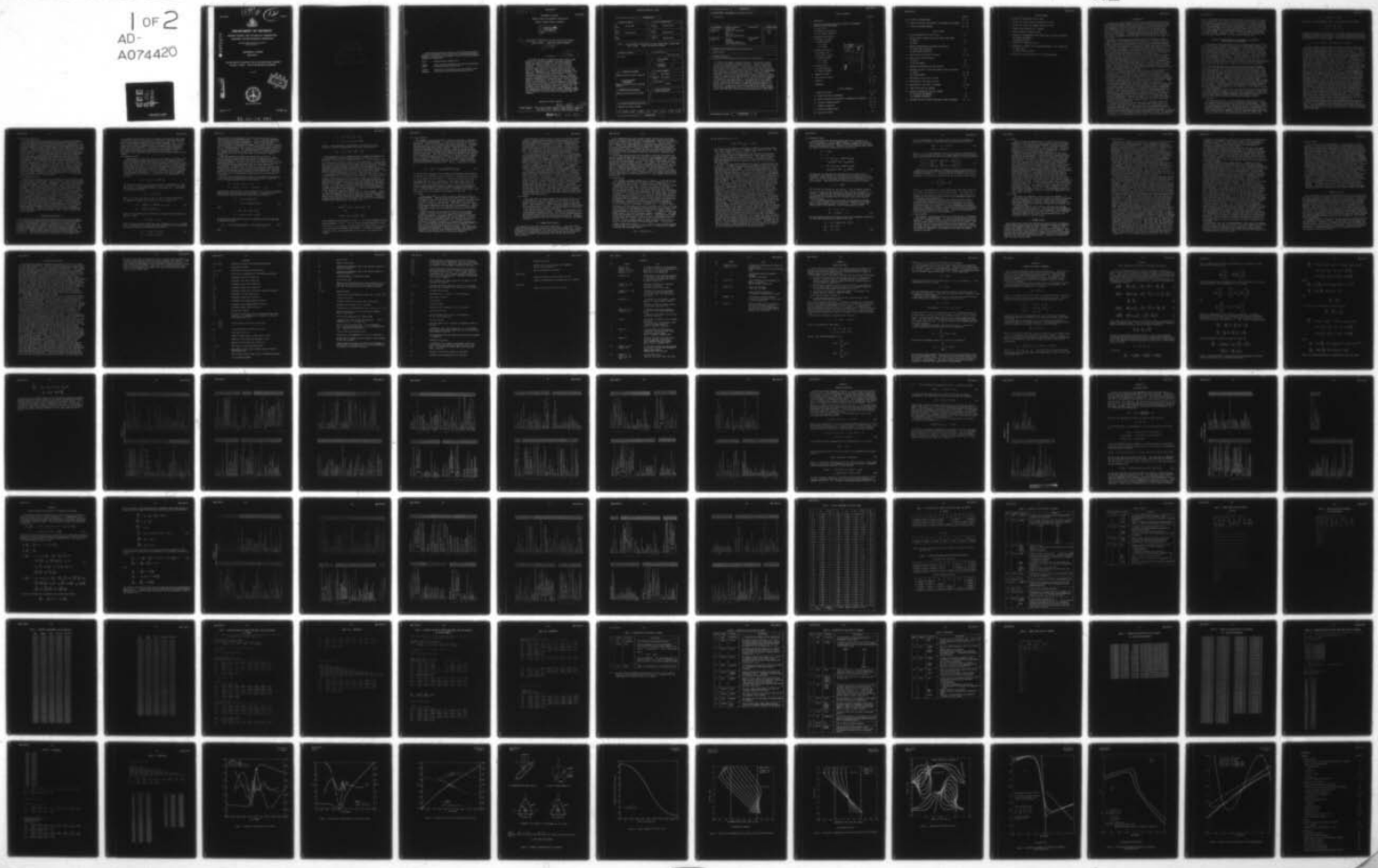
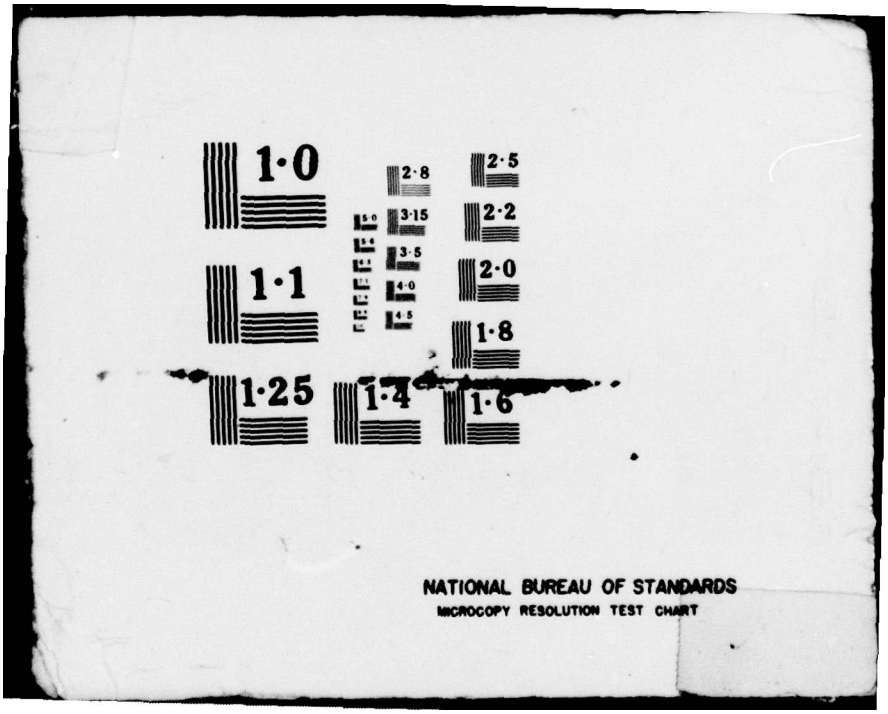
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TECHNICAL REPORT

WSRL-0039-TR

THE ANALYSIS OF TRAJECTORY AND SOLAR ASPECT ANGLE RECORDS OF SHELL FLIGHTS. THEORY AND COMPUTER PROGRAMS.

R.L. POPE

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The theory and computer programs described in this report form part of a research programme in nonlinear dynamics. Methods and computer programs for obtaining aerodynamic coefficients from free flight trials of shells have been developed. The data from the trials consists of both trajectory data from either radar tracking of the shell or ballistic camera records of a flashing light carried in the shell, and complementary solar aspect angle measurements from yawsondes. The data analysis techniques will be used to investigate the exterior ballistics of shells. As well as a tool for investigating dynamic stability problems concerning standard or modified shells they offer a new and more efficient approach to developing a Fire Control Model. An interesting facet of the data analysis is the representation of the aerodynamic force coefficients as analytic functions of Mach number, an approach which has potential for greater accuracy than the piecewise linear representation which is in general use. To facilitate use of the programs by the reader, program listings are supplemented by tables of input variables and sample output. Results from test runs using artificially generated data are discussed.

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1. INTRODUCTION

In recent years the development of new flight instrumentation for shells and new sophisticated methods of analysing free flight data has resulted in a substantial increase in the possible applications of artillery shell flight trials. The developments which have resulted in important gains in the study of the flight behaviour of shells include the invention of the solar aspect angle sensor or yawsonde(ref.1) and the many improvements to it, some of which are described in reference 2. Other developments of note include the modified point mass or four degree of freedom shell trajectory model(ref.3), and the application of parameter estimation methods to the analysis of free flight data (ref.4,5,6).

One potential application of parameter estimation techniques is in the preparation of a Fire Control Model for a shell(ref.7). Currently, the model is made as accurate as possible using wind tunnel data, then modifications to produce a final version of the model are made, using scaling factors obtained by calibrating the four degree of freedom model against impact point data from test firings. The method is costly because the number of firings is very large and the calibration complex. By using parameter estimation techniques to analyse trajectory data derived from radar or other projectile tracking systems such as the flashing light methods described in reference 8, it is possible to substantially reduce the cost of developing a Fire Control Model.

Another major area of application for the yawsonde and associated parameter estimation algorithms is investigation of shell stability. There are many occasions in the development and testing of shells when there is a requirement for free flight investigation of dynamic stability. Such a requirement may result from changes to the external appearance of the shell to meet the need for a long range, low drag shell or from the need for terminal guidance by canard control. In addition, there are often unexpected problems with shells which cannot be explained without using yawsondes to gather further information on the free flight behaviour of such "rogue" shells.

The U.K. is currently developing a method which uses radar tracking data to produce the Fire Control Model from which aiming tables for shells are constructed(ref.7). The USA has been using yawsonde and radar data for some time now to study dynamic stability and anomalous flight behaviour of shells. The potential savings of these techniques in all areas of research, development and usage of artillery shells are so large that Defence Research Centre Salisbury has begun a program which will lead to the development of these skills. Such skills can then be applied with substantial benefits in many areas.

The present report describes the analysis methods and computer programs which will be used to process the data from the preliminary trials. The parameter estimation algorithm used presently is a simple minimum variance application (ref.4,5). This could be improved considerably by using maximum likelihood estimators(ref.7) and Bayesian methods. However, the minimum variance estimator should work satisfactorily in preliminary trials. The first major use of these programs will be a study to quantify the advantages of using parameter identification techniques to construct a Fire Control Model from trajectory measurements. Such an assessment will also involve estimating the accuracy of various alternative tracking techniques and the effect of such accuracies on the data analysis.

An analytic representation of the variation of aerodynamic coefficients, particularly axial force coefficient, with Mach number should result in a substantial improvement in the speed and accuracy of the algorithm for processing the trajectory data. The results of an investigation of analytic functions used to represent such variations are discussed in Section 2. The results of testing the programs for analysing yawsonde and radar data are discussed in Section 5. The two principal points of interest are firstly, the advantages of using an analytic representation of Mach number variation compared with the

commonly used method of analysing a section of data restricted so that the Mach number variation over that section is not significant, and secondly, the effect on the parameter estimation process of noise in the data. Section 3 discusses the method for analysing the trajectory data and gives details of the operation of the program SHELTRAJ, which is used to implement it. The factors affecting the design of experiments involving yawsondes are summarised in Section 4, which also describes the method of analysing the yawsonde data, and the operation of the program YAWSONDE. The final section discusses the design of experiments using radar tracking, yawsondes, or both. It also looks at the present state of progress and considers which direction future work should take.

2. ANALYTICAL REPRESENTATION OF THE VARIATION OF AERODYNAMIC COEFFICIENTS WITH MACH NUMBER

In analysing data from free flight trials, the methods generally used involve the assumption that aerodynamic coefficients are not Mach number dependent. In a few cases a linear variation is allowed(ref.9). The variation of aerodynamic force and moment coefficients with Mach number is taken into account in using these methods by restricting the data analysed in any one pass to a section in which the Mach number variation is so small that there will not be a significant variation in any of the aerodynamic coefficients extracted. There are several obvious disadvantages of this system. The major drawback is the difficulty of obtaining information in the transonic region where the axial force in particular is changing rapidly. Thus in the transonic range it is difficult if not impossible to find a sufficient length of trajectory over which drag coefficient can be adequately represented by a constant or a linear variation with Mach number.

The slowly varying nature of the tracking data from which the aerodynamic forces are derived and the roll rate data from which roll damping is obtained, necessitates using as long a section of the trajectory data as possible. The rapid oscillatory nature of the angular data means that the aerodynamic moments can be determined from much shorter sections of the trajectory so that it is not necessary to use a sophisticated representation of the variation of aerodynamic moments with Mach number. Therefore an analytic form for the variation of axial force coefficient with Mach number will be used, and in addition analytic representations will be used for other slowly varying quantities, namely, average normal force and roll damping. Then the data analysis method will make optimum use of the data, and much more accurate estimates of the aerodynamic parameters will be obtained. In particular, the best estimates of axial force will be obtained, and this is most important if the technique is to be useful in setting up a Fire Control Model.

In order to choose an analytic form to describe the variation with Mach number of the three coefficients, that is, the axial force coefficient, the normal force coefficient and the roll damping coefficient, some experiments were conducted in fitting analytic forms to typical aerodynamic coefficients for a shell. Tabulated values which were used in this exercise are shown in Table 1.

2.1 Axial force

The transonic drag rise is such that polynomial representations of the Mach number variation of axial force coefficient are always poor. Criteria for choosing a suitable analytic function can be found in reference 10 and have been repeated in Appendix I, which gives details of the functional form proposed in reference 10. The principal advantage of this function lies with the small number of parameters necessary to provide adequate representation of the axial force coefficient, over the whole Mach number range. The function, which will be called the NR3634 drag function, has the form

$$C_x = (1+s)A(r) + (1-s)B(r)$$

where A(r), B(r) are polynomials and r, s are functions of Mach number,

$$r = (M^2 - K^2) / (M^2 + K^2),$$

$$s = r / [(1 - L^2) r^2 + L^2]^{\frac{1}{2}}.$$

A detailed discussion of the function, its properties and the values of the constants can be found in reference 10. A possible alternative is the Gavre drag function(ref.11), which is slightly more complex than the NR3634 function but otherwise has the same advantages. The Gavre drag function has the form

$$C_x = a + [1 + b(M-c)^2] \tan^{-1}\{d(M-c) / [e + f(M-c)^2]\} .$$

In order to choose the more appropriate analytic form for incorporation into the data analysis a short study of the goodness of fit of each form was carried out for the tabulated axial force coefficient in Table 1. A non-linear least squares method was used to fit the curves to the data. The sum of squares of the differences between the tabulated values and those derived from the analytical representation was minimised, using the subroutine given in reference 13, which implements the method described in reference 12. The results of the study are given in Table 2 and figure 1. The fitting was carried out over the Mach number range from 0.65 to 1.4 since this particular shell will not be subject to Mach numbers outside that range except in exceptional circumstances. From part (a) of Table 2 it is clear that the optimum fit is obtained using linear polynomials in r to represent both supersonic and subsonic parts of the curve. This result is optimum in the sense that it involves using the least number of adjustable parameters and yet results in a very small r.m.s. error for the fit. If more parameters are used the root mean square error of the fit does not decrease substantially. Coincidentally the number of parameters for optimum use of the NR3634 drag function is the same as the number of parameters used with the Gavre drag function, namely, six. However, the root mean square error attained with the NR3634 drag function is less than one fifth of that obtained with the Gavre drag function, so that the choice is quite clear cut. Since all drag functions have basically the same shape, which includes in particular a rapid rise in the transonic region, this preference can be expected to hold generally. Figure 1, which shows the drag variation with Mach number and the residues obtained with the various fitted curves supports this choice. It is particularly noticeable that the Gavre drag function has difficulty coping with the transonic drag rise. The error peaks at M = 0.93 and M = 1.02 show an inability to cope with the high curvatures at these points which are a feature of the transonic drag rise. These results contrast with the residues shown for the NR3634 drag curves, of which one uses a constant and the other uses a linear representation to describe the subsonic part of the curve. Both are linear supersonically.

It was concluded from this study that the NR3634 drag curve using linear functions of r both subsonically and supersonically would represent the axial force curve adequately over the Mach number range used by shells in flight. However provision was made in the parameter estimation program for analysing the radar data to use up to quadratic terms to cope with exceptional cases if they arise.

2.2 Normal force derivative

Since the NR3634 drag function provides such an excellent representation of the axial force and since the normal force coefficient derivative has similar characteristics to the axial force coefficient, in particular a rapid increase in magnitude in the transonic region, then the NR3634 function is an appropriate choice for analytic representation of the normal force coefficient derivative. Details of the results of fitting such a function to the values of normal force coefficient derivative given in Table 1 are presented in figure 2 and Table 3. Again fitting has only been tried for the Mach number range 0.65 to 1.4. It is clear from the table and figure that an NR3634 curve using linear representations both subsonically and supersonically provides quite adequate representation of the normal force while using only six parameters. The deviation of the analytic representation from the tabulated curve will be less than half a per cent at its maximum value and generally very much less. Since the effect of normal force on the shell is principally involved with the drift of the shell, such accuracy is more than sufficient. Therefore an NR3634 curve can provide an adequate representation of the normal force coefficient derivative, using a linear form both subsonically and supersonically. Again provision is made in the parameter estimation program SHELTRAJ to use up to quadratic terms if necessary, but such a requirement is unlikely and it would be difficult, if not impossible, to determine values for the extra parameters.

2.3 Roll damping derivative

Unlike the force coefficient, the roll damping coefficient derivative does not change rapidly in the transonic region, but shows a small steady variation over the whole Mach number range. Consequently, a polynomial representation is perfectly adequate. Table 3(b) and figure 3 show the results of fitting polynomials to the tabulated values in Table 1, using a linear regression technique. Figure 3 shows the magnitude of the residuals obtained, using both linear and quadratic variations and Table 3(b) shows the r.m.s. errors for polynomials up to and including the fourth degree. The fitting was done over a Mach number range from 0 to 2.5 using all the points tabulated in Table 1. The decrease in the sum of squares of the residuals as the degree of the polynomial increases is a useful measure of the improvement in fitting to the data of the higher degree polynomial. It follows that the results in Table 3(b) show that no significant improvement can be achieved with polynomials of degree higher than four and that a quadratic representation is quite adequate, since the residuals of the quadratic form are less than one per cent. Hence provision is made in program SHELTRAJ to use up to third degree polynomial representations of the derivative of the roll damping moment coefficient. However, a quadratic will generally be adequate, and in many cases with a restricted Mach number range, a linear representation will be sufficient.

3. TRAJECTORY DATA ANALYSIS

The trajectory data analysis and the yawsonde data analysis can be viewed as entirely separate and unconnected processes and both can be undertaken together or each can be undertaken in isolation. Analysis of the trajectory data obtained by radar tracking or flashing light measurement will be the basis of any method, which might be developed for producing a Fire Control Model. The mathematical model which is used as a basis for the parameter estimation is the four degree of freedom or modified point mass model proposed by Lieske and Rieter(ref.3). The algorithm is similar in many ways to the methods already in use(refs.7,9). The basic difference is that both of the above methods treat

only a length of the trajectory which is limited so that the Mach number range is sufficiently small for the variation of aerodynamic coefficients with Mach number to be adequately represented by constants or straight lines. On the other hand the method described below will process the whole trajectory in one sweep using the analytic representations of aerodynamic forces and rolling moments described in Section 2. In addition to describing the mathematical model used in the program, this section outlines the parameter estimation algorithm, describes the program SHELTRAJ itself and provides a brief description of the method for using it and the input to and the output from the program.

3.1 Mathematical model

The mathematical model allows for motion with respect to the three translational degrees of freedom and the axial spin or roll degree of freedom, and so is generally called a four degree of freedom trajectory model. The model relies on the fact that for a well behaved shell, an accurate prediction of the shell drift, that is, the lateral displacement of the shell's impact position from that given by a point mass trajectory model, can be made by assuming that the shell flies at the yaw of response. The yaw of response of a shell is the equilibrium value of incidence when transient yawing and pitching moments are damped out. For most shells the yaw of response is close to the horizontal plane and its magnitude can be approximated by the expression

$$\zeta_R = p_s I_x V_1 g_3 / QSdV^2 C_{m\zeta} \qquad (1)$$

The axes system in which the equations of motion are formulated is a range axes system. It has OX downrange along the line of fire, OY to the right and OZ vertically downwards to complete a right handed set. The velocity in the range axes system is

$$\underset{\sim}{U} = \dot{x}\underset{\sim}{i} + \dot{y}\underset{\sim}{j} + \dot{z}\underset{\sim}{k} ,$$

where $\underset{\sim}{i}$, $\underset{\sim}{j}$, $\underset{\sim}{k}$ are unit vectors in the X, Y and Z directions respectively. The equations of motion for the shell at the yaw of repose, are

$$\dot{\underset{\sim}{U}} = -(QSC_x/m)\ \underset{\sim}{v} + (QSC_L/m)\ \underset{\sim}{j} + \underset{\sim}{g} - \underset{\sim}{\Lambda} \qquad (2)$$

$$\dot{p}_s = (QSd/I_x)\ (p_s d/2V)\ C_{\ell_p} ,$$

where $\underset{\sim}{v}$ is a unit vector in the direction of the shell velocity relative to the air, so that

$$\underset{\sim}{V} = |\underset{\sim}{V}|\underset{\sim}{v} = \underset{\sim}{U} - \underset{\sim}{W}$$

where $\underset{\sim}{W}$ is the local wind velocity and $\underset{\sim}{V}$ has components (V_1, V_2, V_3) in range axes. The drag and lift coefficients shown in equation (2) can be expressed in terms of axial force and normal force as follows,

$$C_D = -C_x \cos\zeta - C_Z \sin\zeta , \qquad (3)$$

$$C_L = C_x \sin\zeta - C_Z \cos\zeta .$$

The resolution of the aerodynamic forces of lift and drag into axial and normal force is not generally necessary. In fact, if the sole purpose of the data analysis were the development of a Fire Control Model the use of lift and drag forces would be preferable. However, since the more significant aspect of the present development is a general study of stability and the program is intended for use in the yawsonde data analysis program as well, it is advantageous at this stage to use a formulation which is in terms of axial and normal force. The remaining symbols in the above equations are standard and are defined in the list of notation at the end of the text. The gravity and Coriolis acceleration terms are discussed at length in Appendix II.

Small approximations have been used in equations (1) and (2). In equation (1) the vertical component of the yaw of repose has been ignored and an approximate expression has been used for the magnitude of the horizontal component. The errors arising from both approximations will have maximum values of only one or two per cent of the total yaw of repose and so it is expected that this approximation will not affect the results significantly. In equation (2), the direction of the lift force has been taken along the vector $\underset{\sim}{j}$. This is true at the beginning of the trajectory if there is no wind, but the direction will alter slightly with cross wind present and as the shell drifts slightly from the line of fire. Once again the overall errors arising from this approximation will not generally be significant.

The final part of the mathematical model is the parametric representation which enables the implementation of the parameter estimation algorithm. Firstly, we consider initial values of the variables

$$P_1 = x_0 , \; P_2 = y_0 , \; P_3 = z_0 \qquad (4)$$

$$P_4 = \dot{x}_0 , \; P_5 = \dot{y}_0 , \; P_6 = \dot{z}_0 \text{ and } P_{27} = P_{SO}.$$

The parametric representations of the aerodynamic forces and the roll damping moment include the axial force coefficient which is represented in the following way, using the functional form presented in Section 2,

$$C_x = (1+s)\,(P_9+P_{10}r+P_{11}r^2) + (1-s)\,(P_{12}+P_{13}\,r+P_{14}\,r^2) \qquad (5)$$

where

$$r = (M^2 - P_7^2) \,/\, (M^2 + P_7^2) ,$$

$$s = r \,/\, [\,(1 - P_8^2)\, r^2 + P_8^2\,]^{\frac{1}{2}} .$$

The normal force coefficient derivative is represented using the same type of function as follows,

$$C_{Z_\delta} = (1+f)\,(P_{17}+P_{18}\,g+P_{19}\,g^2) + (1-f)\,(P_{20}+P_{21}\,g+P_{22}\,g^2) \qquad (6)$$

where

$$g = (M^2 - P_{i5}^2) / (M^2 + P_{i5}^2) ,$$

$$f = g / [(1 - P_{i6}^2) g^2 + P_{i6}^2]^{\frac{1}{2}}$$

Finally, a simple polynomial representation is satisfactory for the derivative of the roll damping moment coefficient, having the form

$$C_{\ell_p} = P_{23} + P_{24}M + P_{25}M^2 + P_{26}M^3 \qquad (7)$$

Thus equations (1) to (7) completely define a mathematical model describing a four degree of freedom or modified point mass model of a shell trajectory in terms of the 27 unknown parameters P_i. In most cases it will not be necessary to determine values for all the parameters. Firstly, some or all of the initial conditions may be sufficiently well determined by other means and so they can be kept fixed throughout the data analysis. Secondly, the variation of the aerodynamic coefficients with Mach number may be adequately represented by fewer parameters than are available, particularly when the available data covers only a restricted range of Mach number. Then the less significant parameters can be set to zero and held constant.

The parameter estimation process which is used to assign values to the unknown parameters P_i is a minimum variance technique. The minimum variance estimator aims to find values of the parameters which will minimise the weighted sum of the squares of the residuals, of the measured values of the three position coordinates and the roll rate when compared with the values predicted by the model. The method is described in detail in references 4, 5 and 8. It is basically a modified Newton-Raphson algorithm and as such it needs an estimate of the partial derivatives of each of the measured variables with respect to the parameters. A means of derivation for these partial derivatives is given in Appendix III.

The interaction between position and roll rate data is minimal. Since the only connecting variable is yaw of repose it is very difficult and often impossible to determine simultaneously, parameters affecting aerodynamic forces directly and those affecting rolling moments directly. In order to determine them separately, use can be made of the fact that in determining minimum variance the residuals are weighted, that is,

$$\sigma^2 = (1/4N) \sum_{j=1}^{N} [w_1 (\hat{x}_j - x_j)^2 + w_2(\hat{y}_j - y_j)^2$$

$$+ w_3 (\hat{z}_j - z_j)^2 + w_4 (\hat{p}_j - p_j)^2] .$$

Then aerodynamic force parameters, P_i for i = 1 to 22 and aerodynamic rolling moment parameters, P_i for i = 23 to 27 can be determined separately, using $w_k \neq 0$ for k = 1, 2, 3 and $w_4 = 0$, when forces are required and $w_1 = w_2 = w_3 = 0$, $w_4 \neq 0$ when rolling moments are required. The choice of appropriate values for the w_i is discussed further in Section 3.2.1 on input to the program.

3.2 Program SHELTRAJ

The mathematical model and parameter estimation algorithm are implemented by the computer program SHELTRAJ. A listing of the program is given in Appendix IV. This section discusses input, output and usage of the program. The program consists of a main program and thirteen subroutines. The main program controls the input, output, solution of equations of motion and adjustment of parameter values by iteration. There are sets of routines controlling input/output, setting up and integrating the mathematical model equations by a fourth order Runge Kutta method, and implementing the parameter estimation algorithm. Function routines define the Kronecker delta and the pitching moment derivative at zero incidence, for the shell under consideration. The pitching moment derivative is used in equation (1) to estimate the yaw of repose. The need for this can be avoided by formulating the parameterised mathematical model equations (2) to (7) in terms of drag and lift rather than axial and normal force. Then C_L in equation (2) is given by

$$C_L = C_{L\zeta}\zeta = (p_s I_x V_1 g_3/QSdV^2)(C_{L\zeta}/C_{M\zeta}) \qquad (8)$$

so that the ratio $(C_{L\zeta}/C_{M\zeta})$ can be included in the parametric representation in equation (6) in place of $C_{Z\zeta}$ and the parametric representation in equation (5) can be used for C_D rather than C_X. This course would be preferable if the program were for use in constructing a Fire Control Model, where the separation of C_X, $C_{Z\zeta}$, $C_{M\zeta}$ is not necessary, since shell trajectories are calculated simply using the mathematical model and the estimated parameter values. When yawsonde data, which will provide independent estimates of $C_{M\zeta}$, is available then the approach employed in SHELTRAJ is more useful, since it provides a better description of the behaviour of the shell.

3.2.1 Input

Three logical units, 3, 5 and 8 are used to input all the data to program SHELTRAJ. The input required from unit 5 either as cards or card images is described in Table 4, and two examples of such input decks are shown in Table 5. The variables LENTER and LCASE are used to control a facility for multiple runs. When LCASE is non-zero control returns to the beginning of the card input on run completion. Then by using LENTER, control can be transferred to any one of several points in the input stream and the run repeated with new values for selected variables.

Several facilities are available in choosing values for input variables to provide the user with fine control on the parameter estimation algorithm. The array JORDER, allows the user to set the order of priority of the parameters so that if JORDER(i)=j then the ith parameter in the mathematical model defined above will be placed in the jth position in the reordered set. The most reliable strategy for obtaining convergence is to approach the solution piecemeal. The technique involves allowing only a few parameter values to vary initially and then repeating the run several times with a gradually increasing number of parameters varying. This is achieved by arranging the parameters in order of importance using JORDER, then gradually increasing the value of NPARAM with repeated runs until values have been estimated for all the desired parameters.

During execution, after the first iteration, the measured variable values are compared with the mathematical model outputs and if the differences between the two are greater than four standard deviations for any element of the measurement vector then that measurement is rejected because it is considered to be a rogue. During the first iteration no estimate of the standard deviation is available and so a value for the data rejection level, REJECT, must be supplied by the user. It is generally good policy to choose a large value so that few, if any, points will be rejected on the first iteration.

As we will discover below it is often difficult, if not impossible, to determine parameters defining aerodynamic forces and those defining rolling moments simultaneously. Separating the two problems can be achieved by means of the weighting factors XS. To estimate force parameters we set XS(1), XS(2), XS(3) at non-zero values and XS(4) = 0, while to estimate the rolling moment parameters we set XS (1) = XS (2) = XS (3) = 0 and XS (4) = 1. Tables 5(a) and 5(b) show data decks designed to implement the two types of calculation. In general values of the weighting factors XS(I) are chosen in proportion to the relative accuracies of each component of the measurement vector.

In addition to the input from unit 5, the program uses experimental data from unit 3 and meteorological data from unit 8. The experimental measurements should be in 5E16.8 format and each record should contain one value of each of time, x, y and z coordinates in range axes, and roll rate. Input begins at the first data point with time greater than TO and every ISKIP th point is read until a total of NPTS points is reached. Finally meteorological data is input as described by the control variable IND. There are two types of table for the meteorological data and they use a 3F10.0 format. Each record contains only one table entry consisting of three items and each table may have up to 150 entries. Each entry in the wind table contains altitude above range origin, range wind, which is positive downrange and crosswind, which is positive to the right, while each entry in the pressure temperature table contains altitude, pressure in pascals and temperature in degrees Kelvin.

3.2.2 Output

The output from the program SHELTRAJ consists basically of a record of the input data, successive values of the parameters as they are adjusted at each iteration, together with r.m.s. errors in each parameter. When convergence is achieved the final parameter values and the covariance matrix are printed. The model outputs and the residuals of the measured values can be both printed and plotted if requested by using a non-zero value of NPLOT in the input.

Artificially produced data from a six degree of freedom model of the shell behaviour is shown in Table 6. This data has been used in conjunction with the data decks in Table 5 to test the program extensively. The results of these tests will be discussed in Section 5. Excerpts from the beginning and the end of the outputs for each data deck appear in Table 7.

4. YAWSONDE DATA ANALYSIS

The yawsonde was developed in the early 1960s(ref.1). Since then several significant improvements have been made to the original design(ref.2). The yawsonde measures the complementary solar aspect angle, which is the angle between the normal to the longitudinal axis of the shell and the direction of the sun. Figure 4 shows a schematic representation of the yawsonde and the angles associated with it.

As the yawsonde measures only the angle between the shell axis and the sun direction it provides essentially only a one-dimensional picture of the angular motion of the shell, which is basically two-dimensional. For this reason the use of yawsondes to study flight behaviour of shells was quite limited for many years. However, the recent application of parameter estimation to the problem has greatly increased the potential of yawsondes with regard to analysing the flight behaviour of shells. Provided that there is a significant component of the angular motion in the plane defined by the instantaneous velocity vector and the direction of the sun, parameter estimation can be used to reconstruct the total motion.

The following subsections deal with the factors which need to be taken into account in designing experiments with yawsondes, the mathematical model on which the data analysis is based, and the use of the program YAWSONDE which is designed to carry out that data analysis. An important aspect of the analysis of the yawsonde data is the piecemeal approach. This change is due to the much higher frequencies of those components of the motion measured by the yawsonde, that is, nutation and precession, compared with the components discussed in the previous section. Because of the higher frequencies the required parameters can be determined from much shorter lengths of data. Hence the restriction of data analysis runs to sections of the trajectory over which variations in Mach number are not significant, is no longer a problem so that the piecemeal approach, which severely limits analysis of the trajectory data, is quite adequate for analysing yawsonde data.

4.1 Yawsonde usage

The yawsonde is depicted schematically in figure 4. It consists of two slits containing sun sensors at two points on the circumference of the shell. The longitudinal plane of each slit makes a specified angle with the longitudinal axis of the shell. As the shell rotates, the yawsonde produces a pulse train of the type shown in figure 4, with pulses of different polarity from each sensor. The relationship between the pulse train and the angle between the direction of the sun and the normal to the longitudinal axis of the shell, that is, the complementary solar aspect angle, is derived in Appendix V. Since the angles γ_1, γ_2 and β are nominal values only and cannot be reproduced exactly in the real yawsonde, and since the theoretical relationship derived in Appendix V does not take account of the finite displacement of the slits from the origin of the axes system, the yawsonde must be calibrated. One method of calibration is described in reference 2. Any such method involves a series of measurements of the corresponding values of τ/T and σ_N as they appear in equation (22). In order to smooth measurement errors and obtain an analytic calibration curve which can be used later to process the experimentally measured values of τ/T, we obtain values of the parameters γ_1, γ_2 and β using a nonlinear least squares fitting technique similar to that described in Section 2. A listing of the program YAWCAL, which is used to do the fitting, appears in Appendix VI. The input required by the program is described in Table 8. The program uses a minimisation routine from reference 13 which is based on the method of Fletcher and Powell(ref.12). Figure 5 shows a typical calibration curve for a yawsonde. The main features which are worth noticing are the sensitivity at $\sigma_N = 0$ and the limitations on the measurement range. If we use the commonly chosen nominal values $\gamma_1 = -\gamma_2 = \gamma$ and $\beta = \pi$ for the angles defined in figure 4 where γ is usually chosen near $\pi/4$, the expressions in Appendix V show that the measurement range is

$$-(\pi/2 - \gamma) < \sigma_N < \pi/2 - \gamma$$

and the sensitivity at $\sigma_N = 0$ is

$$d\,\sigma_N/d\,(\tau/T)\,|_{\tau/T=\frac{1}{2}} = \pi/\tan\gamma\ .$$

This means that the sensitivity deteriorates rapidly as γ increases beyond $\pi/4$ so that a useful value of γ is somewhere in the range $\pi/6$ to $\pi/4$. The value in the examples in figures 5, 6 and 7 is $\pi/4$.

The limited range of measurement means that care must be exercised in trials planning to keep the complementary solar aspect angle within the range that the yawsonde can measure accurately. Care should be taken to avoid approaching the limits $\pm(\pi/2-\gamma)$ too closely because the finite physical dimensions of the slits render the calibration curves unreliable near those limits.

In order to use the yawsonde effectively we need to estimate the position of the sun. The method for estimating the azimuth and elevation of the sun in earth axes is given in Appendix VII together with an expression for the complementary solar aspect angle. For trials planning purposes we assume that the azimuth of the shell is the same as the bearing of the line of fire, since the difference between the two is generally quite small. The program ASPECT uses the results quoted in Appendix VII to estimate the complementary solar aspect angles at a range of times and elevation angles of the shell for a given day, place and line of fire. A listing of the program ASPECT may be found in Appendix VIII. Details of the input required by the program are given in Table 9. Two sets of results are given in figure 6 for this type of calculation for a shell fired at the Woomera Range on the 15th of March 1978. The results are given for directions of firing with bearings of 300 degrees and 0 degrees from true north. The range of angle which can be measured with the yawsonde, $-\pi/4$ to $\pi/4$ in this example, is shown by the dashed line. It is clear from these two figures that in order to obtain complete coverage of the trajectory, particularly for high elevations, it is necessary to fire nearly due north or nearly due south early in the day. In order to get a clearer picture of the optimum firing conditions the results have been presented slightly differently in figure 7. This figure shows an example of the range of elevation angles of the shell for which the yawsonde can detect the sun, as a function of time of day and bearing of the line of fire. The elevation angles on the shaded side of the curve are outside the range. It is apparent from the figure that the whole range of elevation angles can only be covered by firing sufficiently close to due north before 0930 or near enough to due south before 1100. Only the morning hours are covered in this figure, but similar conditions prevail in the late afternoon, when the whole range of elevation angles can be covered again by firing sufficiently near due north or due south. If we are prepared to sacrifice most of the downward leg of the trajectory, that is, negative elevation angles, and the firing elevation is not too high, then there is a much wider choice of firing conditions. For example, if a shell is fired at an elevation of 40 degrees and the trajectory is only required down to -10 degrees, then the firing can take place any time between 1100 hours and 1300 hours on a bearing between 110 degrees and 280 degrees. Alternatively, if the line of fire is fixed and some part of the trajectory can be sacrificed, there is generally a period during the day when the firing can be carried out and a satisfactory coverage of the trajectory obtained.

4.2 Mathematical model

In analysing the data from measurements made with a yawsonde we are interested primarily in the two angular degrees of freedom, pitch and yaw, which are neglected in the four degree of freedom mathematical model used in analysing the trajectory data. The four first order equations which define the angular pitching and yawing motion of the shell are

$$\begin{aligned}
\dot{\psi} &= r \sec\theta \ , \\
\dot{\theta} &= q \ , \\
\dot{q} &= pr - (I_x/I)\, p_s r + (QSd/I)\,[C_m \cos\phi \\
&\quad - C_{np}\,(p_s d/2V)\sin\phi + C_{mq}\,(qd/2V)] \ , \\
\dot{r} &= -pq + (I_x/I)\, p_s q + (QSd/I)\,[C_m \sin\phi \\
&\quad + C_{np}\,(p_s d/2V)\cos\phi + C_{mq}\,(rd/2V)] \ . \qquad (9)
\end{aligned}$$

The equations are formulated in an aeroballistic axes system, which is constrained to pitch and yaw with the vehicle and to maintain the GY' axis horizontal. Hence the GX' axis is forward along the horizontal axis of the shell; the GY' axis is horizontal and the GZ' axis is such that GX'Z' is a vertical plane. Then the roll rate of the axes system must be given by

$$p = - r \tan\theta \qquad (10)$$

The roll rate of the shell, p_s, the velocity, V, and the dynamic pressure, Q, can all be obtained from the trajectory data analysis, discussed in Section 3. Apart from the aerodynamic coefficients and the angle ϕ, which are discussed further below, the remaining parameters in equation (9) all represent physical characteristics of the vehicle which can be measured before the trial.

If this mathematical model is to be used in the parameter estimation process, then a parametric representation of the unknown quantities which appear in these equations has to be formulated. Firstly, consider the initial values of the variables; they are represented as follows,

$$\begin{aligned}
\psi_0 &= P_1 \ , \ \theta = P_2 \ , \\
q_0 &= P_3 \text{ and } r_0 = P_4 \ . \qquad (11)
\end{aligned}$$

The other unknown quantities in equation (9) are the aerodynamic coefficients. The following representation has been used for them

$$\begin{aligned}
C_m &= (P_5 + P_{12}M)\zeta + P_6\zeta^3 + P_7\zeta^5 \ , \\
C_{np} &= P_8\zeta + P_9\zeta^3 \ , \\
C_{mq} &= P_{10} + P_{11}\zeta^2 \ . \qquad (12)
\end{aligned}$$

If, ζ is the magnitude of the angle of attack and ϕ is the orientation of the plane of the angle of attack then they are defined by the relations

$$\tan \zeta = (v + w)^{\frac{1}{2}} / |u| , \qquad (13)$$

$$\tan \phi = (-v) / w ,$$

where (u, v, w) are the components of the true air velocity in aeroballistic axes. They are defined in terms of the range components of vehicle velocity which are obtained from analysis of the trajectory data, by the transformation

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \cos\theta \cos\psi & \cos\theta \sin\psi & -\sin\theta \\ -\sin\psi & \cos\psi & 0 \\ \sin\theta \cos\psi & \sin\theta \sin\psi & \cos\theta \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \qquad (14)$$

Equations (9) to (12) comprise a mathematical model of the portion of the behaviour of the shell which affects the yawsonde measurements, in terms of the unknown parameters, P_i. The criterion used to estimate values for the parameters is to minimise the average of the sum of squares of the residuals

$$\sigma^2 = \frac{1}{N} \sum_{j=1}^{N} (\hat{\sigma}_{Nj} - \sigma_{Nj})^2$$

where $\hat{\sigma}_{Nj}$ is the measured value of the complementary solar aspect angle and σ_{Nj} is the value predicted by the mathematical model. The value of σ_{Nj} is estimated by substituting the value of the local time at which the trial occurred and the values of ψ and θ predicted by the mathematical model into the relations given in Appendix VI.

The parameter estimation algorithm is the same as that used in the trajectory analysis and described in detail in references 4, 5, 8 and 9. The algorithm uses partial derivatives of the complementary solar aspect angles with respect to the parameters to obtain adjustments for each parameter value during each iteration. These partial derivatives are obtained by numerical integration of equations which are derived from the mathematical model equations by partial differentiation. Details of the derivation of the partial derivatives are given in Appendix IX.

4.3 Program YAWSONDE

The parameter estimation processing for the yawsonde data is implemented in the computer program YAWSONDE. A listing of the program may be found in Appendix X. The program consists of a main program and eleven subroutines. The main program controls the processing of the data by the routines. The subroutines fall into three categories which serve the functions of input and output, numerical integration of the differential equations to simulate shell behaviour and estimate values of partial derivatives and finally, the calculation of adjustments to the parameter values using the parameter estimation algorithm.

4.3.1 Input

Details of the input needed from unit 5 either as cards or card images are provided in Table 10, and an example of such a deck appears in Table 11. The controls for the parameter estimation process are the same as those used by SHELTRAJ. They are discussed in detail in Section 3.2.1, and therefore will not be discussed here. The meteorological data is also the same as that for SHELTRAJ and a detailed description of this also may be found in Section 3.2.1.

The major differences from the input requirements of the program SHELTRAJ are the specification of sun position and trajectory data and the experimental measurements. There is provision to tabulate variation of the direction cosines of the sun as a function of time along the trajectory. If this is not deemed necessary constant average values can be entered with times corresponding to the beginning and end of the trajectory as has been done in the example in Table 11. The trajectory data which is needed to estimate such things as dynamic pressure, velocity and so on, in the mathematical model equations must be supplied in a separate input dataset on unit 4. The data will usually be obtained from the smoothed trajectory produced by the analysis described in Section 3. The program will read from the dataset a total of NTRAJ points on the trajectory, using only every ISKIP th point in the dataset, where NTRAJ and ISKIP are input from unit 5. The records in the dataset should have 8F10.0 format and each record should contain values of time, range velocity, drift velocity, which is positive to the right, vertical velocity, which is positive downwards, altitude and roll rate. Finally the experimental data is input from a dataset attached as unit 3. The first data point is that for the first time greater than TO. Every ISKIP th point is input until a total of NPTS points is reached. Values for TO, ISKIP and NPTS are all read from the cards described in Table 10. Each experimental record is in 2E16.7 format and contains a value of time and the complementary solar aspect angle in radians.

4.3.2 Output

The output from YAWSONDE is similar to that from SHELTRAJ. It consists principally of a record of the input data, values of parameters and their estimated errors after each iteration and the covariance matrix when convergence is achieved. Model outputs and residuals of the measured values can be both printed and plotted by assigning a non-zero value to NPLOT.

Artificially produced data relating to trajectory and yawsonde measurements is given in Table 12. This has been used in conjunction with the card deck in Table 11 to test the program. The beginning and end of the output from this test run is given in Table 13. The more significant aspects of the results from the program testing are discussed in detail in the next section.

5. PROGRAM TESTING

Both programs were tested using simulated data obtained from a six degree of freedom trajectory model which used linear interpolation amongst the tabulated values of the aerodynamic coefficient given in Table 1. Listings of the simulated data appear in Tables 6 and 12. Test cases were run both with and without Gaussian white noise superimposed on the data for each program. Tests were also run with the trajectory data to compare the efficiency of the program SHELTRAJ with a method which allowed only a linear variation of drag with Mach number and kept all other coefficients constant. Details of the results of these test runs are given below.

5.1 Program SHELTRAJ

The testing of SHELTRAJ fell naturally into two areas, one involving the aerodynamic forces and the other involving the aerodynamic moments, specifically the roll damping moment. The determination of aerodynamic forces will be discussed first. It proved impossible to determine all the parameters defining the forces even using the clean data. However, only the subsonic part of the curve for the normal force derivative was troublesome. The initial conditions, the drag force coefficient and the supersonic part of the normal force derivative, a total of 16 parameters in all, were well determined. These parameters were the first 16 defined by the data deck in Table 5(a). The whole run occupied some 10 minutes of CPU time on the IBM 370/168 computer. However, it should be remembered that the whole trajectory was processed in that one run, whereas other methods would require at least 25 runs to process the same data.

The same test case was rerun with Gaussian white noise superimposed on each component of the data. The noise had a standard deviation of 5 m. The information extracted from this case was very limited. Only the axial force was obtained with any reliability at all, and the error in that approached 10 per cent in many places. This means that the r.m.s. noise level in each position coordinate would need to be much lower than 5 m if the application of parameter estimation methods to the analysis of trajectory data were to be used in constructing a Fire Control Model.

