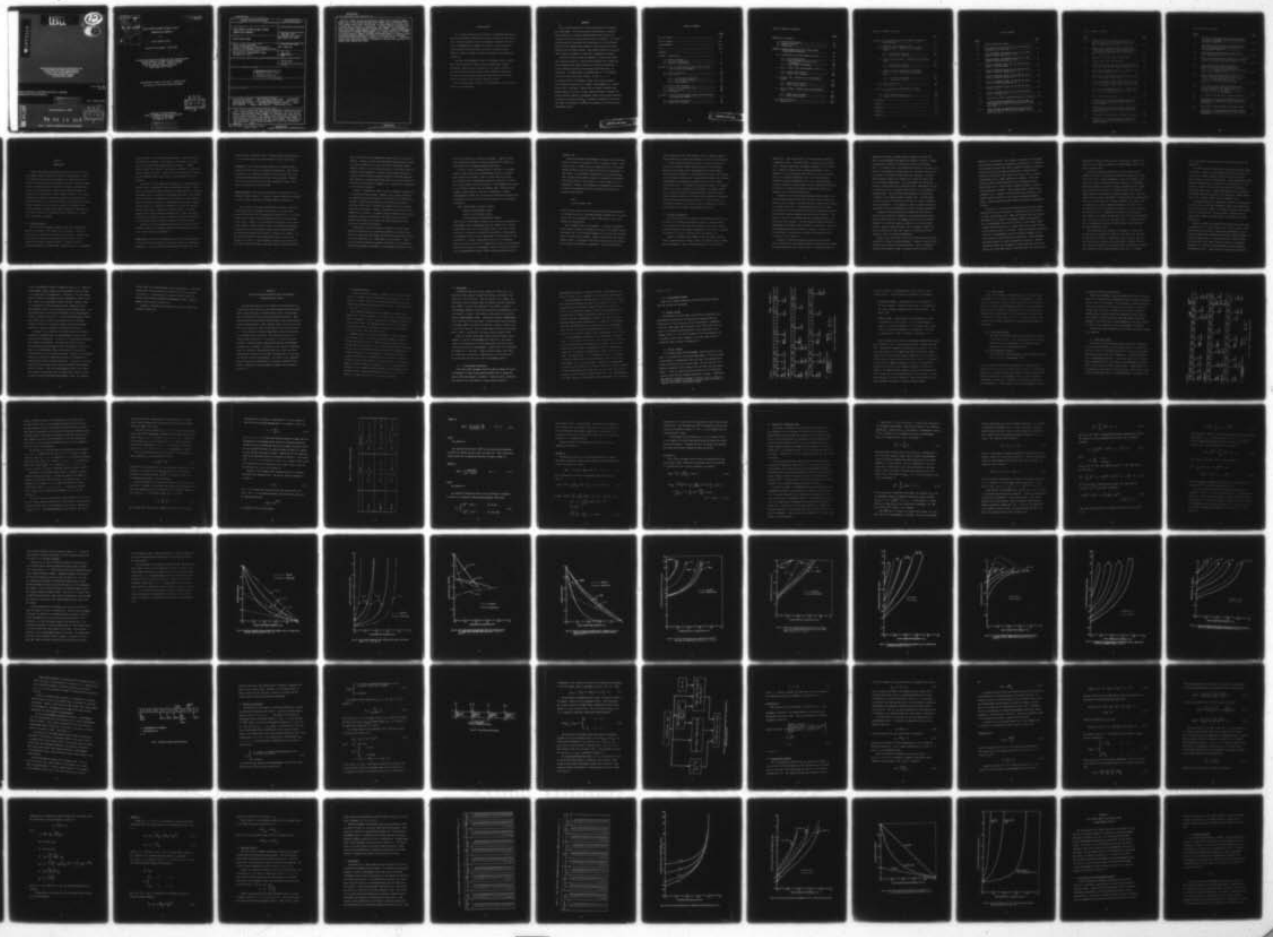
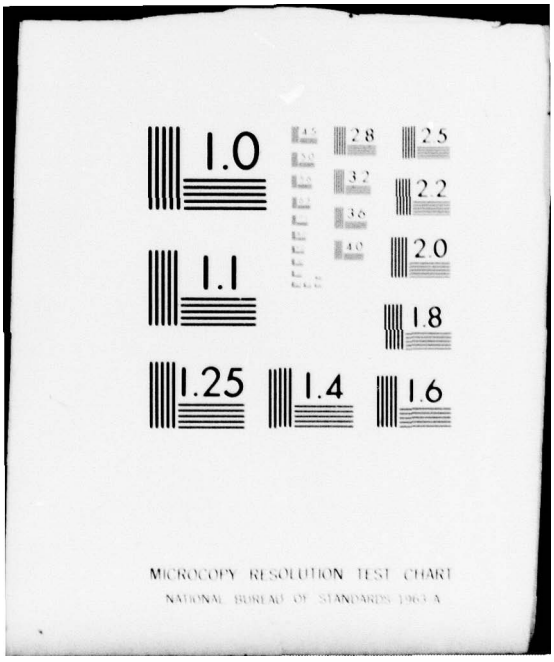
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(6) # ERROR CONTROL SYSTEMS FOR MULTI-ACCESS COMMUNICATION CHANNELS

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by

(10) Billy Hitoshi Saeki

Principal Investigator: Izhak Rubin

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Error control systems for multi-access communication channels are investigated. Error detection and retransmission techniques, also referred to as Automatic Request-Repeat (ARQ) schemes, are integrated with basic channel access-control disciplines. The channel access disciplines coordinate the transmissions by the distributed stations of the communication network to avoid simultaneous transmissions over the shared channel. ARQ systems provide the necessary error recovery procedures to insure data transmission integrity. /over/

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ABSTRACT

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TABLE OF CONTENTS

	Page
List of Tables	vii
List of Figures	viii
ACKNOWLEDGMENTS	xiii
VITA	xiv
ABSTRACT	xv
CHAPTER I INTRODUCTION	1
1.1 Problem Statement	1
1.2 Historical Background	7
1.3 Outline of Dissertation	13
CHAPTER II TIME DIVISION MULTIPLE ACCESS USING STOP-AND-WAIT AND BLOCK ARQ SYSTEMS	17
2.1 System Description	18
2.2 ARQ Systems	19
2.2.1 Acknowledgment Mechanisms	19
2.2.2 Stop-and-Wait Schemes	21
2.2.3 Block Schemes	24
2.3 Message Delay Analysis	27
2.4 Evaluation of Completion Time	36
2.5 Numerical Results	42
2.6 Conclusions	55
CHAPTER III TIME DIVISION MULTIPLE ACCESS USING THE SELECT-AND-REPEAT ARQ SYSTEM	56
3.1 Operational Description	56
3.2 Channel State Process	59

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TABLE OF CONTENTS (continued)

		Page
CHAPTER III (continued)		
3.3	Message Delay Analysis	64
3.4	Numerical Results	73
3.5	Conclusions	74
CHAPTER IV	GROUP RANDOM ACCESS USING RANDOM ACCESS ACKNOWLEDGMENT PROTOCOLS	81
4.1	Introduction to the GRA Channel Structure	81
4.1.1	System Description	82
4.1.2	Basic GRA Channel Access-Control Discipline	85
4.1.3	Acknowledgment Mechanisms	88
4.2	Pure - Random Access Acknowledgment Scheme	91
4.2.1	Channel State Process	92
4.2.2	Packet Delay Analysis	102
4.3	Multiple Copy - Random Access Acknowledgment Scheme	109
4.3.1	Channel State Process	110
4.3.2	Packet Delay Analysis	120
4.4	Period Division - Random Access Acknowledgement Scheme	124
4.4.1	Channel State Process	128
4.4.2	Packet Delay Analysis	139
4.5	Numerical Results	144
4.6	Conclusions	147

TABLE OF CONTENTS (continued)

	Page
CHAPTER V GROUP RANDOM ACCESS USING SCHEDULED ACKNOWLEDGMENT PROTOCOLS	153
5.1 Scheduled Acknowledgment Schemes	154
5.2 Period Division - Scheduled Acknowledgment Scheme	155
5.2.1 Channel State Process	157
5.2.2 Packet Delay Analysis	164
5.3 Dynamic Period Division - Scheduled Acknowledgment Scheme	167
5.3.1 Channel State Process	171
5.3.2 Packet Delay Analysis	179
5.4 Random Noise GRA Channel Under the Dynamic Period Division - Scheduled Acknowledgment Scheme	183
5.4.1 Channel State Process	184
5.4.2 Packet Delay Analysis	191
5.5 Numerical Results	196
5.6 Conclusions	199
CHAPTER VI CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH	206
6.1 Summary and Conclusions	206
6.2 Suggestions for Future Research	208
REFERENCES	210
APPENDIX A	216
APPENDIX B	239
APPENDIX C	249
APPENDIX D	260

LIST OF TABLES

Table		Page
2.1	Protocol Completion Times	32
2.2	Mean Completion Times $E(S_n^{(i)} \mid B_\ell)$ for the Stationary Transmission Error Process	41
3.1	Delay-Throughput Performance Function for a TDMA Channel Using SR ARQ with $P_N = 0.1$	75
3.2	Delay-Throughput Performance Function for a TDMA Channel Using SR ARQ with $P_N = 0.2$	76
4.1	Maximum Throughput Values for the GRA Discipline Under the PRA ACK Scheme	102
4.2	Maximum Throughput Values for the GRA Discipline Under the MCRA ACK Scheme with $\tilde{K} = 12$	120
4.3	Maximum Throughput Values for the GRA Discipline Under the PDRA ACK Scheme with $\tilde{K} = 12$, $\eta = 1$	138
4.4	Maximum Throughput Values for the GRA Discipline Under the PDRA ACK Scheme with $\tilde{K} = 12$, $\eta = 9$	138
4.5	Maximum Throughput Values for the GRA Discipline with $\tilde{K} = 12$	145
5.1	Maximum Throughput Values for the GRA Discipline with $\tilde{K} = 12$, $P_N = 0.0$	197
A.1	Solutions to (A.57), Normalized Time-Out Thresholds	233
A.2	Optimum Time-Out Thresholds for the Block ARQ - PP ACK Scheme Under Geometric Distributed Acknowledgment Delays	234
D.1	Delay Performance Comparisons of the GRA Channel Under the FDA, FDS and PDS Acknowledgment Schemes $\tilde{K} = P = R = 12$	275

LIST OF FIGURES (continued)

Figure		Page
2.11	Message Delay versus Packet Noise Error Probability Curves for a TDMA Channel Using Block ARQ - MMS ACK with $B_i = 5$, $K_i = 1, 2$.....	53
2.12	Message Delay Standard Deviation-to-Mean Ratio versus Packet Noise Error Probability Curves for a TDMA Channel Using Block ARQ - MMS ACK with $B_i = 5$, $K_i = 1, 2$.....	54
3.1	Operation of a TDMA Channel Using SR ARQ.....	58
3.2	Transmit Queue Size Relationships.....	61
3.3	Transition $X_n \rightarrow X_{n+1}$ for the Vector Markov Chain X Associated with the TDMA Channel Using SR ARQ.....	63
3.4	Delay versus Throughput Curves for a TDMA Channel Using SR ARQ with $P_N = 0.2$.....	77
3.5	Delay versus Packet Noise Error Probability Curves for a TDMA Channel Using SR ARQ.....	78
3.6	Maximum Throughput versus Packet Noise Error Probability Curves for a TDMA Channel Using SW, Block or SR ARQ with $B_i = 1$, $K_i = 5$.....	79
3.7	Delay versus Throughput Curves for a TDMA Channel Using SW, Block and SR ARQ with $B_i = 1$, $K_i = 5$, $P_N = 0.2$.....	80
4.1	Basic GRA Channel Structure.....	84
4.2	Transition $X_n \rightarrow X_{n+1}$ for the Markov State Chain X Associated with the GRA Discipline Under the PRA ACK scheme.....	97
4.3	Transition $X_n \rightarrow X_{n+1}$ for the Markov State Chain X Associated with the GRA Discipline Under the MCRA ACK Scheme.....	116
4.4	GRA Channel Structure Under the PDRA ACK Scheme.....	126
4.5	Transition $X_n \rightarrow X_{n+1}$ for the Markov State Chain X Associated with the GRA Discipline Under the PDRA ACK Scheme.....	133

LIST OF FIGURES (continued)

Figure		Page
4.6	GRA Channel Structure Under the PDRA ACK Scheme with the Acknowledgment Subperiod Preceding the Information Subperiod.....	140
4.7	GRA Channel Structure Under the PDRA ACK Scheme with the Information Subperiod Preceding the Acknowledgment Subperiod.....	142
4.8	Packet Delay versus Throughput Curves for the Basic GRA Channel with $\tilde{K} = P = R = 12$.....	148
4.9	Packet Delay versus Packet Probability of Rejection Curves for the Basic GRA Channel with $\tilde{K} = P = R = 12$.....	149
4.10	Packet Delay versus Packet Probability of Rejection Curves for a GRA Channel with $\tilde{K} = P = R = 12$ Under the PRA and MCRA ACK Schemes Using Control Function (4.64).....	150
4.11	Packet Delay versus Packet Probability of Rejection Curves for a GRA Channel with $\tilde{K} = P = R = 12$ Under the PDRA ACK Scheme ($\tilde{K}_I = 6$, $\eta = 1$), ($\tilde{K}_I = 10$, $\eta = 9$) Using Control Function (4.15).....	151
4.12	Packet Delay versus Packet Probability of Rejection Curves for a GRA Channel with $\tilde{K} = P = R = 12$ Under the PRA ($\gamma = 1$), MCRA ($\gamma = 2$), and PDRA ($\tilde{K}_I = 6$, $\eta = 1$) ACK Schemes.....	152
5.1	GRA Channel Structure Under the PDS ACK Scheme.....	158
5.2	Transition $X_n \rightarrow X_{n+1}$ for the Markov State Chain X Associated with the GRA Discipline Under the PDS ACK Scheme.....	161
5.3	GRA Channel Structure Under the DPDS ACK Scheme.....	169
5.4	Transition $X_n \rightarrow X_{n+1}$ for the Markov State Chain X Associated with the GRA Discipline Under the DPDS ACK Scheme.....	175

LIST OF FIGURES (continued)

Figure		Page
5.5	Transition $X_n \rightarrow X_{n+1}$ for the Markov State Chain X Associated with the Random Noise GRA Discipline Under the DPDS ACK Scheme.....	189
5.6	Packet Delay versus Packet Probability of Rejection Curves for a GRA Channel with $\tilde{K} = P = R = 12$, $\eta = 1$ Under the PDS and DPDS ACK Schemes Using Control Functions (5.9) and (5.57), Respectively.....	200
5.7	Packet Delay versus Packet Probability of Rejection Curves for a GRA Channel with $\tilde{K} = P = R = 12$, $\eta = 9$ Under the PDS and DPDS ACK Schemes Using Control Functions (5.9) and (5.57), Respectively.....	201
5.8	Packet Delay versus Packet Probability of Rejection Curves for the Basic GRA Channel and a CRA Channel Under the DPDS ACK Scheme Using Control Functions (5.9) and (5.57), Respectively, with $\tilde{K} = P = R = 12$, $P_N = 0.0$.....	202
5.9	Packet Probability of Rejection versus Packet Noise Error Probability Curves for a GRA Channel with $\tilde{K} = P = R = 12$, $\eta = 9$ Under the DPDS ACK Scheme Using Control Function (5.57).....	203
5.10	Packet Delay versus Packet Noise Error Probability Curves for a GRA Channel with $\tilde{K} = P = R = 12$, $\eta = 9$ Under the DPDS ACK Scheme Using Control Function (5.57).....	204
A.1	Maximum Throughput versus Packet Noise Error Probability Curves for a TDMA Channel Using SW ARQ - PP ACK and Block ARQ - PP ACK, Positive Acknowledgment with Time-Out, with Geometric Distributed Acknowledgment Delays.....	235
A.2	Maximum Throughput versus Packet Noise Error Probability Curves for a TDMA Channel Using SW ARQ - PP ACK and Block ARQ - PP ACK, Positive Acknowledgment with Time-Out, with Uniform Distributed Acknowledgment Delays.....	238
C.1	State Transition Diagram for the Markov Chain y.....	251

LIST OF FIGURES (continued)

Figure		Page
D.1	GRA Channel Structure Under the FDA ACK Scheme.....	264
D.2	GRA Channel Structure Under the FDS ACK Scheme.....	265
D.3	Packet Delay versus Packet Probability of Rejection Curves for a GRA Channel with $\tilde{K} = P = R = 12$, $\eta = 1$ Under the FDA and DPDS ACK Schemes Using Control Functions (5.9) and (5.57), Respectively.....	276
D.4	Packet Delay versus Packet Probability of Rejection Curves for a GRA Channel with $\tilde{K} = P = R = 12$, $\eta = 9$ Under the FDA and DPDS ACK Schemes Using Control Functions (5.9) and (5.57), Respectively.....	277

CHAPTER I

INTRODUCTION

Studies concerning the communication over a multi-access broadcast channel normally address the channel accessing problem. Protocols (channel access-control disciplines) are synthesized ad hoc to control the self-inflicted noise generated in a multi-user environment. Control of random noise errors has been a secondary consideration to the multi-accessing problem. Usually error control procedures for random noise are neglected in the performance anlaysis. Error control procedures are normally evaluated for single-access, dedicated communication channels. In this dissertation, error detection and retransmission systems for error control are applied to the multi-access broadcast channel. Channel access-control disciplines under several error control systems are evaluated, and the operation in a random noise environment is examined.

1.1 Problem Statement

The multi-access broadcast channel provides the communication medium for a network of distributed users (stations). The multi-users share the common channel (multi-access feature). Transmissions by the network stations are received by all the stations of the communication network (broadcast feature). Examples include a satellite communication channel where a satellite acts as a transponder,

a radio channel in a terrestrial radio network, a coaxial cable link, and fiber-optics channels. The stations of the network are message sources (and sinks) which generate messages randomly. Examples include timesharing terminals, stock quotation terminals, vocoders and central computer facilities. These sources may generate short interactive type messages and/or long blocks of data associated with file transfers.

The intent of these communication networks is to deliver messages from the source to destination stations rapidly and reliably. Allocation of the available network resources is a major concern. If the user environment was not distributed, the messages generated at each station could be "queued" for transmission over the network to avoid conflicts. However, the spatial isolation of the distributed environment requires network resources to achieve the cooperation necessary for successful message transmissions. Requests for channel capacity to transmit messages must be made through the channel itself. Hence, the design of channel access protocols which efficiently regulate the allocation of the resources among the network stations is a primary concern.

Channel access-control disciplines developed for the multi-access broadcast channel can be broadly classified as random access, reservation or fixed scheduling disciplines.

RANDOM ACCESS: With these contention schemes newly arriving messages at source stations are allowed to access the channel for transmission immediately. If two or more users simultaneously transmit over the

shared channel, collision occurs. Colliding message transmissions are lost and these messages are retransmitted at future random times.

RESERVATION: Each station holding messages for transmission must request or reserve time for its message transmissions. Thus message transmissions are scheduled to avoid the self-inflicted noise (collisions) experienced under the random access disciplines. However, the necessary reservations must be made through the channel; and, hence, they represent network overhead.

FIXED SCHEDULING: Each station is dedicated a fixed portion of the available channel capacity. In essence, separate channels are provided to each station by time and/or frequency sharing the resources.

These channel access-control disciplines are normally evaluated by measuring channel throughput (the average rate of successful message transmissions over the channel) and message delay. In general, random access schemes yield small message delays at low new message arrival rates. However, as the traffic rate increases, these schemes become unstable yielding excessive delays and diminishing throughput. At high traffic rates reservation schemes provide improved channel utilization and message delay relative to random access schemes. But the overhead required to make reservations results in poorer performance at low rates. Fixed scheduling schemes, which furnish each user with a dedicated channel, yield high channel utilization indices

when each station in the communication network emits a steady flow of messages. However, stations with high peak-to-average message arrival rate ratios (bursty) will often not use their preassigned channel capacity, which is an inefficient allocation of the network resources. Therefore, these categories of channel access-control disciplines attempt to solve the multi-accessing problem by different approaches. Each, however, is concerned with controlling the self-inflicted noise created by the multi-user environment (i.e., simultaneous transmissions over the common bandwidth).

Control of random noise errors has been a secondary consideration to the multi-accessing problem. Normally, error control procedures for random noise have been neglected in the analysis of communication networks. Adequate signal energy per bit-to-noise ratios and/or sufficient forward error correction are assumed so that the required error rates are achieved. However, situations arise in which these assumptions are invalid. For example, without additional error control procedures, physical realizability constraints such as power and bandwidth requirements may restrict the system operating point to regions of unacceptable performance.

Random noise errors are caused, for example, by channel implementation mechanisms such as switching transients and thermal noise, and by the operational environment including both man-made interference and natural phenomena (e.g., lightening, solar transients). Errors are also caused by amplitude and delay distortion of the transmitted waveform (intersymbol interference). Non-negligible distortion can be

caused, for example, by a satellite transponder. Improper channel access algorithm execution or channel synchronization problems can cause destructive overlapping message transmissions.

The error control techniques which are presently in use employ either an error detection and retransmission scheme or a forward error correction code (FEC). When a feedback link is available, error detection and retransmission systems also referred to as automatic request-repeat (ARQ) are attractive. Acknowledgments are returned to the source stations over the feedback link. Positive acknowledgments (PACKs) indicate error-free transmissions. Negative acknowledgements (NACKs) are returned by the destination stations when transmission errors are detected and retransmission of the messages are required.

Basic acknowledgment strategies include:

- positive acknowledgment schemes
- negative acknowledgment schemes
- positive-negative acknowledgment schemes.

Under positive acknowledgment schemes, only errorless transmissions are acknowledged by PACK transmissions over the feedback link. Message transmissions which are not acknowledged within a specified time-out interval are interpreted as negatively acknowledged. Under negative acknowledgment schemes, errant transmissions are acknowledged by NACK transmissions. Message transmissions which are not acknowledged within the time-out interval are interpreted as positively acknowledged. Under positive-negative acknowledgment schemes, each message transmission is acknowledged by either a PACK or NACK transmission over the

feedback link.

Using an appropriate acknowledgment strategy, basic ARQ systems operate in either a stop-and-wait (SW) or continuous fashion. Under SW ARQ systems, a message cannot be transmitted until the preceding message has been correctly received and positively acknowledged. A NACK results in the immediate retranmission of the errant message. Under continuous ARQ systems, messages are transmitted continuously. Like SW ARQ systems, a NACK requires the errant message to be retransmitted. However, since messages are transmitted continuously, two basic retransmission policies are normally associated with continuous ARQ systems:

- Block
- Select-and-Repeat (SR).

Under Block ARQ schemes, not only the negatively acknowledged message is retransmitted, but all subsequent messages transmitted by the source station are also retransmitted. Under SR ARQ schemes, only errant messages are retransmitted.

When a feedback link is not available, FEC systems are applied. These systems can reduce the average error rate on the channel; however, individual codes are not effective against all error patterns encountered with the multi-access channel. Indeed, the success of FEC systems depends on apriori knowledge of the error statistics and this information for many real channels is unavailable. Hence, networks which

require essentially error-free operation (such as computer communication networks) must include ARQ techniques in their control procedures.

In this dissertation ARQ error control systems are applied to a multi-access broadcast channel operating under the Time Division Multiple Access (TDMA) and the Group Random Access (GRA) channel access-control disciplines, a fixed scheduling discipline and a random access discipline, respectively. Appropriate ARQ schemes under suitable acknowledgement mechanisms are integrated with the basic channel accessing protocols. The demands on the network resources to implement these error recovery procedures are determined. Performance indices such as channel throughput and message delay are analyzed through a queueing theoretic approach for network stations modeled as independent sources which generate messages according to a renewal process. A stationary transmission error process model is developed to evaluate the operation of the channel under random noise errors.

1.2 Historical Background

Over the past decade the literature concerning communication over the multi-access channel has steadily increased. Numerous channel access-control disciplines have been proposed in each major category. Delay and throughput performance comparisons can be found, for example, in Kleinrock [1], Lam [2], Kleinrock [3], and Schwartz [4].

Examples of random access schemes include pure ALOHA, slotted ALOHA, Group Random Access (GRA), Carrier Sense Multiple Access (CSMA) and Tree Algorithm. Pure or classical ALOHA was first studied by

Abramson [5]. Users are allowed to access the channel immediately on message arrivals. Collisions result in random future retransmissions. Abramson showed that the maximum throughput of pure ALOHA is 1/2e. Roberts [6] showed that the maximum throughput could be improved to 1/e by partitioning the channel into slots of duration equal to the message transmission time. Users are required to synchronize message transmissions with slots. Delay-throughput-stability questions of ALOHA systems have been subsequently addressed by Metcalf [7], Kleinrock and Lam [8], Carleial and Hellman [9], Ferguson [10], and Lam and Kleinrock [11].

Variations of the slotted ALOHA discipline have developed. The GRA discipline studied by Rubin [12] uses only certain channel time periods to allow a group of network stations to transmit their messages on a random access basis. The CSMA discipline studied by Tobagi and Kleinrock [13,14,15] prohibits stations from transmitting whenever other stations' message transmissions are detected ("sensed"). This scheme is effective for networks with short propagation delays (e.g., terrestrial radio channels). The Tree Algorithm scheme developed by Capetanakis [16] prohibits those stations not involved in a collision from transmitting until the conflict is resolved. Conflicts are resolved through a probabilistic procedure whose progress can be described by a tree structure.

Examples of reservation schemes include Reservation-ALOHA, Robert's Reservation, Split-Channel Reservation Multiple Access (SRMA), and Fixed-Reservation Access Control (FRAC). Reservation-ALOHA was

suggested by Crowther, Rettberg, Walden, Ornstein, and Heart [17] and its delay-throughput performance was studied by Lam [18]. Message transmissions are made on a random access basis in non-reserved slots. If the most recent slot experienced collision or was empty, the next slot is designated a random access slot; otherwise, the next slot is reserved for the source station which was successful. The FRAC scheme studied by Rubin [19] requires each station to reserve slots for its message transmissions. Slots are grouped into alternating periods of reserved slots and reservation slots. Reservations are made in reservation slots on a random access basis or on a contentionless scheduled basis. The FRAC scheme using random access reservations is similar to the scheme proposed by Roberts [20]. The SRMA scheme studied by Tobagi and Kleinrock [21] divides the available bandwidth into two subchannels. One subchannel is used to transmit reservation information; the second subchannel is used for message transmissions.

Frequency Division Multiple Access (FDMA) and Time Division Multiple Access (TDMA) are examples of fixed scheduling disciplines. Under the FDMA scheme (see, for example, Rubin [22]), the available bandwidth is divided into separate channels, one for each network station. Under the TDMA discipline (see, for example, Schmidt [23], Lam [24] and Rubin [19,22]), each station is allocated a periodic sequence of slots for message transmissions.

Random access, reservation, and fixed scheduling describe general categories of channel accessing techniques. However, disciplines which straddle these categories have been proposed. Recently integrated schemes which dynamically adapt to changing traffic patterns have

appeared in the literature. Such schemes, for example, switch between random access and reservation strategies (see Rubin [25]). In addition, the disciplines in these three basic categories coordinate the network stations to prevent simultaneous transmissions. Spread spectrum techniques (see Kochevar [26]) which allow several users to transmit simultaneously over a common bandwidth are available. These Code-Division Multiple Access disciplines encode transmissions with a pseudo-random code. Stations extract messages intended for them by cross-correlation techniques. These schemes require a high level of complexity; yet, in general, yield less efficient utilization of the network resources relative to other channel accessing techniques. However, these disciplines provide inherent anti-jam and cryptographic security.

The majority of the research concerning the multi-access channel neglects error recovery procedures. Exceptions include the studies by Binder and Castonguay [27], Tobagi and Kleinrock [28], and Schwartz and Muntner [29]. Binder and Castonguay consider the degradation due to acknowledgment error control traffic on a two channel system: an ALOHA channel from users to a central station and a queued channel from the central station to the users. Tobagi and Kleinrock investigate the effect of acknowledgment traffic on maximum throughput for the slotted ALOHA and CSMA access-control disciplines. Delay-throughput performance results for CSMA are obtained by simulation. Schwartz and Muntner compare the performance of FEC and ARQ error control procedures for a pure ALOHA channel. They show that a pure FEC procedure

yields poorer channel utilization than an ARQ system. However, this study does not consider the overhead of acknowledgment traffic or the impact on message delay.

The major research concerning error control systems has been primarily confined to single-access (terrestrial and satellite) links (see Sastry [30]). Message transmissions over these channels do not require the conflict resolving protocols necessary in a distributed multi-user communication network. Message transmissions over single-access channels can be queued. In addition, most of these studies assume messages are generated on a regular temporal basis. With such message arrival streams, performance is characterized by the uncorrected error rate and the reduction in useable bandwidth. The latter index is transmission efficiency (*maximum throughput*), the ratio of the successful (error-free) information transmission bit rate to the maximum channel bit rate. Transmission efficiency of ARQ systems has been computed, for example, by Benice and Frey [31], Burton and Sullivan [32], Gatfield [33], Kaul [34], and Sastry [35]. The effect of block size on transmission efficiency has been studied by Cacciamani and Kim [36] and McGruther [37].

FEC systems have been evaluated, for example, by Brayer [38]. He examines the performance of 4 rate 1/2 FEC codes on HF, Troposcatter and satellite channels. Hybrid error control systems which use both FEC and ARQ are expected to provide higher throughput than pure ARQ systems and lower error rates than pure FEC systems when the channel must contend with both random and burst errors (see, for example,

Rocher and Pickholtz [39], Sastry [40], Sastry and Kanal [41] and Brayer [42]).

Most error control procedures have been studied assuming random noise errors. However, on real communication channels, errors occur in bursts or clusters separated by fairly long error-free gaps. These channels are said to exhibit memory. Characterizations of these dependent error processes are found in Kanal and Sastry [41]. Performance of hybrid, FEC, and ARQ systems applied to real data is discussed by Brayer [42]. Fujiwara and Yamashita [43] investigate the GO-BACK-N ARQ system applied to the random error channel (memoryless) and to a Gilbert channel (see also Chadwick [44]).

For single-access links, transmission efficiency and uncorrected bit error rate are sufficient performance indices when transmitting large blocks of data. However, for interactive applications response time is a critical factor. The calculation of response time for single-access channels using ARQ systems is addressed by Reed and Smetanka [45].

Recently, studies have appeared which evaluate ARQ systems when the superposition of message arrivals form a renewal process. These investigations are confined to single-access communication links. Gavish and Konheim [46] consider the statistical queueing behavior of a station transmitting to a second station over a satellite link. They obtain queue size results using both analytic and numerical techniques for a continuous ARQ system under a select-and-repeat retransmission policy. These results are extended by Konheim [47]

to continuous ARQ systems under block and select-and-repeat retransmission policies. Buffer queue size and virtual message delay results are derived. Towsley and Wolf [48,49] consider SW and Block ARQ systems. The generating functions for the waiting time distribution are derived. In addition, a wide range of ARQ systems are examined by Towsley [50] with respect to maximum throughput and average queue lengths.

The majority of these previous results are derived for slotted systems. In these systems time is partitioned into equal length slots. The beginning of a message transmission is synchronized to the beginning of a slot. Furthermore, messages are normally assumed to be fixed length packets whose transmission time is equal to a slot duration. Fayolle, Gelenbe and Pujolle [51] evaluate the performance of the SW ARQ system over an unslotted channel. Message arrivals are governed by the Poisson process and message lengths form a sequence of i.i.d. random variables with a general distribution. They examine a positive acknowledgment scheme with random acknowledgment time delays.

1.3 Outline of Dissertation

The main results of this dissertation relate to error control systems for communication over the multi-access broadcast channel. This communication channel serves a network of spatially isolated stations whose transmissions are coordinated through basic channel accessing protocols. Automatic Request-Repeat error control procedures

under suitable acknowledgment mechanisms are integrated with the channel access-control disciplines.

In Chapter II, the SW and Block ARQ systems are incorporated into the TDMA control discipline. The channel and stations are characterized in Section 2.1. In Section 2.2 the ARQ systems are described. Packet-by-packet and message-by-message acknowledgment mechanisms are introduced. In Section 2.3 the concept of completion times is introduced and the generating function of the steady state message delay distribution is derived. The completion times are explicitly evaluated for a stationary transmission error process model in Section 2.4. Using these results, numerical examples are presented in Section 2.5 for the mean and variance of delay.

In Chapter III, the SR ARQ system is applied to a TDMA channel. The channel and the network stations are characterized and the operation of the SR ARQ scheme is described in Section 3.1. The evolution of the channel is described by a vector Markov chain in Section 3.2. The necessary and sufficient conditions for ergodicity are stated. In Section 3.3 upper and lower bounds on the average message delay at steady state are derived, and numerical examples are presented in Section 3.4.

In Chapter IV, the operation of the GRA channel access-control discipline under random access acknowledgment protocols is examined. In Section 4.1 the channel structure and the results for the basic GRA discipline are reviewed; and, the random access acknowledgment protocols are introduced. The GRA channel under the Pure-Random

Access acknowledgment scheme is examined in Section 4.2. Under this acknowledgment scheme, message and acknowledgment traffic contend on an equal basis for transmission. In Section 4.3 the GRA channel under the Multiple Copy-Random Access acknowledgment scheme is studied. Under this acknowledgment scheme, multiple, identical PACKs are transmitted for each successful message transmission; thus acknowledgment traffic is given preference over message traffic. The GRA channel under the Period Division-Random Access acknowledgment scheme is studied in Section 4.4. Under this acknowledgment scheme, each channel access period is partitioned into two subperiods. Messages and PACKs are transmitted random access within separate subperiods. Numerical examples are presented in Section 4.5.

In Chapter V, the operation of the GRA channel access-control discipline is examined under two acknowledgment protocols which schedule PACK transmissions. In Section 5.1 the scheduled acknowledgment schemes are introduced. The GRA channel under the Period Division-Scheduled acknowledgment scheme is examined in Section 5.2. Under this acknowledgment scheme, each channel access period is partitioned into two fixed length subperiods. Messages are transmitted random access within one subperiod and PACKs are transmitted on a scheduled basis within the second subperiod. In Section 5.3 the GRA channel under the Dynamic Period Division-Scheduled acknowledgment scheme is studied. Under this acknowledgment scheme, each channel access period is partitioned into two subperiods. However, unlike the Period Division-Scheduled acknowledgment scheme, the subperiod

lengths adapt to the acknowledgment traffic requirement. A stationary transmission error process model is developed for the GRA channel in Section 5.4. This random noise channel is studied under the Dynamic Period Division-Scheduled acknowledgment scheme. Numerical examples are presented in Section 5.5.

A summary, conclusions and suggestions for future research are provided in Chapter VI.

CHAPTER II

TIME DIVISION MULTIPLE ACCESS USING STOP-AND-WAIT ARQ AND BLOCK ARQ SYSTEMS

The generating function of the steady-state message delay distribution has been derived for the fixed-assignment synchronous TDMA control discipline under a Poisson message arrival stream by Hayes [52]. The mean delay has also been calculated by Lam [24]. Rubin [19] has derived the generating function under a more general message arrival characterization. These studies assume a zero bit error rate and do not consider the effect of the ARQ error recovery procedures.

In this chapter, the SW and Block ARQ systems are incorporated into the TDMA control discipline. The channel and stations are characterized in Section 2.1. In Section 2.2 the ARQ systems are described. Packet-by-packet and message-by-message acknowledgment mechanisms are introduced. In Section 2.3 the concept of completion times is introduced, the generating function of the steady state message delay distribution is derived, and expressions for the mean and variance of delay are presented. In Section 2.4 the completion times are explicitly evaluated for a stationary transmission error process. Using these results numerical examples are presented in Section 2.5.

2.1 System Description

A time synchronized channel is examined. Time (referenced with respect to a master clock) is divided into equal-length slots, each of duration τ seconds. The start of a message transmission across the channel must coincide with the beginning of a slot. Time slots start at each time $t = n\tau$, $n = 0, 1, 2, \ldots$.

The M network stations share the channel in accordance with a TDMA access - control discipline. Contiguous slots are organized into frames of length $M \geq 2$ slots. Each station is assigned a single (service) slot per frame. The queueing behavior of one of the network stations, say station 1, is studied. The behavior of this station is characteristic of any arbitrary station.

Messages arrive at the station according to a batch stochastic point process $\{A_n, n \geq 1\}$, where A_n denotes the number of message arrivals during the n-th slot. Message arrivals at the station are recorded only at the start of a slot. Each arriving message is divided into one or more fixed-length packets and stored for transmission in a buffer with infinite storage capacity. The transmission time of the data (including both information and overhead bits) contained in a packet is equal to the duration of a single slot (τ). The number of packets in the n-th message is B_n and $\{B_n, n \geq 1\}$ is assumed to be a sequence of i.i.d. random variables governed by an arbitrary discrete distribution with a moment-generating function $B^*(z) = E\{z^{B_n}\}$ and moments $b_i = E\{B_n^i\}$ such that $b = b_1$, b_2 and b_3 are finite.

2.2 ARQ Systems

Conventional ARQ error control systems for single-access communication links separate the data into packets, each packet being encoded for error detection. After the reception of a packet, the destination station responds either by sending a positive acknowledgment (PACK) if no errors are detected, or by sending a negative acknowledgment (NACK) if errors are detected and retransmission of the data packet is required. This system is referred to as a positive-negative acknowledgment scheme. Alternately, positive acknowledgment with a time-out period can be used. The destination station responds by sending a PACK if no errors are detected. However, no ACK is sent if errors are detected. After a predetermined time-out period during which a PACK is not returned, the source station assumes a NACK.

Under the SW ARQ system (using either positive-negative acknowledgment or positive acknowledgment with time-out), a data packet cannot be transmitted until the preceding packet has been correctly received and positively acknowledged. A NACK results in the immediate retransmission of the packet. Under the Block scheme, data packets are transmitted continuously; a NACK requires the retransmission of the packet detected with errors and all subsequent packets.

2.2.1 Acknowledgment Mechanisms

With multi-packet messages, the SW and Block schemes are further distinguished by using either packet-by-packet (PP) or message-by-message (MM) acknowledgment procedures. Packet-by-packet acknowledgment requires that each packet of a multi-packet message be

acknowledged individually. Message-by-message acknowledgment allows groups of packets (e.g., a message) to be acknowledged simultaneously.

The acknowledgment mechanism depends on the available acknowledgment feedback link between the source and destination stations. This feedback link can be provided by piggy-backing ACKs on the data traffic, or by dedicating a separate channel to acknowledgment traffic by either time or frequency multiplexing with data traffic. In general, different acknowledgment mechanisms will induce distinctly different response-time characteristics. In addition, the response-time delay (measured from data transmission time to acknowledgment time) can vary for different source-destination pairs. To include these effects a stochastic model for the acknowledgment delay is used.

The acknowledgment frame delay of the i-th message is denoted by K_i. It is measured from the time of data transmission to the time of its earliest possible retransmission (if needed). Thus, a transmission in the m-th frame is acknowledged within the subsequent K_i frames and, if necessary, retransmission is made in the $(m + K_i)$-th frame. The sequence $\{K_i, i \geq 1\}$ is assumed to be an i.i.d. sequence of random variables with discrete distribution such that the first three moments (k_1, k_2, k_3) are finite. Furthermore, acknowledgment transmissions are assumed to be errorless (sufficient coding is applied) so that acknowledgments are never lost. We also assume that time-out periods are never exceeded by PACK transmissions. Thus acknowledgments are always correctly returned within the acknowledgment delay specified

by $\{K_i, i \geq 1\}$.*

2.2.2 Stop-and-Wait Schemes

Under the preceding assumptions, the following three stop-and-wait (SW) schemes are considered.

(1) SW ARQ - PP ACK

The operation of the SW scheme using PP ACK is identical to the SW procedures described for the single-access communication link. Specifically, if the j-th packet of the i-th message is transmitted in the m-th TDMA frame, then in the $(m + K_i)$-th frame either the j-th packet is retransmitted (NACK received) or the next packet in the transmit queue (i.e., the (j+1)-st packet of the i-th message or the 1-st packet of the (i+1)-st message, if any such packet exists) is transmitted (PACK received); see Figure 2.1a.

(2) SW ARQ - MM ACK

The SW scheme using MM acknowledgment requires packets of a message to be transmitted in consecutive TDMA frames. If the last packet of the message is transmitted in the m-th frame, then message transmission stops until the $(m + K_i)$-th frame. By this frame, the i-th message transmission, which ended in the m-th frame, has been acknowledged. A PACK (all packets received correctly by the destination station) allows the next message transmission to begin. A NACK (when

*The reader is referred to Appendix A (Section A.4) for a study of positive acknowledgment with time-out in which PACKs may exceed the time-out period and be interpreted as NACKs.

