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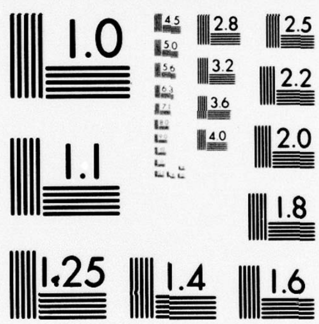

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RESOURCE ALLOCATION WITH DECENTRALIZED INFORMATION IN BALLISTIC MISSILE DEFENSE

Y. Bar-Shalom

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University of Connecticut U-157
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Abstract

This report deals with the way a network of decision makers can control an outside environment. The environment considered is the Ballistic Missile Defense where defense resources are to be allocated. The allocation problem is one of interceptors against an attack. The network of decision makers is described by an arbitrary connection matrix. A decentralized information pattern is considered: each decision maker is a node in the network and has access only to the information of the nodes it is connected to. A decision consists of the allocation of its defense resources to own defense and to support the neighbors. In the dynamic case part of the available resources have to be put aside for future use. The future attack against which these resources will have to be used is known only via a probabilistic description. All these decisions have to be made using the decentralized information available to each decision maker. The framework of solution for this problem is within what is known as team theory. Three decentralized allocations are proposed and compared via simulations with a local strategy (no exchange of information) and the fully centralized strategy. The results show that the most sophisticated decentralized strategy can achieve a performance near the one of the centralized. The implications of these results on the design of a computer network to carry out these functions are also discussed.

Introduction

The problem of real-time allocation and control technology for the BMD environment was dealt with extensively in [R1]. The desirability of using Distributed Computer Systems/Computer Networks (DCS/CN) was pointed out. Some of the reasons for this approach are: reduced vulnerability, more uniform computation load, less reporting and communication problems. The achievement of these goals has been made feasible by the recent advances in microcomputers. On the other hand such systems are harder to analyze and their performance is usually lower when compared to a fully centralized system. Nevertheless, as it is shown in the sequel, a properly decentralized control can achieve a performance very near to a fully cnetralized one.

There are two principal aspects of Distributed Computing Systems:

(i) The internal structure interconnection, switching problems, operating systems, reliability, architecture, distributed data bases, etc.)

(ii) The use of the DCS/CN to control are outside environment.

Our philosophy is that item (ii) is the ultimate goal of the system and it is on this area that our research effort has focused. On the other hand, the tool to carry out (ii) is the DCS/CN. Thus the two items cannot be really separated, as pointed out in [R1]: research on both aspects should be carried out and unified.

This report presents the results of our research on (ii) and points out how the control technology has reached the stage to be connected to the computer network. This connection is discussed further in Section 11.

The goal of our research was to understand how a network of decision makers (in practice a computer network) can be used to control a BMD environment by carrying out the allocation of defense resources.

This report presents a mathematical formulation of a Ballistic Missile Defense resource allocation problem. The motivation of this work stems from the desirability of using a computer network for the real time decisions in the BMD problem. The focus of the report is on how to use such a network, represented here by a set of decision makers, to solve a control problem in a dynamic stochastic environment, i.e., how a network should control an outside environment. This is different from the problem of how the network should be controlled.

The specific problem to be discussed is the allocation of interceptors against an attack. The allocation decisions are made by a set of decision makers. Following the study reported in [R1] which pointed out the desirability of a decentralized/distributed approach, the decision process considered is of the decentralized type. Mathematically, this translates into the fact that the information pattern of the set of decision makers is "nonnested": there is no ordering of the decision makers such that each knows at least as much as its predecessor. A properly designed decentralized decision making system can be close to a centralized system in terms of performance. The present report deals with the mathematical formulation of this problem in the context of team theory [M1]. Another important aspect of a decentralized control is the so-called authority associated with each decision maker: the set of allowable decisions for a DM. From the control point of view, the authority and information pattern define the framework of the defense's optimization problem. The attack side will have to be described in a suitable form such that a well-defined performance index emerges that can then be optimized by the defense within its constraints.

The team theory approach to the BMD resource allocation problem is presented in Section 3, together with its limitations. While this approach is useful as a framework, its limitations suggest that suboptimal solutions have to be sought. The type of suboptimal solution that is developed is of the "member-by-member optimal" type, which, in our case does not guarantee global optimality. This approach is then used on a more general problem formulation than first presented. This more general performance index is presented in Section 4 in the context of an arbitrarily connected network of decision makers. A first and very simple decentralized allocation strategy is described in Section 5. Optimization is carried out within a class of decision functions considered. An arbitrarily connected network of decision making stations is considered with each making a decision based upon its available information. The information available to each decision maker consists of local information as well as the information obtained from the stations to which it has a communication link. The decision by a station affects the performance of that station as well as that of stations it is connected to. Two more sophisticated decentralized strategies are discussed in Sections 6 and 7. These latter strategies have the feature of preferential allocation. Numerical results indicate that the most sophisticated decentralized strategy yields performance close to fully centralized one. The extension to the dynamic situation is presented in Sections 8 and 9 with an example in section 10. Section 11 presents the conclusions and recommendations for future work.

2. Formulation of the Problem

The following resource allocation problem is considered. The context is terminal defense where a set of N defended points (silos) indexed i=1,...,N is assumed to be attacked by a wave of reentry vehicles (RVs). The attack against silo i is by α_i RVs. The overall attack is described by the N-vector $\underline{\alpha}$ and this will be assumed to be a random variable with a known joint distribution. Each silo's defense is decided upon by a local decision maker. At the disposal of the i-th such decision maker there are r_i units of defense resources (interceptors).

A decentralized decision making with the following characteristics is assumed:

1. Authority: each local decision maker (DM) can allocate interceptors for its own defense or its neighbors' (defined for silo i as i-1 and i + 1).
2. Information pattern: the i-th decision maker DM_i knows

$$I_i = \{r_k, \alpha_k; k = i-1, i, i+1\} \tag{2.1}$$

i.e., only the situation of its neighbors in addition to its own. This means that each DM observes a subset of the components of the random variable $\underline{\alpha}$ (the attack).

The probability of survival of silo i in face of the j-th RV attacking it if no interceptor is allocated against it (a_{ij} = 0) is

$$S_{ij}\ (a_{ij}=0) = 1 - \rho_\alpha \triangleq p_0 \tag{2.2}$$

where ρ_α is the reliability of an attacking RV. Denoting by ρ_δ the reliability of an interceptor the probability of survival of a defended silo in face of RV j is

$$S_{ij}\ (a_{ij}=1) = 1 - \rho_\alpha(1-\rho_\delta) \triangleq p_1 \tag{2.3}$$

Thus, for a binary allocation a_{ij} one can write

$$S_{ij}(a_{ij}) = 1 - \rho_\alpha(1-\rho_\delta)^{a_{ij}} \tag{2.4}$$

The probability of survival of silo i conditioned on being attacked by α_i RVs and defended by δ_i interceptors is

$$S_i(\alpha_i,\delta_i)=p_1^{\delta_i}\,p_0^{\alpha_i-\delta_i}=p_0^{\alpha_i}\left(\frac{p_1}{p_0}\right)^{\delta_i} \qquad 0 \le \delta_i \le \alpha_i \tag{2.5}$$

Since $p_1 > p_0$ the above is obviously a convex function of δ_i.

Denote the decision of DM_i by

$$\underline{d}_i = [d_{i,i-1}, d_{i,i}, d_{i,i+1}]' \tag{2.6}$$

where d_{ij} is the allocation made by i to j and

$$d_i = d_{i,i} + d_{i,i-1} + d_{i,i+1} \tag{2.7}$$

is the total number of interceptors initially assigned to DM_i.

