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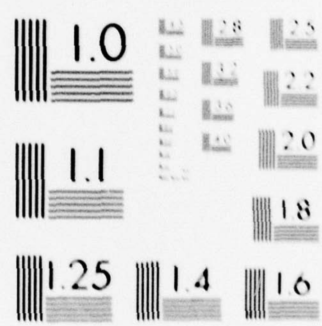

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13. ABSTRACT

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A non-linear method for comparing the efficiency of decision making units (school) is presented and is applied to elementary schools in an urban school district. The method is cross-validated by means of discriminant analysis and a rudimentary sensitivity test is made. The method is found to discriminate between efficient and inefficient schools and is relatively robust under changes in input measures.

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DETERMINING THE ATTRIBUTES OF EFFICIENT AND INEFFICIENT SCHOOLS THROUGH DATA ENVELOPMENT ANALYSIS

by

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August 1979

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ABSTRACT

Conventional methods for comparing the relative productivity of schools employ least squares regression to find expected achievement of schools with the same input characteristics. The result is that one typically contrasts the relative effects of "predictor" variables on achievement rather than comparing school units with respect to their input/output efficiency.

A non-linear method for comparing the efficiency of decision making units (school) is presented and is applied to elementary schools in an urban school district. The method is cross-validated by means of discriminant analysis and a rudimentary sensitivity test is made. The method is found to discriminate between efficient and inefficient schools and is relatively robust under changes in input measures.

KEY WORDS

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Fractional Programming
Linear Programming
Dual Simplex
Production Functions
Estimation Techniques
Least Squares Regression
Discriminant Analysis
School Achievement
Data Envelopment Analysis (DEA)

BACKGROUND

Methods for evaluating the relative productivity of decision making units in the public sector have lagged behind similar applications where production functions were more directly obtainable. Charnes and Cooper in [3] recently reviewed relevant development in economic theory from the standpoint of managerial economics. They also described the methodological developments undertaken with E. Rhodes [4,5] to measure the efficiency of "decision making units" with special reference to not-for-profit enterprise and government agencies. This resulted in a technique that they call Data Envelopment Analysis for measuring and distinguishing different kinds of efficiencies such as "program efficiency" and "managerial efficiency." The utility of the theory has been demonstrated in the secondary analysis of Program Follow-Through evaluation data [4,10]--an important federally-funded intervention aimed at improved education for disadvantaged children.

In the works cited, Charnes, Cooper and Rhodes succeeded in quantifying the relative efficiency of decision making units (DMU's) within a set of like units and, further, conceptualized a method for comparing the relative efficiency of two sets of units classified on some *a priori* basis [4]. Only a limited set of variables were selected for illustrative purposes, the objective being to show how one might compare two sets of schools operating under different programs.

The present paper is directed to a study of the possible use of these measures by management at the individual school level. For this purpose, a comprehensive set of variables will be employed and applied to all schools in a single district.

We want to concentrate on the applicability of DEA in the management of an urban public school district with emphasis on (a) the identification of school units which make better use of input resources in terms of measured outputs, and (b) the obtaining of estimates of the extent to which inputs are underutilized in DMU's which are relatively unproductive.

In addition, we shall contrast the results of DEA with those obtained in the more usual manner, such as least-squares regression models. We shall also cross-validate the resulting classification of DEA by means of discriminant analysis. Finally, a rudimentary sensitivity analysis is performed to test the measure-dependency of DEA.

DATA ENVELOPMENT ANALYSIS

In their work, Charnes, Cooper and Rhodes introduce first a conceptual model which, in ratio form, makes it possible to relate efficiency measurement approaches in engineering, economics, etc. to each other. After exhibiting this property of the conceptual model, Charnes, Cooper and Rhodes then show how this model may be replaced by an equivalent ordinary linear programming problem. The latter, which has all the power and convenience of ordinary linear programming is the one we shall employ, along with the associated definition of efficiency which, in economics, is designated as Pareto Efficiency.* The latter, as given in [3], may be paraphrased here as follows:

> "A DMU (Decision Making Unit) is not efficient in producing its output (from given amounts of input) if it can be shown that some redistribution of resources will result in the same amount of this output with <u>less</u> of some resource and <u>no more</u> of any other resource. Conversely, a firm is efficient if this is not possible."

---

*Also called Pareto-Koopmans Efficiency. See [3].

This is the definition of efficiency we shall employ with a 100% rating being achieved only by an efficient DMU. While we do not here detail the argument from economic theory, it perhaps suffices to say that all of welfare economics rest on this definition of efficiency, i.e. the so-called Pareto optimality condition. In our case this has the advantage of not requiring us to assign weights on an a priori basis to the various educational inputs and outputs. Instead, as we shall see, these are obtained directly from the data and the modes we shall employ in an objective manner.

