direction is assumed to be a constant U (and no longer negligible), yielding the dimensionless axially symmetric reduced wave equation³

$$\begin{aligned}
(1-M^2) \partial^2 p / \partial x^2 + \partial^2 p / \partial r^2 + (1/r) \partial p / \partial r \\
- 4\pi\eta M i \partial p / \partial x + (2\pi\eta)^2 p = 0
\end{aligned} \tag{8}$$

where $M = U/c$ is the Mach number and $\eta = R\omega/(2\pi c)$ is the dimensionless frequency, R being the duct radius.

Boundary Conditions

To absorb acoustic energy within the duct, the outer walls are lined with Helmholtz resonators. The Helmholtz resonator can consist of a perforated sheet placed at distance d from a solid backing sheet. The acoustic impedance of this wall lining is determined by the hole diameters and backing depth.

If Z is the acoustic impedance at the outer wall, then

$$Z = p/n \cdot V$$

where n is the exterior normal to the outer wall. For no flow,

$$i\omega\rho_0 V = -\nabla p, \text{ so}$$

$$iZ/\omega\rho_0 = p/n \cdot \nabla p$$

or

$$\partial p / \partial n + i\omega\rho_0 p / Z = 0$$

(9)

at the wall. [Note that $\partial p / \partial n = n \cdot \nabla p$ by definition.] For the case of uniform flow the continuity of particle displacement is used rather than the continuity of particle velocity to derive the dimensionless boundary condition

$$\partial p / \partial n = -2\pi\eta i p / Z_w - (2M/Z_w) \partial p / \partial x + iM^2 / (2\pi\eta Z_w) \partial^2 p / \partial x^2 \tag{10}$$

at the outer wall (see Ref. 5 for the derivation). For a variable wall lining (such as a three-sectional lining) Z_w is a function of x instead of being constant, as is the case for a uniform lining. The remaining boundary conditions for a cylindrical duct are

$$p(0,r) = f(r) \quad \text{at the entrance} \quad (11a)$$

$$\partial p / \partial n = 0 \quad \text{at } r = 0 \text{ (the centerline)} \quad (11b)$$

$$\partial p / \partial n + 2\pi\eta ip / Z_e = 0 \quad \text{at the exit} \quad (11c)$$

For the rectangular duct the only change in the boundary condition is that $p(0,y) = f(y)$ is specified as an entrance condition.

Sound-Power Attenuation

For both the no-flow and uniform flow cases, the expression used to compute the dimensionless axial intensity^{3,4} is

$$I(x,r) = \text{real} (p^*u)/(2\pi\eta) + M[p^*p + (u^*u + v^*v)/(2\pi\eta)^2]/2 \quad (12)$$

where u is the velocity in the axial direction, v is the velocity in the radial direction, and the asterisk denotes complex conjugation. For the no flow case, of course, $M = 0$ and

$$I = \text{real} (p^*u)/(2\pi\eta) \quad (13)$$

To compute u and v , once p is known, the dimensionless x-momentum and r-momentum equations are used:

$$u = i \partial p / \partial x + i M / (2\pi\eta) \partial u / \partial x \quad (14a)$$

$$v = i \partial p / \partial y + i M / (2\pi\eta) \partial v / \partial x \quad (14b)$$

The initial condition used to solve for u is that $u = 2\pi\eta p$ at the exit. Then the irrotationality condition $\partial u/\partial y = \partial v/\partial x$ and Eqs. (14) are used to solve for v . Once the intensity is known the dimensionless acoustic power is computed as

$$E_x = \int_A I(x,r) dA = \int_0^1 I(x,r) r dr \quad (15)$$

for a cylindrical duct, or

$$E_x = \int_0^1 I(x,y) dy \quad (16)$$

for a rectangular duct. The sound attenuation is then determined as

$$dB = 10 \log_{10} [E_x/E_0] \quad (17)$$

where E_0 is the sound power at the entrance and E_x is the sound power at the axial point x .

SECTION III

CONFORMAL MAPPING

The well-known Riemann Mapping Theorem establishes that an arbitrary simply connected domain in the plane can be mapped conformally onto an open rectangle with vertices $(0,1)$, $(0,-1)$, $(X,-1)$, and $(X,1)$. (For a diagram of such a map, see Fig. 1. Other examples of variables of ducts are given in Figure 2.) If the map is of the form

$$z = z(x,y); \quad r = r(x,y)$$

then, for it to be conformal, the Cauchy-Riemann equations

$$\partial z/\partial x = \partial r/\partial y; \quad \partial z/\partial y = - \partial r/\partial x$$

must be satisfied.

Assuming axial symmetry, Eq. (4) may be rewritten as

$$\partial^2 p / \partial z^2 + \partial^2 p / \partial r^2 + (1/r) \partial p / \partial r + (\omega/c)^2 p = 0$$

where z is the axial coordinate and r is the radial coordinate.

In transformed coordinates this equation becomes

$$\begin{aligned} 0 = & \frac{\partial^2 \hat{p}}{\partial x^2} + \frac{\partial^2 \hat{p}}{\partial y^2} - \frac{1}{r(x,y)} \frac{\partial z(x,y)}{\partial y} \frac{\partial \hat{p}}{\partial x} \\ & + \frac{1}{r(x,y)} \frac{\partial z(x,y)}{\partial x} \frac{\partial \hat{p}}{\partial y} + (2\pi\eta)^2 \left\{ \left(\frac{\partial z(x,y)}{\partial x} \right)^2 \right. \\ & \left. + \left(\frac{\partial z(x,y)}{\partial y} \right)^2 \right\} \hat{p} \end{aligned} \quad (18)$$

where $\hat{p}(x,y) = p(z,r)$ and $\eta = R\omega/(2\pi c)$ is the dimensionless frequency.

The boundary conditions (9) and (11) become

$$\hat{p}(0,y) = f(z(0,y), r(0,y)) = g(y) \quad \text{at the entrance} \quad (19a)$$

$$\frac{\partial \hat{p}}{\partial y} = 0 \quad \text{at } y = 0 \quad (19b)$$

$$\frac{\partial \hat{p}}{\partial y} = -2\pi\eta ip \left[\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 \right]^{1/2} / z_y \quad \text{at } y = 1 \quad (19c)$$

$$\frac{\partial \hat{p}}{\partial x} = -2\pi\eta ip \left[\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 \right]^{1/2} / z_x \quad \text{at } x = X. \quad (19d)$$

Finite difference solutions will be obtained in Sec. IV. (See Ref. 6 for a more complete description.)

Example

For positive real $\alpha < \frac{\pi}{2}$, the cone enclosed in the (z,r) plane by the lines $r = (\sin \alpha)z/\cos \alpha$ and $r = -(\sin \alpha)z/\cos \alpha$ and by arcs of the circles $z^2 + r^2 = R^2$ and $z^2 + r^2 = R^2 e^{2\alpha X}$ is the region considered.

The transformation defined by

$$z = Re^{\alpha x} \cos \alpha y; r = Re^{\alpha x} \sin \alpha y \quad (20)$$

maps the open rectangle onto the prescribed cone. With $z(x,y)$ and $r(x,y)$ defined by Eq. (20), the finite difference equations corresponding to Eqs. (18) and (19) were solved on a rectangle using the method described above.

SECTION IV

FINITE DIFFERENCE SOLUTIONS OF ANALYTIC MODEL

To obtain solutions of Eqs. (6-8) in the case of a rectangular duct, a no-flow cylindrical duct, or a cylindrical duct with uniform flow, respectively, the method of finite differences was used. The boundary conditions (10) and (11) must be included for the problems to be well posed. The finite difference equations may be summarized as follows (see Refs. 2, 3, 6, and especially 7). Set

$$\delta x = X/(n+1)$$

$$\delta r = 1/(m+1)$$

$$x_j = j\delta x \quad j = 1, \dots, n$$

$$r_k = k\delta r \quad k = 1, \dots, m$$

and

$$p_{j,k} = p(x_j, y_k)$$

where m and n are positive integers. Then for a cylindrical duct the difference equation at a point not adjacent to the boundary is

$$\begin{aligned}
& (1-M^2)(p_{j+1,k} - 2p_{j,k} + p_{j-1,k})/(\delta x)^2 \\
& + (p_{j,k+1} - 2p_{j,k} + p_{j,k-1})/(\delta r)^2 + (p_{j,k+1} - p_{j,k-1})/(2(\delta r)r_k) \\
& - 4\pi\eta Mi(p_{j+1,k} - p_{j-1,k})/(2\delta x) + (2\pi\eta)^2 p_{j,k} = 0.
\end{aligned}$$

Collecting terms, the equation becomes

$$\begin{aligned}
& [2 + 2(1-M^2)(\delta r)^2/(\delta x)^2 - (2\pi\eta\delta r)^2] p_{j,k} - p_{j,k+1}[1 + 1/(2k)] \\
& - p_{j,k-1}[1 - 1/(2k)] - p_{j+1,k}[1-M^2 - 2\pi\eta Mi\delta r^2/\delta x] \\
& - p_{j-1,k}[1-M^2 + 2\pi\eta Mi\delta r^2/\delta x] = 0 \text{ for } 1 \leq j \leq n-1, 2 \leq k \leq m-1.
\end{aligned}$$

Similar expressions are obtained for $j = n$, $k = 1$, $k = m$, which correspond to the boundary conditions (10) and (11).

To obtain a solution of Eqs. (18) and (19) on a cylinder, the following finite difference scheme is employed.

Set

$$\begin{aligned}
\delta x &= X/(n+1), \\
\delta y &= 1/(m+1), \\
x_j &= j \delta x, \quad j = 1, \dots, n, \\
y_k &= k \delta y, \quad k = 1, \dots, m, \\
z_{j,k} &= \frac{\partial z}{\partial y}(x_j, y_k) \\
\hat{z}_{j,k} &= \frac{\partial z}{\partial x}(x_j, y_k), \quad j = 1, \dots, n, \\
r_{j,k} &= r(x_j, y_k), \quad k = 1, \dots, m, \\
&\text{and} \\
p_{j,k} &= p(x_j, y_k),
\end{aligned}$$

where m and n are positive integers. Then the difference equation is

$$\begin{aligned}
& (p_{j+1,k} - 2p_{j,k} + p_{j-1,k})/(\delta x)^2 \\
& + (p_{j,k+1} - 2p_{j,k} + p_{j,k-1})/(\delta y)^2 \\
& - \frac{z_{j,k}}{r_{j,k}} (p_{j+1,k} - p_{j-1,k})/(2\delta x) \\
& + \frac{\hat{z}_{j,k}}{r_{j,k}} (p_{j,k+1} - p_{j,k-1})/(2\delta y) \\
& + (2\pi\eta)^2 (\hat{z}_{j,k}^2 + z_{j,k}^2) p_{j,k} = 0.
\end{aligned}$$

Collecting terms, the equation becomes

$$\begin{aligned}
& \left[2 + 2(\delta y)^2/(\delta x)^2 - (2\pi\eta\delta y)^2 (\hat{z}_{j,k}^2 + z_{j,k}^2) \right] p_{j,k} \\
& - p_{j,k+1} \left[1 + \frac{\hat{z}_{j,k}}{r_{j,k}} \delta y/2 \right] - p_{j,k-1} \left[1 - \frac{\hat{z}_{j,k}}{r_{j,k}} \delta y/2 \right] \\
& - p_{j+1,k} \left[(\delta y)^2/(\delta x)^2 - \frac{z_{j,k}}{r_{j,k}} \frac{(\delta y)^2}{2\delta x} \right] \\
& - p_{j-1,k} \left[(\delta y)^2/(\delta x)^2 + \frac{z_{j,k}}{r_{j,k}} \frac{(\delta y)^2}{2\delta x} \right] \\
& = 0 \text{ for } 1 \leq j \leq n-1, 2 \leq k \leq m-1.
\end{aligned}$$

Similar expressions are obtained for $j = n$, $k = 1$, $k = m$, which correspond to the boundary conditions (19).