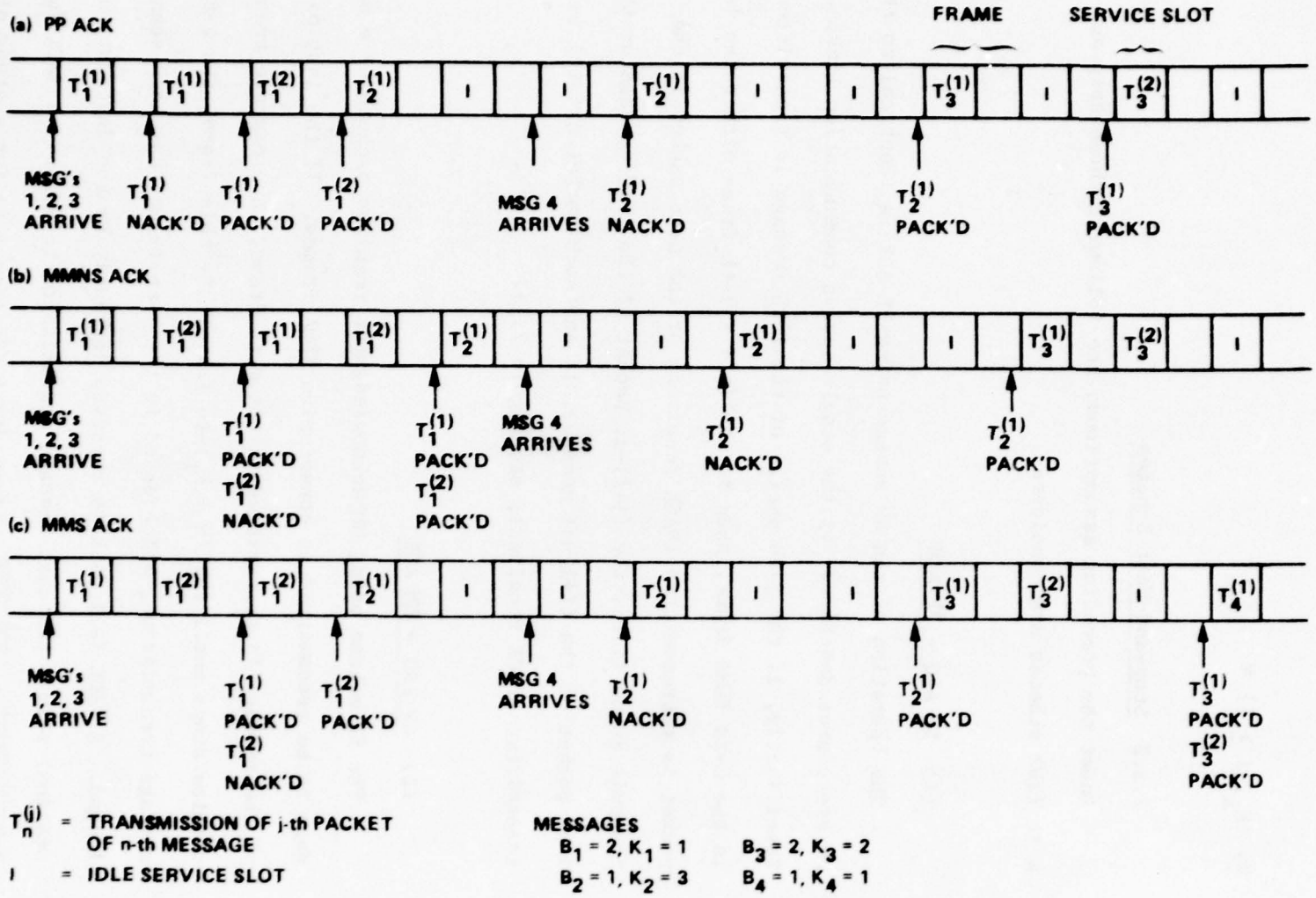


Figure 2.1. Operation of a TDMA Channel Using SW ARQ

one or more packets of the message have errors) induces several possible actions. The following two alternatives are considered:

NON-SELECTIVE (MMNS) - a NACK requires the retransmission of the entire message. A PACK is returned only if all packets in the current message transmission are received correctly. (See Figure 2.1b)

SELECTIVE (MMS) - a NACK requires the retransmission of only the packets that contain errors. If this pseudo-message of packets is subsequently NACK'd, the set of packets that contain errors from this pseudo-message is retransmitted. This cycle is repeated until a PACK is returned. (See Figure 2.1c)

These SW schemes allow only one outstanding unacknowledged message-unit transmission (i.e., a packet under PP ACK or a message under MM ACK). Assuming that messages are transmitted in the order of arrival, the oldest message in the transmit queue must be positively acknowledged before the next message begins transmission. Thus messages are removed from the transmit queue in the order of arrival. In addition, by using the PP or MMNS ACK schemes, the packets of a message are automatically received by the destination station in their proper sequence. The MMS ACK scheme, however, requires the destination station to buffer the errorless packets until all the packets of a multi-packet message are received correctly. Then the destination station must reassemble the packets to form the original message.

2.2.3 Block Schemes

Unlike the SW schemes, the Block schemes do not require packet transmissions to cease while waiting for acknowledgments. Transmissions are continuous as long as packets are waiting for transmission. Hence, more than one unacknowledged transmission can be outstanding. Since our model provides for random message sizes and random acknowledgment delay, transmissions can be acknowledged in an order which is different from their order of arrival. To simplify the Block ARQ analysis, the following assumptions are made (when Block schemes are considered):

(A) At the Source Station

The oldest outstanding transmission determines the operation of the Block protocol. Thus, its NACK requires its retransmission and the retransmission of all subsequent packets, regardless of their acknowledgment status.

(B) At the Destination Station

The destination stations acknowledge each transmission regardless of its past acknowledgment status.

The first assumption guarantees that messages are removed from the buffer of the source station in their order of arrival. By the second assumption, a transmission which has previously been positively acknowledged but which must be retransmitted, due to an older negatively acknowledged transmission, is viewed by the destination station as a new transmission. In other words, the past PACK information for such a

retransmitted message is disregarded.

Under these assumptions, the underlying station service system is modeled as a single-server queueing system where customers are messages and the server is the channel. In this analogous queueing system customers are served on a first-come, first-served basis, so that the message order of arrival is identical to its order of departure. Message delays calculated under these assumptions clearly serve as upper bounds for delays realized by Block ARQ systems which allow messages to be removed from the buffer immediately upon the reception of a PACK, or by systems which retain acknowledgment information.

Incorporating these assumptions, the following three Block schemes are considered:

(1) BLOCK ARQ - PP ACK

The operation of the Block ARQ scheme using PP ACK is identical to the Block ARQ described for the single-access link. Packet transmissions are made in contiguous service slots as long as the transmit queue holds packets waiting for transmission. Each packet transmission is acknowledged separately. If the j-th packet of the i-th message is transmitted in the m-th frame, its acknowledgment is received by the source station by the $(m + K_1)$-th frame. A NACK requires the retransmission of this packet in the $(m + K_i)$-th frame and the retransmission of subsequent packets in succeeding frames. (See Figure 2.2a)

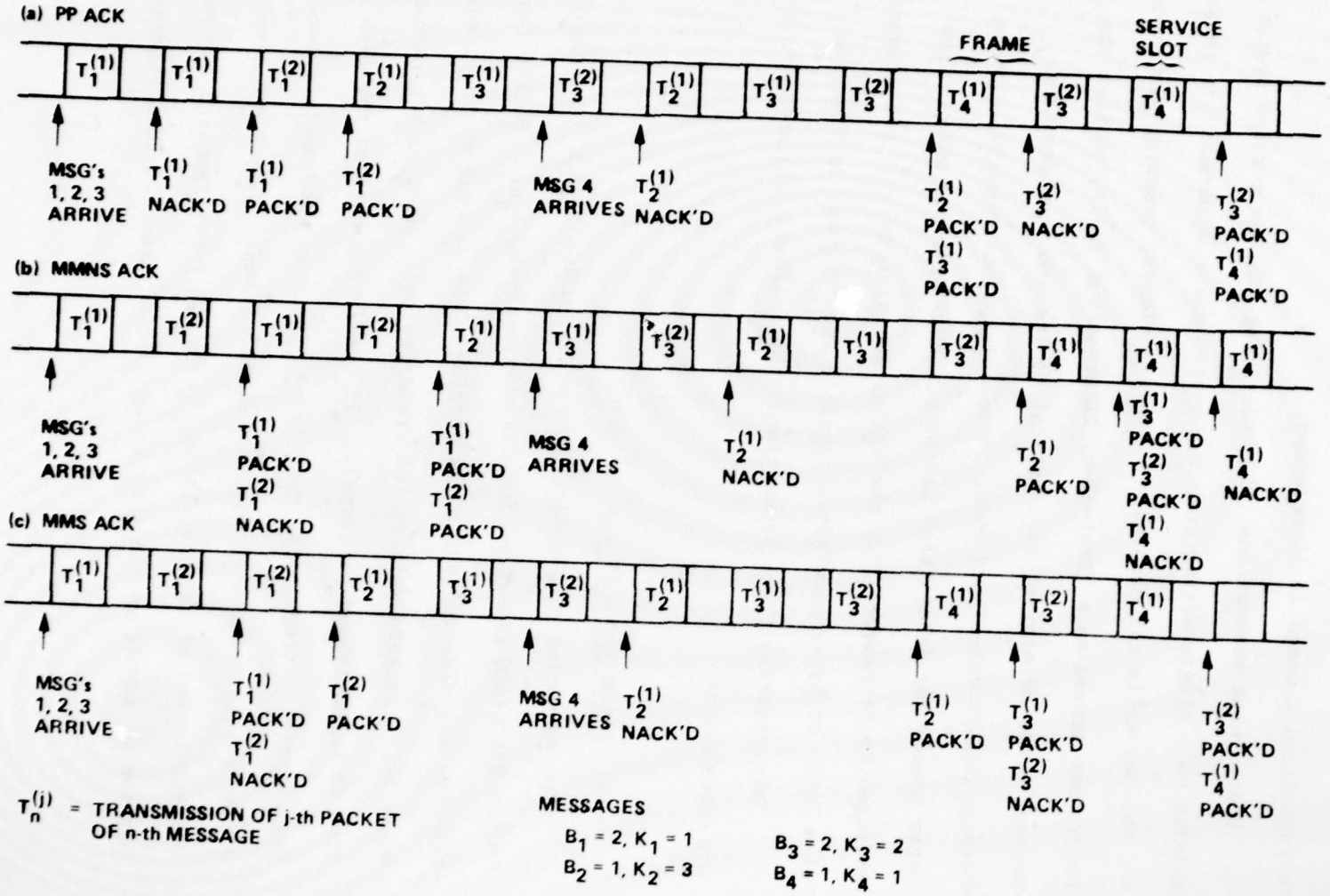


Figure 2.2. Operation of a TDMA Channel Using Block ARQ

(2) BLOCK ARQ - MM ACK

The Block ARQ protocol using MM ACK requires continuous packet transmission in contiguous frames as long as the transmit queue contains packets waiting for transmission. When the last packet of a message is received, the destination station acknowledges the message. A PACK allows the message to be removed from the buffer if it is the oldest message residing in the buffer. If this positively acknowledged message is not the oldest, then its PACK is stored until either

(a) it becomes the oldest and is, thereby, removed from the buffer, or

(b) an older message is negatively acknowledged, in which case the stored PACK is erased and the transmit-acknowledgment sequence for this message is renewed.

Similar to the SW ARQ - MM ACK schemes, a NACK can result in several possible actions. The message-by-message non-selective (MMNS) scheme (see Figure 2.2b) and the message-by-message selective (MMS) scheme (see Figure 2.2c) previously described for the SW ARQ are considered.

Thus, three SW ARQ schemes and three Block ARQ schemes are considered. It can be observed immediately that (1) when $K_i \equiv 1$, the SW and Block ARQ schemes are equivalent, and (2) when $B_i \equiv 1$, the PP and MM ACK schemes are equivalent.

2.3 Message Delay Analysis

Message delay (D_R) is measured (in slots) from the time of message arrival at the source station to the time when the entire message is correctly received by the destination station. For the Block ARQ

protocols under consideration, messages may be transmitted without errors more than once. In these situations, D_R is measured from the message time of arrival to the time of the successful reception of its last copy, following assumptions (A) and (B).

An alternate delay measure (D_S) represents the time period (measured in slots) from the message time of arrival at the source station to the time of its positive acknowledgment and removal from the source station's buffer. Thus, D_S is equal to the required holding time of a message in the station's buffer.

The difference $D_S - D_R$ between the two delay measures is clearly equal to the acknowledgment delay. The acknowledgment delay is the time required to acknowledge the last transmission and remove the corresponding message from the transmission buffer. Since K_i is the acknowledgment frame delay of the i-th message, the difference is bounded by $D_S - D_R \leq K_i M$. This acknowledgment delay can also be modeled by an additional random process related to K_i. Hence, only the delay measure D_R is examined.

The generating function of the steady-state message delay (D_R) distribution for each of the ARQ-ACK protocols under consideration is derived. The techniques employed closely follow those developed by Rubin [19].

The delay of the n-th message is decomposed into the sum

$$D_{R_n} = U_n + W_n + T_n + R \ , \qquad (2.1)$$

where U_n denotes the delay of the message from its arrival slot to the next service slot, W_n is the additional delay until the (n-1)-st message completes its last (errorless) transmission and the n-th message begins transmission, T_n is the effective message transmission time measured from its first time of transmission (following the last errorless transmission of the (n-1)-st message) to the time representing the end of its last errorless transmission, and R is the propagation delay from source to destination.

The variable U_n is characterized by the message arrival sequence $\{A_n, n \geq 1\}$. In particular, if $\{A_n\}$ is an i.i.d. sequence of random variables, then $\{U_n, n \geq 1\}$ is a sequence of i.i.d. uniform distributed random variables, $P(U_n = i) = 1/M$ for $i = 0, 1, 2, \ldots, M-1$ with mean $\frac{1}{2}(M-1)$ and variance $\frac{1}{12}(M^2 - 1)$. Assuming $\{U_n, n \geq 1\}$ is an i.i.d. sequence, designate its moment generating function by $U^*(z) = E(z^{U_n})$.

Furthermore, the message arrival process is assumed to be described by the sequence $\{N_n, n \geq 1\}$ where N_n expresses the number of messages arriving during the n-th frame. The sequence $\{N_n, n \geq 1\}$ is assumed to be a sequence of i.i.d. random variables with discrete distribution $\{P(N_n = j) = n_j, j = 0, 1, \ldots\}$, generating function $N^*(z)$, finite first, second and third moments (n_1, n_2, n_3) and variance σ_n^2.

Since each station is allocated a single transmission (or service) slot per frame, message arrivals can be characterized as group arrivals. The group arrival process is described by the stochastic jump process $\{(\tilde{t}_n, G_n), n \geq 1\}$. The random variables $\tilde{t}_n$ and G_n denote the time of

arrival and the size, respectively, of the n-th group. An arrival group contains messages that arrive within the same frame, provided that their number is positive.

The group arrival sequence $\{\tilde{t}_n, n \geq 1\}$ is a discrete time renewal point process with independent and geometric distributed inter-arrival times, $\{P(\tilde{t}_{n+1} - \tilde{t}_n = j) = \eta_o^{j-1}(1 - \eta_o), j = 1, 2, \ldots\}$. The group-size process $\{G_n, n \geq 1\}$ is a sequence of i.i.d. random variables, statistically independent of $\{\tilde{t}_n, n \geq 1\}$ with discrete distribution $\{P(G_n = j) = \eta_j(1 - \eta_o)^{-1}, j = 1, 2, \ldots\}$ and moments $g_i = E(G_n^i)$.

With this characterization of the message arrival sequence, the delay component W_n can be expressed as the sum

$$W_n = W_n^{(L)} + W_n^{(G)} \quad , \tag{2.2}$$

where $W_n^{(L)}$ is the waiting time of the group leader, which is the first message to be transmitted from among the messages in the group to which the n-th message belongs, and $W_n^{(G)}$ is the waiting time of the n-th message beyond that of its group leader.

The variable $W_n^{(L)}$ has the same distribution as that for the waiting time of a customer in a discrete-time single-server queueing system with bulk independent arrivals, described by the stochastic jump process $\{(\tilde{t}_n, G_n), n \geq 1\}$ and service times $\{S_n, n \geq 1\}$ where

$$S_n = \sum_{i=1}^{G_n} S_n^{(i)} \quad , n \geq 1 \quad . \tag{2.3}$$

The variable $S_n^{(i)}$ represents the completion time of the i-th message

arriving in the n-th group or, equivalently, it is the completion time of the ℓ-th arriving message where ℓ is related to n and i by

$$\ell = i + \sum_{j=1}^{n-1} G_j \tag{2.4}$$

for $n \geq 1$, $i \geq 1$. It is the time interval between the moment when the transmission of the message begins (after the preceding message has finished its last transmission) and the first moment thereafter (after its last transmission) when the channel becomes available to transmit the next message, expressed in frames. Expressions for the completion times of six ARQ-ACK protocols are presented in Table 2.1. The process $\{S_n^{(i)}, n \geq 1, i \geq 1\}$ is assumed to form an i.i.d. sequence, with steady-state generating function denoted by $s^*(z)$, finite first, second and third moments (s_1, s_2, s_3) and variance σ_s^2.

Results for such single-server queueing systems are well-known (see Rubin [19], Meisling [53]). The traffic intensity parameter ρ is equal to

$$\rho = s_1 n_1 \quad . \tag{2.5}$$

For $\rho \geq 1$, the queue size and the waiting time become arbitrarily large. For $\rho < 1$, the limiting waiting time distribution exists and its generating function

$$W_L^*(z) = E(z^{W_n^{(L)}})$$

is presented in the following lemma.

TABLE 2.1: PROTOCOL COMPLETION TIMES

	SW ARQ	Block ARQ
PP ACK	$S_n^{(i)} = K_\ell \sum_{j=1}^{B_\ell} R_{\ell j}$	$S_n^{(i)} = \sum_{j=1}^{B_\ell} [K_\ell (R_{\ell j} - 1) + 1]$
MMNS ACK	$S_n^{(i)} = [B_\ell + K_\ell - 1]\tilde{R}_\ell$	$S_n^{(i)} = B_\ell + [B_\ell + K_\ell - 1](\tilde{R}_\ell - 1)$
MMS ACK	$S_n^{(i)} = \sum_{j=0}^{\hat{R}_\ell - 1} [B_\ell^{(j)} + K_\ell - 1]$	$S_n^{(i)} = B_\ell^{(\hat{R}_\ell - 1)} \sum_{j=0}^{\hat{R}_\ell - 2} [B_\ell^{(j)} + K_\ell - 1]$

Lemma 2.1

$$W_L^*(z) = \frac{(1-\rho)(1-z^M)}{N^*(s^*(z^M)) - z^M} \qquad \begin{matrix} |z| < 1 \\ \rho < 1 \end{matrix} \qquad (2.6)$$

Proof

See Appendix A.

The limiting distribution of $W_n^{(G)}$ has been previously considered by Rubin [19], Towsley and Wolf [49], and Cohen [54]. This distribution always exists and its generating function is given in Lemma 2.2.

Lemma 2.2

$$W_G^*(z) = \frac{1 - N^*(s^*(z^M))}{n_1[1 - s^*(z^M)]} \qquad |z| < 1 \qquad (2.7)$$

Proof

See Appendix A.

The effective tranmission time of the n-th message is expressed in terms of its completion time and acknowledgment frame delay:

$$T_n = \begin{cases} (S_m^{(i)} - K_n)M + 1 & , \quad \text{for SW ARQ} \\ (S_m^{(i)} - 1)M + 1 & , \quad \text{for Block ARQ.} \end{cases} \qquad (2.8)$$

The quantities $(S_m^{(i)} - K_n)M$ and $(S_m^{(i)} - 1)M$ represent the number of slots between the n-th message's first and last packet transmissions for the SW and Block ARQ schemes, respectively. An additional slot is added to account for the last packet transmission. The generating function of T is denoted by $T^*(z)$.

Thus the results for the message delay given by (2.1) are summarized in Theorem 2.1.

Theorem 2.1

The generating function of the limiting message delay under a TDMA control discipline, which uses the SW ARQ or the Block ARQ error recovery procedures, is given by

$$D_R^*(z) = U^*(z)\, W_L^*(z)\, W_G^*(z)\, T^*(z)\, z^R \quad , \quad \text{for } \rho < 1 \quad . \tag{2.9}$$

The corresponding steady-state message-delay mean and variance are given by

$$E(D_R) = E(U) + \frac{M}{2(1-\rho)n_1}\,[n_1^2\sigma_s^2 + s_1\sigma_n^2 - \rho(1-\rho)] + E(T) + R \tag{2.10}$$

$$\begin{aligned} Var(D_R) = Var(U) + \frac{M^2}{12} + \frac{M^2}{3(1-\rho)}\,[s_1^3(n_3 - 3n_2 + 2n_1) + 3s_1s_2(n_2 - n_1) \\ + n_1s_3 - 1] + \frac{M^2}{4(1-\rho)^2}\,[n_2s_1^2 + n_1\sigma_s^2 - 1]^2 \\ + \frac{M^2}{2}\,\sigma_s^2\left(\frac{n_2}{n_1} - 1\right) \\ + \frac{s_1^2M^2}{12}\left[\frac{4n_3}{n_1} - 3\left(\frac{n_2}{n_1}\right)^2 - 1\right] + Var(T). \end{aligned} \tag{2.11}$$

Equations (2.9), (2.10) and (2.11) are valid for each of the six ARQ-ACK protocols. The distribution of $S_n^{(i)}$ is governed by the different statistics presented in Table 2.1. For $\rho \geq 1$, the message delay becomes arbitrarily large.

If the message arrival process $\{A_n, n \geq 1\}$ is assumed to be an i.i.d. sequence of Poisson distributed random variables with average arrival rate equal to λ messages per slot, the steady-state message-delay mean and variance formulas are given as follows.

Corollary 2.1

For a Poisson message arrival process, the message delay mean and variance, under a TDMA control discipline that uses the SW ARQ or the Block ARQ error recovery procedures, are given by

$$E(D_R) = \frac{M-1}{2} + \frac{M\rho s_2}{2(1-\rho)s_1} + E(T) + R \qquad (2.12)$$

$$\mathrm{Var}(D_R) = \frac{M^2(\rho^2+2)}{12} - \frac{1}{12} + \frac{M^2}{3(1-\rho)}\left[\rho^3 + 3\rho^2\frac{s_2}{s_1} + \rho\frac{s_3}{s_1} - 1\right]$$

$$+ \frac{M^2}{4(1-\rho)^2}\left[\rho^2 + \rho\frac{s_2}{s_1} - 1\right]^2 + \frac{M^2 s_2 \rho}{2s_1} + \mathrm{Var}(T) \,,$$

$$\text{for } \rho = \lambda M s_1 < 1. \qquad (2.13)$$

2.4 Evaluation of Completion Time

The concept of completion time was introduced in Section 2.3. The completion time of a message is the time interval measured in frames between the moment when the transmission of the message begins (after the preceding message has finished its last transmission) and the first moment thereafter (after its last transmission) when the channel becomes available to transmit the next message. As previously defined $S_n^{(i)}$ is the completion time of the i-th message arriving in the n-th group. Equivalently, it is the completion time of the ℓ-th arriving message.

The operational differences among the SW and Block ARQ systems are expressed through the completion time. The transmission policies (stop-and-wait versus continuous) and the acknowledgment procedures (packet-by-packet versus message-by-message) together with the retransmission schemes (non-selective versus selective) are mathematically defined by $S_n^{(i)}$. Thus the entries in Table 2.1 delineate the essential features of the six ARQ-ACK schemes described in Section 2.2.

To assess the impact of transmission errors on channel delay and throughput performance, the expressions for completion times in Table 2.1 are evaluated explicitly for a stationary transmission error process. Errors occur as independent events. With probability P_N, errors occur in a single packet transmission, and with probability $1-P_N$, no errors occur. Sufficient error detection capability is assumed such that the probability of an incorrect decision by the destination stations is negligible. Thus P_N is also the probability of a single packet request for retransmission.

Under SW ARQ - PP ACK, each packet of a multi-packet message must be individually acknowledged. Thus the j-th packet of the ℓ-message expends $K_\ell R_{\ell j}$ frames where $R_{\ell j}$ is the number of transmissions required by the j-th packet of the ℓ-message for positive acknowledgment. Hence, the completion time of the ℓ-th message is

$$S_n^{(i)} = K_\ell \sum_{j=1}^{B_\ell} R_{\ell j} \tag{2.14}$$

Under Block ARQ - PP ACK, each of the first $R_{\ell j}-1$ transmissions of the j-th packet expend K_ℓ frames, since a negative acknowledgment requires all packet transmissions made during the wait for acknowledgment to be retransmitted. The last transmission of the j-th packet uses a single service slot since Block ARQ is a continuous system. Hence, the number of service slots required by the j-th packet of the ℓ-th message is $K_\ell(R_{\ell j} - 1) + 1$ and the completion time of the ℓ-th message is

$$S_n^{(i)} = \sum_{j=1}^{B_\ell} [K_\ell(R_{\ell j} - 1) + 1] \tag{2.15}$$

For the stationary transmission error model, the variables $\{R_{\ell j}\}$ form an independent sequence of geometric distributed random variables, $\{P(R_{\ell j} = m) = (1 - P_N)P_N^{m-1}, m = 1, 2, \ldots\}$ with moments $r_i = E(R_{\ell j}^i)$. Since B_ℓ, K_ℓ and the $R_{\ell j}$'s are statistically independent, the completion times $\{S_n^{(i)}\}$ form an i.i.d. sequence.

Under MMNS ACK, all packets of a multi-packet message are transmitted before an acknowledgment is returned. Thus the acknowledgment

delay is experienced only once per message tranmission. If a message is negatively acknowledged (i.e., at least one packet of the multi-packet message is errant), then the entire message is retransmitted. Hence, under SW ARQ, each message transmission expends $B_\ell + K_\ell - 1$ frames. The completion time is given by

$$S_n^{(i)} = [B_\ell + K_\ell - 1]\tilde{R}_\ell \tag{2.16}$$

where $\tilde{R}_\ell$ is the number of message transmissions required by the ℓ-th message. Under Block ARQ, only the first $\tilde{R}_\ell - 1$ message transmissions expend the acknowledgment delay service slots; and, therefore, the completion time is given by

$$S_n^{(i)} = B_\ell + [B_\ell + K_\ell - 1](\tilde{R}_\ell - 1) \tag{2.17}$$

For the stationary transmission error model, the variables $\{\tilde{R}_\ell\}$ form an i.i.d. sequence. The conditional distribution given B_ℓ is geometric, $\{P(\tilde{R}_\ell = m | B_\ell) = (1 - P_N)^{B_\ell}[1 - (1 - P_N)^{B_\ell}]^{m-1}, m = 1, 2, \ldots\}$.

Under MMS ACK, acknowledgments are returned only after all packets of a message/pseudo message are transmitted. A negative acknowledgment requires the retransmission of only errant packets. These errant packets form a pseudo message. Under SW ARQ, each message/pseudo message transmission expends $B_\ell^{(j)} + K_\ell - 1$ service slots where $B_\ell^{(j)}$ is the number of packets in the j-th pseudo message with $B_\ell^{(0)} \triangleq B_\ell$. Hence, the completion time of the ℓ-th message is

$$S_n^{(i)} = \sum_{j=0}^{\hat{R}_\ell - 1} [B_\ell^{(j)} + K_\ell - 1] \tag{2.18}$$

where $\hat{R}_\ell$ is the number of message/pseudo message transmissions required. The variable $\hat{R}_\ell$ can be expressed as the infinite sum of indicator functions:

$$\hat{R}_\ell = \sum_{j=0}^{\infty} I(B_\ell^{(0)} > 0, B_\ell^{(1)} > 0, \ldots, B_\ell^{(j)} > 0) \tag{2.19}$$

where

$$I(x) = \begin{cases} 1 & x \text{ true} \\ 0 & \text{otherwise} \end{cases}$$

Using (2.19) in (2.18), the completion time for SW ARQ - MMS ACK can be rewritten as

$$S_n^{(i)} = \sum_{j=0}^{\infty} [B_\ell^{(j)} + K_\ell - 1] I(B_\ell^{(0)} > 0, B_\ell^{(1)} > 0, \ldots, B_\ell^{(j)} > 0) \tag{2.20}$$

For the stationary transmission error model, the conditional distribution of $B_\ell^{(j+1)}$ given $B_\ell^{(j)}$ is binomial

$$P(B_\ell^{(j+1)} = n | B_\ell^{(j)} = m) = \binom{m}{n} P_N^n (1 - P_N)^{m-n} \tag{2.21}$$

$$j = 0, 1, 2, \ldots$$
$$n = 0, 1, \ldots, m$$

Under this characterization, the conditional expectation of $\hat{R}_\ell$ given B_ℓ is

$$E(\hat{R}_\ell | B_\ell) = \sum_{j=0}^{\infty} [1 - (1 - P_N^j)^{B_\ell}] \tag{2.22}$$

Under Block ARQ - MMS ACK, the j-th message/pseudo message transmission expends $B_\ell^{(j)} + K_\ell - 1$ service slots for $j = 0, 1, 2, \ldots, \hat{R}_\ell - 2$. The last message/pseudo message transmission uses only the $B_\ell^{(\hat{R}_\ell - 1)}$ service slots. Thus the completion time is given by

$$S_n^{(i)} = B_\ell^{(\hat{R}_\ell - 1)} + \sum_{j=0}^{\hat{R}_\ell - 2} [B_\ell^{(j)} + K_\ell - 1] \tag{2.23}$$

Using indicator functions, (2.23) can be rewritten as

$$S_n^{(i)} = \sum_{j=0}^{\infty} [B_\ell^{(j)} + (K_\ell - 1)\, I(B_\ell^{(j+1)} > 0)]$$

$$\cdot\, I(B_\ell^{(0)} > 0,\ B_\ell^{(1)} > 0,\ \ldots,\ B_\ell^{(j)} > 0) \tag{2.24}$$

For the stationary transmission error model, the conditional distribution of $B_\ell^{(j+1)}$ given $B_\ell^{(j)}$ is governed by (2.21).

The conditional expectations of completion time given B_ℓ are summarized in Table 2.2 for the six ARQ-ACK schemes. Maximum throughput (b_1/s_1 packets per service slot) is readily calculated from the table entries. The 2nd and 3rd moments of completion time are presented in Appendix A; these moments are necessary to evaluate the mean and variance of message delay given by (2.10) and (2.11) or (2.12) and (2.13).

TABLE 2.2: MEAN COMPLETION TIMES $E(S_n^{(i)} | B_\ell)$ FOR THE STATIONARY TRANSMISSION ERROR PROCESS

	SW ARQ	Block ARQ
PP ACK	$\frac{k_1 B_\ell}{1 - P_N}$	$B_\ell [\frac{k_1 P_N}{1 - P_N} + 1]$
MMNS ACK	$\frac{B_\ell + k_1 - 1}{(1 - P_N)^{B_\ell}}$	$\frac{B_\ell + (k_1 - 1)[1 - (1 - P_N)^{B_\ell}]}{(1 - P_N)^{B_\ell}}$
MMS ACK	$\frac{B_\ell}{1 - P_N} + (k_1 - 1) \sum_{m=0}^{\infty} [1-(1-P_N^m)^{B_\ell}]$	$\frac{B_\ell}{1 - P_N} + (k_1 - 1) \sum_{m=1}^{\infty} [1-(1-P_N^m)^{B_\ell}]$

2.5 Numerical Results

The delay and throughput performance of the ARQ-ACK schemes are compared assuming Poisson distributed message arrivals. The average message delay is normalized by the TDMA frame size (M). Resulting terms which are proportional in value to 1/M have been neglected. (These terms are negligible for reasonably large M.) The additive fixed propagation delay is set equal to zero ($R \equiv 0$). Results for the stationary transmission error process are presented.

Figures 2.3, 2.4 and 2.5 present throughput and delay results for the SW and Block ARQ systems operating with single-packet messages ($B_i = 1$) and with $K_i = 1, 2, 5$. For single-packet messages the PP ACK and MM ACK schemes are equivalent. In addition, with $K_i = 1$, the SW and Block ARQ systems operate identically.

Maximum throughput versus packet noise error probability curves are shown in Figure 2.3. It is evident that the largest maximum throughput values are provided by $K_i = 1$. As K_i increases the maximum throughput achievable decreases. For each $K_i > 1$, the Block ARQ system performs better than the SW ARQ system. This performance disparity is especially evident at low packet noise error probabilities. The forced idle times required by the SW ARQ system reduce the maximum throughput at $P_N = 0.0$ to 0.5 and 0.2 packets per service slot for $K_i = 2$ and $K_i = 5$, respectively; since the Block ARQ system is a continuous system, it has the maximum throughput value of 1 packet per service slot at $P_N = 0.0$ for all K_i. Average packet delay versus throughput curves are shown in Figure 2.4 with $P_N = 0.2$. The normal delay-throughput behavior is exhibited. Packet delay standard deviation-to-mean (S/M)

ratio versus throughput curves are shown in Figure 2.5. As throughput increases, the mean and variance of delay increase such that the S/M ratio is 1 at maximum throughput.

Figures 2.6, 2.7 and 2.8 present throughput and delay results for the SW and Block systems operating with multi-packet messages ($B_i = 5$) and with $K_i = 2$. These results exhibit the differences among the PP, MMNS and MMS ACK schemes. Maximum throughput versus packet noise error probability curves are shown in Figure 2.6. As found for single-packet messages, the Block ARQ system provides larger maximum throughput values than the SW ARQ system for each acknowledgment scheme. Also for both ARQ systems, the MMS ACK scheme achieves the largest maximum throughput values. This result is expected since the MMS ACK scheme requires fewer packet retransmissions than the MMNS ACK scheme and experiences fewer acknowledgments delays than the PP ACK scheme.

Average message delay and message delay S/M ratio results are shown in Figures 2.7 and 2.8, respectively. The best delay-throughput performance is exhibited by the MMS ACK scheme followed in order by PP ACK and MMNS ACK for both the SW and Block ARQ systems. The advantage of the MMS ACK scheme increases with message size. The performance advantage of the Block ARQ systems over the SW ARQ systems is clearly demonstrated (particularly for PP ACK). This disparity increases as the acknowledgment delay is increased. The message delay S/M ratio versus throughput curves in Figure 2.8 indicate that the Block ARQ - MMS ACK scheme provides the best performance over most

of the throughput range. Hence, the Block ARQ - MMS ACK scheme exhibits the best message delay characteristics from among the SW and Block ARQ systems.

The performance loss sustained by the Block ARQ - MMS ACK scheme under non-zero error rates is indicated in Figures 2.9 and 2.10 for single-packet messages and in Figures 2.11 and 2.12 for multi-packet messages ($B_i = 5$). Average message delay versus packet noise error probability curves are shown in Figures 2.9 and 2.11 for several throughput values with $K_i = 1, 2$. Message delay S/M ratio versus packet noise error probability curves are shown in Figures 2.10 and 2.12. It is evident that the number of packet retransmissions increases with P_N which consequently increases delay. The effect of these retransmissions is magnified by increasing the acknowledgment delay.