In order to gauge the improvement in the data analysis which was achieved by using analytic functions of Mach number to represent aerodynamic forces, tests were run allowing only linear variation of axial force and keeping the normal force derivative constant, an approach similar to that used in reference 9. The results of the testing of capabilities of the two methods are shown in figure 8. Figure 8(a) shows the results for the axial force, comparing results obtained from the parameter estimation methods, using both noisy and clean data, with the curve used to generate the data from the six degree of freedom model. From analysis of uncontaminated data, the method proposed in Section 2 produced an analytic axial force curve which followed the original except at very low Mach numbers where there was no data because the section of trajectory which was analysed only covered Mach numbers from 1.37 down to 0.57. When the method was applied to noisy data the result contained errors as high as 8 per cent but generally followed the original fairly well. It should be noted that the noise model used represents an extremely noisy data collection method and modern tracking radars would produce a very much better signal. The tests with such poor data were aimed at determining the advantages of analytic representations of Mach number, and also the general capabilities of the method when radar data is poor. This aspect of the problem is discussed further in the next section.

By comparing the results from the present approach with those of the method of reference 9 one can elicit very definite conclusions. As far as the axial force is concerned the method of reference 9 produced reasonable results when the data was uncontaminated, although it was unable to represent the high curvatures near Mach numbers of 0.9 and 1.1. However, the results using noisy data were quite hopeless, except around Mach 0.7 where the very large number of data points in a small Mach number range was sufficient to compensate for the effects of noise. Figure 8(b) shows the corresponding results for normal force derivative. Both methods gave reasonable results from uncontaminated data but were completely hopeless when noisy data was used. An interesting feature of the results shown in figure 8(b) is that the normal force derivative was consistently underestimated. The error ranges from two to five per cent and arises because data generated by a six degree of freedom model has been fitted by an approximate four degree of freedom model. It is not apparent whether the principal source of the

discrepancy is the approximate form used for the four degree of freedom model, or whether the discrepancy is mainly due to the use of the four degree of freedom model itself. This aspect of the problem is discussed further in Section 7, in connection with the possible use of program SHELTRAJ in constructing or improving a Fire Control Model.

Finally it should be noted that the average CPU time used in the computer runs implementing the method of reference 9 was 10 s so that if the axial force coefficient can be adequately represented over the range of Mach number covered by the whole trajectory with about thirty straight lines, then one might achieve a fifty percent saving in CPU time compared with the analytic functions method. However, the results for axial force coefficient, derived by the piecemeal approach are less accurate and the method is less robust in the presence of noise. Also, if more runs are needed, the CPU time advantage will rapidly disappear.

Now let us consider the roll damping moments. Experiments with the program rapidly showed that aerodynamic force coefficients and aerodynamic roll damping coefficients could not be satisfactorily determined simultaneously and the method outlined in Section 3.2.1 was used to separate the two processes. Figure 9 shows the results of some test runs to gauge the effect of noisy data on the results obtained for the roll damping moment. The Mach number range covered by the data is from 0.63 to 1.37 and it is apparent from figure 9 that if a cubic expression is used to represent the variation of the roll damping moment coefficient with Mach number there is considerable deviation from the true results near the limits of the range. This deviation increases dramatically as the noise level in the data increases. The examples shown in figure 9 include a quadratic curve obtained by least squares fitting to the original data given in Table 1. This curve, for which coefficients are given in Table 3, is a base against which we can evaluate the other results. Also shown are four curves obtained from applying parameter estimation to artificially generated data from a six degree of freedom model, one obtained using uncontaminated data and the other three using noisy data. The noisy data was obtained from the uncontaminated data by adding Gaussian white noise with r.m.s. levels of 1 and 10 radians respectively. For the data with the higher noise level, results are shown for both cubic and linear variations with Mach number. It is clear from the figure that high noise levels will result in large deviations near the ends of the Mach number range unless the degree of the polynomial is severely restricted. Thus the linear representation results in much lower overall errors when there are high noise levels in the data. It is generally true in all parameter estimation problems, that there exists a critical number of parameters for given noise levels in the data, such that increasing the parameters past that number introduces spurious trends into the results. This applies to most curve fitting problems and is particularly evident here because of the relative simplicity of the problem. Figure 9 shows that the derivation of roll damping moments will be quite accurate despite high noise levels in the data, provided that the degree of polynomial representation is appropriately restricted. The rolling moment is estimated to better than five per cent accuracy when there is a r.m.s. noise level of 10 radians in the data, if a linear representation is used for variation of the moment coefficient with Mach number.

The determination of the roll damping moment is still part of the program SHELTRAJ at this stage, and in this form it takes about 6 minutes of CPU time on an IBM 370/168 to determine one of the cubic curves shown in figure 9. However, it would be a relatively simple undertaking to separate out the portion of the program which deals with rolling motion and this should reduce the computing time to less than a minute.

5.2 Program YAWSONDE

The testing of the program YAWSONDE was rather unrealistic in some respects. The data used, shown in Table 11, was generated from a simple undisturbed trajectory and so represented a very low amplitude oscillation. Hence even the results for uncontaminated data show errors of several per cent in the restoring moment. However, the parameter estimation algorithm converged, even when all twelve parameters were allowed to vary (Table 13). When noise was added to the data with root mean square amplitude about twice that of the oscillation, the algorithm converged with five parameters varying, although the actual results obtained were meaningless. Therefore two important conclusions can be drawn from the testing of program YAWSONDE. Firstly, in designing experiments to obtain aerodynamic moments from yawsonde measurements, care should be taken to produce an oscillation of several degrees amplitude if the moments are to be determined accurately. Secondly the parameter estimation is remarkably robust in this case since it was possible to get convergence with extremely high noise levels in the data.

The program YAWSONDE is restricted to ranges of data where changes in Mach number do not significantly affect aerodynamic coefficients. However, as a result of the high frequencies present in angular oscillations, particularly nutation which can show frequencies around 20 Hz, much shorter ranges of data are needed to determine aerodynamic moment coefficients reliably. As a general rule the amount of data necessary to determine a value for a given parameter is of a length representing about two or three cycles of the component of the motion which is most affected by that parameter. Therefore this restriction will not generally present any problems. In addition, each test run took only about twenty seconds of CPU time. Hence the analysis will be economical of computing time even though many runs will be needed to cover the complete range of Mach number.

6. SUMMARY OF RESULTS

The program SHELTRAJ, which was developed to analyse radar tracking and roll rate data is based on the concept of using analytic functions to represent the Mach number dependence of aerodynamic forces and rolling moments. Testing of the program demonstrated that this approach has potential for providing more accurate estimates of the relevant aerodynamic coefficients than a piecewise approach to the data analysis especially when there is a significant noise component in the data(ref.9). This result is particularly important in the context of using the technique to develop a Fire Control Model because highly accurate estimates of the axial force coefficient of a shell are needed in such a situation. The difficulties with estimating roll damping moments indicate that roll rate data should be analysed independently of the shell position data. This could be accomplished most efficiently using a completely separate computer program and removing altogether the facility for analysing roll rate measurements from SHELTRAJ.

The program YAWSONDE is very similar to other programs for analysing data on the complementary solar aspect angle. This particular version appears to be extremely stable. However, it is important to realise that the program will only provide an accurate estimate of the aerodynamic moments if the data being analysed covers a trajectory segment in which there is a pronounced incidence oscillation. Possible methods for generating the appropriate incidence oscillations are discussed in Section 7.

7. APPLICATION OF RESULTS

Many avenues for further investigation arose during the course of the work which has been reported here. Of these three particular areas are most likely to yield useful results. First let us consider improvements or changes to the programs developed to analyse both trajectory and yawsonde data. It is apparent from the work reported here that two improvements are particularly important. They are the development of a completely separate program to analyse roll rate data, and the formulation of the four degree of freedom model, which is used in the analysis of radar tracking data, in terms of the ratio of lift force derivative to pitching moment derivative. The idea of a separate program to analyse roll rate data was discussed at length in Section 5 where it was indicated that the CPU time used to process roll data could be substantially reduced by this means. Initial trials indicate that the reduction is substantial and that less than ten seconds of CPU time will be needed, rather than several minutes. The use of the ratio of the derivatives of normal force and restoring moment was discussed in detail in Section 3.2 and the key equation (8) given. This formulation of the problem is such that analysis of the trajectory data is completely self contained. Thus a Fire Control Model could be constructed from measurement of several trajectories without the need for wind tunnel data or a large number of range and accuracy firings.

A second avenue which needs further investigation is the connection between the amplitude of angular motions of the shell, accuracy capabilities of the yawsonde, and accuracy requirements for the aerodynamic coefficients which are derived from the measurements. Apart from studies applying the program to artificially manufactured data containing noise of known characteristics, studies will need to be made of methods for exciting angular motion in shells. Application of such methods would need to provide sufficient amplitude of shell yaw to satisfy accuracy requirements for the aerodynamic moment derivatives. Two methods which have been suggested are the introduction of an asymmetric mass distribution(ref.7) and the use of an asymmetrical muzzle brake(ref.18).

Finally, the data analysis method developed in Section 3 could be used extensively in the construction of a Fire Control Model. An investigation of the benefits of introducing parameter estimation techniques into the methodology for constructing a Fire Control Model should elicit many useful results. Several factors would need to be considered, such as the number of firings which would be necessary, the accuracy with which shell position would have to be measured, the possibility of using flashing lights rather than radar to measure position, how accurate the meteorological data would have to be and so on. One aspect of the method of Section 3 which would definitely require further investigation is the use of the four degree of freedom model and the approximations which were used in equations (1) and (2) to make them tractable in the above application. In fact further study of these approximations may be worthwhile in its own right, particularly since they are suspected of causing errors in the estimates of the lift force. The question of the four degree of freedom or modified point mass model is different altogether and it may be a definite advantage of the technique. In general, the Fire Control Model uses a four degree of freedom representation to calculate aiming data, because a more complex model is too difficult to implement on a field computer and too expensive of computing time. The four degree of freedom trajectory model is used in the reverse mode in the Model, from its application in the parameter estimation algorithm. Hence, while parameter estimates, obtained by fitting to several sets of trajectory data, may contain significant errors compared with wind tunnel measurements of the aerodynamic forces represented by them, the overall Model which is derived by this means is likely to be better than one which uses exact wind tunnel data. This follows because the parameter estimation process for construction of the Model would involve fitting the Model to

data from a wide range of trajectories, and so one would expect the Model to be extremely accurate when it is used in reverse to generate trajectories. Using such a method the development of a Fire Control Model would be faster and cheaper than at present, mainly because the number of range and accuracy firings would be substantially reduced. Hence, this is potentially the most useful application of the methods developed in this report, and therefore the most likely to repay further study.

NOTATION

$A(r)$, $B(r)$	Polynomial functions in the NR3634 drag function
a	local speed of sound
a,b,c,d,e,f	coefficients of the Gavre drag function
a_n, b_n	coefficients of polynomials in the NR3634 drag function
C_D	aerodynamic drag force coefficient
C_L	aerodynamic lift force coefficient
C_X	aerodynamic axial force coefficient
$C_{Yp\delta}$	aerodynamic Magnus force derivative
C_Z	aerodynamic normal force coefficient, generally negative
$C_{L\delta}$	aerodynamic lift force derivative
$C_{M\delta}$	aerodynamic normal force derivative
$C\ell_p$	aerodynamic roll damping moment derivative
C_{mq}	aerodynamic pitch damping moment derivative
C_{np}	aerodynamic Magnus moment derivative
d	maximum body diameter
f, g	variables used in NR3634 curve, representing normal force derivative, corresponding to the variables r, s used in representing axial force
$\underset{\sim}{g} = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix}$	acceleration due to gravity in range axes
g_o	acceleration due to gravity at sea level
H	hour angle of the sun
h	height of range origin, 0, above mean sea level
I_x	moment of inertia about the longitudinal axis
I	moment of inertia about the lateral axis
$\underset{\sim}{i}$, $\underset{\sim}{j}$, $\underset{\sim}{k}$	unit vectors in the range axes system
K	Mach number centred on the transonic drag rise for the NR3634 drag curve
L	width of the transonic drag rise for the NR3634 drag curve
M	free stream Mach number

m	mass of shell
N	number of data points
N_a	degree of the polynomial used in the supersonic region of the NR3634 function
N_b	degree of the polynomial used in the subsonic region of the NR3634 function
P_i	unknown parameters in mathematical model
p_s	roll rate of shell
$\begin{bmatrix} p \\ q \\ r \end{bmatrix}$	angular rates describing motion of the aeroballistic axes. Pitch and yaw rates are the same as those for the shell. Roll rate is such that the GY' axis remains horizontal.
$Q = \frac{1}{2}\rho V^2$	dynamic pressure
r	variable used in the NR3634 drag curve, $(M^2 - K^2)/(M^2 + K^2)$.
r_o	radius of the earth
r	distance of the shell from the centre of the earth
S	maximum cross-sectional area of the shell
s	variable used in the NR3634 drag curve, $r/[(1-L^2)\, r^2 + L^2]^{\frac{1}{2}}$
T	period between pulses of like sign from the yawsonde, that is, roll period
t	time from the beginning of the shell flight
$\underset{\sim}{U}$	velocity of the shell relative to range axes. It has components $(\dot{x}, \dot{y}, \dot{z})$ in range axes.
$\underset{\sim}{V}$	true air velocity of the shell. It has components (V_1, V_2, V_3) in range axes and (u, v, w) in aeroballistic axes. Magnitude is denoted by V.
$\underset{\sim}{v}$	a unit vector in the direction of $\underset{\sim}{V}$.
$\underset{\sim}{W}$	wind vector, with components $(w_1, w_2, 0)$ in range axes.
w_i	weights used in forming the sum of squares of the residues of the data variables.
$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$	denotes position in range axes, where OX is downrange, OY is horizontal to the right and OZ is vertically downwards. The origin O is generally at the gun

$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$ denotes position in aeroballistic body axes, where Gx' is forward along the longitudinal axis, Gy' is horizontal to the right, and Gz' completes a right handed axes system. The origin G is at the centre of gravity of the shell.

$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}$ Denotes position in rolling body axes with the origin S on the longitudinal axis of the shell at the location of the centre of the sun slits. Sx" is forward along the longitudinal axis, Sz" is outwards through the centre of sensor 1 in the yawsonde and Sy" completes a right handed set as shown in figure 4.

β the circumferential angle between the two sensors in the yawsonde (see figure 4)

γ_1 , γ_2 the angles which the respective sensors in the yawsonde make with the longitudinal axis of the shell (see figure 4).

δ declination of the sun

δ_{ij} Kronecker delta, = 1 when i = j and 0 otherwise

ζ total angle of attack

ζ_R yaw of repose

Θ latitude of shell

θ elevation of the longitudinal axis of the shell

θ_s elevation of the sun

$\underset{\sim}{\Lambda}$ vector defining Coriolis force, with components (Λ_1 , Λ_2 , Λ_3) in range axes

ρ local air density

σ root mean square error in fitting the parametric model to the data

σ_N complementary solar aspect angle, that is, the complement of the angle between the longitudinal axis of the shell and the direction of the sun.

τ time between positive and negative pulses from the yawsondes (see figure 4)

Φ longitude of the shell.

ϕ orientation of the incidence of the shell relative to a vertical plane containing the longitudinal axis of the body. It is positive clockwise and is zero when the shell is nose up.

Ψ bearing of line of fire relative to true north

ψ azimuth of the longitudinal axis of the shell

ψ_s	azimuth of the sun.
$\underset{\sim}{\Omega}$	angular velocity of the earth, with components $(\Omega_4, \Omega_2, \Omega_3)$ in range axes.
∂	partial differentiation operator.

Superscripts

.	denotes differentiation with respect to time
$\wedge$	denotes an experimentally measured value of a variable

Subscripts

o	denotes an initial value of the variable

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APPENDIX I

NR3634 DRAG CURVE

A brief resume of the properties of the formula, proposed in reference 10, to represent the variation of axial force over the entire Mach number range, is presented here to clarify comments on curve fitting and choice of values for parameters which are made in the main text.

The following criteria were used to construct the drag function defined below,

(1) The function should effect some smoothing when fitted to experimental results. Hence the number of adjustable parameters, while being sufficiently large to provide adequate flexibility should be much less than the number of observations.

(2) The function is empirical, but its behaviour should be consistent with basic characteristics of drag functions, such as asymptotic behaviour at high and low Mach numbers and transonic drag rise.

(3) The analytic form should be simple and compact; transcendental and complex algebraic functions should be avoided.

(4) The function should be analytic.

(5) A simple procedure should be available for obtaining values of the parameters from experimental data.

Each of these conditions constitutes a considerable advantage over conventional methods of coefficient determination which rely on analysing only small portions of data during which the Mach number is almost constant. By repeating the analysis many times on different sections of trajectory such methods produce a table which shows variation of axial force with Mach number. Clearly an analytic representation which satisfies the above five conditions will result in greater accuracy and less computational effort in the analysis of position data for any trial.

The form for the analytic representation of the drag is

$$C_x = (1 + s)A(r) + (1-s)B(r)$$

where r,s are functions of Mach number,

$$r = (M^2 - K^2) / (M^2 + K^2)$$

$$s = r / [(1 - L^2)r^2 + L^2]^{\frac{1}{2}}$$

and A(r), B(r) are both polynomials in r,

$$A(r) = \sum_{n=0}^{N_a} a_n r^n$$

$$B(r) = \sum_{n=0}^{N_b} b_n r^n$$

where a_n, b_n, K, L, N_a and N_b are constants to be determined.

The first property of the axial force coefficient function is explained by the result that K is close to 1 and L is small. Then for subsonic Mach numbers, M = 0 upwards, r begins at -1 and approaches 0, somewhere in the transonic range and s is very close to -1 over the whole subsonic range, only changing appreciably when the transonic range is entered and $r \to 0$. Hence, subsonically

$$C_x \approx 2B(r).$$

Somewhat similar effects occur supersonically, as $M \to \infty$, or for $M > 1+\epsilon$, $s \to 1$ and r progresses from 0 to 1 so that

$$C_x \approx 2A(r).$$

Hence the polynomials A(r) and B(r) represent supersonic and subsonic parts of the drag curve respectively. Relatively few terms are needed; generally, linear or quadratic forms are adequate.

With regard to the constants K and L it is instructive to look at the value at M=K and the slope at that point. When M=K, r=0, s=0, and

$$C_x = a_o + b_o ,$$

$$K(dC_x/dM)|_{M=K} = a_1 + b_1 + (a_o - b_o) / L.$$

Bearing in mind that $2a_o$ is the approximate value of C_x immediately above the transonic drag rise and $2b_o$ is the value immediately below it, then it follows that K is the value of Mach number when the drag is midway between these two levels. If $L \ll 1$, then 2L is the effective interval of Mach number over which the transonic drag rise, $2(a_o - b_o)$ would be attained if the maximum slope were maintained throughout.

Finally, consider the asymptotic behaviour of the curve. Firstly, for subsonic incompressible flow, $M \to 0$, the axial force coefficient,

$$C_x = 2 \sum_{n=0}^{N_b} (-)^n b_n + 0 (M^2)$$

and secondly for hypersonic flow, $M \to \infty$, axial force coefficient,

$$C_x = 2 \sum_{n=0}^{N_a} a_n + 0 (M^{-2}).$$

These are the correct asymptotic forms for the axial force coefficient at both ends of the Mach number range. The properties of the transonic drag rise and the asymptotic behaviour of the curve can be used to make initial estimates for the values of each parameter. The values can then be refined by parameter estimation or nonlinear least squares curve fitting methods.

APPENDIX II

CORIOLIS AND GRAVITY COMPONENTS

High accuracy is required in shell trajectory computation. The effects of Coriolis force and the changes to the gravity vector, occurring along the trajectory, have a significant effect on the trajectory. Therefore these changes must be included in the mathematical model. Both effects are well known and documented, but are included here so that the document is self-contained as far as the description of the mathematical model of the shell trajectory is concerned.

The Coriolis acceleration arises from the rotation of the range axes system, because the range axes system is fixed relative to a rotating earth. The acceleration vector, $\underset{\sim}{\Lambda}$ in equation (2), is given by

$$\underset{\sim}{\Lambda} = 2\,\underset{\sim}{\Omega} \times \underset{\sim}{U}$$

where $\underset{\sim}{U}$ is the velocity of the shell and $\underset{\sim}{\Omega}$ is the angular velocity of the earth, 7.292115×10^{-5} rads/s about an axis from south pole to north pole. Then at latitude Θ, positive south, and line of fire bearing Ψ from true north

$$\begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{bmatrix} = \begin{bmatrix} 2\cos\Theta\cos\Psi \\ 2\cos\Theta\sin\Psi \\ 2\sin\Theta \end{bmatrix} |\,\Omega\,|$$

assuming that the OX axis is along the line of fire, the OZ axis vertically downwards and the OY axis horizontal to the right, thus forming a right handed axes system.

The second part of this appendix looks at the problem of defining the components of gravity in the same axes system. The magnitude of the acceleration due to gravity at a distance r from the centre of the earth is

$$|\underset{\sim}{g}| = g_o\, r_o^2 / r^2$$

where $\underset{\sim}{g}$ is the local acceleration at mean sea level and r_o is the radius of the earth. The direction of the acceleration is radially inwards. Then at a point (x,y,z) in a Cartesian coordinate system with axes as described above and origin at an altitude, h_o, above mean sea level the gravity vector can be represented by

$$\underset{\sim}{g} = -g_o\, r_o^2 \left[x\,\underset{\sim}{i} + y\,\underset{\sim}{j} - (h_o + r_o - z)\,\underset{\sim}{k}\right] / r^3$$

where $r^2 = x^2 + y^2 + (h_o + r_o - z)^2$. This result can be easily derived from the geometry of the situation, using the knowledge that x, y and z are all very small compared with r_o.

APPENDIX III

PARTIAL DERIVATIVES WITH RESPECT TO PARAMETERS FOR SHELTRAJ

In order to implement a parameter estimation algorithm we need to know partial derivatives of the measured variables with respect to the unknown parameters. These partial derivatives are obtained by numerical solution of a set of simultaneous ordinary differential equations obtained by partial differentiation of the mathematical model equations presented in Section 3.1. By separating out the components of velocity and acceleration in equation (2) and differentiating with respect to the parameter, P_i, we obtain the following set of differential equations

$$\frac{d^2}{dt^2}\left(\frac{\partial x}{\partial P_i}\right) = \left(\frac{\rho S}{2m}\right)\left(C_D V_1 \frac{\partial V}{\partial P_i} + C_D \frac{\partial V_1}{\partial P_i} V + \frac{\partial C_D}{\partial P_i} V_1 V\right) + \frac{\partial g_1}{\partial P_i} - \frac{\partial \Lambda_1}{\partial P_i} \tag{15}$$

$$\frac{d^2}{dt^2}\left(\frac{\partial y}{\partial P_i}\right) = \left(\frac{\rho S}{2m}\right)\left(C_D V_2 \frac{\partial V}{\partial P_i} + C_D \frac{\partial V_2}{\partial P_i} V + \frac{\partial C_D}{\partial P_i} V_2 V + 2C_L V \frac{\partial V}{\partial P_i} + \frac{\partial C_L}{\partial P_i} V^2\right)$$

$$+ \frac{\partial g_2}{\partial P_i} - \frac{\partial \Lambda_2}{\partial P_i} \tag{16}$$

$$\frac{d^2}{dt^2}\left(\frac{\partial z}{\partial P_i}\right) = \left(\frac{\rho S}{2m}\right)\left(C_D V_3 \frac{\partial V}{\partial P_i} + C_D \frac{\partial V_3}{\partial P_i} V + \frac{\partial C_D}{\partial P_i} V_3 V\right) + \frac{\partial g_3}{\partial P_i} - \frac{\partial \Lambda_3}{\partial P_i} \tag{17}$$

$$\frac{d}{dt}\left(\frac{\partial p_s}{\partial P_i}\right) = \left(\frac{\rho S d^2}{4I_x}\right)\left(C_{\ell_p} p_s \frac{\partial V}{\partial P_i} + C_{\ell_p} \frac{\partial p_s}{\partial P_i} V + \frac{\partial C_{\ell_p}}{\partial P_i} p_s V\right) \tag{18}$$

if we assume that local air density, ρ and speed of sound a, are insensitive to changes in parameter values. These then are the basic differential equations which can be solved to provide values of the partial derivatives,

$$\frac{\partial x}{\partial P_i}, \frac{\partial y}{\partial P_i}, \frac{\partial z}{\partial P_i} \text{ and } \frac{\partial p_s}{\partial P_i}$$

required by the parameter estimation algorithm. However, these equations include many partial derivatives which are not available from the solution of the equations but must be obtained by partial differentiation of other relationships. The method of calculating these partial derivatives is outlined below. Using the relation

$$V^2 = V_1^2 + V_2^2 + V_3^2 .$$

we find that

$$V \frac{\partial V}{\partial P_i} = V_1 \frac{d}{dt}\left(\frac{\partial x}{\partial P_i}\right) + V_2 \frac{d}{dt}\left(\frac{\partial y}{\partial P_i}\right) + V_3 \frac{d}{dt}\left(\frac{\partial z}{\partial P_i}\right) ,$$

since it follows easily from the relation between velocity relative to range axes and true air velocity that

$$\frac{\partial}{\partial P_i}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \frac{d}{dt}\left\{\frac{\partial}{\partial P_i}\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right\}.$$

Using the expressions given in Appendix II for the accelerations due to Coriolis and gravity effects, the following results can be derived;

$$\frac{\partial}{\partial P_i}\begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{bmatrix} = 2\begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{bmatrix}\frac{d}{dt}\left\{\frac{\partial}{\partial P_i}\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right\}$$

and

$$\frac{\partial}{\partial P_i}\begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} \approx -g\,(r_o/r)^2\,\frac{1}{r}\,\frac{\partial}{\partial P_i}\begin{bmatrix} x \\ y \\ z \end{bmatrix},$$

although the approximation in this equation is negligible. The partial derivatives of the aerodynamic force and moment coefficients can be obtained from equations (1), (3), (5), (6) and (7) which can be combined to define the coefficients in terms of the parameters, P_i. This yields the following results,

$$\frac{\partial C_D}{\partial P_i} = -\frac{\partial Cx}{\partial P_i}\cos\zeta - \frac{\partial Cz}{\partial P_i}\sin\zeta + C_L\frac{\partial \zeta}{\partial P_i}$$

$$\frac{\partial C_L}{\partial P_i} = \frac{\partial Cx}{\partial P_i}\sin\zeta - \frac{\partial Cz}{\partial P_i}\cos\zeta - C_D\frac{\partial \zeta}{\partial P_i}$$

From the expression for the yaw of repose it follows that

$$\frac{\partial \zeta_R}{\partial P_i} = (2I_x/\rho SdC_{m\zeta}V^5)\left\{Vp_s V_1\frac{\partial g_3}{\partial P_i} + Vp_s\frac{\partial V_1}{\partial P_i}g_3 + V\frac{\partial p_s}{\partial P_i}V_1 g_3 - 4\frac{\partial V}{\partial P_i}p_s V_1 g_3\right\}.$$

Finally, using the parametric representations of the aerodynamic coefficients, we obtain the results, for the axial force coefficient,

$$\frac{\partial Cx}{\partial P_i} = (1+s)(\delta_{9i} + \delta_{10i}r + \delta_{11i}r^2) + (1-s)(\delta_{12i} + \delta_{13i}r + \delta_{14i}r^2)$$

$$+ (P_q + P_{10}r + P_{11}r^2 - P_{12} - P_{13}r - P_{14}r^2)\frac{\partial s}{\partial P_i}$$

$$+ [(1+s)(P_{10} + 2P_{11}r) + (1-s)(P_{13} + 2P_{14}r)]\frac{\partial r}{\partial P_i}$$

where

$$\frac{\partial s}{\partial P_i} = [P_8^2 \frac{\partial r}{\partial P_i} + r(r^2-1)\, P_8\, \delta_{8i}]\, [(1-P_8^2)\, r^2 + P_8^2]^{-3/2},$$

$$\frac{\partial r}{\partial P_i} = [2MP_7^2 \frac{\partial M}{\partial P_i} - 2M^2 P_7 \delta_{7i}]\, [M^2 + P_7^2]^{-2}$$

and

$$\frac{\partial M}{\partial P_i} \approx \frac{1}{a}\frac{\partial V}{\partial P_i}$$

Similarly for the normal force coefficient, we obtain

$$\frac{\partial C_z}{\partial P_i} = \frac{\partial C_{z\zeta}}{\partial P_i} + C_{z\zeta}\frac{\partial \zeta}{\partial P_i}$$

$$\frac{\partial C_{z\zeta}}{\partial P_i} = (1+f)(\delta_{17i} + \delta_{18i}g^2) + (1-f)(\delta_{20i} + \delta_{21i}g + \delta_{22i}g^2)$$

$$+ (P_{17} + P_{18}g + P_{19}g^2 - P_{20} - P_{21}g - P_{22}g^2)\frac{\partial f}{\partial P_i}$$

$$+ [(1+f)(P_{18} + 2P_{19}g) + (1-f)(P_{21} + 2P_{22}g)]\frac{\partial g}{\partial P_i}$$

where

$$\frac{\partial f}{\partial P_i} = [P_{16}^2 \frac{\partial g}{\partial P_i} + g(g^2-1)\, P_{16}\delta_{16i}]\, [(1 - P_{16}^2)g^2 + P_{16}^2]^{-3/2}$$

$$\frac{\partial f}{\partial P_i} = [2MP_{15}^2 \frac{\partial M}{\partial P_i} - 2M^2\, P_{15}\, \delta_{15i}]\, [M^2 + P_{15}^2]^{-2}$$

While the rolling moment coefficient representation yields the result,

$$\frac{\partial C_\ell}{\partial P_i}_P = (\delta_{23i} + \delta_{24i} M + \delta_{25i} M^2 + \delta_{26i} M^3)$$

$$+ (P_{24} + 2P_{25}M + 3P_{26}M^2) \frac{\partial M}{\partial P_i} \quad .$$

The equations in this appendix embody all the information necessary for calculating those partial derivatives required by the parameter estimation algorithm to estimate corrections for the initial estimates of the parameter values. If all 27 parameters are allowed to vary, the equations (15) to (18) represent 189 simultaneous, first order, ordinary, differential equations. So, including the equations (2), which define the mathematical model, we find that for each iteration of the parameter estimation algorithm 197 first order equations must be solved.