For a reasonable number of mesh points, say 50 in the x direction and 20 in the y direction, the linear system of equations to be solved consists of 1000 unknown $p_{j,k}$'s to be determined. Moreover, the $p_{j,k}$'s are complex numbers because of the boundary

conditions, so there are 2000 unknown real numbers to compute. Standard iterative methods will not work for this problem, for the system is neither positive definite nor symmetric. Therefore, a direct method was used to solve the system, taking advantage of its sparseness.

The set of difference equations forms a system of linear equations which can be written in matrix form as $MP = F$ where M is the finite difference matrix, P is the unknown pressure matrix and F is the matrix containing the prescribed pressures.

M has the following form:

$$\begin{array}{cc} 0 & \dots & 0 & -a & 0 & 0 & 0 & 0 & \dots & 0 & -1 & b & -1 & 0 & 0 & 0 & 0 & \dots & 0 & -c & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & -a & 0 & 0 & 0 & \dots & 0 & 0 & -1 & b & -1 & 0 & 0 & 0 & \dots & 0 & 0 & -c & 0 & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & 0 & -a & 0 & 0 & \dots & 0 & 0 & 0 & -1 & b & -1 & 0 & 0 & \dots & 0 & 0 & 0 & -c & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & 0 & 0 & -a & 0 & \dots & 0 & 0 & 0 & 0 & -1 & b & -1 & 0 & \dots & 0 & 0 & 0 & 0 & -c & 0 & \dots & 0 \end{array}$$

which can be written as a block-tridiagonal matrix

$$\begin{array}{cccccc} B_1 & C_1 & 0 & 0 & \dots & 0 \\ A_2 & B_2 & C_2 & 0 & \dots & 0 \\ 0 & A_3 & B_3 & C_3 & & 0 \\ 0 & \dots & A_n & B_n & & \end{array} \quad \begin{array}{l} \text{where } A, B, C \text{ are square matrices} \\ \text{which fit into the pattern.} \end{array}$$

M can now be factored as

$$\text{or } M = L * U$$

$$\begin{pmatrix} B_1 & C_1 & 0 & 0 & \dots & 0 \\ A_2 & B_2 & C_2 & 0 & \dots & 0 \\ 0 & A_3 & B_3 & C_3 & & 0 \\ 0 & \dots & A_n & B_n & & \end{pmatrix} = \begin{pmatrix} I & 0 & 0 & \dots & 0 \\ L_2 & I & 0 & \dots & 0 \\ 0 & L_3 & I & \dots & 0 \\ 0 & 0 & \dots & L_n & I \end{pmatrix} \begin{pmatrix} U_1 & C_1 & 0 & 0 & \dots & 0 \\ 0 & U_2 & C_2 & 0 & \dots & 0 \\ 0 & 0 & U_3 & C_3 & \dots & 0 \\ 0 & 0 & \dots & 0 & U_n \end{pmatrix}$$

where

$$U_1 = B_1$$

$$L_m = A_m U_{m-1}^{-1} \quad \text{for } m = 2, \dots, n$$

$$U_m = B_m - L_m C_{m-1}$$

Therefore, the system to be solved may be written as

$$L * U * P = F$$

Solving $L * Y = F$ for Y and then solving $U * P = Y$ for P gives the desired pressures.

SECTION V

PROGRAM DESCRIPTIONS

Program DUCT computes the pressure within cylindrical, rectangular, and nonuniform ducts using the finite difference method described in Section IV. Using these values for the pressure, DUCT then calculates the acoustic power and attenuation.

The logical flow of DUCT is represented schematically in Figure 3. A summary of the various subroutines called by DUCT is now given.

DUCT The main routine which controls the reading of input and the initialization of variables. DUCT calls FUNCT directly or, if the function is to be minimized, calls an optimization routine ZXMIN.

FUNCT A statement function which does the calling to the subroutines which set up the system to be solved,

solve the system, compute the attenuation of sound, and print the output. By having a statement function do the calling, standard optimization routines can be used which minimize a given function.

ZXMIN A library subroutine which finds the minimum of a function.

SETUP1 This subroutine uses a finite difference method to transform the wave equation for a rectangular duct

$$P_{xx} + P_{yy} + \lambda^2 P = 0$$

into centered difference equations at each point. For a point not on the boundary, the equation has the following form:

$$\begin{aligned} & [P_{i+1,j} - 2P_{i,j} + P_{i-1,j}]/Dx^2 \\ & + [P_{i,j+1} - 2P_{i,j} + P_{i,j-1}]/Dy^2 + \lambda^2 P_{i,j} = 0 \end{aligned}$$

Collecting terms, the equation becomes

$$\begin{aligned} & (2 + 2 \frac{Dy^2}{Dx^2} - \lambda^2 Dy^2) P_{i,j} - P_{i,j+1} - P_{i,j-1} \\ & - \frac{Dy^2}{Dx^2} P_{i+1,j} - \frac{Dy^2}{Dx^2} P_{i-1,j} = 0 \end{aligned}$$

This system of equations is then written in matrix form.

SETUP Using the methods and equations presented in Section IV, SETUP produces the finite difference for a nonuniform duct.

SETUP2 This subroutine sets up the matrix for a cylindrical duct as described in Section IV.

BLOCK To increase the efficiency of the program, the matrix produced in the SETUP routines is stored in block-tridiagonal matrix form. BLOCK calls LU3 to factor the matrix into bidiagonal form. LEQS3 is then called to solve the system using back substitution. BLOCK also invokes CLOSMS, OPENMS, WRITMS and READMS which are system subroutines available on Control Data machines.

OUTPUT Prints the real and imaginary parts of the pressure calculated in the BLOCK subroutine.

SOUND Given the pressure, SOUND computes and prints the sound power and attenuation

$$\text{dB} = 10 \log_{10} (E_2/E_1)$$

where E_1 is the acoustic power at the entrance of the duct and E_2 is the acoustic power at the exit.

Input

The information needed to perform the calculations can be entered through cards or disks. A description of required input is presented here.

<u>Card No.</u>	<u>Columns</u>	<u>Variable Name</u>	<u>Description</u>
1	1-3	IDUCT	An integer representing the duct geometry. Enter -1 for a nonuniform duct, 0 for

<u>Card No.</u>	<u>Columns</u>	<u>Variable Name</u>	<u>Description</u>
			a rectangular duct or 1 for a cylindrical duct. (I3)
2	1-10	XM	Mach Number. For the acoustic linearization to be valid $-.5 \leq XM \leq .5$. Set XM equal to zero for no flow case. (F10.0)
3	1-20	ZETAX	Complex number representing the exit impedance (F10.0, F10.0).
4	1-20	ZETAX	Wall impedance at $Y = YMAX$. $ZETAX = 9999999999 + i9999999999$ corresponds to a hard wall (F10.0, F10.0).
5	1-10	XMAX	Length, L, of duct in meters (F10.0).
5	11-20	YMAX	Height, H, of duct in meters. If problem has been made non-dimensional, set $YMAX = 1$ and $XMAX = \frac{L}{H}$. (F10.0)
6	1-10	ETA	Frequency parameter (F10.0).
7	1-3	IEND	Number of pts in x-direction, excluding pts on the boundaries. Maximum of 100 if subroutines READMS, WRITMS are used. If $XM \neq 0$, IEND should be odd. (I3)

<u>Card No.</u>	<u>Columns</u>	<u>Variable Name</u>	<u>Description</u>
7	4-6	IEND	Number of pts. in y-direction, excluding pts on boundaries. Maximum of 20. (I3)
8	1-3	KOUT	Controls the form of the output. Set KOUT = 0 to suppress the printing of pressures and KOUT = 1 for complete output. (I3)

If ZXMIN is to be called to find the minimizing values of ZETAY, the following information is required.

<u>Card No.</u>	<u>Columns</u>	<u>Variable Name</u>	<u>Description</u>
8	4-6	NUMMAX	Maximum number of iterations or calls to FUNCT by ZXMIN. (I3)
8	7-16	IH	Number of digits of accuracy desired in the estimates of ZETAY. (I3)

If KOUT = 0, the following card can be left blank.

<u>Card No.</u>	<u>Columns</u>	<u>Variable Name</u>	<u>Description</u>
9	1-3	IOUT	Increment value. The pressure will be printed for pts with abscissas corresponding to the values of I = 1 through IEND in steps of IOUT. (I3)

<u>Card No.</u>	<u>Columns</u>	<u>Variable Name</u>	<u>Description</u>
9	4-6	JOUT	Increment value. The program will print the pressure at pts located on grid lines corresponding to $J = 1$ through JEND in steps of JOUT. (I3)
9	7-9	IT	The pressure will be printed for pts located on grid lines corresponding to $J = 1$ through IT. Maximum value is $IT = JEND$. (Headings will be distorted if $JOUT = 1$ and $IT > 15$).
10	1-10	ALPHA	Generating angle in radians of a nonuniform duct. Required only if $IDUCT = -1$. (F10.0)

Multisectional Linings

If the user desires a multisectional lining, several changes must be made to the program to accommodate the additional impedance parameters (See Section IV). The following changes were used to modify the program for a three-sectional lining in a nonuniform duct. Modifications for rectangular and cylindrical ducts are enclosed in brackets.

Recall that the impedance is a complex number so that an n-sectional lining means ZET, the array containing these values, will be of dimension $2n$. To adjust the dimension of ZET, insert the appropriate value in line 7 of DUCT. In DUCT, arrays WA, HH, and G are parameters required by ZXMIN and their dimension depends on the dimension of ZET. Set the

dimension of WA equal to $N(N+4)$. The dimension of HH equal to $N(N+1)/2$, and the dimension of G equal to N where N is the dimension of ZET ($N=2n$). For example, for a 3 section lining, DUCT.7 should now read:

```
DIMENSION ZET(6), WA(18), HH(21), G(6)
```

In addition, a dimension statement will need to be inserted after line 8 in SETUP [SETUP1, SETUP2] which reads DIMENSION ZET (6). Also, in line 55 of DUCT, the number following FUNCT in the parameter list is the dimension of ZET, thus it should be changed to

```
ZXMIN (FUNCT, 6, IH, NUMMAX,  $\phi$ , ZET, HH, G, FMIN, WA, IER)
```

To initialize the values of ZET to those of a uniform lining, the following cards should be inserted after line 51 in DUCT

```
ZET(3) = ZET(1)
```

```
ZET(4) = ZET(2)
```

```
ZET(5) = ZET(1)
```

```
ZET(6) = ZET(2)
```

If the program is restarted, the expressions on the right side of the equations can be replaced with the minimizing values from the previous run.

In order to pass the additional values to the subroutines, lines 10 through 12 in FUNCT should be replaced with

```
IF(IDUCT.LT.0) CALL SETUP (ZET)
```

```
IF(IDUCT.EQ.0) CALL SETUP1(ZET)
```

```
IF(IDUCT.GT.0) CALL SETUP2(ZET)
```

and the first line of SETUP [SETUP1, SETUP2] should be replaced with

```

SUBROUTINE SETUP (ZET)
[ SUBROUTINE SETUP1(ZET) ]
[ SUBROUTINE SETUP2(ZET) ]

```

To print the values of ZET before each pass, insert these cards after line 9 in FUNCT

```

PRINT 999, (ZET(I), I = 1,6)
999 FORMAT (1X, 6F16.8)

```

where I ranges from 1 to the dimension of ZET.

Finally, the location of the various linings are inserted after SETUP .34 [SETUP1.22, SETUP2.22]. For example, if each lining occupies one third of the duct the following lines are added:

```

ZETAY = ZET(1)+AI * ZET(2)
IF (K.GT.IEND/3)ZETAY=ZET(3)+AI*ZET(4)
IF (K.GT.2*IEND/3) ZETAY=ZET(5)+AI*ZET(6)

```

Radial Modes

If the function to be evaluated is not equal to a constant (one) at the entrance, the pressure at those points must be inputted. Several changes are required in the main program and the SETUP subroutines in order to modify the program for this case.