In our example, let us consider four hypothetical schools whose median percentile achievement scores for beginning and end of year are as follows:

School	A	B	C	D
Posttest Percentile	60	70	65	82
Pretest Percentile	50	60	65	80

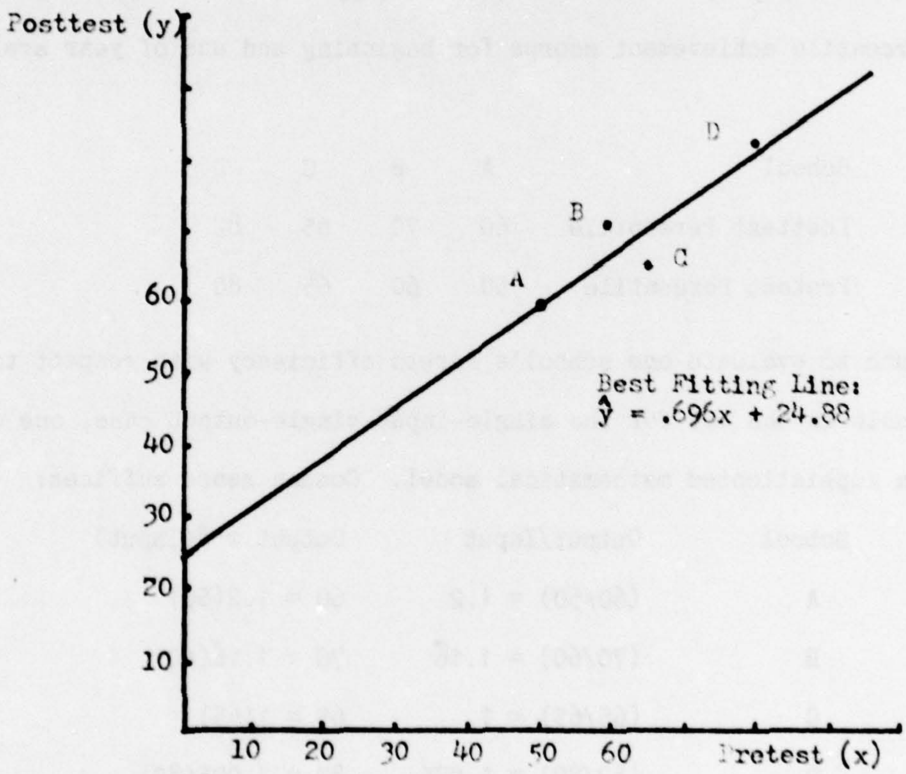
If one wants to evaluate one school's Pareto efficiency with respect to the other schools in the set for the single-input-single-output case, one does not need a sophisticated mathematical model. Common sense suffices:

School	Output/Input	Output = f(Input)
A	(60/50) = 1.2	60 = 1.2(50)
B	(70/60) = $1.1\bar{6}$	70 = $1.1\bar{6}$(60)
C	(65/65) = 1	65 = 1(65)
D	(82/80) = 1.025	82 = 1.025(80)

Clearly school A gets more output units per unit of input than the other schools. Note that results here are descriptive. What is "causing" the different "production" rates is not known.

It should be noted that regression of posttest on pretest is a frequently used descriptive technique and, as is shown below, results from such analysis for the same data are quite different.

Fig. 1
Least Squares Regression of
Posttest on Pretest Achievement Scores



From the graph, note that

(a) For school A, end of year scores were as predicted.

(b) For schools B and D, end of year scores were better than predicted, and

(c) For school C, end of year scores were worse than predicted.

Observing the data, one might reasonably conclude that there is a strong linear trend, and schools B and D both have better achievement than the "average school" with the same beginning achievement. However, the production perspective yields a different conclusion: School A's achievement was better than either B or D, given their respective beginning scores.

Of course, one does not wish to be restricted to the single-input-single-output case. Schools are very complex and a model which simultaneously takes into account multiple inputs and multiple outputs does require a sophisticated mathematical model. The model will be explained in the single-input-single-output case just described and then extended to the multiple-input-multiple-output case.

We may summarize the advantages to the production perspective compared to regression methods in three points:

(a) DEA does not require either distribution assumptions or measurement unit restrictions.

(b) Multiple outputs (criteria) can be employed simultaneously.

(c) Interpretation of production rates is very straightforward while interpretation of "better than predicted" or "better than the average school" is not so clear.