APPENDIX IV

LISTING OF PROGRAM SHELTRAJ

```
C     PROGRAM - SHELL,TRAJ,PARAM,EST - MAIN PROGRAM (SHELTRAJ)                  00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                                00000020
C                                                                               00000030
C     A PARAMETER ESTIMATION PROGRAM TO ANALYSE TRAJECTORY DATA                 00000040
C     ON SHELLS USING THE MODIFIED POINT MASS MODEL FOR SHELL                   00000050
C     TRAJECTORIES. THE PROGRAM USES PARAMETRIC FORMS OF THE                    00000060
C     CURVES PROPOSED IN NAVORD REPORT 3434 FOR AXIAL FORCE                     00000070
C     COEFFICIENT AND NORMAL FORCE DERIVATIVE VARIATION WITH                    00000080
C     MACH NUMBER. A SIMPLE POLYNOMIAL DESCRIPTION IS USED FOR                  00000090
C     THE ROLL DAMPING DERIVATIVE. THE DATA USED IN THE ANALYSIS                00000100
C     ARE THE THREE COORDINATES DESCRIBING THE SHELL POSITION                   00000110
C     IN RANGE AXES AND THE ROLL RATE MEASURED BY YAWSONDE.                     00000120
C     THE MAIN PROGRAM LINKS ROUTINES AND CONTROLS OUTPUT.                      00000130
C                                                                               00000140
      DIMENSION PARAM(27),ORDER(27)                                             00000150
C                                                                               00000160
      COMMON /MODEL/  AMASS,ROLLI,REFS,RODIAM,                                  00000170
     $                XF(27),X(27),XX(27),JORDER(27)                            00000180
     $       /COMP/   TO,REJECT,ACCFAC,HMAX,NPARAM,ITERN,LCASE,NPLOT           00000190
     $       /DATA/   OBS(5,1000),KS(4),PI(5),NPTS,ITER,IEND                   00000200
     $       /INT/    Y(197),DY(197),H0(197),YA(197),Z(10H),N,IOBS,MAX         00000210
     $       /NORME/  PSI(378),CWD(27),E(27)                                   00000220
C                                                                               00000230
      DATA PARAM /'   X0   ','   YD   ','   ZD   ','  XDOTO ',                 00000240
     $ '  YDOTO ','  ZDOTO ','  CDK   ','  CDL   ','  CDAO  ',                 00000250
     $ '  CDA1  ','  CDA2  ','  CDBO  ','  CDB1  ','  CDB2  ',                 00000260
     $ ' CZALFAK','CZALFAL','CZALFA0','CZALFA1','CZALFA2',                      00000270
     $ 'CZALFB0','CZALFH1','CZALFB2','  CLPO  ','  CLP1  ',                    00000280
     $ '  CLP2  ','  CLP3  ','   HO   '/                                       00000290
C                                                                               00000300
C     PROGRAM NAME AND OUTPUT HEADINGS                                          00000310
      WRITE (6,1)                                                               00000320
C                                                                               00000330
C     INPUT DATA                                                                00000340
  100 CALL INPUT                                                                00000350
      ITER=0                                                                    00000360
      IEND=0                                                                    00000370
C                                                                               00000380
C     PREPARE TO PLOT OUTPUT IF REQUESTED                                       00000390
      IF(NPLOT.NE.0) CALL PLOTT                                                 00000400
C                                                                               00000410
C     RECORD PARAMETERS AND ACCURACY VALUES IN ORDER REQUESTED                  00000420
C                                                                               00000430
      DO 110 I=1,27                                                             00000440
      K=JORDER(I)                                                               00000450
      ORDER(K)=PARAM(I)                                                         00000460
      XX(K)=X(I)                                                                00000470
  110 CWD(K)=XF(I)                                                              00000480
C                                                                               00000490
  120 DO 130 I=1,4                                                              00000500
      IA=8*(I-1)+1                                                              00000510
      IB=8*I                                                                    00000520
      IF(IB.GT.27) IB=27                                                        00000530
      WRITE (6,2) (ORDER(J),J=IA,IB)                                            00000540
      WRITE (6,3) (XX(J),J=IA,IB)                                               00000550
  130 WRITE (6,4) (CWD(J),J=IA,IB)                                              00000560
C                                                                               00000570
C     CHECK WHETHER EACH INTEGRATION STEP WILL BE PRINTED                       00000580
      IF(((IEND.EQ.1).OR.(ITER.GE.ITERN)).AND.(NPLOT.NE.0)) WRITE (6,5)         00000590
C                                                                               00000600
C     INITIALISE VARIABLES FOR INTEGRATION                                      00000610
      CALL INCOND                                                               00000620
C                                                                               00000630
C     CLEAR MATRICES AND COUNTERS                                               00000640
C                                                                               00000650
      ISD=0                                                                     00000660
      J=NPARAM*(NPARAM+1)/2                                                     00000670
      DO 140 I=1,J                                                              00000680
  140 PSI(I)=0.                                                                 00000690
      DO 150 I=1,NPARAM                                                         00000700
  150 CWD(I)=0.                                                                 00000710
      SIGMA=0.                                                                  00000720
C                                                                               00000730
C     DO LOOP TO SET UP NORMAL EQUATIONS                                        00000740
C                                                                               00000750
      DO 210 IOBS=2,NPTS                                                        00000760
C                                                                               00000770
C     ESTIMATE MODEL PREDICTIONS AND PARTIAL DERIVATIVES                        00000780
      CALL SIMUL                                                                00000790
C                                                                               00000800
C     TEST FOR OUTLIERS AND ADD IN CONTRIBUTION TO SQUARE ERROR                 00000810
      DO 160 I=2,5                                                              00000820
      E(I)=(OBS(I,IOBS)-PI(I))*XS(I-1)                                          00000830
      IF(DABS(E(I)).GT.REJECT) GO TO 210                                        00000840
  160 SIGMA=SIGMA+E(I)*E(I)                                                     00000850
      ISD=ISD+4                                                                 00000860
C                                                                               00000870
C     IF LAST PRINT - PRINT RESIDUALS, AVOID NORMAL EQUATIONS AND               00000880
C     PLOT DATA IF REQUIRED                                                     00000890
C                                                                               00000900
      IF(((IEND.NE.1).AND.(ITER.LT.ITERN)).OR.(NPLOT.EQ.0)) GO TO 170           00000910
      WRITE (6,6) (E(I),I=2,5)                                                  00000920
      IF(NPLOT.NE.0) CALL PLOTA                                                 00000930
      GO TO 210                                                                 00000940
C                                                                               00000950
C     ADD CONTRIBUTIONS TO NORMAL EQUATIONS                                     00000960
C                                                                               00000970
  170 DO 200 K=1,4                                                              00000980
      KD=0                                                                      00000990
      LD=0                                                                      00001000
C                                                                               00001010
      DO 190 I=1,NPARAM                                                         00001020
      LD=LD+1                                                                   00001030
C                                                                               00001040
      DO 180 J=1,I                                                              00001050
      KD=KD+1                                                                   00001060
  180 PSI(KD)=PSI(KD)+Z(4*I+K-4)*Z(4*J+K-4)*XS(K)*XS(K)                         00001070
C                                                                               00001080
  190 CWD(LD)=CWD(LD)+Z(4*I+K-4)*E(K+1)*XS(K)                                   00001090
  200 CONTINUE                                                                  00001100
C                                                                               00001110
  210 CONTINUE                                                                  00001120
C                                                                               00001130
C     IF THIS IS THE FINAL PRINT STEP - PLOT AS REQUIRED AND FINISH             00001140
C     THIS STEP                                                                 00001150
C                                                                               00001160
      IF(((IEND.NE.1).AND.(ITER.LT.ITERN)) GO TO 220                            00001170
      IF(NPLOT.NE.0) CALL PLOTB                                                 00001180
      GO TO 290                                                                 00001190
C                                                                               00001200
C     CHECK WHETHER SUFFICIENT DATA HAS BEEN ACCEPTED                           00001210
  220 IF(NPARAM.LT.ISD) GO TO 230                                               00001220
      WRITE (6,7) ISD                                                           00001230
      GO TO 290                                                                 00001240
C                                                                               00001250
C     RESET DATA REJECTION LEVEL                                                00001260
  230 SD=ISD                                                                    00001270
      REJECT=4.*DSQRT(SIGMA/SD)                                                 00001280
C                                                                               00001290
C     SOLVE NORMAL EQUATIONS                                                    00001300
      CALL SOLVE(NPARAM,SIGMA,ISD)                                              00001310
C                                                                               00001320
C     ITERATION COUNTER UPDATE                                                  00001330
      ITER=ITER+1                                                               00001340
C                                                                               00001350
C     UPDATE PARAMETER VALUES                                                   00001360
      DO 240 I=1,NPARAM                                                         00001370
  240 XX(I)=XX(I)+E(I)                                                          00001380
      DO 250 I=1,27                                                             00001390
  250 X(I)=XX(JORDER(I))                                                        00001400
C                                                                               00001410
C     WRITE STANDARD DEVIATION AND DEGREES OF FREEDOM                           00001420
      WRITE (6,8) SIGMA,ISD                                                     00001430
C                                                                               00001440
C     CHECK PARAMETER CHANGES FOR CONVERGENCE                                   00001450
C                                                                               00001460
      DO 270 J=1,NPARAM                                                         00001470
      DO 260 I=1,27                                                             00001480
      IF(JORDER(I).NE.J) GO TO 260                                              00001490
      IF(DABS(F(J)).GT.(ACCFAC*XF(I))) GO TO 120                                00001500
  260 CONTINUE                                                                  00001510
C                                                                               00001520
  270 CONTINUE                                                                  00001530
      IEND=1                                                                    00001540
C                                                                               00001550
C     RECORD COVARIANCE MATRIX ON ACHIEVING CONVERGENCE                         00001560
C                                                                               00001570
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      WRITE (6,9)                                                       00001580
      KR=0                                                              00001590
      DO 280 I=1,NPARAM                                                 00001600
      KA=KR+1                                                           00001610
      KR=KA+I-1                                                         00001620
  280 WRITE (6,10) (PSI(J),J=KA,KR)                                     00001630
      WRITE (6,11)                                                      00001640
      GO TO 120                                                         00001650
C                                                                       00001660
C     IF ANOTHER CASE IS TO BE RUN RESTART PROCEDURE                    00001670
  290 IF(ICASE.NE.0) GO TO 100                                          00001680
C                                                                       00001690
      STOP                                                              00001700
C                                                                       00001710
    1 FORMAT('1PARAMETER ESTIMATION FOR ANALYSIS OF TRAJECTORY AND ROLL 00001720
     $DATA FROM RADAR AND YAWSONDE MEASUREMENTS MADE DURING THE FLIGHT O00001730
     $F A SHELL')                                                       00001740
    2 FORMAT('0',10X,A(5X,A8))                                          00001750
    3 FORMAT(' PARAM.     ',8E13.4)                                     00001760
    4 FORMAT(' DELTA      ',8E13.4)                                     00001770
    5 FORMAT('1TIME(S) RANGE(M)  AZI(M)  ALT(M)  ROLL(R/S)    DELX     00001780
     $DELY       DELZ       DELP ')                                     00001790
    6 FORMAT('+',43X,1P8E10.2)                                          00001800
    7 FORMAT('0NOT ENOUGH DATA LEFT, ISD=',I3)                          00001810
    8 FORMAT('0SIGMA=',E13.5,' DEGREES OF FREEDOM=',I4)                 00001820
    9 FORMAT('1'///' COVARIANCE MATRIX (MATRIX IS SYMMETRIC)'/////)     00001830
   10 FORMAT(1X,1P13E10.2)                                              00001840
   11 FORMAT('0 CHANGES TO THE FOLLOWING PARAMETERS ALL SMALL')         00001850
      END                                                               00001860


      SUBROUTINE ATMOS(Z,RHO,VSOUND,TEMPK,JERR)                         00000010
C                                                                       00000020
C     DEFINING THE STANDARD ICAO ATMOSPHERE WITH COESA EXTENSION TO     00000030
C     32 KILOMETRES AS DEFINED IN ARDU REPORT 1334(13) BY RUNDOCK AND   00000040
C     DIETZ (JULY 19AX)                                                 00000050
C                                                                       00000060
C       ALT=GEOMETRICAL ALTITUDE (METRES)                               00000070
C       PAMBIENT  PRESSURE (PASCALS)                                    00000080
C  A=VELOCITY OF SOUND (METRES/SECOND)                                  00000090
C       RHO=DENSITY(KILOGRAMS/CUBIC METRE)                              00000100
C       XMU=COEFFICIENT OF VISCOSITY (NEWTON-SECONDS/SQUARE METRE)      00000110
C       THETA=RELATIVE TEMPERATURE )                                    00000120
C       DELTA=RELATIVE PRESSURE     ) RELATIVE TO CONDITIONS AT SEA LEVEL00000130
C       SIGMA=RELATIVE DENSITY      )                                   00000140
C       JERR=1 IF ALTITUDE IS OUTSIDE THE ALLOWABLE RANGE               00000150
C            =0                                                         00000160
C                                                                       00000170
C*****UPDATED VERSION*****SEPTEMBER 1977*****                          00000180
C                                                                       00000190
      JERR=0                                                            00000200
      ALT=0.001*Z                                                       00000210
      IF((ALT.GT.-1.5).AND.(ALT.LT.32.)) GO TO 100                      00000220
      JERR=1                                                            00000230
      WRITE (A,1) ALT                                                   00000240
    1 FORMAT('0ALTITUDE=',E12.4,'(M) IS OUTSIDE ALLOWABLE RANGE IN ATMOS00000250
     $')                                                                00000260
      RETURN                                                            00000270
C                                                                       00000280
  100 IF(ALT.GT.11.) GO TO 200                                          00000290
      THETA=1.-0.0225577*ALT                                            00000300
      DELTA=THETA**5.24537                                              00000310
      GO TO 400                                                         00000320
C                                                                       00000330
  200 IF(ALT.GT.20.) GO TO 300                                          00000340
      THETA=0.751865                                                    00000350
      DELTA=0.224031*EXP(-0.156922*(ALT-11.))                           00000360
      GO TO 400                                                         00000370
  300 THETA=0.751865+0.0034242*(ALT-20.)                                00000380
      DELTA=1.47923A*(1.+0.0054447A*ALT)**(-31.9241)                    00000390
C                                                                       00000400
  400 TEMP=288.15*THETA                                                 00000410
      SIGMA=DELTA/THETA                                                 00000420
      RHO=1.225*SIGMA                                                   00000430
      VSOUND=20.0467*SQRT(TEMP)                                         00000440
C     PA=101325.*DELTA                                                  00000450
C     XMU=(1.458E-06)*TEMP**1.5/(TEMP+110.4)                            00000460
C                                                                       00000470
      RETURN                                                            00000480
      END                                                               00000490


      FUNCTION CMALFA(MACH)                                             00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000015
C                                                                       00000020
C     A FUNCTION TO INTERPOLATE AMONGST TABULATED VALUES OF STATIC      00000030
C  PITCHING MOMENT DERIVATIVE FOR THE M1 SHELL, TABULATED AS A          00000040
C  FUNCTION OF MACH NUMBER                                              00000050
C                                                                       00000060
      REAL *8 MACH,MACHT                                                00000070
C                                                                       00000080
      DIMENSION CMAT(38),MACHT(38)                                      00000090
C                                                                       00000100
      DATA CMAT /3.5730,3.6756,3.6931,3.7222,3.7575,3.7946,3.8330,      00000110
     $    3.8762,3.9018,3.9318,3.9678,4.0124,4.0878,4.1497,4.2338,      00000120
     $    4.2916,4.3258,4.3161,4.2807,4.2341,4.1882,4.1415,4.1046,      00000130
     $    4.0749,4.0454,3.9922,3.9579,3.9369,3.9246,3.9120,3.9019,      00000140
     $    3.8920,3.8730,3.8342,3.7560,3.7232,3.6785,3.5650/,            00000150
     $    MACHT /0.,0.5,0.55,0.60,0.65,0.7,0.75,0.8,0.825,0.85,0.875,   00000160
     $    0.9,0.925,0.94,0.95,0.96,0.97,0.98,0.99,1.,1.01,1.02,1.03,    00000170
     $    1.04,1.05,1.075,1.1,1.125,1.15,1.2,1.25,1.3,1.4,1.5,1.75,     00000180
     $    2.,2.25,2.5/                                                  00000190
C                                                                       00000200
      DO 100 I=2,38                                                     00000210
      IF(MACH.GT.MACHT(I)) GO TO 100                                    00000220
C                                                                       00000230
      CMALFA=CMAT(I-1)+(MACH-MACHT(I-1))                                00000240
     $      *(CMAT(I)-CMAT(I-1))/(MACHT(I)-MACHT(I-1))                  00000250
      GO TO 200                                                         00000260
  100 CONTINUE                                                          00000270
C                                                                       00000280
  200 RETURN                                                            00000290
      END                                                               00000300


      FUNCTION DELT(I,J)                                                00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000015
C                                                                       00000020
C     REPRESENTING THE KRONECKER DELTA SYMBOL WHICH IS                  00000030
C           =1  WHEN I=J, AND                                           00000040
C           =0  OTHERWISE.                                              00000050
C                                                                       00000060
      DELT=0.                                                           00000070
      IF(I.EQ.J) DELT=1.                                                00000080
C                                                                       00000090
      RETURN                                                            00000100
      END                                                               00000110


      SUBROUTINE DERIV                                                  00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000020
C                                                                       00000030
C     SUPPLIES VALUES OF DERIVATIVES TO INTEGRATION ROUTINES            00000040
C     MODEL USES CURVED EARTH AXES                                      00000050
C                                                                       00000060
      REAL *8 LAMBDA,MACH,MSQ                                           00000070
C                                                                       00000080
      COMMON /MODEL/  AMASS,ROLLI,REFS,RODIAM,                          00000090
     $                XE(27),X(27),XX(27),JORDER(27)                   00000100
     $       /INT/    Y(197),DY(197),HQ(197),YA(197),Z(108),N,IOBS,MAX 00000110
     $       /EARTH/  LAMBDA(3),OMEGA,GRAV,HTASLD,RO                   00000120
C                                                                       00000130
C     REAL *4 MACH                                                      00000140
C     PARAMETERS ARE STORED AS FOLLOWS                                  00000150
C        X(1)=XO,  X(2)=YO,  X(3)=ZO,  X(4)=XDOTO,X(5)=YDOTO,          00000160
C        X(6)=ZDOTO                                                     00000170
C        CX=(1+5)*(X(9)+X(10)*R+X(11)*R*R)                              00000180
C          +(1-5)*(X(12)+X(13)*R+X(14)*R*R)                             00000190
C        R=(M*M-X(7)*X(7))/(M*M+X(7)*X(7))                              00000200
C        S=R/DSQRT((1-X(8)*X(8))*R*R+X(8)*X(8))                         00000210
C        CZALPHA=(1+5)*(X(17)+X(18)*R+X(19)*R*R)                        00000220
C              +(1-5)*(X(20)+X(21)*R+X(22)*R*R)                         00000230
C        R=(M*M-X(15)*X(15))/(M*M+X(15)*X(15))                          00000240
C        S=R/DSQRT((1-X(16)*X(16))*R*R+X(16)*X(16))                     00000250
C        CLP=X(23)+MACH*(X(24)+MACH*(X(25)+MACH*X(26))                  00000260
C        X(27)=RO                                                       00000270
C                                                                       00000280
C     MODEL VARIABLES (IN RANGE AXES) HAVE THE FOLLOWING ALLOCATIONS    00000290
C        Y(1)=TIME,Y(2)=X,   Y(3)=Y,   Y(4)=Z,   Y(5)=P,                00000300
C        Y(6)=XDOT,Y(7)=YDOT,Y(8)=ZDOT,Y(9)=DX/DP(1),                   00000310
C        Y(10)=DY/DP(1),.....,Y(15)=D/DT(DZ/DP(1)),Y(16)=DX/DP(2),..    00000320
C        .......,Y(197)=D/DT(DZ/DP(27))                                 00000330
C                                                                       00000340
C        IN GENERAL  Y(I+7*J)=DY(I)/DP(J) WHERE I=2,3,....,8            00000350
C                                             J=1,2,....,27             00000360
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C          METEOROLOGICAL AND ATMOSPHERIC DATA                                  00000370
C                                                                               00000380
      ALT=HTASLO-Y(4)+0.5*(Y(2)*Y(2)+Y(3)*Y(3))/RO                              00000390
      CALL MET(ALT,XWIND,YWIND,RHO,VSOUND)                                      00000400
C                                                                               00000410
      U=Y(6)-XWIND                                                              00000420
      V=Y(7)-YWIND                                                              00000430
      W=Y(8)                                                                    00000440
      VELSQ=U*U+V*V+W*W                                                         00000450
      VEL=DSQRT(VELSQ)                                                          00000460
      MACH=VEL/VSOUND                                                           00000470
      RSM=0.5*RHO*REFS/AMASS                                                    00000480
      RSDI=0.5*RHO*REFS*RODIAM/ROLLI                                            00000490
C                                                                               00000500
C       GRAVITY AND CORIOLIS COMPONENTS                                         00000510
C                                                                               00000520
      CORX=LAMBDA(2)*Y(8)-LAMBDA(3)*Y(7)                                        00000530
      CORY=LAMBDA(3)*Y(6)-LAMBDA(1)*Y(8)                                        00000540
      CORZ=LAMBDA(1)*Y(7)-LAMBDA(2)*Y(6)                                        00000550
      RA=RO+HTASLO-Y(4)                                                         00000560
      RSQ=Y(2)*Y(2)+Y(3)*Y(3)+RA*RA                                             00000570
      REARTH=DSQRT(RSQ)                                                         00000580
      RATIO=(RO*RO/RSQ)**1.5                                                    00000590
      GX=-GRAV*RATIO*Y(2)/RO                                                    00000600
      GY=-GRAV*RATIO*Y(3)/RO                                                    00000610
      GZ=GRAV*RATIO*RA/RO                                                       00000620
C                                                                               00000630
C     STATIC MOMENT DERIVATIVE                                                  00000640
      CMA=CMALFA(MACH)                                                          00000650
C                                                                               00000660
C     INCIDENCE - YAW OF REPOSE SIMPLIFIED,AXIAL FORCE,NORMAL FORCE,            00000670
C     ROLL DAMPING MOMENT,DRAG AND LIFT                                         00000680
C                                                                               00000690
      ALPHA=Y(5)*U+GZ/(RSDI*VELSQ*VELSQ*CMA)                                    00000700
C                                                                               00000710
      MSQ=MACH*MACH                                                             00000720
      SQK=X(7)*X(7)                                                             00000730
      SQL=X(8)*X(8)                                                             00000740
      R=(MSQ-SQK)/(MSQ+SQK)                                                     00000750
      S=R/DSQRT((1.-SQL)*R*R+SQL)                                               00000760
      CX=(1.+S)*(X(9)+R*(X(10)+R*X(11)))                                        00000770
     $  +(1.-S)*(X(12)+R*(X(13)+R*X(14)))                                       00000780
C                                                                               00000790
      SQK=X(15)*X(15)                                                           00000800
      SQL=X(16)*X(16)                                                           00000810
      R=(MSQ-SQK)/(MSQ+SQK)                                                     00000820
      S=R/DSQRT((1.-SQL)*R*R+SQL)                                               00000830
      CZALFA=(1.+S)*(X(17)+R*(X(18)+R*X(19)))                                   00000840
     $       +(1.-S)*(X(20)+R*(X(21)+R*X(22)))                                  00000850
C                                                                               00000860
      CLP=X(23)*MACH*(X(24)+MACH*(X(25)+MACH*X(26)))                            00000870
C                                                                               00000880
      COSA=DCOS(ALPHA)                                                          00000890
      SINA=DSIN(ALPHA)                                                          00000900
      CD=CX*COSA+CZALFA*SINA*SINA                                               00000910
      CLALFA=CX-CZALFA*COSA                                                     00000920
C                                                                               00000930
C     EQUATIONS OF MOTION                                                       00000940
C                                                                               00000950
      DO 100 I=1,3                                                              00000960
  100 DY(I+1)=Y(I+5)                                                            00000970
      DY(5)=RSDI*0.5*Y(5)*RODIAM*VEL*CLP                                        00000980
C                                                                               00000990
      DY(6)=RSM*(CD*U*VEL+GX+CORX                                               00001000
      DY(7)=RSM*(CD*V*VEL+CLALFA*SINA*VELSQ)+GY+CORY                            00001010
      DY(8)=RSM*CD*W*VEL+GZ+CORZ                                                00001020
C                                                                               00001030
C     DERIVATIVES WITH RESPECT TO TIME OF PARTIAL DERIVATIVES WITH              00001040
C     RESPECT TO PARAMETERS                                                     00001050
C                                                                               00001060
      DO 200 J=1,MAX                                                            00001070
C                                                                               00001080
      CORXDP=LAMBDA(2)*Y(7*J+8)-LAMBDA(3)*Y(7*J+7)                              00001090
      CORYDP=LAMBDA(3)*Y(7*J+6)-LAMBDA(1)*Y(7*J+8)                              00001100
      CORZDP=LAMBDA(1)*Y(7*J+7)-LAMBDA(2)*Y(7*J+6)                              00001110
C                                                                               00001120
      DGXDP=-GRAV*RATIO*Y(7*J+2)/REARTH                                         00001130
      DGYDP=-GRAV*RATIO*Y(7*J+3)/REARTH                                         00001140
      DGZDP=-GRAV*RATIO*Y(7*J+4)/REARTH                                         00001150
C                                                                               00001160
      DUDP=Y(7*J+6)                                                             00001170
      DVDP=Y(7*J+7)                                                             00001180
      DWDP=Y(7*J+8)                                                             00001190
      DVELDP=(U*DUDP+V*DVDP+W*DWDP)/VEL                                         00001200
      DMDP=DVELDP/VSOUND                                                        00001210
C                                                                               00001220
C     WE ASSUME THAT DRHODP AND DVSOUNDDP ARE NEGLIGIBLE                        00001230
C                                                                               00001240
C     AXIAL FORCE                                                               00001250
C                                                                               00001260
      SQK=X(7)*X(7)                                                             00001270
      SQL=X(8)*X(8)                                                             00001280
      SQM=MACH*MACH                                                             00001290
      QUOT=SQM+SQK                                                              00001300
      R=(SQM-SQK)/QUOT                                                          00001310
      DRDP=2.*MACH*X(7)*(X(7)*DMDP-MACH*DELT(J,7))/(QUOT*QUOT)                  00001320
      QUOTSQ=(1.-SQL)*R*R+SQL                                                   00001330
      QUOT=DSQRT(QUOTSQ)                                                        00001340
      S=R/QUOT                                                                  00001350
      DSDP=X(8)*(X(8)*DRDP+R*(R+1.)*(R-1.)*DELT(J,8))/(QUOTSQ*QUOT)             00001360
      DCXDP=(1.+S)*(DELT(J,9)+R*(DELT(J,10)+R*DELT(J,11)))                      00001370
     $      +(1.-S)*(DELT(J,12)+R*(DELT(J,13)+R*DELT(J,14)))                    00001380
     $      +(X(9)-X(12)+R*(X(10)-X(13)+R*(X(11)-X(14))))*DSDP                  00001390
     $      +((1.+S)*(X(10)+2.*R*X(11))+(1.-S)*(X(13)+2.*R*X(14)))*DRDP         00001400
C                                                                               00001410
C     NORMAL FORCE                                                              00001420
C                                                                               00001430
      SQK=X(15)*X(15)                                                           00001440
      SQL=X(16)*X(16)                                                           00001450
      QUOT=SQM+SQK                                                              00001460
      R=(SQM-SQK)/QUOT                                                          00001470
      DRDP=2.*MACH*X(15)*(X(15)*DMDP-MACH*DELT(J,15))/(QUOT*QUOT)               00001480
      QUOTSQ=(1.-SQL)*R*R+SQL                                                   00001490
      QUOT=DSQRT(QUOTSQ)                                                        00001500
      S=R/QUOT                                                                  00001510
      DSDP=X(16)*(X(16)*DRDP+R*(R-1.)*(R+1.)*DELT(J,16))/(QUOTSQ*QUOT)          00001520
      DCZADP=(1.+S)*(DELT(J,17)+R*(DELT(J,18)+R*DELT(J,19)))                    00001530
     $      +(1.-S)*(DELT(J,20)+R*(DELT(J,21)+R*DELT(J,22)))                    00001540
     $      +(X(17)-X(20)+R*(X(18)-X(21)+R*(X(19)-X(22))))*DSDP                 00001550
     $      +((1.+S)*(X(18)+2.*R*X(19))+(1.-S)*(X(21)+2.*R*X(22)))*DRDP         00001560
C                                                                               00001570
C     ROLL DAMPING                                                              00001580
C                                                                               00001590
      DCLPDP=DELT(J,23)+MACH*(DELT(J,24)+MACH*(DELT(J,25)                       00001600
     $                                        +MACH*DELT(J,26)))                00001610
     $      +(X(24)+MACH*(2.*X(25)+MACH*3.*X(26)))*DMDP                         00001620
C                                                                               00001630
C     INCIDENCE                                                                 00001640
C                                                                               00001650
      DINCDP=(Y(5)*U*DGZDP+Y(5)*DUDP*GZ+Y(7*J+5)*U*GZ-4.*Y(5)*...*GZ            00001660
     $      *DVELDP/VEL)/(RSDI*VELSQ*VELSQ*CMA)                                 00001670
C                                                                               00001680
C     DRAG AND LIFT                                                             00001690
C                                                                               00001700
      DCDDP=(-CX*SINA+2.*CZALFA*SINA*COSA)*DINCDP                               00001710
     $      +DCXDP*COSA+DCZADP*SINA*SINA                                        00001720
      DCLADP=CZALFA*SINA*DINCDP+DCXDP-DCZADP*COSA                               00001730
C                                                                               00001740
C     DERIVATIVES                                                               00001750
C                                                                               00001760
      DO 150 I=2,4                                                              00001770
  150 DY(7*J+I)=Y(7*J+I+4)                                                      00001780
      DY(7*J+5)=0.5*RSDI*RODIAM*(CLP*Y(5)*DVELDP+CLP*Y(7*J+5)*VEL               00001790
     $          +DCLPDP*Y(5)*VEL)                                               00001800
      DY(7*J+6) =RSM*(CD*U*DVELDP+CD*DUDP*VEL+DCDDP*U*VEL)+DGXDP+CORXDP         00001810
      DY(7*J+7)=RSM*(CD*V*DVELDP+CD*DVDP*VEL+DCDDP*V*VEL                        00001820
     $          +2.*CLALFA*VEL*SINA*DVELDP+CLALFA*VELSQ*COSA*DINCDP             00001830
     $          +DCLADP*SINA*VELSQ)+DGYDP+CORYDP                                00001840
      DY(7*J+8)=RSM*(CD*W*DVELDP+CD*DWDP*VEL+DCDDP*W*VEL)+DGZDP+CORZDP          00001850
C                                                                               00001860
  200 CONTINUE                                                                  00001870
C                                                                               00001880
      RETURN                                                                    00001890
      END                                                                       00001900
```

```
      SUBROUTINE INCOND                                                 00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000020
C                                                                       00000030
C     INITIALISE VALUES OF ALL VARIABLES FOR INTEGRATION BY DERV        00000040
C     AND RUNGE4 AT THE BEGINNING OF EACH ITERATION OF THE PARAMETER    00000050
C     ESTIMATION ALGORITHM.                                             00000060
C                                                                       00000070
      COMMON /INT/    Y(197),DY(197),HO(197),YA(197),Z(10R),N,IORS,MAX  00000080
     $       /MODEL/  AMASS,ROLLI,REFS,RODIAM,                          00000090
     $                XE(27),X(27),XX(27),JORDER(27)                    00000100
     $       /COMP/   TO,REJECT,ACCFAC,HMAX,NPARAM,ITERN,LCASE         00000110
     $       /DATA/   ORS(5,1000),XS(4),PI(5),NPTS,ITER,IEND           00000120
C                                                                       00000130
      DO 100 I=1,197                                                    00000140
  100 Y(I)=0.                                                           00000150
C                                                                       00000160
      Y(1)=TO                                                           00000170
      DO 200 I=1,3                                                      00000180
      Y(I+1)=X(I)                                                       00000190
      Y(I+5)=X(I+3)                                                     00000200
      Y(8*I+1)=1.                                                       00000210
  200 Y(8*I+26)=1.                                                      00000220
      Y(5)=X(27)                                                        00000230
      Y(196)=1.                                                         00000240
C                                                                       00000250
C     MAXIMUM NUMBER OF EQUATIONS WHICH MUST BE INTEGRATED              00000260
C                                                                       00000270
      MAX=0                                                             00000280
      DO 300 J=1,27                                                     00000290
      IF(JORDER(J).LE.NPARAM) MAX=J                                     00000300
  300 CONTINUE                                                          00000310
      N=7*MAX+8                                                         00000320
      IF(IEND.EQ.1) N=8                                                 00000330
C                                                                       00000340
      RETURN                                                            00000350
      END                                                               00000360



      SUBROUTINE INPUT                                                  00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000020
C                                                                       00000030
C                                                                       00000040
      DIMENSION TITLE(20),DUMMY(4)                                      00000050
C                                                                       00000060
      REAL *8 LAMBDA                                                    00000070
C                                                                       00000080
      COMMON /MODEL/ AMASS,ROLLI,REFS,RODIAM,                           00000090
     $               XE(27),X(27),XX(27),JORDER(27)                     00000100
     $       /COMP/  TO,REJECT,ACCFAC,HMAX,NPARAM,ITERN,LCASE,NPLOT     00000110
     $       /DATA/  ORS(5,1000),XS(4),PI(5),NPTS,ITER,IEND             00000120
     $       /EARTH/ LAMBDA(3),OMEGA,GRAV,HTASLO,RO                     00000130
C                                                                       00000140
C                                                                       00000150
C     THIS ROUTINE HANDLES INPUT AND RECORDING OF ALL DATA FOR THE      00000160
C     PROGRAM                                                           00000170
C     IT IS POSSIBLE TO READ DATA SELECTIVELY USING THE LENTER PARAMETER00000180
C     WHICH IS INPUT TWICE. FIRSTLY AFTER THE RUN TITLE AND SECONDLY    00000190
C     JUST BEFORE THE TRIALS DATA                                       00000200
C                                                                       00000210
      REWIND 3                                                          00000220
C                                                                       00000230
C                                                                       00000240
C                                                                       00000250
C     READ DATA TITLE                                                   00000260
      READ (5,1) TITLE                                                  00000270
      WRITE (6,2) TITLE                                                 00000280
C                                                                       00000290
C     READ ENTRY LABEL FOR INPUT OF NEW DATA                            00000300
      READ (5,3) LENTER                                                 00000310
   90 GO TO (100,200,300,400,500,600,700,800,900),LENTER                00000320
C                                                                       00000330
C     MODEL CONSTANTS                                                   00000340
  100 READ (5,4) AMASS,ROLLI,REFS,RODIAM                                00000350
      WRITE (6,5) AMASS,ROLLI,REFS,RODIAM                               00000360
C                                                                       00000370
C     READ EARTH CONSTANTS                                              00000380
      READ (5,4) OMEGA,ALAT,BEAR,GRAV,HTASLO,RO                         00000390
      WRITE (6,14) OMEGA,ALAT,BEAR,GRAV,HTASLO,RO                       00000400
      ALAT=0.01745329*ALAT                                              00000410
      BEAR=0.01745329*BEAR                                              00000420
      COSL=DCOS(ALAT)                                                   00000430
      SINL=DSIN(ALAT)                                                   00000440
      COSAZ=DCOS(BEAR)                                                  00000450
      SINAZ=DSIN(BEAR)                                                  00000460
      LAMBDA(1)=2.*OMEGA*COSL*COSAZ                                     00000470
      LAMBDA(2)=2.*OMEGA*COSL*SINAZ                                     00000480
      LAMBDA(3)=2.*OMEGA*SINL                                           00000490
C                                                                       00000500
C     RELATIVE ACCURACIES IN PARAMETERS WHICH SIGNIFY CONVERGENCE       00000510
  200 READ (5,4) (XE(I),I=1,27)                                         00000520
C                                                                       00000530
C     INITIAL VALUES FOR PARAMETERS                                     00000540
  400 READ (5,4) (X(I),I=1,27)                                          00000550
C                                                                       00000560
C     REQUIRED ORDER OF PARAMETERS                                      00000570
  500 READ (5,3) (JORDER(I),I=1,27)                                     00000580
      WRITE (6,7) (JORDER(I),I=1,27)                                    00000590
      DO 510 I=1,27                                                     00000600
  510 XX(JORDER(I))=X(I)                                                00000610
C                                                                       00000620
C     INITIAL TIME                                                      00000630
  300 READ (5,4) TO                                                     00000640
      WRITE (6,6) TO                                                    00000650
C                                                                       00000660
C     SCALE FACTORS ON INSTRUMENT READINGS                              00000670
      READ (5,4) (XS(I),I=1,4)                                          00000680
      WRITE (6,9) (XS(I),I=1,4)                                         00000690
C                                                                       00000700
C     PROGRAM CONTROL PARAMETERS                                        00000710
  600 READ (5,4) REJECT,ACCFAC,HMAX                                     00000720
  700 READ (5,3) NPARAM,ITERN,NPLOT                                     00000730
      WRITE (6,8) REJECT,ACCFAC,HMAX,NPARAM,ITERN                       00000740
      IF(NPLOT.EQ.0) WRITE (6,12)                                       00000750
      IF(NPLOT.NE.0) WRITE (6,13) NPLOT                                 00000760
C                                                                       00000770
C     DATA INPUT - IF REQUIRED AND WHICH SECTIONS                       00000780
      READ (5,3) LENTER,LCASE,IFOUT                                     00000790
      GO TO 90                                                          00000800
C                                                                       00000810
C     INPUT FLIGHT RESULTS ORS(I,J)                                     00000820
C     ORS(1,J) =TIME                ORS(2,J)=RANGE                    00000830
C     ORS(3,J) =DEVIATION           ORS(4,J)=ALTITUDE                 00000840
C     ORS(5,J) =ROLL RATE                                             00000850
  800 READ (5,3) NPTS,ISKIP                                             00000860
C                                                                       00000870
C     READ METEOROLOGICAL DATA TABLES AS REQUIRED                       00000880
      CALL READMT                                                       00000890
C                                                                       00000900
      IF(IFOUT.NE.0) WRITE (6,10)                                       00000910
  805 READ (3,15) (ORS(I,1),I=1,5)                                      00000920
      IF(ORS(1,1).LT.(TO-0.0001)) GO TO 805                             00000930
      IF(IFOUT.NE.0) WRITE (6,11) (ORS(I,1),I=1,5)                      00000940
      DO 820 J=2,NPTS                                                   00000950
      DO 810 JJ=1,ISKIP                                                 00000960
  810 READ (3,15) (ORS(I,J),I=1,5)                                      00000970
      IF(IFOUT.NE.0) WRITE (6,11) (ORS(I,J),I=1,5)                      00000980
  820 CONTINUE                                                          00000990
C                                                                       00001000
C     IF NO NEW DATA IS REQUIRED OR IF DATA AS REQUIRED HAS BEEN READ   00001010
  900 RETURN                                                            00001020
    1 FORMAT(20A4)                                                      00001030
    2 FORMAT(1H0,25X,20A4)                                              00001040
    3 FORMAT(16I5)                                                      00001050
    4 FORMAT(8F10.0)                                                    00001060
    5 FORMAT(///' VEHICLE PHYSICAL CONSTANTS'/' MASS ',F8.4,'KG',5X,'MOME00001070
     $NT OF INERTIA IN ROLL',F9.6,'KG M2'/' REFERENCE AREA',F9.6,'M2',  00001080
     $    5X,'REFERENCE LENGTH',F7.4,'M')                               00001090
    6 FORMAT(/' INITIAL TIME',F7.3,'(S)'/)                              00001100
    7 FORMAT(/ ' ORDER OF PARAMETERS'/5X,27I3)                          00001110
    8 FORMAT(///' REJECTION LEVEL ON DATA',F8.2//                       00001120
     $' ACCURACY FACTOR FOR PARAMETER ERROR LEVELS',F7.3/               00001130
     $' MAXIMUM INTEGRATION STEP',F7.4,5X,'NO. OF PARAMETERS VARIED',   00001140
     $I3,5X,'MAXIMUM NO. OF ITERATIONS',I3)                             00001150
    9 FORMAT('0SCALING OF     RANGE     AZIMUTH   ALTITUDE  ROLL RATE'/ 00001160
     $10X,4F10.2//)                                                     00001170
   10 FORMAT('1    TIME(S)   RANGE     AZIMUTH  ALTITUDE ROLL RATE')     00001180
   11 FORMAT(1X,5F10.2)                                                 00001190
   12 FORMAT(' NO PLOTS REQUESTED')                                     00001200
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   13 FORMAT(' PLOTS REQUIRED FOR EVERY',I2,' POINTS')                    00001210
   14 FORMAT('0EARTH MODEL - ANGULAR VELOCITY  LATITUDE  FIRING LINE  GR00001220
     $AVITATION  HEIGHT ABOVE  RADIUS OF '/                               00001230
     $      '                  (RAD/SEC)          (DEG)      (DEG)      00001240
     $ (MEAN SEA LEVEL(M)  THE EARTH(M)'/15X,E14.7,F10.3,F13.3,          00001250
     $F13.5,F12.3,E16.7//)                                              00001260
   15 FORMAT(5F16.7)                                                    00001270
      END                                                               00001280

C                                                                       00000010
C     SUBROUTINE WFT                                                    00000020
C                                                                       00000030
C     SETS UP WIND TABLE (IND=2) IN WDTAB(150,3) WHERE                  00000040
C        WDTAB(I,1) CONTAINS ALTITUDE (M) ABOVE MEAN SEA-LEVEL          00000050
C        WDTAB(I,2) CONTAINS CROSS WIND (M/S),+VE FROM LEFT TO RIGHT    00000060
C        WDTAB(I,3) CONTAINS RANGE WIND (M/S),+VE DOWNRANGE             00000070
C     ALSO SETS UP PRESSURE-TEMPERATURE TABLE (IND=3)                   00000080
C     IN PT(150,3) WHERE                                                00000090
C        PTTAB(I,1) CONTAINS ALTITUDE(M) ABOVE MEAN SEA-LEVEL           00000100
C        PTTAB(I,2) CONTAINS PRESSURE (P)                               00000110
C        PTTAB(I,3) CONTAINS TEMPERATURE (K)                            00000120
C     OR BOTH TABLES IF IND=4                                           00000130
C                                                                       00000140
C     IF A STANDARD ATMOSPHERE WITH NO WINDS IS DESIRED THEN SET IND=1  00000150
C                                                                       00000160
C     IF INPUT SHOULD BE RECORDED THEN IFOUT SHOULD BE NON-ZERO         00000170
C                                                                       00000180
C     THE FIRST PART OF THE ROUTINE INTERPOLATES AMONGST THE TABLES     00000190
C     OR USES THE STANDARD ATMOSPHERE ROUTINE TO ASSIGN VALUES TO THE   00000200
C     ATMOSPHERIC VARIABLES                                             00000210
C                                                                       00000220
      SUBROUTINE WFT(ALT,XWIND,YWIND,RHO,VSOUND)                        00000230
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000240
C                                                                       00000250
      DIMENSION WDTAB(150,3),PTTAB(150,3)                               00000260
C                                                                       00000270
C     ESTIMATE WIND COMPONENTS,ATMOSPHERIC DENSITY AND SOUND SPEED      00000280
      IF(IND.LE.2) CALL ATMOS(ALT,RHO,VSOUND,TEMP,JERR)                 00000290
      XWIND=0.                                                          00000300
      YWIND=0.                                                          00000310
      IF(IND.EQ.1) RETURN                                               00000320
C                                                                       00000330
C     INTERPOLATE FOR WINDS IF TABULATED                                00000340
      IF(IND.EQ.3) GO TO 200                                            00000350
C                                                                       00000360
      DO 100 I=2,NWD                                                    00000370
      IF(ALT.GT.WDTAB(I,1)) GO TO 100                                   00000380
      LWD=I                                                             00000390
      GO TO 110                                                         00000400
  100 CONTINUE                                                          00000410
C                                                                       00000420
      WRITE (6,1) ALT                                                   00000430
    1 FORMAT('1 ALTITUDE TOO LARGE FOR TABULATED WINDS',E16.6,'(M)')    00000440
      STOP                                                              00000450
C                                                                       00000460
  110 DEL=(ALT-WDTAB(LWD-1,1))/(WDTAB(LWD,1)-WDTAB(LWD-1,1))            00000470
      YWIND=WDTAB(LWD-1,2)+DEL*(WDTAB(LWD,2)-WDTAB(LWD-1,2))            00000480
      XWIND=WDTAB(LWD-1,3)+DEL*(WDTAB(LWD,3)-WDTAB(LWD-1,3))            00000490
C                                                                       00000500
C     INTERPOLATE FOR PRESSURE AND TEMPERATURE IF TABULATED             00000510
  200 IF(IND.LE.2) RETURN                                               00000520
C                                                                       00000530
      DO 210 I=2,NPT                                                    00000540
      IF(ALT.GT.PTTAB(I,1)) GO TO 210                                   00000550
      LPT=I                                                             00000560
      GO TO 220                                                         00000570
  210 CONTINUE                                                          00000580
C                                                                       00000590
      WRITE (6,2) ALT                                                   00000600
    2 FORMAT('1ALTITUDE TOO LARGE FOR TABULATE TEMPERATURE AND PRESSURE'00000610
     $       ,E16.6,'(M)')                                              00000620
      STOP                                                              00000630
C                                                                       00000640
  220 DEL=(ALT-PTTAB(LPT-1,1))/(PTTAB(LPT,1)-PTTAB(LPT-1,1))            00000650
      PRESS=PTTAB(LPT-1,2)+DEL*(PTTAB(LPT,2)-PTTAB(LPT-1,2))            00000660
      TEMPK=PTTAB(LPT-1,3)+DEL*(PTTAB(LPT,3)-PTTAB(LPT-1,3))            00000670
      RHO=0.0034836**PRESS/TEMPK                                        00000680
      VSOUND=20.0467*DSQRT(TEMPK)                                       00000690
      RETURN                                                            00000700
C                                                                       00000710
C     ENTRY POINT TO INPUT TABULAR DATA                                 00000720
      ENTRY READWT                                                      00000730
C                                                                       00000740
C     DATA CONTROLS                                                     00000750
      READ (5,4) IND,NWD,NPT,IFOUT                                      00000760
      IF((IND.GE.1).AND.(IND.LE.4)) GO TO 300                           00000770
C                                                                       00000780
      WRITE (6,3) IND                                                   00000790
    3 FORMAT('1IND=',I10,' OUTSIDE ALLOWABLE RANGE IN WET')             00000800
C                                                                       00000810
  300 IF(IND.EQ.1) RETURN                                               00000820
C                                                                       00000830
C     TABLES OF WIND DATA                                               00000840
      IF(IND.EQ.3) GO TO 400                                            00000850
      DO 310 I=1,NWD                                                    00000860
  310 READ (5,5) (WDTAB(I,J),J=1,3)                                     00000870
      IF(IFOUT.EQ.0) GO TO 400                                          00000880
      WRITE (6,6)                                                       00000890
      DO 320 I=1,NWD                                                    00000900
  320 WRITE (6,7) (WDTAB(I,J),J=1,3)                                    00000910
C                                                                       00000920
C     TABLES OF TEMPERATURE AND PRESSURE                                00000930
  400 IF(IND.EQ.2) RETURN                                               00000940
      DO 410 I=1,NPT                                                    00000950
  410 READ (5,5) (PTTAB(I,J),J=1,3)                                     00000960
      IF(IFOUT.EQ.0) RETURN                                             00000970
      WRITE (6,8)                                                       00000980
      DO 420 I=1,NPT                                                    00000990
  420 WRITE (6,9) (PTTAB(I,J),J=1,3)                                    00001000
C                                                                       00001010
      RETURN                                                            00001020
    4 FORMAT(16I5)                                                      00001030
    5 FORMAT(3F10.0)                                                    00001040
    6 FORMAT('0WIND TABLES    HEIGHT    RANGE WIND     CROSS WIND'/     00001050
     $      '                (METRES)        METRES/SECOND)')          00001060
    7 FORMAT(11X,F12.1,F13.2,F15.2)                                     00001070
    8 FORMAT('0PRESSURE AND TEMPERATURE TABLES    HEIGHT    PRESSURE  TE00001080
     $MPER  ATURE'/35X,'(METRES)  (PASCALS)    (KELVIN)')               00001090
    9 FORMAT(33X,F10.1,F11.1,F13.2)                                     00001100
      END


      SUBROUTINE MFSD(A,N,EPS,IER)                                      00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000020
C     ROUTINE FROM THE SYSTEM/360 SCIENTIFIC SUBROUTINE PACKAGE         00000030
C     TO FACTORISE A POSITIVE DEFINITE SYMMETRIC MATRIX INTO A          00000040
C     TRIANGULAR FACTORISATION. THAT IS  A=R'R                          00000050
      DIMENSION A(1)                                                    00000060
      IER=0                                                             00000070
      KPIV=0                                                            00000080
      DO 110 K=1,N                                                      00000090
      KPIV=KPIV+K                                                       00000100
      IND=KPIV                                                          00000110
      LEND=K-1                                                          00000120
      TOL=DABS(EPS*A(KPIV))                                             00000130
      DO 110 I=K,N                                                      00000140
      DSUM=0.D0                                                         00000150
      IF(LEND.EQ.0) GO TO 40                                            00000160
      DO 30 L=1,LEND                                                    00000170
      LANF=KPIV-L                                                       00000180
      LIND=IND-L                                                        00000190
   30 DSUM=DSUM+A(LANF)*A(LIND)                                         00000200
   40 DSUM=A(IND)-DSUM                                                  00000210
      IF(I-NF+K) GO TO 100                                              00000220
      IF((DSUM-TOL).GT.0.) GO TO 90                                     00000230
      IF(DSUM.LE.0.) GO TO 120                                          00000240
      IF(IER.LE.0)IER=-K-1                                              00000250
   90 DPIV=DSQRT(DSUM)                                                  00000260
      A(KPIV)=DPIV                                                      00000270
      DPIV=1.D0/DPIV                                                    00000280
      GO TO 110                                                         00000290
  100 A(IND)=DSUM*DPIV                                                  00000300
  110 IND=IND+I                                                         00000310
      RETURN                                                            00000320
  120 IER=-1                                                            00000330
      RETURN                                                            00000340
      END                                                               00000350
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      SUBROUTINE PLOTT                                                  00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000020
      REAL *4 TITLE,XAXIS,YAXIS,TIME,A,B                                00000030
C                                                                       00000040
C     A ROUTINE WHICH PLOTS RESULTS OF FINAL FITTING TO SHOW RESULTS    00000050
C     FOR VISUAL SCANNING                                               00000060
C     ENTRY POINTS ARE                                                  00000070
C         PLOTA - TO RECORD THE DATA, AND                               00000080
C         PLOTR - TO PLOT THE GRAPHS                                    00000090
C                                                                       00000100
      COMMON /MODEL/  AMASS,ROLLI,REFS, RODIAM,                         00000110
     $                XE(27),X(27),XX(27),JORDER(27)                   00000120
     $       /COMP/   TO,REJECT,ACCFAC,HMAX,NPARAM,ITERN,LCASE,NPLOT   00000130
     $       /DATA/   OBS(5,1000),XS(4),PI(5),NPTS,ITER,IEND           00000140
     $       /INT/    Y(197),DY(197),HO(197),YA(197),Z(108),N,IOBS,MAX  00000150
     $       /NORME/  PSI(378),CWD(27),E(27)                           00000160
C                                                                       00000170
      DIMENSION TITLE(20),TIME(105),A(105),B(105),XY(8,105),XAXIS(6),   00000180
     $          YAXIS(9),DUMMY(9,4)                                     00000190
C                                                                       00000200
      DATA TITLE /44*,'GOOD','NESS',' OF ','FIT ','FOR ','SHEL','L TR', 00000210
     $          'AJEC','TORI','DAT','A   '/,                            00000220
     $      XAXIS /0*.2*.10*.7*,'TIME','(S) '/,                         00000230
     $      DUMMY /0*.0,5*.10*.9*,'RANG','E(KM)','    ',20*'    ',      00000240
     $             0*.10*.10*.12*,'DEVI','ATIO','N(M)',20*'    ',       00000250
     $             0*.0,5*.10*.12*,'ALTI','TUDE','(KM)',20*'    ',       00000260
     $             0*.50*.10*.19*,'ROLL',' RAT','E(RA','D/SE','C)  '/   00000270
C                                                                       00000280
C     INITIALISE                                                        00000290
C                                                                       00000300
      I=0                                                               00000310
      IF=NPLOT                                                          00000320
      RETURN                                                            00000330
C                                                                       00000340
C     STORE DATA POINTS FOR PLOTTING                                    00000350
C                                                                       00000360
      ENTRY PLOTA                                                       00000370
      IF=IF+1                                                           00000380
      IF(IF.LT.NPLOT) RETURN                                            00000390
      IF=0                                                              00000400
      I=I+1                                                             00000410
      TIME(I)=OBS(1,IOBS)                                               00000420
      XY(1,I)=0.001*PI(2)                                               00000430
      XY(2,I)=0.1*E(2)+2.5                                              00000440
      XY(3,I)=PI(3)                                                     00000450
      XY(4,I)=2.*E(3)+50.                                               00000460
      XY(5,I)=-0.001*PI(4)                                              00000470
      XY(6,I)=0.1*E(4)+2.5                                              00000480
      XY(7,I)=PI(5)                                                     00000490
      XY(8,I)=100.*E(5)+250.                                            00000500
      RETURN                                                            00000510
C                                                                       00000520
C     PLOT DATA POINTS WHICH HAVE BEEN STORED                           00000530
C                                                                       00000540
      ENTRY PLOTR                                                       00000550
      MI=-I                                                             00000560
      MR=XY(7,1)*0.01                                                   00000570
      DUMMY(1,4)=100*(MR-4)                                             00000580
      DO 90 K=1,I                                                       00000590
   90 XY(8,K)=XY(8,K)+DUMMY(1,4)                                        00000600
      DO 200 J=1,4                                                      00000610
C                                                                       00000620
      DO 100 K=1,9                                                      00000630
  100 YAXIS(K)=DUMMY(K,J)                                               00000640
      DO 110 K=1,I                                                      00000650
      A(K)=XY(2*J-1,K)                                                  00000660
  110 B(K)=XY(2*J,K)                                                    00000670
C                                                                       00000680
      CALL GRAPH(TITLE,XAXIS,YAXIS,I,TIME,A,MI,TIME,B)                  00000690
C                                                                       00000700
  200 CONTINUE                                                          00000710
      IF(LCASE.EQ.0) CALL ENDGR                                         00000720
      RETURN                                                            00000730
      END                                                               00000740
```

```
      SUBROUTINE RUNGE4 (IND,N,H,Y,DY,HO,YA)                            00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000015
C                                                                       00000020
C     FOURTH ORDER,FIXED STEP RUNGE KUTTA ROUTINE TO INTEGRATE          00000030
C     (N-1) FIRST ORDER ORDINARY DIFFERENTIAL EQUATIONS                 00000040
C     Y(1)=TIME  Y(2).....Y(N) ARE DEPENDENT VARIABLES                  00000050
C        DY(2).....Y(N) ARE CORRESPONDING DERIVATIVES CALCULATED        00000060
C                       IN DERIV                                        00000070
C     NOTE *****  Y,DY MUST APPEAR IN COMMON STATEMENTS IN BOTH DERIV   00000080
C        AND THE CALLING ROUTINE                                        00000090
C          *****  Y,DY,HO,YA MUST HAVE DIMENSION AT LEAST N IN THE     00000100
C            CALLING ROUTINE                                           00000110
C     *****  IND=0 FOR THE FIRST PASS TO INITIALISE DERIVATIVES         00000120
C          =1 TO INTERGRATE ONE TIME STEP                               00000130
C                                                                       00000140
C                                                                       00000150
      DIMENSION Y(1),DY(1),HO(1),YA(1)                                  00000160
      IF(IND.EQ.0) GO TO 500                                            00000170
C                                                                       00000180
C     INITIALISE FOR INTEGRATION                                        00000190
      DO 100 I=2,N                                                      00000200
      YA(I)=Y(I)                                                        00000210
  100 HO(I)=0.                                                          00000220
      HA=Y(1)                                                           00000230
C                                                                       00000240
C     ESTIMATE INTERMEDIATE DERIVATIVES                                 00000250
      DO 300 J=3,5                                                      00000260
      A=(5-J)/2                                                         00000270
      B=J/2                                                             00000280
      BH=B*H                                                            00000290
      AH=(H/A)                                                          00000300
      DO 200 I=2,N                                                      00000310
      HO(I)=HO(I)+BH*DY(I)                                              00000320
  200 Y(I)=YA(I)+AH*DY(I)                                               00000330
      Y(1)=AH+HA                                                        00000340
C                                                                       00000350
C     ESTIMATE DERIVATIVES                                              00000360
      CALL DERIV                                                        00000370
  300 CONTINUE                                                          00000380
C                                                                       00000390
C     COMPLETE INTEGRATION AND UPDATE DERIVATIVES                       00000400
      DO 400 I=2,N                                                      00000410
  400 Y(I)=YA(I)+0.1666667*(HO(I)+H*DY(I))                              00000420
      Y(1)=HA+H                                                         00000430
  500 CALL DERIV                                                        00000440
C                                                                       00000450
      RETURN                                                            00000460
      END                                                               00000470
```

```
      SUBROUTINE SIMUL                                                  00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000020
C                                                                       00000030
C     THIS ROUTINE CONTROLS THE INTEGRATION PROCESS AND PROVIDES        00000040
C     SIMULATED VARIABLE VALUES AND PARTIAL DERIVATIVES IN THE REQUIRED 00000050
C     ORDER                                                             00000060
C                                                                       00000070
      COMMON /COMP/   TO,REJECT,ACCFAC,HMAX,NPARAM,ITERN,LCASE,NPLOT    00000080
     $       /DATA/   OBS(5,1000),XS(4),PI(5),NPTS,ITER,IEND           00000090
     $       /INT/    Y(197),DY(197),HO(197),YA(197),Z(108),N,IOBS,MAX  00000100
     $       /MODEL/  AMASS,ROLLI,REFS,RODIAM,                         00000110
     $                XE(27),X(27),XX(27),JORDER(27)                   00000120
C                                                                       00000130
      IF(IOBS.EQ.2) H=OBS(1,NPTS)-Y(1)                                  00000140
  100 IND=1                                                             00000150
C                                                                       00000160
C     SET UP INTEGRATION STEP LENGTH                                    00000170
C                                                                       00000180
      A=OBS(1,IOBS)-Y(1)                                                00000190
      B=A                                                               00000200
      IF(A.GT.HMAX)A=HMAX                                               00000210
      IF(DABS(A-H).GT.0.00001) IND=0                                    00000220
      H=A                                                               00000230
C                                                                       00000240
C     INTEGRATE ONE STEP                                                00000250
C                                                                       00000260
  110 CALL RUNGE4(IND,N,H,Y,DY,HO,YA)                                   00000270
```

```
C                                                                       00000280
      IF(IND.EQ.1) GO TO 120                                            00000290
      IND=1                                                             00000300
      GO TO 110                                                         00000310
C                                                                       00000320
C     HAS THE NEXT DATA POINT BEEN REACHED                              00000330
C                                                                       00000340
  120 IF(R.GT.HMAX) GO TO 100                                           00000350
C                                                                       00000360
C     ESTIMATE STATE VARIABLES AND PARTIAL DERIVATIVES                  00000370
C                                                                       00000380
      DO 130 J=1,5                                                      00000390
  130 PI(J)=Y(J)                                                        00000400
C                                                                       00000410
      DO 150 I=1,27                                                     00000420
0     IF(JORDER(I).GT.NPARAM) GO TO 150                                 00000430
C                                                                       00000440
      DO 140 J=2,5                                                      00000450
  140 Z(4*JORDER(I)+J-5)=Y(7*I+J)                                       00000460
C                                                                       00000470
  150 CONTINUE                                                          00000480
C                                                                       00000490
C     OUTPUT RESULTS OF INTEGRATION STEP IF REQUIRED                    00000500
C                                                                       00000510
      IF(((IEND.EQ.1).OR.((ITER.GE.ITERN)).AND.(NPLOT.NE.0))            00000520
     $                              WRITE (6,1) (PI(I),I=1,5)           00000530
      RETURN                                                            00000540
    1 FORMAT(1X,F7.3,F9.1,2F8.1,F11.2)                                  00000550
      END                                                               00000560
```

```
      SUBROUTINE SINV(A,N,EPS,IER)                                      00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000015
C     ROUTINE FROM SYSTEM/360 SCIENTIFIC SUBROUTINE PACKAGE             00000020
C     FOR INVERTING A POSITIVE DEFINITE SYMMETRIC MATRIX                00000030
      DIMENSION A(1)                                                    00000040
      CALL MFSD (A,N,EPS,IER)                                           00000060
      IF(IER.LT.0) RETURN                                               00000070
      IPIV=N*(N+1)/2                                                    00000080
      IND=IPIV                                                          00000090
      DO 600 I=1,N                                                      00000100
      DIN=1.0D0/A(IPIV)                                                 00000110
      A(IPIV)=DIN                                                       00000120
      MIN=N                                                             00000130
      KEND=I-1                                                          00000140
      LANF=N-KEND                                                       00000150
      IF(KEND.LE.0) GO TO 500                                           00000160
      J=IND                                                             00000165
      DO 400 K=1,KEND                                                   00000170
      WORK=0.D0                                                         00000180
      MIN=MIN-1                                                         00000190
      LHOR=IPIV                                                         00000200
      LVER=J                                                            00000210
      DO 300 L=LANF,MIN                                                 00000220
      LVER=LVER+1                                                       00000230
      LHOR=LHOR+L                                                       00000240
  300 WORK=WORK+A(LVER)*A(LHOR)                                         00000250
      A(J)=-WORK*DIN                                                    00000260
  400 J=J-MIN                                                           00000270
  500 IPIV=IPIV-MIN                                                     00000280
  600 IND=IND-I                                                         00000290
      DO 800 I=1,N                                                      00000300
      IPIV=IPIV+I                                                       00000310
      J=IPIV                                                            00000320
      DO 800 K=I,N                                                      00000330
      WORK=0.D0                                                         00000340
      LHOR=J                                                            00000350
      DO 700 L=K,N                                                      00000360
      LVER=LHOR+K-I                                                     00000370
      WORK=WORK+A(LHOR)*A(LVER)                                         00000380
  700 LHOR=LHOR+L                                                       00000390
      A(J)=WORK                                                         00000400
  800 J=J+K                                                             00000410
      RETURN                                                            00000420
      END                                                               00000430
```

```
      SUBROUTINE SOLVE(NP,SIGMA,NPTS)                                   00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000015
C                                                                       00000020
C     USES IBM SCIENTIFIC SUBROUTINE PACKAGES 'SINV' AND 'MFSD' TO      00000030
C     SOLVE THE NORMAL EQUATIONS                                        00000040
C     ASSUMES MATRIX PSI IS POSITIVE DEFINITE,SYMMETRIC                 00000050
C                                                                       00000060
C                                                                       00000070
      COMMON /NORME/ PSI(378),CWD(27),E(27)                             00000080
C                                                                       00000090
C                                                                       00000100
C                                                                       00000110
C     INVERT MATRIX                                                     00000120
      EPS=1.0D-15                                                       00000130
      CALL SINV(PSI,NP,EPS,IER)                                         00000140
      IF(IER.EQ.0) GO TO 100                                            00000150
      WRITE (6,1) IER                                                   00000160
      STOP                                                              00000170
C                                                                       00000180
C                                                                       00000190
C     ESTIMATE CORRECTIONS AND PROBABLE ERRORS                          00000200
  100 DO 110 I=1,NP                                                     00000210
  110 E(I)=0.                                                           00000220
      KD=0                                                              00000230
      DO 130 I=1,NP                                                     00000240
      IA=I-1                                                            00000250
      IF(IA.EQ.0) GO TO 125                                             00000260
C                                                                       00000270
      DO 120 J=1,IA                                                     00000280
      KD=KD+1                                                           00000290
      E(I)=E(I)+PSI(KD)*CWD(J)                                          00000300
  120 E(J)=E(J)+PSI(KD)*CWD(I)                                          00000310
  125 KD=KD+1                                                           00000320
  130 E(I)=E(I)+PSI(KD)*CWD(I)                                          00000330
C                                                                       00000340
C     PROBABLE ERRORS - FIRST CORRECTED VALUE OF STANDARD DEVIATION     00000350
      DO 140 I=1,NP                                                     00000360
  140 SIGMA=SIGMA-E(I)*CWD(I)                                           00000370
      DIV=NPTS-NP                                                       00000380
      SIGMA=DSQRT(SIGMA/DIV)                                            00000390
      DO 150 I=1,NP                                                     00000400
      KD=I*(I+1)/2                                                      00000410
  150 CWD(I)=DSQRT(PSI(KD))*SIGMA                                       00000420
C                                                                       00000430
C       EVALUATE COVARIANCE MATRIX                                      00000440
      SIGSQ=SIGMA*SIGMA                                                 00000450
      DO 160 I=1,KD                                                     00000460
  160 PSI(I)=PSI(I)*SIGSQ                                               00000470
C                                                                       00000480
      RETURN                                                            00000490
    1 FORMAT(' DOUBTFUL WHETHER MATRIX FOR INVERSION IS POSITIVE DEFINIT00000500
     $E , IER=',I2)                                                     00000510
      END                                                               00000520
```