The following lines should be inserted after line 28 in DUCT,

```

READ 993 (Y(I), I=1, JEND)
993 FORMAT (12F6.0)
PRINT 994 (Y(I), I=1, JEND)
994 FORMAT (1X, 16F8.3)

```

In addition, line 47 in SETUP2 and line 51 in SETUP1 should be replaced with

$$10 \ Y(I) = - AA(I) * Y(I).$$

In SETUP, line 60 should be replaced by

$$10 \ Y(I) = - A * Y(I).$$

Computations for ZETAX for a Nonuniform Duct

Recall that

$$P = \frac{e^{-2\pi\eta Ri}}{R}$$

and

$$P_R = \frac{-2\pi\eta iP}{Z_x}$$

where η is the frequency and $R = .5e^{-ALPHA \cdot XMAX}$

Therefore

$$\begin{aligned} Z_x &= \frac{-2\pi\eta iP}{P_R} \\ &= \frac{-2\pi\eta i e^{-2\pi\eta Ri}}{R} \cdot \frac{e^{-2\pi\eta Ri}}{R} \left(\frac{1}{R} + 2\pi\eta i \right) \\ &= \frac{R^2 (2\pi\eta)^2 + 2\pi\eta i R}{1 + (2\pi\eta R)^2} \end{aligned}$$

OUTPUT

The output from DUCT consists of the following:

1) The solutions to the wave equation at the indicated points.

Note that in reading the printout from left to right, the y-coordinate varies while the x-coordinate is held constant. The last

column of numbers corresponds to the x-coordinate of the points.

The headings for the program have been set up for printing a maximum of fifteen points in the y-direction. If desired, the pressure can be printed at addition points, but the headings will be distorted.

2) The computed values for the acoustic power at the entrance and exit of the duct and the resultant attenuation.

3) If ZXMIN is called and a multisectional lining is used, the values of ZET will be printed before the headings for each iteration. When all the iterations have been performed, the minimizing value of ZETAY for the first lining, the attenuation, the convergence criterion, the number of iterations, and IER, an error condition will be printed.

IER = 0 means no errors occurred and convergence has been achieved.

IER = 130 means the iteration was terminated due to excessive rounding errors.

IER = 131 means the iteration was terminated because NUMMAX has been reached.

Section VI

SAMPLE RUNS

In this section, the input and output for a variety of sample cases are presented.

Case I

Cylindrical duct: no flow, uniform lining, pressure printed at every other point in both directions.

<u>Case I</u>	<u>Input</u>
1	
0.	
1.	0.
.7	-.6
2.	1.
.5	
50 20	
1	
2 2 20	

Case II

Conical duct: no flow, uniform lining, pressure printed for first 15 points in y-direction.

<u>Case II</u>	<u>Input</u>
-1	
0	
.9641	.1861
.3	-.5
1.	1.
1.	
49 20	
1	
2 1 11	
.75	

Case III

Rectangular duct: no flow, radial modes, pressure printed for first 15 points in y-direction.

<u>Case III</u>	<u>Input</u>	
0		
0.		
1.0453	.332	
.9287	-.7442	
1.	1.	
1.		
50 20		
1		
2 2 20		
-.775 -1.18 -.440 -1.11 ...		Pressures for the
.950 -.075 .990 -.020 ...		20 nodes at the
.170 -.820 -.110 .990 ...		entrance

Case IV

Cylindrical duct: no flow, uniform lining, ZXMIN called to find minimizing value of ZETAY.

<u>Case IV</u>	<u>Input</u>
1	
0.	
1.	0.
.6	-1.5
2.	1.
1.25	
60 20	
0 15 2	
0	

Case V

Rectangular Duct: uniform flow, hard wall, pressure printed at every other point in y-direction.

<u>Case V</u>	<u>Input</u>
0	
.2	
1.	0.
99999999999999999999	(20 times)
1.	1.
1.	
25	20
1	
1	2 20

ENTER XMAX,YMAX F10.0,F10.0
 ENTER ETA F10.0
 ENTER IEND(X),JEND(Y) 2I3
 50 20
 DO YOU WANT COMPLETE OUTPUT? 0 OR 1
 ENTER IOUT,JOUT,IT/3I3

CASE 1
 1 OF 2

CYLINDRICAL DUCT

DX= .0392 DY= .0476 ZETAY= .7000+I -.6000 ZETAX= 1.0000+I 0.0000 ETA= .5000 M= 0.000

REAL PART

Y= .05	Y= .14	Y= .24	Y= .33	Y= .43	Y= .52	Y= .62	Y= .71	Y= .81	Y= .90	X=
-.054	-.050	-.041	-.029	-.014	.002	.017	.030	.037	.038	1.961
-.067	-.063	-.054	-.040	-.023	-.004	.015	.033	.046	.053	1.882
-.084	-.079	-.069	-.053	-.034	-.012	.012	.035	.055	.070	1.804
-.103	-.098	-.087	-.070	-.048	-.022	.006	.035	.063	.087	1.725
-.124	-.119	-.107	-.089	-.065	-.037	-.005	.030	.067	.102	1.647
-.146	-.141	-.129	-.110	-.086	-.056	-.020	.020	.064	.110	1.569
-.167	-.162	-.151	-.133	-.109	-.079	-.041	.003	.053	.109	1.490
-.186	-.182	-.173	-.157	-.135	-.106	-.069	-.023	.031	.094	1.412
-.201	-.198	-.191	-.179	-.161	-.135	-.102	-.058	-.004	.063	1.333
-.209	-.208	-.204	-.196	-.185	-.166	-.140	-.102	-.051	.015	1.255
-.207	-.207	-.208	-.207	-.204	-.196	-.179	-.153	-.111	-.052	1.176
-.191	-.194	-.200	-.207	-.215	-.219	-.218	-.206	-.180	-.135	1.098
-.159	-.164	-.176	-.194	-.214	-.234	-.250	-.259	-.255	-.231	1.020
-.108	-.116	-.135	-.163	-.197	-.234	-.272	-.304	-.327	-.333	.941
-.036	-.047	-.073	-.111	-.160	-.216	-.276	-.336	-.390	-.432	.863
.056	.043	.010	-.038	-.101	-.175	-.257	-.345	-.433	-.516	.784
.169	.153	.115	.058	-.017	-.107	-.211	-.325	-.447	-.571	.706
.298	.280	.239	.175	.091	-.012	-.133	-.270	-.421	-.585	.627
.439	.421	.377	.309	.220	.109	-.023	-.176	-.349	-.545	.549
.585	.567	.523	.455	.365	.252	.117	-.042	-.226	-.441	.471
.727	.709	.668	.604	.518	.411	.281	.127	-.054	-.269	.392
.854	.839	.802	.745	.669	.574	.458	.321	.159	-.035	.314
.955	.943	.913	.868	.806	.729	.635	.525	.397	.248	.235
1.020	1.011	.990	.958	.915	.861	.795	.720	.635	.548	.157
1.038	1.033	1.022	1.006	.983	.956	.922	.884	.845	.820	.078

CASE 1
2 OF 2

IMAGINARY PART

Y= .05	Y= .14	Y= .24	Y= .33	Y= .43	Y= .52	Y= .62	Y= .71	Y= .81	Y= .90	X=
.048	.047	.044	.038	.030	.019	.005	-.012	-.031	-.052	1.961
.035	.034	.033	.031	.027	.021	.011	-.002	-.019	-.040	1.882
.020	.020	.022	.024	.025	.024	.020	.011	-.002	-.022	1.804
.002	.004	.009	.016	.023	.028	.031	.029	.020	.004	1.725
-.020	-.016	-.007	.005	.019	.032	.043	.049	.048	.037	1.647
-.047	-.041	-.028	-.010	.012	.034	.056	.072	.080	.077	1.569
-.080	-.073	-.056	-.031	-.000	.033	.066	.094	.115	.123	1.490
-.122	-.112	-.091	-.059	-.019	.025	.071	.114	.150	.173	1.412
-.172	-.161	-.134	-.095	-.046	.010	.069	.128	.181	.223	1.333
-.231	-.218	-.187	-.141	-.083	-.016	.058	.133	.205	.266	1.255
-.299	-.284	-.249	-.197	-.131	-.053	.034	.125	.217	.303	1.176
-.374	-.357	-.320	-.263	-.190	-.103	-.004	.102	.211	.321	1.098
-.453	-.436	-.396	-.336	-.259	-.165	-.058	.059	.185	.315	1.020
-.533	-.516	-.476	-.415	-.335	-.239	-.128	-.003	.133	.280	.941
-.610	-.594	-.554	-.495	-.416	-.321	-.210	-.085	.055	.211	.863
-.679	-.663	-.626	-.570	-.497	-.407	-.303	-.183	-.048	.106	.784
-.733	-.718	-.685	-.635	-.570	-.490	-.398	-.292	-.172	-.032	.706
-.766	-.754	-.726	-.683	-.629	-.563	-.488	-.404	-.308	-.196	.627
-.774	-.764	-.741	-.708	-.666	-.617	-.564	-.507	-.445	-.374	.549
-.751	-.744	-.727	-.703	-.674	-.643	-.613	-.587	-.566	-.547	.471
-.696	-.691	-.679	-.664	-.647	-.633	-.625	-.630	-.652	-.692	.392
-.607	-.604	-.597	-.589	-.582	-.581	-.592	-.622	-.682	-.762	.314
-.487	-.485	-.482	-.479	-.479	-.486	-.508	-.553	-.639	-.784	.235
-.341	-.340	-.339	-.338	-.341	-.351	-.373	-.419	-.507	-.671	.157
-.176	-.175	-.175	-.175	-.178	-.184	-.198	-.227	-.286	-.412	.078

ACOUSTIC POWER AT ENTRANCE = 1.31163
ACOUSTIC POWER AT EXIT = .00301
ATTENUATION = -26.38735

ENTER ETA F10.0
 ENTER IEND(X),JEND(Y) 2I3
 49 20
 DO YOU WANT COMPLETE OUTPUT? 0 OR 1
 ENTER IOUT,JOUT,IT/3I3
 ENTER ALPHA,F10.0

CASE 2
 1 OF 2

NON-UNIFORM DUCT
 DX= .0200 DY= .0476 ZETAY= .3000+I --.5000 ZETAX= .9641+I .1861 ETA= 1.0000 M= 0.0000

REAL PART

Y= .05	Y= .10	Y= .14	Y= .19	Y= .24	Y= .29	Y= .33	Y= .38	Y= .43	Y= .48	Y= .52
-.264	-.262	-.258	-.252	-.244	-.234	-.222	-.206	-.193	-.175	-.156
-.235	-.234	-.230	-.225	-.219	-.211	-.201	-.190	-.178	-.163	-.147
-.199	-.198	-.196	-.192	-.187	-.182	-.175	-.167	-.158	-.147	-.135
-.156	-.156	-.154	-.152	-.150	-.147	-.143	-.139	-.134	-.128	-.120
-.108	-.108	-.108	-.107	-.107	-.107	-.107	-.106	-.106	-.105	-.103
-.055	-.055	-.056	-.058	-.060	-.063	-.066	-.070	-.074	-.076	-.082
.003	.002	-.001	-.004	-.009	-.015	-.021	-.029	-.038	-.046	-.059
.063	.061	.058	.053	.046	.037	.027	.015	.001	-.014	-.031
.126	.124	.119	.112	.103	.092	.078	.062	.044	.023	-.000
.190	.188	.182	.174	.163	.149	.132	.112	.090	.064	.035
.256	.253	.247	.237	.224	.208	.189	.166	.139	.109	.075
.323	.319	.312	.301	.287	.269	.247	.222	.192	.158	.119
.389	.386	.378	.366	.351	.332	.308	.280	.248	.211	.169
.455	.452	.444	.432	.415	.395	.370	.341	.307	.269	.224
.521	.517	.509	.496	.480	.459	.434	.404	.369	.330	.284
.585	.581	.573	.560	.544	.523	.498	.468	.434	.394	.348
.647	.643	.635	.623	.607	.587	.562	.533	.500	.461	.417
.706	.702	.695	.683	.668	.649	.626	.598	.566	.530	.488
.762	.759	.752	.741	.727	.709	.688	.663	.633	.600	.561
.814	.811	.805	.795	.783	.767	.748	.725	.699	.669	.635
.861	.859	.853	.845	.835	.821	.805	.786	.763	.736	.709
.904	.902	.897	.891	.882	.871	.858	.842	.825	.804	.781
.940	.938	.935	.930	.924	.916	.906	.895	.882	.867	.850
.970	.969	.967	.964	.960	.955	.949	.942	.934	.925	.914
.992	.992	.991	.990	.989	.987	.985	.982	.980	.977	.973