The number of interceptors available for the defense of silo i is

$$\delta_i = d_{i,i} + d_{i-1,i} + d_{i+1,i} \tag{2.8}$$

The problem is to maximize the expected number of surviving silos

$$J^* = \max_{\substack{\underline{d}_i(I_i) \\ i=1,\ldots,N}} \sum_{i=1}^{N} E[S_i(\alpha_i, \delta_i)] \tag{2.9}$$

where the decentralized information pattern (2.1) is the key point. The expectation in (2.9) is over $\underline{\alpha}$ while the decision variables depend on the local information.

3. The Team Theory Approach to Solution

The stochastic optimization problem formulated in the previous section, as summarized by (2.9), falls in the category of static team problems. However, despite its being a rather simplified BMD problem, it is substantially more complex than the team problems whose solutions are known [H1, M1].

The general form of static team problems is as follows. The performance index is

$$\max_{\underline{d}} \; E\{S[\underline{d}_1(I_1), \ldots, \underline{d}_N(I_N), \alpha]\} \tag{3.1}$$

where

$$\underline{d}' = [\underline{d}_1'(I_1), \ldots, \underline{d}_N'(I_N)] \tag{3.2}$$

The member-by-member optimality conditions are

$$\max_{\underline{d}_i} \; E\left\{E\{S[\underline{d}_1^*(I_1), \ldots, \underline{d}_{i-1}^*(I_{i-1}), \underline{d}_i(I_i), \underline{d}_{i+1}^*(I_{i+1}), \ldots, \underline{d}_N^*(I_N), \underline{\alpha}] \,|\, I_i\}\right\} \tag{3.3}$$

where $\underline{d}_j^*$, $j \neq i$, are fixed decision rules. The above is equivalent to

$$\max_{\underline{d}_i(I_i)} \; E\{S[\underline{d}_1^*(I_1), \ldots, d_{i-1}^*(I_{i-1}), d_i(I_i), d_{i+1}^*(I_{i+1}), \ldots, d_N^*(I_n), \underline{\alpha}\} \tag{3.4}$$

The main result pertaining to this probelm, which is due to Radner [M1] states that if the function S is differentiable and concave in $\underline{d}$ then the member-by-member optimality conditions give a unique solution which is the optimum for the overall probelm. The differentiability requirement requires implicitly that the decision variable be continuous.

Note that in order to carry out the maximization in (3.3) to obtain the optimum decision for DM_i one needs to know the forms of the other members' optimal decisions, i.e., the functions

$$\{d_j^*(I_j),\ j = 1,\ldots,i-1,\ i+1,\ldots,N\} \tag{3.5}$$

and the conditional densities

$$\{p(I_j|I_i),\ u = 1,\ldots,i-1,\ i+1,\ldots,N\} \tag{3.6}$$

are needed to carry out the expectation in (3.3). In other words, each decision maker must have a picture of what the others are doing and based on what information they act.

In the case where the performance index is quadratic in the decision variables and the information available to each DM is a subset of the random vector $\underline{\alpha}$, assumed normally distributed, the conditions of Radner are satisfied. The optimal solution for each decision variable is then linear in its observation. The key requirement (3.5) that each DM know the form of the decision rule of all the other DMs is satisfied due to the linearity of the solution. The conditional densities (3.6) are all normal and they follow from the density of the vector $\underline{\alpha}$.

The main differences between this problem and the BMD formulation described in the previous section are:

1) The decision variables are discrete
2) The random vector $\underline{\alpha}$ is discrete-valued
3) The performance index (2.15) requires a maximization of a sum of functions which are convex (rather than concave, which is the more desired case in maximizations).

The implications of the above properties of our problem are that new, discrete optimization techniques will have to be used. Furthermore, the need to know the forms of the decisions (3.5) cannot be satisfied as easily as in the linear-quadratic-gaussian case because the decision rules cannot be linear. A second problem is the specification of the conditional probability densities (or rather, probability functions) for the pertinent random variables.

The main difficulty is posed by the nonnestedness of the information sets, which precludes the use of the Principle of Optimality. This follows from the fact the interleaved (nested) extremizations and expectations in the stochastic version of the Principle of Optimality require nested information sets (e.g., Bellman [B1], Bar-Shalom and Tse [B2]). In contradistinction with this, the team theory problem formulation does incorporate the informational constraint of our problem. However, this theory has provided an explicit solution only to the Linear-Quadratic-Gaussian Problem (see McGuire and Radner [M1], Ho and Chu [H1]). The two properties required from the cost function that were crucial in obtaining the solution to the Linear-Quadratic-Gaussian team problems are concavity (for maximization; convexity for minimization) and differentiability. These two properties led to the equivalence of the global optimization problem to the so-called member-by-member optimization problem.

The unique aspects of the Ballistic Missile Defense problem pointed out in [R1] put it in a class by itself and our work had as starting point this very important observation. The preliminary problem formulation for a Ballistic Missile Defense Resource Allocation Problem presented in Section 2 arrived at a performance index to be maximized; this performance index consisted, however, of a sum of convex, rather than concave functions of the resources. It seems that proof of the global optimality of the member-by-member optimization is not feasible.

The next section presents a performance index that is more general than the one developed in Section 2. This generalized performance is more flexible than the earlier one and is used subsequently in the simulations of the various decentralized allocation strategies presented later.

4. A Generalized Performance Index

There are N stations which experience a "demand" (e.g. attack) of α_i units, $i=1,\ldots,N$. The vector

$$\underline{\alpha} = [\alpha_1 ,\ldots, \alpha_N]' \tag{4.1}$$

is a random variable with a known joint probability distribution function. Each station is a decision maker at whose disposal there are r_i , $i=1,\ldots,N$ resource units to satisfy the demand (e.g. defense units that counter the attack).

A symmetric connectivity matrix

$$\Omega = [\omega_{ij}] \tag{4.2}$$

with binary elements is defined. The unity elements indicate which stations are connected. The diagonal elements are unity by definition. Connected stations (and only those) exchange information about their available resources and the demand they face - this is done with the purpose of allowing a decision maker to assign some of its own resources to "help" the neighbors. The set-up considered allows a network of stations with arbitrary connections.

The information structure at station i is

$$I_i = \{r_k, \alpha_k;\ k\epsilon\ C_i\} \tag{4.3}$$

where C_i is the set of stations connected to i

$$C_i = \{j : \omega_{ij} = 1\} \tag{4.4}$$

The performance index (survivavibility of a station) is taken as

$$S_i(\alpha_i,\delta_i) = p_o^{\ \alpha_i}\left(\frac{p_1}{p_o}\right)^{\delta_i} \quad 0\leq\delta_i\leq\alpha_i \tag{4.5}$$

as in Section 2 with the notations

$$p_1 = 1 - \rho_\alpha\ (1-\rho_\delta) \tag{4.6}$$

$$p_o = 1 - \rho_\alpha \tag{4.7}$$

where ρ_α is the reliability of an attacker unit and ρ_δ the reliability of a defense resource unit. Term (4.6) is the probability of survival of a defended silo while (4.7) is the probability of survival of an undefended silo, both in face of a single attacker. We shall allow use of more defense resources than the number of attacker unite, in which case

$$S_i(\alpha_i,\delta_i) = \left[1-\rho_\alpha\,(1-\rho_\delta)^{\frac{\delta_i}{\alpha_i}}\right]^{\alpha_i} \qquad \delta_i \geq \alpha_i \tag{4.8}$$

