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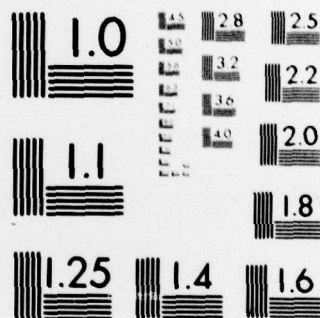
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| <p>The topology of the space of minimal transfer functions has been studied with particular emphasis on symmetric transfer functions and a number of important new results have been obtained. Understanding the topology of the space of transfer functions is important in the development of identification algorithms and symmetric transfer functions arise in the description of many physical systems. The problem of state space realization of transfer functions for systems with delays has also been studied and useful results have been obtained. The problems of deterministic control, stochastic</p> |                                     |                                                             |  |

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20. Abstract continued.

filtering and stochastic control have been studied for linear pure delay time systems. A geometric approach has been used which has simplified previous results and has obtained important new results. In addition, preliminary investigations of some other related research areas has commenced.

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**AFOSR-TR- 79-0981**

**Scientific Report for Grant AFOSR 77-3177**

**Stochastic Filtering and Control**

**Period: 16 May 1978-15 May 1979**

**Principal Investigator: T.E. Duncan**

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## Abstract

During the period 16 May 1978-15 May 1979, the grant year, a number of topics have been investigated with the support of grant AFOSR 77-3177. The topology of the space of minimal transfer functions has been studied with particular emphasis on symmetric transfer functions and a number of important new results have been obtained. Understanding the topology of the space of transfer functions is important in the development of identification algorithms and symmetric transfer functions arise in the description of many physical systems. The problem of state space realization for transfer functions for systems with delays has also been studied and useful results have been obtained. The problems of deterministic control, stochastic filtering and stochastic control have been studied for linear pure delay time systems. A geometric approach has been used which has simplified previous results and has obtained important new results. In addition, preliminary investigations of some other related research areas has commenced.



## Description of Research

Understanding the topology of the space of minimal transfer functions is important for many reasons in system theory. It provides a better understanding of the three descriptions of a linear system (transfer function, state space equations and Hankel matrix), and it is important for identification problems. Brockett (2) has shown that the space of scalar input-output transfer functions contains  $n+1$  topological components where  $n$  is the minimal state space dimension. Glover (8) has shown that the space of multivariable transfer functions with no symmetry properties contains only one topological component. Byrnes-Duncan (4) have studied symmetric transfer functions because of their physical importance and have shown that this space contains  $n+1$  topological components where  $n$  is the minimal state space dimension. The invariant that separates the components is the Maslov-Arnol'd index (1,14) which is a natural generalization of the Cauchy index (5) which Brockett used in his study of scalar input-output transfer functions. Koga (10) and Youla (17) some years ago have studied the problem of the state space realization for network symmetric transfer functions with commensurate pure delays. However, they solved the problem over the fraction field formed from the polynomial ring in the delay so that their realization contained predictors, the inverse of delays, as well as delays. Thus, their realization was not physically useful. Byrnes-Duncan (4) have solved this problem and some generalizations of it over the ring of polynomials using some solutions of the quadratic Serre problem (9,15). Such transfer functions are important for modelling transmission lines where the delays are not negligible.

Duncan (6) has provided a geometric approach to the well known problems of linear quadratic optimal control and dually Gaussian stochastic estimation. In this geometric approach the Riccati equation is an intrinsic object from the Lagrangian Grassmannian. The associated Hamiltonian equations are studied with the action of the symplectic group. The separation of filtering and control in linear stochastic control is related to an indeterminacy of the Hamiltonian.

Using the aforementioned geometric techniques for control and estimation Duncan (7) has obtained the equations for stochastic filtering for linear pure delay time systems where the delays occur only in the state transition matrix. Linear pure delay time systems are important for modelling physical systems because such systems often contain nonnegligible delays. In this geometric approach a linear pure delay time system is viewed as a system over the ring of polynomials in the delays and this description as a finitely generated, projective module (3) is described as an algebraic vector bundle over affine space (16).

Duncan has continued the investigation of linear pure delay time systems and related problems and has obtained some preliminary results generalizing the above work. To solve the filtering problem for linear pure delay time systems it is necessary to use the techniques of the smoothing solution for systems without delays. Since there are a number of different solutions (e.g. (11,13)) to this latter problem it is important to understand these solutions geometrically. Duncan has done this and has shown how the various smoothing formulae arise naturally. Previously, scattering theory had been used to describe these various smoothing



solutions but this approach did not elucidate the various solutions. The equations for stochastic filtering have been obtained for linear pure delay time systems where the delays appear in both the state transition matrix and the observations. These equations describe a Riccati equation which must be augmented by additional equations so that it can be solved. These additional equations are obtained by elementary geometrical considerations. The infinite time problem of filtering for linear pure delay time equations has been studied and algebraic sufficient conditions have been obtained for the stability of the optimal filter. These algebraic conditions are the natural generalization of the conditions for systems without delays. The problem of stability for linear pure delay time systems is more difficult than for linear systems without delays because reachability is not equivalent to coefficient assignability of the characteristic polynomial.

Some preliminary work has been initiated on a geometric approach to the computation of various asymptotic probabilities by the use of stochastic control. This geometric approach uses geometric optics which is important for various optical systems, e.g. lasers.

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## Research Results Supported by this Grant

Papers that have appeared during this grant year:

1. Estimation for jump processes in the tangent bundle of a Riemann manifold, Appl. Math. and Optimization 4 (1978) 265-275
2. Optimal control of stochastic systems in a sphere bundle, Lecture Notes in Math. (1979) Springer-Verlag New York

Papers accepted for publication this year:

1. A geometric approach to linear control and estimation, to appear in Lecture Notes in Control and Information Sciences, Springer-Verlag
2. An algebro-geometric approach to estimation and stochastic control for linear pure delay time systems, to appear in Lecture Notes in Control and Information Sciences, Springer-Verlag

### Other Activities Related to the Grant

The principal investigator was a gastprofessor at the Institute for Applied Mathematics, University of Bonn during the academic year 1978-79

The principal investigator was an invited participant at the following conferences:

1. Conference on System Theory, Bordeaux, France, 11-15 September 1978
2. Conference on Stochastic Control Theory and Stochastic Differential Systems, Bonn, Germany, 4-12 January 1979

The principal investigator attended the following conference:

1. International Congress of Mathematicians, Helsinki, Finland, 17-25 August 1978

The principal investigator is an invited participant at the following conferences:

1. Conference on Algebraic and Geometric Methods in Linear System Theory, Harvard University, 18-29 June 1979
- 2.. Conference on Measure Theory, Oberwolfach, Germany, 1-7 July 1979