IMAGINARY PART
CASE 2
2 OF 2

Y = .05	Y = .10	Y = .14	Y = .19	Y = .24	Y = .29	Y = .33	Y = .38	Y = .43	Y = .46	Y = .52
-.177	-.175	-.171	-.164	-.155	-.144	-.131	-.115	-.101	-.083	-.065
-.235	-.232	-.227	-.218	-.208	-.194	-.179	-.161	-.142	-.121	-.098
-.289	-.286	-.280	-.270	-.258	-.243	-.225	-.205	-.182	-.157	-.131
-.339	-.336	-.329	-.318	-.305	-.288	-.268	-.246	-.220	-.193	-.163
-.384	-.380	-.373	-.362	-.347	-.329	-.308	-.284	-.256	-.226	-.193
-.423	-.420	-.412	-.400	-.385	-.366	-.344	-.319	-.290	-.258	-.223
-.457	-.453	-.445	-.433	-.418	-.399	-.376	-.350	-.321	-.286	-.252
-.484	-.480	-.472	-.461	-.445	-.426	-.403	-.378	-.348	-.316	-.280
-.505	-.501	-.494	-.482	-.467	-.448	-.426	-.401	-.372	-.341	-.306
-.519	-.516	-.508	-.497	-.483	-.465	-.444	-.420	-.393	-.363	-.330
-.527	-.524	-.517	-.506	-.493	-.476	-.457	-.435	-.410	-.382	-.352
-.528	-.525	-.519	-.509	-.497	-.482	-.464	-.444	-.422	-.397	-.371
-.523	-.520	-.514	-.506	-.495	-.481	-.466	-.448	-.429	-.406	-.386
-.511	-.509	-.504	-.496	-.487	-.475	-.462	-.447	-.431	-.414	-.397
-.493	-.491	-.487	-.480	-.472	-.463	-.452	-.440	-.427	-.415	-.402
-.469	-.468	-.464	-.459	-.452	-.444	-.436	-.426	-.417	-.409	-.401
-.440	-.438	-.435	-.431	-.426	-.420	-.413	-.407	-.401	-.396	-.393
-.404	-.403	-.401	-.398	-.394	-.389	-.385	-.380	-.377	-.376	-.376
-.364	-.363	-.361	-.359	-.356	-.353	-.350	-.348	-.347	-.348	-.352
-.318	-.318	-.316	-.315	-.313	-.311	-.309	-.308	-.309	-.312	-.318
-.268	-.268	-.267	-.266	-.265	-.263	-.263	-.263	-.265	-.269	-.276
-.214	-.214	-.214	-.213	-.212	-.211	-.211	-.212	-.214	-.219	-.225
-.157	-.157	-.156	-.156	-.155	-.155	-.155	-.156	-.158	-.162	-.168
-.096	-.096	-.096	-.095	-.095	-.095	-.096	-.096	-.098	-.100	-.104
-.033	-.033	-.032	-.032	-.032	-.032	-.032	-.033	-.033	-.034	-.035

ACOUSTIC POWER AT ENTRANCE = .65421
 ACOUSTIC POWER AT EXIT = .02541
 ATTENUATION = -14.10767

CASE 3
1 OF 2

ENTER IOUT, JOUT, IT/313

-0.775	-1.180	-0.440	-1.110	-0.990	0.170	-0.820	0.460	-0.640	0.620
0.760	-0.305	0.875	-0.170	-0.075	0.990	-0.020	0.990	-0.020	0.950
0.875	-0.170	0.760	0.305	-0.470	0.460	-0.640	0.170	-0.820	-0.110
-0.440	-1.110	-0.775	-1.180						

RECTANGULAR DUCT

DX= 0.0196 DY= 0.0476 ZETAY= 0.9287+I -0.7442 ZETAX= 1.0453+I 0.3320 ETA= 1.0000 M= 0.00

REAL PART

Y=	05	14	24	33	43	52	62	71	81	90	X=
0.097	0.122	0.111	0.085	0.063	0.055	0.065	0.083	0.115	0.127	0.980	
0.149	0.151	0.119	0.077	0.044	0.033	0.048	0.082	0.124	0.156	0.941	
0.201	0.176	0.121	0.062	0.019	0.005	0.024	0.069	0.127	0.180	0.902	
0.250	0.196	0.117	0.041	-0.012	-0.029	-0.006	0.049	0.123	0.198	0.863	
0.294	0.208	0.105	0.013	-0.048	-0.067	-0.042	0.022	0.112	0.206	0.824	
0.329	0.211	0.085	-0.021	-0.089	-0.110	-0.081	-0.010	0.093	0.208	0.784	
0.351	0.202	0.056	-0.060	-0.133	-0.154	-0.123	-0.046	0.066	0.198	0.745	
0.358	0.180	0.019	-0.104	-0.177	-0.197	-0.165	-0.086	0.032	0.176	0.706	
0.345	0.144	-0.026	-0.149	-0.219	-0.237	-0.206	-0.128	-0.008	0.142	0.667	
0.311	0.094	-0.077	-0.194	-0.257	-0.272	-0.241	-0.168	-0.054	0.098	0.627	
0.255	0.031	-0.133	-0.237	-0.288	-0.297	-0.270	-0.206	-0.102	0.043	0.588	
0.175	-0.045	-0.191	-0.274	-0.308	-0.311	-0.288	-0.238	-0.151	-0.020	0.549	
0.073	-0.131	-0.249	-0.302	-0.315	-0.310	-0.293	-0.261	-0.199	-0.091	0.510	
-0.049	-0.224	-0.302	-0.319	-0.307	-0.291	-0.283	-0.274	-0.243	-0.165	0.471	
-0.188	-0.319	-0.348	-0.321	-0.279	-0.254	-0.256	-0.273	-0.279	-0.240	0.431	
-0.339	-0.412	-0.382	-0.305	-0.232	-0.197	-0.210	-0.258	-0.307	-0.313	0.392	
-0.494	-0.498	-0.401	-0.271	-0.165	-0.119	-0.146	-0.227	-0.324	-0.361	0.353	
-0.647	-0.570	-0.401	-0.216	-0.077	-0.022	-0.063	-0.180	-0.327	-0.441	0.314	
-0.790	-0.624	-0.379	-0.139	0.031	0.093	0.037	-0.116	-0.317	-0.492	0.275	
-0.913	-0.652	-0.333	-0.043	0.155	0.222	0.151	-0.038	-0.291	-0.529	0.235	
-1.006	-0.652	-0.262	0.073	0.292	0.362	0.276	0.055	-0.251	-0.554	0.196	
-1.063	-0.617	-0.166	0.203	0.437	0.507	0.408	0.159	-0.195	-0.564	0.157	
-1.073	-0.546	-0.045	0.344	0.583	0.650	0.540	0.271	-0.124	-0.560	0.118	
-1.032	-0.437	0.099	0.490	0.723	0.783	0.667	0.398	-0.036	-0.541	0.078	
-0.935	-0.291	0.265	0.632	0.848	0.899	0.781	0.507	0.062	-0.503	0.039	

CASE 3
2 OF 2

IMAGINARY PART

Y=	.05	.14	.24	.33	.43	.52	.62	.71	.81	Y=	.90	X=
-	.203	-.101	-.013	.053	.092	.103	.088	.048	-.016	-	.099	.980
-	.190	-.078	.012	.075	.111	.121	.108	.071	.009	-	.075	.941
-	.164	-.046	.041	.098	.127	.136	.124	.093	.038	-	.043	.902
-	.125	-.006	.073	.119	.140	.145	.137	.114	.070	-	.004	.863
-	.072	.041	.106	.137	.147	.148	.144	.132	.101	-	.041	.824
-	.006	.093	.139	.150	.146	.142	.143	.144	.132	-	.090	.784
.071	.150	.169	.169	.156	.136	.127	.134	.151	.160	-	.141	.745
.157	.207	.194	.154	.154	.117	.101	.115	.149	.183	-	.192	.706
.249	.261	.213	.143	.086	.086	.064	.085	.138	.200	-	.240	.667
.342	.311	.222	.120	.044	.016	.016	.044	.117	.208	-	.283	.627
.432	.351	.219	.086	-.010	-.043	-.043	-.007	.085	.206	-	.317	.588
.513	.379	.203	.039	-.073	-.110	-.110	-.067	.044	.193	-	.340	.549
.580	.390	.173	-.018	-.145	-.185	-.185	-.135	-.007	.169	-	.351	.510
.627	.382	.128	-.085	-.223	-.264	-.264	-.208	-.065	.134	-	.347	.471
.648	.353	.068	-.161	-.303	-.345	-.345	-.282	-.129	.088	-	.328	.431
.638	.300	-.006	-.241	-.383	-.421	-.421	-.356	-.196	.033	-	.294	.392
.594	.224	-.091	-.322	-.457	-.490	-.490	-.424	-.264	-.030	-	.244	.353
.512	.124	-.185	-.400	-.520	-.547	-.547	-.482	-.328	-.097	-	.180	.314
.393	.003	-.284	-.470	-.567	-.585	-.585	-.526	-.385	-.167	-	.104	.275
.236	-.135	-.383	-.527	-.593	-.601	-.601	-.552	-.433	-.235	-	.018	.235
.045	-.286	-.476	-.565	-.594	-.590	-.590	-.557	-.468	-.298	-	.076	.196
-.176	-.444	-.558	-.578	-.563	-.548	-.548	-.537	-.489	-.352	-	.177	.157
-.419	-.601	-.622	-.563	-.497	-.472	-.472	-.490	-.497	-.392	-	.285	.116
-.674	-.750	-.661	-.514	-.394	-.360	-.360	-.415	-.495	-.421	-	.409	.078
-.932	-.882	-.669	-.428	-.253	-.209	-.209	-.309	-.487	-.481	-	.602	.039

ACOUSTIC POWER AT ENTRANCE = .07034
ACOUSTIC POWER AT EXIT = .00193
ATTENUATION = -15.60759

CASE 4
1 OF 4

ENTER DUCT GEOM\ NONUNIFORM, RECT, CYLIND; -1, 0, 1
 ENTER MACH NO. F10.0
 ENTER ZETAX F10.0, F10.0
 ENTER ZETAY F10.0, F10.0
 ENTER XMAX, YMAX F10.0, F10.0
 ENTER ETA F10.0
 ENTER IEND(X), JEND(Y) 2I3
 60 20
 DO YOU WANT COMPLETE OUTPUT? 0 OR 1
 ENTER IOUT, JOUT, IT/3I3