The performance index given by the piecewise analytical function described by (4.5) and (4.8) is plotted for the purpose of illustration in Fig. 4.1. The first part of the curve is convex (too little resources are of very small use) while the second part is concave (diminishing marginal returns) in δ_i when

$$\delta_i > -\frac{\alpha_i \ell n\, \alpha_i \rho_\alpha}{\ell n\,(1-\rho_\delta)} \tag{4.9}$$

The decision made by the i-th decision maker is the vector

$$\underline{d}_i = [d_{ij}\ ,\ j\epsilon C_i] \qquad i=1,\ldots,N \tag{4.10}$$

where d_{ij} is the allocation from i to j (only if connected). Then

$$\delta_j = \sum_{i\epsilon C_j} d_{ij} \tag{4.11}$$

and the total resource constraint is

$$\sum_{j\epsilon C_i} d_{ij} = d_i \tag{4.12}$$

The problem is then for each decision maker to obtain an allocation of defense units as indicated by (4.10), i.e.,

(i) for itself

(ii) for those stations it is connected to subject to the constraints (4.12), such as to maximize the expected number of surviving stations, or, equivalently, the average survivability. This latter form of the performance is considered further in the next section where a decentralized decision making algorithm is presented and evaluated.

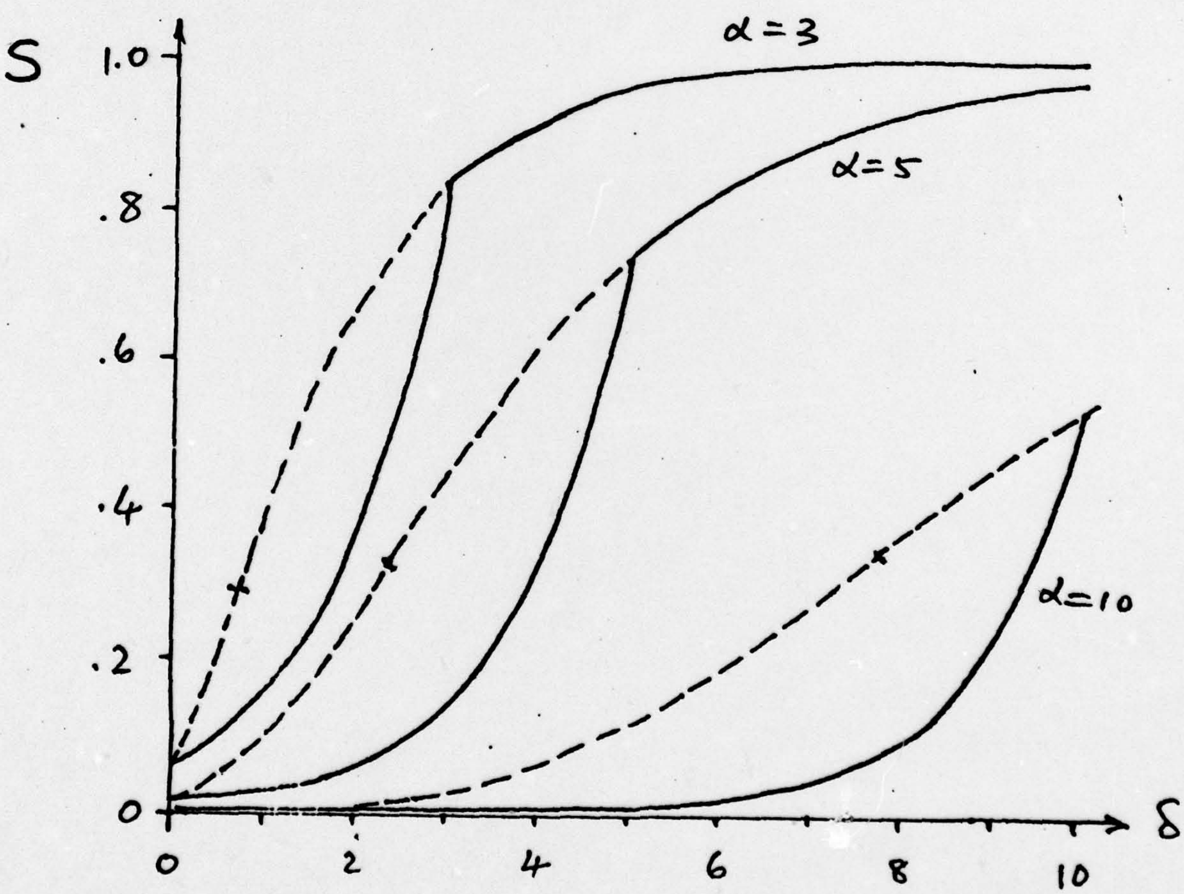


Figure 4.1 Performance indes for $\rho_\alpha = 0.6$, $\rho_\delta = 0.9$.

5. A parametric-optimization decentralized algorithm and simulation results

A problem with N=6 stations was considered with the performance index for each station given by the function plotted in Fig.4.1. The global objective function was the average survivability

$$J = \frac{1}{N} \sum_{i=1}^{N} E\{S_i(\alpha_i, \delta_i)\} \qquad (5.1)$$

where the expectation is over all the possible attack scenarios.

The following connection matrix between the stations was assumed

$$\Omega = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array} \qquad (5.2)$$

The corresponding network is depicted in Fig. 5.1.

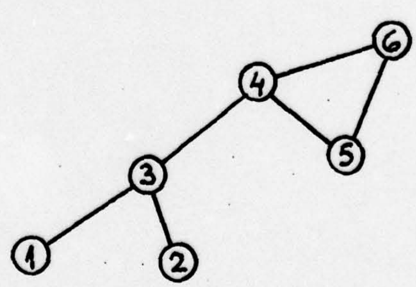


Fig. 5.1 Network of stations in example

A variable, called threat level, has been defined as follows

$$T_i = \frac{\alpha_i}{r_i} \tag{5.3}$$

Through the communication network described by (5.2) each station knows the threat level of all the stations it is connected to. The following decision rule form is assumed

$$d_{ij} = r_i \frac{xT_i}{(1-x)T_i + x \sum_{k\varepsilon C_i} T_k} \qquad j \neq i \tag{5.4}$$

$$d_{ii} = r_i \frac{T_i}{(1-x)T_i + x \sum_{k\varepsilon C_i} T_k} \tag{5.5}$$

Then the problem is to optimize over the parameter x called the transfer factor. The total resources available to station i for its use are then

$$\delta_i = \sum_{j\varepsilon C_i} d_{ji} \tag{5.6}$$

Two example problems were simulated.

Example 1. The available defense resources were

$$\underline{d} = [4 \quad 2 \quad 3 \quad 5 \quad 3 \quad 4] \tag{5.7}$$

The noninteger allocation variables resulting from (5.4), (5.5) were rounded to the nearest integer below and the remainder distributed according to decreasing magnitudes. The overall performance index (5.1) is plotted as function of the transfer factor x in Fig. 5.2. The nonuniqueness (plateaus) of the optimal solution which is in this case $x\varepsilon[.082,.1]$ follows from the integer constraint imposed on the decision variables. Also note the appearance of local extrema - this is a phenomenon typical to nonlinear stochastic control problems as pointed out in Bar-Shalom and Wall [B4].