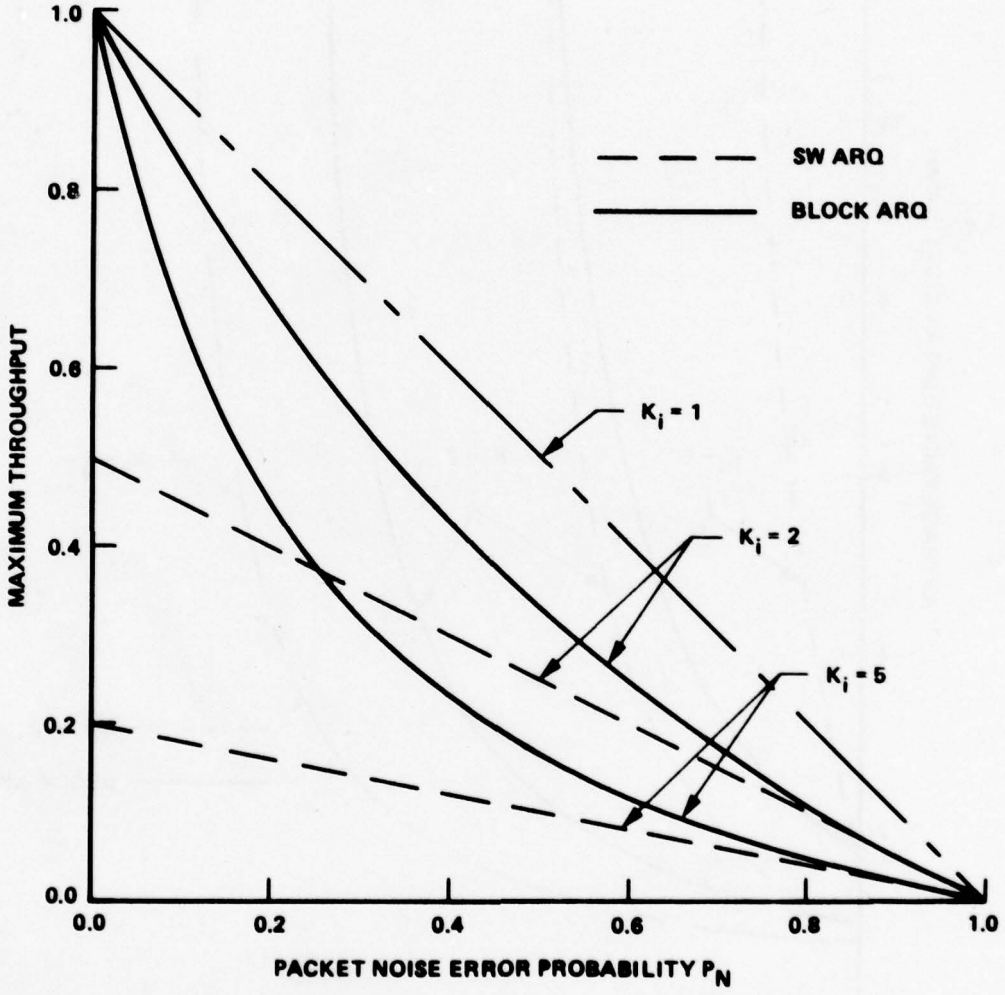


Figure 2.3. Maximum Throughput versus Packet Noise Error Probability Curves for a TDMA Channel Using SW or Block ARQ with $B_i = 1$, $K_i = 1, 2, 5$

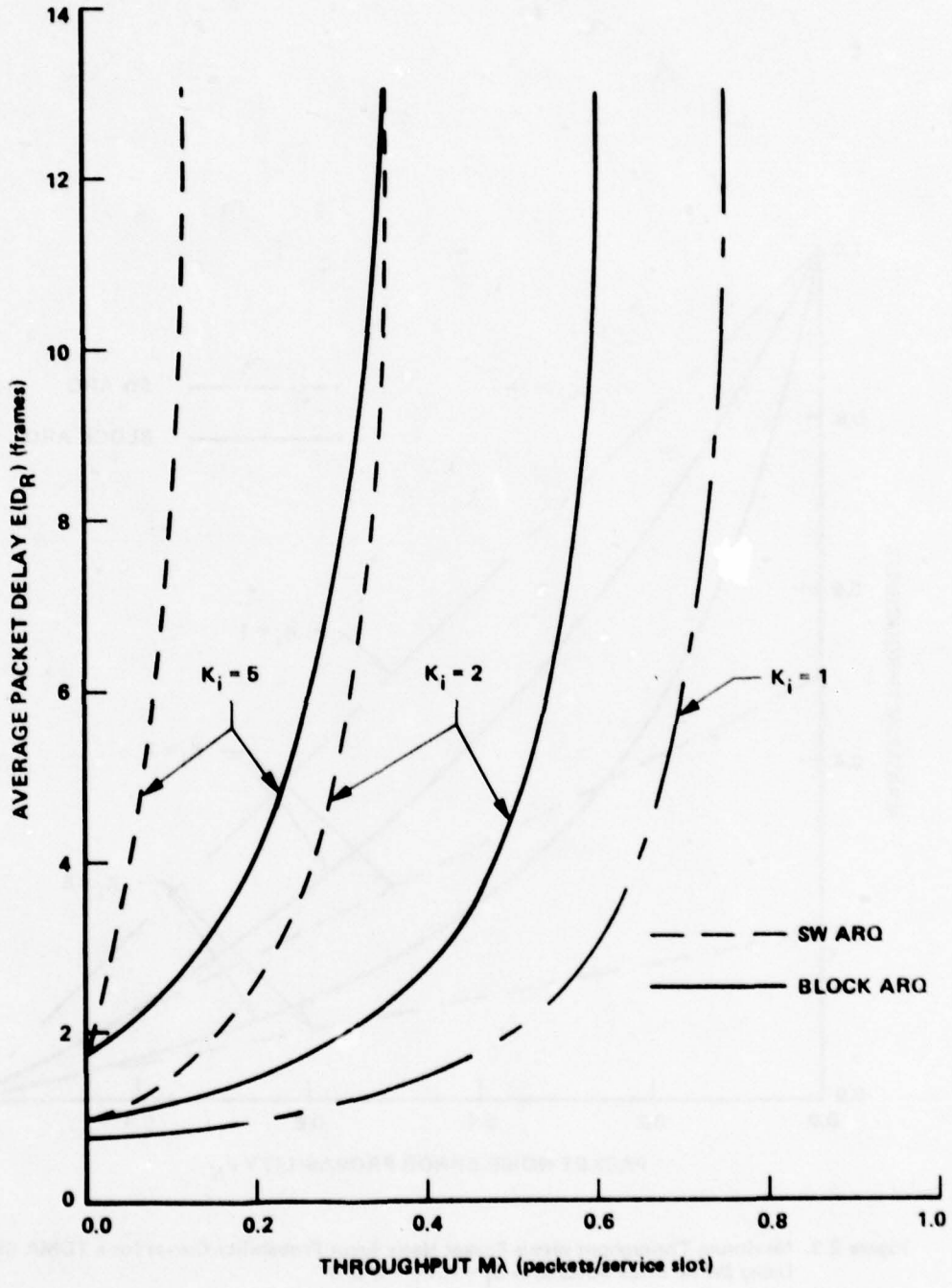


Figure 2.4. Delay versus Throughput Curves for a TDMA Channel Using SW or Block ARQ with $B_i = 1$, $K_i = 1, 2, 5$, $P_N = 0.2$

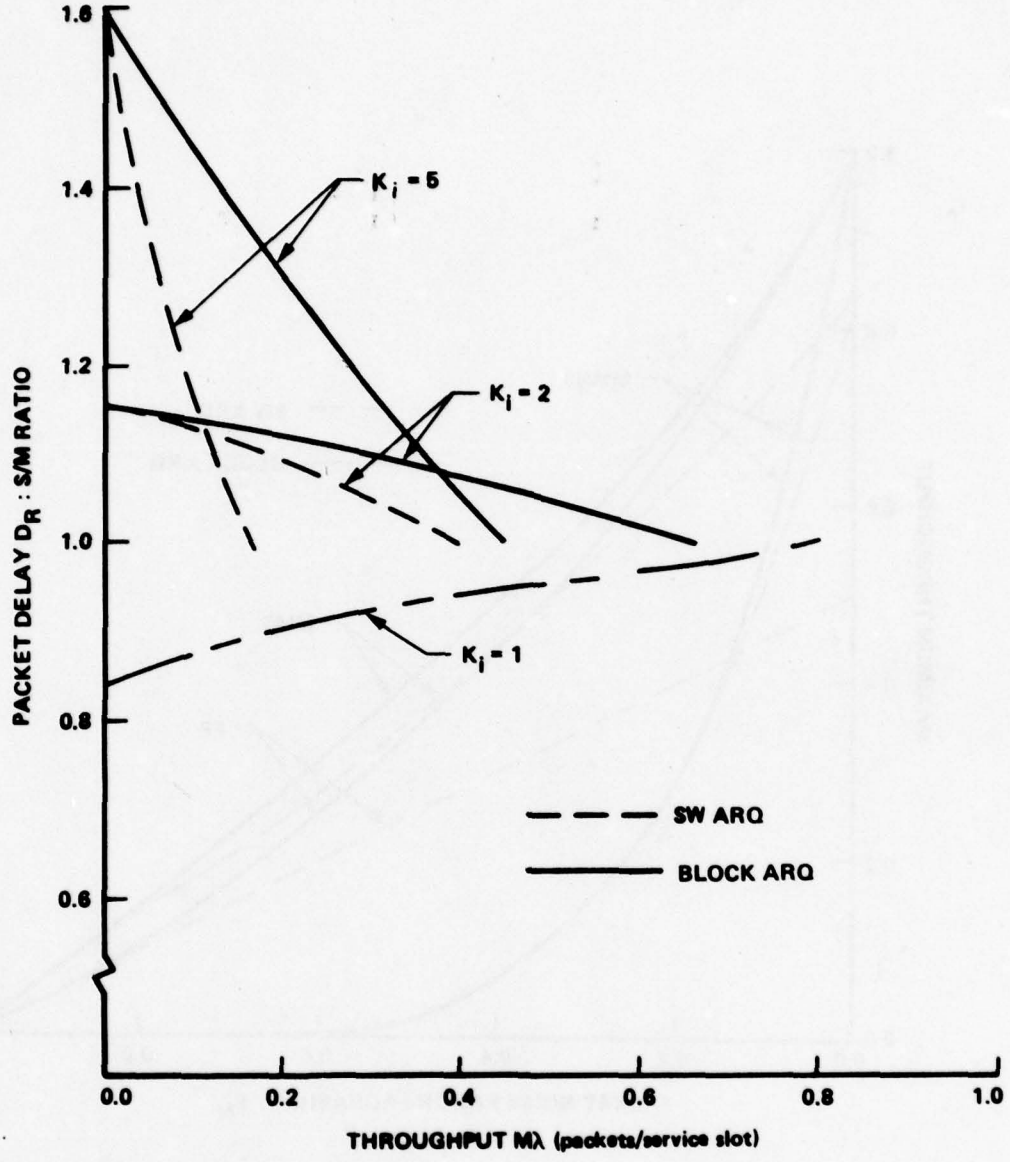


Figure 2.5. Packet Delay Standard Deviation-to-Mean Ratio versus Throughput Curves for a TDMA Channel Using SW or Block ARQ with B_i = 1, K_i = 1, 2, 5, P_N = 0.2

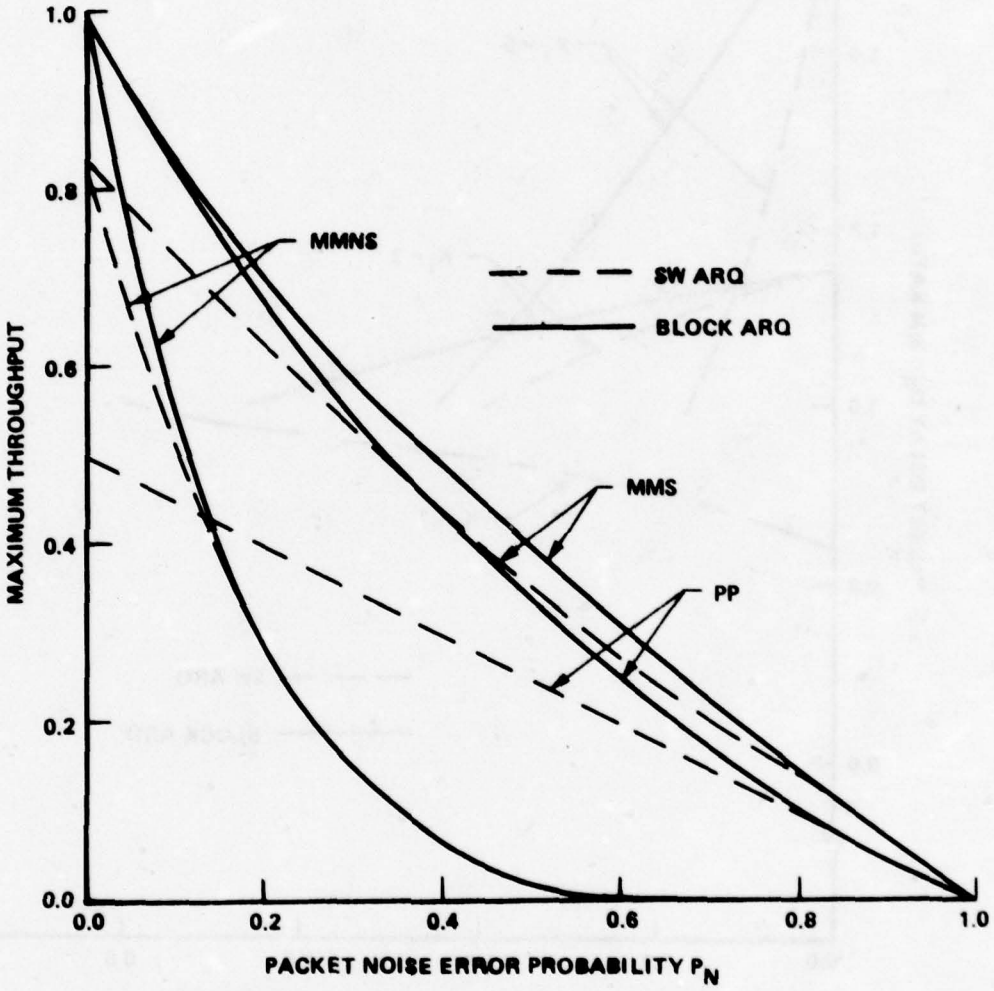


Figure 2.6. Maximum Throughput versus Packet Noise Error Probability Curves for a TDMA Channel Using SW or Block ARQ and PP, MMNS or MMS ACK with $B_i = 5$, $K_i = 2$

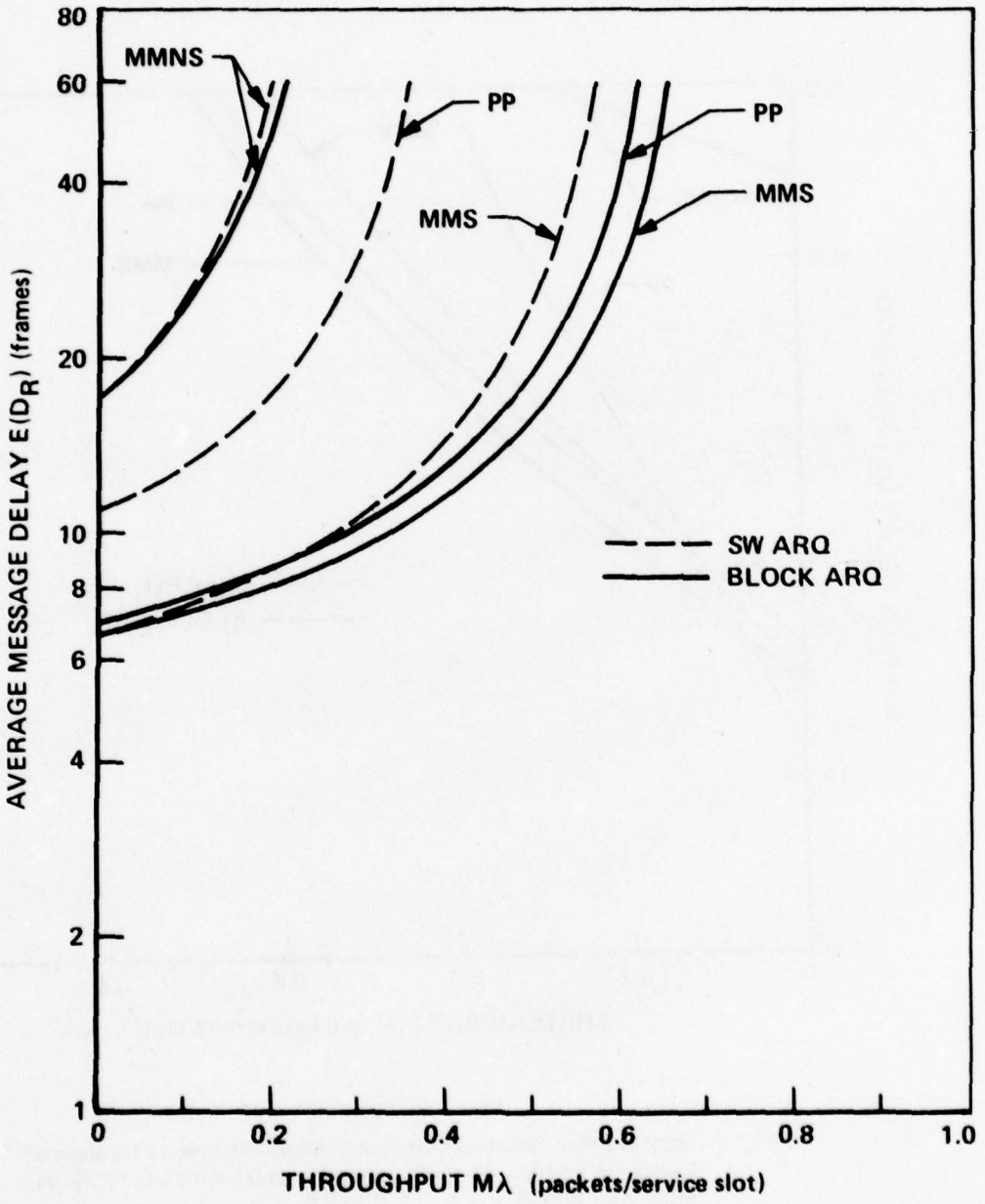


Figure 2.7. Delay versus Throughput Curves for a TDMA Channel Using SW or Block ARQ and PP, MMNS, MMS ACK With $B_i = 5$, $K_i = 2$, $P_N = 0.2$

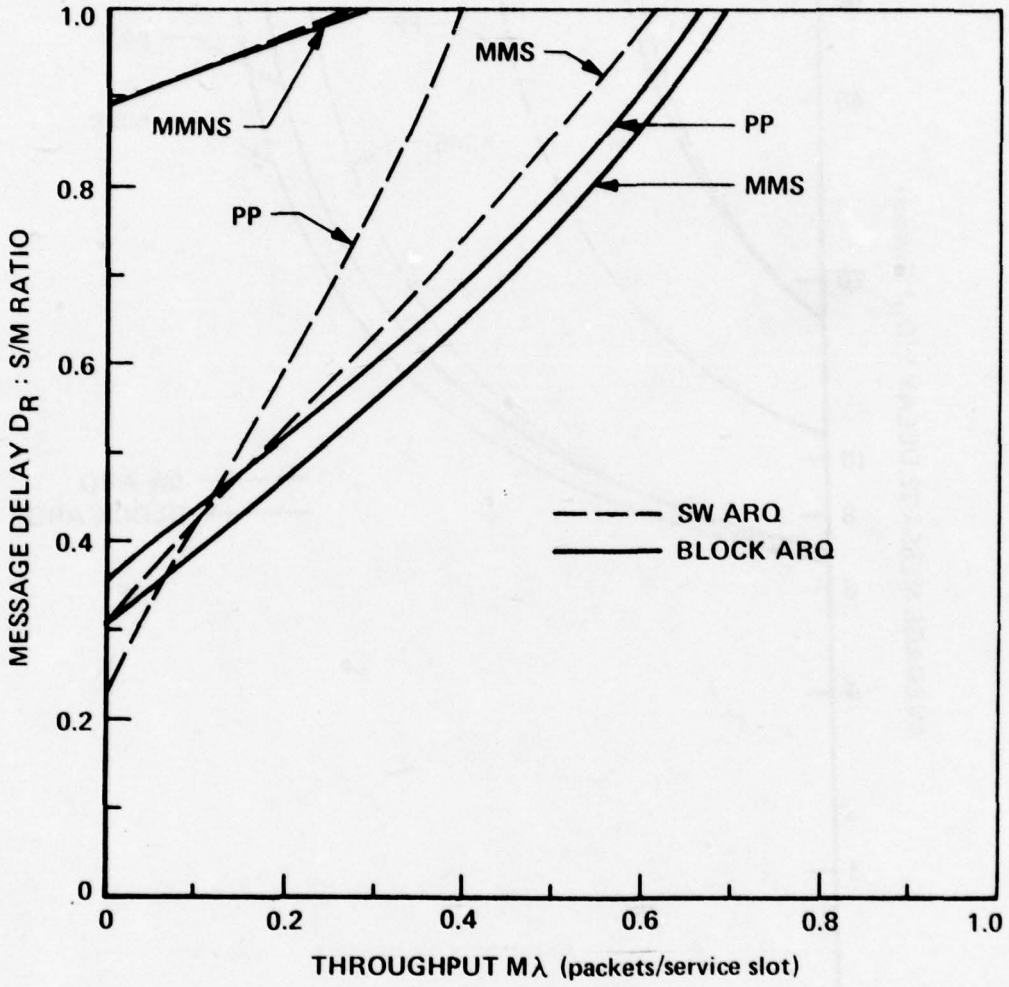


Figure 2.8. Message Delay Standard Deviation-to-Mean Ratio versus Throughput Curves for a TDMA Channel Using SW or Block ARQ and PP, MMNS, MMS ACK With $B_i = 5$, $K_i = 2$, $P_N = 0.2$

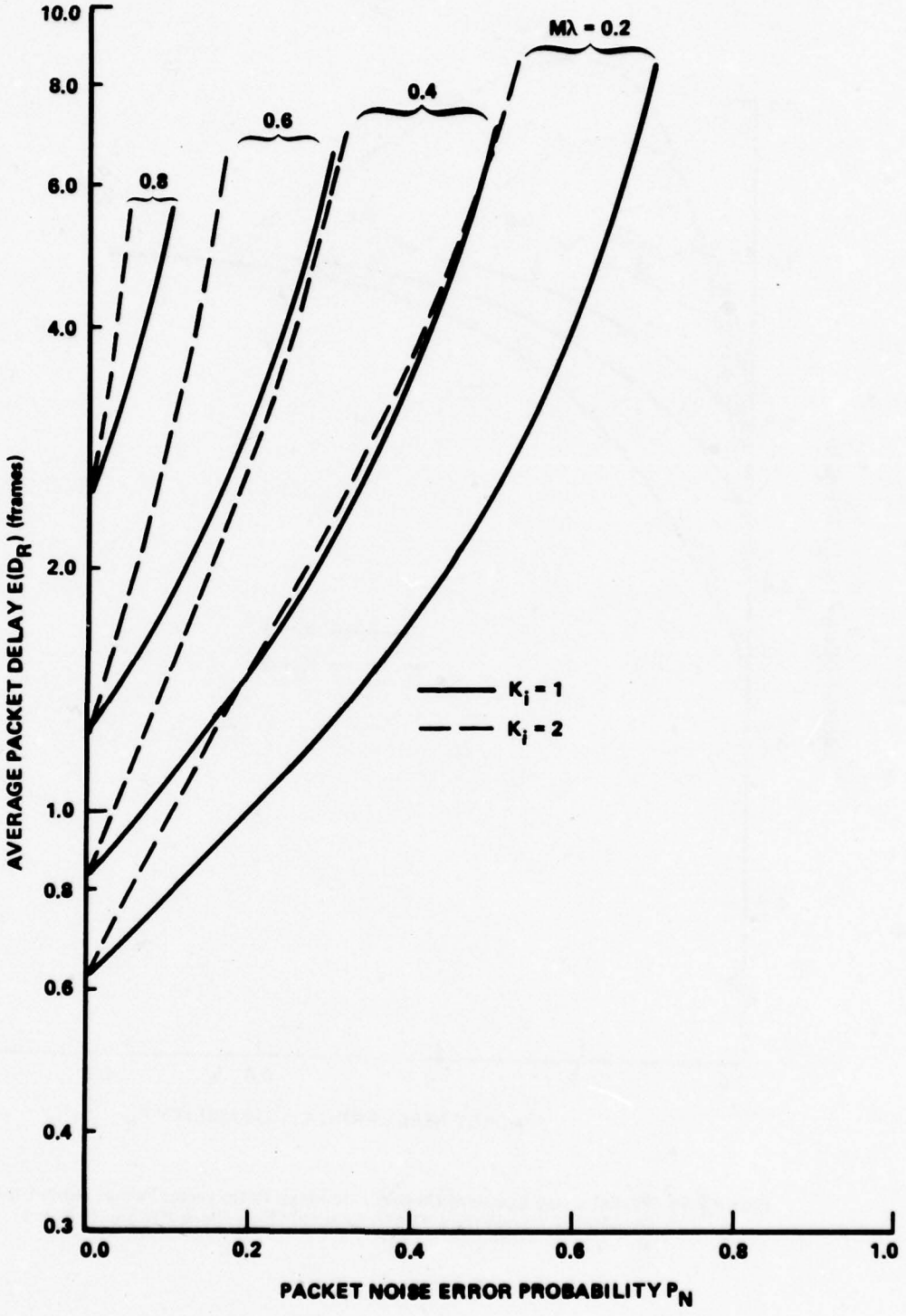


Figure 2.9. Packet Delay versus Packet Noise Error Probability Curves for a TDMA Channel Using Block ARQ with $B_i = 1$, $K_i = 1, 2$

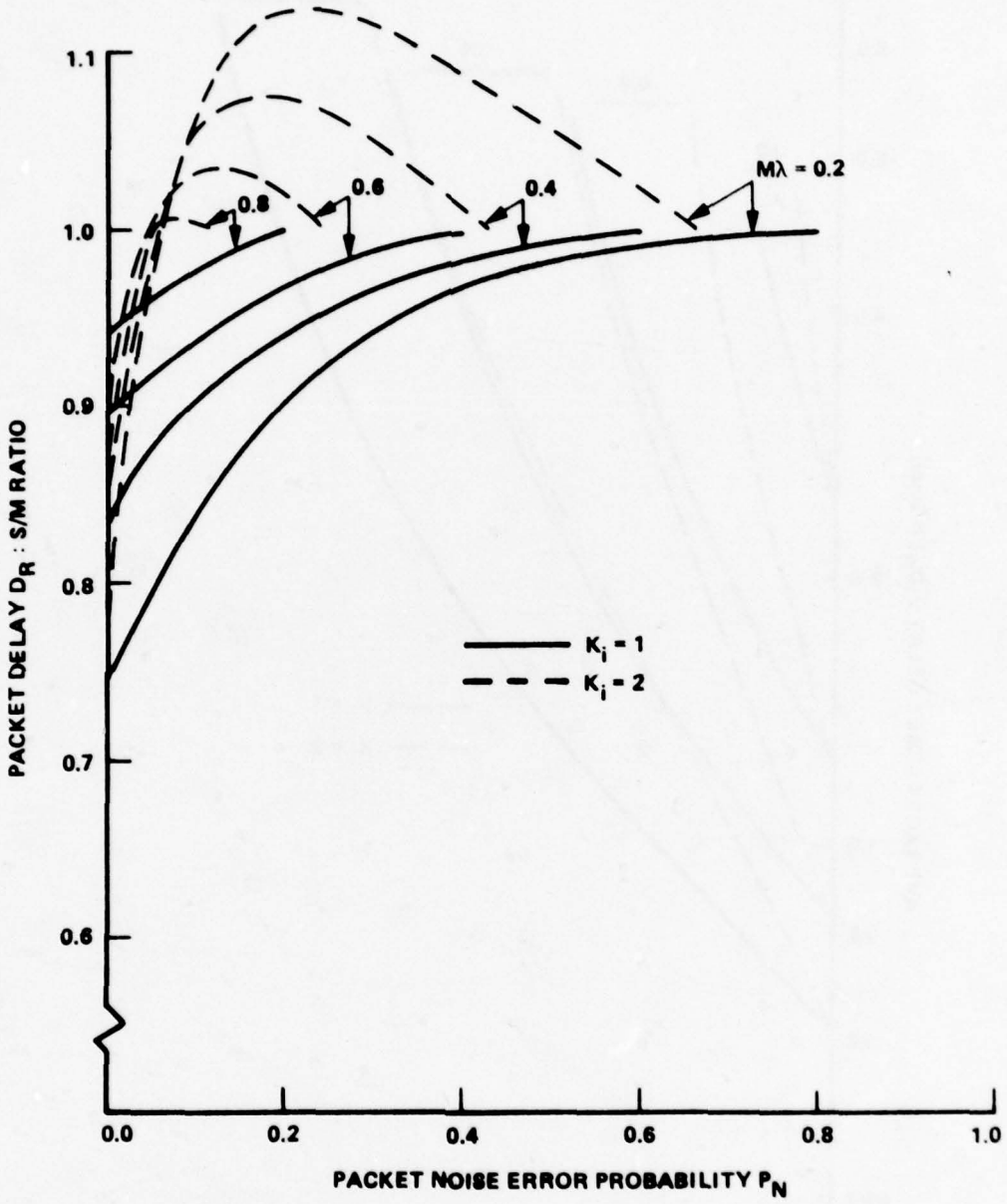


Figure 2.10. Packet Delay Standard Deviation-to-Mean Ratio versus Packet Noise Error Probability Curves for a TDMA Channel Using Block ARQ with $B_i = 1$, $K_i = 1, 2$

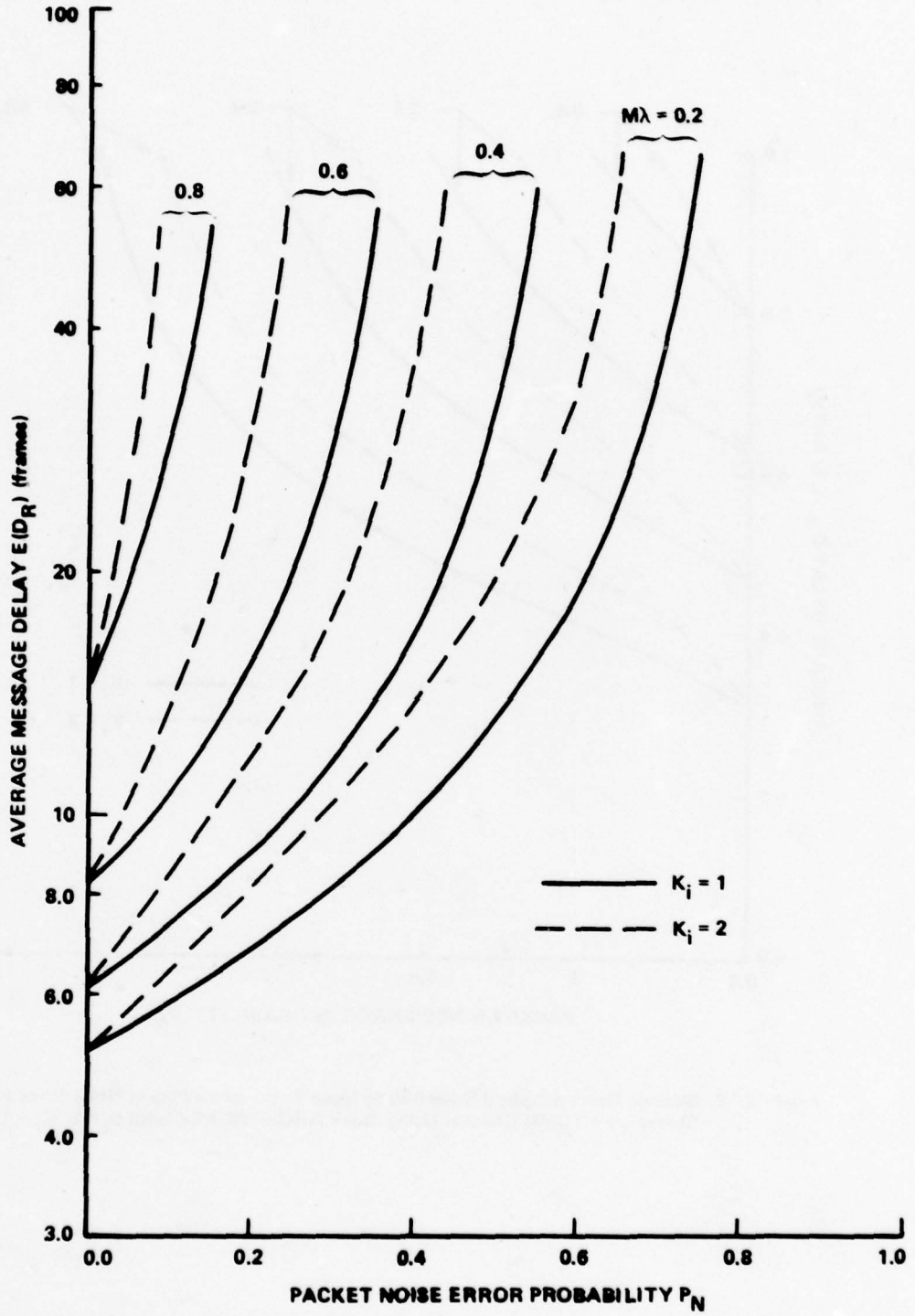


Figure 2.11. Message Delay versus Packet Noise Error Probability Curves for a TDMA Channel Using Block ARQ-MMS ACK with $B_i = 5$, $K_i = 1, 2$

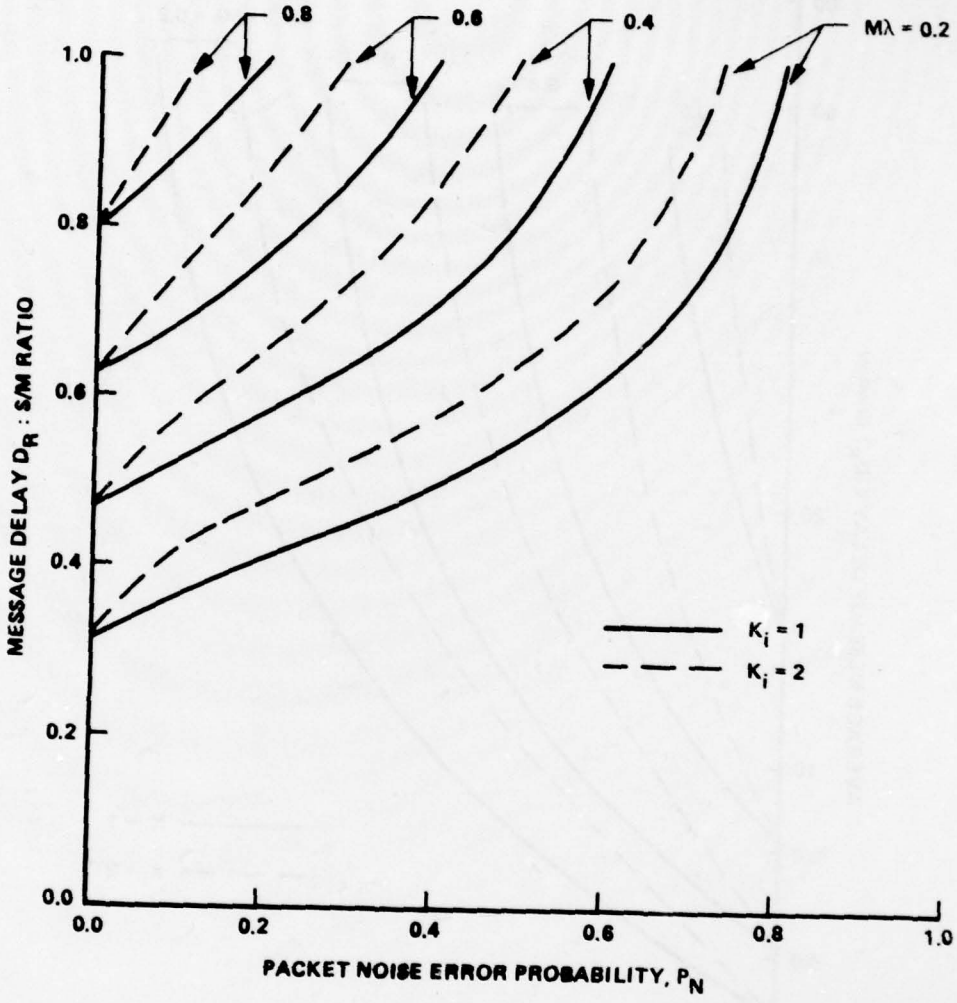


Figure 2.12. Message Delay Standard Deviation-to-Mean Ratio versus Packet Noise Error Probability Curves for a TDMA Channel Using Block ARQ-MMS ACK with $B_i = 5$, $K_i = 1, 2$

2.6 Conclusions

The operation of a TDMA channel using SW and Block ARQ error recovery procedures was examined in this chapter. Acknowledgment mechanisms on a per packet and on a per message basis were considered. By introducing the concept of completion time, the generating function of the message delay distribution at steady state was derived and expressions for the message delay mean and variance were obtained. The completion times for each ARQ - ACK scheme were evaluated for a stationary tranmission error process. Examples of the delay-throughput function were presented assuming a Poisson distributed message arrival process. From the examples, it was observed that the Block ARQ - MMS ACK scheme yields the best message delay characteristics.

CHAPTER III

TIME DIVISION MULTIPLE ACCESS USING THE SELECT-AND-REPEAT ARQ SYSTEM

In this chapter the SR ARQ system is applied to a TDMA channel. Like the Block ARQ schemes examined in Chapter II, the SR ARQ scheme is a continuous system; however, the SR ARQ strategy requires only errant packet transmissions to be retransmitted. Thus the SR ARQ system avoids the unnecessary retransmissions of the Block ARQ schemes and the wasteful idle periods of the SW ARQ schemes. In Section 3.1 the channel and network stations are characterized and the operation of the SR ARQ scheme is described. The evolution of the channel is described by a vector Markov chain in Section 3.2. The necessary and sufficient conditions for ergodicity are stated. In Section 3.3 upper and lower bounds on the average message delay at steady state are derived, and numerical examples are presented in Section 3.4.

3.1 Operational Description

The time synchronized channel structure described in Chapter II is also considered in this chapter. Time is divided into equal length slots. The n-th slot is the interval $[(n-1)\tau, n\tau)$, $n = 1,2,\ldots$. The M network stations share the channel in accordance with a TDMA access-control discipline. Each station is assigned a single slot per frame (a service slot) and the queueing behavior of one network station is examined.

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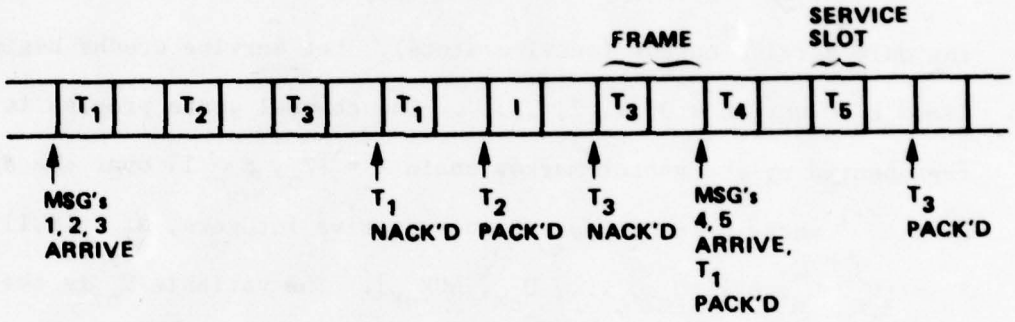
Single-packet messages are assumed with the transmission time of a packet set equal to the duration of a single slot (τ). Messages arrive at the station according to the batch stochastic point process $\{A_n, n \geq 1\}$, where A_n denotes the number of message arrivals during the n-th slot.

Under the SR ARQ strategy, packets are transmitted in contiguous service slots as long as the transmit queue is non-empty (a continuous ARQ system). However, unlike the Block ARQ systems, only errant packets are retransmitted. Retransmissions are made in the service slots immediately following negative acknowledgments.

An ideal acknowledgment mechanism is assumed. Acknowledgments are never misinterpreted and are returned after a fixed acknowledgment delay (K frames). Thus a packet transmission in the m-th frame is acknowledged within K frames and, if necessary, the packet is retransmitted in the m+K)-th frame. (See Figure 3.1).

The operation of this TDMA channel using SR ARQ is conveniently described by visualizing two separate buffers at the source station: a transmit buffer and a transit buffer. The infinite capacity transmit buffer holds those messages waiting for their initial transmission. The transit buffer holds those packets waiting for acknowledgment or retransmission; hence, there can exist at most K such packets in the transit buffer.

Arriving messages are placed in the transmit queue. Prior to each service slot, the transit buffer is interrogated for a packet ready for retransmission. If such a packet exists, it is transmitted in the immediate service slot; if no such packet exists, then the



T_j = TRANSMISSION OF j-th MESSAGE

I = IDLE SERVICE SLOT

K = 3

Figure 3.1. Operation of a TDMA Channel Using SR ARQ

packet at the head of the transmit queue is removed, transmitted and placed in the transit buffer. However, if the transmit buffer is empty, the service slot goes idle. Packets are removed from the transit buffer as they are positively acknowledged.

3.2 Channel State Process

The evolution of the channel is completely described by considering only service epochs (service slots). Let service epochs begin at times $t_n = nM\tau$, $n = 0, 1, 2, \ldots$. The channel state process is represented by the vector Markov chain $Z = \{Z_n, n \geq 1\}$ over the space $d^2 \times d_1^{K+1}$ where d is the set of non-negative integers, $d_1 = \{0,1\}$ and $Z_n = \{N_n, Q_n^-, U_{n1}, U_{n2}, \ldots, U_{nK}, ACK_{nK}\}$. The variable N_n is the number of new message arrivals between the (n-1)-st and n-th service epochs. The transmit queue size (those packets waiting for their first transmission) at $t_n^- = nM\tau - 0$ is denoted by Q_n^-. The variables $\{U_{ni}, i = 1, 2, \ldots, K\}$ indicate packet transmissions in the (n-i+1)-st service slots:

$$U_{ni} = \begin{cases} 1 & \text{if a packet is transmitted/retransmitted in the (n-i+1)-st service slot} \\ 0 & \text{otherwise} \end{cases} \qquad (3.1)$$

The variable ACK_{nK} indicates the acknowledgment status of the transmission made in the (n-K+1)-st service slot:

$$ACK_{nK} = \begin{cases} 1 & \text{if a packet transmitted/retransmitted in the } (n-K+1)\text{-st service slot is PACK'd} \\ 0 & \text{otherwise} \end{cases} \tag{3.2}$$

The message arrival sequences $\{A_n, n \geq 1\}$ and $\{N_n, n \geq 1\}$ are related by

$$N_n = \sum_{i=(n-1)M+1}^{nM} A_i \tag{3.3}$$

The process $\{N_n, n \geq 1\}$ is assumed to be an i.i.d. sequence of random variables with finite first and second moments (n_1, n_2), variance σ_n^2 and generating function $E(z^{N_n}) = N^*(z)$.

The transmit queue size is governed by the following recursive relationship (see Figure 3.2):

$$Q_{n+1}^- = N_{n+1} + [Q_n^- - I(\epsilon_n)]^+ \tag{3.4}$$

where $[x]^+ = \max(0,x)$

$$I(x) = \begin{cases} 1 & x \text{ true} \\ 0 & \text{otherwise} \end{cases}$$

$$\epsilon_n = \{U_{nK} = 1, ACK_{nK} = 1) \text{ or } (U_{nK} = 0)\}.$$

If the event ϵ_n is true, a new packet transmission may begin in the (n+1)-st service slot and the transmit queue size is reduced by 1. A transmission is made in the (n+1)-st service slot if either (1) a

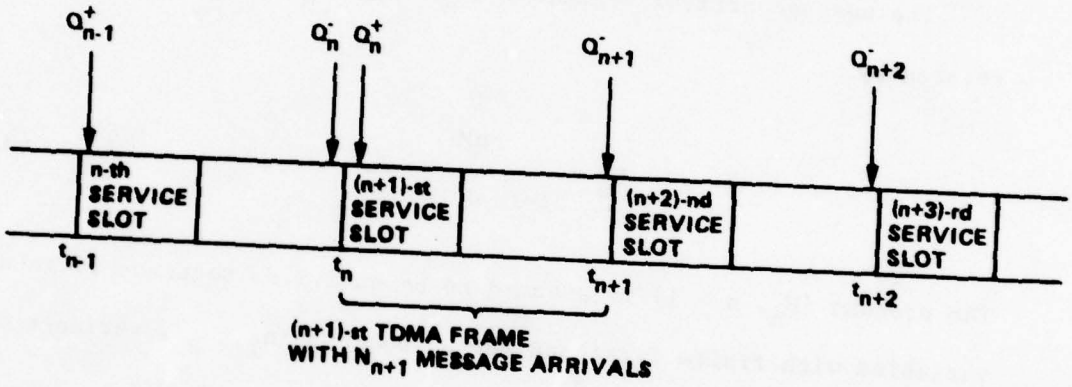


Figure 3.2. Transmit Queue Size Relationships

transmission in the (n-K+1)-st service slot is negatively acknowledged or (2) the transmit queue is non-empty at t_n^- (i.e., $Q_n^- > 0$). Hence,

$$U_{n+1,1} = I(\{U_{nK} = 1,\ ACK_{nK} = 0\} \text{ or } \{Q_n^- > 0\}) \tag{3.5}$$

The stationary transmission error process introduced in Chapter II is assumed. Errors occur as independent events. With probability P_N, an error occurs in a single packet transmission, and with probability $(1 - P_N)$, no errors occur. Thus, the process $\{ACK_{nK}\}$ is an i.i.d. sequence of Bernoulli distributed random variables:

$$P(ACK_{nK} = j \mid U_{nK} = 1) = \begin{cases} P_N & \text{if } j = 0 \\ 1 - P_N & \text{if } j = 1 \end{cases} \tag{3.6}$$

The preceding relationships yield the transition probability function for the vector Markov chain Z. Since $ACK_{n+1,K}$ and N_{n+1} are statistically independent of Z_n, the vector $X_n = \{Q_n^-, U_{n1}, U_{n2}, \ldots, U_{nK}\}$ forms the state sequence of interest and $X = \{X_n,\ n \geq 1\}$ is a vector Markov chain over the space $d \times d_1^K$. A flow diagram indicating the transition $X_n \rightarrow X_{n+1}$ is presented in Figure 3.3.