CYLINDRICAL DUCT
 DX= .0328 DY= .0476 ZETAY= .6000+I-1.5000 ZETAX= 1.0000+I 0.0000 ETA= 1.2500 M= 0.000

ACOUSTIC POWER AT ENTRANCE = 2.66222
 ACOUSTIC POWER AT EXIT = .64423
 ATTENUATION = -5.16201

CASE 4
2 OF 4

CYLINDRICAL DUCT
DX= .0328 DY= .0475 ZETAY= .6000+I-1.5000 ZETAX= 1.0000+I 0.0000 ETA= 1.2500 M= 0.000

ACOUSTIC POWER AT ENTRANCE = 2.66222
ACOUSTIC POWER AT EXIT = .64423
ATTENUATION = -6.16201

CYLINDRICAL DUCT
DX= .0328 DY= .0475 ZETAY= .6000+I-1.5000 ZETAX= 1.0000+I 0.0000 ETA= 1.2500 M= 0.000

ACOUSTIC POWER AT ENTRANCE = 2.66222
ACOUSTIC POWER AT EXIT = .64423
ATTENUATION = -6.16201

CYLINDRICAL DUCT
DX= .0328 DY= .0475 ZETAY= 1.0129+I-1.4825 ZETAX= 1.0000+I 0.0000 ETA= 1.2500 M= 0.000

ACOUSTIC POWER AT ENTRANCE = 2.60974
ACOUSTIC POWER AT EXIT = .67955
ATTENUATION = -5.84377

CASE 4
3 OF 4

CYLINDRICAL DUCT
DX= .0328 DY= .0476 ZETAY= .8065+I-1.4912 ZETAX= 1.0000+I 0.0000 ETA= 1.2500 M= 0.000

ACOUSTIC POWER AT ENTRANCE = 2.63518
ACOUSTIC POWER AT EXIT = .64948
ATTENUATION = -5.08246

CYLINDRICAL DUCT
DX= .0328 DY= .0476 ZETAY= .6206+I-1.4991 ZETAX= 1.0000+I 0.0000 ETA= 1.2500 M= 0.000

ACOUSTIC POWER AT ENTRANCE = 2.65959
ACOUSTIC POWER AT EXIT = .64267
ATTENUATION = -6.16827

CYLINDRICAL DUCT
DX= .0328 DY= .0476 ZETAY= .6206+I-1.4991 ZETAX= 1.0000+I 0.0000 ETA= 1.2500 M= 0.000

ACOUSTIC POWER AT ENTRANCE = 2.65959
ACOUSTIC POWER AT EXIT = .64267
ATTENUATION = -6.16827

CASE 4
4 OF 4

CYLINDRICAL DUCT
DX= .0328 DY= .0475 ZETAY= .6205+I-1.4991 ZETAX= 1.0000+I 0.0000 ETA= 1.2500 M= 0.000

ACOUSTIC POWER AT ENTRANCE = 2.65959
ACOUSTIC POWER AT EXIT = .64267
ATTENUATION = -6.16827

CYLINDRICAL DUCT
DX= .0328 DY= .0475 ZETAY= .6568+I-1.5548 ZETAX= 1.0000+I 0.0000 ETA= 1.2500 M= 0.000

ACOUSTIC POWER AT ENTRANCE = 2.65670
ACOUSTIC POWER AT EXIT = .64260
ATTENUATION = -6.16400

CYLINDRICAL DUCT
DX= .0328 DY= .0475 ZETAY= .6387+I-1.5270 ZETAX= 1.0000+I 0.0000 ETA= 1.2500 M= 0.000

ACOUSTIC POWER AT ENTRANCE = 2.65797
ACOUSTIC POWER AT EXIT = .64199
ATTENUATION = -6.17025

CASE 5
1 OF 2

ENTER IEND(X),JEND(Y) 2I3
25 20
DO YOU WANT COMPLETE OUTPUT? 0 OR 1
ENTER IOUT,JOUT,IT/3I3

RECTANGULAR DUCT
DX= .0385 DY= .0476 ZETAY=*****I***** ZETAX= 1.2000+I 0.0000 ETA= 1.0000 M= .200

REAL PART

Y= .05	Y= .14	Y= .24	Y= .33	Y= .43	Y= .52	Y= .62	Y= .71	Y= .81	Y= .90	X=
.330	.330	.330	.330	.330	.330	.330	.330	.330	.330	.962
.133	.133	.133	.133	.133	.133	.133	.133	.133	.133	.923
-.069	-.069	-.069	-.069	-.069	-.069	-.069	-.069	-.069	-.069	.885
-.269	-.269	-.269	-.269	-.269	-.269	-.269	-.269	-.269	-.269	.846
-.457	-.457	-.457	-.457	-.457	-.457	-.457	-.457	-.457	-.457	.808
-.626	-.626	-.626	-.626	-.626	-.626	-.626	-.626	-.626	-.626	.769
-.771	-.771	-.771	-.771	-.771	-.771	-.771	-.771	-.771	-.771	.731
-.883	-.883	-.883	-.883	-.883	-.883	-.883	-.883	-.883	-.883	.692
-.960	-.960	-.960	-.960	-.960	-.960	-.960	-.960	-.960	-.960	.654
-.998	-.998	-.998	-.998	-.998	-.998	-.998	-.998	-.998	-.998	.615
-.995	-.995	-.995	-.995	-.995	-.995	-.995	-.995	-.995	-.995	.577
-.952	-.952	-.952	-.952	-.952	-.952	-.952	-.952	-.952	-.952	.538
-.870	-.870	-.870	-.870	-.870	-.870	-.870	-.870	-.870	-.870	.500
-.753	-.753	-.753	-.753	-.753	-.753	-.753	-.753	-.753	-.753	.462
-.605	-.605	-.605	-.605	-.605	-.605	-.605	-.605	-.605	-.605	.423
-.433	-.433	-.433	-.433	-.433	-.433	-.433	-.433	-.433	-.433	.385
-.243	-.243	-.243	-.243	-.243	-.243	-.243	-.243	-.243	-.243	.346
-.044	-.044	-.044	-.044	-.044	-.044	-.044	-.044	-.044	-.044	.308
.158	.158	.158	.158	.158	.158	.158	.158	.158	.158	.269
.352	.352	.352	.352	.352	.352	.352	.352	.352	.352	.231
.533	.533	.533	.533	.533	.533	.533	.533	.533	.533	.192
.692	.692	.692	.692	.692	.692	.692	.692	.692	.692	.154
.822	.822	.822	.822	.822	.822	.822	.822	.822	.822	.115
.920	.920	.920	.920	.920	.920	.920	.920	.920	.920	.077
.980	.980	.980	.980	.980	.980	.980	.980	.980	.980	.038

CASE 5
2 OF 2

IMAGINARY PART

Y= .05	Y= .14	Y= .24	Y= .33	Y= .43	Y= .52	Y= .62	Y= .71	Y= .81	Y= .90	X=
.944	.944	.944	.944	.944	.944	.944	.944	.944	.944	.962
.991	.991	.991	.991	.991	.991	.991	.991	.991	.991	.923
.998	.998	.998	.998	.998	.998	.998	.998	.998	.998	.885
.965	.965	.965	.965	.965	.965	.965	.965	.965	.965	.846
.892	.892	.892	.892	.892	.892	.892	.892	.892	.892	.808
.783	.783	.783	.783	.783	.783	.783	.783	.783	.783	.769
.642	.642	.642	.642	.642	.642	.642	.642	.642	.642	.731
.476	.476	.476	.476	.476	.476	.476	.476	.476	.476	.692
.289	.289	.289	.289	.289	.289	.289	.289	.289	.289	.654
.091	.091	.091	.091	.091	.091	.091	.091	.091	.091	.615
-.111	-.111	-.111	-.111	-.111	-.111	-.111	-.111	-.111	-.111	.577
-.308	-.308	-.308	-.308	-.308	-.308	-.308	-.308	-.308	-.308	.538
-.493	-.493	-.493	-.493	-.493	-.493	-.493	-.493	-.493	-.493	.500
-.659	-.659	-.659	-.659	-.659	-.659	-.659	-.659	-.659	-.659	.462
-.797	-.797	-.797	-.797	-.797	-.797	-.797	-.797	-.797	-.797	.423
-.903	-.903	-.903	-.903	-.903	-.903	-.903	-.903	-.903	-.903	.385
-.972	-.972	-.972	-.972	-.972	-.972	-.972	-.972	-.972	-.972	.346
-1.002	-1.002	-1.002	-1.002	-1.002	-1.002	-1.002	-1.002	-1.002	-1.002	.308
-.991	-.991	-.991	-.991	-.991	-.991	-.991	-.991	-.991	-.991	.269
-.939	-.939	-.939	-.939	-.939	-.939	-.939	-.939	-.939	-.939	.231
-.850	-.850	-.850	-.850	-.850	-.850	-.850	-.850	-.850	-.850	.192
-.725	-.725	-.725	-.725	-.725	-.725	-.725	-.725	-.725	-.725	.154
-.572	-.572	-.572	-.572	-.572	-.572	-.572	-.572	-.572	-.572	.115
-.395	-.395	-.395	-.395	-.395	-.395	-.395	-.395	-.395	-.395	.077
-.201	-.201	-.201	-.201	-.201	-.201	-.201	-.201	-.201	-.201	.038

ACOUSTIC POWER AT ENTRANCE = 1.12897
ACOUSTIC POWER AT EXIT = 1.14285
ATTENUATION = .05309

GOVERNING EQ.

$$\frac{\partial^2 P}{\partial \xi^2} + \frac{\partial^2 P}{\partial \eta^2} + (2\pi\lambda)^2 P = 0$$

TRANSFORMS VIA

$$\xi = \frac{e^x - e^{-x}}{2} \cos y$$

$$\eta = \frac{e^x + e^{-x}}{2} \sin y$$

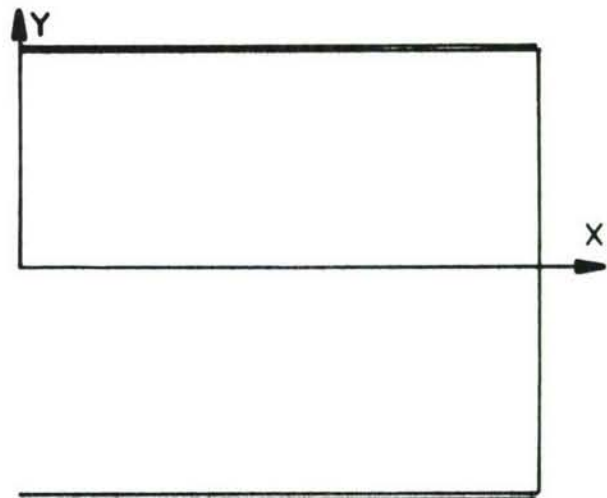
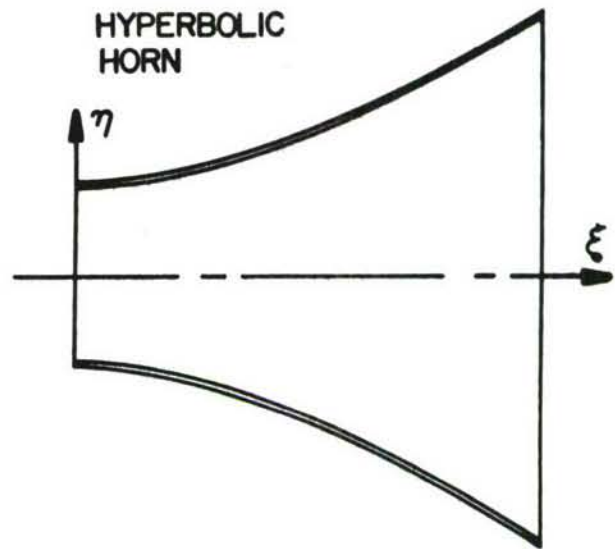
TO

AND THE GOVERNING EQ.
BECOMES

$$\frac{\partial^2 P}{\partial X^2} + \frac{\partial^2 P}{\partial Y^2} + (2\pi\lambda)^2 \cdot F(X,Y) \cdot P = 0$$

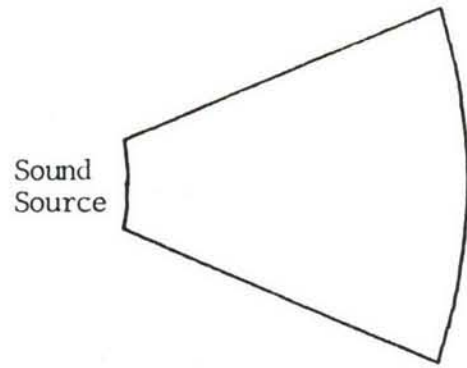
WHERE FOR THE
HYPERBOLIC HORN

$$F(X,Y) = \frac{e^{2X} - e^{-2X} + 2(\cos^2 y - \sin^2 y)}{4}$$

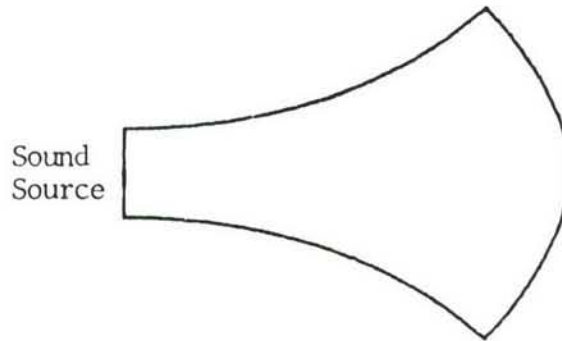


Example of conformal map.