The three equiprobable scenarios considered in this first example were as follows:

Scenario 1:

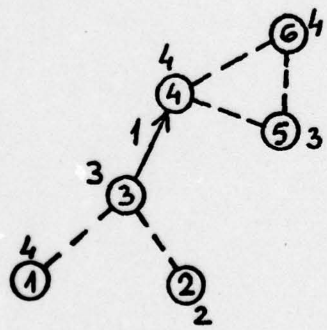
Attack = (4 2 3 5 3 4), worst case

Optimum Average Survivability = 0.7511

Allocation matrix from station i to j:

	1	2	3	4	5	6
1	4	0	0	0	0	0
2	0	2	0	0	0	0
3	0	0	3	0	0	0
4	0	0	1	4	0	0
5	0	0	0	0	3	0
6	0	0	0	0	0	4
	4	2	4	4	3	4
			vs.			
	4	2	3	5	3	4

Exchanges between stations:



Scenario 2:

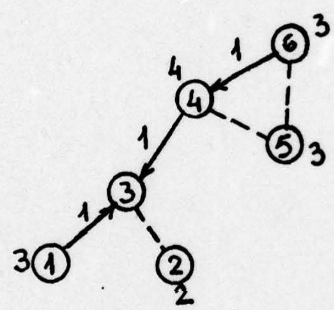
Attack = (2 2 8 5 2 2), best case

Optimum Average Survivability = 0.7586

Allocation matrix from station i to j:

	1	2	3	4	5	6
1	3	0	1	0	0	0
2	0	2	0	0	0	0
3	0	0	3	0	0	0
4	0	0	1	4	0	0
5	0	0	0	0	3	0
6	0	0	0	1	0	3
	|	|	|	|	|	|
	3	2	5	5	3	3
			vs.			
	2	2	8	5	2	2

Exchanges between stations:



Scenario 3

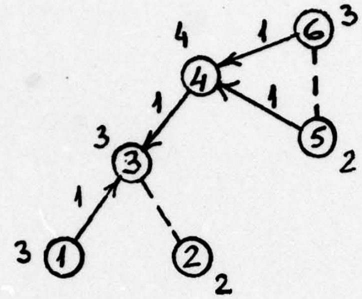
Attack = (2 2 5 8 2 2)

Optimum Average Survivability = 0.7560

Allocation matrix from station i to j

	1	2	3	4	5	6
1	3	0	1	0	0	0
2	0	2	0	0	0	0
3	0	0	3	0	0	0
4	0	0	1	4	0	0
5	0	0	0	1	2	0
6	0	0	0	1	0	3
	|	|	|	|	|	|
	3	2	5	6	2	3
			vs.			
	2	2	5	8	2	2

Exchange between stations:



A major issue is where does the proposed scheme stand in terms of its performance with respect to the following alternative strategies

(i) Local - each decision maker is using strictly local information, i.e., no communication between the stations and, therefore, no resources can be exchanged

(ii) Fully centralized - all the information is available at a single location where a global allocation of all the available resources is carried out.

The local scheme described in (i) above will yield a lower bound of the performance of the decentralized algorithm. On the other hand, a fully centralized algorithm will yield an upper bound. Despite the fact that this upper bound is not achievable, a major question of interest pointed out in [R1] is how close the decentralized algorithm's performance can be to the one of the centralized algorithm. From the point of view of robustness or cost of implementation one would favor a decentralized algorithm [R1], even though quantitative studies have not yet been made in this area to obtain measures of robustness.

The performance of the local decision scheme can be obtained from Fig. 5.2 - it corresponds to no exchanges of defense resources between the stations, i.e., x=o. The result is $J_L = 0.745$ (L = local) and is lower than $J^*_{DC} = 0.755$ (DC - decentralized)

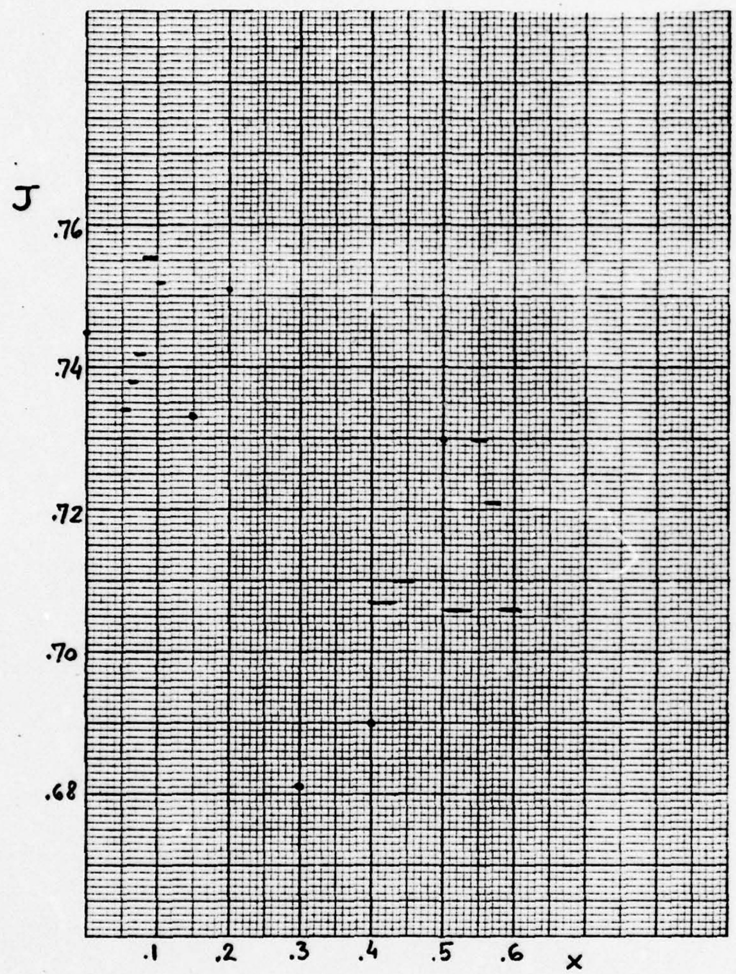


Fig. 3.2 Performance of decentralized decision scheme vs. transfer factor for Example 1.

even though, the difference is very small.

The fully centralized scheme will completely redistribute the total amount of resources in an optimal way to meet the demand (counter the attack). Note that this set-up assumes not only communication from each station to the central decision maker but also the implicit assumption that any resource can be used anywhere in the network without limitations. The exact solution to the fully centralized problem is a general integer programming algorithm, so a simple iterative approach has been adopted. The original assignment (5.7) was taken as initial allocation. An iteration scheme consisting of switching units of resources to obtain the largest improvement was then carried out. This was done subject to the nonnegativity constraint. Such a procedure will always converge and lead to a locally optimal allocation. The final solution will be the global optimum if the individual performance indices are concave. Using this procedure the average survivability with the centralized decision scheme in Example 1 was

Scenario 1: $J = 0.807$

Scenario 2: $J = 0.813$

Scenario 3: $J = 0.813$

In all three scenarios the centralized allocation turned out to be an exact match of the attack pattern. This was possible because the total amount of defense resources was assumed the same as the total number of attackers.

Thus the average survivability over the three scenarios was $J_C = 0.811$ (C - centralized). As expected one has the ordering

$$J_L < J_{DC} < J_C \qquad (5.8)$$

Nevertheless in this example the sensitivity of the performance is not very high. In view of this a different case is considered next.

Example 2

In this case the available resources are

$\underline{d}$ = [4 2 3 5 1 1] (5.9)

while the attack scenarios are (4 2 3 5 3 4), (2 2 8 5 2 2) and (2 2 5 8 2 2), all equiprobable. Note that, unlike in Example 1, here the total defense resources are only 16 vs. attack size 21. It is expected that in the case where the defense is outnumbered the sensitivity of the performance to the defense strategy is higher.