For single-packet messages with $K_i = K = 1$, the operation of the SR, SW and Block ARQ schemes is identical; and, therefore, these systems are described by equivalent channel state processes. Thus, using the results from Chapter II, the processes X and Z are ergodic if and only if

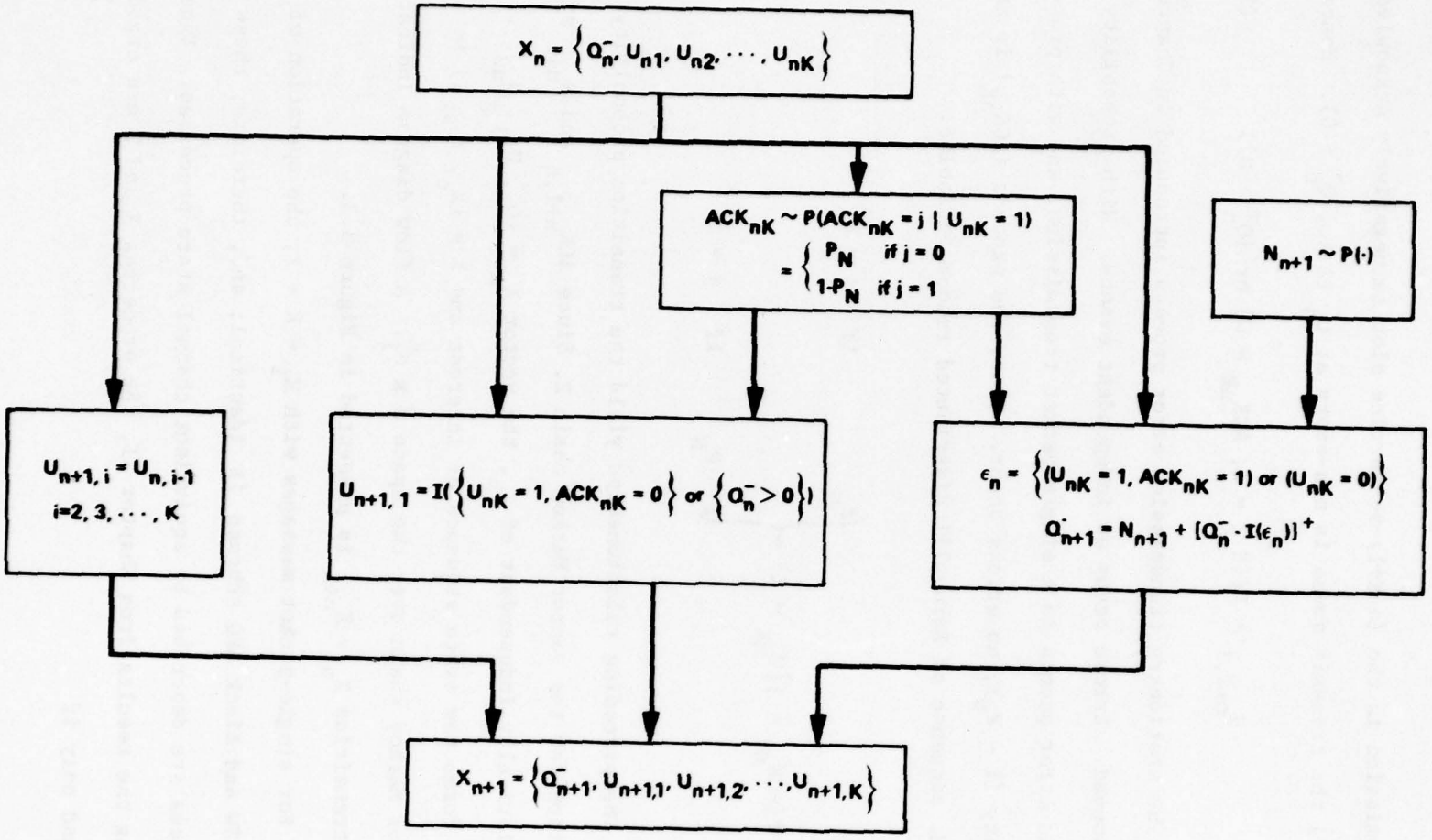


Figure 3.3. Transition $X_n \rightarrow X_{n+1}$ for the Vector Markov Chain X Associated with the TDMA Channel Using SR ARQ

$$n_1 < 1 - P_N \tag{3.7}$$

when K = 1. Moreover, Konheim [47] shows that (3.7) is the necessary and sufficient condition for ergodicity for arbitrary $K \geq 1$.

Proposition 3.1

The processes Z and X are ergodic if and only if $n_1 < 1 - P_N$.

Throughput is defined as the average number of successful packet transmissions per service slot. Thus from Proposition 3.1 maximum throughput is given by

$$\text{maximum throughput} = \frac{1}{K} E\left\{\begin{array}{l|l}\text{number of successful packet transmissions in K contiguous service slots} & \text{a packet is transmitted in each service slot}\end{array}\right\}$$

$$= \frac{(1 - P_N)K}{K}$$

$$= 1 - P_N \tag{3.8}$$

for each $K \geq 1$.

3.3 Message Delay Analysis

The two message delay measures (D_R, D_S) discussed in Chapter II for the Block and SW ARQ systems are also appropriate for the SR ARQ system. The data transfer delay (D_R) from source to destination is expressed by (2.1). The required holding time (measured in slots)

of the n-th message at the source station is decomposed into the sum

$$D_{S_n} = U_n + W_n + H_n \tag{3.9}$$

where U_n denotes the delay of the message from its arrival slot to the next service slot and W_n is the additional delay to its first transmission as previously described in Section 2.3. The variable H_n represents the remaining time until the n-th message is positively acknowledged and subsequently discarded from the source station's transit buffer. Thus $U_n + W_n$ and H_n represent the times spent by the n-th message in the transmit buffer and transit buffer, respectively.

The difference $D_S - D_R$ between the two delay measures is the acknowledgment delay. The effective transmission time of the n-th message is given by

$$T_n = MK\,\tilde{R}_n + 1 \tag{3.10}$$

and the holding time in the transit buffer is bounded by

$$H_n \leq MK(\tilde{R}_n + 1) \tag{3.11}$$

where $\tilde{R}_n$ is the number of errant transmissions of the n-th message. Thus the difference $D_S - D_R$ is clearly bounded by $D_S - D_R \leq MK - R - 1$ where R is the propagation delay.

Under the stationary transmission error process model, $\{\tilde{R}_n,\ n \geq 1\}$ is an i.i.d. sequence of geometric distributed random variables with mean $E(\tilde{R}_n) = P_N(1 - P_N)^{-1}$. Therefore,

$$E(T) = \frac{MK\,P_N}{1 - P_N} + 1 \tag{3.12}$$

and

$$E(H) \leq \frac{MK}{1 - P_N} \tag{3.13}$$

To completely specify either delay measure requires the statistical characterization of the waiting time in the transmit queue W_n. An efficient solution for the distribution of W_n is thwarted by the store-and-forward manner in which packets traverse the channel. Enforced service idle times and the simultaneous service of up to K messages (either in transmission or waiting acknowledgment) make exact, useful mathematical characterizations difficult. Therefore, manageable bounds on the average waiting time at steady state are derived.

At steady state, the average waiting time in the transmit queue is related to the average queue size by Little's result [55].

Proposition 3.2

$$E(W_n) = \frac{ME(Q_n^+)}{n_1} \tag{3.14}$$

where the expectations are w.r.t. the steady state distributions. The variable Q_n^+ is the transmit queue size at $t_n^+ = nM\tau + 0$. It is related to Q_n^- by

$$Q_n^- = Q_{n-1}^+ + N_n \tag{3.15}$$

Squaring both sides of (3.4), taking expectations w.r.t. the steady state distributions, and rearranging terms, we find that

$$E\{Q_n^-[I(\varepsilon_n, Q_n^- > 0) - N_{n+1}]\} = \frac{1}{2}[n_1(1 - n_1) + \sigma_n^2] \tag{3.16}$$

The first term on the left hand side of (3.16) is evaluated by considering the following conditional expectation:

$$E\{Q_n^- I(\varepsilon_n, Q_n^- > 0) | Q_n^-, U_{nK}\} = Q_n^-[(1 - P_N) I(U_{nK} = 1) + I(U_{nK} = 0)] \tag{3.17}$$

Using the inequality $Q_n^- \geq N_n$ yields

$$E\{Q_n^- I(\varepsilon_n, Q_n^- > 0)\} \geq (1 - P_N) E(Q_n^-) + P_N n_1 P(U_{nK} = 0) \tag{3.18}$$

with equality when K = 1. The steady state distribution of U_{nK} is derived in Appendix B:

$$P(U_{nK} = j) = \begin{cases} 1 - \dfrac{n_1}{1 - P_N} & \text{if} \quad j = 0 \\ \dfrac{n_1}{1 - P_N} & \text{if} \quad j = 1 \end{cases} \tag{3.19}$$

Since Q_n^- and N_{n+1} are statistically independent, (3.16), (3.18) and (3.19) yield the following upper bound on the average transmit queue size:

$$E(Q_n^-) \leq \frac{n_1(1 - n_1) + \sigma_n^2}{2(1 - P_N - n_1)} - \frac{P_N n_1}{1 - P_N} \tag{3.20}$$

This result together with (3.14) and (3.15) provides an upper bound on the average waiting time in the transmit queue at steady state:

$$\frac{1}{M} E(W) \leq \frac{(n_2 - n_1)\ (1 - P_N) + 2n_1^2\ P_N}{2n_1(1 - P_N - n_1)\ (1 - P_N)} \triangleq W_U \tag{3.21}$$

A similar development leads to the following lower bounds:

$$E(Q_n^-) \geq \frac{n_1(1 - n_1) + \sigma_n^2}{2(1 - n_1)} + \frac{n_1^2\ P_N}{(1 - P_N)(1 - n_1)} \tag{3.22}$$

$$\frac{1}{M} E(W) \geq \frac{(n_2 - n_1)\ (1 - P_N) + 2n_1^2\ P_N}{2n_1(1 - n_1)\ (1 - P_N)} \triangleq W_L \tag{3.23}$$

The upper bound W_U is an exact result for K = 1; and, therefore, the largest average waiting times in the transmit queue at steady state are experienced with single-frame acknowledgment delays. Furthermore, W_U agrees with the result derived in Chapter II for the Block and SW ARQ systems with $B_i = 1$, $K_i = 1$, $i \geq 1$. However, these waiting time bounds are independent of K and they deviate from each other as the average message arrival rate approaches the maximum throughput value as demonstrated by the following ratio:

$$\frac{W_U}{W_L} = \frac{1 - n_1}{1 - P_N - n_1} \tag{3.24}$$

Slightly more complex bounds are derived in Appendix B:

$$\frac{1}{M} E(W) \geq W_U - \frac{(K-1)P_N}{1 - P_N} \triangleq W_{LK} \tag{3.25}$$

$$\frac{1}{M} E(W) \leq W_{LK} + \frac{P_N N_K}{n_1(1 - P_N)} \triangleq W_{UK} \tag{3.26}$$

where

$$N_K = \begin{cases} \sum_{i=1}^{K-1} P(N_1 + N_2 + \ldots + N_i > 0) & \text{if} \quad K > 1 \\ 0 & \text{if} \quad K = 1 \end{cases}$$

These results are summarized in Lemma 3.1.

Lemma 3.1

For the TDMA channel using SR ARQ, the average (single-packet) message waiting time in the transmit queue at steady state is bounded by

$$M\bar{W}_L \leq E(W) \leq M\bar{W}_U \tag{3.27}$$

where

$$\bar{W}_L = \max (W_L, W_{LK})$$
$$\bar{W}_U = \min (W_U, W_{UK}) \qquad \text{if} \quad n_1 < 1 - P_N.$$

The difference between these bounds is itself noted to be bounded by

$$\bar{W}_U - \bar{W}_L \leq \frac{(K - 1)P_N}{1 - P_N} \tag{3.28}$$

Thus by using the bounds on the average waiting time stated in Lemma 1 together with (2.1) and (3.12), bounds on the average message delay at steady state are established.

Theorem 3.1

Under the TDMA access-control discipline using SR ARQ error control, the average steady state message (single-packet) delay is finite and bounded by

$$D_L \leq E(D_R) \leq D_U \tag{3.29}$$

where

$$D_X = E(U) + M\bar{W}_X + \frac{MK\ P_N}{1 - P_N} + 1 + R$$

when $n_1 < 1 - P_N$. When $n_1 \geq 1 - P_N$, the limiting message delay is infinite.

If the message arrival process $\{A_n,\ n \geq 1\}$ is an i.i.d. sequence of Poisson distributed random variables with average arrival rate equal to λ messages per slot, the steady state message delay bounds are specified as follows.

Corollary 3.1

For a Poisson message arrival process, the bounds on the average

message delay at steady state, under a TDMA control discipline using the SR ARQ error recovery procedure, are given by

$$D_L \leq E(D_R) \leq D_U$$

where

$$D_X = \frac{M+1}{2} + M\bar{W}_X + \frac{MK\ P_N}{1 - P_N} + R$$

$$\bar{W}_L = \max\ (W_L,\ W_{LK})$$

$$\bar{W}_U = \min\ (W_U,\ W_{UK})$$

$$W_U = \frac{\lambda M(1 + P_N)}{2(1 - P_N - \lambda M)(1 - P_N)}$$

$$W_{UK} = W_U - \frac{(K-1)P_N}{1 - P_N} + \frac{P_N}{\lambda M(1 - P_N)} \left[\frac{K(1 - e^{-\lambda M}) - (1 - e^{-\lambda MK})}{1 - e^{-\lambda M}}\right]$$

$$W_L = \frac{\lambda M(1 + P_N)}{2(1 - \lambda M)\ (1 - P_N)}$$

$$W_{LK} = W_U - \frac{(K-1)P_N}{1 - P_N}$$

when $\lambda M < 1 - P_N$. When $\lambda M \geq 1 - P_N$, the limiting message delay is infinite.

To demonstrate the utilization of the preceding bounds consider the following example.

EXAMPLE 3.1

Assume $\{A_n, n \geq 1\}$ is an i.i.d. sequence of Poisson distributed random variables with average arrival rate λ messages per slot. With $K = 2$,

$$W_{U2} = W_U - \frac{P_N}{1 - P_N} + \frac{P_N}{1 - P_N} \left[\frac{1 - e^{-\lambda M}}{\lambda M}\right] \tag{3.30}$$

$$W_{L2} = W_U - \frac{P_N}{1 - P_N} \tag{3.31}$$

Since $0 < (1 - e^{-\lambda M})/\lambda M \leq 1$ for $0 \leq \lambda M < 1$, (3.30) implies that $W_{U2} \leq W_U$. Equation (3.31) implies that there exists a λ_0 such that $W_{L2} < W_L$ for $\lambda < \lambda_0$ and $W_{L2} \geq W_L$ for $\lambda \geq \lambda_0$. Hence, for $K = 2$ and for a Poisson distributed message arrival process,

$$\bar{W}_U = W_{U2}$$

$$\bar{W}_L = \begin{cases} W_L & \text{if} \quad \lambda < \lambda_0 \\ W_{L2} & \text{if} \quad \lambda \geq \lambda_0 \end{cases}$$

where $\lambda_0 M = \frac{1}{2}(1 - P_N)$. In addition, the difference between the bounds is itself bounded by

$$\bar{W}_U - \bar{W}_L \leq \frac{P_N}{1 - P_N} \left[\frac{1 - e^{-\lambda M}}{\lambda M}\right] . \tag{3.32}$$

Equality is attained in (3.32) when $\lambda \geq \lambda_0$.

Hence, when K = 2, the average waiting time in the transmit queue is less than W_U (an exact result for K = 1):

$$E(W)\Big|_{K=2} \leq E(W)\Big|_{K=1} \quad .$$

However, due to the larger average effective transmission time,

$$E(D_R)\Big|_{K=2} \geq E(D_R)\Big|_{K=1} \quad .$$

3.4 Numerical Results

The delay versus throughput performance functions are evaluated assuming Poisson distributed message arrivals. The average packet delay bounds are normalized by the TDMA frame size (M). Resulting terms which are proportional in value to 1/M have been neglected. The additive fixed propagation delay is set equal to zero (R = 0).

Results are presented in Tables 3.1 and 3.2 for packet noise error probabilities P_N = 0.1 and 0.2, respectively, with K = 1, 2, 5, 10. These results clearly demonstrate the robust nature of the bounds. Over most of the throughput region, the difference between the upper and lower bounds is strictly less than

$$D_U - D_L < \frac{(K-1)P_N}{1 - P_N} \quad .$$

Packet delay upper bound D_U versus throughput curves are shown in Figure 3.4 with K = 1, 2, 5, 10 and P_N = 0.2. These curves exhibit the characteristic delay-throughput behavior. Upper bound D_U versus

packet noise error probability curves are shown in Figure 3.5 for constant throughputs $\lambda M = 0.2, 0.4, 0.6$.

Maximum throughput versus packet noise error probability curves are shown in Figure 3.6 for the SW, Block and SR ARQ systems. These results are derived for single-packet messages and a five-frame acknowledgment delay. As expected, the SR scheme provides the largest maximum throughput values for all error probabilities. Average packet delay versus throughput curves are shown in Figure 3.7. The SR ARQ curve is the upper bound on the actual steady state delay. These results clearly demonstrate the advantage possible with the SR ARQ scheme for communication channels with non-zero error rates.

3.5 Conclusions

The operation of a TDMA channel using the SR ARQ error recovery procedures was examined in this chapter. By considering single-packet messages, constant acknowledgment frame delays, and a stationary transmission error process, the evolution of the channel was described by a vector Markov chain. The conditions for ergodicity were stated. Upper and lower bounds on the average message delay at steady state were derived. The difference between these bounds was itself bounded and shown to be tight over the entire throughput range. The utilization of these bounds was demonstrated by numerical examples and the performance was compared to the Block and SW ARQ systems. These results clearly demonstrate the performance advantage possible with the SR ARQ scheme for communication channels with non-zero error rates.

TABLE 3.1. DELAY-THROUGHPUT PERFORMANCE FUNCTION FOR A TDMA CHANNEL USING SR ARQ WITH $P_N = 0.1$

Throughput λM	K=1 $E(D_R)$	K=2			K=5			K=10		
		D_L	D_U	D_U-D_L	D_L	D_U	D_U-D_L	D_L	D_U	D_U-D_L
0.0	0.611	0.722	0.722	0.0	1.056	1.056	0.0	1.611	1.611	0.0
0.05	0.647	0.754	0.755	0.001	1.088	1.092	0.004	1.643	1.647	0.004
0.10	0.687	0.790	0.793	0.003	1.123	1.132	0.008	1.679	1.687	0.008
0.15	0.733	0.830	0.837	0.006	1.163	1.178	0.014	1.719	1.733	0.014
0.20	0.786	0.875	0.886	0.011	1.208	1.230	0.022	1.764	1.786	0.022
0.25	0.846	0.926	0.944	0.019	1.259	1.291	0.031	1.815	1.846	0.031
0.30	0.917	0.984	1.013	0.029	1.317	1.361	0.044	1.873	1.917	0.044
0.35	1.000	1.051	1.094	0.042	1.385	1.444	0.060	1.940	2.000	0.060
0.40	1.100	1.130	1.192	0.062	1.463	1.544	0.081	2.019	2.100	0.081
0.45	1.222	1.222	1.312	0.089	1.556	1.667	0.111	2.111	2.222	0.111
0.50	1.375	1.375	1.462	0.087	1.667	1.819	0.153	2.222	2.375	0.153
0.55	1.571	1.571	1.657	0.085	1.802	2.016	0.213	2.358	2.571	0.213
0.60	1.833	1.833	1.917	0.084	1.972	2.278	0.306	2.528	2.833	0.306
0.65	2.200	2.200	2.282	0.082	2.200	2.644	0.444	2.746	3.200	0.454
0.70	2.750	2.750	2.830	0.080	2.750	3.194	0.444	3.037	3.750	0.713
0.75	3.667	3.667	3.745	0.078	3.667	4.111	0.444	3.667	4.667	1.000
0.80	5.500	5.500	5.576	0.076	5.500	5.944	0.444	5.500	6.500	1.000
0.85	11.000	11.000	11.075	0.075	11.000	11.429	0.429	11.000	12.000	1.000
0.86	13.750	13.750	13.825	0.075	13.750	14.175	0.425	13.750	14.750	1.000
0.87	18.333	18.333	18.408	0.074	18.333	18.755	0.422	18.333	19.333	1.000
0.88	27.500	27.500	27.574	0.074	27.500	27.918	0.418	27.500	28.500	1.000
0.89	55.000	55.000	55.074	0.074	55.000	55.415	0.415	55.000	56.000	1.000

TABLE 3.2. DELAY-THROUGHPUT PERFORMANCE FUNCTION FOR A TDMA CHANNEL USING SR ARQ WITH $P_N = 0.2$

Throughput λM	K=1 $E(D_R)$	K=2			K=5			K=10		
		D_L	D_U	D_U-D_L	D_L	D_U	D_U-D_L	D_L	D_U	D_U-D_L
0.0	0.750	1.000	1.000	0.0	1.750	1.750	0.0	3.000	3.000	0.0
0.05	0.800	1.039	1.044	0.004	1.789	1.800	0.011	3.039	3.050	0.011
0.10	0.857	1.083	1.095	0.012	1.833	1.857	0.024	3.083	3.107	0.024
0.15	0.923	1.132	1.155	0.023	1.882	1.923	0.041	3.132	3.172	0.041
0.20	1.000	1.187	1.227	0.039	1.937	2.000	0.063	3.187	3.250	0.063
0.25	1.091	1.250	1.312	0.062	2.000	2.091	0.091	3.250	3.341	0.091
0.30	1.200	1.321	1.416	0.095	2.071	2.200	0.129	3.321	3.450	0.129
0.35	1.333	1.404	1.544	0.140	2.154	2.333	0.179	3.404	3.583	0.179
0.40	1.500	1.500	1.706	0.206	2.250	2.500	0.250	3.500	3.750	0.250
0.45	1.714	1.714	1.916	0.201	2.364	2.714	0.351	3.614	3.964	0.351
0.50	2.000	2.000	2.197	0.197	2.500	3.000	0.500	3.750	4.250	0.500
0.55	2.400	2.400	2.592	0.192	2.667	3.400	0.733	3.917	4.650	0.733
0.60	3.000	3.000	3.188	0.188	3.000	4.000	1.000	4.125	5.250	1.125
0.65	4.000	4.000	4.184	0.184	4.000	5.000	1.000	4.393	6.250	1.857
0.70	6.000	6.000	6.180	0.180	6.000	7.000	1.000	6.000	8.250	2.250
0.75	12.000	12.000	12.176	0.176	12.000	13.000	1.000	12.000	14.250	2.250
0.76	15.000	15.000	15.175	0.175	15.000	16.000	1.000	15.000	17.250	2.250
0.77	20.000	20.000	20.174	0.174	20.000	21.000	1.000	20.000	22.250	2.250
0.78	30.000	30.000	30.173	0.174	30.000	31.000	1.000	30.000	32.250	2.250
0.79	60.000	60.000	60.173	0.173	60.000	61.000	1.000	60.000	62.250	2.250

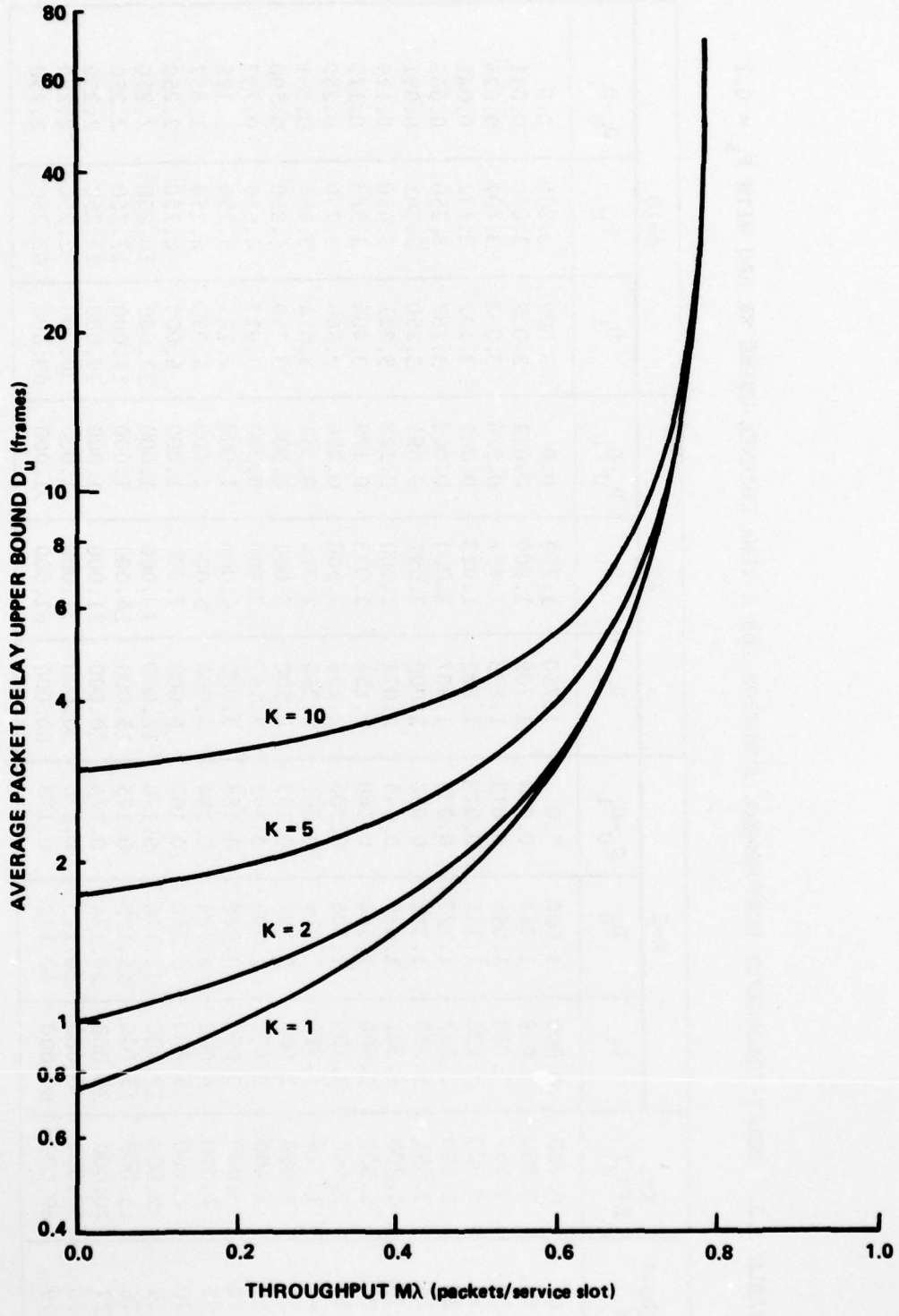


Figure 3.4. Delay versus Throughput Curves for a TDMA Channel Using SR ARQ with $P_N = 0.2$

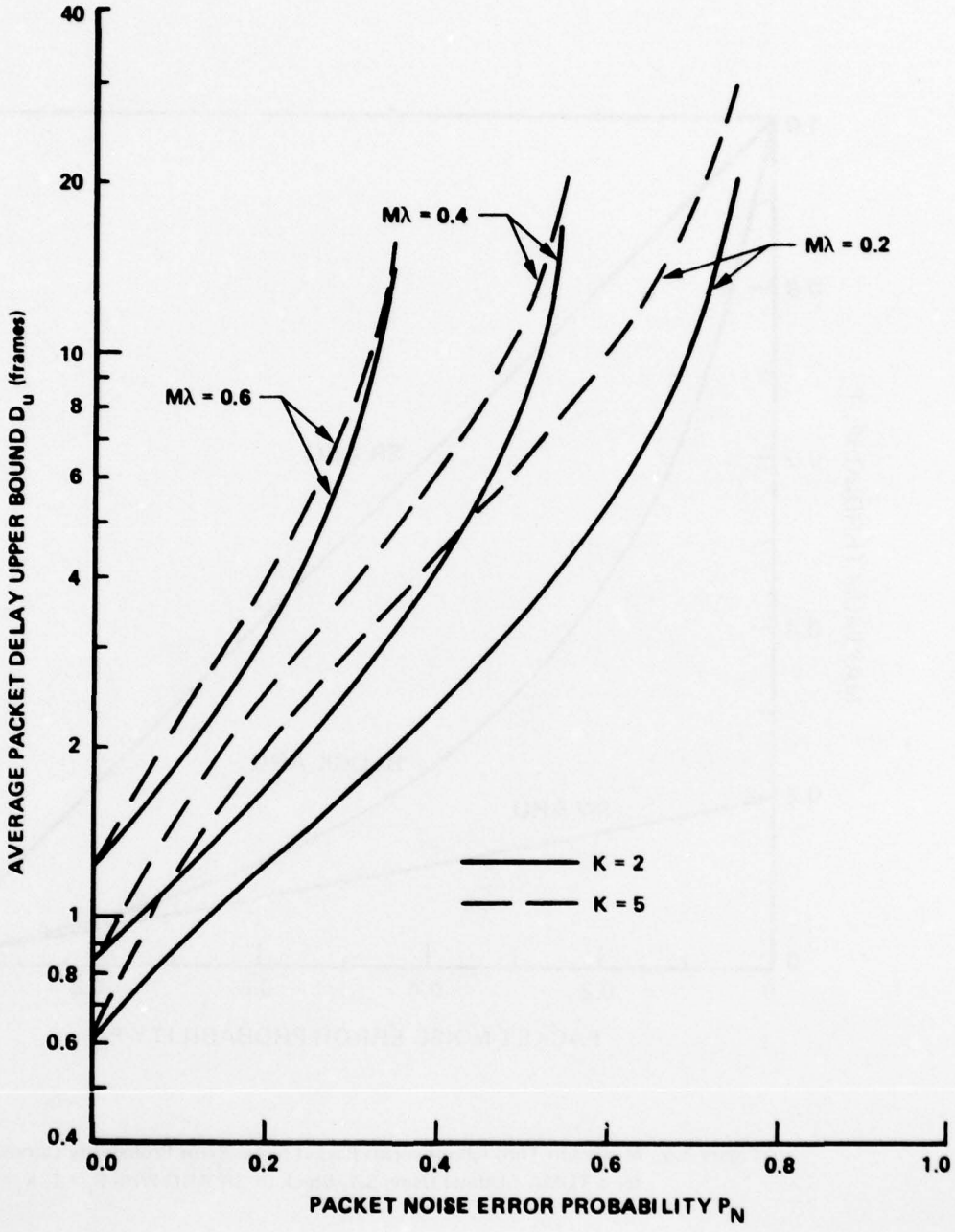


Figure 3.5. Delay versus Packet Noise Error Probability Curves for a TDMA Channel Using SR ARQ

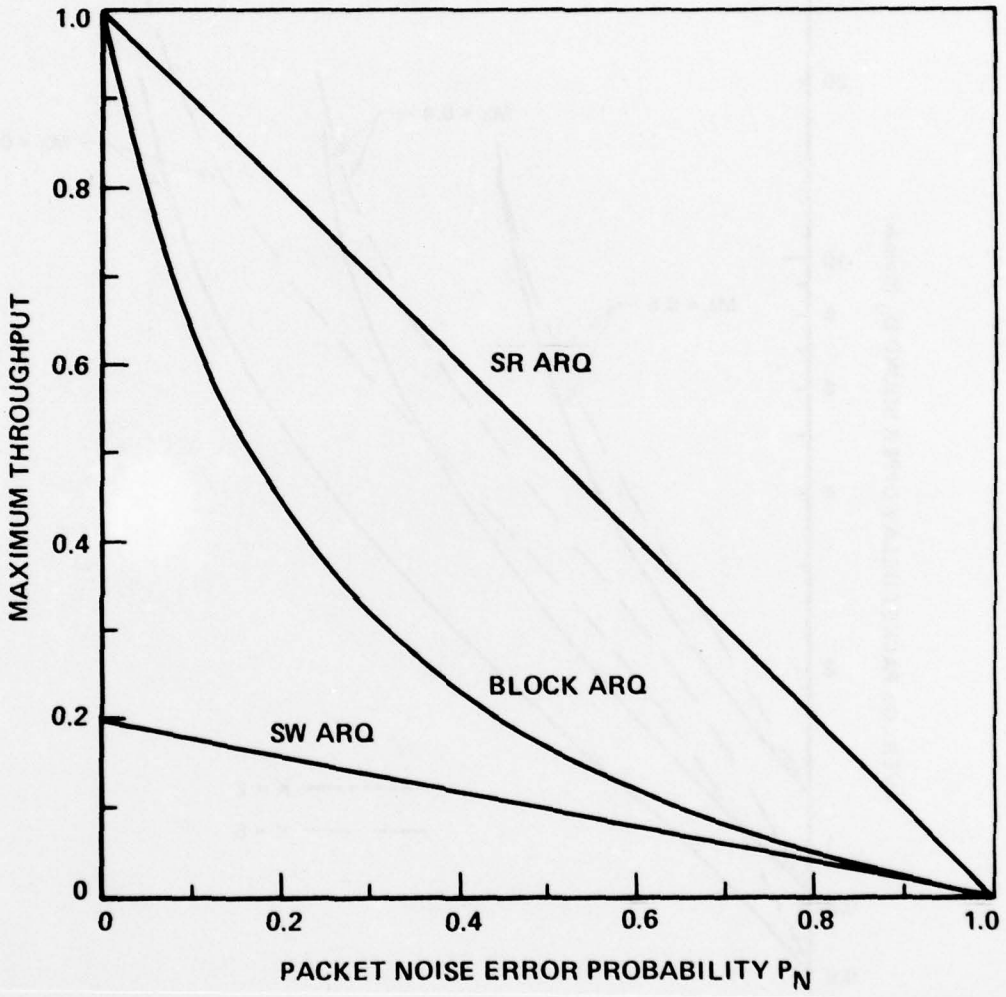


Figure 3.6. Maximum Throughput versus Packet Noise Error Probability Curves for a TDMA Channel Using SW, Block or SR ARQ With $B_i = 1$, $K_i = 5$

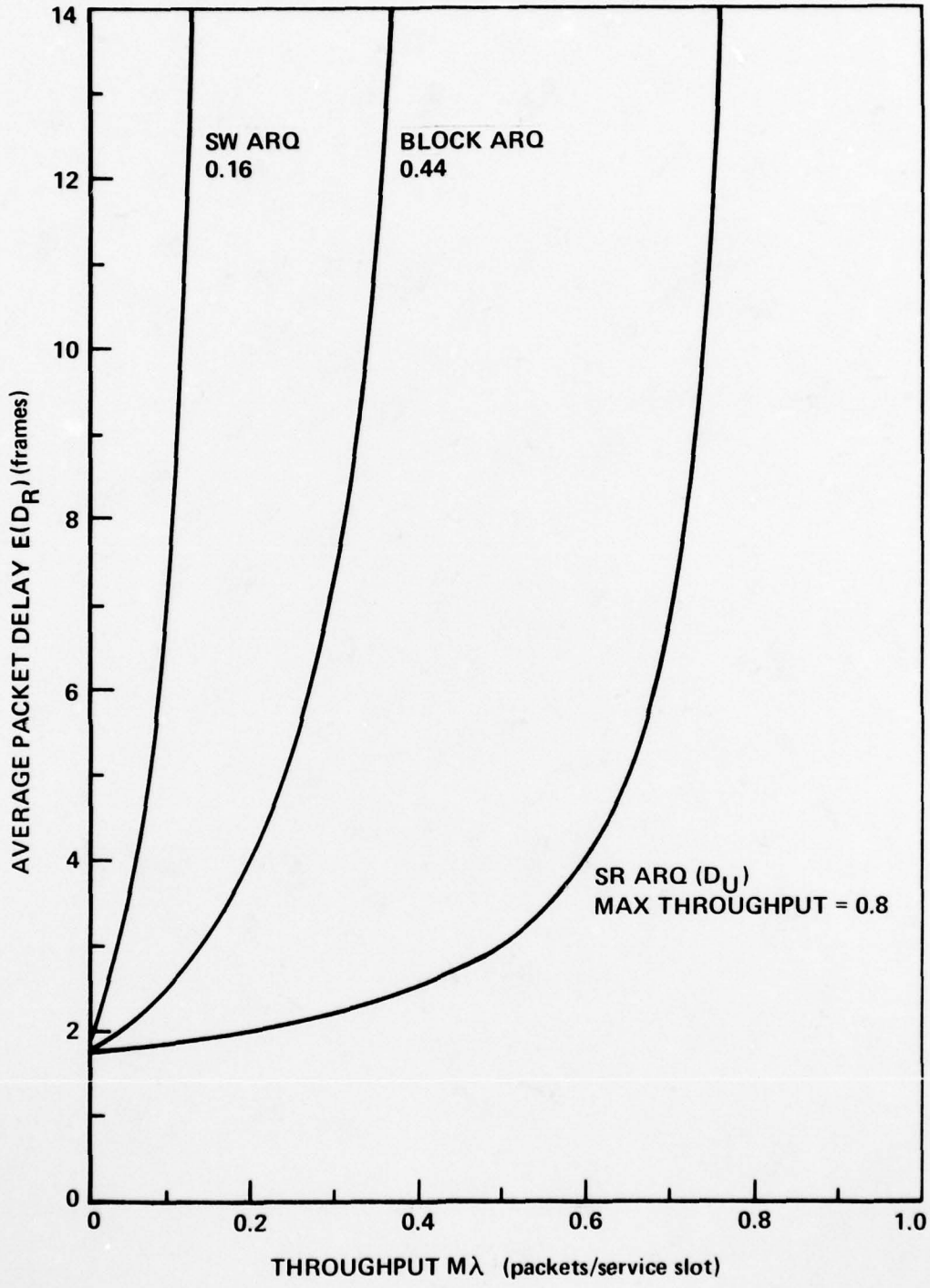


Figure 3.7. Delay versus Throughput Curves for a TDMA Channel Using SW, Block and SR ARQ With $B_i = 1$, $K_i = 5$, $P_N = 0.2$

CHAPTER IV

GROUP RANDOM ACCESS USING RANDOM ACCESS ACKNOWLEDGMENT PROTOCOLS

The Group Random Access (GRA) access-control discipline has been developed and studied by Rubin [12] assuming an ideal acknowledgment mechanism is provided at no cost. In this chapter three random access acknowledgment mechanisms are incorporated into the GRA discipline. In Section 4.1 the GRA channel structure and the results obtained by Rubin are reviewed and the acknowledgment protocols are introduced. The GRA channel under the Pure-Random Access acknowledgment scheme is examined in Section 4.2. In Section 4.3 the GRA channel under the Multiple Copy-Random Access acknowledgment scheme is studied, and the GRA channel under the Period Division-Random Access acknowledgment scheme is studied in Section 4.4. Numerical examples are presented in Section 4.5.