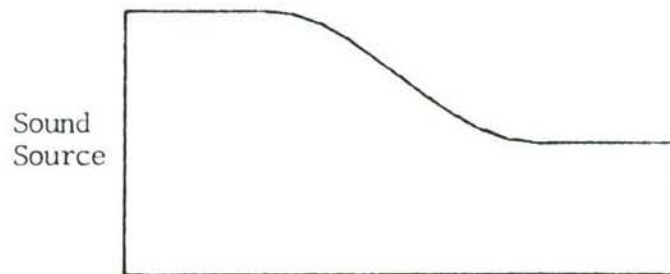
FIGURE 1



Conical Duct



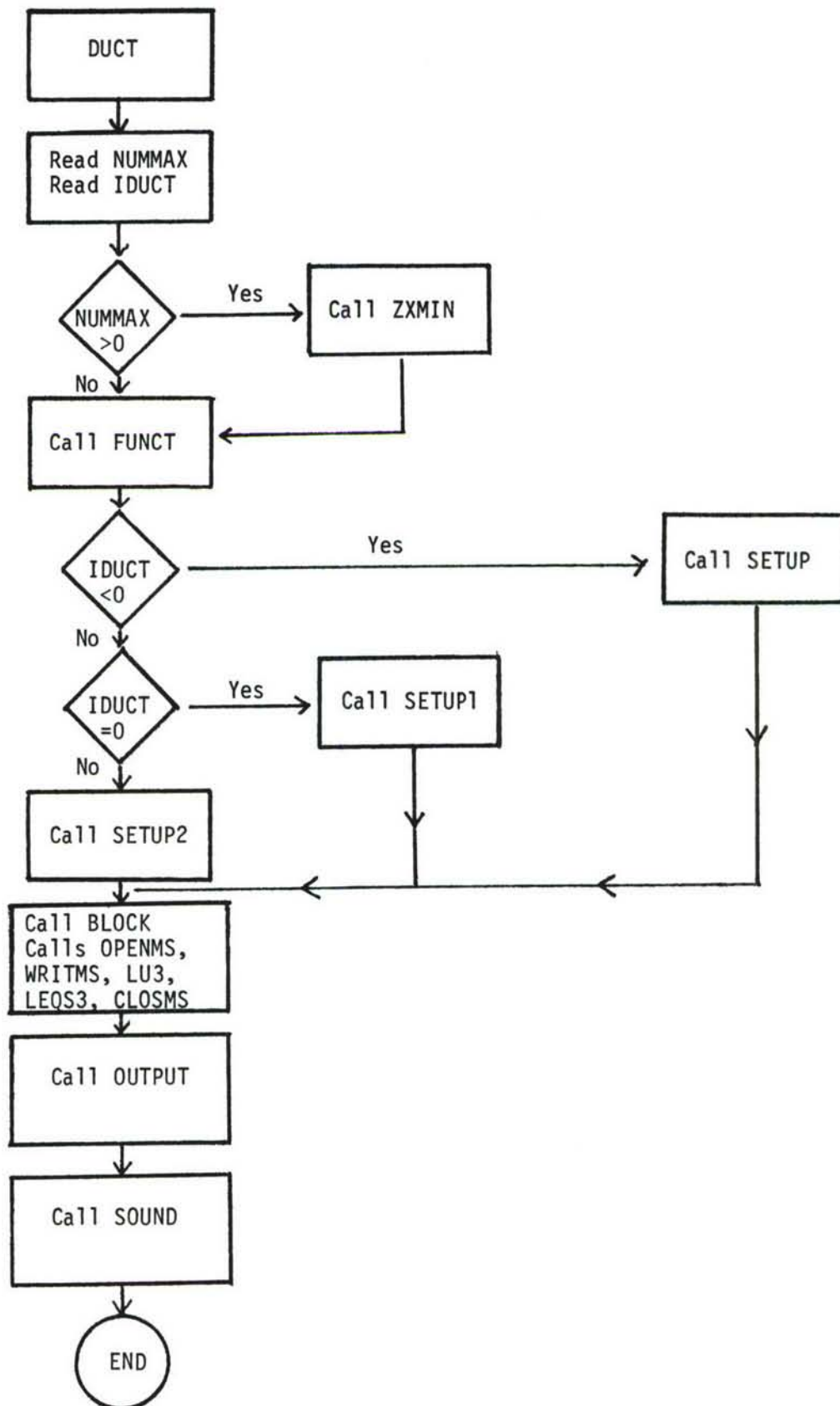
Catenoidal Duct



Tapered Duct

Examples of Variable Area Ducts

FIGURE 2



Logical Flow of Duct

FIGURE 3

APPENDIX

```

PROGRAM DUCT( INPUT,OUTPUT, TAPE 1, TAPE 2, TAPE 3, TAPE 4 )
COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
2 AA(20),OLT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY ,C1, A,A1,B1,AA
EXTERNAL FUNCT
DIMENSION ZET(2),G(2),HH(3),WA(6)
M=20
PRINT 981
READ 983, IDUCT
981 FORMAT(* ENTER DUCT GEOM\ NONUNIFORM,RECT,CYLIND;-1,0,1 *)
PRINT 991
READ 990, XM
PRINT 988
READ 990, ZETAX
PRINT 987
READ 990, ZETAY
PRINT 986
READ 990, XMAX, YMAX
PRINT 985
READ 990, ETA
PRINT 984
READ 992,IEND,JEND
PRINT 992,IEND,JEND
PRINT 982
READ 992,KOUT,NUMMAX,IH
PRINT 989
READ 992, IOUT,JOUT,IT
989 FORMAT(1X,*ENTER IOUT,JOUT,IT/3I3*)
992 FORMAT(3I3)
982 FORMAT(* DO YOU WANT COMPLETE OUTPUT? 0 OR 1 *)
983 FORMAT(3I3,F10.0)
984 FORMAT(* ENTER IEND(X),JEND(Y) 2I3 *)
985 FORMAT(* ENTER ETA F10.0 *)
986 FORMAT(* ENTER XMAX,YMAX F10.0,F10.0 *)
987 FORMAT(* ENTER ZETAY F10.0,F10.0 *)
988 FORMAT(* ENTER ZETAX F10.0,F10.0 *)
990 FORMAT(2F10.0)
991 FORMAT(* ENTER MACH NO. F10.0 *)
AI=(0.,1.)
ZETAX=ZETAX*(1.+XM)
NN=JEND
DX=XMAX/(IEND+1)
DY=YMAX/(JEND+1)
DYDX=DY/DX
CON=2.*3.14159265358979*ETA
1 A=-(1.-XM*XM)*DYDX**2-CON*XM*AI*DYDX*DY
BBBB=2.+2.*DYDX**2-CON**2*DY**2
BBBB=BBBB-2.*XM*XM*DYDX**2
ZET (1)=REAL(ZETAY)
ZET (2)=AIMAG(ZETAY)

```

```

        IF (NUMMAX.GT.0) GO TO 100
        CALL FUNCT(2,ZET,FMIN)
        GO TO 500
100    NVAR=2
        IOPT=0
        CALL ZXMIN(FUNCT,NVAR,IH,NUMMAX,IOPT,ZET,HH,G,FMIN,WA,IER)

        PRINT 999,(ZET(I),I=1,2),FMIN,IH,NUMMAX,IER
999    FORMAT(1X,3F16.8,3I5)
500    STOP
        END

```

SUBROUTINE SETUP

```

C
C CYLINDRICAL NON-UNIFORM DUCT USING CONFORMAL MAP
C
COMMON/BIG/ AI, ZETAX, ZETAY, XMAX, YMAX, ETA, XM, IEND, JEND, DYDX, M, NN,
1 CON, A, B, B, B, B(20,20), C(20,20), T(20,20), D, Y(20), IP(20), V(20), DX, DY,
2 AA(20), OUT(20), IOUT, JOUT, KOUT, IT, IDUCT, KEND, DB
COMPLEX B, C, T, D, Y, V, AT, ZETAX, ZETAY, C1, A, A1, B1, AA
REWIND 1
PRINT 999
READ 998, ALPHA
998 FORMAT(2F10.0)
999 FORMAT(1X, * ENTER ALPHA, F10.0 *)
A = -DYDX**2 + ALPHA*DY*DYDX/2.
DO 100 J=1, JEND
100 AA(J)=A
DO 6 K=1, IEND
DO 5 I=1, JEND
C(I,I) = -DYDX**2 - ALPHA*DY*DYDX/2.
DO 5 J=1, JEND
IF(J.NE.I) C(I,J)=0.
B(I,J)=0.
XK=.5
ARG1=DX*FLOAT(K)
ARG2=ALPHA*FLOAT(I)*DY
FOFY=-COS(ARG2)/SIN(ARG2)
SHAPE=ALPHA*ALPHA*EXP(2.*ALPHA*ARG1)*XK*XK
IF(I.EQ.J) B(I,J)=B, B, B, B + (1.-SHAPE)*CON**2*DY**2
IF(J.EQ.I-1) B(I,J)=-1.-ALPHA*FOFY*DY/2.
IF(J.EQ.I+1) B(I,J)=-1.+ALPHA*FOFY*DY/2.
5 CONTINUE
C
C INCLUDE BOUNDARY DATA CORRESPONDING TO Y=0 AND Y=1
C
B1=-2.*CON*AI*DY*ALPHA*XK*EXP(ALPHA*ARG1)/ZETAY
B(JEND, JEND-1)=-2.
P(1,2)=-2.
ARG2=ALPHA*FLOAT(JEND)*DY
FOFY=-COS(ARG2)/SIN(ARG2)
B(JEND, JEND)=B(JEND, JEND)+(ALPHA*FOFY*DY/2. - 1.)*B1
IF(K.EQ.IEND) GO TO 6
WRITE(1) AA, B, C
6 CONTINUE
C
C INCLUDE BOUNDARY DATA CORRESPONDING TO X=1
C
SHAPE=ALPHA*XK*EXP(ALPHA*ARG1)
CON1=CON
CON=CON*SHAPE
DO 7 I=1, JEND
7 B(I,I)=B(I,I)-C(I,I)*2.*CON*AI*DX/ZETAX
CON=CON1
WRITE(1) AA, B, C
DO 8 J=1, JEND
8 AA(J)=A
DO 10 I=1, JEND
10 Y(I)=-A
REWIND 1
RETURN
END