APPENDIX V

YAWSONDE CALIBRATION

Consider the yawsonde depicted in figure 4. It consists of two sun slits at angles γ_1 and γ_2 to the longitudinal axis of the shell as shown in figure 4(c). The centres of the slits are separated by rotation about the shell longitudinal axis through an angle β in figure 4(b). When the shell rotates during flight a train of pulses of opposite polarity like the train in figure 4(d) with period T, the roll period, is emitted by each sensor. The relationship connecting the time, τ, between pulses of opposite polarity and the complementary solar aspect angle, σ_N in figure 4(a), is the calibration curve of the yawsonde. This appendix is concerned with the derivation of that curve. It is assumed during the derivation that the shell does not pitch or yaw so that the direction of the longitudinal axis remains constant. The errors arising from this assumption will not be significant if the roll rate is much greater than the nutation and precession rates, as is generally the case.

If the sun is detected by sensor 1 of the yawsonde then it lies in the half plane defined by the relations,

$$x'' \sin\gamma_1 - y'' \cos\gamma_1 = 0, \quad z'' \geq 0 \tag{19}$$

where the axes system Sx"y"z" is fixed in the shell and rotates with it; Sx" is forward along the shell axis, Sz" is outwards through the centre of sensor 1 and Sy" completes a right handed set as shown in figure 4. The sun is likewise detected by sensor 2 when it lies in the half plane defined by the relations

$$x'' \sin\gamma_2 - y'' \cos\gamma_2 \cos\beta - z'' \cos\gamma_2 \sin\beta = 0,$$

$$z'' \cos\beta - y'' \sin\beta \geq 0. \tag{20}$$

If the angular velocity component about the longitudinal axis is P_s, then

$$2\pi p_s = 1/T,$$

and the direction cosines of the sun in terms of the complementary solar aspect angle are

$$(\sin\sigma_N, \cos\sigma_N \sin p_s t, \cos\sigma_N \cos p_s t) \tag{21}$$

where t is time measured from when the sun is in the Sx"z" plane. Hence using equations (19) to (21), the complementary solar aspect angle can be determined from the measured time interval, τ, by the relation

$$\tan\sigma_N = -\sin(2\pi\tau/T + \beta) \,/\, [\tan^2\gamma_1 + \tan^2\gamma_2 - 2\tan\gamma_1 \tan\gamma_2 \cos(2\pi\tau/T + \beta)]^{\frac{1}{2}} \tag{22}$$

In order to obtain a symmetrical calibration curve and make optimum use of the yawsonde, the nominal values which are usually chosen for the parameters in equation (22) are $\gamma_1 = -\gamma_2 = \gamma_1$ where γ is generally between $\pi/6$ and $\pi/4$, and

$\beta = \pi$. If we substitute into equation (22) for γ_1, γ_2 and β we find that

$$\tan \sigma_N = \cos (\pi\tau/T) \; / \; \tan \gamma. \tag{23}$$

It follows from the expression (23), since $0 \leqslant \tau \leqslant T$, that the range of complementary solar aspect angles which can be measured by the yawsonde is

$$-(\pi/2 - \gamma) \leqslant \sigma_N \leqslant +(\pi/2-\gamma). \tag{24}$$

Figure 5 shows a typical yawsonde calibration curve of the type described by equation (23) with $\gamma = \pi/4$. The curve is approximately linear over much of its range. The extremities of the curve are not of much interest since experiments will generally be designed to avoid this area for two reasons. Firstly pulses from the two sensors will tend to interfere and the experimental record will be difficult to decipher. Secondly, the finite dimensions of the sun slits, which have been ignored in deriving the calibration curve (22), will make the calibration invalid for values of τ/T approaching 0 or 1.

The slope of the curve at $\tau/T = 1/2$ will be a good measure of the overall sensitivity of the yawsonde, that is,

$$d\sigma_N \Big| d(\tau/T) \Big|_{\tau/T=1/2} = -\pi/\tan\gamma.$$

Clearly it is necessary to compromise when choosing a value of γ, the angle each slit makes with the longitudinal axis of the shell. If γ is much greater than $\pi/4$ the sensitivity given in this equation begins to fall away rapidly. However, the value of γ should be as large as possible to provide an adequate range of measurement as defined by relation (24), particularly since it is not practical to use the total range for τ, from 0 to T.

APPENDIX VI

LISTING OF PROGRAM YAWCAL

```
C     RLP,FORT(YAWCAL)                                                  00000010
C     A PROGRAM TO FIT A THEORETICAL YAWSONDE CALIBRATION CURVE TO      00000020
C     THE MEASURED CALIBRATION CURVE .                                  00000030
C                                                                       00000040
      EXTERNAL FUNCT                                                    00000050
C                                                                       00000060
      DIMENSION X(3),G(3),H(15),TITLE(18)                               00000070
C                                                                       00000080
      COMMON /DATA/ T(100),S(100),NPTS,RES(100)                         00000090
C                                                                       00000100
      N=3                                                               00000110
      EPS=0.000001                                                      00000120
      LIMIT=100                                                         00000130
C                                                                       00000140
      READ (5,1) TITLE                                                  00000150
      READ (5,2) X                                                      00000160
      READ (5,3) NPTS                                                   00000170
C                                                                       00000180
      DO 100 J=1,NPTS                                                   00000190
      READ (5,7) T(J),S(J)                                              00000200
  100 S(J)=TAN(0.01745329*S(J))                                         00000210
C                                                                       00000220
      CALL RMFP(FUNCT,N,X,F,G,EST,EPS,LIMIT,IER,H)                      00000230
C                                                                       00000240
      WRITE (6,4) TITLE                                                 00000250
      WRITE (6,5) X,EST,IER                                             00000260
      WRITE (6,6) (T(I),S(I),RES(I),I=1,NPTS)                           00000270
      WRITE (8,6) (T(I),S(I),RES(I),I=1,NPTS)                           00000280
C                                                                       00000290
      STOP                                                              00000300
    1 FORMAT(18A4)                                                      00000310
    2 FORMAT(4F10.0)                                                    00000320
    3 FORMAT(I10)                                                       00000330
    4 FORMAT(1H1,18A4)                                                  00000340
    5 FORMAT('0PARAMETERS',3E15.6/' ESTIMATED MINIMUM',E15.6/' IER=',I3)00000350
    6 FORMAT(' TIME RATIO   TAN(SIGMAN)  RESIDUE'/100(1X,F8.2,F12.3,   00000360
     $      F15.6/))                                                   00000370
    7 FORMAT(2F10.0)                                                    00000380
      END                                                               00000390
      SUBROUTINE FUNCT(N,X,F,G)                                         00000400
C                                                                       00000410
C     A ROUTINE TO ESTIMATE THE VALUE AND GRADIENT OF THE SUM OF THE    00000420
C     SQUARES OF THE RESIDUALS                                          00000430
C                                                                       00000440
      DIMENSION X(1),G(1)                                               00000450
C                                                                       00000460
      COMMON /DATA/ T(100),S(100),NPTS,RES(100)                         00000470
C                                                                       00000480
      DATA PI /3.14159277/                                              00000490
C                                                                       00000500
      F=0                                                               00000510
      DO 100 I=1,3                                                      00000520
  100 G(I)=0.                                                           00000530
C                                                                       00000540
      DO 200 I=1,NPTS                                                   00000550
      XX=PI*T(I)-X(3)                                                   00000560
C                                                                       00000570
      IF(X(2).NE.-1.) GO TO 150                                         00000580
      SINX=SIN(XX)                                                      00000590
      COSX=COS(XX)                                                      00000600
      TSN=SINX/X(1)                                                     00000610
      G2=2.*SINX*COSX*COSX                                              00000620
      G3=-COSX/X(1)                                                     00000630
      GO TO 170                                                         00000640
C                                                                       00000650
  150 COSX=COS(2.*XX)                                                   00000660
      SINX=SIN(2.*XX)                                                   00000670
      ROOT=SQRT(1.+X(2)*X(2)-2.*X(2)*COSX)                              00000680
      TSN=SINX/(X(1)*ROOT)                                              00000690
      G2=SINX*(COSX-X(2))/(X(1)*ROOT)                                   00000700
      G3=2.*(X(2)-(1.+X(2)*X(2)-X(2)*COSX))/(X(1)*ROOT*ROOT*ROOT)       00000710
C                                                                       00000720
  170 RES(I)=TSN-S(I)                                                   00000730
      F=F+RES(I)*RES(I)                                                 00000740
      G(1)=G(1)-2.*RES(I)*TSN/X(1)                                      00000750
      G(2)=G(2)+2.*RES(I)*G2                                            00000760
      G(3)=G(3)+2.*RES(I)*G3                                            00000770
C                                                                       00000780
  200 CONTINUE                                                          00000790
      WRITE (6,*) F                                                     00000800
C                                                                       00000810
      RETURN                                                            00000820
      END                                                               00000830
```


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APPENDIX VII

SOLAR ASPECT ANGLE

In order to use the data on the complementary solar aspect angle which is measured with the yawsonde one must know the position of the sun. The solution for the azimuth and elevation of the sun in range axes is given in reference 14, obtained using the method given in reference 15. The derivation uses the sidereal time, $(ST)_o$, corresponding to the Universal or Greenwich mean time at which the position of the sun is required. The sidereal time is the Hour Angle of the first point of Aries. The right ascension, RA, and the declination, δ, of the sun are also used. All three can be obtained from the Astronomical Ephemeris(ref.16) by interpolating between values given for each variable at 0 hours Universal Time, UT. Then, the local sidereal time,

$$LST = (ST)_o + \frac{366.2422}{365.2422} UT - \Phi,$$

where Φ is the longitude of the shell, and the hour angle of the sun,

$$H = LST - RA,$$

so that the azimuth, ψ_s, and elevation, θ_s, of the sun in earth axes are given by

$$\sin \theta_s = \sin \delta \sin \Theta + \cos \delta \cos \Theta \cos H$$

$$\cos \psi_s \cos \theta_s = \sin \delta \cos \Theta - \cos \delta \sin \Theta \cos H$$

$$\sin \psi_s \cos \theta_s = \sin H \cos \delta.$$

It must be remembered that the azimuth, ψ_s, is measured relative to true north, so that if Ψ is the azimuth of the shell relative to true north, the complementary solar aspect angle is given by

$$\sin \sigma_N = \cos \theta_s \cos \psi_s \cos \theta \cos \Psi + \cos \theta_s \sin \psi_s \cos \theta \sin \Psi + \sin \theta_s \sin \theta$$

where θ is the angle of elevation of the shell. The angle Ψ has two components, Ψ_o, the bearing of the line of fire of the shell and ψ, the angle of azimuth of the shell relative to the line of fire, so that $\Psi = \Psi_o + \psi$. The above expression can be rearranged to give

$$\sin \sigma_N = \cos \theta_s \cos \theta \cos (\psi_s - \Psi) + \sin \theta \sin \theta_s \qquad (25)$$

so that the origin used to measure the angles of azimuth is no longer relevant and the most convenient origin can be used. The expression (25) is needed both for trials planning and data analysis. For trials planning purposes, it is generally convenient to refer both azimuth angles to true north because the bearing of the line of fire is used to approximate the azimuth of the shell. However, in analysing flight data it is more convenient to refer azimuth measurements to range axes. Details of how the expression (25) is used are given in Section 4.

APPENDIX VIII

LISTING OF PROGRAM ASPECT

```
C     RLP,FORT(ASPECT)                                                        00000010
C     A FORTRAN PROGRAM TO ESTIMATE THE COMPLEMENTRAY SOLAR ASPECT ANGLE      00000020
C     FROM DATA SUPPLIED BY THE EPHEMERIS TABLES                              00000030
C                                                                             00000040
      REAL *4 LAT,LONG,LAMBDA,L,M,N,II,JJ                                     00000050
C                                                                             00000060
      DIMENSION HA(2,3),RA(2,3),DE(2,3),HANG(2),RASC(2),DECL(2),              00000070
     $          TITLE(18),DATES(5),GLABEL(6),XAXIS(12),YAXIS(12),             00000080
     $          LAT(3),LONG(3),BEAR(3),SIGMAN(102,10),L(100),M(100),          00000090
     $          N(100),THETA(100),TIME(10),NTYPE(10),ITIME(10),               00000100
     $          AA(102),BB(102),CC(102),DD(102),EE(102),FF(102),GG(102),      00000110
     $          HH(102),II(102),JJ(102)                                       00000120
C                                                                             00000130
      DATA  AAA,BBB,CCC,DDD,TDIFF,TRATIO,PI,EEE /15.,0.0166667,               00000140
     $        0.0174532,57.2958,9.5,1.00273791,3.1415927,0.04166667/,         00000150
     $     XAXIS /-90.,10.,18.,32.,'COMP','LEME','NTAR','Y SO','LAR ',        00000160
     $           'ASPE','CT A','NGLE'/,                                        00000170
     $     YAXIS /-90.,18.,10.,24.,'ELEV','ATIO','N OF',' SHE','LL A',         00000180
     $           'XIS '/                                                       00000190
C                                                                             00000200
      EQUIVALENCE (AA(1),SIGMAN(1)),(BB(1),SIGMAN(103)),                      00000210
     $    (CC(1),SIGMAN(205)),(DD(1),SIGMAN(307)),(EE(1),SIGMAN(409)),        00000220
     $    (FF(1),SIGMAN(511)),(GG(1),SIGMAN(613)),(HH(1),SIGMAN(715)),        00000230
     $    (II(1),SIGMAN(817)),(JJ(1),SIGMAN(919)),                            00000240
     $          (NA,NTYPE(1)),(NB,NTYPE(2)),(NC,NTYPE(3)),(ND,NTYPE(4)),     00000250
     $          (NE,NTYPE(5)),(NF,NTYPE(6)),(NG,NTYPE(7)),(NH,NTYPE(8)),     00000260
     $          (NI,NTYPE(9)),(NJ,NTYPE(10))                                 00000270
C                                                                             00000280
C     INPUT DATA                                                              00000290
C     THE FOLLOWING INPUT DATA IS REQUIRED                                    00000300
C          - JOB TITLE                                                        00000310
C          - HOUR ANGLE OF ARIES IN HOURS,MINUTES AND SECONDS                 00000320
C          - APPARENT RIGHT ASCENSION IN HOURS,MINUTES AND SECONDS            00000330
C          - APPARENT DECLINATION IN DEGREES,MINUTES AND SECONDS              00000340
C          - LATITUDE AND LONGITUDE OF RANGE ORIGIN IN DEGREES,               00000350
C     MINUTES AND SECONDS, WHERE LATITUDE IS POSITIVE NORTH AND               00000360
C     LONGITUDE IS POSITIVE WEST                                              00000370
C          - NUMBER OF VALUES OF LOCAL TIME TO BE USED TOGETHER WITH          00000380
C     INITIAL TIME AND INCREMENT IN HOURS                                     00000390
C          - BEARING OF FIRING LINE IN DEGREES,MINUTES AND SECONDS            00000400
C          - NUMBER OF VALUES OF ELEVATION TOGETHER WITH INITIAL VALUE        00000410
C     AND INCREMENT IN DEGREES                                                00000420
C                                                                             00000430
      READ (5,1) TITLE                                                        00000440
      WRITE (6,2) TITLE                                                       00000450
C                                                                             00000460
      READ (5,3) ((HA(I,J),J=1,3),I=1,2),((RA(I,J),J=1,3),I=1,2)              00000470
     $          ,((DE(I,J),J=1,3),I=1,2)                                      00000480
      READ (5,1) DATES                                                        00000490
      DO 100 I=1,2                                                            00000500
      HANG(I)=AAA*(HA(I,1)+BBB*(HA(I,2)+BBB*HA(I,3)))                         00000510
      RASC(I)=AAA*(RA(I,1)+BBB*(RA(I,2)+BBB*RA(I,3)))                         00000520
  100 DECL(I)=(DE(I,1)+BBB*(DE(I,2)+BBB*DE(I,3)))                             00000530
      WRITE (6,4) DATES,HANG,RASC,DECL                                        00000540
      DO 200 I=1,2                                                            00000550
      HANG(I)=HANG(I)*CCC                                                     00000560
      RASC(I)=RASC(I)*CCC                                                     00000570
  200 DECL(I)=DECL(I)*CCC                                                     00000580
C                                                                             00000590
      READ (5,3) LAT,LONG                                                     00000600
      LAMBDA=LAT(1)+BBB*(LAT(2)+BBB*LAT(3))                                   00000610
      PHI=LONG(1)+BBB*(LONG(2)+BBB*LONG(3))                                   00000620
      WRITE (6,5) LAMBDA,PHI                                                  00000630
      LAMBDA=LAMBDA*CCC                                                       00000640
      PHI=PHI*CCC                                                             00000650
C                                                                             00000660
      READ (5,6) NTIMES                                                       00000670
      READ (5,3) TO,DT                                                        00000680
C                                                                             00000690
      READ (5,3) BEAR                                                         00000700
      PSI=BEAR(1)+BBB*(BEAR(2)+BBB*BEAR(3))                                   00000710
      WRITE (6,7) PSI                                                         00000720
      PSI=PSI*CCC                                                             00000730
      COSPSI=COS(PSI)                                                         00000740
      SINPSI=SIN(PSI)                                                         00000750
C                                                                             00000760
      READ (5,6) NELEVN                                                       00000770
      READ (5,3) ELO,DEL                                                      00000780
C                                                                             00000790
C     COMMENCE CALCULATION - FIRST FIND AZIMUTH AND ELEVATION OF THE          00000800
C     SUN AT A GIVEN TIME                                                     00000810
C                                                                             00000820
      DO 300 J=1,NELEVN                                                       00000830
      THETA(J)=CCC*(ELO+(J-1)*DEL)                                            00000840
      COSTHE=COS(THETA(J))                                                    00000850
      SINTHE=SIN(THETA(J))                                                    00000860
      L(J)=COSTHE*COSPSI                                                      00000870
      M(J)=COSTHE*SINPSI                                                      00000880
      N(J)=-SINTHE                                                            00000890
  300 CONTINUE                                                                00000900
      WRITE (6,8)                                                             00000910
C                                                                             00000920
      DO 500 I=1,NTIMES                                                       00000930
      TIME(I)=TO+(I-1)*DT                                                     00000940
      UT=TIME(I)-TDIFF                                                        00000950
      DELTA=0.0416666667*UT                                                   00000960
      UT=AAA*UT*CCC                                                           00000970
      MARIES=HANG(1)+DELTA*(HANG(2)-HANG(1))                                  00000980
      D=DECL(1)+DELTA*(DECL(2)-DECL(1))                                       00000990
      R=RASC(1)+DELTA*(RASC(2)-RASC(1))                                       00001000
C                                                                             00001010
C     LOCAL SIDERIAL TIME                                                     00001020
      ALST=MARIES+TRATIO*UT-PHI                                               00001030
C                                                                             00001040
C     HOUR ANGLE                                                              00001050
      H=ALST-R                                                                00001060
C                                                                             00001070
```

```
C     ELEVATION AND AZIMUTH                                              00001080
C                                                                        00001090
      SINH=SIN(H)                                                        00001100
      COSH=COS(H)                                                        00001110
      SINL=SIN(LAMBDA)                                                   00001120
      COSL=COS(LAMBDA)                                                   00001130
      SIND=SIN(D)                                                        00001140
      COSD=COS(D)                                                        00001150
      SUNL=SIND*COSL-COSD*SINL*COSH                                      00001160
      SUNM=-SIN*COSD                                                     00001170
      SUNN=-SIND*SINL-COSD*COSL*COSH                                     00001180
      EL=ASIN(-SUNN)*DDD                                                 00001190
      AZ=ATAN2(SUNM,SUNL)*DDD                                            00001200
      ITIME(I)=TIME(I)                                                   00001210
      X=TIME(I)-ITIME(I)                                                 00001220
      ITIME(I)=100.*ITIME(I)+60.*X+0.5                                   00001230
      WRITE (6,9) ITIME(I),AZ,EL,SUNL,SUNM,SUNN                          00001240
C                                                                        00001250
C     ESTIMATE COMPLEMENTARY SOLAR ASPECT ANGLE                          00001260
C                                                                        00001270
      DO 400 J=1,NELEVN                                                  00001280
      COSSIG=SUNL*EL(J)+SUNM*M(J)+SUNN*N(J)                              00001290
  400 SIGMAN(J,I)=(ACOS(COSSIG)-0.5*PI)*DDD                              00001300
C                                                                        00001310
  500 CONTINUE                                                           00001320
C                                                                        00001330
      DO 550 J=1,NELEVN                                                  00001340
  550 THETA(J)=THETA(J)*DDD                                              00001350
      WRITE (6,10)                                                       00001360
      WRITE (6,11) (ITIME(I),I=1,NTIMES)                                 00001370
      WRITE (6,12)                                                       00001380
      DO 600 J=1,NELEVN                                                  00001390
  600 WRITE (6,13) THETA(J),(SIGMAN(J,I),I=1,NTIMES)                     00001400
C                                                                        00001410
      DO 700 J=1,10                                                      00001420
  700 NTYPE(J)=0                                                         00001430
      DO 800 J=1,NTIMES                                                  00001440
  800 NTYPE(J)=NELEVN                                                    00001450
      DO 900 J=1,5                                                       00001460
  900 GLABEL(J+1)=DATES(J)                                               00001470
      GLABEL(1)=20.                                                      00001480
      CALL GRAPH(GLABEL,XAXIS,YAXIS,NA,AA,THETA,NB,BB,THETA,NC,CC,THETA,00001490
     $           ND,DD,THETA,NE,EE,THETA,NF,FF,THETA,NG,GG,THETA,        00001500
     $           NH,HH,THETA,NI,II,THETA,NJ,JJ,THETA)                    00001510
      CALL ENDGR                                                         00001520
C                                                                        00001530
      STOP                                                               00001540
C                                                                        00001550
    1 FORMAT(18A4)                                                       00001560
    2 FORMAT('1 SOLAR ASPECT ANGLE FOR ',18A4)                           00001570
    3 FORMAT(6F10.6)                                                     00001580
    4 FORMAT('0ON THE ',5A4,' AT 0HR, U.T. THE HOUR ANGLES OF ARIES ARE'00001590
     $     ,2F10.4,' DEGREES'/41X,'THE APPARENT RIGHT ASCENSIONS ARE',   00001600
     $      2F10.4,' DEGREES'/41X,' AND THE APPARENT DECLINATIONS ARE',  00001610
     $      2F10.4,' DEGREES')                                          00001620
    5 FORMAT('ORANGE ORIGIN IS AT LATITUDE',F10.4,' DEGREES AND LONGITUD00001630
     $E',F10.4,' DEGREES')                                               00001640
    6 FORMAT(12I5)                                                       00001650
    7 FORMAT('OBEARING OF LINE OF FIRE IS',F10.4,' DEGREES TRUE')        00001660
    8 FORMAT('0TIME(STD)  AZIMUTH  ELEVATION       L           M        N'00001670
     $      /'   (HRS)      (DEG)      (DEG) ')                         00001680
    9 FORMAT(1X,I8,F10.2,F11.3,3F10.6)                                   00001690
   10 FORMAT('1 COMPLEMENTARY SOLAR ASPECT ANGLES')                      00001700
   11 FORMAT('0TIME(HRS)',20I6)                                          00001710
   12 FORMAT(' ELEVATION'/'   (DEG)')                                    00001720
   13 FORMAT(1X,F9.3,20F6.1)                                             00001730
      END                                                                00001740
```

APPENDIX IX

PARTIAL DERIVATIVES WITH RESPECT TO PARAMETERS FOR YAWSONDE

The parameter estimation algorithm which is used to analyse the yawsonde data requires, as input, values of partial derivatives of the complementary solar aspect angle with respect to each of the parameters. These partial derivatives are obtained by partial differentiation of the expression (25) which defines the complementary solar aspect angle. Thus the partial derivatives are defined by the relation

$$\cos\sigma_N \frac{\partial\sigma_N}{\partial P_i} = -[\cos\theta_s \sin\theta \cos(\psi_s - \psi) + \sin\theta_s \cos\theta] \frac{\partial\theta}{\partial P_i} + \cos\theta_s \cos\theta \sin(\psi_s - \psi) \frac{\partial\psi}{\partial P_i} \tag{26}$$

The partial derivatives needed to evaluate equation (26) are obtained by numerical integration of a set of ordinary differential equations which are derived from the mathematical model equations (9) by partial differentiation. This results in the following equations being obtained

$$\frac{d}{dt}\left(\frac{\partial\psi}{\partial Pi}\right) = \frac{\partial r}{\partial Pi} \sec\theta + r \sec\theta \tan\theta \frac{\partial\theta}{\partial Pi},$$

$$\frac{d}{dt}\left(\frac{\partial\theta}{\partial Pi}\right) = \frac{\partial q}{\partial Pi}$$

$$\frac{d}{dt}\left(\frac{\partial q}{\partial Pi}\right) = [p - (I_x/I)p_s] \frac{\partial r}{\partial Pi} + r \frac{\partial p}{\partial Pi} + \left(\frac{QSd}{I}\right)\Bigg\{\left[\frac{\partial Cm}{\partial\zeta}\cos\phi - \left(\frac{p_s d}{2V}\right)\frac{\partial Cnp}{\partial\delta}\sin\phi + \frac{\partial C_{m_q}}{\partial\zeta}\left(\frac{qd}{2V}\right)\right]\frac{\partial\zeta}{\partial Pi} - \left[C_m \sin\phi + C_{m_p}\left(\frac{p_s d}{2V}\right)\cos\phi\right]\frac{\partial\phi}{\partial Pi} + C_{m_q}\left(\frac{d}{2V}\right)\frac{\partial q}{\partial Pi} + \frac{\partial C_m}{\partial Pi}\cos\phi - \frac{\partial C_{n_p}}{\partial Pi}\left(\frac{p_s d}{2V}\right)\sin\phi + \frac{\partial C_{m_q}}{\partial Pi}\left(\frac{qd}{2V}\right)\Bigg\}, \tag{27}$$

$$\frac{d}{dt}\left(\frac{\partial r}{\partial Pi}\right) = [-p + (I_x/I)p_s]\frac{\partial q}{\partial Pi} - q\frac{\partial p}{\partial Pi} + \left(\frac{QSd}{I}\right)\Bigg\{\left[\frac{\partial C_m}{\partial\zeta}\sin\phi + \frac{\partial C_{n_p}}{\partial\zeta}\left(\frac{p_s d}{2V}\right)\cos\phi + \frac{\partial C_{m_q}}{\partial\zeta}\left(\frac{rd}{2V}\right)\right]\frac{\partial\zeta}{\partial Pi} + \left[C_m\cos\phi - C_{n_p}\left(\frac{p_s d}{2V}\right)\sin\phi\right]\frac{\partial\phi}{\partial Pi} + C_{m_q}\left(\frac{d}{2V}\right)\frac{\partial q}{\partial Pi} + \frac{\partial C_m}{\partial Pi}\sin\phi + \frac{\partial C_{n_p}}{\partial Pi}\left(\frac{p_s d}{2V}\right)\cos\phi + \frac{\partial C_{m_q}}{\partial Pi}\left(\frac{rd}{2V}\right)\Bigg\}.$$

By partial differentiation of equation (10) we obtain the relation

$$\frac{\partial p}{\partial Pi} = -\frac{\partial r}{\partial Pi}\tan\theta - r\sec^2\theta\frac{\partial\theta}{\partial Pi}$$

Using the parametric representations of the aerodynamic coefficients defined in equation (12) we can obtain the following relations by partial differentiation

$$\frac{\partial C_m}{\partial \zeta} = P_5 + P_{12}M + 3P_6 \zeta^2 + 5P_7 \zeta^4 ,$$

$$\frac{\partial C_{n_p}}{\partial \zeta} = P_8 + 3P_9 \zeta^2$$

$$\frac{\partial C_{m_q}}{\partial \zeta} = 2P_{11} \zeta ,$$

$$\frac{\partial C_m}{\partial Pi} = (\delta_{5i} + \delta_{12i} M) \zeta + \delta_{6i} \zeta^3 + \delta_{7i} \zeta^5 , \qquad (28)$$

$$\frac{\partial C_{n_p}}{\partial Pi} = \delta_{8i} \zeta + \delta_{9i} \zeta^3 ,$$

$$\frac{\partial C_{m_q}}{\partial Pi} = \delta_{10i} + \delta_{11i} \zeta^2 .$$

Finally the partial derivatives of the magnitude and the orientation of the incidence can be obtained from the definitions given in equations (13) and (14), so that

$$\frac{\partial \zeta}{\partial Pi} = u \left(v \frac{\partial v}{\partial Pi} + w \frac{\partial w}{\partial Pi}\right) / V^2 (v^2+w^2)^{\frac{1}{2}} - (v^2+w^2)^{\frac{1}{2}} \frac{\partial u}{\partial Pi} / V^2 \qquad (29)$$

$$\frac{\partial \phi}{\partial Pi} = \left(v \frac{\partial w}{\partial Pi} - w \frac{\partial v}{\partial Pi}\right) / (v + w^2)$$

where

$$\frac{\partial u}{\partial Pi} = -w \frac{\partial \theta}{\partial Pi} + v \cos\theta \frac{\partial \psi}{\partial Pi}$$

$$\frac{\partial v}{\partial Pi} = -(V_1 \cos\psi + V_2 \sin\psi) \frac{\partial \psi}{\partial Pi}$$

$$\frac{\partial w}{\partial Pi} = u \frac{\partial \theta}{\partial Pi} + v \sin\theta \frac{\partial \psi}{\partial Pi}$$

The relations (28) and (29) can be used to evaluate the terms in the differential equations (27). It then becomes possible to integrate the differential equations numerically and so find the values of the partial derivatives needed to evaluate the equations (26).