Model for Hypothetical Schools:

	Fractional Model	Linear Equivalent
Objective (School A):		
	Max $h_A = (60u/50v)$	$v = (6/5h_A)u$ for fixed h where $0 < h_A \leq 1$
Constraints:		
	School A Constraint $(60u/50v) \leq 1$	$v \geq (6/5)u \Longleftrightarrow u \leq (5/6)v$
	School B Constraint $(70u/60v) \leq 1$	$v \geq (7/6)u \Longleftrightarrow u \leq (6/7)v$
	School C Constraint $(65u/65v) \leq 1$	$v \geq u \Longleftrightarrow u \leq v$
	School D Constraint $(82u/80v) \leq 1$	$v \geq (8.2/8)u \Longleftrightarrow u \leq (8/8.2)v$

$u,v > 0$; weights employed to produce Pareto-efficiency comparisons with other units in set.

Note that school A, the school being evaluated, is given the most favorable weighting that the constraints allow; that is, the most favorable weighting possible, given that all schools in the set are compared on the same basis. The constraint set insures that the schools are evaluated relative to each other by providing a maximum value for the objective (right-hand-side constant of inequality constraints) to take for the school or schools doing best relative to the set of schools. The choice of this constant is arbitrary in that whatever the maximum is, all schools will be compared to that same value. A maximum of one was selected for modeling and computational convenience.

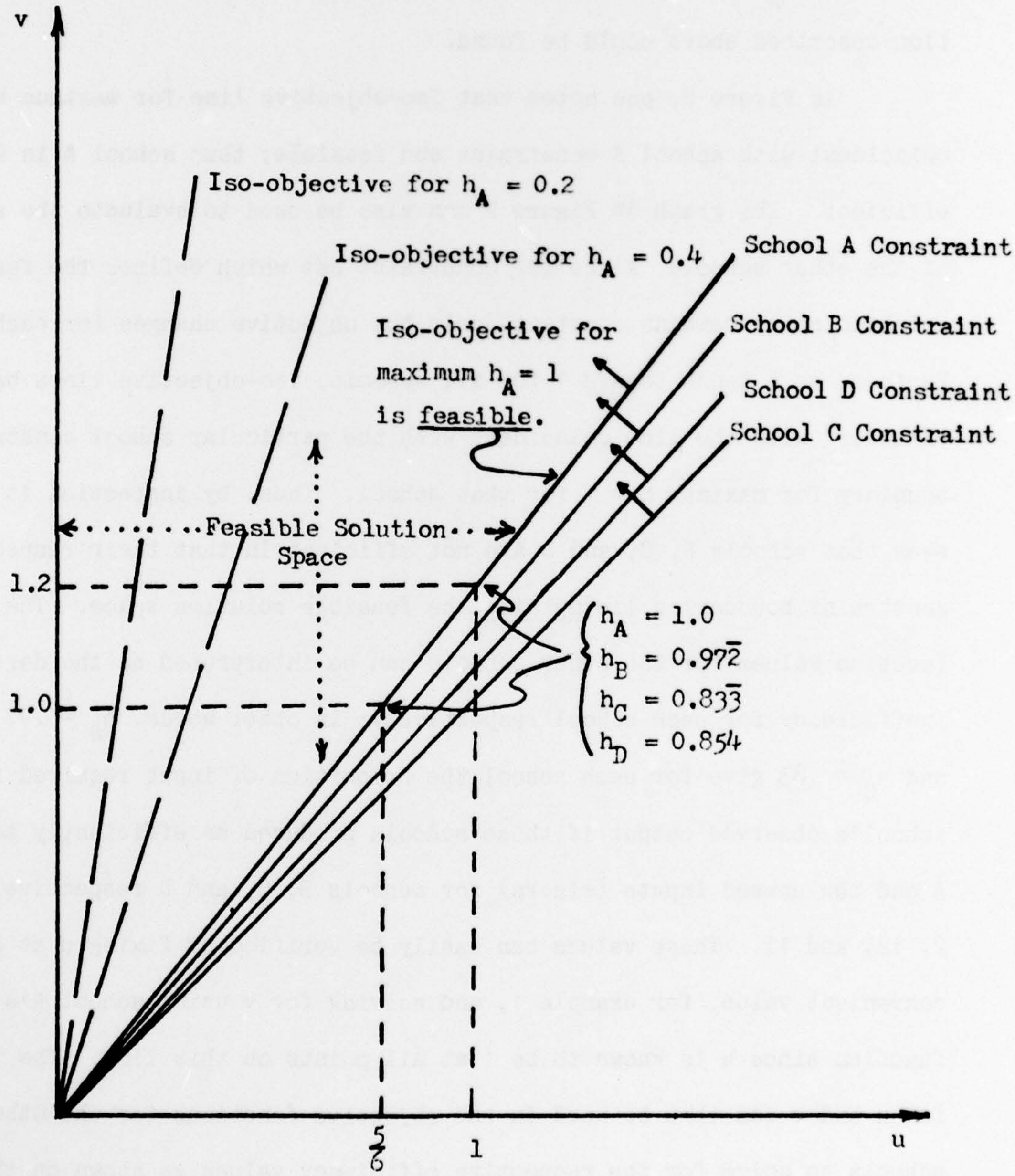
Once the above model is solved, one can apply the Pareto optimality criterion to determine whether or not school A is Pareto efficient or inefficient:

If $h_A < 1$, then school A is Pareto inefficient because a school or combination of schools has been found which can produce the same amount of output with less input.

If $H_A = 1$, then school A is Pareto efficient in that no such combination described above could be found.

In Figure 2, one notes that iso-objective line for maximum $h_A = 1$ is coincident with school A constraint and feasible; thus school A is Pareto efficient. The graph in Figure 2 can also be used to evaluate the efficiency of the other schools, since the constraint set which defines the feasible solution space remains constant--only the objective changes for each school. Further, as h tends toward 1 for all schools, iso-objective lines become "flatter" with the line coincident with the particular school constraint boundary for maximum h = 1 for that school. Thus, by inspection it can be seen that schools B, C, and D are not efficient in that their respective constraint boundaries lie outside the feasible solution space. The objective function values for the other schools can be interpreted as the degree of inefficiency for each school respectively; in other words, $h_B \doteq .97$, $h_D \doteq .85$ and $h_C \doteq .83$ give for each school the proportion of input required for each school's observed output if these schools produced as efficiently as school A and the unused inputs (slacks) for schools B, C, and D respectively are 2, 12, and 11. These values can easily be verified by fixing u at some convenient value, for example 1, and solving for v using school A's objective function since h is known to be 1 at all points on this line. The values for u and v can then be used in the objective functions for the other schools to solve for the respective efficiency values as shown on the graph in Figure 2. Slack values can be obtained by subtracting h times measured input from the measured input.

Fig. 2
Graphic Solution to Linear Programming Model Equivalent



A more complex graphic illustration and associated simplex tableaus for a two input-single output example can be found in [3] along with the generalized form of the model for multiple inputs and outputs. This model is reproduced below for reader convenience.

General Model--Multiple inputs and outputs.

Objective: Maximize $h_0 = (\sum_{r=1}^{s} u_r y_{r0} / \sum_{i=1}^{m} v_i x_{i0})$, where y_{r0} designates the rth output measure for school 0--the school being evaluated. There are s output measures. x_{i0} designates the ith input measure for school 0. There are m input measures. u_r and v_i are the desired weights.

Constraints: $(\sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij}) \quad 1$ for $j = 1, \ldots, n$

Note that there are n constraints, one for each of the $j = 1, \ldots, n$ schools in the set.

$$u_r, v_i > 0, \; y_{rj} > 0, \; x_{ij} > 0 \text{ for all } i, j, \text{ and } r.$$

Note that the analysis requires a model for each school in the set as schools must be evaluated one at a time. Computational efficiency can be gained by using dual-simplex methods as described in [10], pp. 13-16. The linear programming model equivalent to the fractional programming model given above which was used for the analysis under review is fully described in [5], pp. 431-432, model numbered (3). The particular linear programming model used was readily solvable by existing algorithms and provided (a) relative efficiency-inefficiency values, (b) surpluses of resources under-utilized by inefficient schools and (c) opportunity costs for exhausted resources. MPOS [9] was used to solve the 55 ordinary linear programming models which were reformulations of the fractional programming models described above.

AN URBAN SCHOOL DISTRICT APPLICATION

An urban school district with 60,000 pupils in attendance was chosen for the application because a recent study by Jennings [8] provided measures of input from school, community and pupils along with output measures of achievement. Further, a close working relationship with the upper level administration of the school district provided a means for pursuing administrative response to the information provided by the analysis.

The 55 elementary schools in the district were taken as the decision making units (DMU's) and the level of aggregation of all data was the school unit--outputs expressed as median percentile achievement scores for the school and inputs expressed as school totals, ratios, or percents as was appropriate.

As is usual for such schools, large amounts of federal funds were allocated to schools on the basis of compensatory education efforts for disadvantaged pupils. In other respects, state and local funds were allocated to schools on a uniform basis except for some special program funds available to schools for subventing attempts to improve instruction on an R & D basis.