For the local strategy the performance was:

Scenario 1: J = 0.573

Scenario 2: J = 0.561

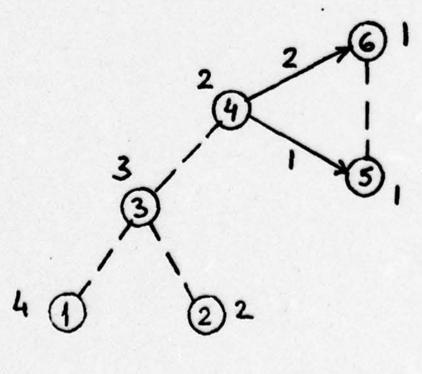
Scenario 3: J = 0.467

which yields J_L = 0.534

For the decentralized strategy specified earlier in this section the average survivability is plotted in Fig. 5.3. The optimum decentralized performance turned out to be J_{DC} = 0.58. The optimum x is in the interval [0.181, 0.2].

The shifting of defense units by the decentralized algorithm was as follows

Scenario 1: J = 0.540



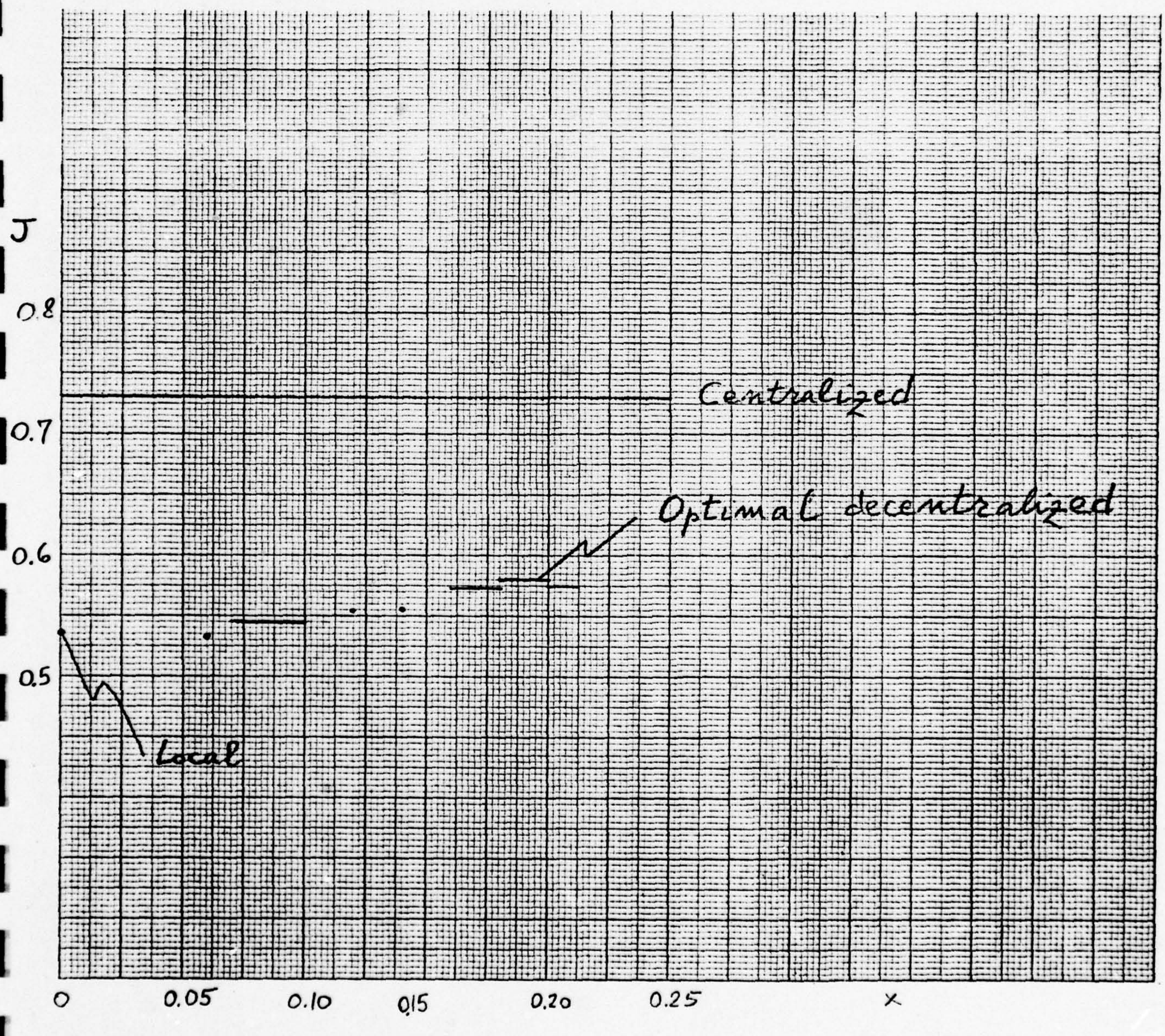
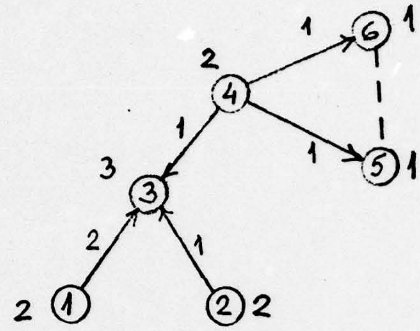
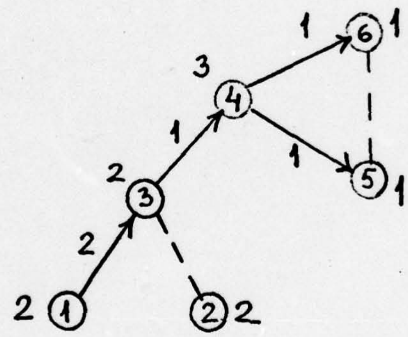


Fig. 5.3 Performance of decentralized decision scheme vs. transfer factor for Example 2.

Scenario 2: J = 0.557



Scenario 3: J = 0.644



The completely centralized strategy yielded the following results

Scenario 1: J = 0.686

Allocation: (4 2 3 0 3 4)

vs. Attack: (4 2 3 5 3 4)

The fourth site is given up.

Scenario 2: J = 0.753

Allocation: (3 3 0 6 2 2)

vs. Attack: (2 2 8 5 2 2)

The third site is given up.

Scenario 3: $J = 0.753$

Allocation: (3 3 6 0 2 2)

vs. Attack: (2 2 5 8 2 2)

The fourth site is given up.

The average survivability with the centralized strategy was in this case $J_C = 0.731$.

Comparing the three strategies it can be seen that in this example the decentralized outperformed the local by about 10% while the centralized stragy had a performance about 25% higher than the decentralized one. The reason that the centralized strategy was substantially better than the others is that it adopted a perferential defense mode - this is a consequence of the availability of global information at the location of the central decision maker.

6. <u>Decentralized strategies with preferential allocation.</u>

This section presents the description of a defense strategy that, when outnumbered by the offense, should be able to adopt a preferential mode according to some priorities to be computed in real time.

As the previous section's computation results indicated, the partially decentralized strategy based on the balanced threat level (we call it <u>strategy T</u>) is slightly better than the isolated strategy. Somehow it still lacks the feature to do the defense preferentially when the total attack is more than the total available defense resource,i.e., there is a need to give up certain sites so that other sites may have more resources.