APPENDIX X

LISTING OF PROGRAM YAWSONDE

```
C     PROGRAM = SHELL,YAWSONDE,PARAM,RST - MAIN PROGRAM (YAWSONDE)
      IMPLICIT REAL *8 (A-H,O-Z)
C
C     THE PROGRAMME USES SIMPLE NON-LINEAR POLYNOMIAL REPRESENTATIONS
C     OF PITCHING MOMENT, PITCH DAMPING MOMENT AND MAGNUS MOMENT.
C     ONLY THE ANGULAR MOTION IS ANALYSED.
C
      DIMENSION PARAM(12),ORDER(12)
C
      COMMON /MODEL/  REFS,RODIAM,PITCHI,RATIOI,
     $                XF(12),X(12),XX(12),JORDER(12)
     $        /COMM/  TO,REJECT,ACCFAC,MMAX,NPARAM,ITERM,LCASE,NPLOT
     $        /DATA/  CMS(2,100),PI(2),NPTS,ITER,IEND
     $        /INT/   Y(53),DY(53),MO(53),YA(53),Z(12),N,IOBS,MAX
     $        /NORMS/ ES(17),CMO(12),F(12)
C
      DATA PARAM /'  PSID  ','  THETAD','    QD  ','    RD  ','   CM1  '
     $          ','   CM3  ','   CM5  ','  CNP1  ','  CNP3  ','  CM00  '
     $          ','  CM02  ','  CMMACH '/
C
C     PROGRAM NAME AND OUTPUT HEADINGS
      WRITE (6,1)
C
C     INPUT DATA
  100 CALL INPUT
      ITER=0
      IEND=0
C
C     PREPARE TO PLOT OUTPUT IF REQUESTED
      IF(NPLOT.NE.0) CALL PLOTT
C
C     RECORD PARAMETERS AND ACCURACY VALUES IN ORDER REQUESTED
C
      DO 110 I=1,12
      K=JORDER(I)
      ORDER(K)=PARAM(I)
      XX(K)=X(I)
  110 CMO(K)=XF(I)
C
  120 DO 130 I=1,2
      IA=12*(I-1)+1
      IB=12*I
      IF(IB.GT.12) IB=12
      WRITE (6,2) (ORDER(J),J=IA,IB)
      WRITE (6,3) (XX(J),J=IA,IB)
  130 WRITE (6,4) (CMO(J),J=IA,IB)
C
C     CHECK WHETHER EACH INTEGRATION STEP WILL BE PRINTED
      IF(((IEND.EQ.1).OR.(ITER.GE.ITERM)).AND.(NPLOT.NE.0)) WRITE (6,5)
C
C     INITIALISE VARIABLES FOR INTEGRATION
      CALL INCOND
C
C     CLEAR MATRICES AND COUNTERS
C
      ISO=0
      J=NPARAM*(NPARAM+1)/2
      DO 140 I=1,J
  140 PSI(I)=0.
      DO 150 I=1,NPARAM
  150 CMO(I)=0.
      SIGMA=0.
C
C     DO LOOP TO SET UP NORMAL EQUATIONS
C
      DO 210 IOBS=2,NPTS
C
C     ESTIMATE MODEL PREDICTIONS AND PARTIAL DERIVATIVES
      CALL SIMUL
C
C     TEST FOR OUTLIERS AND ADD IN CONTRIBUTION TO SQUARE ERROR
      F(2)=OBS(2,IOBS)-Y(2)
      IF(DABS(F(2)).GT.*REJECT) GO TO 210
      SIGMA=SIGMA+F(2)*F(2)
      ISO=ISO+1
C
C     IF LAST PRINT - PRINT RESIDUALS, AVOID NORMAL EQUATIONS AND
C     PLOT DATA IF REQUIRED
C
      IF(((IEND.NE.1).AND.(ITER.LT.ITERM)).OR.(NPLOT.EQ.0)) GO TO 170
      WRITE (6,6) F(2)
      IF(NPLOT.NE.0) CALL PLOTA
      GO TO 210
C
C     ADD CONTRIBUTIONS TO NORMAL EQUATIONS
C
  170 KO=0
      LO=0
C
      DO 190 I=1,NPARAM
      LO=LO+1
C
      DO 180 J=1,I
      KO=KO+1
  180 PSI(KO)=PSI(KO)+Z(I)*Z(J)
C
  190 CMO(LO)=CMO(LO)+Z(I)*F(2)
  200 CONTINUE
C
  210 CONTINUE
C
C     IF THIS IS THE FINAL PRINT STEP - PLOT AS REQUIRED AND FINISH
C     THIS STEP
C
      IF(((IEND.NE.1).AND.(ITER.LT.ITERM)) GO TO 220
      IF(NPLOT.NE.0) CALL PLOTA
      GO TO 290
C
C     CHECK WHETHER SUFFICIENT DATA HAS BEEN ACCEPTED
  220 IF(NPARAM.LT.ISO) GO TO 230
      WRITE (6,7) ISO
      GO TO 240
C
C     RESET DATA REJECTION LEVEL
  230 SO=ISO
      REJECT=4.*DSQRT(SIGMA/SO)
C
C     SOLVE NORMAL EQUATIONS
      CALL SOLVE(NPARAM,SIGMA,ISO)
C
C     ITERATION COUNTER UPDATE
      ITER=ITER+1
C
C     UPDATE PARAMETER VALUES
      DO 240 I=1,NPARAM
  240 XX(I)=XX(I)+F(I)
      DO 250 I=1,12
  250 X(I)=XX(JORDER(I))
C
C     WRITE STANDARD DEVIATION AND DEGREES OF FREEDOM
      WRITE (6,8) SIGMA,ISO
C
C     CHECK PARAMETER CHANGES FOR CONVERGENCE
C
      DO 270 J=1,NPARAM
      DO 260 I=1,12
      IF(JORDER(I).NE.J) GO TO 260
      IF(DABS(F(J)).GT.(ACCFAC*XF(I))) GO TO 120
  260 CONTINUE
C
  270 CONTINUE
      IEND=1
C
C     RECORD COVARIANCE MATRIX ON ACHIEVING CONVERGENCE
C
      WRITE (6,9)
      KR=0
      DO 280 I=1,NPARAM
      KA=KR+1
      KR=KA+I-1
  280 WRITE (6,10) (PSI(J),J=KA,KR)
      WRITE (6,11)
      GO TO 120
C
C     IF ANOTHER CASE IS TO BE RUN RESTART PROCEDURE
  290 IF(LCASE.NE.0) GO TO 100
```

```
C                                                                       00001570
      STOP                                                              00001580
C                                                                       00001590
    1 FORMAT('1PARAMETER ESTIMATION FOR ANALYSIS OF YAWSONDE DATA FROM R00001600
     $ADAR AND YAWSONDE MEASUREMENTS MADE DURING THE FLIGHT OF A SHELL')00001610
    2 FORMAT('0',10X,A(5X,A8))                                          00001620
    3 FORMAT(' PARAM.     ',4E13.4)                                     00001630
    4 FORMAT(' DELTA      ',4E13.4)                                     00001640
    5 FORMAT('1TIME(S)  COMPLEMENTARY SOLAR   RESIDUE'/                 00001650
     $       '           SOLAR ASPECT ANGLE   (RADIANS)')               00001660
    6 FORMAT('+',27X,1PE13.4)                                           00001670
    7 FORMAT('0NOT ENOUGH DATA LEFT, ISD=',I3)                          00001680
    8 FORMAT('0SIGMA=',F13.5,' DEGREES OF FREEDOM=',I4)                 00001690
    9 FORMAT('1'///' COVARIANCE MATRIX (MATRIX IS SYMMETRIC)'/////)     00001700
   10 FORMAT(1X,1P13E10.2)                                              00001710
   11 FORMAT('0 CHANGES TO THE FOLLOWING PARAMETERS ALL SMALL')         00001720
      END                                                               00001730


      SUBROUTINE ATMOS(Z,RHO,VSOUND,TEMPK,JERR)                         00000010
C                                                                       00000020
C     DEFINING THE STANDARD ICAO ATMOSPHERE WITH COESA EXTENSION TO     00000030
C     32 KILOMETRES AS DEFINED IN ARDU REPORT 1354(13) BY RUNDOCK AND   00000040
C     DIETZ (JULY 196X)                                                 00000050
C                                                                       00000060
C       ALT=GEOMETRICAL ALTITUDE (METRES)                               00000070
C       PA=AMBIENT  PRESSURE (PASCALS)                                  00000080
C  A=VELOCITY OF SOUND (METRES/SECOND)                                  00000090
C       RHO=DENSITY(KILOGRAMS/CUBIC METRE)                              00000100
C       XMU=COEFFICIENT OF VISCOSITY (NEWTON-SECONDS/SQUARE METRE)      00000110
C       THETA=RELATIVE TEMPERATURE )                                    00000120
C       DELTA=RELATIVE PRESSURE     ) RELATIVE TO CONDITIONS AT SEA LEVEL00000130
C       SIGMA=RELATIVE DENSITY      )                                   00000140
C       JERR=1 IF ALTITUDE IS OUTSIDE THE ALLOWABLE RANGE               00000150
C            =0                                                         00000160
C                                                                       00000170
C*****UPDATED VERSION*****SEPTEMBER 1977*****                           00000180
C                                                                       00000190
      JERR=0                                                            00000200
      ALT=0.001*Z                                                       00000210
      IF((ALT.GT.-1.5).AND.(ALT.LT.32.)) GO TO 100                      00000220
      JERR=1                                                            00000230
      WRITE (6,1) ALT                                                   00000240
    1 FORMAT('0ALTITUDE=',E12.4,'(M) IS OUTSIDE ALLOWABLE RANGE IN ATMOS00000250
     $')                                                                00000260
      RETURN                                                            00000270
C                                                                       00000280
  100 IF(ALT.GT.11.) GO TO 200                                          00000290
      THETA=1.-0.0225577*ALT                                            00000300
      DELTA=THETA**5.24537                                              00000310
      GO TO 400                                                         00000320
C                                                                       00000330
  200 IF(ALT.GT.20.) GO TO 300                                          00000340
      THETA=0.751865                                                    00000350
      DELTA=0.224031*EXP(-0.156922*(ALT-11.))                           00000360
      GO TO 400                                                         00000370
  300 THETA=0.751865+0.0036242*(ALT-20.)                                00000380
      DELTA=1.479236*(1.+0.00544474*ALT)**(-31.9241)                    00000390
C                                                                       00000400
  400 TEMP=288.15*THETA                                                 00000410
      SIGMA=DELTA/THETA                                                 00000420
      RHO=1.225*SIGMA                                                   00000430
      VSOUND=20.0467*SQRT(TEMP)                                         00000440
C     PA=101325.*DELTA                                                  00000450
C     XMU=(1.458E-06)*TEMP**1.5/(TEMP+110.4)                            00000460
C                                                                       00000470
      RETURN                                                            00000480
      END                                                               00000490


      SUBROUTINE DERIV                                                  00000010
      IMPLICIT REAL*8 (A-H,O-Z)                                         00000020
C                                                                       00000030
C       SUPPLIES DERIVATIVES TO INTEGRATION ROUTINE                     00000040
C                                                                       00000050
      COMMON /XCOM/L, NEES,LOGIAC,PITCH1,RATIO1,                        00000060
     $            XF(12),X(12),XX(12),JORDER(12)                        00000070
     *       /INT/   Y(6,6),DY(63),H0(63),YA(63),Z(12),N,ICRS,MAX      00000080
     *       /TRAJ/  TIME(1..),XDOT(100),YDOT(100),ZDOT(100),ALT(100),  00000090
     *               ROLL(1:2),NTRAJ                                    00000100
C                                                                       00000110
C     PARAMETERS ARE STORED AS FOLLOWS                                  00000120
C     X(1)=TIME, X(2)=R1, X(3)=THETA, X(4)=R, X(5)=R, X(6)=PHSI/CHX(1), 00000130
C     AND SO ON. DY(I)/ X(J)=Y(6*J+I), WHERE I=2,3,4,5, AND J=1,2,...,12 00000140
C                                                                       00000150
C     X(1)=PSI0, X(2)=THETAC, X(3)=RC, X(4)=R0,                         00000160
C     CH=(X(5)+X(12)*(CH)*ALPHA**(A)*ALPHA**3+X(7)*ALPHA**5             00000170
C     CNR=X(8)*ALPHA**(9)*ALPHA**3                                      00000180
C     CH0=X(10)+X(11)*ALPHA**2                                          00000190
C                                                                       00000200
C     INITIALISE                                                        00000210
C                                                                       00000220
      COSTHE=DCOS(Y(3))                                                 00000230
      SINTHE=DSIN(Y(3))                                                 00000240
      COSPSI=DCOS(Y(2))                                                 00000250
      SINPSI=DSIN(Y(2))                                                 00000260
C                                                                       00000270
C       INTERPOLATE FOR TRAJECTORY VARIABLES                            00000280
      DO 100 I=2,NTRAJ                                                  00000290
      IF(Y(1).GT.TIME(I)) GO TO 100                                     00000300
      DELR=(Y(1)-TIME(I-1))/(TIME(I)-TIME(I-1))                         00000310
      VX=XDOT(I-1)+DELR*(XDOT(I)-XDOT(I-1))                             00000320
      VY=YDOT(I-1)+DELR*(YDOT(I)-YDOT(I-1))                             00000330
      VZ=ZDOT(I-1)+DELR*(ZDOT(I)-ZDOT(I-1))                             00000340
      HEIGHT=ALT(I-1)+DELR*(ALT(I)-ALT(I-1))                            00000350
      PSHELL=ROLL(I-1)+DELR*(ROLL(I)-ROLL(I-1))                         00000360
      GO TO 2                                                           00000370
  100 CONTINUE                                                          00000380
      WRITE (6,1)                                                       00000390
    1 FORMAT('1TIME OUTSIDE TABULATED RANGE IN TRAJECTORY DATA')        00000400
C                                                                       00000410
    2 CALL ATMOS(HEIGHT,XMU,PA,TEMPK,RHO,VSOUND)                        00000420
C                                                                       00000430
C     VELOCITY COMPONENTS AND INCIDENCE                                 00000440
      VX=INDEX-X(1)                                                     00000450
      VY=INDEX-Y(1)                                                     00000460
      VZ=INDEX                                                          00000470
      U=VX*(INDEX*COSTHE*COSPSI+VY*(-INDEX*COSTHE*SINPSI-VZ*INDEX*SINTHE 00000480
      V=-VX*(INDEX*SINPSI+VY*(-INDEX*COSPSI                             00000490
      W=VX*(INDEX*SINTHE*COSPSI+VY*(-INDEX*SINTHE*SINPSI+VZ*INDEX*COSTHE 00000500
      VEL=SQRT(U*U+V*V+W*W)                                             00000510
      PDYNEL=.5*RHO*VEL**2                                              00000520
      VEL=DSQRT(VEL**2)                                                 00000530
      MACH=VEL/VSOUND                                                   00000540
      PSI=PDYN*PREFS*COS(ALPHA)/PITCH1                                  00000550
      RN2=PSHELL*RHO1*ALPHA*.5/VEL                                      00000560
      RN2=.5*RHO1*ALPHA/VEL                                             00000570
      SINZ=DSQRT((V*V+W*W)/VEL                                          00000580
      COSZ=U/VEL                                                        00000590
      ZETA=DABS(SINZ)                                                   00000600
      ZSQ=ZETA*ZETA                                                     00000610
      SINPHI=-V/(VEL*SINZ)                                              00000620
      COSPHI=W/(VEL*SINZ)                                               00000630
C                                                                       00000640
C     AERODYNAMIC MOMENTS AND EQUATIONS OF MOTION                       00000650
      CM=ZETA*(X(5)+X(13)*PMACH+ZSQ*(X(A)+ZSQ*X(7)))                    00000660
      CNR=ZETA*(X(A)+ZSQ*X(9))                                          00000670
      CH0=X(10)+ZSQ*X(11)                                               00000680
      DCHDZ=X(5)+X(12)*PMACH+ZSQ*(3.*X(A)+ZSQ*5.*X(7))                  00000690
      DCHDZ=X(A)+ZSQ*3.*X(9)                                            00000700
      DCH0DZ=2.*X(11)*ZETA                                              00000710
C                                                                       00000720
C     EQUATIONS OF MOTION                                               00000730
      DY(2)=Y(5)/COSTHE                                                 00000740
      DY(3)=Y(6)                                                        00000750
      PAXES=-DY(2)*SINTHE                                               00000760
      DY(A)=PAXES*Y(5)-RATIO1*PSHELL*Y(5)+QSO1*(CM*COSPHI               00000770
     $     -CNR*RD2*Y(5)*SINPHI+CN0*Y(A)*D2V)                           00000780
      DY(5)=-PAXES*Y(A)+RATIO1*PSHELL*Y(A)*QSO1*(CM*SINPHI              00000790
     $     +CNR*RD2*Y(5)*COSPHI+CN0*Y(5)*D2V)                           00000800
C                                                                       00000810
C     PARTIAL DERIVATIVES WITH RESPECT TO PARAMETERS                    00000820
C                                                                       00000830
      DO 500 J=1,12                                                     00000840
      DP=-(Y(6*J+5)*SINTHE+Y(5)*Y(6*J+3)/COSTHE)/COSTHE                 00000850
      DU=+6*Y(6*J+3)+VP*COSTHE*Y(6*J+2)                                 00000860
      DV=-(VX*(INDEX*COSPSI+VY*(-INDEX*SINPSI)*Y(6*J+2)                 00000870
```

```
      DU=U*Y(A*J+3)+V*SINTHE*Y(A*J+2)                                          00000880
      DZETA=(V*DV+W*DW)*COSZ/(VELSQ*SINZ)-DU*SINZ/VEL                          00000890
      DPHI=(V*DW-W*DV)/(V**2+V**2)                                             00000900
      DY(A*J+2)=(Y(A*J+5)+Y(5)*SINTHE*Y(A*J+3)/COSTHE)/COSTHE                  00000910
      DY(A*J+3)=Y(A*J+4)                                                       00000920
      DY(A*J+4)=Y(5)*DP+PAXES*Y(A*J+5)-RATIO1*PSHELL*Y(A*J+5)                  00000930
     $          +QSQI*((DCMDZ*COSPHI-DCNPDZ*PD2V*SINPHI+DCMDDZ*Y(A)*D2V)       00000940
     $          *DZETA+(-CM*SINPHI-CNP*PD2V*COSPHI)*DPHI                       00000950
     $          *CM0*D2V*Y(A*J+4))                                             00000960
      DY(A*J+5)=-Y(A)*DP-PAXES*Y(A*J+4)+RATIO1*PSHELL*Y(A*J+4)                 00000970
     $          +QSQI*((DCMDZ*SINPHI+DCNPCZ*PD2V*COSPHI+DCMDDZ*Y(5)*D2V)       00000980
     $          *DZETA+(CM*COSPHI-CNP*SINPHI*PD2V)*DPHI                        00000990
     $          *CM0*D2V*Y(A*J+5))                                             00001000
  400 CONTINUE                                                                 00001010
C                                                                              00001020
C     KRONECKER DELTA TERMS                                                    00001030
C                                                                              00001040
      AAA=QSQI*ZETA                                                            00001050
      DY(24)=DY(24)+AAA*COSPHI                                                 00001060
      DY(25)=DY(25)+AAA*SINPHI                                                 00001070
      DY(52)=DY(52)+AAA*PHACM*COSPHI                                           00001080
      DY(53)=DY(53)+AAA*PHACM*SINPHI                                           00001090
      AAA=AAA*ZSQ                                                              00001100
      DY(28)=DY(28)+AAA*COSPHI                                                 00001110
      DY(29)=DY(29)+AAA*SINPHI                                                 00001120
      AAA=AAA*ZSQ                                                              00001130
      DY(32)=DY(32)+AAA*COSPHI                                                 00001140
      DY(33)=DY(33)+AAA*SINPHI                                                 00001150
C                                                                              00001160
      AAA=QSQI*PD2V*ZETA                                                       00001170
      DY(36)=DY(36)-AAA*SINPHI                                                 00001180
      DY(37)=DY(37)+AAA*COSPHI                                                 00001190
      AAA=AAA*ZSQ                                                              00001200
      DY(40)=DY(40)-AAA*SINPHI                                                 00001210
      DY(41)=DY(41)+AAA*COSPHI                                                 00001220
C                                                                              00001230
      AAA=QSQI*D2V                                                             00001240
      DY(44)=DY(44)+AAA*Y(4)                                                   00001250
      DY(45)=DY(45)+AAA*Y(5)                                                   00001260
      AAA=AAA*ZSQ                                                              00001270
      DY(48)=DY(48)+AAA*Y(4)                                                   00001280
      DY(49)=DY(49)+AAA*Y(5)                                                   00001290
C                                                                              00001300
      RETURN                                                                   00001310
      END                                                                      00001320
```

```
      SUBROUTINE INCOND                                                        00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                               00000020
C                                                                              00000030
C     INITIALISE VALUES OF ALL VARIABLES FOR INTEGRATION BY DERY               00000040
C     AND RUNGE4 AT THE BEGINNING OF EACH ITERATION OF THE PARAMETER           00000050
C     ESTIMATION ALGORITHM.                                                    00000060
C                                                                              00000070
      COMMON /INT/    Y(53),DY(53),HO(53),YA(53),Z(12),N,IORS,MAX              00000080
     $       /MODEL/  REFS,RODIAM,PITCHI,RATIOI,                               00000090
     $                XF(12),X(12),XX(12),JORDER(12)                          00000100
     $       /COMP/   TO,REJECT,ACCFAC,HMAX,NPARAM,ITERN,LCASE                00000110
     $       /DATA/   ORS(2,1000),PI(2),NPTS,ITER,IEND                        00000120
C                                                                              00000130
      DO 100 I=1,53                                                            00000140
  100 Y(I)=0.                                                                  00000150
C                                                                              00000160
      Y(1)=TO                                                                  00000170
      DO 200 I=1,4                                                             00000180
      Y(I+1)=X(I)                                                              00000190
  200 Y(5+I+1)=1.                                                              00000200
C                                                                              00000210
C     MAXIMUM NUMBER OF EQUATIONS WHICH MUST BE INTEGRATED                     00000220
C                                                                              00000230
      MAX=0                                                                    00000240
      DO 300 J=1,12                                                            00000250
      IF(JORDER(J).LE.NPARAM) MAX=J                                            00000260
  300 CONTINUE                                                                 00000270
      N=6*MAX+5                                                                00000280
      IF(IEND.EQ.1) N=0                                                        00000290
C                                                                              00000300
      RETURN                                                                   00000310
      END                                                                      00000320
```

```
      SUBROUTINE INPUT                                                         00000010
      IMPLICIT REAL*8 (A-H,O-Z)                                                00000020
C                                                                              00000030
      DIMENSION TITLE(20)                                                      00000040
C                                                                              00000050
      COMMON /MODEL/ REFS,RODIAM,PITCHI,RATIOI,                                00000060
     $               XF(12),X(12),XX(12),JORDER(12)                            00000070
     $       /TRAJ/  TIME(100),XDOT(100),YDOT(100),ZDOT(100),ALT(100),        00000080
     $               ROLL(100),NTRAJ                                           00000090
     $       /DATA/  ORS(2,1000),PI(2),NPTS,ITER,IEND                         00000100
     $       /DIRCOS/TCOS(10),COSL(10),COSM(10),COSN(10),NCOS                  00000110
     $       /COMP/  TO,REJECT,ACCFAC,HMAX,NPARAM,ITERN,LCASE,NPLOT           00000120
C                                                                              00000130
C     THIS ROUTINE HANDLES INPUT AND RECORDING OF ALL DATA FOR THE             00000140
C     PROGRAM                                                                  00000150
C     IT IS POSSIBLE TO READ DATA SELECTIVELY USING THE PARAMETER,             00000160
C     LENTER, WHICH IS INPUT TWICE, FIRSTLY AFTER THE RUN TITLE AND            00000170
C     SECONDLY JUST BEFORE THE TRIALS DATA                                     00000180
C                                                                              00000190
      REWIND 3                                                                 00000200
C                                                                              00000210
C     READ RUN TITLE                                                           00000220
      READ (5,1) TITLE                                                         00000230
      WRITE (6,2) TITLE                                                        00000240
C                                                                              00000250
C     ENTRY POINT FOR INPUT OF NEW DATA                                        00000260
      READ (5,3) LENTER                                                        00000270
   90 GO TO (100,200,300,400,500,600,700,800,900,1000),LENTER                  00000280
C                                                                              00000290
C     DIRECTION COSINES OF THE SUN TABULATED AS A FUNCTION OF TIME             00000300
C     ALONG THE TRAJECTORY                                                     00000310
  100 READ (5,3) NCOS                                                          00000320
      DO 110 I=1,NCOS                                                          00000330
  110 READ (5,4) TCOS(I),COSL(I),COSM(I),COSN(I)                               00000340
      WRITE (6,5) (TCOS(I),COSL(I),COSM(I),COSN(I),I=1,NCOS)                   00000350
C                                                                              00000360
C     TRAJECTORY DATA TABULATED OVER THE WHOLE TRAJECTORY FROM UNIT 4           00000370
      READ (5,3) NTRAJ,IFOUT,ISKIP                                             00000380
      DO 130 I=1,NTRAJ                                                         00000390
      DO 120 II=1,ISKIP                                                        00000400
  120 READ (4,6) TIME(I),XDOT(I),YDOT(I),ZDOT(I),ALT(I),ROLL(I)               00000410
  130 CONTINUE                                                                 00000420
      IF(IFOUT.NE.6) WRITE (A,7) (TIME(I),XDOT(I),YDOT(I),ZDOT(I),             00000430
     $                            ALT(I),ROLL(I),I=1,NTRAJ)                    00000440
C                                                                              00000450
C     ACCURACIES IN PARAMETERS WHICH SIGNIFY CONVERGENCE                       00000460
  200 READ (5,A) (XF(I),I=1,12)                                                00000470
C                                                                              00000480
C     INITIAL TIME AND MODEL CONSTANTS                                         00000490
  300 READ (5,6) REFS,RODIAM,PITCHI,ROLLI                                      00000500
      WRITE (6,A) REFS,RODIAM,PITCHI,ROLLI                                     00000510
      RATIOI=ROLLI/PITCHI                                                      00000520
C                                                                              00000530
C     INITIAL VALUES FOR PARAMETERS                                            00000540
  400 READ (5,A) (X(I),I=1,12)                                                 00000550
C                                                                              00000560
C     REQUIRED ORDER OF PARAMETERS                                             00000570
  500 READ (5,3) (JORDER(I),I=1,12)                                            00000580
      WRITE (6,9) (JORDER(I),I=1,12)                                           00000590
      DO 510 I=1,12                                                            00000600
  510 XX(JORDER(I))=X(I)                                                       00000610
C                                                                              00000620
C     TIME OF THE FIRST DATA POINT                                             00000630
  600 READ (5,6) TO                                                            00000640
      WRITE (6,13) TO                                                          00000650
C                                                                              00000660
C     PROGRAM CONTROL PARAMETERS                                               00000670
  700 READ(5,6) REJECT,ACCFAC,HMAX                                             00000680
  800 READ (5,3) NPARAM,ITERN,NPLOT                                            00000690
      WRITE (6,10) REJECT,ACCFAC,HMAX,NPARAM,ITERN                             00000700
      IF(NPLOT.EQ.0) WRITE (A,11)                                              00000710
      IF(NPLOT.NE.0) WRITE (A,12) NPLOT                                        00000720
C                                                                              00000730
C     EXPERIMENTAL DATA INPUT - IF REQUIRED                                    00000740
      READ (5,3) LENTER,LCASE,IFOUT                                            00000750
      GO TO 90                                                                 00000760
C                                                                              00000770
C     INPUT FLIGHT RESULTS ORS(I,J)                                            00000780
C     ORS(1,J)=TIME            ORS(2,J)=COMPLEMENTARY SOLAR ASPECT ANGLE       00000790
C                                                                              00000800
  900 READ (5,3) NPTS,ISKIP                                                    00000810
```

```
C                                                                       00000825
C     READ METEOROLOGICAL DATA TABLES                                   00000830
      CALL READMT                                                       00000840
      IF((IFOUT.NE.0) WRITE (A,14)                                      00000850
C                                                                       00000860
  910 READ (3,4)  ((ORS(I,J),I=1,2)                                     00000870
      IF(ORS(1,1).LT.(170-6.0001)) GO TO 910                            00000880
      IF((IFOUT.NE.0) WRITE (A,15) (ORS(I,J),I=1,2)                     00000890
C                                                                       00000900
      DO 930 J=2,NPTS                                                   00000910
C                                                                       00000920
      DO 920 II=1,ISKIP                                                 00000930
  920 READ (3,4)  (ORS(I,J),I=1,2)                                      00000940
C                                                                       00000950
      IF((IFOUT.NE.0) WRITE (A,15) (ORS(I,J),I=1,2)                     00000960
  930 CONTINUE                                                          00000970
C                                                                       00000980
C     IF NO NEW EXPERIMENTAL DATA IS REQUIRED THEN RETURN               00000990
 1000 RETURN                                                            00001000
C                                                                       00001010
    1 FORMAT(20A4)                                                      00001020
    2 FORMAT(1H0,25X,20A4)                                              00001030
    3 FORMAT(16I5)                                                      00001040
    4 FORMAT(5E16.7)                                                    00001050
    5 FORMAT('0DIRECTION COSINES FOR THE SUN IN RANGE AXES AS A FUNCTION00001060
     $ OF TIME ALONG THE TRAJECTORY'/' TIME(S)',5X,'L',9X,'M',9X,'N'/   00001070
     $         10(1X,F7.2,3F10.6/))                                     00001080
    6 FORMAT(8F10.6)                                                    00001090
    7 FORMAT('0TARULATED TRAJECTORY DATA'//                             00001100
     $       '   TIME    XDOT    YDOT    ZDOT     ALT     ROLL'/       00001110
     $       ' (SEC)    (METRES/SECOND)       (M)   (RAD/SEC)'/       00001120
     $       100(1X,FA.1,3F7.2,2F8.2/))                                 00001130
    8 FORMAT('0VEHICLE PHYSICAL CONSTANTS'//10X,'REFERENCE AREA=',F9.4, 00001140
     $       '(M**2)',10X,'BODY DIAMETER=',F7.4,'(M)'/10X,              00001150
     $       'PITCH INERTIA=',F9.4,'(KG M**2)',10X,'ROLL INERTIA=',     00001160
     $       F10.7,'(KG M**2)')                                         00001170
    9 FORMAT('0ORDER OF PARAMETERS'/5X,1213)                            00001180
   10 FORMAT('0INITIAL REJECTION LEVEL ON DATA',FA.2/                   00001190
     $       ' ACCURACY FACTOR FOR PARAMETER ERROR LEVELS',F7.3/        00001200
     $       ' MAXIMUM INTEGRATION STEP',F7.4/                          00001210
     $       ' NUMBER OF PARAMETERS VARIED',I3/                         00001220
     $       ' MAXIMUM NUMBER OF ITERATIONS',I3)                        00001230
   11 FORMAT('0NO PLOTS REQUESTED')                                     00001240
   12 FORMAT('0PLOTS REQUESTED EVERY',I3,' POINTS')                     00001250
   13 FORMAT('0STARTING TIME=',F7.3,'(S)')                              00001260
   14 FORMAT('1      TIME   COMPLEMENTARY SOLAR'/                       00001270
     $       '     (SEC)   ASPECT ANGLE(DEG)')                        00001280
   15 FORMAT(1X,F10.4,F16.8)                                            00001290
      END                                                               00001300



C     SUBROUTINE MET                                                    00000010
C                                                                       00000020
C     SETS UP WIND TABLE (IND=2) IN WDTAR(150,3) WHERE                  00000030
C        WDTAR(I,1) CONTAINS ALTITUDE (M) ABOVE MEAN SEA-LEVEL          00000040
C        WDTAR(I,2) CONTAINS CROSS WIND (M/S),+VE FROM LEFT TO RIGHT    00000050
C        WDTAR(I,3) CONTAINS RANGE WIND (M/S),+VE DOWNRANGE             00000060
C     ALSO SETS UP PRESSURE,TEMPERATURE TABLE (IND=3)                   00000070
C     IN PT(150,3) WHERE                                                00000080
C        PTTAR(I,1) CONTAINS ALTITUDE(M) ABOVE MEAN SEA-LEVEL           00000090
C        PTTAR(I,2) CONTAINS PRESSURE (P)                               00000100
C        PTTAR(I,3) CONTAINS TEMPERATURE (K)                            00000110
C     OR BOTH TABLES IF IND=4                                           00000120
C                                                                       00000130
C     IF A STANDARD ATMOSPHERE WITH NO WINDS IS DESIRED THEN SET IND=1  00000140
C                                                                       00000150
C     IF INPUT SHOULD BE RECORDED THEN IFOUT SHOULD BE NON-ZERO         00000160
C                                                                       00000170
C     THE FIRST PART OF THE ROUTINE INTERPOLATES AMONGST THE TABLES     00000180
C     OR USES THE STANDARD ATMOSPHERE ROUTINE TO ASSIGN VALUES TO THE   00000190
C     ATMOSPHERIC VARIABLES                                             00000200
C                                                                       00000210
      SUBROUTINE MET(ALT,XWIND,YWIND,RHO,VSOUND)                        00000220
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000230
C                                                                       00000240
      DIMENSION WDTAR(150,3),PTTAR(150,3)                               00000250
C                                                                       00000260
C     ESTIMATE WIND COMPONENTS,ATMOSPHERIC DENSITY AND SOUND SPEED      00000270
      IF(IND.LE.2) CALL ATMOS(ALT,RHO,VSOUND,TEMP,JERR)                 00000280
      XWIND=0.                                                          00000290
      YWIND=0.                                                          00000300
      IF(IND.EQ.1) RETURN                                               00000310
C                                                                       00000320
C     INTERPOLATE FOR WINDS IF TABULATED                                00000330
      IF(IND.EQ.3) GO TO 200                                            00000340
C                                                                       00000350
      DO 100 I=2,NWD                                                    00000360
      IF(ALT.GT.WDTAR(I,1)) GO TO 100                                   00000370
      LWD=I                                                             00000380
      GO TO 110                                                         00000390
  100 CONTINUE                                                          00000400
C                                                                       00000410
      WRITE (6,1) ALT                                                   00000420
    1 FORMAT('1 ALTITUDE TOO LARGE FOR TABULATED WINDS',E16.6,'(M)')    00000430
      STOP                                                              00000440
C                                                                       00000450
  110 DEL=(ALT-WDTAR(LWD-1,1))/(WDTAR(LWD,1)-WDTAR(LWD-1,1))            00000460
      YWIND=WDTAR(LWD-1,2)+DEL*(WDTAR(LWD,2)-WDTAR(LWD-1,2))            00000470
      XWIND=WDTAR(LWD-1,3)+DEL*(WDTAR(LWD,3)-WDTAR(LWD-1,3))            00000480
C                                                                       00000490
C     INTERPOLATE FOR PRESSURE AND TEMPERATURE IF TABULATED             00000500
  200 IF(IND.LE.2) RETURN                                               00000510
C                                                                       00000520
      DO 210 I=2,NPT                                                    00000530
      IF(ALT.GT.PTTAR(I,1)) GO TO 210                                   00000540
      LPT=I                                                             00000550
      GO TO 220                                                         00000560
  210 CONTINUE                                                          00000570
C                                                                       00000580
      WRITE (6,2) ALT                                                   00000590
    2 FORMAT('1ALTITUDE TOO LARGE FOR TARULATE TEMPERATURE AND PRESSURE'00000600
     $       ,E16.6,'(M)')                                              00000610
      STOP                                                              00000620
C                                                                       00000630
  220 DEL=(ALT-PTTAR(LPT-1,1))/(PTTAR(LPT,1)-PTTAR(LPT-1,1))            00000640
      PRESS=PTTAR(LPT-1,2)+DEL*(PTTAR(LPT,2)-PTTAR(LPT-1,2))            00000650
      TEMPK=PTTAR(LPT-1,3)+DEL*(PTTAR(LPT,3)-PTTAR(LPT-1,3))            00000660
      RHO=0.003AR368*PRESS/TEMPK                                        00000670
      VSOUND=20.0467*DSQRT(TEMPK)                                       00000680
      RETURN                                                            00000690
C                                                                       00000700
C     ENTRY POINT TO INPUT TABULAR DATA                                 00000710
      ENTRY READMT                                                      00000720
C                                                                       00000730
C     DATA CONTROLS                                                     00000740
      READ (5,4) IND,NWD,NPT,IFOUT                                      00000750
      IF((IND.GE.1).AND.(IND.LE.4)) GO TO 300                           00000760
C                                                                       00000770
      WRITE (6,3) IND                                                   00000780
    3 FORMAT('1IND=',I10,' OUTSIDE ALLOWABLE RANGE IN MET')             00000790
C                                                                       00000800
  300 IF(IND.EQ.1) RETURN                                               00000810
C                                                                       00000820
C     TABLES OF WIND DATA                                               00000830
      IF(IND.EQ.3) GO TO 400                                            00000840
      DO 310 I=1,NWD                                                    00000850
  310 READ (A,5) (WDTAR(I,J),J=1,3)                                     00000860
      IF(IFOUT.EQ.0) GO TO 400                                          00000870
      WRITE (6,A)                                                       00000880
      DO 320 I=1,NWD                                                    00000890
  320 WRITE (A,7) (WDTAR(I,J),J=1,3)                                    00000900
C                                                                       00000910
C     TABLES OF TEMPERATURE AND PRESSURE                                00000920
  400 IF(IND.EQ.2) RETURN                                               00000930
      DO 410 I=1,NPT                                                    00000940
  410 READ (A,5) (PTTAR(I,J),J=1,3)                                     00000950
      IF(IFOUT.EQ.0) RETURN                                             00000960
      WRITE (6,A)                                                       00000970
      DO 420 I=1,NPT                                                    00000980
  420 WRITE (6,9) (PTTAR(I,J),J=1,3)                                    00000990
C                                                                       00001000
      RETURN                                                            00001010
    4 FORMAT(16I5)                                                      00001020
    5 FORMAT(3F10.0)                                                    00001030
    A FORMAT('0WIND TABLES    HEIGHT     RANGE WIND      CROSS WIND'/   00001040
     $       '                (METRES)          METRES/SECOND)')       00001050
    7 FORMAT(11X,F12.1,F13.2,F15.2)                                     00001060
    8 FORMAT('0PRESSURE AND TEMPERATURE TABLES     HEIGHT     PRESSURE  TE00001070
     $MPER  ATURE'/35X,'(METRES)  (PASCALS)     (KELVIN)')              00001080
    9 FORMAT(33X,F10.1,F11.1,F13.2)                                     00001090
      END                                                               00001100
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```
      SUBROUTINE MFSD(A,N,EPS,IER)                                      00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000020
C     ROUTINE FROM THE SYSTEM/360 SCIENTIFIC SUBROUTINE PACKAGE         00000030
C     TO FACTORISE A POSITIVE DEFINITE SYMMETRIC MATRIX INTO A          00000040
C     TRIANGULAR FACTORISATION. THAT IS  A=R'R                          00000050
      DIMENSION A(1)                                                    00000060
      IER=0                                                             00000070
      KPIV=0                                                            00000080
      DO 110 K=1,N                                                      00000090
      KPIV=KPIV+K                                                       00000100
      IND=KPIV                                                          00000110
      LEND=K-1                                                          00000120
      TOL=DABS(EPS*A(KPIV))                                             00000130
      DO 110 I=K,N                                                      00000140
      DSUM=0.DO                                                         00000150
      IF(LEND.EQ.0) GO TO 40                                            00000160
      DO 30 L=1,LEND                                                    00000170
      LANF=KPIV-L                                                       00000180
      LIND=IND-L                                                        00000190
   30 DSUM=DSUM+A(LANF)*A(LIND)                                         00000200
   40 DSUM=A(IND)-DSUM                                                  00000210
      IF(I.NE.K) GO TO 100                                              00000220
      IF((DSUM-TOL).GT.0.) GO TO 90                                     00000230
      IF(DSUM.LE.0.) GO TO 120                                          00000240
      IF(IER.LE.0)IER=-1                                                00000250
   90 DPIV=DSQRT(DSUM)                                                  00000260
      A(KPIV)=DPIV                                                      00000270
      DPIV=1.DO/DPIV                                                    00000280
      GO TO 110                                                         00000290
  100 A(IND)=DSUM*DPIV                                                  00000300
  110 IND=IND+I                                                         00000310
      RETURN                                                            00000320
  120 IER=-1                                                            00000330
      RETURN                                                            00000340
      END                                                               00000350
```

```
      SUBROUTINE PLOTT                                                  00000010
      IMPLICIT REAL*8 (A-H,O-Z)                                         00000020
C                                                                       00000030
      REAL *4 TITLE(20),XAXIS(10),YAXIS(15),TIME(505),A(505),R(505),    00000040
     $        C(505)                                                    00000050
C                                                                       00000060
C     A ROUTINE WHICH PLOTS THE FINAL FIT TO THE DATA TO PROVIDE RESULTS00000070
C     FOR VISUAL SCANNING                                               00000080
C     ENTRY POINTS ARE (OTHER THAN PLOTT WHICH INITIALISES THE ROUTINE) 00000090
C          PLOTA - TO RECORD THE DATA, AND                              00000100
C          PLOTR - TO PLOT THE GRAPHS                                   00000110
C                                                                       00000120
      COMMON /COMP/  TO,REJECT,ACCFAC,HMAX,NPARAM,ITERN,LCASE,NPLOT     00000130
     $       /DATA/  OBS(2,1000),PI(2),NPTS,ITER,IEND                   00000140
     $       /INT/   Y(53),DY(53),HO(53),YA(53),Z(12),N,IOBS,MAX        00000150
C                                                                       00000160
      DATA TITLE /40*'GOOD','NESS',' OF ','FIT ','FOR ','SHEL','L YA', 00000170
     $          'WSON','OF D','ATA ',99*'  '/,                          00000180
     $      XAXIS /0.,.0,.10,.7,.'TIME','(S) ',48*'  '/,                00000190
     $      YAXIS /-50.,.10,.10,.37,.'COMP','LEMF','NTAR','Y SO','LAR ', 00000200
     $         'ASPE','CT A','NGLE','(DEG',')  ','  ',' '/              00000210
C                                                                       00000220
C     INITIALISE                                                        00000230
C                                                                       00000240
      I=0                                                               00000250
      IF=NPLOT                                                          00000260
      RETURN                                                            00000270
C                                                                       00000280
C     STORE DATA POINTS FOR PLOTTING                                    00000290
C                                                                       00000300
      ENTRY PLOTA                                                       00000310
      IF=IF+1                                                           00000320
      IF(IF.LT.NPLOT) RETURN                                            00000330
      IF=0                                                              00000340
      I=I+1                                                             00000350
      TIME(I)=OBS(1,IOBS)                                               00000360
      A(I)=PI(2)*57.295A                                                00000370
      R(I)=OBS(2,IOBS)*57.295A                                          00000380
      C(I)=10.*(R(I)-A(I))                                              00000390
      RETURN                                                            00000400
C                                                                       00000410
C     PLOT DATA WHICH HAS BEEN STORED                                   00000420
C                                                                       00000430
```

```
      ENTRY PLOTR                                                       00000440
      MI=-I                                                             00000450
      XAXIS(1)=TIME(1)                                                  00000460
      XAXIS(2)=0.1*(TIME(I)-TIME(1))                                    00000470
C                                                                       00000480
      CALL GRAPH(TITLE,XAXIS,YAXIS,I,TIME,A,I,TIME,R,MI,TIME,C)         00000490
C                                                                       00000500
      IF(LCASE.EQ.0) CALL ENDGR                                         00000510
      RETURN                                                            00000520
      END                                                               00000530
```

```
      SUBROUTINE RUNGE4 (IND,N,H,Y,DY,HO,YA)                            00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000015
C                                                                       00000020
C     FOURTH ORDER,FIXED STEP RUNGE KUTTA ROUTINE TO INTEGRATE          00000030
C     (N-1) FIRST ORDER ORDINARY DIFFERENTIAL EQUATIONS                 00000040
C     Y(1)=TIME  Y(2)......Y(N) ARE DEPENDENT VARIABLES                 00000050
C          DY(2)......Y(N) ARE CORRESPONDING DERIVATIVES CALCULATED     00000060
C                          IN DERIV                                     00000070
C     NOTE *****  Y,DY MUST APPEAR IN COMMON STATEMENTS IN BOTH DERIV   00000080
C          AND THE CALLING ROUTINE                                      00000090
C            *****  Y,DY,HO,YA MUST HAVE DIMENSION AT LEAST N IN THE   00000100
C              CALLING ROUTINE                                          00000110
C     *****  IND=0 FOR THE FIRST PASS TO INITIALISE DERIVATIVES         00000120
C          =1 TO INTERGRATE ONE TIME STEP                               00000130
C                                                                       00000140
C                                                                       00000150
      DIMENSION Y(1),DY(1),HO(1),YA(1)                                  00000160
      IF(IND.EQ.0) GO TO 500                                            00000170
C                                                                       00000180
C     INITIALISE FOR INTEGRATION                                        00000190
      DO 100 I=2,N                                                      00000200
      YA(I)=Y(I)                                                        00000210
  100 HO(I)=0.                                                          00000220
      HA=Y(1)                                                           00000230
C                                                                       00000240
C     ESTIMATE INTERMEDIATE DERIVATIVES                                 00000250
      DO 300 J=3,5                                                      00000260
      A=(5-J)/2                                                         00000270
      B=J/2                                                             00000280
      BH=B*H                                                            00000290
      AH=(H/A)                                                          00000300
      DO 200 I=2,N                                                      00000310
      HO(I)=HO(I)+BH*DY(I)                                              00000320
  200 Y(I)=YA(I)+AH*DY(I)                                               00000330
      Y(1)=AH+HA                                                        00000340
C                                                                       00000350
C     ESTIMATE DERIVATIVES                                              00000360
      CALL DERIV                                                        00000370
  300 CONTINUE                                                          00000380
C                                                                       00000390
C     COMPLETE INTEGRATION AND UPDATE DERIVATIVES                       00000400
      DO 400 I=2,N                                                      00000410
  400 Y(I)=YA(I)+0.1666667*(HO(I)+H*DY(I))                              00000420
      Y(1)=HA+H                                                         00000430
  500 CALL DERIV                                                        00000440
C                                                                       00000450
      RETURN                                                            00000460
      END                                                               00000470
```

```
      SUBROUTINE SIMUL                                                  00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000020
C                                                                       00000030
C     THIS ROUTINE CONTROLS THE INTEGRATION PROCESS AND PROVIDES        00000040
C     SIMULATED VARIABLE VALUES AND PARTIAL DERIVATIVES OF COMPLEMENTARY00000050
C     SOLAR ASPECT ANGLE WITH RESPECT TO EACH PARAMETER IN THE ORDER    00000060
C     SPECIFIED IN THE JORDER ARRAY.                                    00000070
C                                                                       00000080
      COMMON /COMP/   TO,REJECT,ACCFAC,HMAX,NPARAM,ITERN,LCASE,NPLOT    00000090
     $       /DATA/   OBS(2,1000),PI(2),NPTS,ITER,IEND                  00000100
     $       /INT/    Y(53),DY(53),HO(53),YA(53),Z(12),N,IOBS,MAX       00000110
     $       /MODEL/  RHFS,RODIAM,PITCHI,RATIOI,                       00000120
     $                XF(12),X(12),XX(12),JORDER(12)                   00000130
     $       /DIRCOS/ TCOS(10),COSL(10),COSM(10),COSN(10),NCOS         00000140
C                                                                       00000150
C     INTEGRATION CONTROL                                               00000160
C                                                                       00000170
```

```
      IF(IOBS.EQ.2) H=OBS(1,NPTS)-Y(1)                                  00000180
  100 IND=1                                                             00000190
C                                                                       00000200
C     SET UP INTEGRATION STEP LENGTH                                    00000210
C                                                                       00000220
      A=OBS(1,IOBS)-Y(1)                                                00000230
      R=A                                                               00000240
      IF(A.GT.HMAX) A=HMAX                                              00000250
      IF(DABS(A-H).GT.0.00001) IND=0                                    00000260
      H=A                                                               00000270
C                                                                       00000280
C     INTEGRATE ONE STEP                                                00000290
C                                                                       00000300
  110 CALL RUNGE4(IND,N,H,Y,DY,HQ,YA)                                   00000310
C                                                                       00000320
      IF(IND.EQ.1) GO TO 120                                            00000330
      IND=1                                                             00000340
      GO TO 110                                                         00000350
C                                                                       00000360
C     TEST WHETHER INTEGRATION HAS REACHED NEXT DATA POINT              00000370
C                                                                       00000380
  120 IF(R.GT.HMAX) GO TO 100                                           00000390
C                                                                       00000400
C     ESTIMATE STATE VARIABLES AND PARTIAL DERIVATIVES                  00000410
C                                                                       00000420
      DO 130 I=2,NCOS                                                   00000430
      IF(Y(1).GT.TCOS(I)) GO TO 130                                     00000440
      DELT=(Y(1)-TCOS(I-1))/(TCOS(I)-TCOS(I-1))                         00000450
      L=I                                                               00000460
      GO TO 140                                                         00000470
  130 CONTINUE                                                          00000480
  140 DIRL=COSL(L-1)+DELT*(COSL(L)-COSL(L-1))                           00000490
      DIRM=COSM(L-1)+DELT*(COSM(L)-COSM(L-1))                           00000500
      DIRN=COSN(L-1)+DELT*(COSN(L)-COSN(L-1))                           00000510
      SINPSI=DSIN(Y(2))                                                 00000520
      COSPSI=DCOS(Y(2))                                                 00000530
      SINTHE=DSIN(Y(3))                                                 00000540
      COSTHE=DCOS(Y(3))                                                 00000550
      COSSIG=DIRL*COSTHE*COSPSI+DIRM*COSTHE*SINPSI-DIRN*SINTHE          00000560
      SIGMAN=1.570796-DARCOS(COSSIG)                                    00000570
      PI(1)=Y(1)                                                        00000580
      PI(2)=SIGMAN                                                      00000590
      SECSN=1./DCOS(SIGMAN)                                             00000600
C                                                                       00000610
      DO 190 I=1,12                                                     00000620
      IF(JORDER(I).GT.NPARAM) GO TO 190                                 00000630
      Z(JORDER(I))=SECSN*(-(DIRL*SINTHE*COSPSI+DIRM*SINTHE*SINPSI       00000640
     $           +DIRN*COSTHE)*Y(A*I+3)-(DIRL*SINPSI-DIRM*COSPSI)       00000650
     $           *COSTHE*Y(A*I+2))                                      00000660
  190 CONTINUE                                                          00000670
C                                                                       00000680
C     OUTPUT RESULTS OF INTEGRATION STEP IF REQUIRED                    00000690
C                                                                       00000700
      IF(((IEND.EQ.1).OR.(ITER.GE.ITERN)).AND.(NPLOT.NE.0))             00000710
     $                          WRITE (6,1) (PI(I),I=1,2)               00000720
C                                                                       00000730
      RETURN                                                            00000740
    1 FORMAT(1X,F7.3,F16.7)                                             00000750
      END                                                               00000760


      SUBROUTINE SINV(A,N,EPS,IER)                                      00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000015
C     ROUTINE FROM SYSTEM/360 SCIENTIFIC SUBROUTINE PACKAGE             00000020
C     FOR INVERTING A POSITIVE DEFINITE SYMMETRIC MATRIX                00000030
      DIMENSION A(1)                                                    00000040
      CALL MFSD (A,N,EPS,IER)                                           00000050
      IF(IER.LT.0) RETURN                                               00000070
      IPIV=N*(N+1)/2                                                    00000080
      IND=IPIV                                                          00000090
      DO 600 I=1,N                                                      00000100
      DIN=1.000/A(IPIV)                                                 00000110
      A(IPIV)=DIN                                                       00000120
      MIN=N                                                             00000130
      KEND=I-1                                                          00000140
      LANF=N-KEND                                                       00000150
      IF(KEND.LE.0) GO TO 500                                           00000160
      J=IND                                                             00000165
      DO 400 K=1,KEND                                                   00000170
      WORK=0.00                                                         00000180
      MIN=MIN-1                                                         00000190
      LHOR=IPIV                                                         00000200
      LVER=J                                                            00000210
      DO 300 L=LANF,MIN                                                 00000220
      LVER=LVER+1                                                       00000230
      LHOR=LHOR+L                                                       00000240
  300 WORK=WORK+A(LVER)*A(LHOR)                                         00000250
      A(J)=-WORK*DIN                                                    00000260
  400 J=J-MIN                                                           00000270
  500 IPIV=IPIV-MIN                                                     00000280
  600 IND=IND-1                                                         00000290
      DO 800 I=1,N                                                      00000300
      IPIV=IPIV+I                                                       00000310
      J=IPIV                                                            00000320
      DO 800 K=I,N                                                      00000330
      WORK=0.00                                                         00000340
      LHOR=J                                                            00000350
      DO 700 L=K,N                                                      00000360
      LVER=LHOR+K-I                                                     00000370
      WORK=WORK+A(LHOR)*A(LVER)                                         00000380
  700 LHOR=LHOR+L                                                       00000390
      A(J)=WORK                                                         00000400
  800 J=J+K                                                             00000410
      RETURN                                                            00000420
      END                                                               00000430


      SUBROUTINE SOLVE(NP,SIGMA,NPTS)                                   00000010
      IMPLICIT REAL *8 (A-H,O-Z)                                        00000020
C                                                                       00000030
C     USES IBM SCIENTIFIC SUBROUTINE PACKAGES 'SINV' AND 'MFSD' TO      00000040
C     SOLVE THE NORMAL EQUATIONS                                        00000050
C     ASSUMES MATRIX PSI IS POSITIVE DEFINITE,SYMMETRIC                 00000060
C                                                                       00000070
C                                                                       00000080
C                                                                       00000090
      COMMON /NORME/ PSI(78),CWD(12),E(12)                              00000100
C                                                                       00000110
C                                                                       00000120
C                                                                       00000130
C     INVERT MATRIX                                                     00000140
      EPS=1.0D-15                                                       00000150
      CALL SINV(PSI,NP,EPS,IER)                                         00000160
      IF(IER.EQ.0) GO TO 100                                            00000170
      WRITE (6,1) IER                                                   00000180
      STOP                                                              00000190
C                                                                       00000200
C                                                                       00000210
C     ESTIMATE CORRECTIONS AND PROBABLE ERRORS                          00000220
  100 DO 110 I=1,NP                                                     00000230
  110 E(I)=0.                                                           00000240
      KD=0                                                              00000250
      DO 130 I=1,NP                                                     00000260
      IA=I-1                                                            00000270
      IF(IA.EQ.0) GO TO 125                                             00000280
C                                                                       00000290
      DO 120 J=1,IA                                                     00000300
      KD=KD+1                                                           00000310
      E(I)=E(I)+PSI(KD)*CWD(J)                                          00000320
  120 E(J)=E(J)+PSI(KD)*CWD(I)                                          00000330
  125 KD=KD+1                                                           00000340
  130 E(I)=E(I)+PSI(KD)*CWD(I)                                          00000350
C                                                                       00000360
C     PROBABLE ERRORS - FIRST CORRECTED VALUE OF STANDARD DEVIATION     00000370
      DO 140 I=1,NP                                                     00000380
  140 SIGMA=SIGMA-E(I)*CWD(I)                                           00000390
      DIV=NPTS-NP                                                       00000400
      SIGMA=DSQRT(SIGMA/DIV)                                            00000410
      DO 150 I=1,NP                                                     00000420
      KD=I*(I+1)/2                                                      00000430
  150 CWD(I)=DSQRT(PSI(KD))*SIGMA                                       00000440
C                                                                       00000450
C       EVALUATE COVARIANCE MATRIX                                      00000460
      SIGSQ=SIGMA*SIGMA                                                 00000470
      DO 160 I=1,KD                                                     00000480
  160 PSI(I)=PSI(I)*SIGSQ                                               00000490
C                                                                       00000500
      RETURN                                                            00000510
    1 FORMAT(' DOUBTFUL WHETHER MATRIX FOR INVERSION IS POSITIVE DEFINIT00000520
     $E , IER=',I2)                                                     00000530
      END
```