The measures obtained for the analysis were categorized as follows:

<u>Output Measures</u>

Four outputs measured by the California Achievement Test in May, 1977.*

y_1 median percentile reading achievement for all pupils at the school

y_2 median percentile reading achievement for only those pupils in attendance at the school for a full year

*Two different medians were computed for reading and mathematics achievement in order to have an indication of the "school effect" without the influence of pupil mobility. Thus y_2 and y_4 achievement medians are based on only those pupils who attended no other school during the year, whereas y_1 and y_3 scores included pupils who attended other schools before transferring into the school.

y_3 median percentile mathematics achievement test score for all students in attendance

y_4 median percentile mathematics achievement test score for only those pupils in attendance for a full year.

Input Measures

Pupil inputs measured by the California Achievement Test in May, 1976.

x_1 median percentile reading achievement for all pupils at the school

x_2 median percentile reading achievement for only those pupils in attendance at the school for a full year

x_3 median percentile mathematics achievement test score for all students in attendance

x_4 median percentile mathematics achievement test score for only those pupils in attendance for a full year.

Proxy measures for neighborhood and home conditions (obtained from school district records)

x_5 percent Mexican-American students

x_6 percent Black students

x_7 percent Anglo-American students

x_8 percent students from low income families

x_9 total enrollment in schoool

x_{10} percent in average daily attendance

x_{11} mobility index: (total enrollment - number entered late or withdrawn)/total enrollment

Proxy measures for within school conditions (obtained from school district records)

x_{12} pupil-teacher ratio

x_{13} age of school building

x_{14} per pupil expenditures for instructional supplies

x_{15} total expenditure for instruction

x_{16} total federal funds for disadvantaged children

x_{17} total funds for all major special programs in the school

x_{18} percent of funds for instruction derived from local sources

School organizational climate indicators obtained from Organizational Climate Description Questionnaire [5]; a high score on each dimension indicates the following:

x_{19} disengagement--an indicator of lack of cooperation among teachers

x_{20} hindrance--an indicator of the extent to which teachers believe they are burdened with unnecessary tasks

x_{21} esprit--an indicator of job satisfaction

x_{22} intimacy--an indicator of how much social interaction exists among teachers

x_{23} aloofness--an indicator of impersonal treatment of teachers by the principal

x_{24} production emphasis--an indicator that the principal is task oriented and directive in supervision of teachers

x_{25} thrust--principal motivates teachers by personal example of work orientation

x_{26} consideration--measure of the principal's friendliness and cooperativeness with teachers

Measures of classroom instructional processes (obtained from Individualization of Instruction Inventory [8]; higher scores indicate greater degree of individual rather than group oriented teaching methods)

x_{27} intraclass grouping

x_{28} variety of teaching materials

x_{29} pupil autonomy

x_{30} differentiated assignments

x_{31} tutoring

x_{32} total individualized instruction index

Fifty-five different models were solved--one for each school and the comparative efficiency rating, h_0, was obtained for each school. An h_0 value < 1 indicates that the associated school is inefficient with respect to the measures and the objective process employed in that a combination of schools have been found which can produce the same amount of output with less input. An h_0 value = 1 means that no such combination could be found and thus the associated school is Pareto efficient. Slack and dual variable values* were also obtained in order to (a) inspect the extent to which inputs were being under-utilized by DMU's and (b) inspect the opportunity costs for exhausted resources. The obtained h_0 values can be found in Table 1 and slack values for inefficient schools in Table 2.

Variable Dependency of Results

It is clear that our intent has been to achieve a descriptive treatment, but the question still remains--how good a description has been obtained? Since the rationale has been very straightforward, with no distributional assumptions about the data, the major threats to descriptive validity are the adequacy of the variable set to fully describe the production system and the sensitivity of the classification of DMU's to the inclusion or exclusion of inputs which are observed to correlate with outputs.

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*Note that for efficient schools all slack variable values = 0.

Table 1
Comparative Efficiency Values for Schools

DMU Number	h_0	DMU Number	h_0	DMU Number	h_0
1	1.00	20	1.00	38	1.00
2	1.00	21	1.00	39	1.00
3	0.77*	22	1.00	40	1.00
4	0.96*	23	1.00	41	1.00
5	1.00	24	1.00	42	1.00
6	0.97*	25	1.00	43	1.00
7	0.96*	26	1.00	44	1.00
8	1.00	27	0.95*	45	1.00
9	1.00	28	1.00	46	1.00
10	0.97*	29	1.00	47	1.00
11	1.00	30	1.00	48	1.00
12	0.92*	31	1.00	49	1.00
13	1.00	32	1.00	50	1.00
14	1.00	33	1.00	51	1.00
15	1.00	34	1.00	52	1.00
16	1.00	35	1.00	53	1.00
17	1.00	36	1.00	54	1.00
18	1.00	37	1.00	55	1.00
19	1.00				

* Denotes an inefficient school.