We would like to define a decentralized strategy with which the decision node will be able to carry out assignment of priorities to the nodes within its information domain (and support domain). The priority order in each domain should be determined based on the total available resource in the domain, and on the attack pattern the entire domain is facing. Hopefully, the mechanism to generate these priority orders should guarantee the consistency of these orders between any two overlapping domains, at least approximately. The results of the priority order should not be known to the opponent, and the opponent should be prevented from being able to generate such information himself. Therefore, for real implementation, some sort of random coding known only to the defense nodes is necessary to generate such priority orders.

The following is a scheme of decentralized strategy with priority.

<u>Strategy P1</u>:

For each decision node i, collect information from its domain, (Domain i), and do the following:

1. From the attack pattern on its domain, and the total available defense resource within the domain, compute a priority order for the nodes in the

domain according to some rule.

2. Determine a desirable defense pattern for all the nodes in the domain by redistributing the total resources in the domain. The rule will be detailed later. Let the desirable resource for node j as viewed by node i be y(j,i).
3. Compare y(i,i) with the actual resource that node i has. If y(i,i) is smaller, node i does nothing but wait for, hopefully, support from its neighbors.
4. If y(i,i) is larger than what node i actually has, it will send away those extra units of resources to support certain nodes in Domain i. Those to be supported are the ones that have their respective y(j,i) more than their actual assignments. Such support transfers are made proportional to the deficency in resource. Example: Let the actual resource in Domain 1 be (4 5 1 1), and the desirable defense allocation as viewed by node 1 is (1 2 3 5). Node 1 will send 3 units away to nodes 3 and 4, because the latter two have 2 and 4 units in deficiency respectively. One unit will be sent to node 3, and two units to node 4 (1:2=2:4, 1+2=3). (Convert to integer solutions where needed)
5. The priority order in a domain will be tentatively decided in the reverse order of the amount of attack in the domain. Namely, the node which has the minimum amount of attack gets the highest priority, etc.
6. A tentative scheme to decide on the desirable defense pattern given the attack pattern and the priority order is the following: From the total available resource in Domain i, assign as many as available and to the extent of the amount of the attacks, to the sites according to the priority order. Example: There are totally 14 units of defense resource in Domain 1. The attack pattern is (1 2 3 9 11). Then the desirable defense pattern is (1 2 3 5 0). When the total amount of resources is more than the total attack, allocate proportionally according to the attacks.

Another decentralized strategy with a certain feature of priority is the following:

Strategy PO

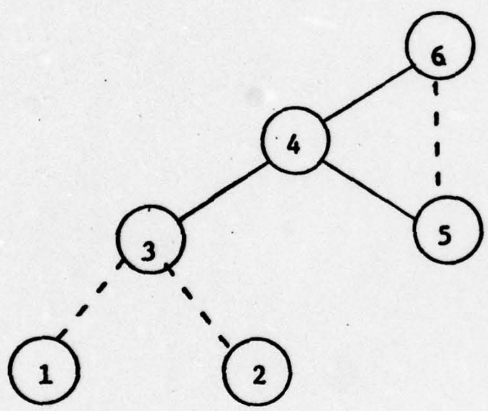
Similar to strategy P1, except the determination of priority and the desirable defense pattern in each domain is replaced by a "centralized optimization" within that domain (replace 6 above).

Strategy PO is more complicated, but presumably performs better compared to Strategy P1.

Further details and simulation results are presented in the next sections.

7. Application of the decentralized strategies with preferential allocation.

Strategy P1 described in Section 6 is applied to Example 2 from Section 5. To illustrate this strategy, consider site 4 with scenario 1. This site is connected to 3, 5 and 6, as illustrated below



The defense available in this 4th domain is (3 5 1 1), which is a total of 10, while the attack is (3 5 3 4). The priorities in domain 4 are, in the order of increasing attack, as follows: site 3, site 5, site 6, site 4. Thus site 4 reallocates the total of 10 defense resources according to this priority order: site 3 gets 3 units, site 5 gets 3 units, site 6 gets 4 units (which total already to 10) and site 4 gets nothing. The desirable allocation pattern in domain 4 is then, in view of what DM_4 knows, (3 0 3 4), compared to the initial allocation of (3 5 1 1). Thus site 4 will deliver 2 units to site 5 and 3 units to site 6.

All the DM's carry out a similar assessment as above. The results are summarized below in Tables 7.1 - 7.3.

for site / as viewed by DM	1	2	3	4	5	6	Action by DM
1	4		3				None
2		2	3				None
3	4	2	3	5			None
4			3	0	3	4	send 2 to #5 and 3 to #6.
5				0	3	4	(Expects help)
6				0	3	4	(Expects help)
Attack	4	2	3	5	3	4	
Initial allocation	4	2	3	5	1	1	
Final allocation	4	2	3	0	3	4	

Table 7.1 Desirable allocations as viewed by the DMs with strategy P1 and the resulting allocations for Scenario 1.

The fourth row in Table 7.1 is the one discussed in detail earlier. Note the discrepancy between the desirable allocations for site 4 as viewed by the various DMs - this is a consequence of the decentralized information pattern. The final allocation is thus (4 2 3 0 3 4) for which the performance is $J = 0.686$.

Table 7.2 shows the desirable allocations and the corresponding actions by the DMs for scenario 2. The performance in this case is $J = 0.550$.

for site / as viewed by DM	1	2	3	4	5	6	Action by DM
1	2		5				Send 2 to #3
2		2	3				None
3	2	2	5	5			(Expects help)
4			1	5	2	2	None
5				3	2	2	(Expects help)
6				3	2	2	(Expects help)
Attack	2	2	8	5	2	2	
Initial allocation	4	2	3	5	1	1	
Final allocation	2	2	5	5	1	1	

Table 7.2 Desirable allocations as viewed by the DMs with strategy P1 and the resulting allocations for Scenario 2.

The results for Scenario 3 are shown in Table 7.3. The performance for this scenario is $J = 0.636$.

for site / as viewed by DM	1	2	3	4	5	6	Action by DM
1	2		5				Send 2 to #3
2		2	3				None
3	2	2	5	5			(Expects help)
4			5	1	2	2	Send 2 to #3; 1 to #5; 1 to #6
5				3	2	2	(Expects help)
6				3	2	2	(Expects help)
Attack	2	2	5	8	2	2	
Initial allocation	4	2	3	5	1	1	
Final allocation	2	2	7	1	2	2	

Table 7.3 Desirable allocations as viewed by the DMs with strategy P1 and the resulting allocations for Scenario 3.

The average performance for this example when using Strategy P1 is $J = 0.624$.

The second, more sophisticated decentralized strategy P0, also described earlier, is considered next. Instead of prioritizing the sites within a domain, an integer programming type optimization is carried out by each DM for allocation within its domain.

The result for Scenario 1 was the same as for strategy P1, which is shown in Table 7.1, and will not be repeated. For the other two scenarios the allocations were different and the performance better.