TABLE 1. TYPICAL AERODYNAMIC DATA FOR A SHELL

M	C_x	$C_{y_{p\zeta}}$	$C_{z\zeta}$	C_p	$C_{m\zeta}$	$C_{n_{p\zeta}}$	C_{m_q}
0.0	-0.1299	-1.5	-1.7851	-0.0357	3.5730	0.0	-12.0
0.5	-0.1299	-1.5	-1.7851	-0.0290	3.6756	0.0	-13.0
0.55	-0.1299	-1.5	-1.7851	-0.0284	3.6931	0.0	-13.0
0.60	-0.1299	-1.5	-1.7851	-0.0279	3.7222	0.0	-13.0
0.70	-0.1281	-1.5	-1.7831	-0.0270	3.7946	0.012	-15.0
0.75	-0.1250	-1.5	-1.7738	-0.0265	3.8330	0.025	-17.0
0.80	-0.1225	-1.5	-1.7563	-0.0261	3.8762	0.038	-18.0
0.825	-0.1217	-1.5	-1.7449	-0.0259	3.9018	0.044	-19.0
0.850	-0.1214	-1.5	-1.7319	-0.0257	3.9318	0.05	-20.0
0.875	-0.1249	-1.5	-1.7206	-0.0255	3.9678	0.4	-21.0
0.90	-0.1338	-1.5	-1.7126	-0.0253	4.0124	0.75	-23.0
0.925	-0.1533	-1.5	-1.7172	-0.0252	4.0876	1.1	-24.0
0.94	-0.1769	-1.5	-1.7348	-0.0250	4.1697	1.1	-25.0
0.95	-0.1998	-1.5	-1.7551	-0.0250	4.2338	1.1	-25.0
0.96	-0.2266	-1.5	-1.7804	-0.0249	4.2916	1.05	-25.0
0.97	-0.2555	-1.5	-1.8090	-0.0248	4.3258	1.0	-24.0
0.98	-0.2849	-1.5	-1.8392	-0.0248	4.3161	0.94	-23.0
0.99	-0.3132	-1.5	-1.8695	-0.0247	4.2807	0.89	-23.0
1.00	-0.3391	-1.5	-1.8986	-0.0246	4.2341	0.85	-22.0
1.01	-0.3612	-1.5	-1.9250	-0.0245	4.1862	0.79	-21.0
1.02	-0.3784	-1.5	-1.9477	-0.0245	4.1415	0.75	-21.0
1.03	-0.3892	-1.5	-1.9651	-0.0244	4.1066	0.77	-21.0
1.04	-0.3958	-1.5	-1.9794	-0.0243	4.0749	0.65	-20.0
1.05	-0.4008	-1.5	-1.9932	-0.0243	4.0454	0.60	-20.0
1.075	-0.4085	-1.5	-2.0271	-0.0241	3.9922	0.52	-19.0
1.1	-0.4118	-1.5	-2.0620	-0.0239	3.9579	0.44	-18.0
1.125	-0.4124	-1.5	-2.0964	-0.0238	3.9369	0.37	-17.0
1.15	-0.4102	-1.5	-2.1281	-0.0236	3.9246	0.30	-16.0
1.20	-0.4007	-1.5	-2.1863	-0.0233	3.9120	0.20	-16.0
1.25	-0.3921	-1.5	-2.2422	-0.0230	3.9019	0.17	-15.0
1.30	-0.3839	-1.5	-2.3000	-0.0227	3.8920	0.13	-15.0
1.40	-0.3693	-1.5	-2.4061	-0.0221	3.8730	0.07	-14.0
1.50	-0.3566	-1.5	-2.5042	-0.0215	3.8342	0.0	-13.0
1.75	-0.3309	-1.5	-2.7107	-0.0203	3.7560	0.0	-13.0
2.00	-0.3104	-1.5	-2.8569	-0.0192	3.7232	0.0	-13.0
2.25	-0.2941	-1.5	-2.9393	-0.0182	3.6785	0.0	-13.0
2.50	-0.2841	-1.5	-2.9579	-0.0173	3.5650	0.0	-13.0

d = 0.105 S = 0.008659 m = 14.9688 moment reference = 0.3115
I_x = 0.02326 I = 0.22557

TABLE 2. FITTING ANALYTIC CURVES TO THE AXIAL FORCE COEFFICIENT

(a) NR3634

K	L	a_o	a_1	a_2	b_o	b_1	r.m.s.
0.97956	0.04979	-0.22346	0.10800	-	-0.05871	-	0.00484
0.97719	0.05633	-0.22897	0.12629	-	-0.04734	0.04665	0.00165
0.97719	0.05634	-0.22899	0.12635	-0.00005	-0.04734	0.04666	0.00167

(b) Gavre

a	b	c	d	e	f	r.m.s.
-0.26447	0.09913	0.97206	19.963	-7.5430	-30.023	0.00903

NOTE: The Mach number range used in the nonlinear least squares fitting was 0.65 to 1.4

TABLE 3. ANALYTIC REPRESENTATIONS FOR OTHER COEFFICIENTS

(a) Normal force derivative

K	L	a_o	a_1	a_2	b_o	b_1	r.m.s.
0.97321	0.02156	-0.94111	-0.74737	-	-0.87307	-	0.0180
0.96413	0.03441	-0.93980	-0.73030	-	-0.85000	0.13116	0.0060
0.96564	0.03584	-0.94492	-0.68620	-0.10380	-0.84917	0.13321	0.0059

(b) Roll damping derivative

a_o	a_1	a_2	a_3	a_4	r.m.s.
-0.03149	0.00648	-	-	-	0.000891
-0.03485	0.01251	-0.002279	-	-	0.000243
-0.03559	0.01569	-0.005577	0.000899	-	0.000068
-0.03570	0.01702	-0.008138	0.002558	-0.000339	0.000034

TABLE 4. DESCRIPTION OF CARD INPUT TO SHELTRAJ

Card	Format	Variable	Comments
1	20A4	TITLE(20)	A title card to describe the job being processed, reproduced on the output.
2	16I5	LENTER	This variable controls a facility which allows the user to begin entering data at any one of several different points in the input routine. LENTER → CARD 1 → 3 2 → 5 3 → 15 4 → 9 5 → 13 6 → 17 7 → 18 8 → 20 9 → return
3	8F10.0	AMASS ROLLI REFS BODIAM	Shell mass (kg) Moment of inertia about the longitudinal axis (kg m^2) Reference area (m^2) Reference length (m)
4	8F10.0	OMEGA ALAT BEAR GRAV HTASLO RO	Angular velocity of the earth. Since this is added to acceleration components by the program instead of subtracted from them, as in equation (2) a negative value should be entered. ($-7.292115.10^{-5}$ rad/s) Latitude of the shell, positive south (degrees) Bearing of the line of fire relative to true north (degrees) Acceleration due to gravity (ms^{-2}) Height of range origin above mean sea-level (m) Radius of the earth (m)
5-8	8F10.0	XE(27)	The maximum changes allowed in each parameter, when testing for convergence.
9-12	8F10.0	X(27)	Initial estimates for values of the parameters Pi.
13-14	16I5	JORDER(27)	Order of the parameters for internal use by the program particularly when determining the first NPARAM parameters.
15	8F10.0	TO	Time at which integration begins.
16	8F10.0	XS(4)	Weighting factors proportional to the emphasis to be placed on each of the four measured quantities, range, deviation, altitude and roll rate.
17	8F10.0	REJECT ACCFAC HMAX	Initial data rejection level. Scaling factor multiplying all XE. Can be used for wholesale adjustment of levels for convergence. Maximum value allowed for integration step size. This value will be used unless time to reach next data point is smaller.

TABLE 4 (CONTD.).

Card	Format	Variable	Comments
18	16I5	NPARAM	The first NPARAM parameters only, as specified in the JORDER array, will be allowed to vary.
		ITERN	Maximum number of iterations of parameter estimation algorithm
		NPLOT	Plot every NPLOT th point of the simulated variable values. A maximum of 200 points can be plotted.
19	16I5	LENTER	A repeat of the options allowed by card 2. Principally to avoid inputting new experimental data if none is needed.
		LCASE	Non-zero if data for another run follows.
		IFOUT	Non-zero if a record of the experimental data points is required.
20	16I5	NPTS	Number of data points to be used in this run.
		ISKIP	Use only every ISKIP point from the dataset containing the experimental measurements.
21	16I5	IND	Control variable, describing the nature of the meteorological data, which is available = 1 none available = 2 wind table only = 3 pressure and temperature table only = 4 both wind table and pressure temperature table
		NWD	Number of points in wind tabulation
		NPT	Number of points in tabulation of temperature and pressure
		IFOUT	Non-zero if meteorological data should be tabulated as it is read.

TABLE 5. SAMPLE INPUT DECK FOR SHELTRAJ

(a) Forces

```
A TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA - NR3634 DRAG CURVE
    1
14.9688    0.02326    0.008659   0.105
7.2722E-05 35.        315.       9.807      0.          6373368.
1.         1.         1.         1.         1.          1.          0.01        0.001
0.005      0.005      0.001      0.005      0.005       0.001       0.01        0.001
0.01       0.01       0.01       0.01       0.01        0.01        0.0001      0.0001
0.00005    0.00001    1.
0.         0.         0.         289.40     0.          -363.71     0.977187    0.0563317
-0.228973  0.126289   0.         -0.04734240.0466457    0.          0.964132    0.0344147
-0.939802  -0.730303  0.         -0.849998  0.131162    0.          -0.03559420.0156919
-0.00557720.000898891390.
   10   11   12    7    8    9    3    6    1    4   18    2    5   19   15   22
   13   16   20   14   17   21   23   24   25   26   27
0.
1.         1.         1.         0.
10.        1.         0.2
    2    6    0
    8    1    0
  200    1
    1
A TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA FROM SHELL TRAJECTORIES - A
    7
    3    6    0
    9    1    0
TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA FROM SHELL TRAJECTORIES - B
    7
    4    6    0
    9    1    0
TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA FROM SHELL TRAJECTORIES - C
    7
    5    6    0
    9    1    0
TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA FROM SHELL TRAJECTORIES - D
    7
    6    6    0
    9    1    0
TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA FROM SHELL TRAJECTORIES - E
    7
    7    6    0
    9    1    0
TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA FROM SHELL TRAJECTORIES - F
    7
    8    6    0
    9    1    0
TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA FROM SHELL TRAJECTORIES - G
    7
    9    6    0
    9    1    0
TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA FROM SHELL TRAJECTORIES - H
    7
   10    6    0
    9    1    0
TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA FROM SHELL TRAJECTORIES - I
    7
   11    6    0
    9    1    0
TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA FROM SHELL TRAJECTORIES - J
    7
   12    6    0
    9    1    0
TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA FROM SHELL TRAJECTORIES - K
    7
   13    6    0
    9    1    0
TEST EXERCISE - L
    7
   14    6    0
    9    1    0
TEST EXERCISE - M
    7
   15    6    0
    9    1    0
TEST EXERCISE - N
    7
   16    6    0
    9    1    0
TEST EXERCISE - O
    7
   17    6    0
    9    0    0
```

TABLE 5. SAMPLE INPUT DECK FOR SHELTRAJ

(b) Rolling moments

```
A DECK TO TEST THE ROLL FITTING CAPABILITIES OF THIS PROGRAM
    1
14.9688    0.02326    0.0086659  0.105
7.2722E-05 35.        9.807      0.         637338.
1.         1.         1.         1.         1.         0.1        0.005
0.02       0.01       0.0001     0.005      0.005      0.0001     0.1        0.002
0.1        0.05       0.0001     0.05       0.01       0.0001     0.005      0.001
0.0005     0.0001     1.
1.27       0.         1.02       289.       0.         -364.      0.9768     0.05731
-0.2296    0.1289     0.         -0.04444   0.06547    0.         0.9841     0.03441
-0.9162    -0.7303    0.         -0.8307    0.1312     0.         -0.036     0.016
0.         0.         1390.
    6    7    8    9   10   11   12   13   14   15   16   17   18   19   20   21
   22   23   24   25   26   27    1    2    4    5    3
0.
0.         0.         0.         1.
10.        1.         0.2
    1   20    0
    8    1    0
  200    1
    1
ROLL TESTING - 6A
    7
    2   20    0
    9    1    0
 ROLL TESTING - 6B
    7
    3   20    0
    9    1    0
 ROLLTESTING - 6C
    7
    4   20    0
    9    1    0
 ROLL TESTING - 6D
    7
    5   20    2
    9    0    0
```

TABLE 6. SIMULATED EXPERIMENTAL DATA FOR SHELTRAJ

Time	Range	Drift	Altitude	Roll rate
0.0000000E+00	0.0000000E+00	0.0000000E+00	0.0000000E+00	1.1899733E+03
1.0000000E-01	2.4851295E+01	3.5840267E-04	-3.4210655E+01	1.1881710E+03
2.0000000E-01	5.7525882E+01	2.4925130E-03	-7.2103222E+01	1.1863863E+03
3.0000000E-01	8.6026847E+01	7.5523392E-03	-1.0764065E+02	1.1846191E+03
4.0000000E-01	1.1435735E+02	1.6637133E-02	-1.4294657E+02	1.1828691E+03
5.0000000E-01	1.4251963E+02	2.2687447E-02	-1.7790248E+02	1.1811360E+03
6.0000000E-01	1.7051333E+02	3.2380756E-02	-2.1255476E+02	1.1794197E+03
7.0000000E-01	1.9834287E+02	4.4712344E-02	-2.4690562E+02	1.1777200E+03
8.0000000E-01	2.2601057E+02	5.9258958E-02	-2.8095763E+02	1.1760365E+03
9.0000000E-01	2.5351827E+02	7.5030253E-02	-3.1471407E+02	1.1743692E+03
1.0000000E+00	2.8086729E+02	9.2988361E-02	-3.4817485E+02	1.1727179E+03
1.1000000E+00	3.0805951E+02	1.1129729E-01	-3.8133424E+02	1.1710820E+03
1.2000000E+00	3.3509750E+02	1.3291705E-01	-4.1420352E+02	1.1694613E+03
1.3000000E+00	3.6198352E+02	1.5629176E-01	-4.4677905E+02	1.1678558E+03
1.4000000E+00	3.8871907E+02	1.8094988E-01	-4.7907739E+02	1.1662652E+03
1.5000000E+00	4.1530551E+02	2.0692057E-01	-5.1122149E+02	1.1646920E+03
1.6000000E+00	4.4174473E+02	2.3505940E-01	-5.4299427E+02	1.1631318E+03
1.7000000E+00	4.6803894E+02	2.6540128E-01	-5.7449465E+02	1.1615860E+03
1.8000000E+00	4.9419014E+02	2.9747885E-01	-6.0572510E+02	1.1600547E+03
1.9000000E+00	5.2019982E+02	3.3085925E-01	-6.3668841E+02	1.1585374E+03
2.0000000E+00	5.4606929E+02	3.6583329E-01	-6.6738742E+02	1.1570342E+03
2.1000000E+00	5.7180011E+02	4.0222944E-01	-6.9782465E+02	1.1555447E+03
2.2000000E+00	5.9739415E+02	4.4089090E-01	-7.2800233E+02	1.1540686E+03
2.3000000E+00	6.2285325E+02	4.8143283E-01	-7.5792264E+02	1.1526060E+03
2.4000000E+00	6.4817897E+02	5.2347995E-01	-7.8758793E+02	1.1511567E+03
2.5000000E+00	6.7337266E+02	5.6685575E-01	-8.1700074E+02	1.1497214E+03
2.6000000E+00	6.9843578E+02	6.1171614E-01	-8.4616375E+02	1.1482974E+03
2.7000000E+00	7.2337006E+02	6.5838476E-01	-8.7507947E+02	1.1468852E+03
2.8000000E+00	7.4817741E+02	7.0688520E-01	-9.0373025E+02	1.1454850E+03
2.9000000E+00	7.7285977E+02	7.5719923E-01	-9.3217852E+02	1.1440967E+03
3.0000000E+00	7.9741906E+02	8.0904335E-01	-9.6038649E+02	1.1427234E+03
3.1000000E+00	8.2185698E+02	8.6226391E-01	-9.8833821E+02	1.1413628E+03
3.2000000E+00	8.4617524E+02	9.1690524E-01	-1.1603514E+03	1.1400137E+03
3.3000000E+00	8.7037572E+02	9.7314069E-01	-1.0435204E+03	1.1386756E+03
3.4000000E+00	8.9446045E+02	1.0311149E+00	-1.1207766E+03	1.1373483E+03
3.5000000E+00	9.1843143E+02	1.0908336E+00	-1.0978063E+03	1.1360317E+03
3.6000000E+00	9.4229075E+02	1.1521740E+00	-1.1246122E+03	1.1347257E+03
3.7000000E+00	9.6604050E+02	1.2149849E+00	-1.1511971E+03	1.1334302E+03
3.8000000E+00	9.8968265E+02	1.2791899E+00	-1.1775633E+03	1.1321449E+03
3.9000000E+00	1.0132191E+03	1.3448243E+00	-1.2037151E+03	1.1308697E+03
4.0000000E+00	1.0366520E+03	1.4119929E+00	-1.2296553E+03	1.1296045E+03
4.1000000E+00	1.0599837E+03	1.4807879E+00	-1.2553830E+03	1.1283573E+03
4.2000000E+00	1.0832166E+03	1.5512281E+00	-1.2809056E+03	1.1271147E+03
4.3000000E+00	1.1063535E+03	1.6232554E+00	-1.3062248E+03	1.1258811E+03
4.4000000E+00	1.1293973E+03	1.6967783E+00	-1.3313444E+03	1.1246577E+03
4.5000000E+00	1.1523515E+03	1.7717325E+00	-1.3562688E+03	1.1234445E+03
4.6000000E+00	1.1752197E+03	1.8481148E+00	-1.3810018E+03	1.1222427E+03
4.7000000E+00	1.1980058E+03	1.9259772E+00	-1.4055498E+03	1.1210480E+03
4.8000000E+00	1.2207136E+03	2.0053908E+00	-1.4299136E+03	1.1198613E+03
4.9000000E+00	1.2433473E+03	2.0862052E+00	-1.4541015E+03	1.1186825E+03
5.0000000E+00	1.2659109E+03	2.1684262E+00	-1.4781166E+03	1.1175114E+03
5.1000000E+00	1.2884081E+03	2.2521216E+00	-1.5019634E+03	1.1163481E+03
5.2000000E+00	1.3108427E+03	2.3372487E+00	-1.5256441E+03	1.1151940E+03
5.3000000E+00	1.3332182E+03	2.4238134E+00	-1.5491687E+03	1.1140491E+03
5.4000000E+00	1.3555379E+03	2.5118236E+00	-1.5725350E+03	1.1129111E+03
5.5000000E+00	1.3778050E+03	2.6052513E+00	-1.5957487E+03	1.1117814E+03
5.6000000E+00	1.4000225E+03	2.6971834E+00	-1.6188131E+03	1.1106577E+03
5.7000000E+00	1.4221930E+03	2.7907828E+00	-1.6417312E+03	1.1095406E+03
5.8000000E+00	1.4443192E+03	2.8860787E+00	-1.6645059E+03	1.1084301E+03
5.9000000E+00	1.4664035E+03	2.9830790E+00	-1.6871398E+03	1.1073271E+03
6.0000000E+00	1.4884480E+03	3.0817794E+00	-1.7096353E+03	1.1062319E+03
6.1000000E+00	1.5104546E+03	3.1821762E+00	-1.7319944E+03	1.1051441E+03
6.2000000E+00	1.5324250E+03	3.2842759E+00	-1.7542193E+03	1.1040623E+03
6.3000000E+00	1.5543605E+03	3.3880984E+00	-1.7763113E+03	1.1029860E+03
6.4000000E+00	1.5762624E+03	3.4936752E+00	-1.7982720E+03	1.1019151E+03
6.5000000E+00	1.5981321E+03	3.6010417E+00	-1.8201028E+03	1.1008496E+03
6.6000000E+00	1.6199706E+03	3.7102299E+00	-1.8418050E+03	1.0997898E+03
6.7000000E+00	1.6417789E+03	3.8212635E+00	-1.8633797E+03	1.0987364E+03
6.8000000E+00	1.6635576E+03	3.9341579E+00	-1.8848275E+03	1.0976894E+03
6.9000000E+00	1.6853075E+03	4.0489215E+00	-1.9061494E+03	1.0966488E+03
7.0000000E+00	1.7070291E+03	4.1655588E+00	-1.9273461E+03	1.0956141E+03
7.1000000E+00	1.7287231E+03	4.2840744E+00	-1.9484183E+03	1.0945855E+03
7.2000000E+00	1.7503901E+03	4.4044758E+00	-1.9693668E+03	1.0935635E+03
7.3000000E+00	1.7720308E+03	4.5267760E+00	-1.9901923E+03	1.0925473E+03
7.4000000E+00	1.7936457E+03	4.6509827E+00	-2.0108956E+03	1.0915369E+03
7.5000000E+00	1.8152354E+03	4.7771471E+00	-2.0314771E+03	1.0905331E+03
7.6000000E+00	1.8368001E+03	4.9052807E+00	-2.0519373E+03	1.0895325E+03
7.7000000E+00	1.8583402E+03	5.0353534E+00	-2.0722786E+03	1.0885384E+03
7.8000000E+00	1.8798561E+03	5.1674416E+00	-2.0924958E+03	1.0875494E+03
7.9000000E+00	1.9013481E+03	5.3015370E+00	-2.1125950E+03	1.0865657E+03
8.0000000E+00	1.9228165E+03	5.4376479E+00	-2.1325765E+03	1.0855872E+03
8.1000000E+00	1.9442617E+03	5.5757804E+00	-2.1524350E+03	1.0846138E+03
8.2000000E+00	1.9656840E+03	5.7159406E+00	-2.1721747E+03	1.0836455E+03
8.3000000E+00	1.9870837E+03	5.8581364E+00	-2.1918001E+03	1.0826823E+03
8.4000000E+00	2.0084610E+03	6.0023782E+00	-2.2113056E+03	1.0817240E+03
8.5000000E+00	2.0298162E+03	6.1486793E+00	-2.2306933E+03	1.0807707E+03
8.6000000E+00	2.0511495E+03	6.2970557E+00	-2.2499637E+03	1.0798222E+03
8.7000000E+00	2.0724610E+03	6.4475247E+00	-2.2691169E+03	1.0788787E+03
8.8000000E+00	2.0937510E+03	6.6001039E+00	-2.2881533E+03	1.0779400E+03
8.9000000E+00	2.1150196E+03	6.7548094E+00	-2.3070732E+03	1.0770060E+03
9.0000000E+00	2.1362670E+03	6.9116551E+00	-2.3258778E+03	1.0760768E+03
9.1000000E+00	2.1574935E+03	7.0706528E+00	-2.3445663E+03	1.0751524E+03
9.2000000E+00	2.1786992E+03	7.2318120E+00	-2.3631361E+03	1.0742328E+03
9.3000000E+00	2.1998843E+03	7.3951468E+00	-2.3815923E+03	1.0733174E+03
9.4000000E+00	2.2210489E+03	7.5606455E+00	-2.3999333E+03	1.0724066E+03
9.5000000E+00	2.2421932E+03	7.7283310E+00	-2.4181591E+03	1.0715000E+03
9.6000000E+00	2.2633172E+03	7.8982020E+00	-2.4362700E+03	1.0705993E+03
9.7000000E+00	2.2844211E+03	8.0702643E+00	-2.4542662E+03	1.0697023E+03
9.8000000E+00	2.3055049E+03	8.2445254E+00	-2.4721478E+03	1.0688097E+03
9.9000000E+00	2.3265686E+03	8.4209951E+00	-2.4899151E+03	1.0679216E+03
1.0000000E+01	2.3476129E+03	8.5996856E+00	-2.5075683E+03	1.0670378E+03
1.0100000E+01	2.3686373E+03	8.7806101E+00	-2.5251074E+03	1.0661584E+03
1.0200000E+01	2.3896420E+03	8.9637838E+00	-2.5425331E+03	1.0652833E+03
1.0300000E+01	2.4106273E+03	9.1492211E+00	-2.5598456E+03	1.0644125E+03
1.0400000E+01	2.4315931E+03	9.3369357E+00	-2.5770435E+03	1.0635459E+03
1.0500000E+01	2.4525397E+03	9.5269394E+00	-2.5941288E+03	1.0626835E+03
1.0600000E+01	2.4734670E+03	9.7192442E+00	-2.6111011E+03	1.0618254E+03

Time	Range	Drift	Altitude	Roll rate
1.070C000E+01	2.4943752E+03	9.913R575E+00	-2.627960SE+03	1.260971JE+03
1.0800000E+01	2.515264AE+03	1.0110787E+01	-2.6447072E+03	1.2601214E+03
1.0900000E+01	2.536134AE+03	1.0310641E+01	-2.6613413E+03	1.2592756E+03
1.1000000E+01	2.5569A61E+03	1.0511624E+01	-2.6778631E+03	1.2584238E+03
1.1100000E+01	2.577818?E+03	1.0715544E+01	-2.6942726E+03	1.2575961E+03
1.1200000E+01	2.598632AE+03	1.0921809E+01	-2.7105701E+03	1.2567623E+03
1.1300000E+01	2.6194?8?E+03	1.1130429E+01	-2.7267558E+03	1.2559325E+03
1.1400000E+01	2.6402C52E+03	1.1341413E+01	-2.7428297E+03	1.2551066E+03
1.1500000E+01	2.6609A37E+03	1.1554775E+01	-2.7587921E+03	1.2542846E+03
1.1600000E+01	2.6817C30E+03	1.1770527E+01	-2.7746431E+03	1.2534665E+03
1.1700000E+01	2.7024259E+03	1.1988681E+01	-2.7903829E+03	1.2526522E+03
1.1800000E+01	2.7231297E+03	1.2209251E+01	-2.8060116E+03	1.2518417E+03
1.1900000E+01	2.743815AE+03	1.2432250E+01	-2.8215293E+03	1.2510350E+03
1.2000000E+01	2.7644A31E+03	1.2657689E+01	-2.8369363E+03	1.2502320E+03
1.2100000E+01	2.7851329E+03	1.2885585E+01	-2.8522326E+03	1.2494328E+03
1.2200000E+01	2.8057A4AE+03	1.3115932E+01	-2.8674185E+03	1.2486372E+03
1.2300000E+01	2.826379DE+03	1.3348754E+01	-2.8824940E+03	1.2478453E+03
1.2400000E+01	2.8469754E+03	1.3584054E+01	-2.8974593E+03	1.2470570E+03
1.2500000E+01	2.8675543E+03	1.3821841E+01	-2.9123145E+03	1.2462724E+03
1.2600000E+01	2.888115AE+03	1.4062122E+01	-2.9270599E+03	1.2454912E+03
1.2700000E+01	2.9086A59E+03	1.4304905E+01	-2.9416956E+03	1.2447137E+03
1.2800000E+01	2.9291A5AE+03	1.4550201E+01	-2.9562216E+03	1.2439396E+03
1.2900000E+01	2.9496969E+03	1.4798019E+01	-2.9706379E+03	1.2431691E+03
1.3000000E+01	2.9701A6AE+03	1.5048376E+01	-2.9849450E+03	1.2424020E+03
1.3100000E+01	2.9906A14E+03	1.5301262E+01	-2.9991430E+03	1.2416383E+03
1.3200000E+01	3.0111190E+03	1.5556711E+01	-3.0132319E+03	1.2408780E+03
1.3300000E+01	3.0315595E+03	1.5814730E+01	-3.0272119E+03	1.2401211E+03
1.3400000E+01	3.0519B30E+03	1.6075330E+01	-3.0410831E+03	1.2393676E+03
1.3500000E+01	3.0723B6EE+03	1.6338525E+01	-3.0548456E+03	1.2386173E+03
1.3600000E+01	3.092779AE+03	1.6604326E+01	-3.0684997E+03	1.2378705E+03
1.3700000E+01	3.1131524E+03	1.6872746E+01	-3.0820454E+03	1.2371269E+03
1.3800000E+01	3.1335B7E+03	1.7143796E+01	-3.0954828E+03	1.2363865E+03
1.3900000E+01	3.1539A8AE+03	1.7417485E+01	-3.1088121E+03	1.2356494E+03
1.4000000E+01	3.1741715E+03	1.7693825E+01	-3.1220334E+03	1.2349153E+03
1.4100000E+01	3.1946781E+03	1.7972824E+01	-3.1351468E+03	1.2341843E+03
1.4200000E+01	3.2147A2EE+03	1.8254491E+01	-3.1481524E+03	1.2334563E+03
1.4300000E+01	3.2350A2CE+03	1.8538835E+01	-3.1610505E+03	1.2327314E+03
1.4400000E+01	3.2552994E+03	1.8825865E+01	-3.1738410E+03	1.2320095E+03
1.4500000E+01	3.2755AC4E+03	1.9115590E+01	-3.1865242E+03	1.2312905E+03
1.4600000E+01	3.2957655E+03	1.9408019E+01	-3.1991001E+03	1.2305744E+03
1.4700000E+01	3.3159744E+03	1.9703163E+01	-3.2115688E+03	1.2298612E+03
1.4800000E+01	3.3361671E+03	2.0001030E+01	-3.2239307E+03	1.2291512E+03
1.4900000E+01	3.3563438E+03	2.0301632E+01	-3.2361856E+03	1.2284439E+03
1.5000000E+01	3.3765046E+03	2.0604980E+01	-3.2483338E+03	1.2277396E+03
1.5100000E+01	3.3966496E+03	2.0911086E+01	-3.2603753E+03	1.2270382E+03
1.5200003E+01	3.4167785E+03	2.1219966E+01	-3.2723103E+03	1.2263397E+03
1.5300000E+01	3.4368917E+03	2.1531617E+01	-3.2841387E+03	1.2256439E+03
1.5400000E+01	3.4569891E+03	2.1846067E+01	-3.2958611E+03	1.2249512E+03
1.5500000E+01	3.4770710E+03	2.2163323E+01	-3.3074773E+03	1.2242612E+03
1.5600000E+01	3.4971371E+03	2.2483396E+01	-3.3189873E+03	1.2235740E+03
1.5700000E+01	3.5171878E+01	2.2806296E+01	-3.3303915E+03	1.2228897E+03
1.5800000E+01	3.5372229E+03	2.3132035E+01	-3.3416898E+03	1.2222081E+03
1.5900000E+01	3.5572425E+01	2.3460634E+01	-3.3528823E+03	1.2215294E+03
1.6000000E+01	3.5772465E+03	2.3792090E+01	-3.3639693E+03	1.2208534E+03
1.6100000E+01	3.5972357E+01	2.4126421E+01	-3.3749508E+03	1.2201801E+03
1.6200000E+01	3.6172094E+01	2.4463639E+01	-3.3858269E+03	1.2195096E+03
1.6300000E+01	3.6371687E+01	2.4803755E+01	-3.3965977E+03	1.2188418E+03
1.6400000E+01	3.6571111E+01	2.5146779E+01	-3.4072634E+03	1.2181768E+03
1.6500000E+01	3.6770392E+01	2.5492724E+01	-3.4178240E+03	1.2175145E+03
1.6600000E+01	3.6969523E+01	2.5841607E+01	-3.4282796E+03	1.2168549E+03
1.6700000E+01	3.7168508E+01	2.6193441E+01	-3.4386305E+03	1.2161980E+03
1.6800000E+01	3.7367335E+03	2.6548244E+01	-3.4488766E+03	1.2155438E+03
1.6900000E+01	3.756A017E+03	2.6905997E+01	-3.4590180E+03	1.2148922E+03
1.7000000E+01	3.7764551E+03	2.7266721E+01	-3.4690550E+03	1.2142432E+03
1.7100000E+01	3.7962937E+03	2.7629436E+01	-3.4789876E+03	1.2135968E+03
1.7200000E+01	3.8161175E+03	2.7996160E+01	-3.4888158E+03	1.2129530E+03
1.7300000E+01	3.8359267E+03	2.8365910E+01	-3.4985399E+03	1.2123119E+03
1.7400000E+01	3.8557212E+03	2.8738682E+01	-3.5081598E+03	1.2116734E+03
1.7500000E+01	3.8755011E+03	2.9114506E+01	-3.5176758E+03	1.2110374E+03
1.7600000E+01	3.8952665E+03	2.9492665E+01	-3.5270879E+03	1.2104039E+03
1.7700000E+01	3.9150174E+03	2.9874309E+01	-3.5363962E+03	1.2097730E+03
1.7800000E+01	3.9347538E+03	3.0259000E+01	-3.5456008E+03	1.2091446E+03
1.7900000E+01	3.954475AE+03	3.0646752E+01	-3.5547019E+03	1.2085187E+03
1.8000000E+01	3.9741834E+03	3.1037575E+01	-3.5636993E+03	1.2078952E+03
1.8100000E+01	3.993877?E+03	3.1431479E+01	-3.5725935E+03	1.2072743E+03
1.8200000E+01	4.0135562E+03	3.1828476E+01	-3.5813843E+03	1.2066558E+03
1.8300000E+01	4.0332212E+03	3.2228576E+01	-3.5900720E+03	1.2060398E+03
1.8400000E+01	4.052A72QE+03	3.2631788E+01	-3.5986565E+03	1.2054263E+03
1.8500000E+01	4.0725087E+03	3.3038123E+01	-3.6071380E+03	1.2048152E+03
1.8600000E+01	4.0921314E+03	3.3447591E+01	-3.6155167E+03	1.2042066E+03
1.8700000E+01	4.1117401E+03	3.3860202E+01	-3.6237925E+03	1.2036004E+03
1.8800000E+01	4.131334AE+03	3.4275965E+01	-3.6319655E+03	1.2029967E+03
1.8900000E+01	4.1509154E+03	3.4694889E+01	-3.6400360E+03	1.2023954E+03
1.9000000E+01	4.1704A2SE+03	3.5116988E+01	-3.6480039E+03	1.2017965E+03
1.9100000E+01	4.1900135E+03	3.5542276E+01	-3.6558693E+03	1.2012001E+03
1.9200000E+01	4.2095749E+03	3.5970733E+01	-3.6636325E+03	1.2006061E+03
1.9300000E+01	4.2291100E+03	3.6402404E+01	-3.6712933E+03	1.1999236E+03
1.9400000E+01	4.2486125E+03	3.6837297E+01	-3.6788521E+03	1.1992235E+03
1.9500000E+01	4.268110?E+03	3.7275391E+01	-3.6863087E+03	1.1985256E+03
1.9600000E+01	4.2875959E+03	3.7716727E+01	-3.6936636E+03	1.1978298E+03
1.9700000E+01	4.3070671E+03	3.8161304E+01	-3.7009162E+03	1.1971361E+03
1.9800000E+01	4.3265250E+03	3.8609136E+01	-3.7080672E+03	1.1964446E+03
1.9900000E+01	4.3459694E+03	3.9060230E+01	-3.7151165E+03	1.1957552E+03

TABLE 7. BEGINNING AND END OF OUTPUT FROM SAMPLE INPUT FOR SHELTRAJ

(A) FORCES

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PARAMETER ESTIMATION FOR ANALYSIS OF TRAJECTORY AND ROLL DATA FROM RADAR AND YAWSONDE MEASUREMENTS MADE DURING THE FLIGHT OF A SHELL

                    A TEST EXERCISE FOR THE ANALYSIS OF RADAR DATA - NR3634 DRAG CURVE


VEHICLE PHYSICAL CONSTANTS
MASS  14.96RRKG      MOMENT OF INERTIA IN ROLL 0.023260KG M2
REFERENCE AREA 0.C0R659M2      REFERENCE LENGTH 0.1050M

EARTH MODEL - ANGULAR VELOCITY  LATITUDE  FIRING LINE  GRAVITATION  HEIGHT ABOVE  RADIUS OF
                    (RAD/SEC)      (DEG)      (DEG)        (M/S/S)   SEA LEVEL(M)  THE EARTH(M)
                  0.7272200D-04    35.000     315.000      9.80700       0.0     0.63733680+07


ORDER OF PARAMETERS
     10 11 12  7  8  9  3  6  1  4 18  2  5 19 15 22 13 16 20 14 17 21 23 24 25 26 27

INITIAL TIME  0.0  (S)

SCALING OF     RANGE     AZIMUTH    ALTITUDE  ROLL RATE
                1.00        1.00        1.00       0.0


REJECTION LEVEL ON DATA 10.00

ACCURACY FACTOR FOR PARAMETER ERROR LEVELS  1.000
MAXIMUM INTEGRATION STEP 0.2000      NO. OF PARAMETERS VARIED  2      MAXIMUM NO. OF ITERATIONS  6
NO PLOTS REQUESTED

              CDA0        CDR0        CDK         CDA1        CDR1        CDL         XDOT0       YDOT0
PARAM.   -0.2290D+00 -0.4734D-01  0.9772D+00  0.1263D+00  0.4665D-01  0.5633D-01  0.2894D+03  0.0
DELTA     0.5000D-02  0.5000D-02  0.1000D-01  0.5000D-02  0.5000D-02  0.1000D-02  0.1000D+01  0.1000D+01

              ZDOT0       X0          Y0          Z0          CZALFA0     CZALFR0     CZALFAK     CZALFA1
PARAM.   -0.3637D+03  0.0         0.0         0.0        -0.9398D+00 -0.8500D+00  0.9641D+00 -0.7303D+00
DELTA     0.1000D+01  0.1000D+01  0.1000D+01  0.1000D+01  0.1000D-01  0.1000D-01  0.1000D-01  0.1000D-01

              CZALFR1     CDA2        CDR2        CZALFA2     CZALFR2     CZALFAL     CLP0        CLP1
PARAM.    0.1312D+00  0.0         0.0         0.0         0.0         0.3441D-01 -0.3559D-01  0.1569D-01
DELTA     0.1000D-01  0.1000D-02  0.1000D-02  0.1000D-01  0.1000D-01  0.1000D-02  0.1000D-03  0.1000D-03

              CLP2        CLP3        PO
PARAM.   -0.5577D-02  0.8989D-03  0.1390D+04
DELTA     0.5000D-04  0.1000D-04  0.1000D+01
SIGMA=  0.1R207D+01 DEGREES OF FREEDOM= 796


COVARIANCE MATRIX (MATRIX IS SYMMETRIC)



  1.250-0R
 -1.980-0R  3.RR0-0R

 CHANGES TO THE FOLLOWING PARAMETERS ALL SMALL

              CDA0        CDR0        CDK         CDA1        CDR1        CDL         XDOT0       YDOT0
PARAM.   -0.2270D+00 -0.4954D-01  0.9772D+00  0.1263D+00  0.4665D-01  0.5633D-01  0.2894D+03  0.0
DELTA     0.1118D-03  0.1918D-03  0.1000D-01  0.5000D-02  0.5000D-02  0.1000D-02  0.1000D+01  0.1000D+01

              ZDOT0       X0          Y0          Z0          CZALFA0     CZALFR0     CZALFAK     CZALFA1
PARAM.   -0.3637D+03  0.0         0.0         0.0        -0.9398D+00 -0.8500D+00  0.9641D+00 -0.7303D+00
DELTA     0.1000D+01  0.1000D+01  0.1000D+01  0.1000D+01  0.1000D-01  0.1000D-01  0.1000D-01  0.1000D-01

              CZALFR1     CDA2        CDR2        CZALFA2     CZALFR2     CZALFAL     CLP0        CLP1
PARAM.    0.1312D+00  0.0         0.0         0.0         0.0         0.3441D-01 -0.3559D-01  0.1569D-01
DELTA     0.1000D-01  0.1000D-02  0.1000D-02  0.1000D-01  0.1000D-01  0.1000D-02  0.1000D-03  0.1000D-03

              CLP2        CLP3        PO
PARAM.   -0.5577D-02  0.8989D-03  0.1390D+04
DELTA     0.5000D-04  0.1000D-04  0.1000D+01


REJECTION LEVEL ON DATA  1.R0

ACCURACY FACTOR FOR PARAMETER ERROR LEVELS  1.000
MAXIMUM INTEGRATION STEP 0.2000      NO. OF PARAMETERS VARIED 14      MAXIMUM NO. OF ITERATIONS  6
NO PLOTS REQUESTED

              CDA0        CDR0        CDK         CDA1        CDR1        CDL         XDOT0       YDOT0
PARAM.   -0.2290D+00 -0.4433D-01  0.9769D+00  0.1295D+00  0.6534D-01  0.5747D-01  0.2888D+03  0.6914D-01
DELTA     0.5000D-02  0.5000D-02  0.1000D-01  0.5000D-02  0.5000D-02  0.1000D-02  0.1000D+01  0.1000D+01

              ZDOT0       X0          Y0          Z0          CZALFAL     CZALFR0     CZALFAK     CZALFA1
PARAM.   -0.3640D+03  0.1382D+01 -0.7498D-01  0.1108D+01 -0.8232D+00 -0.8500D+00  0.9641D+00 -0.7303D+00
DELTA     0.1000D+01  0.1000D+01  0.1000D+01  0.1000D+01  0.1000D-01  0.1000D-01  0.1000D-01  0.1000D-01

              CZALFR1     CDA2        CDR2        CZALFA2     CZALFR2     CZALFAL     CLP0        CLP1
PARAM.    0.1312D+00  0.0         0.0         0.0         0.0         0.3441D-01 -0.3559D-01  0.1569D-01
DELTA     0.1000D-01  0.1000D-02  0.1000D-02  0.1000D-01  0.1000D-01  0.1000D-02  0.1000D-03  0.1000D-03


              CLP2        CLP3        PO
PARAM.   -0.5577D-02  0.8989D-03  0.1390D+04
DELTA     0.5000D-04  0.1000D-04  0.1000D+01
SIGMA=  0.A3726D+00 DEGREES OF FREEDOM= 752