4.1 Introduction to the GRA Channel Structure

In large communication networks, situations may arise which indicate an advantage by time-sharing the channel among families of network stations. Families may be distinguished by their priorities, performance requirements or by the statistics and nature of their communications. Each family of network stations can access the channel under an appropriate access-control discipline during their allocated

channel access periods. Under a GRA discipline, a group of network stations is provided with a periodic sequence of channel-access periods to gain access into the channel using a random access discipline

4.1.1 System Description

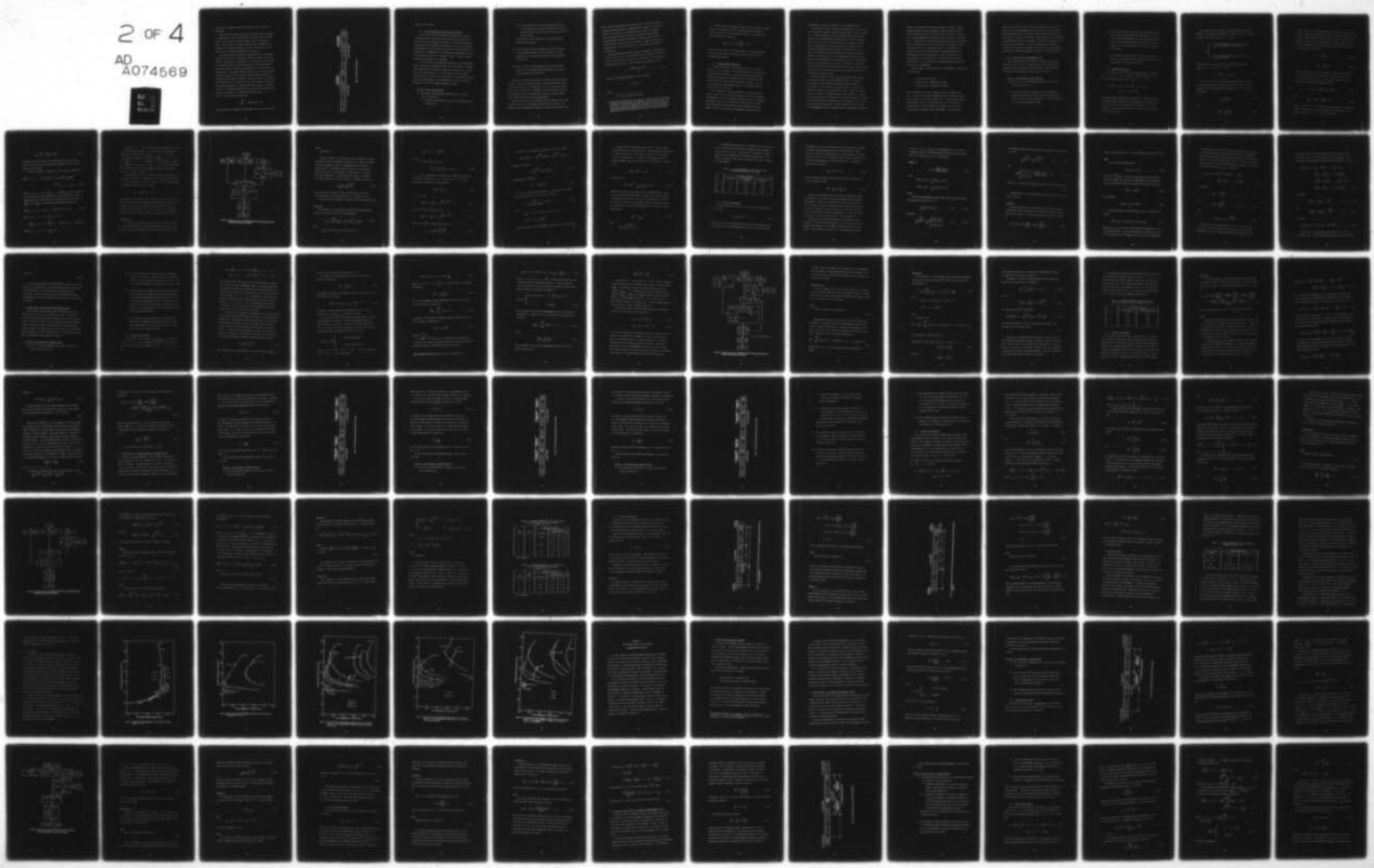
A time synchronized channel is examined. Time (referenced with respect to a master clock) is divided into equal-length slots, each of duration τ seconds. The start of a message transmission across the channel must coincide with the beginning of a slot. In addition, each information-bearing (data) message is a single fixed-length packet of b bits (including both information and overhead bits). The transmission rate across the channel is C bits per second and the transmission time of a data packet is set equal to the duration of a single slot so that

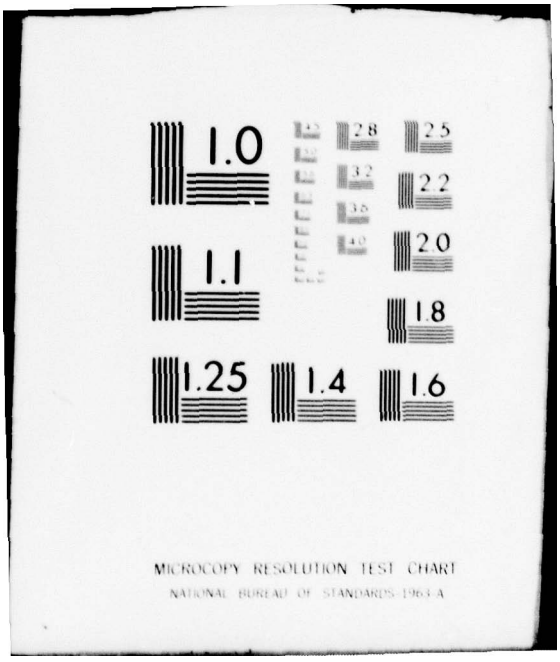
$$\tau = b/C.$$

The network stations are organized into families (or groups) which share the channel on a time multiplex basis. Contiguous slots are organized into frames and each family is allocated a fixed portion of a frame for message transmissions. Each family of network stations can access the channel in their allocated service slots under an appropriate channel access-control discipline. In particular, a group of $\tilde{M}$ network stations which uses a random access discipline to gain

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access into the channel during its allocated service periods is considered.

Under this Group Random Access channel access-control discipline, the frame length is fixed at $\tilde{K}+P$ slots. The channel access periods contain $\tilde{K}$ slots and the interval between successive periods is P slots. Thus the group of network stations under consideration is provided with a periodic sequence of channel access periods to gain access into the channel as illustrated in Figure 4.1.

The group of network stations is characterized as a large population of low duty-cycle bursty users for whom a random access discipline is appropriate. Furthermore, it is assumed that each station will essentially hold at most one message in its transmit buffer at any given time; and, therefore, queueing effects at each station are insignificant. Alternately, buffer storage for only a single message could be provided at each station so that new message arrivals are blocked when the buffer is occupied. Message arrivals at each station are recorded only at the start of a slot. New messages arrive at the i-th station according to a Poisson stream of intensity λ_i messages per slot. The overall message arrival stream for the group of network stations is described by a Poisson point process $\{A_{ni}, i = 1, 2, \ldots, \tilde{K}+P, n \geq 1\}$ with average arrival rate

$$\lambda = \sum_{i=1}^{\tilde{M}} \lambda_i \quad \text{messages per slot}$$

where A_{ni} denotes the number of new message arrivals in the i-th slot

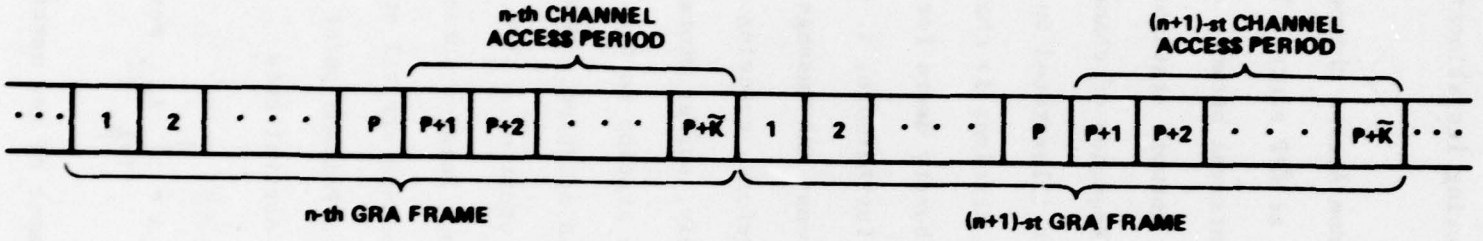


Figure 4.1 Basic GRA Channel Structure

within the n-th frame.

4.1.2 Basic GRA Channel Access-Control Discipline

The basic GRA discipline has been studied by Rubin assuming transmission errors result only from transmission collisions (i.e., two or more users simultaneously transmit over the same bandwidth). Transmission errors caused by localized noise sources are neglected. Thus assuming sufficient coding is applied to detect collisions with probability 1, transmissions are automatically acknowledged by virtue of the broadcast feature of the channel. Hence, a separate acknowledgment mechanism may not be necessary.

For this automatic acknowledgement mechanism, the acknowledgment delay is equal to the propagation delay (R) of the channel. Assuming the interval between successive channel access periods (P) is larger than the propagation delay $P \geq R$, collisions in the n-th channel access period are detected prior to the (n+1)-st period (NACK received) and the colliding packets are retransmitted in the (n+1)-st period. Therefore, the basic GRA channel operates as follows.

Protocol: Basic GRA Discipline

(1) New message arrivals which are admitted by the network control procedure are

   i) transmitted immediately, if they arrive during a channel access period

ii) transmitted in the next channel access period in a slot determined by a uniform distribution over the $\tilde{K}$ available service slots, if they arrive in the interval between periods.

Messages which are not admitted by the network control procedure are rejected.

(2) Each information packet colliding within the n-th period is retransmitted within the (n+1)-st period in a slot determined by a uniform distribution over the $\tilde{K}$ available service slots.

(3) Each information packet admitted by the network control procedure is transmitted and retransmitted until successfully transmitted collision-free.

The basic GRA protocol provides for a network control procedure to reject certain new message arrivals. Rejected messages are lost or they must attempt to gain admittance at a later time under appropriate arrival statistics. This packet rejection procedure provides the necessary control to stabilize the inherently unstable random access channels and yield finite average delays.

Rubin considers a class of binary control strategies that either accept or reject all new message arrivals within each frame. The optimal control strategy which yields the minimal average packet delay

under a prescribed maximal value of probability of rejection involves a single-threshold structure. Assuming the total number of colliding packets in a period can be observed by each station*, the optimal control procedure accepts or rejects new message arrivals within a frame depending on whether the number of colliding packets in the previous period is below or above a threshold. Thus an important performance index for the GRA discipline is the probability of packet rejection (P_R).

Another performance measure is channel throughput δ, the average (steady state) rate of successful packet transmissions per service slot. Since all message arrivals admitted by the network control procedure are transmitted/retransmitted until successful, throughput is given by

$$\tilde{\delta} = \frac{\lambda(\tilde{K} + P)\ (1 - P_R)}{\tilde{K}} \tag{4.1}$$

Moreover, the maximum throughput is upper bounded by

$$\tilde{\delta} \leq e^{-1}\ \tilde{G}_{\tilde{K}} \simeq e^{-1} \tag{4.2}$$

where

$$\tilde{G}_{\tilde{K}} = \frac{1}{(\tilde{K} - 1)\ \ell n[1 + (\tilde{K} - 1)^{-1}]} .$$

*In general, the network stations may not be able to observe the total number of colliding packets in a period. The stations may only observe the number of collision-free packet transmissions and the number of slots which experience collision in each period. Within the range of acceptable delay and probability of rejection values, Rubin [12] has demonstrated that these observations provide equivalent performance.

Packet delay D_R is measured (in slots) from the moment of message arrival at the source station to the moment when the message is correctly received by the destination station. The average steady-state packet delay is given by

$$E(D_R) = \frac{P}{2} + (P + \tilde{K}) \frac{E(R_n)}{\tilde{R}\,\tilde{\delta}} + R + 1 \qquad (4.3)$$

where R_n is the number of collisions (number of colliding packets) in the n-th period and the expectations are with respect to the stationary distribution.

4.1.3 Acknowledgment Mechanisms

The basic GRA channel access-control discipline reviewed in Section 4.1.2 is appropriate when transmission errors evolve strictly from transmission collisions. Since the network stations receive all packet transmissions (broadcast feature), the source stations can detect these collisions and retransmit accordingly. Thus, a positive-negative acknowledgment system is provided automatically by the nature of the channel.

Actual networks, however, can experience random transmission errors. These errors can be local in nature such that the automatic acknowledgment system provided by the broadcast feature is inadequate. For example, errors can be introduced at the destination station, while the source station receives its own transmission errorless (PACK). Therefore, a separate acknowledgment mechanism should be

incorporated to insure the integrity of the transmitted data.

Many previous studies, concerning communications over the multi-access broadcast channel, assume the acknowledgment traffic is carried over a channel separate from the data channel under investigation. Moreover, the acknowledgments are assumed to be delivered reliably and at no cost. But it has become increasingly apparent in recent years that bandwidth is a limited resource; and, therefore, the required acknowledgment traffic cannot be carried without cost. Hence, the operation of the GRA channel using ARQ error control procedures is examined under several acknowledgment mechanisms which adhere to the philosophy of the GRA channel structure. Specifically, positive acknowledgment with time-out is considered. The required PACKs are transmitted only during the allocated channel access periods; and, hence, they compete with the information packet transmissions for the available channel resources.

Random transmission errors in PACK transmissions are assumed negligible and single-frame acknowledgment delays are assumed. In this chapter, however, random access acknowledgment schemes are investigated. PACKs contend for channel access on a random access basis; thus PACKs can be destroyed by collision. Hence, the channel operates as follows. Information packets which experience collision in the n-th period are retransmitted in the (n+1)-st period. Collision-free information packet transmissions are acknowledged in the (n+1)-st period with PACK transmissions. PACKs returned successfully within the acknowledgment time-out interval allow the acknowledged information

packets to be dropped from the source stations' buffers. Unacknowledged information packet transmissions (PACK collisions) require their retransmission after the time-out interval expires. In particular, the acknowledgement time-out interval is fixed at two GRA frames (i.e., a PACK collision in the (n+1)-st period requires the information packet retransmission in the (n+2)-nd period). Alternative schemes with longer time-out intervals are possible. Such schemes, for example, could allow each collision-free information packet to be acknowledged by PACK transmissions in successive channel access periods within the time-out interval. These methods may increase the probability that a collision-free information packet transmission is successfully acknowledged.

In this chapter, three random access acknowledgment implementations are investigated:

- Pure-Random Access (PRA)
- Multiple Copy - Random Access (MCRA)
- Period Division - Random Access (PDRA).

PACK transmissions contend for channel access on a random access basis. Under the PRA scheme, information packets and PACKs contend in each period for the same $\tilde{K}$ service slots. Under the MCRA scheme, acknowledgment traffic is given preference over data traffic by sending multiple, identical PACKs (in a single period) for each collision-free information packet transmission. Although these

multiple copies may increase the total load on the channel, the preference given to acknowledgment traffic can increase the successful PACK transmission rate and, thereby, provide larger throughput values than achievable under the PRA scheme. Another method to give preference to acknowledgment traffic is to properly partition each channel access period into two subperiods. Under this PDRA scheme, information packets are transmitted on a random access basis within one subperiod, and PACKs are transmitted random access within the second subperiod.

4.2 Pure - Random Access Acknowledgment Scheme

Under the PRA acknowledgment scheme, information packets and PACKs gain channel access on a random access basis. These packets contend over the $\check{K}$ service slots in each period. The GRA channel using the PRA acknowledgment scheme operates as follows.

Protocol: GRA Discipline - PRA ACK Scheme

(1) and (2) See the corresponding steps under the basic GRA discipline (Section 4.1.2).

(3) For each collision-free information packet transmission within the n-th period, a PACK is transmitted within the (n+1)-st period in a slot determined by a uniform distribution over the $\check{K}$ available service slots.

(4) Each collision-free information packet transmission within the n-th period which remains unacknowledged at the end of the two frame acknowledgment time-out interval (PACK collision) is retransmitted within the (n+2)-nd period in a slot determined by a uniform distribution over the $\tilde{K}$ available service slots.

(5) Each information packet admitted by the network control procedure is transmitted and retransmitted until successfully acknowledged by a collision-free PACK.

4.2.1 Channel State Process

The evolution of this channel is described by a vector Markov chain $Z = \{Z_n, n \geq 1\}$ over the space $d^{\tilde{K}} \times d_{\tilde{K}}^{\tilde{K}} \times d^{P+\tilde{K}}$ where d is the set of non-negative integers, $d_{\tilde{K}} = \{0, 1, 2, \ldots, \tilde{K}\}$ and

$$Z_n = \{(T_{ni}^{(I)}, i = 1,2,\ldots,\tilde{K}), (T_{ni}^{(A)}, n = 1,2,\ldots,\tilde{K}),$$

$$(A_{ni}, i = 1,2,\ldots,\tilde{K}+P)\}.$$

The variables $T_{ni}^{(I)}$ and $T_{ni}^{(A)}$ denote the number of information packet retransmissions and the number of PACK transmissions, respectively, allocated to the i-th slot within the n-th channel access period. The number of new message arrivals in the i-th slot within the n-th frame is denoted by A_{ni}.

The network control procedure on new message arrivals, which stabilizes this random access channel, is characterized by a sequence of binary control functions $\{\tilde{U}_n, n \geq 1\}$ where

$$\tilde{U}_n = \begin{cases} 0 & \text{if all new message arrivals within the n-th frame are accepted for channel access} \\ 1 & \text{if all new message arrivals within the n-th frame are rejected} \end{cases}$$

These control variables induce the controlled arrival process $\{A_{ni}^{(C)}, i = 1, 2, \ldots, \tilde{K}+P, n \geq 1\}$ where

$$A_{ni}^{(C)} = (1 - \tilde{U}_n) A_{ni} \tag{4.4}$$

Let the variables R_n and $\tilde{S}_n^{(I)}$ denote the number of collisions (information packets and PACKs) and the number of collision-free information packet transmissions, respectively, in the n-th period. By definition,

$$R_n = \sum_{i=1}^{\tilde{K}} T_{n+1,i}^{(I)} \tag{4.5}$$

and

$$\tilde{S}_n^{(I)} = \sum_{i=1}^{\tilde{K}} T_{n+1,i}^{(A)} \tag{4.6}$$

since the number of information packet retransmissions in the (n+1)-st period is equal to the number of collisions in the n-th period, and the number of PACK transmissions in the (n+1)-st period is equal to the number of collision-free information packet transmissions in the n-th period. Furthermore, the variables R_n and $\tilde{S}_n^{(I)}$ are determined by the following relationships:

$$R_n = \sum_{i=1}^{\tilde{K}} R_{ni} \tag{4.7}$$

$$\tilde{S}_n^{(I)} = \sum_{i=1}^{\tilde{K}} \tilde{S}_{ni}^{(I)} \tag{4.8}$$

where R_{ni} and $\tilde{S}_{ni}^{(I)}$ represent the number of collisions and collision-free information packet transmissions, respectively, in the i-th slot within the n-th period. A collision occurs when two or more packets are simultaneously transmitted; hence,

$$R_{ni} = (N_{ni} + T_{ni}^{(A)})\ I(N_{ni} + T_{ni}^{(A)} > 1) \tag{4.9}$$

and

$$\tilde{S}_{ni}^{(I)} = I(N_{ni} = 1,\ T_{ni}^{(A)} = 0) \tag{4.10}$$

where $n \geq 1$, $i = 1, 2, \ldots, \tilde{K}$ and N_{ni} denotes the number of information packet transmissions in the i-th slot within the n-th period. The sequence $\{N_{ni},\ 1 \leq i \leq \tilde{K},\ n \geq 1\}$ is determined by

$$N_{ni} = T_{ni}^{(I)} + A_{n,P+i}^{(C)} + \tilde{A}_{ni}^{(C)} \tag{4.11}$$

where $\tilde{A}_{ni}^{(C)}$ denotes the number of new message arrivals in the first P slots of the n-th frame allocated for transmission in the i-th slot within the n-th period.

The sequence $\{\tilde{A}_{ni}^{(C)}\}$ is governed by the multinomial distribution

$$P\{\tilde{A}_{ni}^{(C)} = \alpha_i,\ 1 \le i \le \tilde{K} \mid \sum_{i=1}^{P} A_{ni}^{(C)} = j\} = \frac{j!}{\alpha_1!\ \alpha_2! \ \ldots\ \alpha_{\tilde{K}}!} \left(\frac{1}{\tilde{K}}\right)^j$$

$$= g_j^{(\tilde{K})}(\alpha_1, \alpha_2, \ldots, \alpha_{\tilde{K}}) \tag{4.12}$$

where $0 \le \alpha_i \le j$, $1 \le i \le \tilde{K}$, $\sum_{i=1}^{\tilde{K}} \alpha_i = j$. Similarly, since the position of an information packet retransmission and the position of a PACK transmission are both determined by uniform distributions over $[1, \tilde{K}]$, $\{T_{n+1,i}^{(I)}\}$ and $\{T_{n+1,i}^{(A)}\}$ are governed by multinomial distributions:

$$P\{T_{n+1,i}^{(I)} = \alpha_i,\ 1 \le i \le \tilde{K} \mid R_n = j\} = g_j^{(\tilde{K})}(\alpha_1, \alpha_2, \ldots, \alpha_{\tilde{K}}) \tag{4.13}$$

where $0 \le \alpha_i \le j$, $1 \le i \le \tilde{K}$, $\sum_{i=1}^{\tilde{K}} \alpha_i = j$ and

$$P\{T_{n+1,i}^{(A)} = \theta_i,\ 1 \le i \le \tilde{K} \mid \tilde{S}_n^{(I)} = j\} = g_j^{(\tilde{K})}(\theta_1, \theta_2, \ldots, \theta_{\tilde{K}}) \tag{4.14}$$

where $0 \le \theta_i \le j$, $1 \le i \le \tilde{K}$, $\sum_{i=1}^{\tilde{K}} \theta_i = j$.

Equations (4.4) - (4.14) yield the transition probability function for the vector Markov chain Z. One observes that the uncontrolled new message arrival variables $\{A_{n+1,i}, i = 1, 2, \ldots, \tilde{K}+P\}$ are statistically independent of Z_n and that $\{T^{(I)}_{n+1,i}, i = 1, 2, \ldots, \tilde{K}\}$ and $\{T^{(A)}_{n+1,i}, i = 1, 2, \ldots, \tilde{K}\}$ depend on Z_n only through $X_n = (\tilde{S}^{(I)}_n, R_n)$. Moreover, the sequence $X = \{X_n, n \geq 1\}$ is a vector Markov chain over the space $d_{\tilde{K}} \times d$. A flow diagram indicating the transition $X_n \rightarrow X_{n+1}$ is shown in Figure 4.2.

An appropriate choice for the binary control functions $\{\tilde{U}_n, n \geq 1\}$ is one which stabilizes the channel so that the Markov state sequences Z and X are irreducible, positive recurrent. Consider the following single-threshold binary control function:

$$\tilde{U}_{n+1} = I(\tilde{S}^{(I)}_n + R_n \geq N_T) \tag{4.15}$$

New message arrivals in the (n+1)-st frame are rejected if the total number of information packet retransmissions and PACK transmissions scheduled for the (n+1)-st period is greater than or equal to the (finite) fixed threshold N_T. Thus this control function admits new message arrivals only when the load on the channel is less than N_T.

Proposition 4.1

The vector Markov chains Z and X, which describe the evolution of the GRA channel under the PRA acknowledgement scheme, are ergodic using the single-threshold binary control function defined by (4.15).

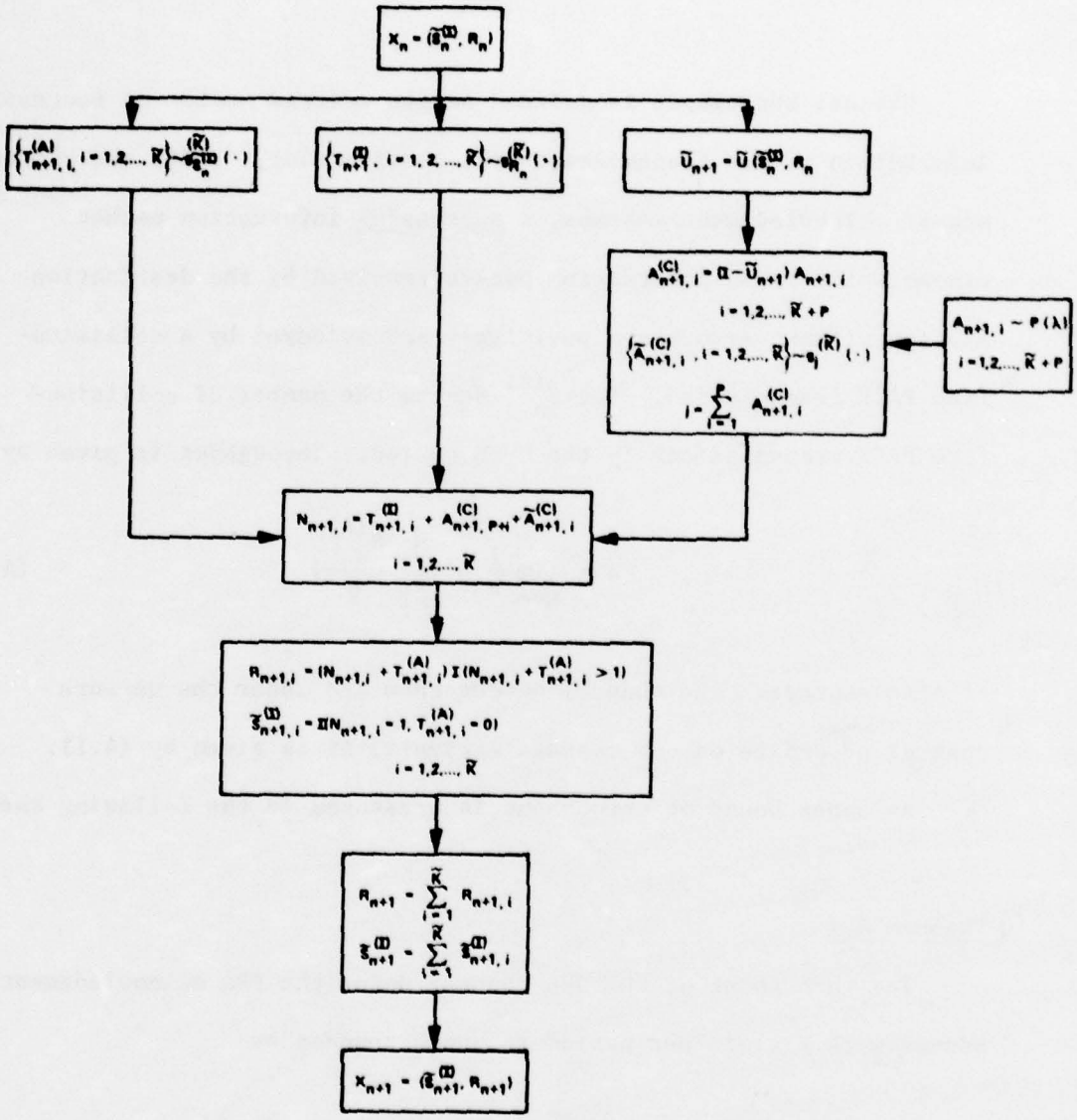


Figure 4.2 Transition $X_n \rightarrow X_{n+1}$ for the Markov State Chain X Associated with the GRA Discipline Under the PRA ACK Scheme

Proof

See Appendix C.

Channel throughput is defined as the average number of successful information packet transmissions per service slot. Under the random access acknowledgment schemes, a successful information packet transmission is an information packet received by the destination station without errors and positively acknowledged by a collision-free PACK transmission. Let $\tilde{S}_n^{(A)}$ denote the number of collision-free PACK transmissions in the n-th period. Throughput is given by

$$\tilde{\delta} = \lim_{N\to\infty} \frac{1}{N} E\{ \sum_{n=1}^{N} \frac{\tilde{S}_n^{(A)}}{\tilde{K}} \} \tag{4.16}$$

It also expresses the channel output rate and under the network control procedure on new message arrivals, it is given by (4.1).

An upper bound on throughput is presented in the following theorem.

Theorem 4.1

The throughput of the GRA channel under the PRA acknowledgment scheme with $\tilde{K}$ slots per period is upper bounded by

$$\tilde{\delta} < \frac{V_o(\tilde{K})}{\tilde{K}[1 + V_o(\tilde{K})\ V_1(\tilde{K})]} \approx \frac{e^{-1}}{1 + e^{-1}[1 - (1 - \frac{1}{\tilde{K}})^{\tilde{K}}]} \tag{4.17}$$

where

$$V_o(\tilde{K}) = e^{-1}\ \tilde{K}\{\ell n[1 + (\tilde{K} - 1)^{-1}]\ (\tilde{K} - 1)\}^{-1}$$

$$V_1(\tilde{K}) = [1 - (1 - \frac{1}{\tilde{K}})^{\tilde{K}}]\tilde{K}^{-1} .$$

Proof

The variable $\tilde{S}_n^{(A)}$ is given by

$$\tilde{S}_n^{(A)} = \sum_{i=1}^{\tilde{K}} I(T_{ni}^{(A)} = 1, N_{ni} = 0) , \tag{4.18}$$

$n \geq 1$. By the multinomial distribution (4.14), $T_{ni}^{(A)}$ is governed by a binomial distribution, $1 \leq i \leq \tilde{K}$. From (4.11) - (4.13), N_{ni} is governed by a binomial distribution given

$$N_n^{(I)} = \sum_{i=1}^{\tilde{K}} N_{ni} , \tag{4.19}$$

$1 \leq i \leq \tilde{K}$. Hence,

$$E(\tilde{S}_n^{(A)}) = E\{\tilde{S}_{n-1}^{(I)} (1 - \frac{1}{\tilde{K}})^{\tilde{S}_{n-1}^{(I)} + N_n^{(I)} - 1}\} . \tag{4.20}$$

From (4.10), similar arguments yield

$$E(\tilde{S}_n^{(I)}) = E\{N_n^{(I)} (1 - \frac{1}{\tilde{K}})^{\tilde{S}_{n-1}^{(I)} + N_n^{(I)} - 1}\} \tag{4.21}$$

From (4.20), $E(\tilde{S}_n^{(A)}) < E(\tilde{S}_{n-1}^{(I)})$ and, therefore,

$$\tilde{s} < \tilde{s}_I = \lim_{N \to \infty} \frac{1}{N} E\{\sum_{n=1}^{N} \frac{\tilde{S}_n^{(I)}}{\tilde{K}}\} \tag{4.22}$$

From (4.21), the conditional expectation of $\tilde{S}_n^{(I)}$ given $\tilde{S}_{n-1}^{(I)}$ is

$$E(\tilde{S}_n^{(I)} | \tilde{S}_{n-1}^{(I)}) = (1 - \frac{1}{\tilde{K}})^{\tilde{S}_{n-1}^{(I)}} E\{N_n^{(I)} (1 - \frac{1}{\tilde{K}})^{N_n^{(I)}-1} | \tilde{S}_{n-1}^{(I)}\} \quad (4.23)$$

Using the linear bound

$$(1 - \frac{1}{\tilde{K}})^{\tilde{S}_{n-1}^{(I)}} \leq V_1(\tilde{K})\ \tilde{S}_{n-1}^{(I)} + 1 \quad (4.24)$$

and applying the inequality

$$xa^x \leq e^{-1}[\ell n(a^{-1})]^{-1} \quad (4.25)$$

to (4.23), the limiting average number of collision-free information packet transmissions per service slot is bounded by

$$\tilde{S}_I \leq \frac{V_o(\tilde{K})}{[1 + V_o(\tilde{K})\ V_1(\tilde{K})]\tilde{K}} < V_o(\tilde{K})\ \tilde{K}^{-1} \quad (4.26)$$

where

$$V_o(\tilde{K}) = e^{-1}\ \tilde{K}\{\ell n[1 + (\tilde{K} - 1)^{-1}]\ (\tilde{K} - 1)\}^{-1}$$

$$V_1(\tilde{K}) = [1 - (1 - \frac{1}{\tilde{K}})^{\tilde{K}}]\tilde{K}^{-1}.$$

The desired upper bound on throughput follows from (4.22) and (4.26)

Q.E.D.

The upper bound on throughput given by (4.17) is not expected to be tight since $\tilde{\delta}_I$ is used to bound δ. Because PACKs are transmitted on a random access basis, this bound is expected to be excessive by a factor of approximately e. To derive an approximation for the maximum throughput $\tilde{\delta}_{max}$, add (4.20) and (4.21) and apply (4.25) which yields

$$E\{\tilde{S}_n^{(A)} + \tilde{S}_n^{(I)}\} \leq V_o(\tilde{K}) \tag{4.27}$$

with equality if

$$\tilde{S}_{n-1}^{(I)} + N_n^{(I)} = \frac{1}{\ell n[(1 - \frac{1}{\tilde{K}})^{-1}]} \triangleq V_2(\tilde{K}) \quad . \tag{4.28}$$

Equations (4.27) and (4.28) suggest that the maximum number of collision-free transmissions per period is achieved when the total number of transmissions per period is equal to the integer part of $V_2(\tilde{K})$. Assuming (4.28) provides an approximate condition for maximum throughput, substituting (4.28) into (4.20) and (4.21) yields the following approximation:

$$\tilde{\delta}_{max} \approx \tilde{\delta}_{I_{max}} e^{-1} \tag{4.29}$$

where

$$\tilde{\delta}_{I_{max}} \triangleq \frac{e^{-1} V_2(\tilde{K})}{\tilde{K}(1 + e^{-1}) - 1} \quad .$$

A straightforward simulation of the Markov chain X under the control function (4.15) has been used to compute maximum throughput values for $\tilde{K}$ = 6, 12, 18. These results are presented in Table 4.1. Results for the upper bound (4.17) and the approximation (4.29) are also tabulated. These results indicate that (4.29) is a fairly good approximation.

TABLE 4.1. MAXIMUM THROUGHPUT VALUES FOR THE GRA DISCIPLINE UNDER THE PRA ACK SCHEME

$\tilde{K}$	Maximum Throughput		
	Simulation	Approximation (4.29)	Upper Bound (4.17)
6	0.13	0.124	0.318
12	0.12	0.110	0.308
18	0.11	0.106	0.305

4.2.2 Packet Delay Analysis

The delay (measured in slots) of the n-th message is decomposed into the sum

$$D_{S_n} = \tilde{W}_n + R + 1 \tag{4.30}$$

where $\tilde{W}_n$ is the system waiting time of the n-th message measured from its arrival slot to the transmission slot of its collision-free PACK.

The single slot and R slot durations in (4.30) represent the transmission time and the propagation delay of the collision-free PACK. This delay measures the holding time of the n-th message in the source station's buffer; it is the interval between message arrival and its positive acknowledgement by a collision-free PACK. The limiting average delay is given by

$$E(D_S) = E(\tilde{W}) + R + 1 \tag{4.31}$$

where the limiting average system waiting time can be expressed as the limit (when it exists)

$$E(\tilde{W}) = \lim_{N\to\infty} \frac{1}{N} E\{ \sum_{n=1}^{N} \tilde{W}_n \} \quad . \tag{4.32}$$

Under an uncontrolled GRA access-control discipline, the channel state sequences are transient, yielding unbounded limiting average packet delay. Therefore, an appropriate network control function $\tilde{U}_n = \tilde{U}(\cdot)$ such as (4.15) is chosen so that the vector Markov chains Z and X are irreducible, positive recurrent. The transition probability functions and stationary distributions of Z and X are denoted by $\{P_Z(\underline{i}, \underline{j})\}$, $\{\pi_Z(\underline{i})\}$ and $\{P_X(\underline{i}, \underline{j})\}$, $\{\pi_X(\underline{i})\}$, respectively.

Let $N(Z_n, Z_{n+1})$ represent the number of new message arrivals admitted in the n-th frame and let $\tilde{W}(Z_n, Z_{n+1})$ denote the sum of the waiting-time components of all packets transmitted during the n-th period. The functions $N(\cdot)$ and $\tilde{W}(\cdot)$ are time-homogeneous functions

of $\{Y_n, n \geq 1\}$, $Y_n = (Z_n, Z_{n+1})$. The sequence $\{Y_n, n \geq 1\}$ is an irreducible, positive recurrent Markov chain with stationary distribution $\{\pi(\underline{i}, \underline{j})\}$ where $\pi(\underline{i}, \underline{j}) = \pi_Z(\underline{i})\, P_Z(\underline{i}, \underline{j})$.

Lemma 4.1

$$E(\tilde{W}) = \frac{E\{\tilde{W}(Z_n, Z_{n+1})\}}{E\{N(Z_n, Z_{n+1})\}} \tag{4.33}$$

where

$$E\{\tilde{W}(Z_n, Z_{n+1})\} = \sum_{\underline{i},\underline{j}} \tilde{W}(\underline{i}, \underline{j})\, \pi(\underline{i}, \underline{j})$$

$$E\{N(Z_n, Z_{n+1})\} = \sum_{\underline{i},\underline{j}} N(\underline{i}, \underline{j})\, \pi(\underline{i}, \underline{j}).$$

Proof

Considering nondegenerate controls that yield a packet rejection probability less than one ($P_R < 1$),

$$\lim_{N\to\infty} \sum_{n=1}^{N} N(Z_n, Z_{n+1}) \to \infty \qquad \text{w.p.1} \quad . \tag{4.34}$$

Therefore,

$$\lim_{N\to\infty} \frac{\sum_{n=1}^{N} \tilde{W}_n}{N} = \lim_{N\to\infty} \frac{\sum_{n=1}^{N} \tilde{W}(Z_n, Z_{n+1})}{\sum_{n=1}^{N} N(Z_n, Z_{n+1})} \qquad \text{w.p.1} \quad . \tag{4.35}$$

By applying a Markov ratio limit theorem in Chung [56] to both sides of (4.35),

$$\lim_{N\to\infty} \frac{\sum_{n=1}^{N} \tilde{W}_n}{N} = \lim_{N\to\infty} \frac{\sum_{n=1}^{N} E(\tilde{W}_n)}{N} \qquad \text{w.p.1} \qquad (4.36)$$

and

$$\lim_{N\to\infty} \frac{\sum_{n=1}^{N} \tilde{W}(Z_n, Z_{n+1})}{\sum_{n=1}^{N} N(z_n, Z_{n+1})} = \frac{E\{\tilde{W}(Z_n, Z_{n+1})\}}{E\{N(Z_n, Z_{n+1})\}} \qquad \text{w.p.1} \qquad (4.37)$$

Equation (4.33) follows by substituting (4.36) and (4.37) into (4.35).

Q.E.D.

Applying Lemma 4.1, the limiting average packet delay can be computed as follows.

Theorem 4.2

With the operation of a controlled GRA channel under the PRA acknowledgment scheme described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delay is given by

$$E(D_S) = \frac{P}{2} + (\tilde{K}+P)\frac{E(R_n)}{E(\tilde{S}_n^{(A)})} + (\tilde{K}+P)\frac{E(\tilde{S}_n^{(I)})}{E(\tilde{S}_n^{(A)})} + R + 1 \qquad (4.38)$$

where the expectations are w.r.t. the stationary distribution $\{\pi(\underline{i},\underline{j})\}$.