Table 2
Slack Variable values for Schools with Low Comparative Efficiency Index

Variable/DMU #	3	4	6	7	10	12	27
h_0	.77	.96	.97	.96	.97	.92	.95
x_1	5.3	5.2	2.2	1.8	2.5	5.9	1.0
x_2	1.4	2.5	0.0	0.0	0.0	4.4	0.2
x_3	8.6	0.0	7.1	11.5	4.6	4.6	0.0
x_4	7.1	3.3	6.4	15.2	2.6	0.1	0.0
x_5	0.0	10.8	10.3	0.0	0.0	21.2	23.7
x_6	29.3	21.3	6.6	25.7	70.6	0.0	0.0
x_7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_8	34.0	0.0	0.0	25.8	58.5	30.8	12.4
x_9	2.4	1.6	0.0	0.0	0.0	3.7	6.0
x_{10}	21.6	26.4	12.7	17.6	52.4	15.3	20.5
x_{11}	23.2	7.8	0.0	17.3	69.3	0.0	6.6
x_{12}	7.2	8.2	2.5	3.2	13.6	2.7	6.7
x_{13}	28.8	7.7	17.5	5.1	35.1	21.9	0.0
x_{14}	0.0	2.0	0.1	2.5	3.2	0.9	1.2
x_{15}	2.8	47.8	14.6	17.8	36.2	261.5	376.8
x_{16}	42.5	0.2	0.8	74.1	0.0	45.2	0.9
x_{17}	72.7	0.0	17.0	91.0	19.2	75.8	0.0
x_{18}	21.4	28.0	14.8	5.9	57.8	12.5	22.1
x_{19}	37.4	32.6	33.5	37.9	56.9	9.4	37.5
x_{20}	32.8	28.2	19.2	20.0	44.6	16.3	13.6
x_{21}	41.2	31.2	35.2	36.2	53.4	24.5	17.4
x_{22}	30.6	27.1	26.3	32.6	50.9	6.6	17.7
x_{23}	31.3	19.6	22.2	24.9	43.2	0.0	12.5
x_{24}	38.3	28.3	34.2	25.6	56.7	12.5	10.9
x_{25}	39.4	22.2	30.3	4.7	50.6	33.8	0.0
x_{26}	41.6	27.3	32.7	14.5	56.2	33.5	11.7
x_{27}	27.7	35.0	18.1	2.3	47.6	8.7	19.0
x_{28}	18.4	39.2	12.7	10.2	38.0	9.3	15.7
x_{29}	13.9	27.1	15.7	7.6	35.4	7.8	12.4
x_{30}	23.8	30.2	19.2	0.0	40.7	15.7	9.7
x_{31}	35.4	34.5	22.0	27.5	40.1	20.9	3.3
x_{32}	24.3	33.0	17.9	10.1	40.0	12.5	12.1

[a] unit = \$100 [b] unit = \$1000

Our procedure for this rudimentary sensitivity analysis was as follows:

a. A least squares regression of each output variable on the set of input variables to establish whether a predictive relationship existed in the data set.

b. Given an affirmative result in (a) above, a test of units found to be in the less productive set to see if their classification changed with a reduced set of input variables--excluding those having high correlations with outputs.

c. A discriminant analysis to cross-validate the classification and to provide management information on the input-output dependencies (in the data, if not in the field).

In test one, the regression models showed a high relationsip of inputs to outputs ($R^2 > 0.92$) for each output used one at a time as the criterion variable and the complete set of input variables as predictors. The standard error of estimate was ≤ 6.0. As might be guessed, achievement input (pretest) scores were the best predictors of output achievement, thus in step 6 we resolved the linear programming models for all ten schools in administrative area 1 of the district with entering achievement deleted from the input set. Five of these ten schools had initially been classified as efficient ($h_0 = 1$) and five were inefficient ($h_0 < 1$). The changes resulting from the reanalysis are shown in Table 3.

Table 3
Comparison of DEA Results for Area 1
Schools, Controlling for Entering Achievement

DMU #	Initial h_0 Values (all input variables)	Reduced h_0 Values (entering achievement deleted)	Shrinkage
1	1.00	1.00	0.00
2	1.00	1.00	0.00
3*	0.77	0.77	0.00
4*	0.96	0.94	0.02
5	1.00	1.00	0.00
6*	0.97	0.95	0.02
7*	0.96	0.85	0.11
8	1.00	1.00	0.00
9	1.00	1.00	0.00
10*	0.97	0.91	0.06

*Inefficient schools

We see in Table 3 that dropping the input measures most highly correlated with output did not change the classification of school units, but did result in a shrinkage in h_0 for less productive schools. This is the result one would expect if the analysis is not highly sensitive to changes in the input variable set (no change in classification). Shrinkage in h_0 values makes schools appear to be only slightly more inefficient when a major pupil input was not considered.