              CDA0        CDR0        CDK         CDA1        CDR1        CDL         XDOT0       YDOT0
PARAM.   -0.2297D+00 -0.4442D-01  0.9768D+00  0.1290D+00  0.6530D-01  0.5735D-01  0.2890D+03  0.5198D-02
DELTA     0.8972D-02  0.4209D-02  0.7017D-02  0.4435D-01  0.1371D-01  0.6838D-02  0.4321D+00  0.7904D-01
```

TABLE 7(A). (CONTINUED)

	ZDOTD	XD	YD	ZD	CZALFA0	CZALFR0	CZALFAK	CZALFA1
PARAM.	-0.3A4D+03	0.1326D+01	-0.5810D-02	0.1065D+01	-0.916AD+00	-0.830D+00	0.9641D+00	-0.7303D+00
DELTA	0.53A0+00	0.2027D+00	0.1492D+00	0.2467D+00	0.945D-01	0.152D-01	0.1000D-01	0.1000D-01

	CZALFR1	CDA2	CDR2	CZALFA2	CZALFR2	CZALFAL	CLPD	CLP1
PARAM.	0.1312D+00	0.0	0.0	0.0	0.0	0.3441D-01	-0.3569D-01	0.1569D-01
DELTA	0.1000D-01	0.1000D-02	0.1000D-02	0.1000D-01	0.1000D-01	0.1000D-02	0.1000D-03	0.1000D-03

	CLP2	CLP3	PD
PARAM.	-0.5577D-02	0.A99D-03	0.1390D+04
DELTA	0.50C0D-04	0.1000D-04	0.1000D+01

SIGMA= 0.A2139D+00 DEGREES OF FREEDOM= 74C

COVARIANCE MATRIX (MATRIX IS SYMMETRIC)

```
 7.65D-05
-3.07D-05  1.80D-05
-5.49D-05  1.67D-05  4.57D-05
-3.72D-04  1.38D-04  2.73D-04  1.88D-03
-9.11D-05  5.90D-05  4.35D-05  4.02D-04  1.99D-04
-5.24D-05  2.77D-05  3.26D-05  2.38D-04  8.49D-05  4.58D-05
 2.98D-03 -9.07D-04 -2.23D-03 -1.46D-02 -2.53D-03 -1.52D-03  1.74D-01
 4.58D-06 -6.23D-07 -4.14D-04 -2.66D-05 -5.28D-07 -1.75D-05  3.30D-04  5.89D-03
-1.73D-03  1.13D-03  2.79D-03  2.00D-02  3.17D-03  1.90D-03 -2.17D-01 -4.10D-04  2.71D-01
-9.39D-04  2.63D-04  6.95D-04  5.35D-03  7.18D-04  4.30D-04 -6.99D-02 -1.35D-04  8.83D-02  3.83D-02
-5.36D-07 -6.19D-07  1.14D-06  5.20D-06 -2.95D-06 -3.24D-07 -1.14D-04 -9.93D-03  1.41D-04  5.71D-05  2.08D-02
 1.18D-03 -3.33D-04 -8.74D-04 -6.72D-03 -9.20D-04 -6.43D-04  8.72D-02  1.67D-04 -1.09D-01 -6.20D-02 -6.92D-05  5.66D-02
-1.17D-06  1.57D-06 -2.84D-07  3.98D-06  6.36D-06  1.47D-06 -4.89D-06  6.93D-03  6.91D-06  6.61D-06 -1.67D-02 -1.45D-05  8.52D-03
 2.70D-07 -3.69D-07  5.83D-08 -9.91D-07 -1.63D-06 -3.08D-07  3.45D-06 -7.17D-04 -4.77D-06  8.83D-08  7.69D-04  2.54D-06 -1.06D-03
 2.41D-04
```

CHANGES TO THE FOLLOWING PARAMETERS ALL SMALL

	CDA0	CDR0	CDK	CDA1	CDP1	CDL	XDOTD	YDOTD
PARAM.	-0.229A0+00	-0.4444D-01	0.97AAD+00	0.1289D+00	0.5547D-01	0.5731D-01	0.2890D+03	0.5780D-02
DELTA	0.A749D-02	0.4247D-02	0.47A3D-02	0.4308D-01	0.1410D-01	0.676AD-02	0.4175D+00	0.7674D-01

	ZDOTD	XD	YD	ZD	CZALFA0	CZALFR0	CZALFAK	CZALFA1
PARAM.	-0.3A00+03	0.1272D+01	-0.5992D-02	0.1023D+01	-0.916A0+00	-0.8307D+00	0.9641D+00	-0.7303D+00
DELTA	0.5203D+00	0.195AD+00	0.1441D+00	0.2380D+00	0.923C0-01	0.1554D-01	0.1000D-01	0.1000D-01

	CZALFR1	CDA2	CDR2	CZALFA2	CZALFR2	CZALFAL	CLPD	CLP1
PARAM.	0.1312D+00	0.0	0.0	0.0	0.0	0.3441D-01	-0.3569D-01	0.1569D-01
DELTA	0.1000D-01	0.1000D-02	0.1000D-02	0.1000D-01	0.1000D-01	0.1000D-02	0.1000D-03	0.1000D-03

	CLP2	CLP3	PD
PARAM.	-0.5577D-02	0.A99D-03	0.1390D+04
DELTA	0.5CC0D-04	0.1000D-04	0.1000D+01

TABLE 7. BEGINNING AND END OF OUTPUT FROM SAMPLE INPUT FOR SHELTRAJ

(B) ROLLING MOMENTS

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PARAMETER ESTIMATION FOR ANALYSIS OF TRAJECTORY AND ROLL DATA FROM RADAR AND YAWSONDE MEASUREMENTS MADE DURING THE FLIGHT OF A SHELL

                    A DECK TO TEST THE ROLL FITTING CAPABILITIES OF THIS PROGRAM


VEHICLE PHYSICAL CONSTANTS
MASS  14.96RRKG      MOMENT OF INERTIA IN ROLL 0.02326GKG M2
REFERENCE AREA 0.00R659M2      REFERENCE LENGTH 0.1050M

EARTH MODEL - ANGULAR VELOCITY  LATITUDE  FIRING LINE  GRAVITATION  HEIGHT ABOVE  RADIUS OF
                  (RAD/SEC)        (DEG)      (DEG)        (M/S/S)   SEA LEVEL(M)  THE EARTH(M)
                 0.7272200D-04    35.000     315.000      9.80700       0.0      0.6373368D+07


ORDER OF PARAMETERS
     A  7  R  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 2A 27  1  2  4  5  3

INITIAL TIME  0.0  (S)

SCALING OF    RANGE    AZIMUTH   ALTITUDE  ROLL RATE
                0.0       0.0       0.0       1.00


REJECTION LEVEL ON DATA 10.00

ACCURACY FACTOR FOR PARAMETER ERROR LEVELS  1.000
MAXIMUM INTEGRATION STEP 0.2000      NO. OF PARAMETERS VARIED  1      MAXIMUM NO. OF ITERATIONS 20
NO PLOTS REQUESTED

              CLP0          CLP1          P0            CLP2          CLP3          X0            Y0            Z0
PARAM.    -0.3600D-01   0.1800D-01   0.1390D+04    0.0           0.0           0.1270D+01    0.0           0.1020D+01
DELTA      0.5000D-02    0.1000D-02   0.1000D+01    0.5000D-03    0.1000D-03    0.1000D+01    0.1000D+01    0.1000D+01

              XDOT0         YDOT0         ZDOT0         CDK           CDL           CDA0          CDA1          CDA2
PARAM.    0.2890D+03    0.0          -0.3440D+03    0.9780D+00    0.5731D-01   -0.2290D+00    0.1290D+00    0.0
DELTA      0.1000D+01    0.1000D+01    0.1000D+01    0.1000D+00    0.5000D-02    0.2000D-01    0.1000D-01    0.1000D-03

              CDR0          CDR1          CDR2          CZALFAK       CZALFAL       CZALFA0       CZALFA1       CZALFA2
PARAM.    -0.4444D-01   0.4547D-01    0.0           0.9841D+00    0.3441D-01   -0.9162D+00   -0.7303D+00    0.0
DELTA      0.5000D-02    0.5000D-02    0.1000D-03    0.1000D+00    0.2000D-02    0.1000D+00    0.5000D-01    0.1000D-03

              CZALFR0       CZALFR1       CZALFR2
PARAM.    -0.8307D+00   0.1312D+00    0.0
DELTA      0.5000D-01    0.1000D-01    0.1000D-03
SIGMA=  0.789470-01 DEGREES OF FREEDOM=  7A

              CLP0          CLP1          P0            CLP2          CLP3          X0            Y0            Z0
PARAM.    -0.4149D-01   0.1400D-01   0.1390D+04    0.0           0.0           0.1270D+01    0.0           0.1020D+01
DELTA      0.2142D-04    0.1000D-02   0.1000D+01    0.5000D-03    0.1000D-03    0.1000D+01    0.1000D+01    0.1000D+01

              XDOT0         YDOT0         ZDOT0         CDK           CDL           CDA0          CDA1          CDA2
PARAM.    0.2890D+03    0.0          -0.3440D+03    0.9780D+00    0.5731D-01   -0.2290D+00    0.1290D+00    0.0
DELTA      0.1300D+01    0.1000D+01    0.1000D+01    0.1000D+00    0.5000D-02    0.2000D-01    0.1000D-01    0.1000D-03

              CDR0          CDR1          CDR2          CZALFAK       CZALFAL       CZALFA0       CZALFA1       CZALFA2
PARAM.    -0.4444D-01   0.4547D-01    0.0           0.9841D+00    0.3441D-01   -0.9162D+00   -0.7303D+00    0.0
DELTA      0.5000D-02    0.5000D-02    0.1000D-03    0.1000D+00    0.2000D-02    0.1000D+00    0.5000D-01    0.1000D-03




              CZALFR0       CZALFR1       CZALFR2
PARAM.    -0.8307D+00   0.1312D+00    0.0
DELTA      0.5000D-01    0.1000D-01    0.1000D-03
SIGMA=  0.8905AD+00 DEGREES OF FREEDOM=  44R


COVARIANCE MATRIX (MATRIX IS SYMMETRIC)


  A.200-16

 CHANGES TO THE FOLLOWING PARAMETERS ALL SMALL

              CLP0          CLP1          P0            CLP2          CLP3          X0            Y0            Z0
PARAM.    -0.4149D-01   0.1400D-01   0.1390D+04    0.0           0.0           0.1270D+01    0.0           0.1020D+01
DELTA      0.2490D-04    0.1000D-02   0.1000D+01    0.5000D-03    0.1000D-03    0.1000D+01    0.1000D+01    0.1000D+01

              XDOT0         YDOT0         ZDOT0         CDK           CDL           CDA0          CDA1          CDA2
PARAM.    0.2890D+03    0.0          -0.3440D+03    0.9780D+00    0.5731D-01   -0.2290D+00    0.1290D+00    0.0
DELTA      0.1000D+01    0.1000D+01    0.1000D+01    0.1000D+00    0.5000D-02    0.2000D-01    0.1000D-01    0.1000D-03

              CDR0          CDR1          CDR2          CZALFAK       CZALFAL       CZALFA0       CZALFA1       CZALFA2
PARAM.    -0.4444D-01   0.4547D-01    0.0           0.9841D+00    0.3441D-01   -0.9162D+00   -0.7303D+00    0.0
DELTA      0.5000D-02    0.5000D-02    0.1000D-03    0.1000D+00    0.2000D-02    0.1000D+00    0.5000D-01    0.1000D-03

              CZALFR0       CZALFR1       CZALFR2
PARAM.    -0.8307D+00   0.1312D+00    0.0
DELTA      0.5000D-01    0.1000D-01    0.1000D-03
```

TABLE 7(B). (CONTINUED)

ROLLTESTING - AC

REJECTION LEVEL ON DATA 0.1R

ACCURACY FACTOR FOR PARAMETER ERROR LEVELS 1.000
MAXIMUM INTEGRATION STEP 0.2000 NO. OF PARAMETERS VARIED 4 MAXIMUM NO. OF ITERATIONS 20
NO PLOTS REQUESTED

	CLP0	CLP1	P0	CLP2	CLP3	X0	Y0	Z0
PARAM.	-0.3192D-01	0.7277D-02	0.1390D+04	0.0	0.0	0.1270D+01	0.0	0.1020D+01
DELTA	0.5000D-02	0.1000D-02	0.1000D+01	0.5000D-03	0.1000D-03	0.1000D+01	0.1000D+01	0.1000D+01

	XDOT0	YDOT0	ZDOT0	CDK	CDL	CDA0	CDA1	CDA2
PARAM.	0.2890D+03	0.0	-0.3640D+03	0.9768D+00	0.5731D-01	-0.2296D+00	0.1289D+00	0.0
DELTA	0.1000D+01	0.1000D+01	0.1000D+01	0.1000D+00	0.5000D-02	0.2000D-01	0.1000D-01	0.1000D-03

	CDB0	CDB1	CDB2	CZALFAK	CZALFAL	CZALFA0	CZALFA1	CZALFA2
PARAM.	-0.4444D-01	0.6547D-01	0.0	0.9841D+00	0.3441D-01	-0.9162D+00	-0.7303D+00	0.0
DELTA	0.5000D-02	0.5000D-02	0.1000D-03	0.1000D+00	0.2000D-02	0.1000D+00	0.5000D-01	0.1000D-03

	CZALFB0	CZALFB1	CZALFB2
PARAM.	-0.8307D+00	0.1312D+00	0.0
DELTA	0.5000D-01	0.1000D-01	0.1000D-03

SIGMA= 0.A0572D-02 DEGREES OF FREEDOM= 7RR

	CLP0	CLP1	P0	CLP2	CLP3	X0	Y0	Z0
PARAM.	-0.3442D-01	0.1270D-01	0.1390D+04	-0.2867D-02	0.0	0.1270D+01	0.0	0.1020D+01
DELTA	0.1689D-04	0.3684D-04	0.3091D-02	0.1965D-04	0.1000D-03	0.1000D+01	0.1000D+01	0.1000D+01

	XDOT0	YDOT0	ZDOT0	CDK	CDL	CDA0	CDA1	CDA2
PARAM.	0.2890D+03	0.0	-0.3640D+03	0.9768D+00	0.5731D-01	-0.2296D+00	0.1289D+00	0.0
DELTA	0.1000D+01	0.1000D+01	0.1000D+01	0.1000D+00	0.5000D-02	0.2000D-01	0.1000D-01	0.1000D-03

	CDB0	CDB1	CDB2	CZALFAK	CZALFAL	CZALFA0	CZALFA1	CZALFA2
PARAM.	-0.4444D-01	0.6547D-01	0.0	0.9841D+00	0.3441D-01	-0.9162D+00	-0.7303D+00	0.0
DELTA	0.5000D-02	0.5000D-02	0.1000D-03	0.1000D+00	0.2000D-02	0.1000D+00	0.5000D-01	0.1000D-03

	CZALFB0	CZALFB1	CZALFB2
PARAM.	-0.8307D+00	0.1312D+00	0.0
DELTA	0.5000D-01	0.1000D-01	0.1000D-03

SIGMA= 0.A3CA1D-02 DEGREES OF FREEDOM= 79A

COVARIANCE MATRIX (MATRIX IS SYMMETRIC)

```
 2.72D-10
-5.87D-10  1.27D-09
-2.83D-08  6.48D-08  7.34D-06
 1.08D-10 -6.71D-10 -3.63D-08  3.55D-10
```

CHANGES TO THE FOLLOWING PARAMETERS ALL SMALL

	CLP0	CLP1	P0	CLP2	CLP3	X0	Y0	Z0
PARAM.	-0.3436D-01	0.1258D-01	0.1390D+04	-0.2801D-02	0.0	0.1270D+01	0.0	0.1020D+01
DELTA	0.1649D-04	0.3567D-04	0.2710D-02	0.1884D-04	0.1000D-03	0.1000D+01	0.1000D+01	0.1000D+01

	XDOT0	YDOT0	ZDOT0	CDK	CDL	CDA0	CDA1	CDA2
PARAM.	0.2890D+03	0.0	-0.3640D+03	0.9768D+00	0.5731D-01	-0.2296D+00	0.1289D+00	0.0
DELTA	0.1000D+01	0.1000D+01	0.1000D+01	0.1000D+00	0.5000D-02	0.2000D-01	0.1000D-01	0.1000D-03

	CDB0	CDB1	CDB2	CZALFAK	CZALFAL	CZALFA0	CZALFA1	CZALFA2
PARAM.	-0.4444D-01	0.6547D-01	0.0	0.9841D+00	0.3441D-01	-0.9162D+00	-0.7303D+00	0.0
DELTA	0.5000D-02	0.5000D-02	0.1000D-03	0.1000D+00	0.2000D-02	0.1000D+00	0.5000D-01	0.1000D-03

	CZALFB0	CZALFB1	CZALFB2
PARAM.	-0.8307D+00	0.1312D+00	0.0
DELTA	0.5000D-01	0.1000D-01	0.1000D-03

TABLE 8. DESCRIPTION OF CARD INPUT TO YAWCAL

Card	Format	Variable	Description
1	18A4	TITLE(18)	A 72 character description of the job.
2	6F10.0	X(3)	Initial estimates of the three parameters describing the calibration curve, equation (22) $\tan \sigma_N = -[\sin\alpha/X(1)]\ (1 + X(2)^2 - 2X(2)\cos\alpha)^{-\frac{1}{2}}$ where $\alpha = 2(\tau/T + X(3))$ X(3) is in radians. The three parameters are defined by $X(1) = \tan\gamma_1$, $X(2) = \tan\gamma_2/\tan\gamma_1$, and $X(3) = \beta$, where γ_1, γ_2 and β are shown in figure 4.
3	16I5	NPTS	Number of measured points on calibration curve.

NOTE: The three cards are followed by NPTS records on unit 4, each in 2F10.0 format, and each containing simultaneous values of time ratio, τ/T, and complementary solar aspect angle, σ_N, in degrees.

TABLE 9. DESCRIPTION OF CARD INPUT TO ASPECT

Card	Format	Variable	Description
1	18A4	TITLE(18)	A 72 character description of the current job.
2	6F10.0	HA(I,J)	The hours, minutes and seconds, J=1, 2 and 3, of the hour angle of the first point of Aries at 0 hours Universal Time for the date being considered and the following day, I = 1, 2.
3	6F10.0	RA(I,J)	The hours, minutes and seconds, J=1, 2 and 3, of the right ascension of the sun at 0 hours Universal Time for the day in question and the following day, I = 1, 2.
4	6F10.0	DE(I,J)	The degrees, minutes and seconds, J=1, 2 and 3, of the declination of the sun at 0 hours Universal Time for the day in question and the following day, I=1,2.
5	18A4	DATES(5)	A 20 character description of the date for which the complementary solar aspect angle is estimated.
6	6F10.0	LAT(3), LONG(3)	Degrees, minutes and seconds for latitude and longitude of shell. Latitude is positive north and longitude is positive west.
7	12I5	NTIMES	Number of local times (local South Australian time) for which results are required. The local times used are corrected to Universal or Greenwich Mean Time in the program by subtracting nine and a half hours.
8	6F10.0	TO,DT	The local times should begin at TO hours and increment regularly by DT hours.
9	6F10.0	BEAR(3)	The degrees, minutes and seconds of the line of fire relative to true north.
10	12I5	NELEVN	The number of elevation angles for which results are required.
11	6F10.0	ELO, DEL	The elevation angles should begin with ELO degrees and increment regularly by DEL degrees.

TABLE 10. DESCRIPTION OF CARD INPUT TO YAWSONDE

Read	Format	Variable	Description
1	20A4	TITLE(20)	An 80 character job description which is reproduced on the output.
2	16I5	LENTER	A control variable which can be used to begin entering data at any one of the following points in the input stream. LENTER → READ 1 → 3 2 → 6 3 → 7 4 → 8 5 → 9 6 → 10 7 → 11 8 → 12 9 → 14 10 → return
3	16I5	NCOS	Number of entries in a table which provides direction cosines of the sun in range axes as a function of time along the trajectory.
4	5E16.7	TIME(I), COSL(I), COSM(I), COSN(I)	Entries in the table for I=1, NCOS with one set per card.
5	16I5	NTRAJ, IFOUT, ISKIP	Controls for the entry of trajectory data from a dataset constructed from the results of SHELTRAJ, further details of trajectory data are given in Section 4.2.1. This run uses every ISKIP th point up to a total of NTRAJ points and the points used are recorded on the printed output if IFOUT is non-zero.
6	8F10.0	XE(12)	Maximum changes in each parameter value, which are used in testing for convergence.
7	8F10.0	REFS BODIAM PITCHI ROLLI	Reference area for aerodynamic coefficients. Reference length for aerodynamic coefficients. Moment of inertia of shell in pitch and yaw Moment of inertia of shell in roll.
8	8F10.0	X(12)	Initial estimates for the values of the parameters P_i.
9	16I5	JORDER(12)	Order of the parameters for internal use by the program particularly when determining values for the first NPARAM parameters.
10	8F10.0	TO	Time at which integration begins.
11	8F10.0	REJECT ACCFAC	Initial value for data rejection level. Scaling factor for all the XE. Can be used for wholesale adjustment of the convergence levels.

TABLE 10 (CONTINUED)

Read	Format	Variable	Description
		HMAX	Maximum value for integration step. This value is used unless time to next data point is smaller.
12	16I5	NPARAM	The first NPARAM parameters only are allowed to vary.
		ITERN	Maximum number of iterations.
		NPLOT	Plot every NPLOT th point of the simulated measurement values. A maximum of 500 points can be plotted.
13	16I5	LENTER	A repeat of the options allowed in read 2. Principally to avoid inputting new experimental data if none is required.
		LCASE	Non-zero if data for another run follows.
		IFOUT	Non-zero if a record of the experimental data points is required in the print out.
14	16I5	NPTS	Number of experimentally measured data points to be used in this run.
		ISKIP	Uses every ISKIP th point from the dataset containing the experimental measurements.
15	16I5	IND	Control variable, describing the nature of the meteorological data which is available. =1 none available. =2 wind table only, =3 pressure and temperature table only, =4 both wind table and pressure and temperature table.
		NWD	Number of points in wind table.
		NPT	Number of points in pressure and temperature table.
		IFOUT	NON-zero if meteorological data should be recorded on printed output.

TABLE 11. SAMPLE INPUT DECK FOR YAWSONDE

```
TEST DECK FOR YAWSONDE DATA ANALYSIS PROGRAMME
    1
    2
0.              -0.690534       0.011401        -0.723210
40.             -0.690534       0.011401        -0.723210
   10    1    1
0.001     0.001     0.01      0.01      0.1       0.1       1.        0.1
1.        1.        1.        0.01
0.008569  0.105     0.22557   0.02326
0.1       0.427431  0.        0.        3.76      0.        0.        0.
0.        -14.      0.        0.
    2    1    3    4    5    8   10    7   11    9   12    6
18.
1.        1.        0.01
    1   20    0
    9    1    1
  100    1
    1
PART II
    A
    2   20    0
   10    1
PART III
    A
    3   20    0
   10    1
PART IV
    A
    4   20    0
   10    1
PART V
    A
    5   20    0
   10    1
PART VI
    A
    6   20    0
   10    1
PART VII
    A
    7   20    0
   10    1
PART VIII
    A
    8   20    0
   10    1
PART IX
    A
    9   20    0
   10    1
PART X
    A
   10   20    0

   10    1
PART XI
    A
   11   20    0
   10    1
PART XII
    A
   12   20    1
   10    0
```

TABLE 12. SIMULATED EXPERIMENTAL DATA FOR YAWSONDE

(A) TRAJECTORY MEASUREMENTS

18.000000	196.94579	9.71028	-89.705223566.139161207.80000
18.100000	196.79242	9.79641	-88.676693575.070691207.17249
18.200000	196.64590	9.88202	-87.635533583.898771206.54720
18.300000	196.50600	9.96349	-86.582633592.622201205.92411
18.400000	196.37213	10.03762	-85.519623601.239851205.30319
18.500000	196.24341	10.10182	-84.448733609.750841204.68442
18.600000	196.11874	10.15421	-83.372633618.154511204.06778
18.700000	195.99688	10.19369	-82.294233626.450471203.45325
18.800000	195.87654	10.21999	-81.216543634.638661202.84079
18.900000	195.75645	10.23362	-80.142433642.719291202.23040
19.000000	195.63549	10.23578	-79.074563650.692851201.62198
19.100000	195.51265	10.22829	-78.015163658.560091201.01551
19.200000	195.38711	10.21342	-76.965943666.321951200.41097
19.300000	195.25824	10.19376	-75.928033673.979491199.80833
19.400000	195.12568	10.17204	-74.901963681.533891199.20757
19.500000	194.98932	10.15102	-73.887643688.986331198.60868
19.600000	194.84927	10.13328	-72.884413696.337941198.01163
19.700000	194.70586	10.12116	-71.891093703.589791197.41640
19.800000	194.55960	10.11659	-70.906083710.742781196.82298
19.900000	194.41113	10.12105	-69.927453717.797651196.23134
20.000000	194.26120	10.13549	-68.953063724.754931195.64148

TABLE 12. SIMULATED EXPERIMENTAL DATA FOR YAWSONDE

(B) YAWSONDE MEASUREMENTS

1.8000000E+01	-3.3041852E-01	1.8265000E+01	-3.5929316E-01
1.8005000E+01	-3.3044029E-01	1.8270000E+01	-3.5955638E-01
1.8010000E+01	-3.3063585E-01	1.8275000E+01	-3.6012715E-01
1.8015000E+01	-3.3114431E-01	1.8280000E+01	-3.6097965E-01
1.8020000E+01	-3.3199186E-01	1.8285000E+01	-3.6198605E-01
1.8025000E+01	-3.3308270E-01	1.8290000E+01	-3.6296266E-01
1.8030000E+01	-3.3423385E-01	1.8295000E+01	-3.6373594E-01
1.8035000E+01	-3.3524110E-01	1.8300000E+01	-3.6420481E-01
1.8040000E+01	-3.3595251E-01	1.8305000E+01	-3.6437677E-01
1.8045000E+01	-3.3632292E-01	1.8310000E+01	-3.6436477E-01
1.8050000E+01	-3.3642983E-01	1.8315000E+01	-3.6434621E-01
1.8055000E+01	-3.3644504E-01	1.8320000E+01	-3.6449900E-01
1.8060000E+01	-3.3657236E-01	1.8325000E+01	-3.6493775E-01
1.8065000E+01	-3.3697433E-01	1.8330000E+01	-3.6567298E-01
1.8070000E+01	-3.3771395E-01	1.8335000E+01	-3.6660771E-01
1.8075000E+01	-3.3873251E-01	1.8340000E+01	-3.6757249E-01
1.8080000E+01	-3.3987099E-01	1.8345000E+01	-3.6838643E-01
1.8085000E+01	-3.4092745E-01	1.8350000E+01	-3.6892209E-01
1.8090000E+01	-3.4172973E-01	1.8355000E+01	-3.6915110E-01
1.8095000E+01	-3.4219726E-01	1.8360000E+01	-3.6915405E-01
1.8100000E+01	-3.4236978E-01	1.8365000E+01	-3.6909107E-01
1.8105000E+01	-3.4239251E-01	1.8370000E+01	-3.6914402E-01
1.8110000E+01	-3.4246355E-01	1.8375000E+01	-3.6945122E-01
1.8115000E+01	-3.4276237E-01	1.8380000E+01	-3.7005806E-01
1.8120000E+01	-3.4338553E-01	1.8385000E+01	-3.7090111E-01
1.8125000E+01	-3.4431235E-01	1.8390000E+01	-3.7183115E-01
1.8130000E+01	-3.4541267E-01	1.8395000E+01	-3.7266689E-01
1.8135000E+01	-3.4649392E-01	1.8400000E+01	-3.7326010E-01
1.8140000E+01	-3.4737044E-01	1.8405000E+01	-3.7354870E-01
1.8145000E+01	-3.4793015E-01	1.8410000E+01	-3.7357878E-01
1.8150000E+01	-3.4817470E-01	1.8415000E+01	-3.7348765E-01
1.8155000E+01	-3.4821840E-01	1.8420000E+01	-3.7345417E-01
1.8160000E+01	-3.4824668E-01	1.8425000E+01	-3.7363445E-01
1.8165000E+01	-3.4844911E-01	1.8430000E+01	-3.7410604E-01
1.8170000E+01	-3.4895135E-01	1.8435000E+01	-3.7484046E-01
1.8175000E+01	-3.4977035E-01	1.8440000E+01	-3.7571377E-01
1.8180000E+01	-3.5080864E-01	1.8445000E+01	-3.7655119E-01
1.8185000E+01	-3.5188960E-01	1.8450000E+01	-3.7718995E-01
1.8190000E+01	-3.5282113E-01	1.8455000E+01	-3.7753757E-01
1.8195000E+01	-3.5346462E-01	1.8460000E+01	-3.7760493E-01
1.8200000E+01	-3.5378470E-01	1.8465000E+01	-3.7750192E-01
1.8205000E+01	-3.5386154E-01	1.8470000E+01	-3.7739764E-01
1.8210000E+01	-3.5386160E-01	1.8475000E+01	-3.7745953E-01
1.8215000E+01	-3.5397740E-01	1.8480000E+01	-3.7779339E-01
1.8220000E+01	-3.5435834E-01	1.8485000E+01	-3.7840581E-01
1.8225000E+01	-3.5505729E-01	1.8490000E+01	-3.7920209E-01
1.8230000E+01	-3.5601197E-01	1.8495000E+01	-3.8002052E-01
1.8235000E+01	-3.5706769E-01		
1.8240000E+01	-3.5803301E-01		
1.8245000E+01	-3.5874866E-01		
1.8250000E+01	-3.5914460E-01		
1.8255000E+01	-3.5926495E-01		
1.8260000E+01	-3.5925168E-01		

TABLE 13. BEGINNING AND END OF OUTPUT FROM SAMPLE INPUT TO YAWSONDE

```
PARAMETER ESTIMATION FOR ANALYSIS OF YAWSONDE DATA FROM RADAR AND YAWSONDE MEASUREMENTS MADE DURING THE FLIGHT OF A SHELL

                        TEST DECK FOR YAWSONDE DATA ANALYSIS PROGRAMME

DIRECTION COSINES FOR THE SUN IN RANGE AXES AS A FUNCTION OF TIME ALONG THE TRAJECTORY
TIME(S)      L             M             N
   0.0   -0.A90534  0.011401 -0.723210
  40.00  -0.A90534  0.011401 -0.723210


TABULATED TRAJECTORY DATA

   TIME    XDOT    YDOT    ZDOT     ALT     ROLL
  (SEC)     (METRES/SECOND)          (M)  (RAD/SEC)
  1A.0 19A.95   9.71 -A9.71 3566.14 1207.A0
  1A.1 19A.79   9.80 -A8.68 3575.07 1207.17
  1A.2 19A.65   9.88 -A7.64 3583.90 1206.55
  1A.3 19A.51   9.96 -A6.58 3592.62 1205.92
  1A.4 19A.37  10.04 -A5.52 3601.24 1205.30
  1A.5 19A.24  10.10 -A4.45 3609.75 1204.68
  1A.6 19A.12  10.15 -A3.37 3618.15 1204.07
  1A.7 19A.00  10.19 -A2.29 3626.45 1203.45
  1A.8 19A.88  10.22 -A1.22 3634.64 1202.84
  1A.9 19A.76  10.23 -A0.14 3642.72 1202.23
  19.0 19A.64  10.24 -79.07 3650.69 1201.62
  19.1 19A.51  10.23 -78.02 3658.56 1201.02
  19.2 19A.39  10.21 -76.97 3666.32 1200.41
  19.3 19A.26  10.19 -75.93 3673.98 1199.81
  19.4 19A.13  10.17 -74.90 3681.53 1199.21
  19.5 19A.00  10.15 -73.89 3688.99 1198.61
  19.6 19A.85  10.13 -72.88 3696.34 1198.01
  19.7 19A.71  10.12 -71.89 3703.59 1197.42
  19.8 19A.56  10.12 -70.91 3710.74 1196.82


VEHICLE PHYSICAL CONSTANTS

        REFERENCE AREA= 0.0085A9(M**2)          BODY DIAMETER= 0.1050(M)
        PITCH INERTIA= 0.225570(KG M**2)          ROLL INERTIA= 0.0232A00(KG M**2)

ORDER OF PARAMETERS
      2  1  3  4  5  8 10  7 11  9 12  6

STARTING TIME= 1A.000(S)

INITIAL REJECTION LEVEL ON DATA   .00
ACCURACY FACTOR FOR PARAMETER ERROR LEVELS  1.000
MAXIMUM INTEGRATION STEP 0.0100
NUMBER OF PARAMETERS VARIED  1
MAXIMUM NUMBER OF ITERATIONS 20

NO PLOTS REQUESTED


   TIME   COMPLEMENTARY SOLAR
  (SEC)   ASPECT ANGLE(DEG)
 1A.0000     -0.33041852
 1A.0050     -0.33044029
 1A.0100     -0.33063585
 1A.0150     -0.33114431
 1A.0200     -0.33199186
 1A.0250     -0.33308270
 1A.0300     -0.33423385
 1A.0350     -0.33524110
 1A.0400     -0.33595251
 1A.0450     -0.33632292
 1A.0500     -0.33642983
 1A.0550     -0.33644504
 1A.0600     -0.33657236
 1A.0650     -0.33697433
 1A.0700     -0.33771395
 1A.0750     -0.33873251
 1A.0800     -0.33987099
 1A.0850     -0.34092745
 1A.0900     -0.34172973
 1A.0950     -0.34219726
 1A.1000     -0.3423697R
 1A.1050     -0.34239251
 1A.1100     -0.3424A355
 1A.1150     -0.3427A237
 1A.1200     -0.3433R553
 1A.1250     -0.34431235
 1A.1300     -0.34541267
 1A.1350     -0.34649392
 1A.1400     -0.34737044
 1A.1450     -0.34793015
 1A.1500     -0.34R17470
 1A.1550     -0.34R21R46
 1A.1600     -0.34R24A6R
 1A.1650     -0.34R44911
 1A.1700     -0.34R95135
 1A.1750     -0.34977635
 1A.1800     -0.350R0RA4
 1A.1850     -0.351RR960
 1A.1900     -0.352R2113
 1A.1950     -0.3536A462
 1A.2000     -0.3537RA70
 1A.2050     -0.353RA15A
 1A.2100     -0.353R6160
 1A.2150     -0.35397740
 1A.2200     -0.3543SR34
 1A.2250     -0.35505729
 1A.2300     -0.35601197
 1A.2350     -0.3570A7A9
 1A.2400     -0.35R03301
 1A.2450     -0.35R74R66
 1A.2500     -0.35914460
 1A.2550     -0.3592A495
 1A.2600     -0.3592516R
 1A.2650     -0.3592031A
 1A.2700     -0.3595563R
 1A.2750     -0.3A012715
 1A.2800     -0.3A097065
 1A.2850     -0.3A19R605
 1A.2900     -0.3A20A7AA
```

TABLE 13. (CONTINUED)

```
 1A.2950      -0.36373594
 1A.3000      -0.36420481
 1A.3050      -0.36437677
 1A.3100      -0.36436477
 1A.3150      -0.36434621
 1A.3200      -0.36449900
 1A.3250      -0.36493775
 1A.3300      -0.36567294
 1A.3350      -0.36660771
 1A.3400      -0.36757249
 1A.3450      -0.36834643
 1A.3500      -0.36892209
 1A.3550      -0.36915110
 1A.3600      -0.36915405
 1A.3650      -0.36909107
 1A.3700      -0.36914402
 1A.3750      -0.36945122
 1A.3800      -0.37005866
 1A.3850      -0.37090111
 1A.3900      -0.37183115
 1A.3950      -0.37266689
 1A.4000      -0.37326010
 1A.4050      -0.37354870
 1A.4100      -0.37357878
 1A.4150      -0.37348765
 1A.4200      -0.37345417
 1A.4250      -0.37363445
 1A.4300      -0.37410604
 1A.4350      -0.37488046
 1A.4400      -0.37571377
 1A.4450      -0.37655119
 1A.4500      -0.37719995
 1A.4550      -0.37753767
 1A.4600      -0.37760493
 1A.4650      -0.37750192
 1A.4700      -0.37739764
 1A.4750      -0.37745953
 1A.4800      -0.37779339
 1A.4850      -0.37846581
 1A.4900      -0.37920209
 1A.4950      -0.38002652
```

	THETA0	PSI0	00	R0	CM1	CMMACH	CNP1	CM3
PARAM.	0.4274D+00	0.1000D+00	0.0	0.0	0.3760D+01	0.0	0.0	0.0
DELTA	0.1000D-02	0.1000D-02	0.1000D-01	0.1000D-01	0.1000D+00	0.1000D-01	0.1000D+00	0.1000D+00

	CM00	CM5	CNP3	CM02
PARAM.	-0.1400D+02	0.0	0.0	0.0
DELTA	0.1000D+01	0.1000D+01	0.1000D+01	0.1000D+01

SIGMA= 0.180100-03 DEGREES OF FREEDOM= 99

COVARIANCE MATRIX (MATRIX IS SYMMETRIC)

4.550-10

CHANGES TO THE FOLLOWING PARAMETERS ALL SMALL

	THETA0	PSI0	00	R0	CM1	CMMACH	CNP1	CM3
PARAM.	0.4271D+00	0.1000D+00	0.0	0.0	0.3760D+01	0.0	0.0	0.0
DELTA	0.2133D-04	0.1000D-02	0.1000D-01	0.1000D-01	0.1000D+00	0.1000D-01	0.1000D+00	0.1000D+00

	CM00	CM5	CNP3	CM02
PARAM.	-0.1400D+02	0.0	0.0	0.0
DELTA	0.1000D+01	0.1000D+01	0.1000D+01	0.1000D+01

PART X1.

```
INITIAL REJECTION LEVEL ON DATA  0.00
ACCURACY FACTOR FOR PARAMETER ERROR LEVELS  1.000
MAXIMUM INTEGRATION STEP 0.0100
NUMBER OF PARAMETERS VARIED 12
MAXIMUM NUMBER OF ITERATIONS 20

PLOTS REQUESTED EVERY  1 POINTS
```

	THETA0	PSI0	00	R0	CM1	CMMACH	CNP1	CM3
PARAM.	0.4274D+00	0.1001D+00	0.4530D-05	-0.1067D-03	0.3719D+01	0.1345D+00	0.1684D-01	-0.4937D+01
DELTA	0.1000D-02	0.1000D-02	0.1000D-01	0.1000D-01	0.1000D+00	0.1000D-01	0.1000D+00	0.1000D+00

	CM00	CM5	CNP3	CM02
PARAM.	-0.1434D+02	0.1727D+04	-0.1323D+01	0.0
DELTA	0.1000D+01	0.1000D+01	0.1000D+01	0.1000D+01

SIGMA= 0.786100-07 DEGREES OF FREEDOM= 99

	THETA0	PSI0	00	R0	CM1	CMMACH	CNP1	CM3
PARAM.	0.4274D+00	0.1000D+00	0.4294D-05	-0.4976D-04	0.3590D+01	0.3411D+00	0.7043D-03	-0.7754D+01
DELTA	0.4478D-05	0.5812D-04	0.4356D-05	0.5078D-04	0.1941D+00	0.3033D+00	0.1940D-01	0.4396D+01

	CM00	CM5	CNP3	CM02
PARAM.	-0.1416D+02	0.2046D+04	0.4721D+01	-0.1266D+03
DELTA	0.2070D+00	0.7811D+03	0.7248D+01	0.1446D+03

SIGMA= 0.786160-07 DEGREES OF FREEDOM= 99

TABLE 13. (CONTINUED)

COVARIANCE MATRIX (MATRIX IS SYMMETRIC)

```
 1.990D-11
-2.580D-10  3.350D-09
-5.380D-12  6.910D-11  1.890D-11
 2.120D-10 -2.760D-09 -5.730D-11  2.570D-09
-7.530D-07  9.810D-06  2.100D-07 -9.580D-06  3.730D-02
 1.200D-06 -1.560D-05 -3.340D-07  1.510D-05 -5.830D-02  9.120D-02
-8.610D-08  1.120D-06  2.280D-08 -8.960D-07  3.170D-03 -5.060D-03  3.760D-04
-1.440D-05  1.850D-04  3.860D-06 -1.130D-04  2.890D-01 -4.960D-01  6.340D-02  1.930D+01
 8.340D-07 -1.080D-05  6.480D-09  9.000D-06 -3.240D-02  5.140D-02 -3.660D-03 -5.590D-01  4.280D-02
 1.530D-03 -1.960D-02 -4.110D-04  7.210D-03  9.500D-01  6.900D+00 -6.850D+00 -3.210D+03  5.370D+01  6.130D+05
 3.160D-05 -4.100D-04 -8.280D-06  3.100D-04 -1.060D+00  1.710D+00 -1.390D-01 -2.590D+01  1.340D+00  3.100D+03  5.290D+01
-6.060D-04  7.870D-03  3.760D-05 -6.120D-03  2.130D+01 -3.410D+01  2.670D+00  4.670D+02 -2.940D+01 -5.290D+04 -1.010D+03  2.100D+04
```

CHANGES TO THE FOLLOWING PARAMETERS ALL SMALL

	THETA0	PSI0	D0	R0	CM1	CMMACH	CNP1	CM3
PARAM.	0.4274D+00	0.1000D+00	0.4307D-05	-0.6990D-04	0.3591D+01	0.3403D+00	0.7220D-03	-0.7753D+01
DELTA	0.4457D-05	0.5788D-04	0.4352D-05	0.5065D-04	0.1932D+00	0.3020D+00	0.1939D-01	0.4397D+01

	CM00	CM5	CNP3	CM02
PARAM.	-0.1416D+02	0.2046D+04	0.4730D+01	-0.1265D+03
DELTA	0.2068D+00	0.7830D+03	0.7271D+01	0.1450D+03

TIME(S)	COMPLEMENTARY SOLAR SOLAR ASPECT ANGLE	RESIDUE (RADIANS)
1R.005	-0.3304404	6.6066D-08
1R.010	-0.3304359	4.9169D-08
1R.015	-0.3311443	3.4623D-08
1R.020	-0.3319919	8.0963D-09
1R.025	-0.3330827	-1.1051D-08
1R.030	-0.3342338	-2.4883D-08
1R.035	-0.3352411	-3.2460D-08
1R.040	-0.3359525	-4.1568D-08
1R.045	-0.3363229	-5.4128D-08
1R.050	-0.3364298	-6.3506D-08
1R.055	-0.3364456	-6.9345D-08
1R.060	-0.3365723	-9.0966D-08
1R.065	-0.3369742	-8.3816D-08
1R.070	-0.3377139	-6.8245D-08
1R.075	-0.3387325	-7.0179D-09
1R.080	-0.3398710	3.4756D-08
1R.085	-0.3409275	7.4592D-08
1R.090	-0.3417298	9.7658D-08
1R.095	-0.3421974	1.0705D-07
1R.100	-0.3423699	9.1618D-08
1R.105	-0.3423926	8.6648D-08
1R.110	-0.3424636	8.4362D-08
1R.115	-0.3427624	4.6150D-08
1R.120	-0.3433856	3.0343D-08
1R.125	-0.3443124	5.7059D-10
1R.130	-0.3454126	-2.2518D-08
1R.135	-0.3464939	-3.8359D-08
1R.140	-0.3473704	-6.1747D-08
1R.145	-0.3479301	-6.8258D-08
1R.150	-0.3481746	-7.0701D-08
1R.155	-0.3482183	-5.9730D-08
1R.160	-0.3482666	-4.7456D-08
1R.165	-0.3484491	-2.6918D-08
1R.170	-0.3489514	8.4553D-09
1R.175	-0.3497704	4.5964D-08
1R.180	-0.3508087	6.8321D-08
1R.185	-0.3518897	8.9261D-08
1R.190	-0.3528212	8.2788D-08
1R.195	-0.3534647	7.5785D-08
1R.200	-0.3537868	6.1253D-08
1R.205	-0.3538616	5.4191D-08
1R.210	-0.3538616	3.3716D-08
1R.215	-0.3539774	7.4703D-09
1R.220	-0.3543583	-2.5954D-08
1R.225	-0.3550572	-6.6804D-08
1R.230	-0.3560119	-9.3172D-08
1R.235	-0.3570676	-1.2250D-07
1R.240	-0.3580329	-1.2351D-07
1R.245	-0.3587685	-1.1814D-07
1R.250	-0.3591445	-9.4587D-08
1R.255	-0.3592649	-5.9012D-08
1R.260	-0.3592517	-2.5731D-08
1R.265	-0.3592932	1.3421D-08
1R.270	-0.3595566	5.0677D-08
1R.275	-0.3601272	7.4485D-08
1R.280	-0.3609797	8.5077D-08
1R.285	-0.3619861	9.2789D-08
1R.290	-0.3629627	8.1593D-08
1R.295	-0.3637360	8.1031D-08
1R.300	-0.3642049	4.7920D-08
1R.305	-0.3643768	3.2164D-08
1R.310	-0.3643648	8.6391D-09
1R.315	-0.3643642	-2.0909D-08
1R.320	-0.3644989	-5.8531D-08
1R.325	-0.3649377	-9.0814D-08
1R.330	-0.3656729	-1.1381D-07
1R.335	-0.3666076	-1.3127D-07
1R.340	-0.3675724	-1.1506D-07
1R.345	-0.3683863	-9.0036D-08
1R.350	-0.3689220	-6.0469D-08
1R.355	-0.3691511	-1.4271D-08
1R.360	-0.3691541	2.9567D-08
1R.365	-0.3690911	6.6248D-08
1R.370	-0.3691441	1.0376D-07
1R.375	-0.3694514	1.3400D-07
1R.380	-0.3700582	1.5749D-07
1R.385	-0.3709013	1.6033D-07
1R.390	-0.3718313	1.4572D-07
1R.395	-0.3726670	1.2297D-07
1R.400	-0.3732602	8.5574D-08
1R.405	-0.3735487	4.0156D-08
1R.410	-0.3735788	-7.5489D-09
1R.415	-0.3734876	-5.6746D-08
1R.420	-0.3734541	-9.9098D-08
1R.425	-0.3736343	-1.2498D-07
1R.430	-0.3741059	-1.3228D-07
1R.435	-0.3748403	-1.1512D-07
1R.440	-0.3757137	-9.0512D-08
1R.445	-0.3765511	-5.0904D-08
1R.450	-0.3771899	-3.0777D-08
1R.455	-0.3775378	-7.9339D-09
1R.460	-0.3776049	-1.3314D-09
1R.465	-0.3775019	3.4939D-09
1R.470	-0.3773977	1.8150D-08
1R.475	-0.3774596	3.2472D-08
1R.480	-0.3777934	5.2132D-08
1R.485	-0.3784059	6.8984D-08
1R.490	-0.3792021	8.2143D-08
1R.495	-0.3800205	-1.8839D-08