```

```

SUBROUTINE BLOCK
COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY ,C1, A,A1,B1,AA,CC
DIMENSION CC(20,20)
DIMENSION INDEX(301)
CALL OPENMS(2,INDEX,301,0)
REWIND 1
REWIND 3
REWIND 4
READ(4) (Y(I),I=1,JEND)
READ(1) AA,B,CC
CALL WRITMS(2,B,800,1,-1,0)
CALL WRITMS(2,CC,800,2,-1,0)
CALL WRITMS(2,Y,40,3,-1,0)
DO 100 K=2,IEND
DO 12 I=1,JEND
12 V(I)=Y(I)
READ(4) (Y(I),I=1,JEND)
DO 15 I=1,JEND
DO 15 J=1,JEND
C(I,J)=CC(I,J)
15 T(I,J)=B(I,J)
CALL LU3(NN,T,M,IP)
CALL LEQSR(NN,T,M,IP,V)
DO 20 I=1,JEND
A=AA(I)
IF(K.EQ. IEND) A = A + C(I,I)
D=Y(I)
20 Y(I)=D-A*V(I)
READ(1) AA,B,CC
DO 50 I=1,JEND
DO 30 J=1,JEND
30 V(J)=C(J,I)
CALL LEQSR(NN,T,M,IP,V)
DO 40 J=1,JEND
A=AA(J)
IF(K.EQ. IEND) A = A + C(J,J)
40 B(J,I)=B(J,I)-A*V(J)
50 CONTINUE
KK=3*K-3
CALL WRITMS(2,B,800,KK+1,-1,0)
CALL WRITMS(2,C,800,KK+2,-1,0)
CALL WRITMS(2,Y,40,KK+3,-1,0)
100 CONTINUE

```

```

C
C  INVERT UPPER TRIANGULAR MATRIX TO OBTAIN SOLUTION
C

```

```

      DO 110 I=1,JEND
      V(I)=Y(I)
      DO 110 J=1,JEND
110  T(I,J)=B(I,J)
      CALL LU3(NN,T,M,IP)
      CALL LEQSR3(NN,T,M,IP,V)
      DO 120 I=1,JEND
120  Y(I)=V(I)
      REWIND 1
      WRITE(3) (Y(I),I=1,JEND)
      IFEND1=IFEND-1
      DO 200 K=1,IFEND1
      KK=5*IFEND-3*K-3
      CALL READMS(2,B,800,KK+1)
      CALL READMS(2,C,800,KK+2)
      CALL READMS(2,AA,40,KK+3)
      DO 140 I=1,JEND
      V(I)=1.
      DO 130 J=1,JEND
130  V(I)=C(I,J)*Y(J)+V(I)
140  V(I)=AA(I)-V(I)
      CALL LU2(NN,B,M,IP)
      CALL LEQSR3(NN,B,M,IP,V)
      DO 160 I=1,JEND
160  Y(I)=V(I)
      WRITE(3) (Y(I),I=1,JEND)
200  CONTINUE
      REWIND 1
      CALL CLOSMS(2)
      RETURN
      END

```

```

SUBROUTINE OUTPUT
COMMON/3IG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
2AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY,C1,A,A1,B1,AA
PRINT 998
DATA YEQ/3H Y=/,XEQ/3H X=/
IF(IDUCT .GT. 0) PRINT 993
IF(IDUCT .EQ. 0) PRINT 992
IF(IDUCT .LT. 0) PRINT 991
991 FORMAT(* NON-UNIFORM DUCT *)
992 FORMAT(* RECTANGULAR DUCT *)
993 FORMAT(* CYLINDRICAL DUCT *)
PRINT 996, DX, DY, ZETAY, ZETAX, ETA ,XM
IF(KOUT .EQ. 0) RETURN
PRINT 989
REWIND 3
DO 200 J=1,IT,JOUT
200 OUT(J)=J*DY
PRINT 990,((YEQ,OUT(J)),J=1,IT,JOUT),XEQ
990 FORMAT(1X,15(1X,A3,F4.2),3X,A3)
L=0
DO 210 I=1,IEND
READ(3) (Y(J),J=1,JEND)
L=L+1
IF(L .LT. IOUT .AND. I .GT. 1) GO TO 210
DO 209 J=1,JEND,JOUT
209 OUT(J)=REAL(Y(J))
X=(IEND-I+1)*DX
PRINT 999,(OUT(J),J=1,IT,JOUT),X
L=0
210 CONTINUE
REWIND 3
PRINT 998
PRINT 995
PRINT 998
DO 201 J=1,IT,JOUT
201 OUT(J)=J*DY
PRINT 990,((YEQ,OUT(J)),J=1,IT,JOUT),XEQ
L=0
DO 212 I=1,IEND
READ(3) (Y(J),J=1,JEND)
L=L+1
IF(L .LT. IOUT .AND. I .GT. 1) GO TO 212
DO 211 J=1,JEND,JOUT
211 OUT(J)=AIMAG(Y(J))
X=(IEND-I+1)*DX
PRINT 999,(OUT(J),J=1,IT,JOUT),X
L=0
212 CONTINUE

```

```

934 FORMAT(1X,12F10.5)
935 FORMAT( 20X, *IMAGINARY PART*)
936 FORMAT(1X,*DX=*,F6.4,* DY=*,F6.4,* ZETAY=*,F7.4,*+I*,F7.4,
1 * ZETAX=*,F7.4,*+I*,F7.4,* ETA=*,F7.4,* N=*,F7.4,/)
989 FORMAT(20X,*REAL PART*,/)
997 FORMAT(1H1)
998 FORMAT(1X,///)
999 FORMAT(1X,16F8.3)
      RETURN
      END

```

```

SUBROUTINE SOUND
  COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
1 CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
  COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY,C1,A,A1,B1,AA
  REWIND 3
  DA=DY
  IF ( XM .NE. 0. ) GO TO 299
  SUM1=0.
  READ(3) (Y(I),I=1,JEND)
  READ(3) (V(I),I=1,JEND)
  DO 220 J=1,JEND
  IF(IDUCT .NE. 0) DA=J*DY
220 SUM1=SUM1+REAL(AI*CONJG(Y(J))*(Y(J)-V(J)))
  1 *DA
  SUM2=0.
  KLAST=IEND-3
  DO 225 K=1,KLAST
  READ(3) (Y(I),I=1,JEND)
225 CONTINUE
  READ(3) (V(I),I=1,JEND)
  DO 230 J=1,JEND
  IF(IDUCT .NE. 0) DA=J*DY
230 SUM2=SUM2+REAL(AI*CONJG(V(J))*(Y(J)-V(J)))
  1 *DA
  PRINT 998
  PRINT 994,SUM2,SUM1
  IF(SUM2.LT.0.) RETURN
  DB=10.*ALOG10(SUM1/SUM2)
  PRINT 999,DB
  RETURN
299 IEND2=IEND-2
  READ(3) (Y(J),J=1,JEND)
  READ(3) (V(J),J=1,JEND)
  X=XMAX-DX
  DO 300 J=1,JEND
  T(1,J)=Y(J)
300 B(1,J)=CEXP(CON*AI*X/XM)*(Y(J)-V(J))/DX
  DO 400 I=1,IEND2
  FACT=FLOAT(I)-2.*FLOAT(I/2)
  WT=4.
  IF(FACT .LT. .5) WT=2.
  X=XMAX-DX-DX*FLOAT(I)
  READ(3) (AA(J),J=1,JEND)
  DO 350 J=1,JEND
  C1=(Y(J)-AA(J))/(2.*DX)
  B(1,J)=B(1,J)+WT*C1*CEXP(CON*AI*X/XM)
  Y(J)=V(J)
350 V(J)=AA(J)
400 CONTINUE

```

```

DO 450 J=1,JEND
C1=(Y(J)-AA(J))/DY
R(1,J)=R(1,J)+CEXP(CON*AI*DX/XM)*C1
Y(J)=T(1,J)
B1=DX*B(1,J)/3.
A1=CON*B1/XM
X=XMAX-DX
C1=CON*Y(J)*CEXP(CON*AI*X/XM)
R(1,J)=(A1+C1)*CEXP(-CON*AI*DX/XM)
451 CONTINUE
C1=CON*AI/(XM*XM+YM)
X=XMAX-DX
R1=-XM*CEXP(C1*X)+XM*CEXP(C1*DX)
IF(NT.EQ.2.) PRINT 993
993 FORMAT(* IEND IS EVEN YIELDING ERROR IN QUADRATURE*)
SUM2=0.
DO 510 J=1,JEND
IF(J.EQ.1) A1=(Y(J+1)-Y(J))/DY
IF(J.EQ.JEND) A1=(Y(J)-Y(J-1))/DY
IF(J.GT.1.AND.J.LT.JEND) A1=(Y(J+1)-Y(J-1))/(2.*DY)
XI=(1.+XM)*CONJG(Y(J))*Y(J)+XM*(1.+XM)*(1.+XM)*A1*CONJG(A1)/
1 (2.*CON*CON)
IF(IDUCT.NE.0) DA=J*DY
SUM2=SUM2+DA*XI
501 CONTINUE
SUM1=0.
DO 600 J=1,JEND
IF(J.EQ.1) A1=(AA(J+1)-AA(J))/DY
IF(J.EQ.JEND) A1=(AA(J)-AA(J-1))/DY
IF(J.GT.1.AND.J.LT.JEND) A1=(AA(J+1)-AA(J-1))/(2.*DY)
IF(J.EQ.1) B1=(R(1,2)-R(1,1))/DY
IF(J.EQ.JEND) B1=(R(1,JEND)-R(1,JEND-1))/DY
IF(J.GT.1.AND.J.LT.JEND) B1=(R(1,J+1)-R(1,J-1))/(2.*DY)
C1=AI*A1+AI*XM*B1/CON
XI=REAL(CONJG(AA(J))*R(1,J)/CON)
XI=XI+XM*CONJG(AA(J))*AA(J)/2.
XI=XI+XM*(CONJG(R(1,J))*R(1,J)+CONJG(C1)*C1)/(2.*CON**2)
IF(IDUCT.NE.0) DA=J*DY
SUM1=SUM1+DA*XI
601 CONTINUE
PRINT 998
PRINT 994,SUM1,SUM2
IF(SUM1.LT.0.) RETURN
DR=10.*ALOG10(SUM2/SUM1)
PRINT 992,DR
994 FORMAT(1X,*ACOUSTIC POWER AT ENTRANCE = *,F10.5,/,
1 1X,*ACOUSTIC POWER AT EXIT = *,F10.5)
997 FORMAT(1H1)
998 FORMAT(1X,///)
999 FORMAT(1X,*ATTENUATION = *,F10.5)
RETURN
END

```

SUBROUTINE SETUP1

C THIS SUBROUTINE SETS-UP THE FINITE DIFFERENCE MATRIX FOR A RECTANGULAR
C

```
COMMON/BIG/BI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IOUST,KEND,DB
COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY ,C1, A,A1,B1,AA
REWIND 1
DO 6 K=1,IEND
DO 5 I=1,JEND
C( I,I)=A+2.*CON*XM*AI*DYDX*DY
DO 5 J=1,JEND
IF(J.NE.I) C( I,J)=0.
B( I,J)=0.
IF(I.EQ.J) B( I,J)=BBBB
IF(J.EQ.I+1) B( I,J)=-1.
IF(J.EQ.I-1) B( I,J)=-1.
5 CONTINUE
```

C INCLUDE BOUNDARY DATA CORRESPONDING TO Y=0 AND Y=1
C

```
A1= 2.*XM*XM*AI*DYDX/(CON*ZETAY*DX)
B1=-2.*A1-2.*CON*AI*DY/ZETAY
C1=A1-2.*XM*DYDX/ZETAY
A1=A1+2.*XM*DYDX/ZETAY
B( 1,2)=-2.
B( JEND,JEND-1)=-2.
B(1,1)=B(1,1)-B1
B(JEND,JEND)=B(JEND,JEND)- B1
C(1,1)=C(1,1)-C1
C(JEND,JEND)=C(JEND,JEND)- C1
IF(K.EQ.IEND) GO TO 6
DO 100 J=1,JEND
100 AA(J)=A
AA(1)=AA(1)-A1
AA(JEND)=AA(JEND)-A1
WRITE(1) AA,B,C
6 CONTINUE
```

C INCLUDE BOUNDARY DATA CORRESPONDING TO X=1
C

```
DO 7 I=1,JEND
7 B(I,I)=B(I,I)-C(I,I)*2.*CON*AI*DX/ZETAX
DO 8 J=1,JEND
8 AA(J)=A
AA(1)=AA(1)-A1
AA(JEND)=AA(JEND)-A1
WRITE(1) AA,B,C
DO 10 I=1,JEND
10 Y(I)=-AA(I)
REWIND 1
RETURN
END
```