Proof

For the controlled GRA channel,

$$N(Z_n, Z_{n+1}) = A_n^{(C)} \tag{4.39}$$

where $A_n^{(C)} = \sum_{i=1}^{\tilde{K}+P} A_{ni}^{(C)}$. Since each message admitted by the network control procedure is served until acknowledged by a collision-free PACK, in equilibrium applying the stationary distribution $\{\pi(\underline{i},\underline{j})\}$,

$$E(A_n^{(C)}) = E(\tilde{S}_n^{(A)}) \tag{4.40}$$

and, therefore,

$$E\{N(Z_n, Z_{n+1})\} = E(\tilde{S}_n^{(A)}) \tag{4.41}$$

The system waiting time function $\tilde{W}(Z_n, Z_{n+1})$ is decomposed into the sum

$$\tilde{W}(Z_n, Z_{n+1}) = \tilde{W}_o(Z_n, Z_{n+1}) + \tilde{W}_1(Z_n, Z_{n+1}) \tag{4.41}$$

where $\tilde{W}_o(Z_n, Z_{n+1})$ is the sum of the waiting time components of all new message arrivals in the n-th frame measured from the arrival slot to the transmission slot and $\tilde{W}_1(Z_n, Z_{n+1})$ is the sum of the waiting

time components of all packets (both information packets and PACKs) transmitted during the n-th period measured from the transmission slot in the n-th period to the resultant action slot in the (n+1)-st period. Since new message arrivals in the n-th period are transmitted immediately, only those messages which arrive during the P non-service slots contribute to $\tilde{W}_o(Z_n, Z_{n+1})$. Therefore,

$$\tilde{W}_o(Z_n, Z_{n+1}) = PA_{n1}^{(C)} + (P-1)A_{n2}^{(C)} + \ldots + A_{nP}^{(C)}$$

$$+ \tilde{A}_{n2}^{(C)} + 2\tilde{A}_{n3}^{(C)} + \ldots + (\tilde{K}-1)\tilde{A}_{n\tilde{K}}^{(C)} \qquad (4.43)$$

By symmetry

$$E(A_{ni}^{(C)}) = \frac{E(A_n^{(C)})}{\tilde{K} + P} \quad , \quad 1 \leq i \leq P+\tilde{K} \qquad (4.44)$$

and

$$E(\tilde{A}_{ni}^{(C)}) = \frac{PE(A_n^{(C)})}{\tilde{K}(\tilde{K} + P)} \quad , \quad 1 \leq i \leq \tilde{K} . \qquad (4.45)$$

Hence,

$$E\{\tilde{W}_o(Z_n, Z_{n+1})\} = \frac{PE(A_n^{(C)})}{2} \qquad (4.46)$$

Collision-free PACK transmissions in the n-th period, require no action in the (n+1)-st period. Collision-free information packet transmissions in the n-th period require PACK transmissions within

the (n+1)-st period. Each information packet collision or PACK collision in the n-th period, requires the retransmission of the appropriate information packet within the (n+1)-st period. Therefore,

$$\tilde{W}_1(Z_n, Z_{n+1}) = \tilde{K}\, R_{n1} + (\tilde{K}-1)R_{n2} + \ldots + R_{n\tilde{K}} + P\, R_n$$

$$+ T^{(I)}_{n+1,2} + 2T^{(I)}_{n+1,3} + \ldots + (\tilde{K}-1)T^{(I)}_{n+1,\tilde{K}}$$

$$+ \tilde{K}\, \tilde{S}^{(I)}_{n1} + (\tilde{K}-1)\tilde{S}^{(I)}_{n2} + \ldots + \tilde{S}^{(I)}_{n\tilde{K}} + P\, \tilde{S}^{(I)}_n$$

$$+ T^{(A)}_{n+1,2} + 2T^{(A)}_{n+1,3} + \ldots + (\tilde{K}-1)T^{(A)}_{n+1,\tilde{K}} \quad . \qquad (4.47)$$

By symmetry

$$E(R_{ni}) = E(T^{(I)}_{n+1,i}) = \frac{E(R_n)}{\tilde{K}} \quad , \quad 1 \le i \le \tilde{K} \qquad (4.48)$$

and

$$E(\tilde{S}^{(I)}_{ni}) = E(T^{(A)}_{n+1,i}) = \frac{E(\tilde{S}^{(I)}_n)}{\tilde{K}} \quad , \quad 1 \le i \le \tilde{K} \; . \qquad (4.49)$$

Therefore,

$$E\{\tilde{W}_1(Z_n, Z_{n+1})\} = (\tilde{K}+P)\; E(R_n) + (\tilde{K}+P)\; E(\tilde{S}^{(I)}_n) \quad . \qquad (4.50)$$

Hence, $E(\tilde{W})$ is computed by applying Lemma 4.1 using (4.41), (4.46) and (4.50). The limiting average packet delay (4.38) follows

from (4.31).

Q.E.D.

The terms $E(R_n)/E(\tilde{S}_n^{(A)})$ and $E(\tilde{S}_n^{(I)})/E(\tilde{S}_n^{(A)})$ represent the average number of information packet retransmissions per admitted message and the average number of PACK transmissions per admitted message, respectively. Thus (4.38) indicates that each information packet retransmission and each PACK transmission contributes an average delay of $\tilde{K}+P$ slots.

4.3 Multiple Copy - Random Access Acknowledgment Scheme

Under the MCRA acknowledgment scheme, information packets and PACKs gain channel access on a random access basis. Like the PRA scheme both information packets and PACKs contend over the $\tilde{K}$ service slots in each period. However, under the MCRA scheme, acknowledgment traffic is given preference over data traffic by sending multiple, identical PACKs in the same period for each collision-free information packet transmission in the previous period. The GRA channel using the MCRA acknowledgment scheme operates as follows.

Protocol: GRA Discipline - MCRA ACK Scheme

(1) and (2) See the corresponding steps under the basic GRA discipline (Section 4.1.2).

(3) For each collision-free information packet transmitted within the n-th period, γ identical PACKs are transmitted within the (n+1)-st period in distinct service slots. Each of the $\binom{\tilde{K}}{\gamma}$ PACK transmission allocations is equally likely.

(4) Each collision-free information packet transmission within the n-th period which remains unacknowledged at the end of the two frame acknowledgment time-out interval (i.e., all γ identical PACKs transmitted within the (n+1)-st period experience collision) is retransmitted within the (n+2)-nd period in a slot determined by a uniform distribution over the $\tilde{K}$ available service slots.

(5) Each information packet admitted by the network control procedure is transmitted and retransmitted until successfully acknowledged by a collision-free PACK (i.e., by at least one of the γ identical PACKs).

4.3.1 Channel State Process

The evolution of this channel is described by a vector Markov chain $Z = \{Z_n, n \geq 1\}$ over the space $d^{\tilde{K}} \times d_{\tilde{K}}^{\tilde{K}} \times d^{\tilde{K}+P} \times d_1^{\tilde{K}}$ where d is the set of non-negative integers, $d_{\tilde{K}} = \{0, 1, \ldots, \tilde{K}\}$, $d_1 = \{0, 1\}$ and

$$Z_n = \{(T_{ni}^{(I)},\ i = 1, 2, \ldots, \tilde{K}),\ (T_{ni}^{(A)},\ i = 1, 2, \ldots, \tilde{K}),$$

$$(A_{ni},\ i = 1, 2, \ldots, \tilde{K}+P),\ (\tilde{S}_{ni}^{(A)},\ i = 1, 2, \ldots, \tilde{K})\}\ .$$

The variables $T_{ni}^{(I)}$, $T_{ni}^{(A)}$ and A_{ni} were previously defined in Section 4.2.1 under the PRA acknowledgment scheme. The variables $T_{ni}^{(I)}$ and $T_{ni}^{(A)}$ denote the number of information packet retransmissions and the number of PACK transmissions, respectively, in the i-th slot within the n-th period, $n \geq 1$, $1 \leq i \leq \tilde{K}$. The number of uncontrolled new message arrivals in the i-th slot within the n-th frame is denoted by A_{ni}, $n \geq 1$, $1 \leq i \leq \tilde{K}+P$. The 0-1 variable $\tilde{S}_{ni}^{(A)}$ indicates the occurrence of a first collision-free PACK transmission among the γ copies in the i-th slot within the n-th period.

Let $R_n^{(I)}$ denote the number of information packet collisions within the n-th period and let $R_n^{(A)}$ denote the number of information packets unsuccessfully acknowledged within the n-th period. An unsuccessful acknowledgment occurs when each of the γ PACKs (associated with a single collision-free information packet transmission) experiences collision. Then the effective number of collisions within the n-th period or, equivalently, the number of information packets which must be retransmitted within the (n+1)-st period is denoted by the sum

$$R_n = R_n^{(I)} + R_n^{(A)} \tag{4.51}$$

The variable R_n is also expressed by (4.5); and, therefore $\{T_{n+1,i}^{(I)}\}$ is

governed by the multinomial distribution (4.13).

The number of information packet collisions within the n-th period is determined by

$$R_n^{(I)} = \sum_{i=1}^{\tilde{K}} R_{ni}^{(I)} \tag{4.52}$$

where $R_{ni}^{(I)}$ is the number of information packet collisions in the i-th slot within the n-th period:

$$R_{ni}^{(I)} = N_{ni}\, I(N_{ni} \geq 1,\ N_{ni} + T_{ni}^{(A)} > 1) \tag{4.53}$$

The number of information packet transmissions in the i-th slot within the n-th period, N_{ni}, is specified by (4.11).

For each of the $\tilde{S}_n^{(I)}$ collision-free information packet transmissions within the n-th period, γ PACKs are transmitted within the (n+1)-st period. Let T(n+1, i, m) indicate a PACK transmission in the i-th slot within the (n+1)-st period for the m-th collision-free information packet transmission within the n-th period:

$$T(n+1,\ i,\ m) = \begin{cases} 1 & \text{if PACK transmission} \\ 0 & \text{if no PACK transmission} \end{cases}$$

where $n \geq 1$, $1 \leq i \leq \tilde{K}$, $m = 1, 2, \ldots, \tilde{S}_n^{(I)}$. The set of allocations $\{T(n, i, m),\ 1 \leq i \leq \tilde{K}\}$ is governed by

$$P(T(n, i, m) = \alpha_i,\ 1 \le i \le \tilde{K}) = \frac{1}{\binom{\tilde{K}}{\gamma}} \tag{4.54}$$

where $\alpha_i \in \{0, 1\}$, $1 \le i \le \tilde{K}$, $\sum_{i=1}^{\tilde{K}} \alpha_i = \gamma$ and $\binom{\tilde{K}}{\gamma}$ denotes the binomial coefficient

$$\binom{\tilde{K}}{\gamma} = \frac{\tilde{K}!}{\gamma!\ (\tilde{K} - \gamma)!} \quad . \tag{4.55}$$

Thus the total number of PACK transmissions in the i-th slot within the (n+1)-st period is given by*

$$T^{(A)}_{n+1,i} = \sum_{m=1}^{\tilde{S}_n^{(I)}} T(n+1, i, m) \quad . \tag{4.56}$$

The number of collision-free information packet transmissions within the n-th period is specified by

$$\tilde{S}_n^{(I)} = N_n - R_n^{(I)} \tag{4.57}$$

where $N_n = \sum_{i=1}^{\tilde{K}} N_{ni}$.

Let $\tilde{S}(n+1, i, m)$ indicate a collision-free PACK transmission in the i-th slot within the (n+1)-st period for the m-th collision-free information packet transmission within the n-th period:

*Empty summations are set equal to 0 (i.e., when $\tilde{S}_n^{(I)} = 0$).

$$\tilde{S}(n+1,\ i,\ m) = I(T(n+1,\ i,\ m) = 1,\ N_{n+1,i} + T^{(A)}_{n+1,i} = 1) \qquad (4.58)$$

where $1 \leq i \leq \tilde{K}$, $m = 1, 2, \ldots, \tilde{S}^{(I)}_n$. The slot position of the first collision-free PACK transmission (among the γ copies) within the (n+1)-st period for the m-th collision-free information packet transmission within the n-th period is denoted by

$$\tilde{J}(n+1,m) = \begin{cases} \min\{i:\ \tilde{S}(n+1,i,m) = 1\} & \text{if } \sum_{j=1}^{\tilde{K}} \tilde{S}(n+1,j,m) \geq 1 \\ \infty & \text{Otherwise} \end{cases} \qquad (4.59)$$

Thus the number of successful acknowledgments (first collision-free PACK among the γ PACKs) in the i-th slot within the (n+1)-st period is given by

$$\tilde{S}^{(A)}_{n+1,i} = \sum_{m=1}^{\tilde{S}^{(I)}_n} I(\tilde{J}(n+1,\ m) = i)\ . \qquad (4.60)$$

Hence,

$$\tilde{S}^{(A)}_{n+1} = \sum_{i=1}^{\tilde{K}} \tilde{S}^{(A)}_{n+1,i} \qquad (4.61)$$

and the number of unsuccessful acknowledgments within the (n+1)-st period is specified by

$$R_{n+1}^{(A)} = \tilde{S}_n^{(I)} - \tilde{S}_{n+1}^{(A)} \quad . \tag{4.62}$$

Equations (4.4) - (4.13) and (4.51) - (4.62) yield the transition probability function for the Markov state sequence Z under the MCRA scheme. One observes that the uncontrolled new message arrival variables $\{A_{n+1,i},\ 1 \leq i \leq \tilde{K}+P\}$ are statistically independent of Z_n and that $\{T_{n+1,i}^{(I)}\}$, $\{T_{n+1,i}^{(A)}\}$ and $\{\tilde{S}_{n+1,i}^{(A)}\}$ depend on Z_n only through $(\tilde{S}_n^{(I)}, R_n)$. The sequence $X = \{X_n,\ n \geq 1\}$, $X_n = (\tilde{S}_n^{(I)}, R_n)$, is a vector Markov chain over the space $d_{\tilde{K}} \times d$. A flow diagram indicating the transition $X_n \rightarrow X_{n+1}$ is shown in Figure 4.3.

Two possible network control functions which yield irreducible positive recurrent Markov state sequences are

$$\hat{U}_{n+1} = I(R_n + \tilde{S}_n^{(I)} \geq N_T) \tag{4.63}$$

$$\tilde{U}_{n+1} = I(R_n + \gamma\, \tilde{S}_n^{(I)} \geq N_T). \tag{4.64}$$

Both control functions reject new message arrivals within the (n+1)-st frame if the threshold N_T is exceeded. The first control function (4.63) denies channel access to new arrivals whenever the number of information packet retransmissions plus the number of information packets which require acknowledgment exceeds the threshold. Hence, this control essentially limits the outstanding transmission obligations on the channel. The second control function (4.64) rejects new

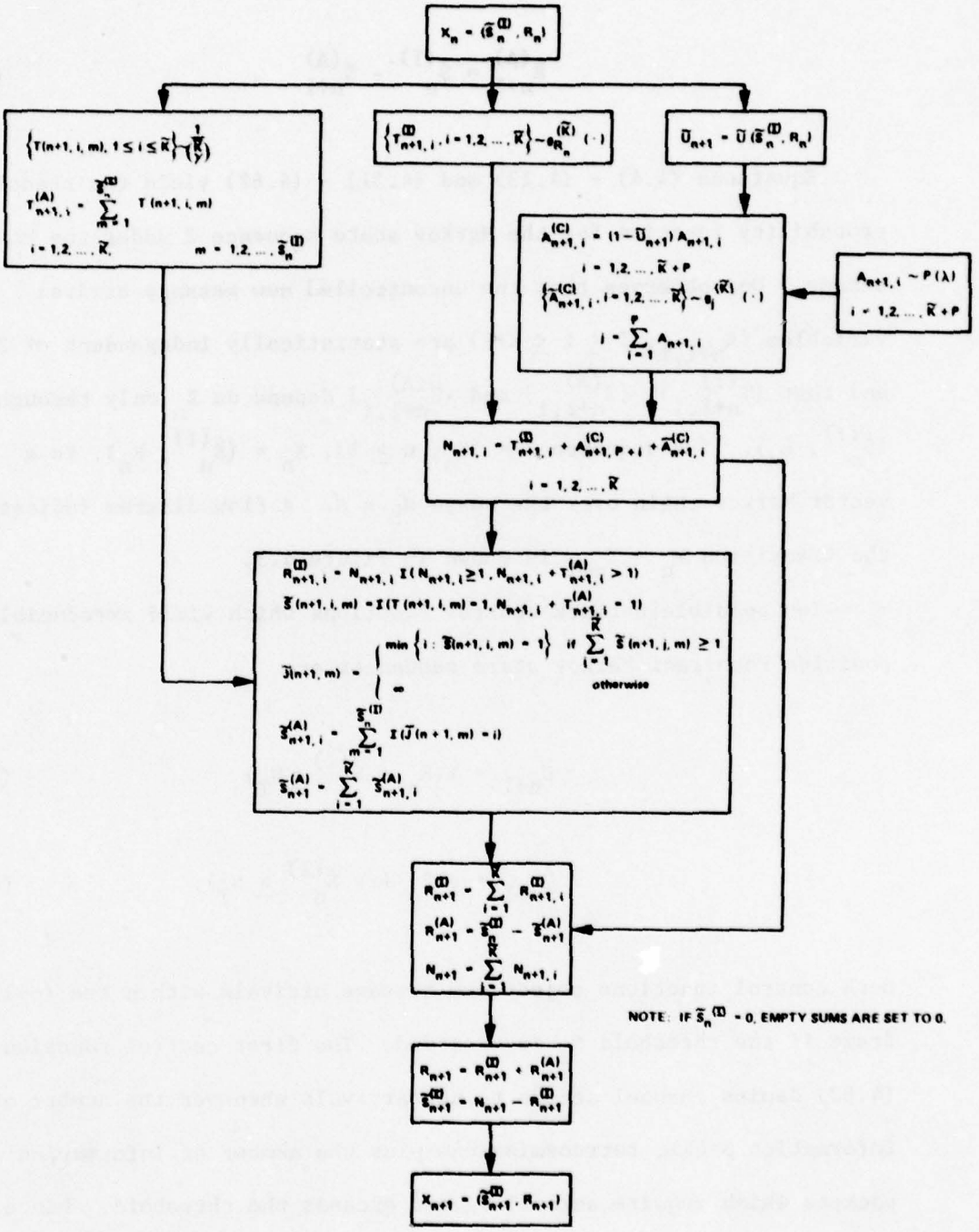


Figure 4.3 Transition $X_n \rightarrow X_{n+1}$ for the Markov State Chain X Associated with the GRA Discipline Under the MCRA ACK Scheme

arrivals, whenever the number of information packet retransmissions plus the number of PACK transmissions exceeds the threshold. Thus this control denies channel access whenever the total channel load exceeds the threshold.

Proposition 4.2

The vector Markov chains Z and X, which describe the evolution of the GRA channel under the MCRA acknowledgment scheme, are irreducible, positive recurrent using either single-threshold binary control function defined by (4.63) and (4.64).

Proof

Similar technique as Proposition 4.1.

Q.E.D.

Throughput is expressed by the limit (4.16) where $\tilde{S}_n^{(A)}$ denotes the number of successful (positive) acknowledgments within the n-th period. Under the MCRA scheme, a successful acknowledgment occurs when at least one of the γ PACKs is transmitted collision-free. Thus $\tilde{S}_n^{(A)}$ is given by (4.61); it can also be expressed as

$$\tilde{S}_n^{(A)} = \sum_{m=1}^{\tilde{S}_{n-1}^{(I)}} I(\tilde{S}(n,1,m) = 1 \text{ or } \tilde{S}(n,2,m) = 1 \text{ or } \ldots \text{ or } \tilde{S}(n,\tilde{K},m) = 1) \; . \tag{4.65}$$

Working with (4.65), the following upper bound on throughput is derived.

Theorem 4.3

The throughput of the GRA channel under the MCRA acknowledgment scheme using γ multiple identical PACKs with $\tilde{K}$ slots per period is upper bounded by

$$\tilde{\delta} < \frac{V_o(\tilde{K})}{\tilde{K}[1 + V_o(\tilde{K})\ V_\gamma(\tilde{K})]} \simeq \frac{e^{-1}}{1 + e^{-1}[1 - (1 - \frac{\gamma}{\tilde{K}})^{\tilde{K}}]} \tag{4.66}$$

where

$$V_o(\tilde{K}) = e^{-1}\ \tilde{K}\{\ell n[1 + (\tilde{K} - 1)^{-1}](\tilde{K} - 1)\}^{-1}$$

$$V_\gamma(\tilde{K}) = [1 - (1 - \tfrac{\gamma}{\tilde{K}})^{\tilde{K}}]\tilde{K}^{-1}$$

Proof

Rewriting (4.65)

$$\tilde{S}_n^{(A)} = \tilde{S}_{n-1}^{(I)} - \sum_{m=1}^{\tilde{S}_{n-1}^{(I)}} I(\tilde{S}(n,1,m) = 0,\ \tilde{S}(n,2,m) = 0,\ \ldots,\ \tilde{S}(n,\tilde{K},m) = 0). \tag{4.67}$$

The expectation of $\tilde{S}_n^{(A)}$ given $\tilde{S}_{n-1}^{(I)}$ is

$$E(\tilde{S}_n^{(A)} | \tilde{S}_{n-1}^{(I)}) = \tilde{S}_{n-1}^{(I)} - \tilde{S}_{n-1}^{(I)}\ P(\tilde{S}(n,1,m) = 0,\ \tilde{S}(n,2,m) = 0,\ \ldots,$$

$$\tilde{S}(n,\tilde{K},m) = 0 | \tilde{S}_{n-1}^{(I)}) . \tag{4.68}$$

Therefore,

$$E(\tilde{S}_n^{(A)}) < E(\tilde{S}_{n-1}^{(I)}) .$$

Thus maximum throughput can be bounded by bounding $\tilde{S}_{n-1}^{(I)}$ following a procedure used in the proof of Theorem 4.1.

The number of collision-free information packet transmissions within the n-th period can be expressed as

$$\tilde{S}_n^{(I)} = \sum_{i=1}^{\tilde{K}} I(T_{ni}^{(A)} = 0,\ N_{ni} = 1) \ . \tag{4.69}$$

Since

$$P(T_{ni}^{(A)} = 0 | \tilde{S}_{n-1}^{(I)}) = (1 - \frac{\gamma}{\tilde{K}})^{\tilde{S}_{n-1}^{(I)}} \ , \tag{4.70}$$

the expectation of $\tilde{S}_n^{(I)}$ given $\tilde{S}_{n-1}^{(I)}$ is

$$E(\tilde{S}_n^{(I)} | \tilde{S}_{n-1}^{(I)}) = (1 - \frac{\gamma}{\tilde{K}})^{\tilde{S}_{n-1}^{(I)}} E\{N_n (1 - \frac{1}{\tilde{K}})^{N_n - 1} | \tilde{S}_{n-1}^{(I)}\} \ . \tag{4.71}$$

The techniques applied to (4.23) in the proof of Theorem 4.1 are applied to (4.71) to obtain (4.66).

Q.E.D.

The bounds on throughput stated in Theorems 4.1 and 4.3 for the PRA and MCRA acknowledgment schemes, respectively, have the same structure. Moreover, since the two schemes are identical when $\gamma = 1$, the bounds are equivalent for this case. Equation (4.66) is not expected to provide a tight bound on maximum throughput, since the second term on the right hand side of (4.68) is not included.

A straightforward simulation of the Markov chain X has been used to compute maximum throughput values for $\tilde{K} = 12$, $\gamma = 1, 2, 3, 4$. These results are presented in Table 4.2 along with the bounds calculated using (4.66). The values computed using (4.66) exceed the simulation results by approximately the factor e. The simulation results indicate that for $\tilde{K} = 12$, a two PACK acknowledgment scheme ($\gamma = 2$) provides the largest maximum throughput value.

TABLE 4.2. MAXIMUM THROUGHPUT VALUES FOR THE GRA DISCIPLINE UNDER THE MCRA ACK SCHEME WITH $\tilde{K} = 12$

γ	Maximum Throughput	
	Simulation	Upper Bound (4.66)
1	0.12	0.308
2	0.135	0.287
3	0.131	0.280
4	0.117	0.278

4.3.2 Packet Delay Analysis

The delay of the n-th message is defined by (4.30). However, under the MCRA acknowledgment scheme, multiple identical PACKs are transmitted for each collision-free information packet transmission. Therefore, the system waiting time $\tilde{W}_n$ is measured from the arrival slot to the transmission slot of its first collision-free PACK. The technique developed in Section 4.2.2 for the PRA scheme is used to calculate the limiting average packet delay under the MCRA scheme.

Theorem 4.4

With the operation of a controlled GRA channel under the MCRA acknowledgment scheme described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delay is given by

$$E(D_S) = \frac{P}{2} + (\tilde{K}+P)\frac{E(R_n^{(I)})}{E(\tilde{S}_n^{(A)})} + (\frac{2P+\tilde{K}+1}{2})\frac{E(\tilde{S}_n^{(I)})}{E(\tilde{S}_n^{(A)})} + (\frac{2P+3\tilde{K}-1}{2})\frac{E(R_n^{(A)})}{E(\tilde{S}_n^{(A)})}$$
$$+ \frac{E(\tilde{S}_{n2}^{(A)}) + 2E(\tilde{S}_{n3}^{(A)}) + \ldots + (\tilde{K}-1)\,E(\tilde{S}_{nK}^{(A)})}{E(\tilde{S}_n^{(A)})} + R + 1 \qquad (4.72)$$

where the expectations are w.r.t. the stationary distribution.

Proof

Assume an appropriate network control function is chosen so that Z is an irreducible, positive recurrent state sequence. Consider the two functionals $N(\cdot)$ and $\tilde{W}(\cdot)$ defined in Section 4.2.2. The time homogeneous function $N(Z_n, Z_{n+1})$ is determined by (4.39) - (4.41). The system waiting time function is given by (4.42) and $\tilde{W}_o(Z_n, Z_{n+1})$ is determined by (4.43) - (4.46).

To evaluate $\tilde{W}_1(Z_n, Z_{n+1})$, consider the waiting time components of each type of transmission separately. For information packet transmissions within the n-th period which experience collision, the interval from the transmission slot within the n-th period to the retransmission slot within the (n+1)-st period is measured:

$$\tilde{W}_{11}(Z_n, Z_{n+1}) = \tilde{K}\, R_{n1}^{(I)} + (\tilde{K}-1)R_{n2}^{(I)} + \ldots + R_{n\tilde{K}}^{(I)} + P\, R_n^{(I)}$$

$$+ T_{n+1,2}^{(I)} + 2T_{n+1,3}^{(I)} + \ldots + (\tilde{K}-1)\, T_{n+1,\tilde{K}}^{(I)} . \qquad (4.73)$$

For collision-free information packet transmissions within the n-th period, the waiting time component is measured from the transmission slot within the n-th period to the beginning of the (n+1)-st period:

$$\tilde{W}_{12}(Z_n, Z_{n+1}) = \tilde{K}\, \tilde{S}_{n1}^{(I)} + (\tilde{K}-1)\, \tilde{S}_{n2}^{(I)} + \ldots + \tilde{S}_{n\tilde{K}}^{(I)} + P\, \tilde{S}_n^{(I)}. \qquad (4.74)$$

For unsuccessful acknowledgments (i.e., all γ PACKs experience collision) within the n-th period, the interval from the beginning of the n-th period to the information packet retransmission slot within the (n+1)-st period is measured:

$$\tilde{W}_{13}(Z_n, Z_{n+1}) = (\tilde{K}+P)R_n^{(A)} + T_{n+1,2}^{(A)} + 2T_{n+1,3}^{(A)} + \ldots + (\tilde{K}-1)\, T_{n+1,\tilde{K}}^{(A)} . \qquad (4.75)$$

For successful acknowledgments (i.e., at least one of the γ PACKs is transmitted collision-free) the waiting time component is measured from the beginning of the n-th period to the transmission slot of the first collision-free PACK:

$$\tilde{W}_{14}(Z_n, Z_{n+1}) = \tilde{S}_{n2}^{(A)} + 2\tilde{S}_{n3}^{(A)} + \ldots + (\tilde{K}-1)\, \tilde{S}_{n\tilde{K}}^{(A)} . \qquad (4.76)$$

Therefore,

$$\tilde{W}_1(Z_n, Z_{n+1}) = \sum_{j=1}^{4} \tilde{W}_{1j}(Z_n, Z_{n+1}) \ . \tag{4.77}$$

By applying Lemma 4.1 and by taking advantage of the symmetry provided by the uniform distributed slot allocations in each period for information packet transmissions, (4.72) is proved.

Q.E.D.

The term $E(R_n^{(I)})/E(\tilde{S}_n^{(A)})$ in (4.72) represents the average number of information packet collisions per admitted message while $E(R_n^{(A)})/E(\tilde{S}_n^{(A)})$ represents the average number of unsuccessful acknowledgment attempts per admitted message. The average number of collision-free information packet transmissions per admitted message is denoted by $E(\tilde{S}_n^{(I)})/E(\tilde{S}_n^{(A)})$. The term $E(\tilde{S}_{ni}^{(A)})/E(\tilde{S}_n^{(A)})$ represents the conditional probability of a successful acknowledgment (the first collision-free PACK among the γ copies) in the i-th slot within a period, $1 \leq i \leq \tilde{K}$, given that at least one successful acknowledgment is made in the period. One observes that the set of probabilities $\{E(\tilde{S}_{ni}^{(A)})\}$ (the probability of a successful acknowledgment in the i-th slot within a period) satisfies

$$E(\tilde{S}_{ni}^{(A)}) \geq E(\tilde{S}_{nj}^{(A)}) \tag{4.78}$$

for $i \leq j$, $1 \leq i \leq \tilde{K}$, $1 \leq j \leq \tilde{K}$.

In equilibrium, applying the stationary distribution to (4.62),

$$\lim_{n\to\infty} E(\tilde{S}_n^{(I)}) = \lim_{n\to\infty} E(R_n^{(A)}) + \lim_{n\to\infty} E(\tilde{S}_n^{(A)}) \ . \tag{4.79}$$

Substituting (4.79) into (4.72), the limiting average delay can be rewritten as

$$E(D_S) = \frac{P}{2} + (\tilde{K}+P)\frac{E(R_n)}{E(\tilde{S}_n^{(A)})} + (\tilde{K}+P)\frac{E(\tilde{S}_n^{(I)})}{E(\tilde{S}_n^{(A)})} + \frac{E(\tilde{S}_{n2}^{(A)}) + 2E(\tilde{S}_{n3}^{(A)}) + \ldots + (\tilde{K}-1)\,E(\tilde{S}_{nK}^{(A)})}{E(\tilde{S}_n^{(A)})} - \frac{\tilde{K}-1}{2} + R + 1 \qquad (4.80)$$

where the expectations are w.r.t. the stationary distribution. In general, $\{\tilde{S}_{ni}^{(A)},\ 1 \leq i \leq \tilde{K}\}$ obey the inequality specified by (4.78). In particular, when the number of PACK copies is set equal to 1 ($\gamma = 1$),

$$E(\tilde{S}_{ni}^{(A)}) = \frac{E(\tilde{S}_n^{(A)})}{\tilde{K}}, \qquad (4.81)$$

$1 \leq i \leq \tilde{K}$, and in this case (4.80) reduces to (4.38).

4.4 Period Division - Random Access Acknowledgment Scheme

Under the PDRA acknowledgment scheme, information packets and PACKs gain channel access on a random access basis. Unlike either the PRA or MCRA schemes, information packets and PACKs do not contend over the same slots. Each period is partitioned into two fixed length subperiods; one subperiod is dedicated to information packet transmissions, while the other subperiod is dedicated to PACK transmissions.

Thus by proper slot allocation, preference to acknowledgment traffic can be given so that the delay and throughput performance is optimized.

Let $\tilde{K}_A$ and $\tilde{K}_I$ denote the number of slots in the acknowledgment subperiod and information subperiod, respectively, such that

$$\tilde{K} = \tilde{K}_A + \tilde{K}_I \tag{4.82}$$

An example of a partitioned period is illustrated in Figure 4.4. The acknowledgment subperiod precedes the information subperiod. In general, the PACK length (in bits) will be less than the information packet length. Let η denote the ratio of information packet length to PACK length. Thus under the PDRA scheme, the $\tilde{K}_A$ slots allocated to PACK transmissions are divided into

$$\tilde{K}_\eta = \lfloor \eta \tilde{K}_A \rfloor \tag{4.83}$$

mini-slots where $\lfloor x \rfloor$ denotes the integer part of x (under the restriction $\eta \geq 1$).

The GRA channel under the PDRA acknowledgment scheme operates as follows.

Protocol: GRA Discipline - PDRA ACK Scheme

(1) New message arrivals which are admitted by the network control procedure are

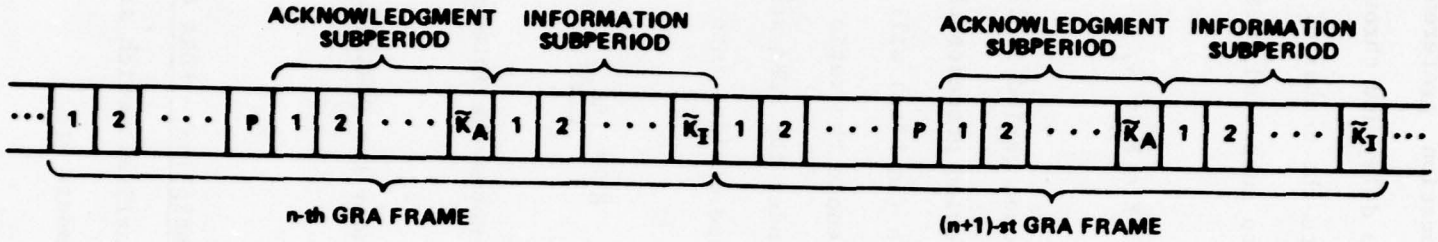


Figure 4.4 GRA Channel Structure Under the PDRA ACK Scheme

Thus by proper slot allocation, preference to acknowledgment traffic can be given so that the delay and throughput performance is optimized.

Let $\tilde{K}_A$ and $\tilde{K}_I$ denote the number of slots in the acknowledgment subperiod and information subperiod, respectively, such that

$$\tilde{K} = \tilde{K}_A + \tilde{K}_I \tag{4.82}$$

An example of a partitioned period is illustrated in Figure 4.4. The acknowledgment subperiod precedes the information subperiod. In general, the PACK length (in bits) will be less than the information packet length. Let η denote the ratio of information packet length to PACK length. Thus under the PDRA scheme, the $\tilde{K}_A$ slots allocated to PACK transmissions are divided into

$$\tilde{K}_\eta = \lfloor \eta \tilde{K}_A \rfloor \tag{4.83}$$

mini-slots where $\lfloor x \rfloor$ denotes the integer part of x (under the restriction $\eta \geq 1$).

The GRA channel under the PDRA acknowledgment scheme operates as follows.

Protocol: GRA Discipline - PDRA ACK Scheme

(1) New message arrivals which are admitted by the network control procedure are

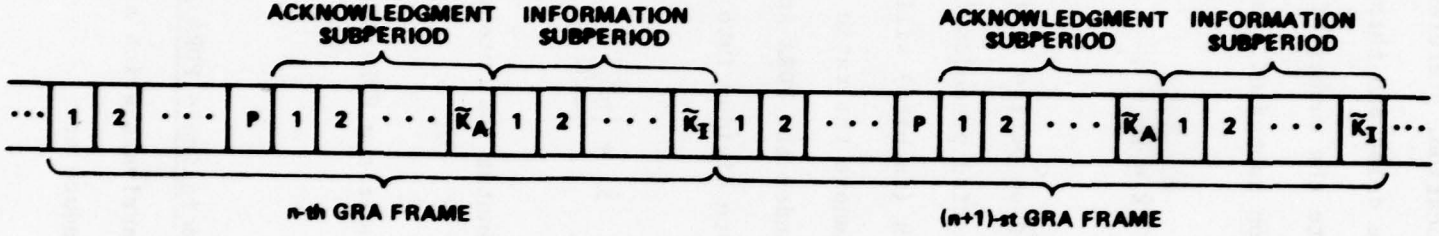


Figure 4.4 GRA Channel Structure Under the PDRA ACK Scheme

Thus by proper slot allocation, preference to acknowledgment traffic can be given so that the delay and throughput performance is optimized.

Let $\tilde{K}_A$ and $\tilde{K}_I$ denote the number of slots in the acknowledgment subperiod and information subperiod, respectively, such that

$$\tilde{K} = \tilde{K}_A + \tilde{K}_I \tag{4.82}$$

An example of a partitioned period is illustrated in Figure 4.4. The acknowledgment subperiod precedes the information subperiod. In general, the PACK length (in bits) will be less than the information packet length. Let η denote the ratio of information packet length to PACK length. Thus under the PDRA scheme, the $\tilde{K}_A$ slots allocated to PACK transmissions are divided into

$$\tilde{K}_\eta = \lfloor \eta \tilde{K}_A \rfloor \tag{4.83}$$

mini-slots where $\lfloor x \rfloor$ denotes the integer part of x (under the restriction $\eta \geq 1$).

The GRA channel under the PDRA acknowledgment scheme operates as follows.

Protocol: GRA Discipline - PDRA ACK Scheme

(1) New message arrivals which are admitted by the network control procedure are

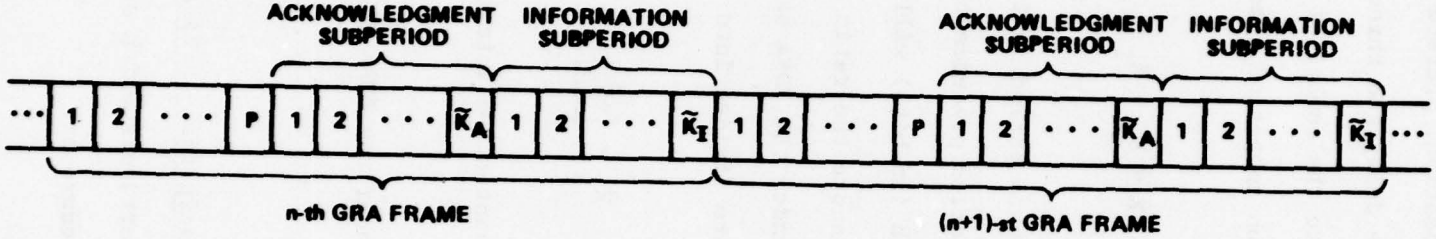


Figure 4.4 GRA Channel Structure Under the PDRA ACK Scheme

i) transmitted immediately, if they arrive during a channel access subperiod allocated to information packet transmissions

ii) transmitted in the next information subperiod, in a slot determined by a uniform distribution over the $\tilde{K}_I$ available service slots, if they arrive in the interval between periods or during the acknowledgment subperiod.

Messages which are not admitted by the network control procedure are rejected.

(2) Each information packet colliding within the n-th period is retransmitted within the (n+1)-st information subperiod in a slot determined by a uniform distribution over the $\tilde{K}_I$ available service slots.

(3) For each collision-free information packet transmission within the n-th period, a PACK is transmitted within the (n+1)-st acknowledgement subperiod in a mini-slot determined by a uniform distribution over the $\tilde{K}_\eta$ available service (mini) slots.

(4) Each collision-free information packet transmission within the n-th period, which remains unacknowledged at the end of the two frame acknowledgment time-out interval, is retransmitted within the (n+2)-nd information subperiod in a slot determined by a uniform distribution over the $\tilde{K}_I$ available service slots.

(5) Each information packet admitted by the network control procedure is transmitted and retransmitted until successfully acknowledged by a collision-free PACK.

4.4.1 Channel State Process

Since the operation of the GRA channel under the PDRA and PRA acknowledgement schemes is similar, their respective Markov state sequences have comparable structures. Therefore, the construction of the transition probability function under the PDRA scheme follows a development parallel to the development under the PRA scheme. The evoluation of the channel under the PDRA scheme is described by a vector Markov chain $Z = \{Z_n, n \geq 1\}$ over the space

$d^{\tilde{K}_I} \times d_{\tilde{K}_I}^{\tilde{K}_\eta} \times d^{P+\tilde{K}}$ where d is the set of non-negative integers, $d_{\tilde{K}_I} = \{0, 1, 2, \ldots, \tilde{K}_I\}$ and

$$Z_n = \{(T_{ni}^{(I)}, i = 1,2,\ldots,\tilde{K}_I), (T_{ni}^{(A)}, i = 1,2,\ldots,\tilde{K}_\eta),$$

$$(A_{ni}, i = 1,2,\ldots,\tilde{K}+P)\} .$$

The variable $T_{ni}^{(I)}$ denotes the number of information packet retransmissions in the i-th slot within the n-th information subperiod ($n \geq 1$, $1 \leq i \leq \tilde{K}_I$), $T_{ni}^{(A)}$ denotes the number of PACK transmissions in the i-th mini-slot within the n-th acknowledgment subperiod ($n \geq 1$, $1 \leq i \leq \tilde{K}_\eta$), and $A_{n,i}$ denotes the number of uncontrolled new message arrivals in the i-th slot within the n-th frame ($n \geq 1$, $1 \leq i \leq \tilde{K}+P$).

Let R_n and $\tilde{S}_n^{(I)}$ denote the number of collisions (information packet and PACK) and the number of collision-free information packet transmissions, respectively, within the n-th period. By definition

$$R_n = \sum_{i=1}^{\tilde{K}_I} T_{n+1,i}^{(I)} \tag{4.84}$$

and

$$\tilde{S}_n^{(I)} = \sum_{i=1}^{\tilde{K}_\eta} T_{n+1,i}^{(A)} \quad . \tag{4.85}$$

Since the retransmission slot of an information packet and the transmission slot of a PACK are determined by uniform distributions over $[1, \tilde{K}_I]$ and $[1, \tilde{K}_\eta]$, respectively, $\{T_{n+1,i}^{(I)}, 1 \leq i \leq \tilde{K}_I\}$ and $\{T_{n+1,i}^{(A)}, 1 \leq i \leq \tilde{K}_\eta\}$ depend on Z_n through R_n and $\tilde{S}_n^{(I)}$. They are governed by the following multinomial distributions:

$$P\{T_{n+1,i}^{(I)} = \alpha_i, 1 \leq i \leq \tilde{K}_I | R_n = j\} = g_j^{(\tilde{K}_I)}(\alpha_1, \alpha_2, \ldots, \alpha_{\tilde{K}_I}) \tag{4.86}$$

where $0 \leq \alpha_i \leq j$, $1 \leq i \leq \tilde{K}_I$, $\sum_{i=1}^{\tilde{K}_I} \alpha_i = j$ and

$$P\{T^{(A)}_{n+1,i} = \theta_i,\ 1 \le i \le \tilde{K}_\eta \mid \tilde{S}^{(I)}_n = j\} = g_j^{(\tilde{K}_\eta)}(\theta_1, \theta_2, \ldots, \theta_{\tilde{K}_\eta}) \qquad (4.87)$$

where $0 \le \theta_i \le j$, $1 \le i \le \tilde{K}_\eta$, $\sum_{i=1}^{\tilde{K}_\eta} \theta_i = j$.