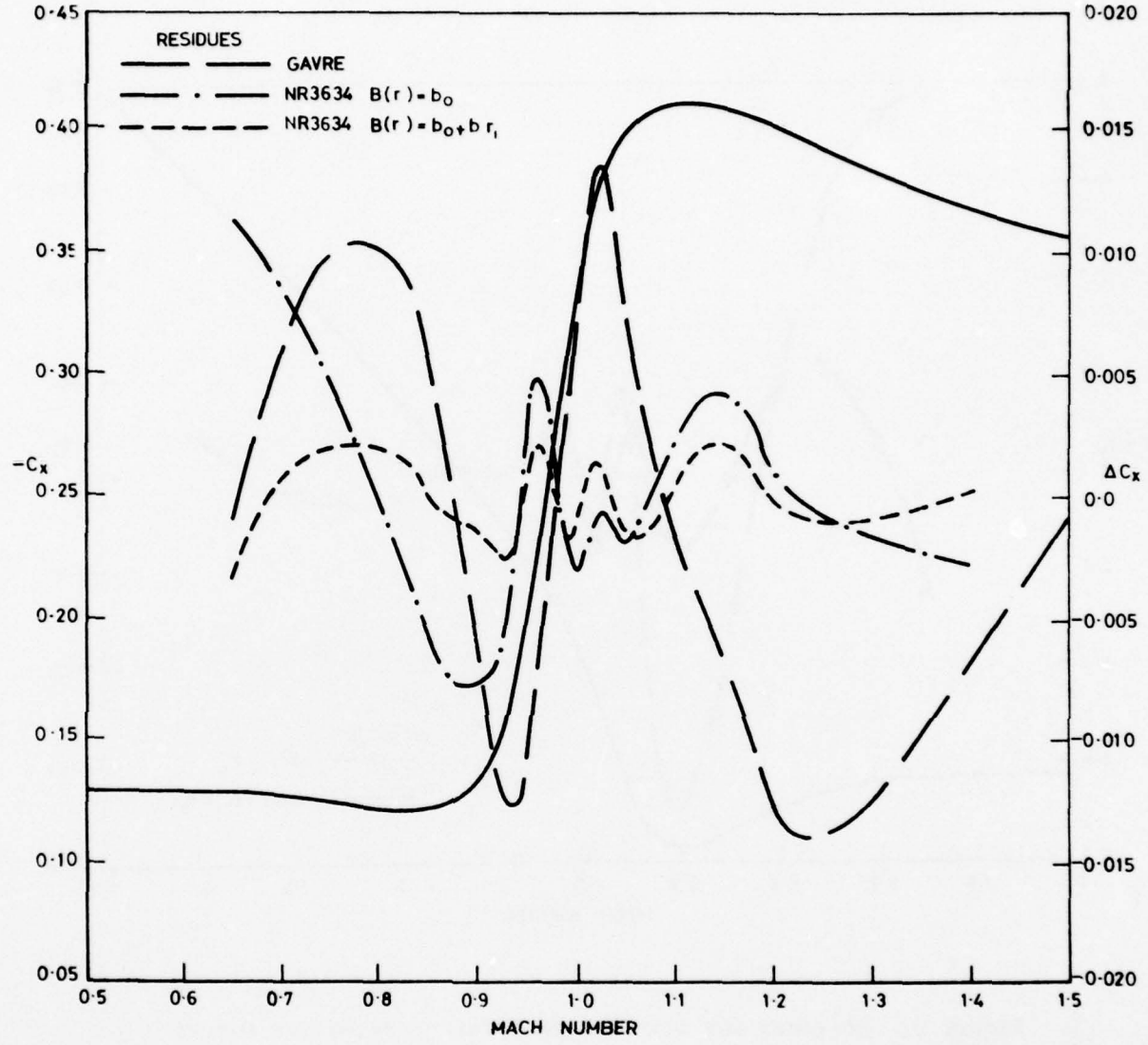


Figure 1. Residues for fitted axial force curves

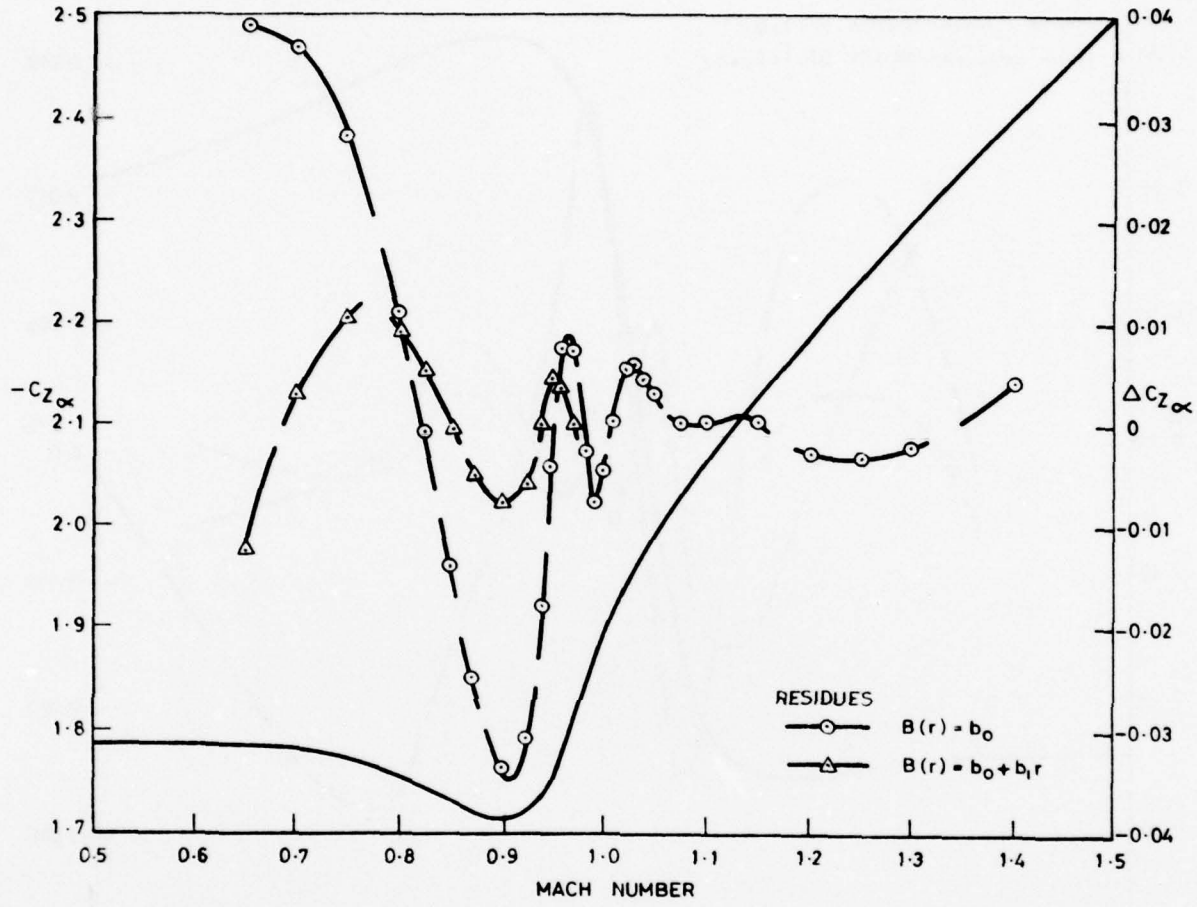


Figure 2. Residues for fitted normal force derivative curves

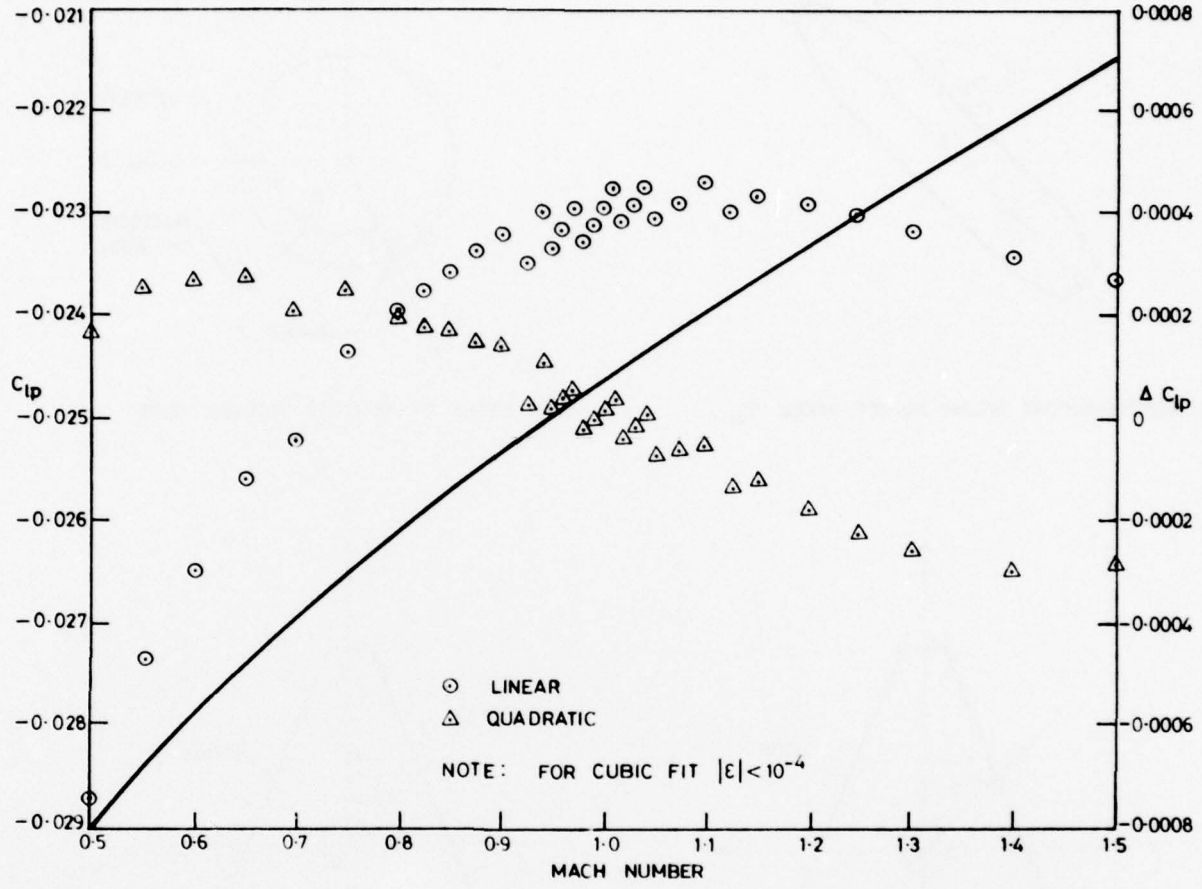
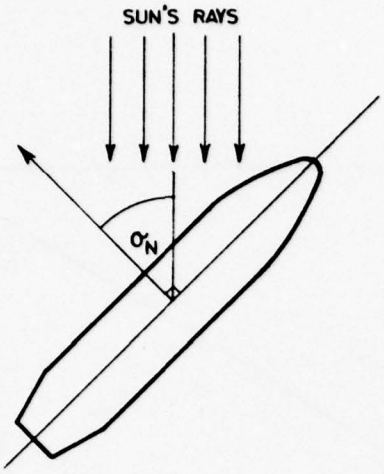
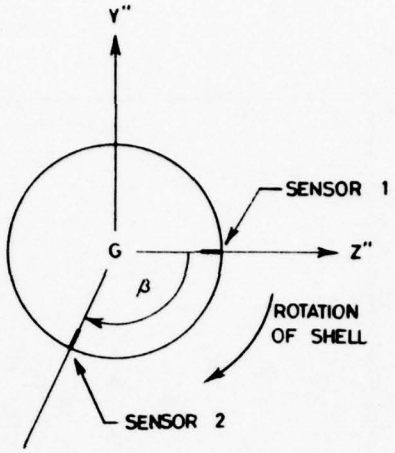


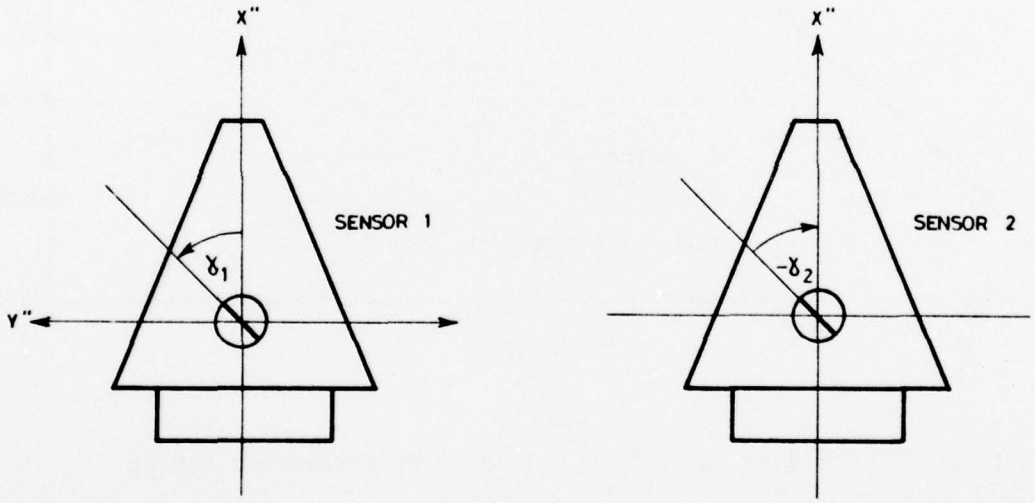
Figure 3. Residues for fitted roll damping derivative curves



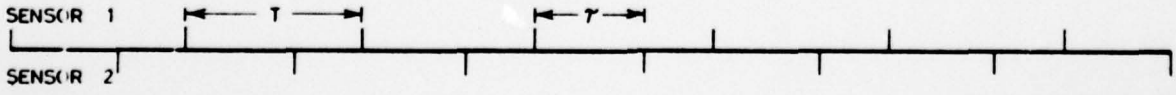
(a) COMPLEMENTARY SOLAR ASPECT ANGLE σ'_N



(b) ANGLE OF ROTATION BETWEEN SLITS



(c) ORIENTATION OF SLITS RELATIVE TO THE LONGITUDINAL AXIS OF THE SHELL



(d) PULSE TRAIN FROM YAWSONDE

Figure 4. Schematic representation of a yawsonde

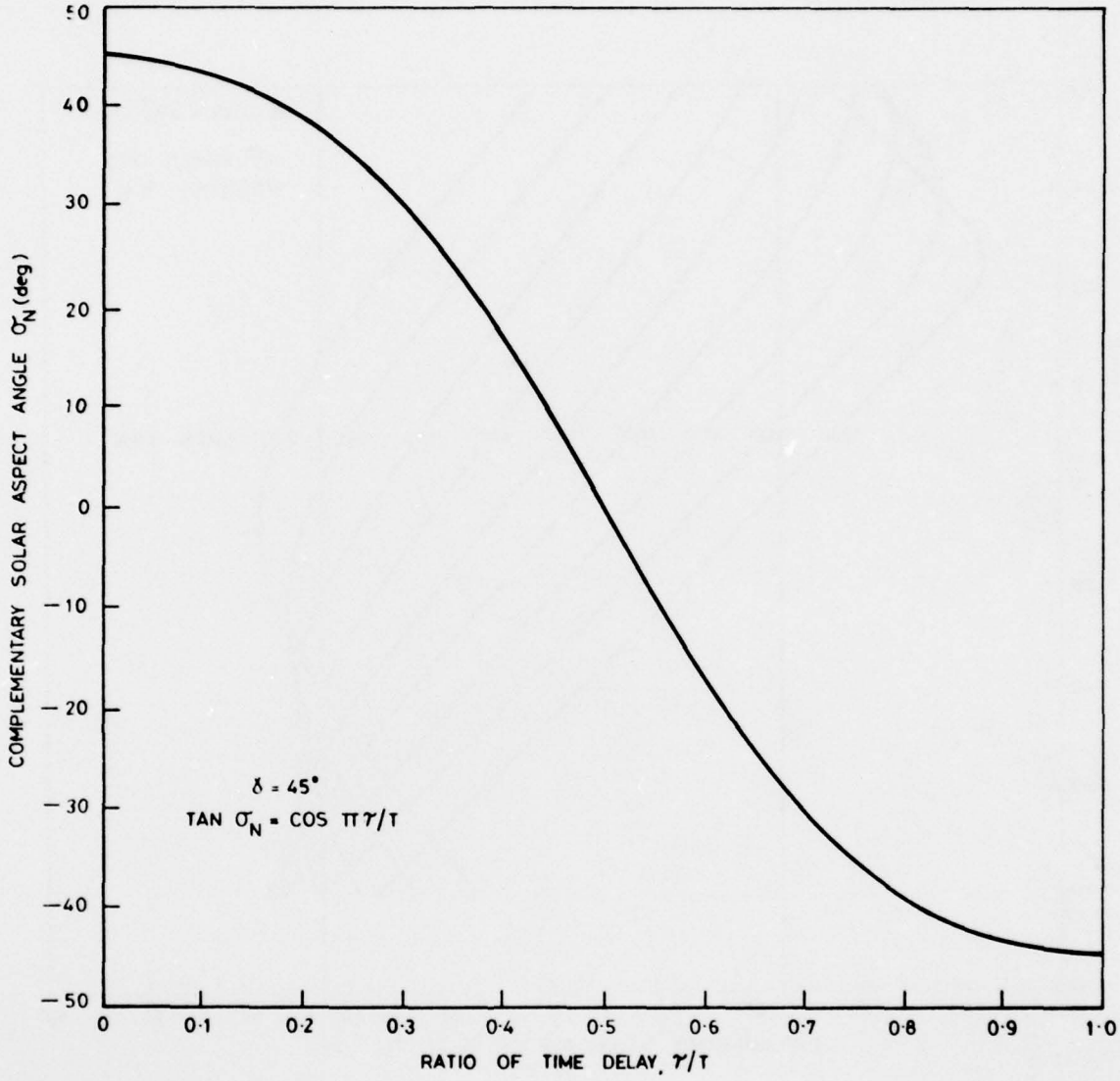
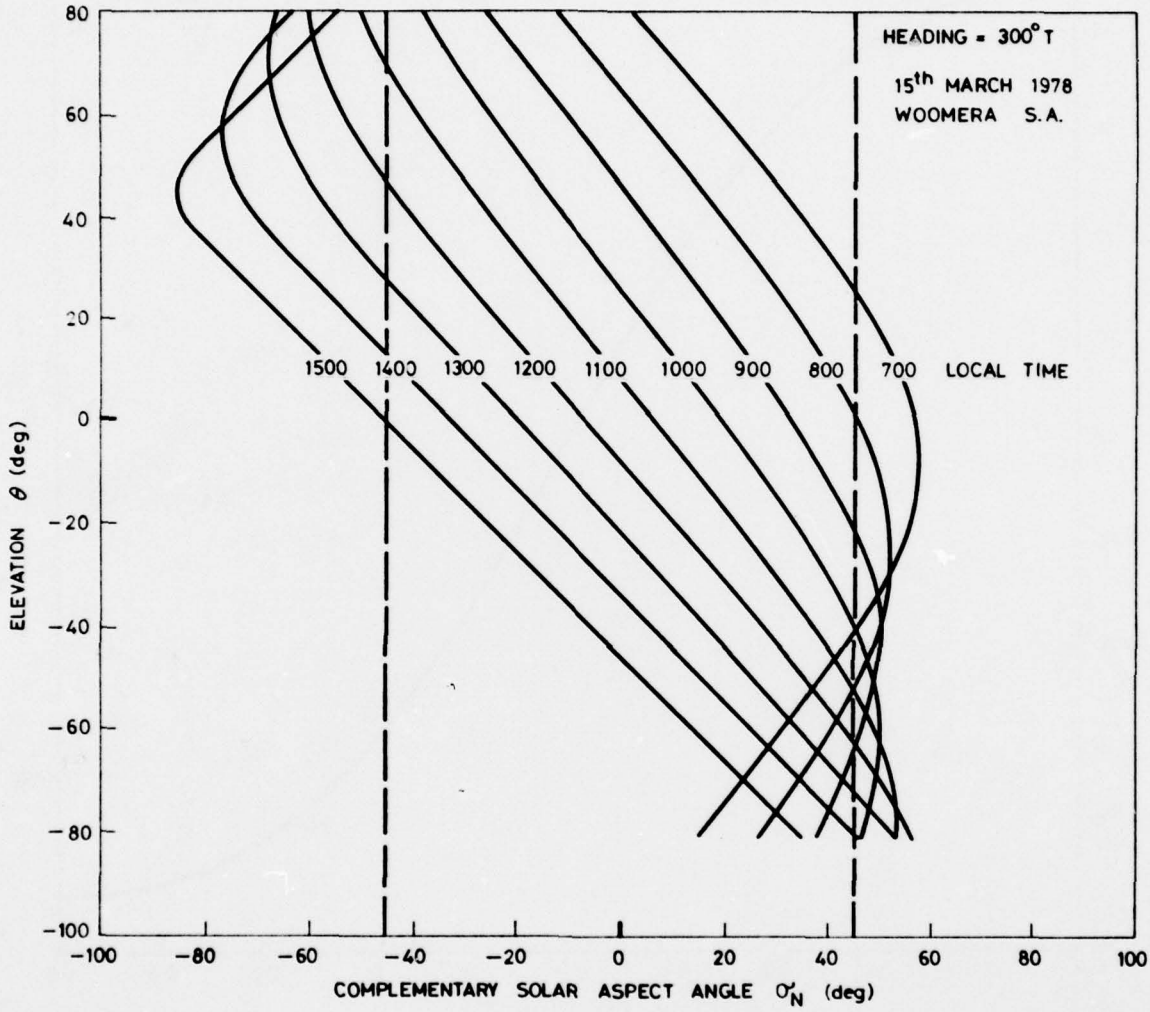
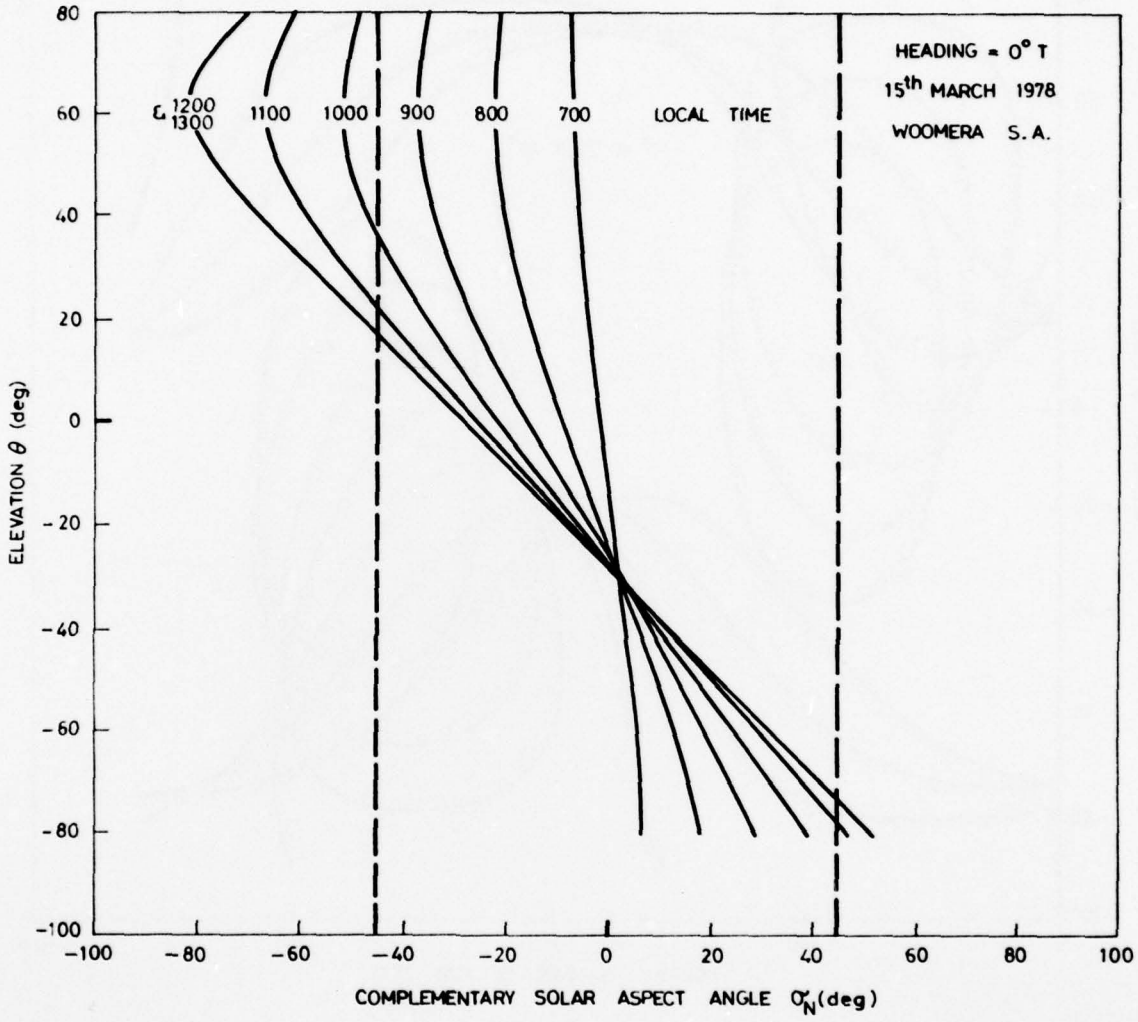


Figure 5. Typical yawsonde calibration curve



(a) heading 300 degrees T

Figure 6. Variation of complementary solar aspect angle with shell elevation



(b) heading due north

Figure 6. Variation of complementary solar aspect angle with shell elevation

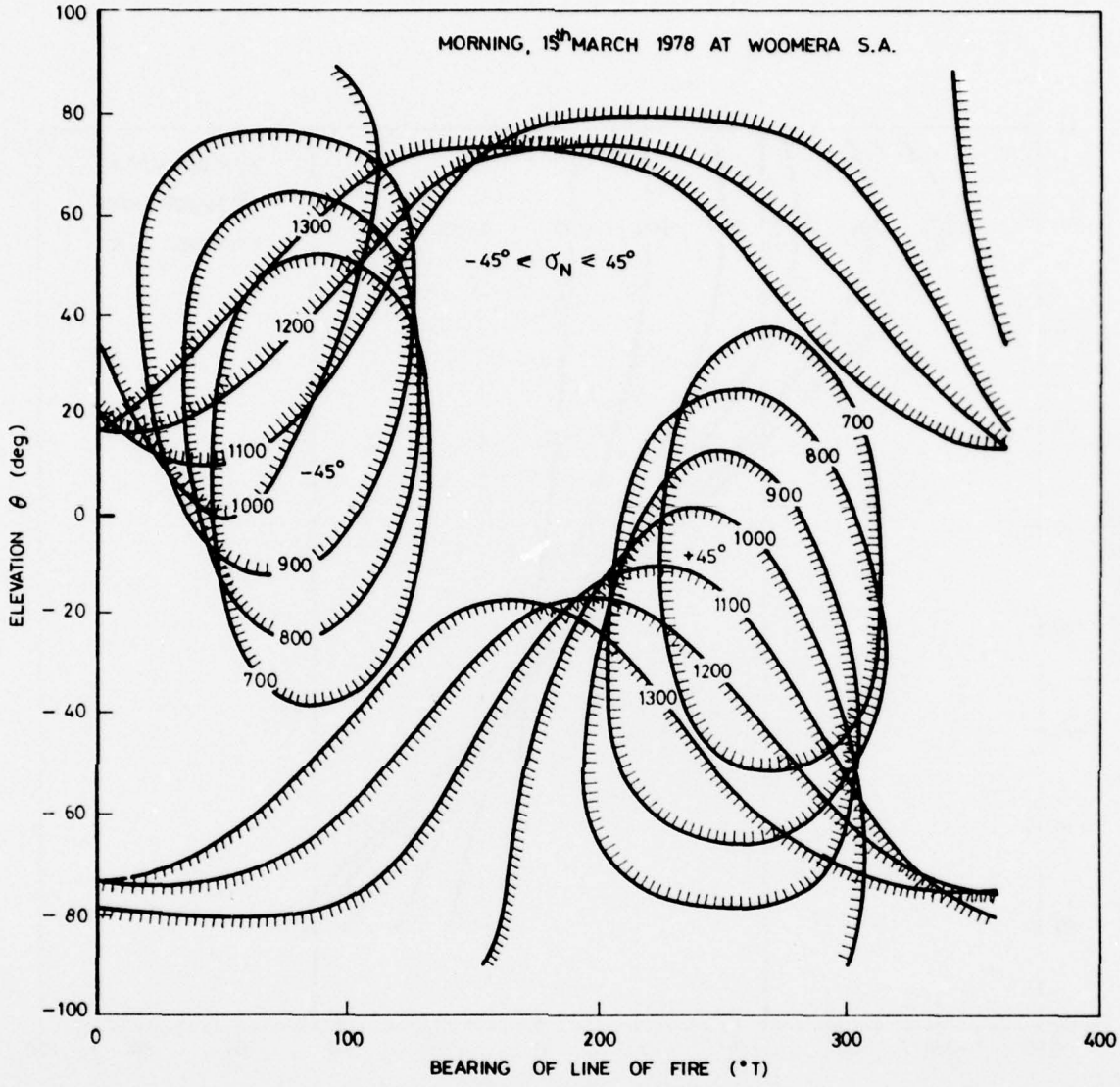
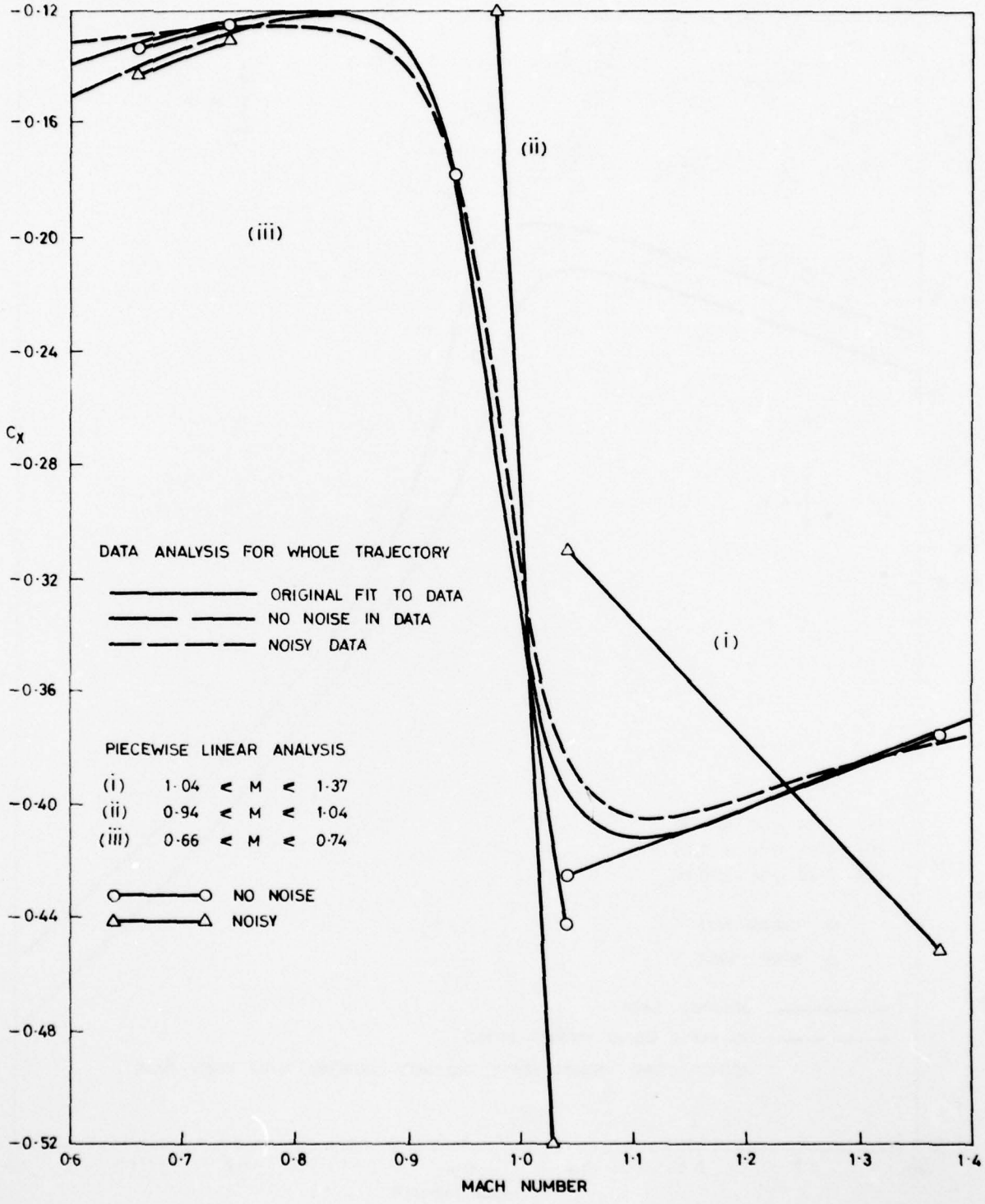
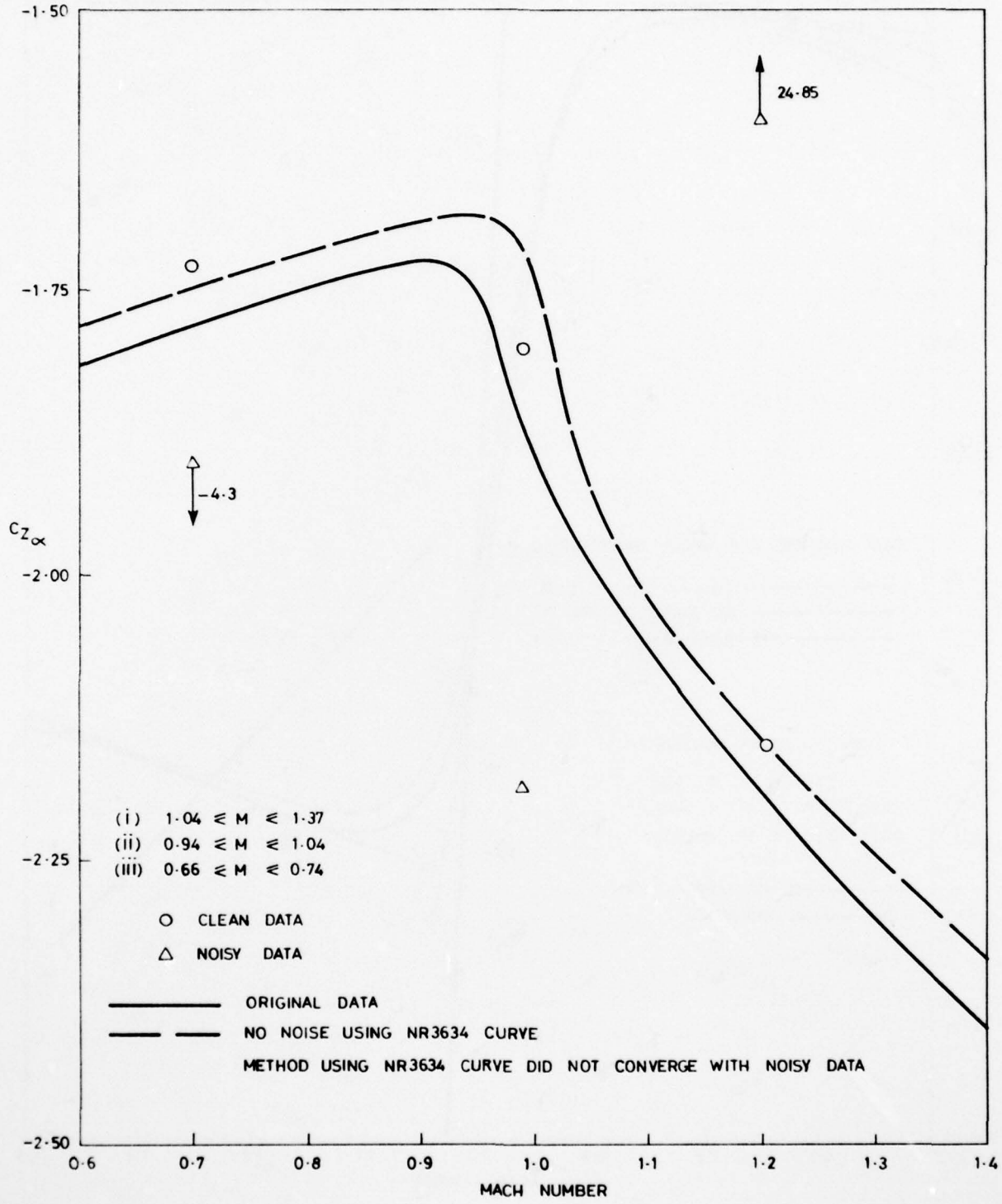


Figure 7. Conditions for detecting the sun



(a) axial force

Figure 8. Comparison of methods for estimating aerodynamic force coefficients



(b) normal force derivative

Figure 8. Comparison of methods for estimating aerodynamic force coefficients

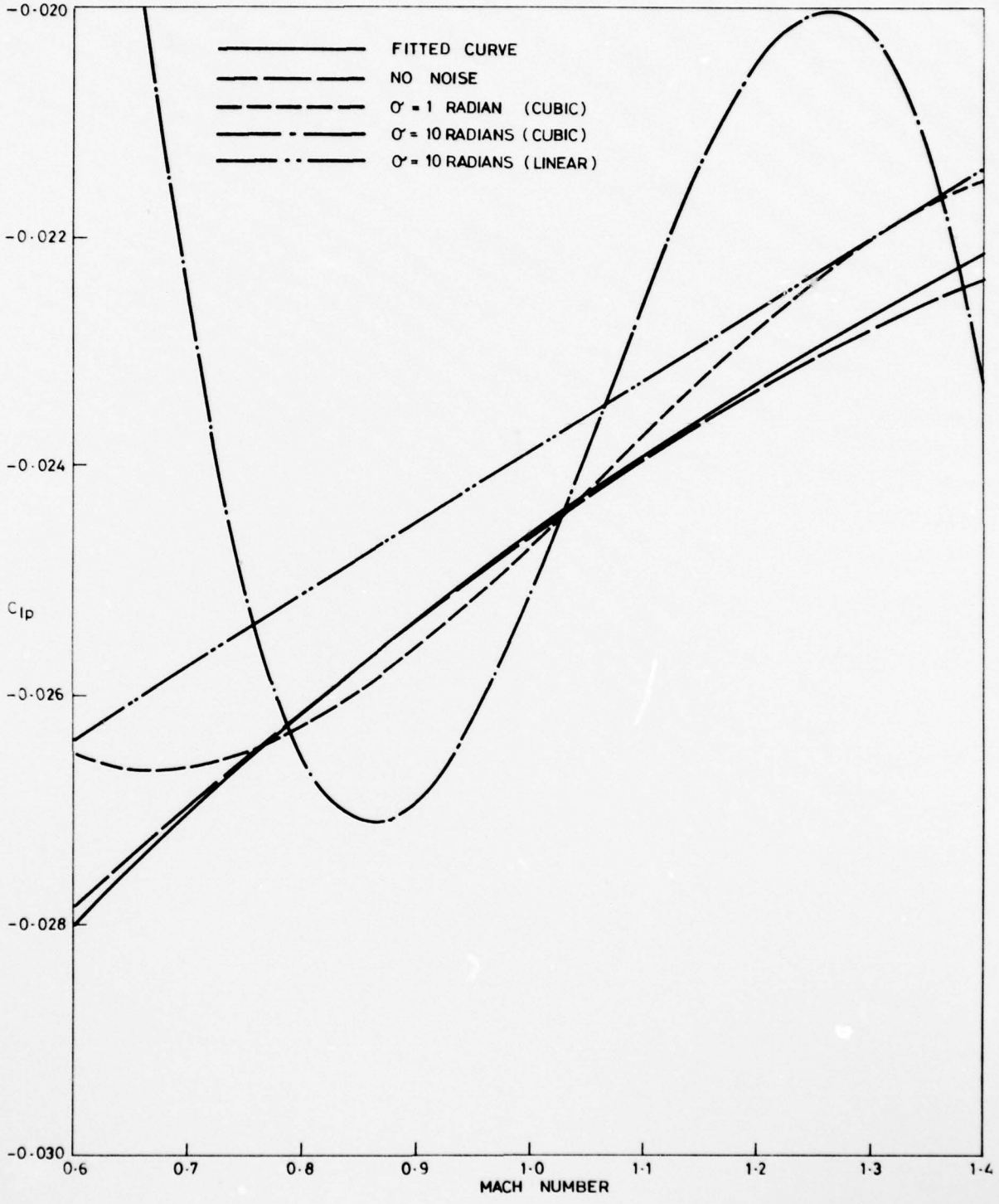


Figure 9. Effects of noise on estimation of roll damping moments

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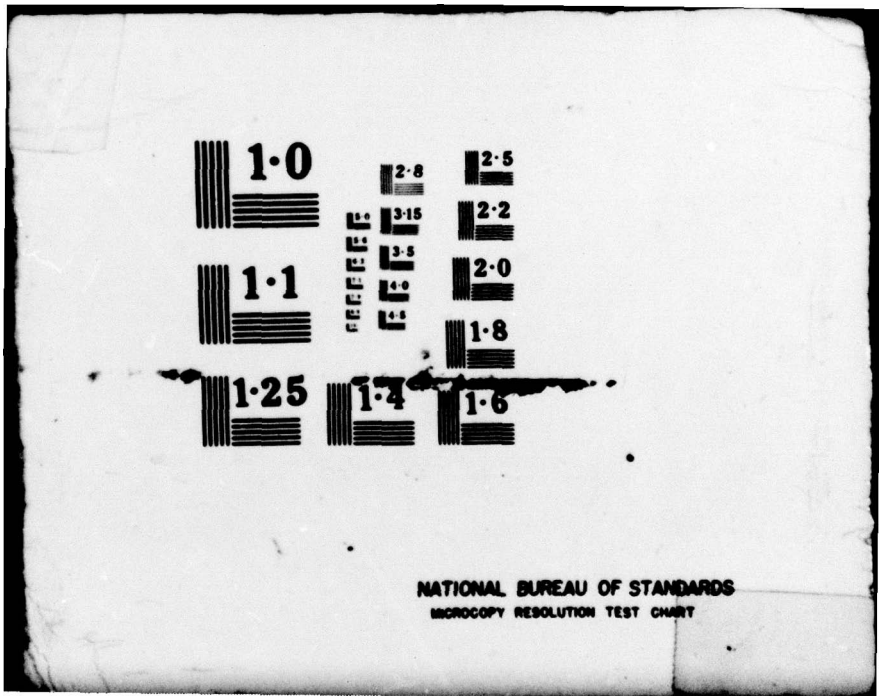
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