The explanation for the robustness of classification offered here is that the input variable set is broad enough to include correlates of the input variable deleted, and the effects of the missing variable are still accounted for.

It would seem, then, that if no correlates of the excluded measure were in the model, a <u>change</u> in classification would likely occur if the

excluded input is an important predictor of output. A test of this speculation was conducted by excluding all six variables correlated ($r \geq .50$) with achievement measures:

a. entering achievement

b. percent minority group enrollment

c. percent students from low income families

d. percent students in attendance

e. total federal funds for disadvantaged children

f. total funds for major special programs

The model for school #1, classified as efficient, was reanalyzed with the six achievement-correlated variables listed above deleted from the input set. The resulting value of h_0 was .999976 which might indicate either round-off error or that school #1 would be reclassified as inefficient. In any event, the change in h_0 was quite small.

The conclusion drawn from this result was that DEA should be preceded by a preliminary analysis to ascertain that input measures are correlated with outputs and that measures representing all important aspects of the school production system are included in the model.

DISCRIMINANT ANALYSIS: EFFICIENT VS INEFFICIENT SETS

A unique contribution of the Data Envelopment Analysis is that it provides a way to classify decision making units into efficient and inefficient sets when no a priori basis exists for such partitioning. That is, beginning with a set of measures on units, those that are equally efficient are distinguished from the others. This is converse to discriminant analysis which begins with a known classification of units into groups and accounts for the grouping by linear combinations of measures obtained on the units. In DEA, no readily available means exists for judging the relative success of the partitioning in terms of the discriminating power of the measures employed to achieve the classification into efficient and inefficient sets. In the use of discriminant analysis, however, one has both statistical indicators of discriminating power and a heuristic check by using the linear discriminant function to replicate the classification.

Thus, it would seem that the two procedures are complementary with discriminant analysis adding a cross-validation of outcomes. Accordingly, a discriminant analysis was performed and interpreted as follows.

Procedure

Using SPSS, version 7.0, subprogram DISCRIMINANT [12], a two-group analysis was performed for efficient ($n_1 = 48$) and inefficient ($n_2 = 7$) schools. Classification was adjusted for unequal size of groups. The same input variables defined earlier (x_1 to x_{32}) were used as discriminating variables with a single exception. Percent of Anglo enrollment was deleted to eliminate the linear dependency of the variable with percent Black and Mexican-American.

A discriminant function with considerable discriminating power was obtained: canonical correlation = 0.88 and Wilk's Lambda = .219.

One hundred percent success of classification of DMU's into the correct set was found. While there was likely some overfitting because the same variables were used in DEA to create the subsets and in discriminant analysis to classify DMU's into the subsets, the results indicate a high discriminating power even allowing for some shrinkage.

To illustrate the marked separation achieved by the discriminant function, the discriminant scores for the ten schools in a single area of the district (same schools as in Table 3) are given in Table 4.

Table 4
Discriminant Scores for Efficient
and Inefficient Schools in Area 1

	DISCRIMINANT SCORES	
DMU	EFFICIENT DMU'S	INEFFICIENT DMU'S
1	0.86	
2	0.51	
3		-7.82
4		-6.72
5	0.16	
6		-6.37
7		-5.23
8	-1.03	
9	-0.54	
10		-7.10
Centroid	0.914	-6.27

DISCUSSION

We have shown that we can identify individual school units that are less efficient than other comparable units in terms of measured achievement scores relative to input factors representing entering achievement, school neighborhood characteristics, expenditures for instruction, type of instruction, and attributes of faculty and principal. It is beyond the scope of the present analysis to determine administrative reallocation of resources in order to achieve greater overall efficiency. What we can do, however, is a significant improvement in presently available management information in the administration of schools--we can identify inefficient units and show what school input factors contribute to their less efficient status by being underutilized. The administrative response to such

information should not necessarily be to reduce the slack resource. In some cases the resource is mandated by law and is allocated on a formula basis and even when local leeway is present, the absence of causal evidence would suggest the need for a cautious management response.

What is suggested then, is a three step procedure for the use of DEA results in school districts: (a) Identification of inefficient units by top-level administration and reporting of results to individual schools, (b) consideration of slack variable values by school unit administrators and their interpretation in terms of changes in the school unit operation targeted to the improvement of efficiency, and (c) reanalysis at the time of the next achievement testing to determine if adjustments indicate improvements.