The performances were for scenarios 2 and 3, respectively, $J = 0.591$ (vs. 0.550) and $J = 0.750$ (vs. 0.636). The detailed results are presented in Tables 7.4 and 7.5.

for site / as viewed by DM	1	2	3	4	5	6	Action by DM
1	6		1				(Expects help)
2		5	0				(Expects help)
3	3	3	0	8			Send 3 to #4
4			0	6	2	2	(Expects help)
5				0	4	3	(Expects help)
6				0	3	4	(Expects help)
Attack	2	2	8	5	2	2	
Initial allocation	4	2	3	5	1	1	
Final allocation	4	2	0	8	1	1	

Table 7.4 Desirable allocations as viewed by the DMs with strategy P0 and the resulting allocations for Scenario 2.

for site / as viewed by DM	1	2	3	4	5	6	Action by DM
1	3		4				Send 1 to #3
2		3	2				(Expects help)
3	3	3	8	0			(Expects help)
4			6	0	2	2	Send 3 to #3; 1 to #5; 1 to #6
5				0	4	3	(Expects help)
6				0	3	4	(Expects help)
Attack	2	2	5	8	2	2	
Initial allocation	4	2	3	5	1	1	
Final allocation	3	2	7	0	2	2	

Table 7.5 Desirable allocations as veiwed by the DMs with strategy PO and the resulting allocations for Scenario 2.

A comparison of the average performances for the strategies considered is presented in Table 7.6

Strategy	Expected Performance
Completely decentralized	0.534
Strategy T (transfer factor)	0.580
Strategy P1 (Preferential with priorities)	0.624
Strategy PO (Preferential with local optimization)	0.676
Fully centralized	0.731

Table 7.6 Comparison of the various strategies.

The local optimization approach taken by strategy PO appears to pay off. The result is quite close to the upper bound given by the centralized strategy.

8. Resource Allocation in a Dynamic Environment

The problem of extending the preferential strategies to the multistage case is discussed next.

The network consists of N nodes connected according to a (symmetric) connectivity matrix.

$$\Omega = [\omega_{ij}] \tag{8.1}$$

with binary elements. The domain of node i (the set of nodes connected to it) is

$$C_i = \{j : \omega_{ij} = 1\} \tag{8.2}$$

The initial defense distributions known to DM_i are d_j, $j \epsilon C_i$.

The network is subjected to K waves of attack. In the k-th wave, the i-th node is attacked by $\alpha_{k,i}$ RVs. DM_i knows at time 1 the variables $\alpha_{1,j}$, $j \epsilon C_i$ as well as the probability distribution of the future attack against its domain.

Based on this information DM_i makes the following decisions:

(i) Decompose d_i into d_i' and $d_i'' = d_i - d_i'$; d_i' to be used against the current wave and d_i'' against future attacks. The neighbors are then notified of the value of d_i' so the next step can be carried out

(ii) Find the allocation of d_i' into d_{ij} , $j \epsilon C_i$ according to a strategy discussed in the previous section.

This process is repeated after every wave. When a site is destroyed, the defense resources associated with it are also assumed to be gone.

9. A Two-wave Problem

The survivability function for a site attacked simultaneously by α RVs and defended by δ units is $S(\alpha,\delta)$. For a two-wave attack this becomes

$$S^{(2)} = S(\alpha',\delta') \; S(\alpha'',\delta'') \tag{9.1}$$

To maximize the above, subject to $\delta' \geq 0$, $\delta'' \geq 0$ and

$$\delta' + \delta'' = \delta \tag{9.2}$$

one has the conditions

$$\frac{\partial S(\alpha',\delta')}{\partial \delta'} S(\alpha',\delta') = \frac{\partial S(\alpha'',\delta'')}{\partial \delta''} S(\alpha'',\delta'')$$

$$\text{if} \quad \delta'>0 \quad \delta''>0 \tag{9.3}$$

$$\frac{\partial S(\alpha',\delta')}{\partial \delta'} S(\alpha',\delta') \geq \frac{\partial S(\alpha'',\delta'')}{\partial \delta''} S(\alpha'',\delta'')$$

$$\text{if} \quad \delta'>0 \quad \delta''=0 \tag{9.4}$$

$$\frac{\partial S(\alpha',\delta')}{\partial \delta'} S(\alpha',\delta') \leq \frac{\partial S(\alpha'',\delta'')}{\partial \delta''} S(\alpha'',\delta'')$$

$$\text{if} \quad \delta'=0 \quad \delta''>0 \tag{9.5}$$

However, in practice the future attack is not known perfectly. The best information one can have about it is a probabilistic description of its size, i.e.,

$$P\{\alpha'' = a_\ell\} = p_\ell \qquad , \ell = 1,\ldots,L \tag{9.6}$$

where L is the number of possible outcomes of the size of the future (second wave in this case) attacks.

With this and the total probability theorem, (9.1) becomes, for an uncertain second wave

$$S^{(2)} = S(\alpha',\delta') \sum_{\ell=1}^{L} S(a_\ell,\delta'') \, p_\ell \tag{9.7}$$

Note that now the optimization problem for this "present-future" allocation is slightly more complicated; nevertheless a numerical search can be employed to maximize (9.7). The algorithm is presented in the next section together with an example.

10. The Dynamic Allocation Algorithm and an Example.

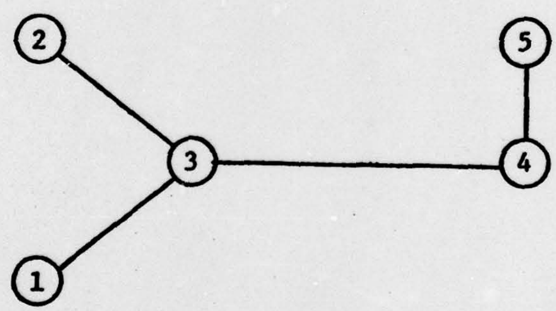
It is assumed that all the DMs know the probability distribution of the equivalent second wave attack ("future attack") against the sites within their domain. Thus DM_i knows

$$P\{\alpha_j'' = a_{j\ell}\} = p_\ell \qquad \forall j \varepsilon C_i \tag{10.1}$$

where $a_{j\ell}$, $\ell=1,\ldots,L$ in the possible future attack against site j within its domain, C_i.

The algorithm is illustrated next on an example.

There are 5 sites connected as follows:



The available defense weapons are

$$\underline{\delta} = (\,2 \;\; 2 \;\; 4 \;\; 3 \;\; 1\,) \tag{10.2}$$

while the first wave of attack is

$$\underline{\alpha}' = (\,1 \;\; 2 \;\; 2 \;\; 4 \;\; 1\,) \tag{10.3}$$

The information about the second wave is

$$P\ \{\underline{\alpha}'' = (1\ \ 1\ \ 1\ \ 2\ \ 1)\} = \frac{1}{3}$$

$$P\ \{\underline{\alpha}'' = (1\ \ 1\ \ 2\ \ 1\ \ 1)\} = \frac{1}{3} \tag{10.4}$$

$$P\ \{\underline{\alpha}'' = (1\ \ 2\ \ 1\ \ 1\ \ 1)\} = \frac{1}{3}$$

The parameters entering into the survivability function are $\rho_\alpha = 0.6$ and $\rho_\alpha = 0.9$, as before.

Each decision maker uses in this algorithm the following variables. The average first wave attack in the domain C_i of DM_i is defined as

$$\overline{\alpha}'_i = \frac{1}{n_i} \sum_{j \in C_i} \alpha'_j \tag{10.5}$$

where n_i is the number of members of the domain of DM_i,

$$n_i = \sum_{j \in C_i} \omega_{ij} \tag{10.6}$$

and ω_{ij} are the (binary) elements of the network's connection matrix.

The average future attack $\overline{\alpha}''_i$ against the domain C_i of DM_i is a random variable that can take the values

$$\overline{a}_{i\ell} = \frac{1}{n_i} \sum_{j \in C_i} a_{j\ell} \tag{10.7}$$

with probability p_ℓ. This follows from the joint distribution of $\alpha''_{i\ell}$, which is illustrated in (10.4). It is assumed that all DMs have the same probabilistic information about the future. However, the algorithm is not affected at all if each DM uses a different distribution.