Let $R_n^{(I)}$ and $R_n^{(A)}$ denote the number of information packet collisions and PACK collisions, respectively, within the n-th period so that

$$R_n = R_n^{(I)} + R_n^{(A)} \quad . \qquad (4.88)$$

The variables $R_n^{(I)}$ and $R_n^{(A)}$ are determined by the following relationships:

$$R_n^{(I)} = \sum_{i=1}^{\tilde{K}_I} R_{ni}^{(I)} \qquad (4.89)$$

and

$$R_n^{(A)} = \sum_{i=1}^{\tilde{K}_\eta} R_{ni}^{(A)} \qquad (4.90)$$

where $R_{ni}^{(I)}$ and $R_{ni}^{(A)}$ denote the number of information packet collisions in the i-th slot within the n-th information subperiod and the number of PACK collisions in the i-th slot within the n-th acknowledgment subperiod, respectively. A collision occurs when two or more packets are simultaneously transmitted; hence,

$$R_{ni}^{(I)} = N_{ni}\, I(N_{ni} > 1) \quad , \quad 1 \le i \le \tilde{K}_I \qquad (4.91)$$

and

$$R_{ni}^{(A)} = T_{ni}^{(A)} \; I(T_{ni}^{(A)} > 1) \quad , \quad 1 \leq i \leq \tilde{K}_\eta \tag{4.92}$$

where N_{ni} denotes the number of information packet transmissions in the i-th slot within the n-th information subperiod:

$$N_{ni} = T_{ni}^{(I)} + A_{n,P+\tilde{K}_A+i}^{(C)} + \tilde{A}_{ni}^{(C)} \quad , \quad 1 \leq i \leq \tilde{K}_I \; . \tag{4.93}$$

The variable $\tilde{A}_{ni}^{(C)}$ denotes the number of controlled new message arrivals allocated for transmission in the i-th slot within the n-th information subperiod from among those arrivals in the $P+\tilde{K}_A$ slots which are not allocated for information packet transmission. The sequence $\{\tilde{A}_{ni}^{(C)}\}$ is governed by the multinomial distribution

$$P\{\tilde{A}_{ni}^{(C)} = \alpha_i, \; 1 \leq i \leq \tilde{K}_I \Big| \sum_{i=1}^{P+\tilde{K}_A} A_{ni}^{(C)} = j\} = g_j^{(\tilde{K}_I)}(\alpha_1, \alpha_2, \ldots, \alpha_{\tilde{K}_I}) \tag{4.94}$$

where $0 \leq \alpha_i \leq j$, $1 \leq i \leq \tilde{K}_I$, $\sum_{i=1}^{\tilde{K}_I} \alpha_i = j$.

Let $\tilde{S}_{ni}^{(I)}$ denote the number of collision-free information packet transmissions in the i-th slot within the n-th information subperiod. By definition

$$\tilde{S}_{ni}^{(I)} = I(N_{ni} = 1) \quad , \quad 1 \leq i \leq \tilde{K}_I \tag{4.95}$$

and

$$\tilde{S}_n^{(I)} = \sum_{i=1}^{\tilde{K}_I} \tilde{S}_{ni}^{(I)} \; . \tag{4.96}$$

Equations (4.4), (4.84) - (4.96) yield the transition probability function for the vector Markov chain Z under the PDRA scheme. The uncontrolled new message arrival variables $\{A_{n+1,i},\ 1 \le i \le \tilde{K}+P\}$ are statistically independent of Z_n; and $\{T^{(I)}_{n+1,i},\ 1 \le i \le \tilde{K}_I\}$ and $\{T^{(A)}_{n+1,i},\ 1 \le i \le \tilde{K}_\eta\}$ depend on Z_n only through $X_n = (\tilde{S}^{(I)}_n,\ R_n)$. The sequence $X = \{X_n,\ n \ge 1\}$ is a vector Markov chain over the space $d_{\tilde{K}_I} \times d$. A flow diagram indicating the transition $X_n \to X_{n+1}$ is shown in Figure 4.5.

The network control function on new message arrivals given by (4.15) yields the following proposition.

Proposition 4.3

The vector Markov chains Z and X, which describe the evolution of the GRA channel under the PDRA acknowledgment scheme, are irreducible, positive recurrent using the single-threshold binary control function defined by (4.15).

Proof

Similar technique as Proposition 4.1.

Q.E.D.

Under the PDRA scheme, the number of collision-free PACK transmissions within the (n+1)-st period is given by

$$\tilde{S}^{(A)}_{n+1} = \sum_{i=1}^{\tilde{K}_\eta} I(T^{(A)}_{n+1,i} = 1) \quad . \tag{4.97}$$

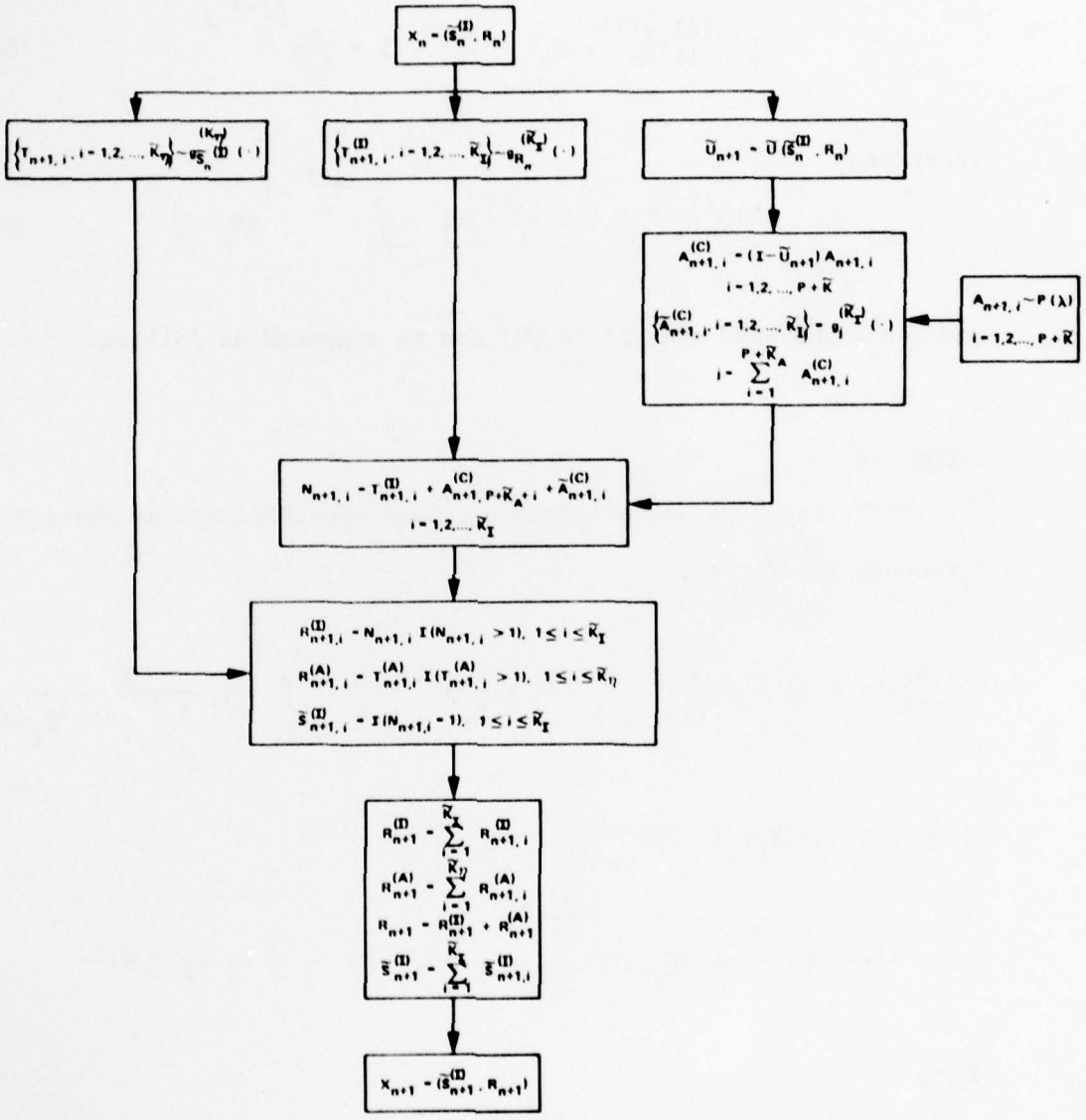


Figure 4.5 *Transition* $X_n \rightarrow X_{n+1}$ **for the Markov State Chain X Associated with the GRA Discipline Under the PDRA ACK Scheme**

Since $\{T^{(A)}_{n+1,i}\}$ is governed by the multinomial distribution (4.87), the expectation of $\tilde{S}^{(A)}_{n+1}$ given $\tilde{S}^{(I)}_n$ and N_n is

$$E(\tilde{S}^{(A)}_{n+1}|\tilde{S}^{(I)}_n, N_n) = \tilde{S}^{(I)}_n \left(1 - \frac{1}{\tilde{K}_\eta}\right)^{\tilde{S}^{(I)}_n - 1} . \tag{4.98}$$

Therefore,

$$E(\tilde{S}^{(A)}_{n+1}|N_n) = E\{\tilde{S}^{(I)}_n \left(1 - \frac{1}{\tilde{K}_\eta}\right)^{\tilde{S}^{(I)}_n - 1} | N_n\} \tag{4.99}$$

and the right hand side of (4.99) can be computed as follows.

Lemma 4.2

Under the PDRA acknowledgment scheme specified by the channel structure $(\tilde{K}, \tilde{K}_I, \eta)$,

$$E(\tilde{S}^{(A)}_{n+1}|N_n = j) = j\left(\frac{1}{\tilde{K}_I}\right)^j \sum_{\ell=0}^{\hat{\ell}} \ell \left(1 - \frac{1}{\tilde{K}_\eta}\right)^{\ell-1} \binom{\tilde{K}_I}{\ell} \sum_{L_\ell} \frac{1}{\ell_1!\, \ell_2! \cdots \ell_{\tilde{K}_I - \ell}!} \tag{4.100}$$

where $\hat{\ell} = \min(\tilde{K}_I, j)$

$$L_\ell = \{(\ell_1, \ell_2, \ldots, \ell_{\tilde{K}_I - \ell}): \sum_{i=1}^{\tilde{K}_I - \ell} \ell_i = j - \ell, \ell_i \neq 1, \ell_i \geq 0\} .$$

Proof

The right hand side of (4.99) can be expressed by

$$E\{\tilde{S}^{(I)}_n \left(1 - \frac{1}{\tilde{K}_\eta}\right)^{\tilde{S}^{(I)}_n - 1} | N_n\} = \sum_{\ell=0}^{\hat{\ell}} \ell \left(1 - \frac{1}{\tilde{K}_\eta}\right)^{\ell-1} P(\tilde{S}^{(I)}_n = \ell | N_n) \tag{4.101}$$

The variables $\{N_{ni},\ 1 \le i \le \tilde{K}_I\}$ are governed by the multinomial distribution

$$P\{N_{ni} = \ell_i,\ 1 \le i \le \tilde{K}_I | N_n = j\} = \frac{j!}{\ell_1!\ \ell_2! \ \ldots\ \ell_{\tilde{K}_I}!} \left(\frac{1}{\tilde{K}_I}\right)^j \tag{4.102}$$

where $0 \le \ell_i \le j$, $1 \le i \le \tilde{K}_I$, $\sum_{i=1}^{\tilde{K}_I} \ell_i = j$. The probability of exactly ℓ collision-free information packet transmissions from N_n transmissions is given by (4.102) summed over the tupples $(\ell_1, \ell_2, \ldots, \ell_{\tilde{K}_I})$ such that exactly ℓ of the ℓ_i's are equal to 1. There are $\binom{\tilde{K}_I}{\ell}$ combinations which can yield ℓ collision-free information packet transmissions in $\tilde{K}_I$ slots. The set L_ℓ represents the transmission slot allocations of the remaining $N_n - \ell$ packets so that either collision occurs or no transmission is made in each of the $\tilde{K}_I - \ell$ slots, Hence,

$$P(\tilde{S}_n^{(I)} = \ell | N_n = j) = \sum_{L_\ell} \binom{\tilde{K}_I}{\ell} \frac{j!}{\ell_1!\ \ell_2! \ \ldots\ \ell_{\tilde{K}_I - \ell}!} \left(\frac{1}{\tilde{K}_I}\right)^j \tag{4.103}$$

where $0 \le \ell \le \hat{\ell}$.

Substituting (4.103) into (4.101) yields (4.100).

Q.E.D.

An upper bound on throughput as a function of the channel structure parameters $(\tilde{K},\ \tilde{K}_I,\ \eta)$ is presented in the following theorem.

Theorem 4.5

The throughput of the GRA channel under the PDRA acknowledgment scheme specified by the channel structure $(\tilde{K}, \tilde{K}_I, \eta)$ is upper bounded by

$$\tilde{\delta} \leq \max_j \{\frac{j!}{\tilde{K}} (\frac{1}{\tilde{K}_I})^j \sum_{\ell=0}^{\hat{\ell}} \ell(1 - \frac{1}{\tilde{K}_\eta})^{\ell-1} (\tilde{K}_\ell^{I}) \sum_{L_\ell} \frac{1}{\ell_1! \ell_2! \cdots \ell_{\tilde{K}_I - \ell}!}\} \quad (4.104)$$

Proof

Replace $E(\tilde{S}_{n+1}^{(A)})$ in (4.16) with $\max_j E(\tilde{S}_{n+1}^{(A)} | N_n = j)$ which is given by (4.100).

Q.E.D.

The summation over L_ℓ makes the evaluation of the throughput bound given by (4.104) inconvenient. An upper bound which is more readily computed is stated in the following theorem.

Theorem 4.6

The throughput of the GRA channel under the PDRA acknowledgment scheme specified by the channel structure $(\tilde{K}, \tilde{K}_I, \eta)$ is upper bounded by

$$\tilde{\delta} \leq \begin{cases} \frac{1}{\tilde{K}} V_o(\tilde{K}_I) \left(1 - \frac{1}{\tilde{K}_\eta}\right)^{V_o(\tilde{K}_I)-1} & \text{if} \quad V_o(\tilde{K}_I) \leq V_2(\tilde{K}_\eta) \\ \frac{1}{\tilde{K}} V_o(\tilde{K}_\eta) & \text{if} \quad V_o(\tilde{K}_I) > V_2(\tilde{K}_\eta) \end{cases} \tag{4.105}$$

where

$$V_o(x) = e^{-1} x\{(x-1) \ln[1 + (x-1)^{-1}]\}^{-1}$$

$$V_2(\tilde{K}_\eta) = \{\ln[(1 - \frac{1}{\tilde{K}_\eta})^{-1}]\}^{-1}.$$

Proof

See Appendix C.

A straightforward simulation of the Markov state sequence X has been used to compute maximum throughput values for $\tilde{K} = 12$ with $\eta = 1, 9$ for several information subperiod lengths ($\tilde{K}_I$). These results are presented in Tables 4.3 and 4.4 for $\eta = 1$ and $\eta = 9$, respectively. The upper bounds computed using (4.104) and (4.105) are also tabulated. These results indicate that (4.104) provides the tighter upper bound; however, (4.105) provides reasonable values without the computational complexity involved in evaluating (4.104).

TABLE 4.3. MAXIMUM THROUGHPUT VALUES FOR THE GRA DISCIPLINE UNDER THE PDRA ACK SCHEME WITH $\tilde{K} = 12$, $\eta = 1$

$\tilde{K}_I$ (slots)	$\tilde{K}_\eta$ (slots)	Maximum Throughput		
		Simulation	Upper Bounds	
			(4.104)	(4.105)
4	8	0.109	0.119	0.129
5	7	*	0.133	0.146
6	6	0.136	0.141	0.156
7	5	*	0.140	0.156
8	4	0.125	0.126	0.141
9	3	*	0.102	0.113

*Not Computed

TABLE 4.4. MAXIMUM THROUGHPUT VALUES FOR THE GRA DISCIPLINE UNDER THE PDRA ACK SCHEME WITH $\tilde{K} = 12$, $\eta = 9$

$\tilde{K}_I$ (slots)	$\tilde{K}_\eta$ (mini-slots)	Maximum Throughput		
		Simulation	Upper Bounds	
			(4.104)	(4.105)
8	36	*	0.242	0.247
9	27	0.251	0.260	0.266
10	18	0.260	0.265	0.274
11	9	*	0.228	0.241

*Not Computed

4.4.2 Packet Delay Analysis

The delay of the n-th message is defined by (4.30) with the single slot duration representing the PACK transmission time replaced by $1/\eta$. The computation of the limiting average packet delay is identical to the technique introduced in Section 4.2.2 for the PRA scheme.

Under the PDRA acknowledgment scheme, the acknowledgment subperiod is partitioned into $\tilde{K}_\eta$ mini-slots per (4.83). The fraction of a mini-slot given by

$$\tilde{K}_\delta = \eta \tilde{K}_A - \tilde{K}_\eta \tag{4.106}$$

cannot be used for PACK transmissions. The placement of this fractional mini-slot and the position of the acknowledgment and information subperiods relative to each other alters the packet delay.

Consider the channel structure illustrated in Figure 4.6 where the fractional mini-slot is placed at the beginning of each period and the acknowledgment subperiod precedes the information subperiod.

Theorem 4.7

With the operation of a controlled GRA channel under the PDRA acknowledgment scheme described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delay under the channel structure illustrated in Figure 4.6 is given by

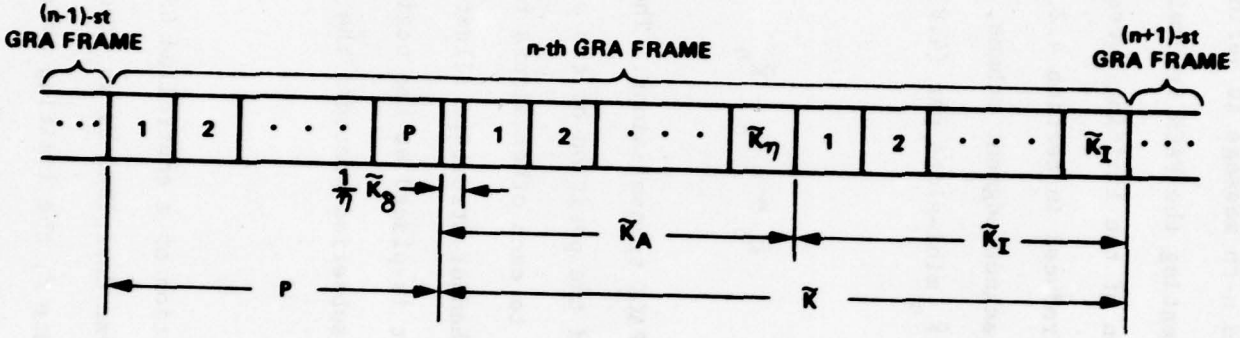


Figure 4.6 GRA Channel Structure Under the PDRA ACK Scheme with the Acknowledgment Subperiod Preceding the Information Subperiod

$$E(D_S) = \frac{P + \tilde{K}_A}{2} + (\tilde{K}+P)\,\frac{E(R_n^{(I)})}{E(\tilde{S}_n^{(A)})}$$

$$+ \frac{1}{2}\left[2P + 3\tilde{K} - \frac{1}{\eta}\,\tilde{K}_\delta - (1 - \frac{1}{\eta})\right]\frac{E(R_n^{(A)})}{E(\tilde{S}_n^{(A)})}$$

$$+ \frac{1}{2}\left[2P + \tilde{K} + \frac{1}{\eta}\,\tilde{K}_\delta + (1 - \frac{1}{\eta})\right]\frac{E(\tilde{S}_n^{(I)})}{E(\tilde{S}_n^{(A)})}$$

$$+ R + \frac{1}{\eta} \qquad (4.107)$$

where the expectations are w.r.t. the stationary distribution.

Proof

Similar technique as Theorem 4.2.

Q.E.D.

Consider an alternate channel structure illustrated in Figure 4.7. The information subperiod precedes the acknowledgment subperiod and the fractional mini-slot is placed at the end of each period. The limiting average packet delay under this structure is presented in the following lemma.

Lemma 4.3

With the operation of a controlled GRA channel under the PDRA acknowledgment scheme described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delay under the channel structure illustrated in Figure 4.7 is given by

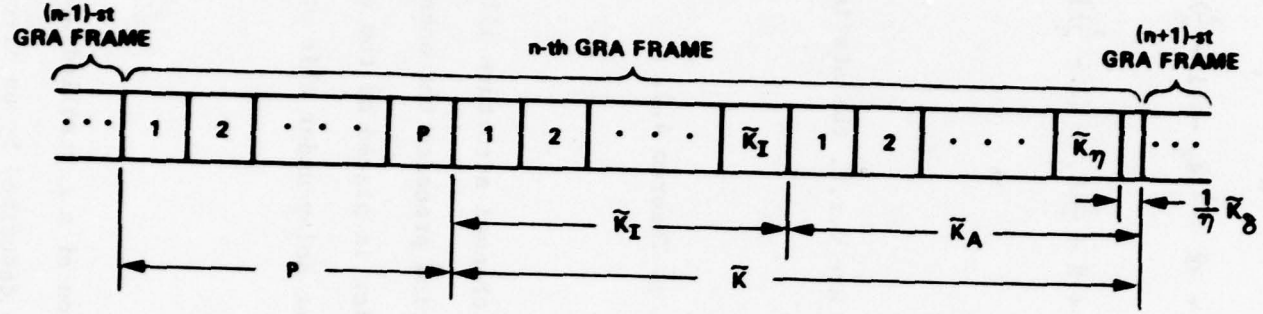


Figure 4.7 GRA Channel Structure Under the PDRA ACK Scheme with the Information Subperiod Preceding the Acknowledgment Subperiod

$$E(D_S) = \frac{P + \tilde{K}_A}{2} + (\tilde{K}+P)\frac{E(R_n^{(I)})}{E(\tilde{S}_n^{(A)})}$$

$$+ \frac{1}{2}\left[2P + \tilde{K} + \frac{1}{\eta}\tilde{K}_\delta - (1 - \frac{1}{\eta})\right]\frac{E(R_n^{(A)})}{E(\tilde{S}_n^{(A)})}$$

$$+ \frac{1}{2}\left[2P + 3\tilde{K} - \frac{1}{\eta}\tilde{K}_\delta + (1 - \frac{1}{\eta})\right]\frac{E(\tilde{S}_n^{(I)})}{E(\tilde{S}_n^{(A)})}$$

$$+ R + \frac{1}{\eta} \qquad (4.108)$$

where the expectations are w.r.t. the stationary distribution.

Proof

Similar technique as Theorem 4.2.

Q.E.D.

The difference between the limiting average packet delays given by (4.107) and (4.108) is

$$E(D_S)_{(4.108)} - E(D_S)_{(4.107)} = (\tilde{K} - \frac{1}{\eta}\tilde{K}_\delta)\left[\frac{E(\tilde{S}_n^{(I)})}{E(\tilde{S}_n^{(A)})} - \frac{E(R_n^{(A)})}{E(\tilde{S}_n^{(A)})}\right]. \qquad (4.109)$$

The term $E(\tilde{S}_n^{(I)})/E(\tilde{S}_n^{(A)})$ is the average number of collision-free information packet transmissions per admitted message and $E(R_n^{(A)})/E(\tilde{S}_n^{(A)})$ is the average number of PACK collisions per admitted message. Since

$$\tilde{S}_n^{(I)} = R_{n+1}^{(A)} + \tilde{S}_{n+1}^{(A)} \quad , \tag{4.110}$$

$E(\tilde{S}_n^{(I)}) \geq E(\tilde{R}_n^{(A)})$; and, hence,

$$E(D_S)_{(4.108)} \geq E(D_S)_{(4.107)} \quad .$$

Thus the channel structure with the acknowledgment subperiod preceding the information subperiod (Figure 4.6) provides the smaller limiting average packet delay.

4.5 Numerical Results

The delay and throughput performance of the GRA channel under the PRA, MCRA and PDRA acknowledgment schemes has been determined by simulating the appropriate Markov state sequences. The Markov chain simulations are run for a large but finite number of slots; hence, these results represent estimates which indicate performance trends and should not be interpreted as absolute.

Maximum throughput results with $\tilde{K} = 12$ are summarized in Table 4.5. The basic GRA discipline relies on the broadcast feature of the channel to provide automatic acknowledgments. A separate acknowledgment mechanism is not included; and, hence, the basic GRA channel achieves the maximum throughput value e^{-1} packets per service slot. Under the random access acknowledgment schemes, the maximum throughput is severely reduced. The PRA scheme achieves a maximum throughput value

of only 0.12 packets per service slot. Under the MCRA scheme, the maximum throughput (0.14 packets per service slot) is achieved with two identical PACK transmissions for each collision-free information packet transmission ($\gamma = 2$). This maximum throughput value is also achieved under the PDRA scheme with $\tilde{K}_I = 6$ and $\eta = 1$. With $\eta = 9$, the maximum throughput is increased to 0.26 packets per service slot with $\tilde{K}_I = 10$.

TABLE 4.5. MAXIMUM THROUGHPUT VALUES FOR THE GRA DISCIPLINE WITH $\tilde{K} = 12$

Acknowledgment Scheme	Maximum Throughput	
	$\eta = 1$	$\eta = 9$
PRA	0.12	---
MCRA ($\gamma = 2$)	0.14	---
PDRA	0.14 ($\tilde{K}_I = 6$)	0.26 ($\tilde{K}_I = 10$)
Basic GRA	e^{-1} (≈ 0.368)	e^{-1} (≈ 0.368)

The delay performance D_R of the basic GRA channel is presented in Figures 4.8 and 4.9 with $\tilde{K} = P = R = 12$. Average packet delay versus throughput curves are shown in Figure 4.8 for three threshold values of the single-threshold network control function. Both throughput and delay increase as the average message arrival rate (λ packets per slot) increases until a maximum throughput value is reached. As λ is increased further, throughput decreases (i.e., new message arrivals are rejected) and delay increases rapidly. Average packet

delay versus packet probability of rejection curves are shown in Figure 4.9. These constant arrival rate curves indicate that the minimum probability of rejection with $\tilde{K} = 12$ is effectively zero for $\lambda = 0.05, 0.1$. For $\lambda = 0.15, 0.2$ large rejection probabilities and small delays are experienced with low threshold values. As the threshold is increased, the probability of rejection decreases; however, the number of collisions increases which increases delay. A minimum rejection probability is attained, beyond which raising the threshold increases both the delay and the probability of rejection, since collisions predominate.

Average packet delay D_S versus probability of rejection curves are shown in Figure 4.10 for the GRA channel under the PRA ($\gamma = 1$) and MCRA ($\gamma = 2, 3$) acknowledgment schemes with $\tilde{K} = P = R = 12$. These curves exhibit the same behavior displayed under the basic GRA channel. The results indicate that the MCRA scheme with $\gamma = 2$ provides the best overall performance, minimun packet delay under a prescribed probability of rejection.

Average packet delay versus probability of rejection curves are shown in Figure 4.11 for the GRA channel under the PDRA acknowledgment scheme with $\tilde{K} = P = R = 12$ and $\eta = 1,9$. These curves exhibit the characteristic behavior and indicate the performance attainable under unequal PACK and information packet lengths ($\eta > 1$).

The performance of the GRA channel under the random access acknowledgment schemes are summarized in Figure 4.12. Average packet delay versus probability of rejection curves are shown with $\tilde{K} = P = R = 12$. For $\lambda = 0.025$ and $\lambda = 0.05$, the MCRA scheme ($\gamma = 2$) achieves the

lowest delay for all rejection probabilities. For $\lambda = 0.075$, the PDRA scheme ($\tilde{K}_I = 6$) attains the smallest probability of rejection but with a relatively large delay value.

4.6 Conclusions

The operation of the GRA channel under three random access (positive) acknowledgment schemes was studied in this chapter. Under the PRA scheme, PACKs and information packets contended on a random access basis in each period; under the MCRA scheme, each collision-free information packet transmission was acknowledged by multiple, identical PACK transmissions; and under the PDRA scheme, each period was partitioned into two subperiods with information packets and PACKs transmitted on a random access basis within separate subperiods. The channel state process under each acknowledgment scheme was described by an irreducible, positive recurrent Markov chain. Upper bounds on the maximum throughput and expressions for the limiting average packet delay were derived.

Numerical examples of the delay-throughput (rejection probability) function were presented with $\tilde{K} = P = R = 12$. Under the MCRA scheme, the optimum number of PACK copies was observed to be $\gamma = 2$ and under the PDRA scheme, the optimum period division was $\tilde{K}_I = 6$ with $\eta = 1$ and $\tilde{K}_I = 10$ with $\eta = 9$. With $\eta = 1$, the best overall performance was achieved under the MCRA scheme ($\gamma = 2$); however, both the delay and throughput performance was found to be reduced compared to the basic GRA discipline without acknowledgment.

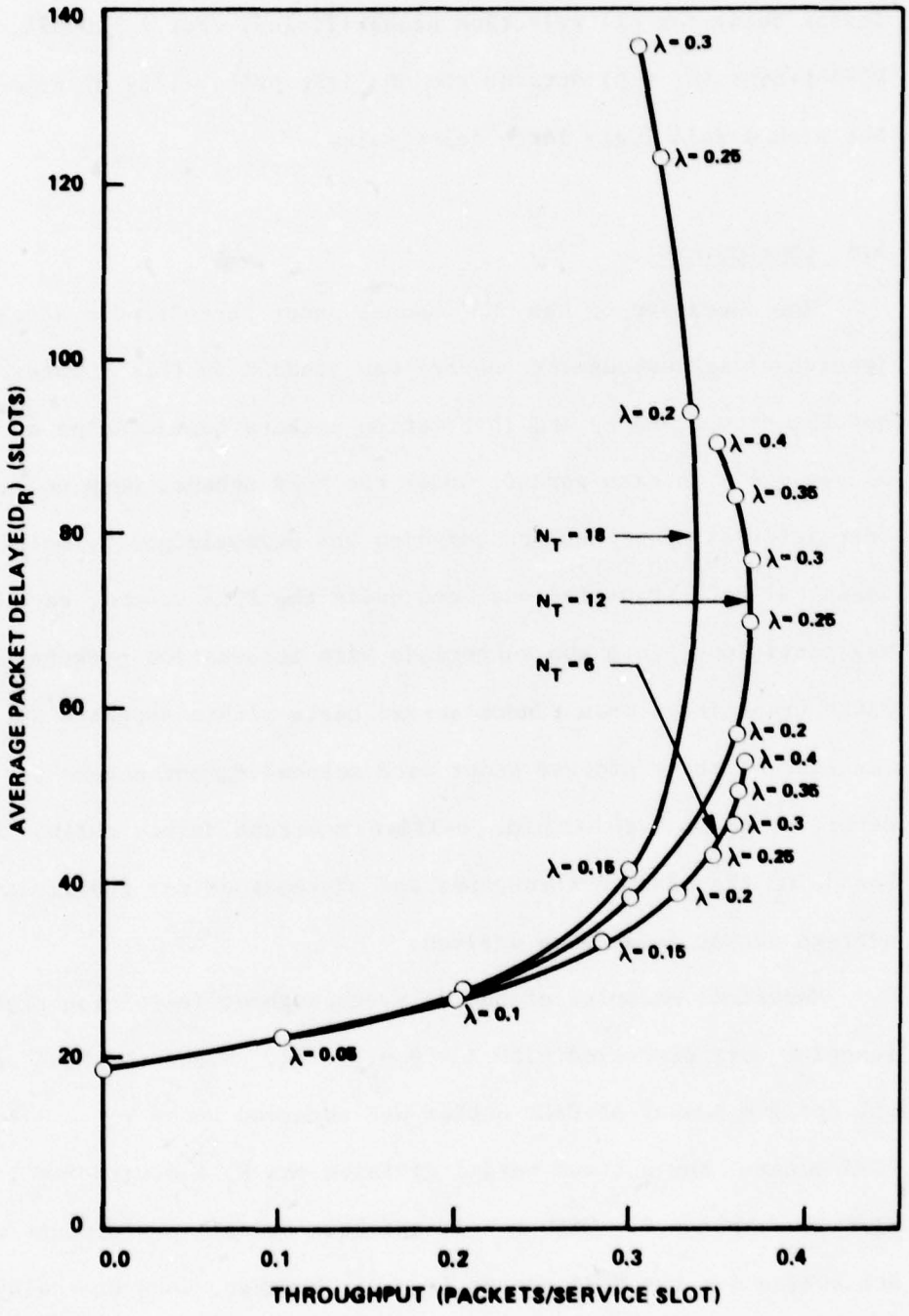


Figure 4.8 Packet Delay versus Throughput Curves for the Basic GRA Channel with $\bar{K}$ = P = R = 12

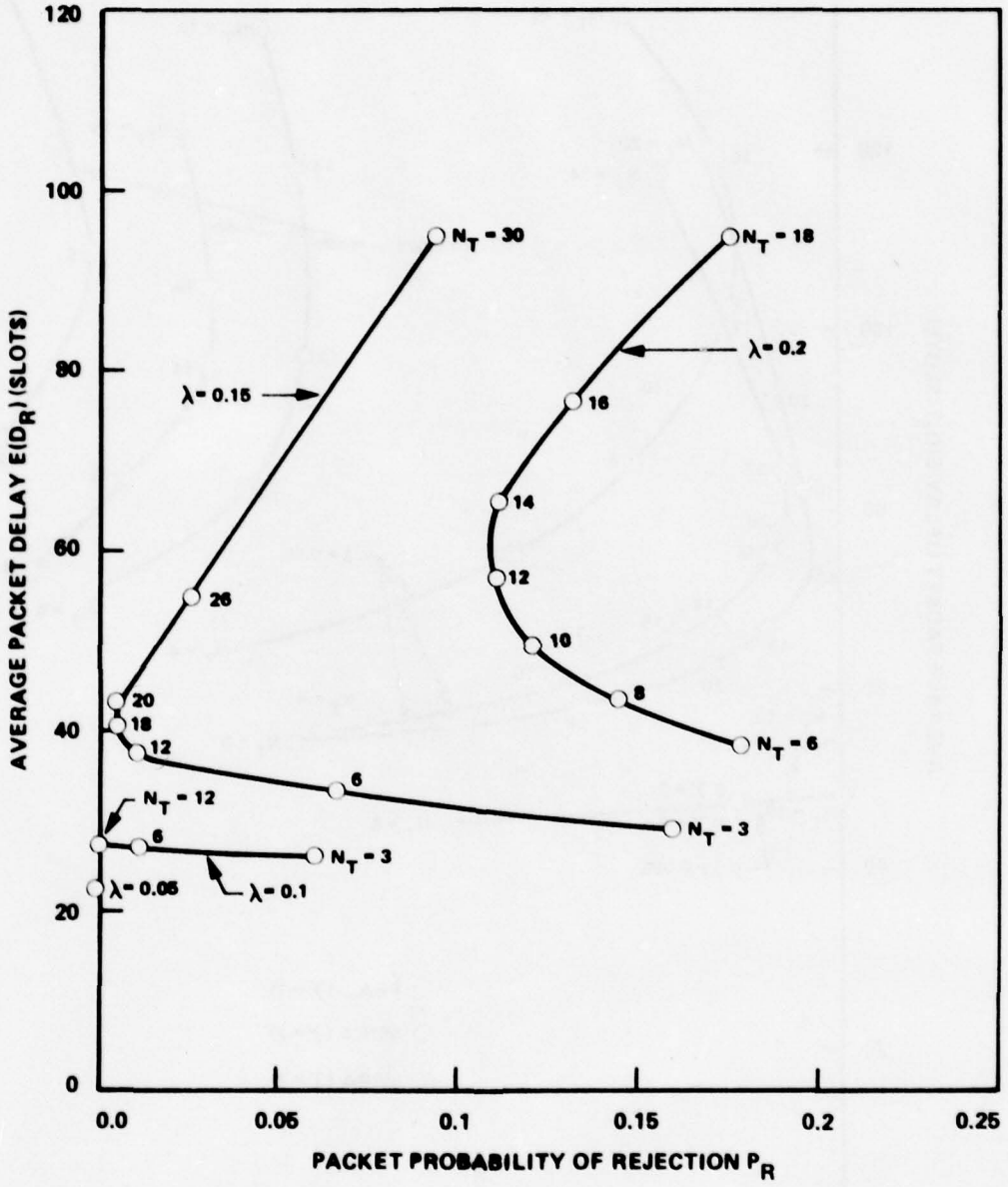


Figure 4.9 Packet Delay versus Packet Probability of Rejection Curves for the Basic GRA Channel with $\widetilde{K} = P = R = 12$

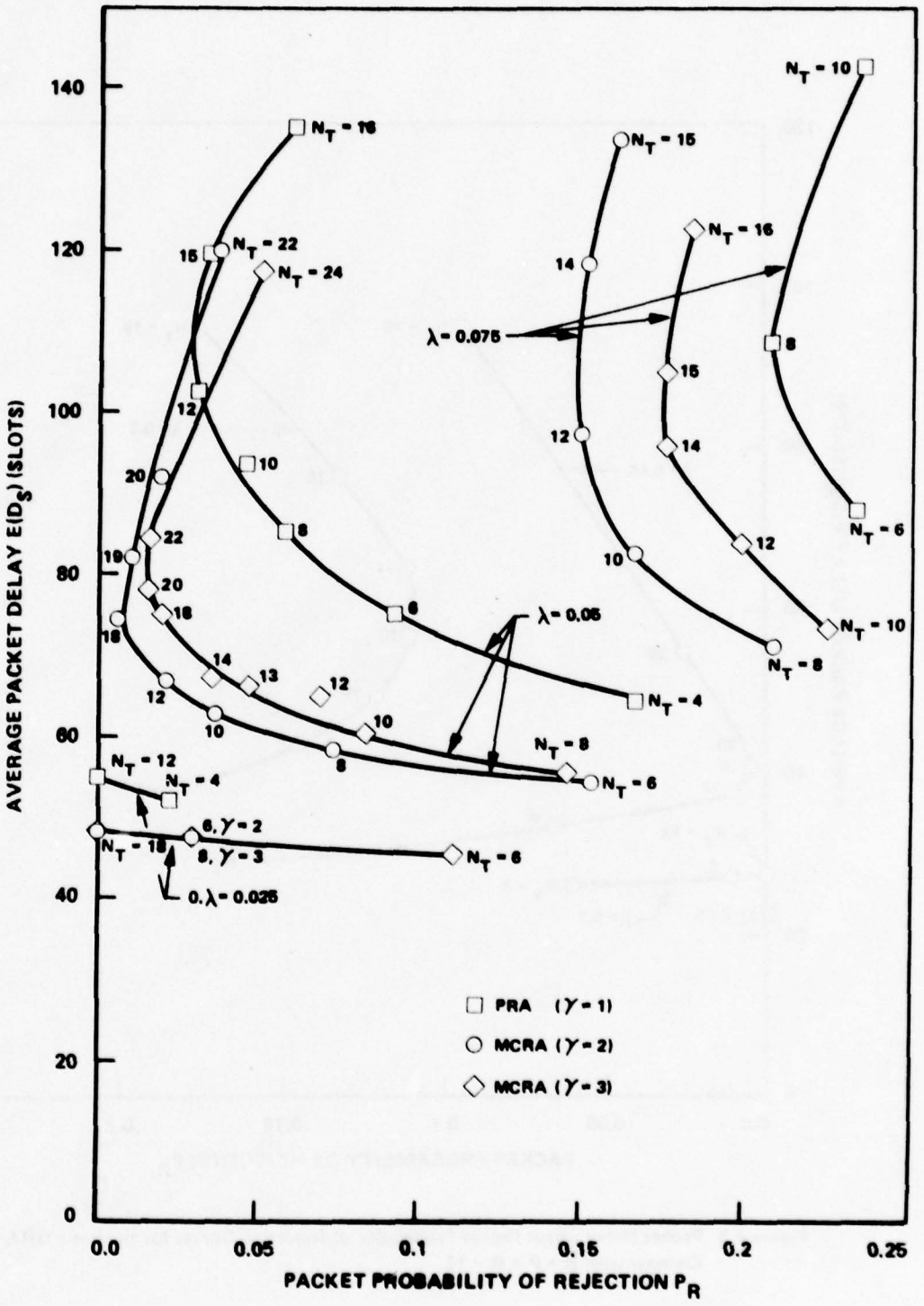


Figure 4.10 Packet Delay versus Packet Probability of Rejection Curves for a GRA Channel with $\widetilde{K} = P = R = 12$ Under the PRA and MCRA ACK Schemes Using Control Function (4.64)