SUBROUTINE SETUP2

```

C
C THIS SUBROUTINE SETS-UP THE FINITE DIFFERENCE MATRIX FOR A CYLINDRICAL
C
COMMON/RIG/ AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY ,C1, A,A1,B1,AA
REWIND 1
DO 6 K=1,IEND
DO 5 I=1,JEND
C( I,I)=A+2.*CON*XM*AI*DYDX*DY
DO 5 J=1,JEND
IF(J.NE.I) C( I,J)=0.
B( I,J)=0.
IF(I.EQ.J) B( I,J)=BBBB
IF(J .EQ. I+1) B(I,J)=-1.-1./FLOAT(2*I)
IF(J .EQ. I-1) B(I,J)=-1.+1./FLOAT(2*I)
5 CONTINUE
C
C INCLUDE BOUNDARY DATA CORRESPONDING TO Y=0 AND Y=1
C
A1= 2.*XM*XM*AI*DYDX/(CON*ZETAY*DX)
B1=-2.*A1-2.*CON*AI*DY/ZETAY
C1=A1-2.*XM*DYDX/ZETAY
A1=A1+2.*XM*DYDX/ZETAY
B( 1,2)=-2.
B( JEND,JEND-1)=-2.
B(JEND,JEND)=B(JEND,JEND)- B1*(1.+1. /FLOAT(2*JEND))
C(JEND,JEND)=C(JEND,JEND)- C1*(1.+1. /FLOAT(2*JEND))
IF(K .EQ. IEND) GO TO 6
DO 100 J=1,JEND
100 AA(J)=A
AA(JEND)=AA(JEND)-A1*(1.+1./FLOAT(2*JEND))
WRITE(1) AA,B,C
6 CONTINUE
C
C INCLUDE BOUNDARY DATA CORRESPONDING TO X=1
C
DO 7 I=1,JEND
7 B(I,I)=B(I,I)-C(I,I)*2.*CON*AI*DX/ZETAX
DO 8 J=1,JEND
8 AA(J)=A
AA(JEND)=AA(JEND)-A1*(1.+1. /FLOAT(2*JEND))
WRITE(1) AA,B,C
DO 10 I=1,JEND
10 Y(I)=-AA(I)
REWIND 1
RETURN
END

```

```

SUBROUTINE FUNCT(N,ZET,F)
  COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
  COMPLEX B,C,T,D,Y,V,AI,ZETAY,ZETAX,C1,A,A1,B1,AA
  DIMENSION ZET(N)
  ZETAY=ZET(1)+AI*ZET(2)
  IF(ZET(1).LT.0.) GO TO 40
  A=-(1.-XM*XM)*DYDX**2-CON*XM*AI*DYDX*DY
  IF(IDUCT.LT.0) CALL SETUP
  IF(IDUCT.EQ.0) CALL SETUP1
  IF(IDUCT.GT.0) CALL SETUP2
  RENTND 4
  WRITE(4) (Y(I),I=1,JEND)
  DO 10 I=1,JEND
10 Y(T)=0.
  TEND1=TEND-1
  DO 15 J=1,IEND1
  WRITE(4) (Y(I),I=1,JEND)
15 CONTINUE
  WRITE(4) AA
  CALL BLOCK
  CALL OUTPUT
  DB=0.
  CALL SOUND
  GO TO 50
40 DB=1.
50 F=DB
  RETURN
END

```

```

C*****
C  IDENTIFICATION
C    LU3 - LU FACTORIZATION OF A REAL SQUARE MATRIX
C    FORTRAN SUBROUTINE SUBPROGRAM
C    AFROSPACE RESEARCH LABORATORIES
C    WRIGHT-PATTERSON AFB, OHIO 45433
C  PURPOSE
C    LU3 COMPUTES TRIANGULAR MATRICES L AND U AND A PERMUTATION
C    MATRIX P SATISFYING  $LU = PA$  GIVEN AN N-SQUARE REAL MATRIX A.
C    LU3 IS INTENDED FOR USE WITH THE ENTRY POINT LEQ3 TO PRODUCE
C    SOLUTIONS OF THE VECTOR EQUATION  $AX = B$ .
C  CONTROL
C
C    DIMENSION A(N,M), IP(N)
C    .
C    .
C    .
C    CALL LU3(N,A,M,IP)
C
C  WHERE
C    N IS AN INTEGER INPUT VARIABLE, THE ORDER OF THE MATRIX A.
C    A AS A REAL INPUT ARRAY IS THE MATRIX TO BE TRIANGULARIZED.
C    A AS A REAL OUTPUT ARRAY IS THE UPPER TRIANGULAR FACTOR U
C    IN A(I,J), I .LE. J, AND THE LOWER TRIANGULAR FACTOR L IN
C    A(I,J), I .GT. J.
C    M IS AN INTEGER INPUT VARIABLE, THE ROW DIMENSION OF A.
C    IP IS AN INTEGER OUTPUT ARRAY, IP(K), K .LT. N, BEING THE IN-
C    DEX OF THE K-TH PIVOT ROW WITH IP(N) BEING ZERO IF A IS
C    SINGULAR AND DET(P) IF A IS NONSINGULAR.
C  OTHER PROGRAMMING INFORMATION
C    LU3 CONTAINS THE FORTRAN STATEMENT
C
C    DATA NUM/200/
C
C    NUM IS INITIAL ARGUMENT SUPPLIED TO THE SYSTEM SUBROUTINE EN-
C    TERED IN CASE LU3 WAS ENTERED WITH A NONPOSITIVE N.
C    DET(A) MAY BE CALCULATED FROM THE FORMULA
C     $DET A = FLOAT(IP(N)) * A(1,1) * \dots * A(N,N)$ 
C    LEQ3 IS CALLED TO COMPUTE SOLUTIONS OF LINEAR SYSTEMS AFTER
C    TRIANGULARIZATION OF A BY LU3.
C  OTHER PROGRAMS REQUIRED
C    THE SYSTEM SUBROUTINE IS CALLED FOR ERROR TRACING AND TERM-
C    INATION.
C  METHOD
C    THE MATRIX A IS REDUCED IN SITU TO TRIANGULAR PRODUCT FORM
C    USING GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING.
C*****

```

IDENTIFICATION

LEQSS3 - SOLUTION OF A LINEAR SYSTEM GIVEN A TRIANGULAR FACTORIZATION OF THE COEFFICIENT MATRIX

FORTRAN SUBROUTINE SUBPROGRAM

AEROSPACE RESEARCH LABORATORIES

WRIGHT-PATTERSON AFB, OHIO 45433

PURPOSE

LEQSS3 COMPUTES THE SOLUTION X OF THE LINEAR SYSTEM $LUX = PAX$
 $= PB$ WHERE L, U, AND P ARE COMPUTED FROM A BY LU3.

CONTROL

DIMENSION A(M,M), IP(N), B(N)

.

.

.

CALL LEQSS3(N,A,M,IP,B)

WHERE

N IS AN INTEGER INPUT VARIABLE, THE ORDER OF THE MATRIX A.

A IS A REAL INPUT ARRAY, THE TRIANGULARIZED COEFFICIENT MATRIX
 COMPUTED BY LU3.

M IS AN INTEGER INPUT VARIABLE, THE ROW DIMENSION OF A.

IP IS AN INTEGER INPUT ARRAY, THE INDEXES OF THE PIVOT ROWS
 COMPUTED BY LU3.

B AS A REAL INPUT ARRAY IS THE COLUMN VECTOR RIGHT-HAND SIDE B

B AS A REAL OUTPUT ARRAY IS THE COLUMN VECTOR SOLUTION X.

OTHER PROGRAMMING INFORMATION

LEQSS3 IS AN ENTRY POINT TO SUBROUTINE LU3.

OTHER PROGRAMS REQUIRED

NONE

METHOD

THE TWO TRIANGULAR SYSTEMS $LY = PB$ AND $UX = Y$ ARE SOLVED IN
 TURN FOR THE SOLUTION X OF $LUX = PB$. NO CHECK IS MADE FOR
 NONSINGULARITY OF THE MATRIX U. THIS CHECK IS MADE BY INTER-
 ROGATING IP(N). AN ATTEMPT TO SOLVE $UX = Y$ WHERE A IS SINGULAR
 WILL RESULT IN AN INFINITE OR INDEFINITE QUOTIENT.

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(1) CLEVE B. MOLER, ALGORITHM 423 - LINEAR EQUATION SOLVER,
 COMM. ACM 15(1972), P.274.

(2) PAUL J. NIKOLAI, THE ARL LINEAR ALGEBRA LIBRARY, ARL
 TECHNICAL REPORT ARL 71-J137 (1971).

 SUBROUTINE LU3(N, A, M, IP, B)

TRIANGULARIZATION OF A REAL SQUARE MATRIX USING GAUSSIAN ELIMINATION

DIMENSION A(M,M), IP(N), B(N)

COMPLEX A, B, T

EQUIVALENCE (KP1, KM1)

DATA NUM /200/

IP(N) = 1

NM1 = N - 1

IF (NM1) 2, 65, 4

CALL SYSTEM(NUM, 12H1ILLEGAL ARG)

```

4      DO 60 K = 1, NM1
      KP1 = K + 1
      L = K
      DO 10 I = KP1, N
        IF ((ABS(REAL(A(I,K))) + ABS(AIMAG(A(I,K)))) .GT.
*      (ABS(REAL(A(L,K))) + ABS(AIMAG(A(L,K))))) L = I
10     CONTINUE
      IP(K) = L
      IP(N) = ISIGN(1, K - L)*IP(N)
      T = A(L,K)
      A(L,K) = A(K,K)
      A(K,K) = T
      IF (ABS(REAL(T)) + ABS(AIMAG(T))) 15, 13, 15
13     IP(N) = 0
      GO TO 60
15     DO 20 I = KP1, N
      A(I,K) = -A(I,K)/T
20     CONTINUE
      DO 40 J = KP1, N
      T = A(L,J)
      A(L,J) = A(K,J)
      A(K,J) = T
      IF (ABS(REAL(T)) + ABS(AIMAG(T))) 25, 40, 25
25     DO 30 I = KP1, N
      A(I,J) = A(I,J) + T*A(I,K)
30     CONTINUE
40     CONTINUE
60     CONTINUE
65     IF (A(N,N) .EQ. (0.0,0.0)) IP(N) = 0
      RETURN
      ENTRY LEQS3
C
C SOLUTION OF A REAL LINEAR SYSTEM TRIANGULARIZED BY SUBROUTINE LU3
C
      NM1 = N - 1
      IF (NM1) 2, 90, 67
67     DO 70 K = 1, NM1
      KP1 = K + 1
      L = IP(K)
      T = B(L)
      B(L) = B(K)
      B(K) = T
      DO 69 I = KP1, N
      B(I) = B(I) + T*A(I,K)
69     CONTINUE
70     CONTINUE
      DO 80 L = 1, NM1
      KM1 = N - L
      K = KM1 + 1
      B(K) = B(K)/A(K,K)
      T = -B(K)
      DO 75 I = 1, KM1
      B(I) = B(I) + T*A(I,K)
75     CONTINUE
80     CONTINUE
90     B(1) = B(1)/A(1,1)
      RETURN
      END

```

REFERENCES

1. Rice, E. J., "Attenuation of Sound in Soft Walled Circular Ducts," AFOSR-UTIAS Symposium on Aerodynamic Noise, University of Toronto Press, 1969.
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3. Baumeister, K. J., and Rice, E. J., "A Difference Theory for Noise Propagation in an Acoustically Lined Duct with Mean Flow," Progress in Astronautics and Aeronautics-Aeroacoustics: Jet and Combustion Noise; Duct Acoustics, Vol. 37, edited by H. T. Nagamatsu, J. V. O'Keefe, and I. R. Schwartz, AIAA, New York, 1975, pp. 435-453.
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