Table 10.1 summarizes the information available to each DM at the time of the first wave.

Decision Maker i	Total Attack 1st wave against domain i	Average 1st wave attack $\overline{\alpha}_i'$	Average second wave attack against domain i and its probability $a_{i,\ell}$; p_ℓ		
1	3	1.5	1; $\frac{1}{3}$	1.5; $\frac{1}{3}$	1; $\frac{1}{3}$
2	4	2	1; $\frac{1}{3}$	1.5; $\frac{1}{3}$	1.5; $\frac{1}{3}$
3	9	2.25	1.25; $\frac{1}{3}$	1.25; $\frac{1}{3}$	1.25; $\frac{1}{3}$
4	7	2.33	1.33; $\frac{1}{3}$	1.33; $\frac{1}{3}$	1; $\frac{1}{3}$
5	5	2.5	1.5; $\frac{1}{3}$	1; $\frac{1}{3}$	1; $\frac{1}{3}$

Table 10.1 Information available to DMs in the dynamic example

The optimization (9.7) is carried out for the resources δ_i using $\overline{\alpha}_i'$ defined in (10.7). Both are illustrated for the example in Table 10.1. The integer solutions resulting from optimizing over the present/future allocation (9.7) by each DM_i as specified above are presented in Table 10.2.

DM i	Available resources δ_i	Allocation against first wave δ_i'	Allocation against second wave δ_i''
1	2	1	1
2	2	2	0 *
3	4	3	1
4	3	2	1
5	1	1	0 **

Table 10.2 Present/future allocations by individual DMs using domain information

* The decomposition is not unique: (1,1) yields same expected performance
** The decomposition (0,1) yields same performance

The allocations within each domain against the first wave as viewed by the various DMs are presented in Table 10.3. They happen to be the same for each of the following strategies: T (transfer), P1 (preferential defense with priorities), PO (preferential defense with optimization and CC (completely centralized). The corresponding performance (average survivability) after first wave is S(1)= 0.796.

for site / As viewed by DM	1	2	3	4	5	Action by DM
1	1		3			none
2		2	3			none
3	1	2	2	3		send 1 to #4
4			2	3	1	(expects help)
5				2	1	none
Attack	1	2	2	4	1	
Initial allocation against first wave	1	2	3	2	1	
Final allocation	1	2	2	3	1	

Table 10.3 Defense against first wave (strategies T, P1, PO, C).

The local (L) strategy which effects no transfers of resources yields performance $S_L(1)= 0.773$ after the first wave.

For the second wave the analysis is done as follows: The effective remaining weapons after the first wave is

$$\underline{\hat{\delta}}'' = S(1)\ \underline{\delta}'' \qquad (10.8)$$

This yields the following (noninteger) values of resources for strategies T, P1, P0, and CC:

$$\underline{\hat{\delta}}'' = (.796 \quad 0 \quad .796 \quad .796 \quad 0) \tag{10.9}$$

and for strategy L

$$\underline{\hat{\delta}}'' = (.773 \quad 0 \quad .773 \quad .773 \quad 0) \tag{10.10}$$

At this point advantage is taken of the fact that the generalized performance index can be used also for noninteger values of resources to obtain the <u>expected performance for the second wave</u> and then the <u>expected overall performance</u>. An alternative method would be to look at all the possible outcomes of the first wave; since this is quite time consuming the simplified approach presented above is considered preferred. The following realization of the (random) second wave is considered

$$\underline{\alpha}'' = (\ 2 \quad 0 \quad 3 \quad 0 \quad 1\ ) \tag{10.11}$$

The local strategy with resources (10.10) against attack (10.11) yields second wave survivability $S_L(2) = 0.567$ which corresponds to an overall performance

$$S_L^{(2)} = S_L(1)\ S_L(2) = 0.773 \cdot 0.567 = 0.438 \tag{10.12}$$

The T strategy has resources (10.9) against attack (10.11) and its optimal transfer factor turns out to be zero. Its second wave performance is $S_T(2) = 0.646$ and the overall performance

$$S_T^{(2)} = S_T(1)\ S_T(2) = 0.796 \cdot 0.646 = 0.514 \tag{10.13}$$

The CC strategy has the following (anticipated) optimal second wave allocation strategy

$$\underline{\delta}''_{CC} = (1.3877 \quad 0 \quad 0 \quad 0 \quad 1) \tag{10.14}$$

which yields second wave performance of $S_{CC}^{(2)} = 0.706$ and overall performance

$$S_{CC}^{(2)} = S_{CC}(1)\ S_{CC}(2) = 0.796 \cdot 0.706 = 0.562 \qquad (10.15)$$

The (anticipated) actions undertaken by stragegy P1 against the second wave are shown in Table 10.4.

As viewed by DM \ for site	1	2	3	4	5	Action by DM
1	1.592		0			(expects help)
2		0	.796			none
3	2	0	.388	0		send .408 to #1
4			.592	0	1	send .796 to #5
5				0	.796	(expects help)
Attack	2	0	3	0	1	
Initial allocation against second wave	.796	0	.796	.796	0	
Final allocation against second wave	1.204	0	.388	0	.796	

Table 10.4 The decisions of the P1 strategy in the second wave.

The second wave performance is $S_{P1}^{(2)} = 0.665$ and the overall performance

$$S_{P1}^{(2)} = S_{P1}(1).\ S_{P1}(2) = 0.665 \cdot 0.796 = 0.529 \qquad (10.16)$$

The P0 stragety's (anticipated) decisions are shown in Table 10.5.

As viewed by DM \ For site	1	2	3	4	5	Action by DM
1	1.592		0			(expects help)
2		0	.796			none
3	2.388	0	0	0		send .796 to #1
4			.301	0	1.291	send .796 to #5
5				0	.796	(expects help)
Attack	2	0	3	0	1	
Initial allocation against second wave	.796	0	.796	.796	0	
Final allocation against second wave	1.592	0	0	0	.796	

Table 10.5 The decisions of the PO strategy in the second wave.

The second wave performance is $S_{PO}(2) = 0.695$ and the overall performance

$$S_{PO}^{(2)} = S_{PO}(1)\ S_{PO}(2) = 0.796 \cdot 0.695 = 0.553 \qquad (10.17)$$

A summary of the overall performances of the five strategies for the dynamic problem is given in Table 10.6.

Strategy	Performance for two-wave problem $S^{(2)}$
L	0.438
T	0.514
P1	0.529
PO	0.553
CC	0.562

Table 10.6 Summary of results for the dynamic problem simulated.

11. Conclusion and Future Work

A set of 5 strategies have been studied for the static problem of allocating defense resources against a single-wave attack. It has been shown that a decentralized preferential strategy with local optimization can yield performance nearly as good as the fully centralized strategy.

The feasibility of implementing decentralized resource allocation strategies for a dynamic problem has also been demonstrated. The performance achieved with the decentralized strategy that uses an optimization method for reallocation of the resources within its domain has been shown to be very close to the performance of the fully centralized strategy.

In this study we have assumed a certain information pattern, which dictates the communication requirements and the available data base for each decision maker, and developed the methodology to evaluate the performance of the overall system. The next natural step following this study is the following: optimize the connections in the computer network (i.e. among the decision makers) to achieve maximum performance subject to a total cost for the connections. This can make the transition from the evaluation to the design stage of a dedicated computer network whose task is to control a BMD environment. As a by-product of such a follow-on study one can obtain the relationship between overall cost and the optimized performance.

Acknowledgement

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