Let us illustrate these steps by a discussion of a possible scenario for school #3, the most inefficient unit.

Step (a)--As was shown in Table 1, the h_0 value for school 3 is 0.77. This may be interpreted to indicate that, compared to other units, school #3 should be able to achieve its productivity rate by using only 77% of the total inputs currently observed in that school. If achievement is low in school 3, this would suggest strong efforts are needed to improve achievement <u>without</u> additional resource allocations. If achievement is high relative to other schools, top administration might consider reallocation of some resources <u>to other low achieving but efficient units</u>. School #3 actually had 68% in reading scores and 76% in mathematics scores. These are in the high average range of productivity. Looking further, however, we see that the school had higher reading percentile achievement scores the previous year (75%ile). Thus the inefficient status of the school would seem to be partly attributable to lowered achievement (output) relative to previous achievement (input). Other factors, of course, are involved and we will pass the results on to the principal and faculty of school #3 for their on-the-scene interpretation.

The implication of the above, however, is that the school needs to regain its previous achievement level without additional school district resources under the threat that, since its achievement is not among the lowest in the district, some reallocatable resources may be removed. Efficiency ratios can also be brought into balance by reducing underutilized inputs.

Step (b)--The staff of DMU #3 must now consider the indicators of how they differ from more efficient units. Table 5 provides a profile of variables that are illustrative of the inputs that the staff of the DMU might wish to consider. First, the mean values for the schools in the efficient and inefficient sets are shown, followed by the observed value for the DMU. Finally, the slack variable value from the DEA results for school #3 indicates "excess" resources. That is, if the production rate of school #3 were the same as schools with the greatest production rate, then input resource i could be reduced by as much as s_i without any associated reduction in y_r (output).

Table 5
Profile of Inefficient School on Under utilized Resources

Input	Mean Efficient	Mean Inefficient	Sch. 3 Value	Slack Value
Attendance	94.1%	92.6%	93.0%	21.5%
Disengagement	54.6%	70.0%	75.0%	37.4%
Hindrance	51.6%	64.6%	75.0%	32.8%
Esprit	76.5%	75.4%	75.0%	41.2%
Intimacy	64.2%	68.6%	71.0%	30.6%
Production Emphasis	64.0%	70.7%	75.0%	38.3%
Thrust	74.2%	72.4%	75.0%	39.4%
Individualized Grouping	63.1%	64.0%	66.0%	27.7%
Pupil Autonomy	56.0%	53.1%	44.0%	13.8%
Differentiated Assignments	61.9%	61.1%	58.0%	23.8%
Tutoring	48.0%	57.7%	64.0%	35.4%
Total $ for Instruction	$336,000	$388,000	$428,300	$2,800
Federal $ for Disadvantaged	21,200	54,900	89,200	42,500

The staff might consider, for example, that two of the factors, Disengagement and Hindrance, are negative attributes which they have in greater measure than other schools. Accordingly, they might propose to reduce the "excess" units by greater commitment to instructional efforts (engaging in the central task of the school) and by reducing the amount of hindrance caused by burdensome non-teaching routines. Supposedly, this gain in staff energy and attention might be employed to increase the use of those underutilized instructional methods detected in more efficient schools. Thus, increased pupil autonomy and individualized instruction might be employed more effectively in an effort to increase achievement.

If some analysis such as the one suggested were made by the staff of school #3, then they would be able to propose a modified operating plan for the subsequent year. This, along with district-level modifications, would become the goal-setting vehicle for the DMU.

Step (c)--After a year of operation under the modified plan, measures would again be obtained and DEA computed for all schools. If the efficiency-increasing steps have been effective, it would be detectable by an increase in h_0 or perhaps, in the best outcome, the DMU would be classed as efficient. Of course, in the unlikely event that all schools in the district improved their effectiveness, no change or even a drop in h_0 could be observed, since it is a comparative measure. However, in this case, an increase in outputs for all the schools should be observed.

In this fashion, the analysis provides the basis for needs identification, program planning and evaluation. These are all needed management tools for school district administration.

SUMMARY

Management of schools has been hindered by lack of appropriate analytical tools. A technique called Data Envelopment Analysis (DEA) has been employed to measure the productivity of individual schools in an urban school district and to identify those that are less efficient than others with respect to the Pareto-Koopmans Optimality Criterion.

The results of DEA were cross-validated by means of discriminant analysis and the results were confirmed. Finally, a discussion of results presented the outline of a procedure for using DEA results as management information for the improved efficiency of schools.

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