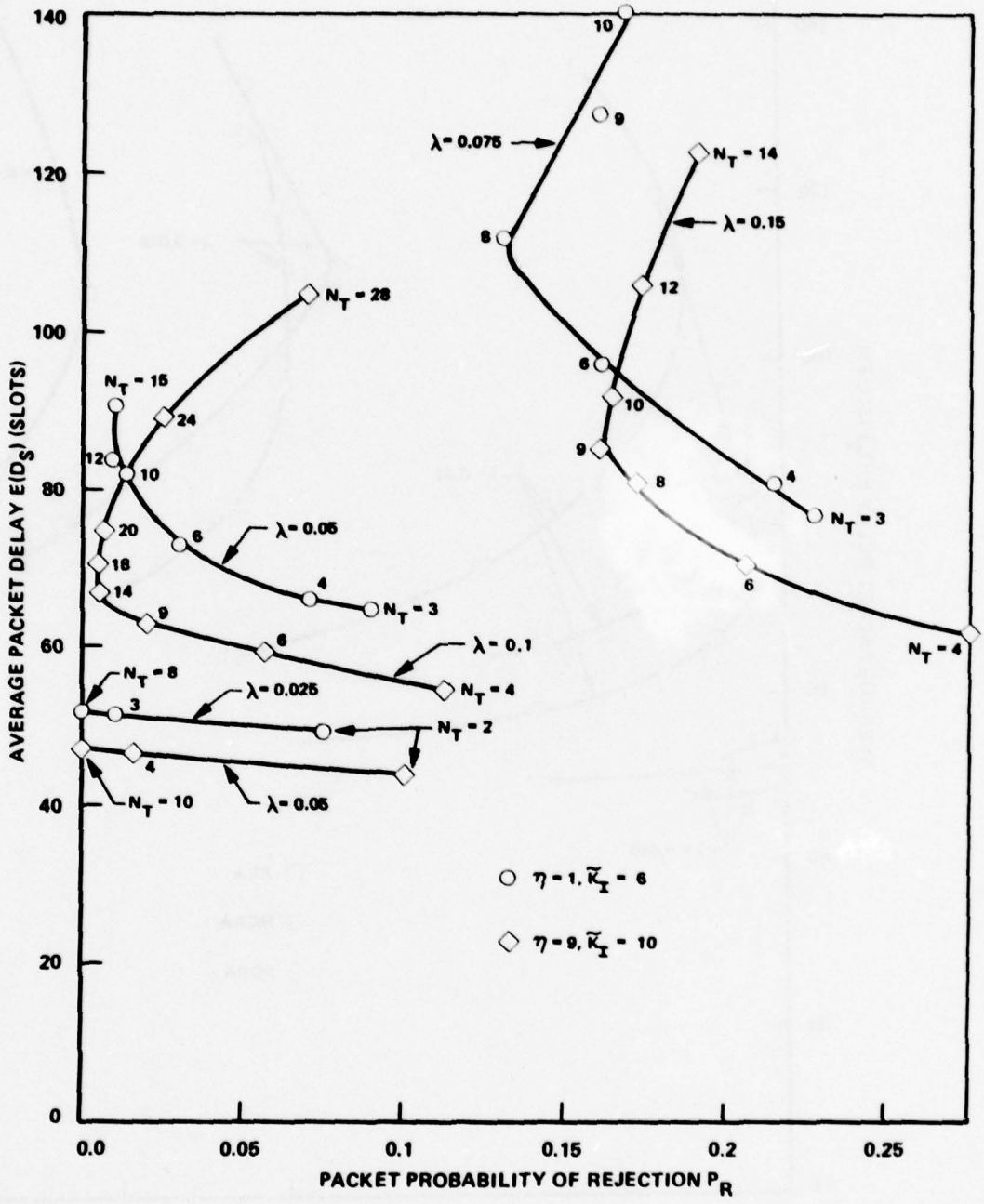


Figure 4.11 Packet Delay versus Packet Probability of Rejection Curves for a GRA Channel with $\bar{K} = P = R = 12$ Under the PDRA ACK Scheme ($\bar{K}_I = 6, \eta = 1$), ($\bar{K}_I = 10, \eta = 9$) Using Control Function (4.15)

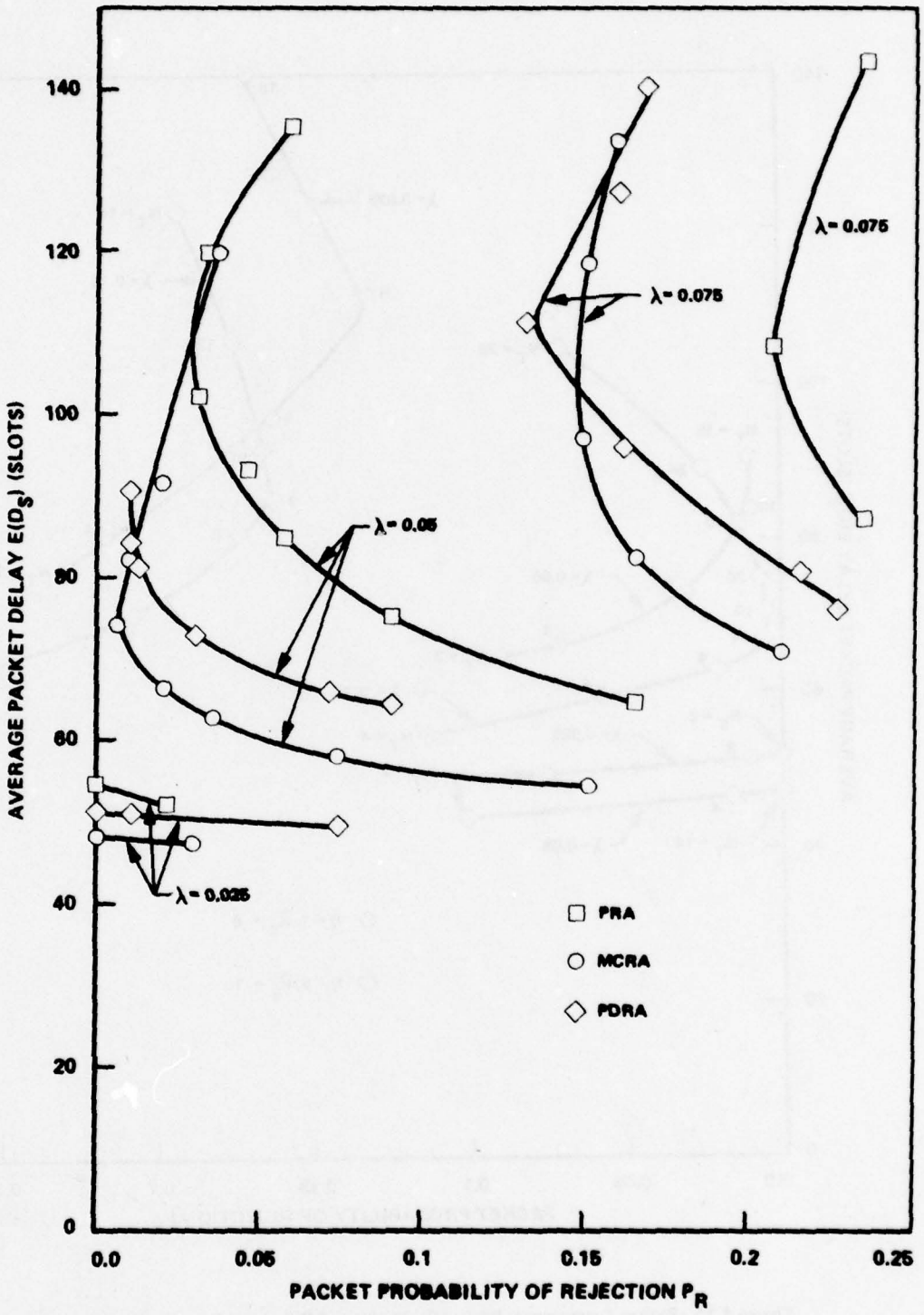


Figure 4.12 Packet Delay versus Packet Probability of Rejection Curves for a GRA Channel with $\bar{K} = P = R = 12$ Under the PRA ($\gamma = 1$), MCRA ($\gamma = 2$), and PDRA ($\bar{K}_I = 6$, $\eta = 1$) ACK Schemes

CHAPTER V

GROUP RANDOM ACCESS USING SCHEDULED ACKNOWLEDGMENT PROTOCOLS

The three random access acknowledgment implementations examined in Chapter IV yield reduced performance capabilities with respect to the basic GRA discipline which operates without a separate acknowledgment mechanism. Although implementation requirements of random access disciplines are modest compared to other disciplines (e.g., reservation), PACK transmission collisions are undesirable. Reduced throughput, longer delays and higher packet rejection probabilities result.

In this chapter the operation of the GRA discipline under two acknowledgment systems which schedule PACK transmissions to avoid the unresolvable collisions is examined. In Section 5.1 the scheduled acknowledgment schemes are introduced. The GRA channel under the Period Division - Scheduled acknowledgment scheme is examined in Section 5.2. In Section 5.3 the GRA channel under the Dynamic Period Division - Scheduled acknowledgment scheme is studied. A stationary transmission error process model is developed for the GRA channel in Section 5.4. This random noise channel is studied under the Dynamic Period Division - Scheduled acknowledgment scheme. Numerical examples are presented in Section 5.5.

5.1 Scheduled Acknowledgment Schemes*

The time slotted channel structure examined in Chapter IV is again considered. The basic GRA channel structure shown in Figure 4.1 is adopted and the same group of network stations characterized as a large population of low duty cycle bursty users is considered. Single-packet messages are assumed and the overall message arrival stream is described by a Poisson point process with average arrival rate λ messages per slot.

Two positive acknowledgment schemes which schedule PACK transmission to avoid collision are studied:

- Period Division - Scheduled (PDS)
- Dynamic Period Division - Scheduled (DPDS).

Both schemes partition the channel access periods into two subperiods: information packets are transmitted in one subperiod on a random access basis while PACKs are transmitted in the second subperiod on a scheduled basis. The subperiod lengths are fixed under the PDS scheme. The period division under the DPDS scheme dynamically adjusts to the current acknowledgment traffic requirement.

*Two frequency division acknowledgment schemes are studied in Appendix D. The allocated bandwidth is partitioned into two channels with distinct non-overlapping bandwidths.

To achieve conflict-free PACK transmission, prior to the start of every channel access period, each network station must be given PACK transmission slot allocations by a network controller (a control station) in a centralized control network, or each station must determine the allocation in a distributed control network. Therefore, compared to the simple distributed control mechanisms involved with the random access acknowledgment schemes, a cost associated with the scheduled acknowledgment schemes is increased complexity.

To be consistent with the GRA philosophy, a distributed control network is studied in this chapter. Each station must recognize collision-free information packet transmissions so that a network scheduling algorithm, which establishes a procedure to avoid PACK transmission conflicts, can be implemented. These protocols must be precisely executed by each station for successful channel operation.

5.2 Period Division - Scheduled Acknowledgment Scheme

Under the PDS scheme, the acknowledgment and information subperiod lengths, $\tilde{K}_A$ and $\tilde{K}_I$, are chosen such that (4.82) is satisfied and the information packet length to PACK length ratio is denoted by $\eta \geq 1$ as defined for the PDRA scheme studied in Section 4.4. The period division is chosen so that collision-free information packet transmissions in a single period are acknowledged in the next period by conflict-free PACK transmissions.

Since the maximum number of collision-free information packet transmissions per period is $\tilde{K}_I$, the number of acknowledgment mini-slots

required is $\tilde{K}_\eta = \tilde{K}_I$. Therefore, the period division must satisfy

$$\eta \tilde{K}_A \geq \tilde{K}_I \quad . \tag{5.1}$$

Thus for a channel structure specified by $(\tilde{K}, \eta)$, the maximum information subperiod length which satisfies (5.1) is

$$\tilde{K}_I = \left\lfloor \frac{\tilde{K}\eta}{1 + \eta} \right\rfloor \qquad \text{slots} \tag{5.2}$$

where $\lfloor x \rfloor$ denotes the integer part of x. The acknowledgment subperiod length which satisfies (4.82) and (5.2) is given by

$$\tilde{K}_A = \left\lfloor \frac{\tilde{K}}{1 + \eta} \right\rfloor_U \qquad \text{slots} \tag{5.3}$$

where

$$\lfloor x \rfloor_U = \begin{cases} x & \text{if } x \text{ is an integer} \\ \lfloor x \rfloor + 1 & \text{otherwise} \end{cases} \quad .$$

The fraction of a slot expressed by

$$\tilde{K}_\Delta = \tilde{K}_A - \frac{1}{\eta} \tilde{K}_I \tag{5.4}$$

cannot be used for information packet transmissions and it is not required for PACK transmissions. This fractional slot is placed

(arbitrarily) at the beginning of each channel access period with the acknowledgment subperiod preceding the information subperiod as illustrated in Figure 5.1.

The GRA channel under the PDS acknowledgment scheme operates as follows.

Protocol: GRA Discipline - PDS ACK Scheme

(1) and (2) See the corresponding steps under the PDRA ACK scheme in Section 4.4.

(3) For a collision-free information packet transmission in the i-th slot within the n-th information subperiod, a PACK is transmitted in the i-th mini-slot within the (n+1)-st acknowledgment subperiod.

(4) Each information packet admitted by the network control procedure is transmitted and retransmitted until successful.

5.2.1 Channel State Process

The evolution of the channel is described by a vector Markov chain $Z = \{Z_n, n \geq 1\}$ over the space $d^{\tilde{K}_I} \times d_1^{\tilde{K}_I} \times d^{\tilde{K}+P}$ where d is the set of non-negative integers, $d_1 = \{0,1\}$ and

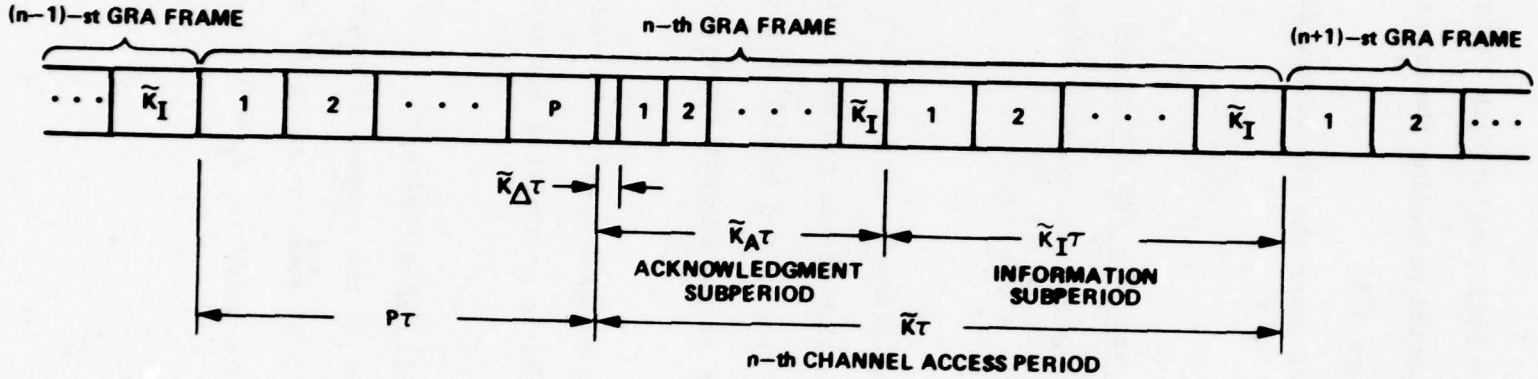


Figure 5.1 GRA Channel Structure Under the PDS ACK Scheme

$$Z_n = \{(T_{ni}^{(I)},\ i = 1, 2, \ldots, \tilde{K}_I),\ (T_{ni}^{(A)},\ i = 1, 2, \ldots, \tilde{K}_I),$$

$$(A_{ni},\ i = 1, 2, \ldots, \tilde{K}+P)\}\ .$$

The variable $T_{ni}^{(I)}$ denotes the number of information packet retransmissions in the i-th slot within the n-th information subperiod ($n \geq 1$, $1 \leq i \leq \tilde{K}_I$), the 0-1 variable $T_{ni}^{(A)}$ indicates a PACK transmission in the i-th mini-slot within the n-th acknowledgment subperiod ($n \geq 1$, $1 \leq i \leq \tilde{K}_I$), and A_{ni} denotes the number of uncontrolled new message arrivals in the i-th slot within the n-th frame ($n \geq 1$, $1 \leq i \leq \tilde{K}+P$).

Let R_n denote the number of information packet collisions within the n-th period. It is expressed by (4.84) and by

$$R_n = \sum_{i=1}^{\tilde{K}_I} R_{ni} \tag{5.5}$$

where R_{ni} is the number of information packet collisions in the i-th slot within the n-th information subperiod. The variable R_{ni} is given by

$$R_{ni} = N_{ni}\ I(N_{ni} > 1) \tag{5.6}$$

where N_{ni} denotes the number of information packet transmissions in the i-th slot within the n-th information subperiod (4.93). The controlled new message arrival variables $\{A_{ni}^{(C)},\ 1 \leq i \leq \tilde{K}+P\}$ and

$\{\tilde{A}^{(C)}_{ni},\ 1 \le i \le \tilde{K}_I\}$ are governed by (4.4) and (4.94), respectively. The sequence $\{T^{(I)}_{n+1,i},\ 1 \le i \le \tilde{K}_I\}$ is governed by the multinomial distribution (4.86).

A collision-free information packet transmission in the i-th slot within the n-th information subperiod is acknowledged by a PACK transmission in the i-th mini-slot within the (n+1)-st acknowledgment subperiod. Let $\tilde{S}^{(I)}_{ni}$ indicate a collision-free information packet transmission in the i-th slot within the n-th information subperiod. Hence,

$$\tilde{S}^{(I)}_{ni} = T^{(A)}_{n+1,i} \quad , \tag{5.7}$$

$n \ge 1$, $1 \le i \le \tilde{K}_I$. The 0-1 variable $\tilde{S}^{(I)}_{ni}$ is given by

$$\tilde{S}^{(I)}_{ni} = I(N_{ni} = 1) \quad . \tag{5.8}$$

Equations (4.4), (4.86), (4.93), (4.94) and (5.5) - (5.8) yield the transition probability function for the vector Markov chain Z under the PDS scheme. The uncontrolled new message arrival variables $\{A_{n+1,i},\ 1 \le i \le \tilde{K}+P\}$ are statistically independent of Z_n; and $\{T^{(I)}_{n+1,i},\ 1 \le i \le \tilde{K}_I\}$ and $\{T^{(A)}_{n+1,i},\ 1 \le i \le \tilde{K}_I\}$ depend on Z_n only through $X_n = \{(\tilde{S}^{(I)}_{ni},\ i = 1, 2, \ldots, \tilde{K}_I),\ R_n\}$. The sequence $X = \{X_n,\ n \ge 1\}$ is a vector Markov chain over the space $d_1^{\tilde{K}_I} \times d$. A flow diagram indicating the transition $X_n \to X_{n+1}$ is shown in Figure 5.2. Furthermore, X_{n+1} depends on X_n only through R_n (if $\tilde{U}_{n+1}$ depends on X_n only through R_n)

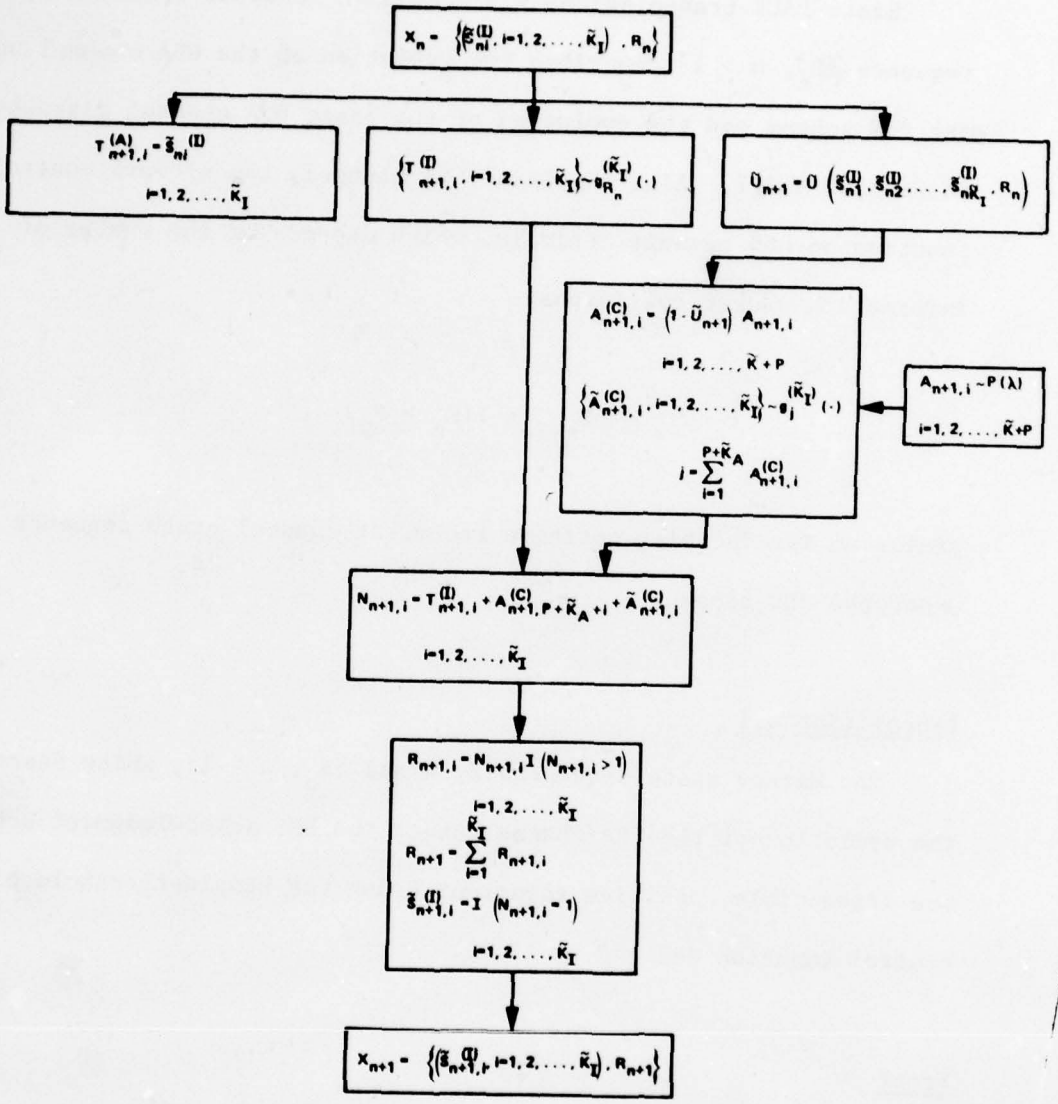


Figure 5.2 Transition $X_n \rightarrow X_{n+1}$ for the Markov State Chain X Associated with the GRA Discipline Under the PDS ACK Scheme

and $\{R_n, n \geq 1\}$ is a scalar Markov chain over the space d.

Since PACK transmissions are scheduled to avoid collisions, the sequence $\{R_n, n \geq 1\}$ describes the evolution of the GRA channel under the PDS scheme and the evolution of the basic GRA channel discussed in Section 4.1.2. Like the basic GRA channel, the network control function on new message arrivals, which depends on the number of information packet collisions

$$U_{n+1} = I(R_n \geq N_T) \quad , \tag{5.9}$$

yields an irreducible, positive recurrent channel state sequence under the PDS scheme.

Proposition 5.1

The Markov state sequences Z, X and $\{R_n, n \geq 1\}$, which describe the evolution of the GRA channel under the PDS acknowledgment scheme, are irreducible, positive recurrent using the single-threshold binary control function defined by (5.9).

Proof

Similar technique as Proposition 4.1.

Q.E.D.

Under the PDS scheme, the limiting average number of collision-free information packet transmissions and PACK transmissions are equal.

Therefore, throughput (measured in packets per service slot) can be expressed as the limit (when it exists)

$$\tilde{\delta} = \lim_{N\to\infty} \frac{1}{N} E\{ \sum_{n=1}^{N} \frac{\tilde{S}_n^{(I)}}{\tilde{K}} \} \tag{5.10}$$

where $\tilde{S}_n^{(I)}$ denotes the number of collision-free information packet transmissions within the n-th period. An upper bound on throughput is stated in the following theorem.

Theorem 5.1

The throughput of the GRA channel under the PDS acknowledgment scheme specified by the channel structure $(\tilde{K}, \eta)$ is upper bounded by

$$\tilde{\delta} \leq \frac{\tilde{K}_I}{\tilde{K}} \; G_{\tilde{K}_I} \; e^{-1} \tag{5.11}$$

where

$$G_{\tilde{K}_I} = \{(\tilde{K}_I - 1) \; \ell n[1 + (\tilde{K}_I - 1)^{-1}]\}^{-1}$$

and $\tilde{K}_I$ is determined by (5.2).

Proof

The expectation of $\tilde{S}_n^{(I)}$ given N_n, the total number of information packet transmissions within the n-th period, is given by

$$E(\tilde{S}_n^{(I)} | N_n) = N_n (1 - \frac{1}{\tilde{K}_I})^{N_n - 1} \quad . \tag{5.12}$$

Applying the inequality (4.25) to (5.12) and using (5.10) proves (5.11).

Q.E.D.

A straightforward simulation of the Markov chain X has been used to compute maximum throughput values for $\tilde{K} = 12$, $\eta = 1,9$. The maximum throughput values are 0.18 and 0.3 packets per service slot for $\eta = 1$ and 9, respectively. The upper bounds computed using (5.11) are 0.202 and 0.323 packets per service slot.

5.2.2 Packet Delay Analysis

The delay (measured in slots) of the n-th message is decomposed into the sum

$$D_{S_n} = \tilde{W}_n + R + 1/\eta \tag{5.13}$$

where $\tilde{W}_n$ is the system waiting time of the n-th message measured from its arrival slot to the transmission mini-slot of its PACK. The R slot and $1/\eta$ slot durations represent the propagation delay and transmission time of the PACK. The computation of the limiting average packet delay parallels the technique introduced in Section 4.2.2. Consider the channel structure illustrated in Figure 5.1 where the

fractional slot is placed at the beginning of each channel access period and the acknowledgment subperiod precedes the information subperiod.

Theorem 5.2

With the operation of a controlled GRA channel under the PDS acknowledgment scheme described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delay is given by

$$E(D_S) = \frac{3}{2} P + \frac{\tilde{K}}{2} \left(\frac{2 + \eta}{1 + \eta}\right) + \frac{\tilde{K}_\Delta}{2} \left(\frac{1 + 2\eta}{1 + \eta}\right) + \frac{1}{2} \left(1 + \frac{1}{\eta}\right)$$

$$+ (P+\tilde{K}) \frac{E(R_n)}{E(\tilde{S}_n^{(1)})} + R \qquad (5.14)$$

where the expectations are w.r.t. the stationary distribution.

Proof

Similar technique as Theorem 4.2.

Q.E.D.

The limiting average packet delay expressed by (5.14) is the average holding time of a message in the source station's buffer. The packet delay measured from arrival at the source station until successful reception at the destination station (data transfer delay) is given by the following corollary.

Corollary 5.1

With the operation of a controlled GRA channel under the PDS acknowledgment scheme described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delay (data transfer delay) is given by

$$E(D_R) = \frac{1}{2}\left[P + \frac{\tilde{K}}{1+\eta} + \frac{\eta\,\tilde{K}_{\Delta}}{1+\eta}\right] + (P+\tilde{K})\,\frac{E(R_n)}{E(\tilde{S}_n^{(I)})} + R + 1 \qquad (5.15)$$

where the expectations are w.r.t. the stationary distribution.

Proof

Since PACK transmissions are scheduled to avoid collisions, the difference between $E(D_S)$ and $E(D_R)$ is given by

$$E(D_S) - E(D_R) = \frac{E\{\tilde{W}_{12}(Z_n, Z_{n+1})\}}{E(\tilde{S}_n^{(I)})} - (1 - \frac{1}{\eta}) \qquad (5.16)$$

where the first term on the right hand side of (5.16) is the limiting average waiting time component measured from the transmission slot of a collision-free information packet to the transmission mini-slot of its PACK. The one slot and $1/\eta$ slot durations represent the information packet and PACK transmission times, respectively. The waiting time component is given by

$$\tilde{W}_{12}(Z_n, Z_{n+1}) = \tilde{K}_I \tilde{S}_{n1}^{(I)} + (\tilde{K}_I - 1)\tilde{S}_{n2}^{(I)} + \ldots + \tilde{S}_{n\tilde{K}_I}^{(I)}$$

$$+ (P+\tilde{K}_\Delta)\tilde{S}_n^{(I)}$$

$$+ \frac{1}{\eta}\left[T_{n+1,2}^{(A)} + 2T_{n+1,3}^{(A)} + \ldots + (\tilde{K}_I - 1)T_{n+1,\tilde{K}_I}^{(A)}\right]. \quad (5.17)$$

Applying Lemma 4.1 to (5.17) with $N(Z_n, Z_{n+1}) = \tilde{S}_n^{(I)}$ yields

$$\frac{E\{\tilde{W}_{12}(Z_n, Z_{n+1})\}}{E(\tilde{S}_n^{(I)})} = \frac{1}{2}\left[2P + \tilde{K} + \tilde{K}_\Delta + (1 - \frac{1}{\eta})\right] \quad (5.18)$$

Substituting (5.14) and (5.18) into (5.16), proves (5.15).

Q.E.D.

5.3 Dynamic Period Division - Scheduled Acknowledgment Scheme

Under the DPDS acknowledgment scheme, each channel access period is partitioned into an acknowledgment subperiod and an information subperiod. Information packets are transmitted on a random access basis while PACK transmissions are scheduled. However, unlike the PDS scheme, the subperiod slot allocations are not fixed. The subperiod slot allocations adapt to the acknowledgment traffic requirements.

Under the PDS scheme, the mini-slot allocation of the acknowledgment subperiod is large enough to accommodate $\tilde{K}_I$ PACK transmissions in each period (5.1). However, the likelihood of $\tilde{K}_I$ collision-free

information packet transmissions within a single period is highly improbable. Thus, in general, a much smaller mini-slot allocation is sufficient. In particular, the number of necessary mini-slots in the n-th acknowledgment subperiod is equal to the number of collision-free information packet transmissions within the (n-1)-st information subperiod, $\tilde{S}_{n-1}^{(I)}$. Therefore, the length of the n-th acknowledgment subperiod (measured in slots) is given by

$$\tilde{K}_n^{(A)} = \underline{\left\lfloor \frac{1}{\eta} S_{n-1}^{(I)} \right\rfloor}_U \tag{5.19}$$

where $\underline{\lfloor x \rfloor}_U = \underline{\lfloor x \rfloor} + I(x - \underline{\lfloor x \rfloor} > 0)$ and the length of the n-th information subperiod is given by

$$\tilde{K}_n^{(I)} = \tilde{K} - \tilde{K}_n^{(A)} \quad . \tag{5.20}$$

The fraction of a slot given by

$$\tilde{K}_n^{(\Delta)} = \tilde{K}_n^{(A)} - \frac{1}{\eta} \tilde{S}_{n-1}^{(I)} \tag{5.21}$$

cannot be used for information packet transmissions and it is not required for PACK transmissions. This fractional slot is placed (arbitrarily) at the end of the acknowledgment subperiod. In addition, the period structure with the acknowledgment subperiod preceding the information subperiod is considered as illustrated in Figure 5.3.

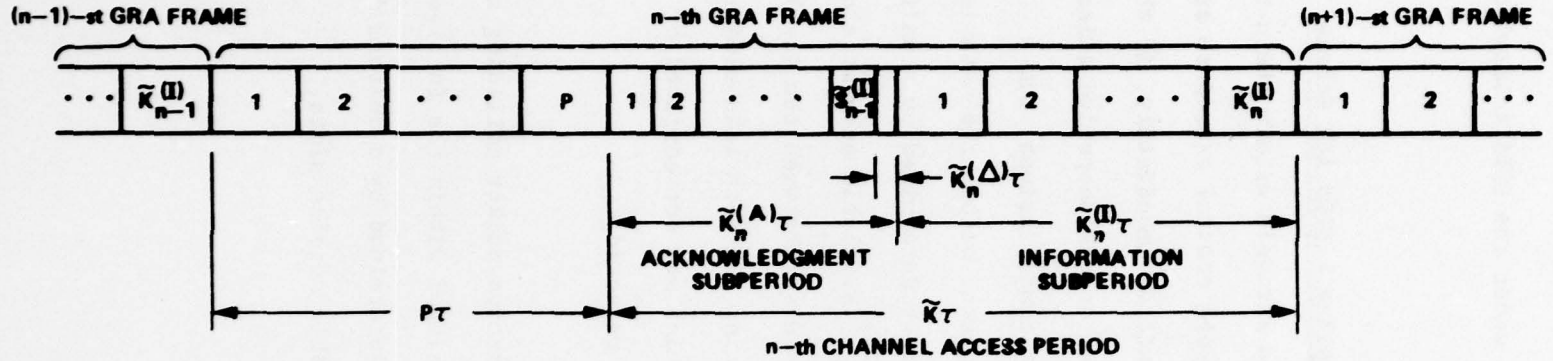


Figure 5.3 GRA Channel Structure Under the DPDS ACK Scheme

The GRA channel under the DPDS acknowledgment scheme operates as follows.

Protocol: GRA Discipline - DPDS ACK Scheme

(1) New message arrivals within the n-th frame which are admitted by the network control procedure are

i) transmitted immediately, if they arrive during the channel access subperiod allocated to information packet transmissions, and

ii) transmitted within the n-th information subperiod in a slot determined by a uniform distribution over the $\tilde{K}_n^{(I)}$ available service slots, if they arrive in the interval between the (n-1)-st and n-th periods or during the n-th acknowledgment subperiod.

Messages which are not admitted by the network control procedure are rejected.

(2) Each information packet colliding within the n-th period is retransmitted within the (n+1)-st information subperiod in a slot determined by a uniform distribution over the $\tilde{K}_{n+1}^{(I)}$ available service slots.

(3) A PACK is transmitted in the i-th mini-slot within the (n+1)-st acknowledgment subperiod for the i-th collision-free information packet transmission within the n-th information subperiod, $1 \leq i \leq \tilde{S}_n^{(I)}$.

(4) Each information packet admitted by the network control procedure is transmitted and retransmitted until successful.

Step (3) defines the algorithm which schedules PACK transmission to avoid conflicts. This protocol requires each network station to recognize and record collision-free information packet transmissions and to compute the period division using (5.19) and (5.20).

5.3.1 Channel State Process

The evolution of the channel is described by a vector Markov chain $Z = \{Z_n, n \geq 1\}$ over the space $d_{\tilde{K}} \times d^{\tilde{K}} \times d_1^{\tilde{K}} \times d^{\tilde{K}+P}$ where d is the set of non-negative integers, $d_{\tilde{K}} = \{0, 1, \ldots, \tilde{K}\}$, $d_1 = \{0,1\}$ and

$$Z_n = \{\tilde{S}_{n-1}^{(I)}, (T_{ni}^{(I)}, i = 1, 2, \ldots, \tilde{K}), (T_{ni}^{(A)}, i = 1, 2, \ldots, \tilde{K}),$$

$$(A_{ni}, i = 1, 2, \ldots, \tilde{K}+P)\} .$$

The variable $\tilde{S}_{n-1}^{(I)}$ denotes the number of collision-free information packet transmissions within the (n-1)-st period, $T_{ni}^{(I)}$ denotes the

number of information packet retransmissions in the i-th slot within the n-th information subperiod, $T_{ni}^{(A)}$ indicates a PACK transmission in the i-th mini-slot within the n-th acknowledgment subperiod, and A_{ni} denotes the number of uncontrolled new message arrivals in the i-th slot within the n-th frame.

By definition, R_n (the number of colliding information packet transmissions within the n-th period) is given by

$$R_n = \sum_{i=1}^{\tilde{K}_n^{(I)}} R_{ni} \tag{5.22}$$

where R_{ni} is the number of information packet collisions in the i-th slot within the n-th information subperiod. It can be expressed as

$$R_{ni} = N_{ni}\, I(N_{ni} > 1) \tag{5.23}$$

where N_{ni} (the number of information packet transmissions in the i-th slot within the n-th information subperiod) is given by

$$N_{ni} = T_{ni}^{(I)} + A^{(C)}_{n,P+\tilde{K}_n^{(A)}+i} + \tilde{A}_{ni}^{(C)} , \tag{5.24}$$

$1 \leq i \leq \tilde{K}_n^{(I)}$.

Since information packet collisions within the n-th period are retransmitted within the (n+1)-st information subperiod,

$$R_n = \sum_{i=1}^{\tilde{K}_{n+1}^{(I)}} T_{n+1,i}^{(I)} . \tag{5.25}$$

The sequence $\{T^{(I)}_{n+1,i},\ 1 \le i \le \tilde{K}^{(I)}_{n+1}\}$ is governed by the following multinomial distribution:

$$P\{T^{(I)}_{n+1,i} = \alpha_i,\ 1 \le i \le \tilde{K}^{(I)}_{n+1} | \tilde{S}^{(I)}_n,\ R_n\}$$

$$= g_{R_n}^{(\tilde{K}^{(I)}_{n+1})}(\alpha_1, \alpha_2, \ldots, \alpha_{\tilde{K}^{(I)}_{n+1}}) \tag{5.26}$$

where $0 \le \alpha_i \le R_n$, $1 \le i \le \tilde{K}^{(I)}_{n+1}$, $\sum_{i=1}^{\tilde{K}^{(I)}_{n+1}} \alpha_i = R_n$.

The controlled new message arrival sequence $\{A^{(C)}_{n+1,i},\ 1 \le i \le \tilde{K}+P\}$ is governed by (4.4) and the sequence $\{\tilde{A}^{(C)}_{n+1,i},\ 1 \le i \le \tilde{K}^{(I)}_{n+1}\}$ is governed by the following multinomial distribution:

$$P\{\tilde{A}^{(C)}_{n+1,i} = \alpha_i,\ 1 \le i \le \tilde{K}^{(I)}_{n+1} | \sum_{i=1}^{P+\tilde{K}^{(A)}_{n+1}} A^{(C)}_{n+1,i} = j,\ \tilde{S}^{(I)}_n\}$$

$$= g_j^{(\tilde{K}^{(I)}_{n+1})}(\alpha_1, \alpha_2, \ldots, \alpha_{\tilde{K}^{(I)}_{n+1}}) \tag{5.27}$$

where $0 \le \alpha_i \le j$, $1 \le i \le \tilde{K}^{(I)}_{n+1}$, $\sum_{i=1}^{\tilde{K}^{(I)}_{n+1}} \alpha_i = j$.

By definition,

$$T^{(A)}_{n+1,i} = \begin{cases} 1 & \text{if} \quad 1 \le i \le \tilde{S}^{(I)}_n \\ 0 & \text{otherwise} \end{cases} \tag{5.28}$$

and $\tilde{S}^{(I)}_n$ is determined by

$$\tilde{S}_n^{(I)} = \sum_{i=1}^{\tilde{K}_n^{(I)}} \tilde{S}_{ni}^{(I)} \tag{5.29}$$

where

$$\tilde{S}_{ni}^{(I)} = I(N_{ni} = 1),\ 1 \leq i \leq \tilde{K}_n^{(I)}.$$

Equations (4.4), (5.19) - (5.29) yield the transition probability function for the vector Markov chain Z under the DPDS scheme. The uncontrolled new message arrival variables $\{A_{n+1,i},\ 1 \leq i \leq \tilde{K}+P\}$ are statistically independent of Z_n; and $\{T^{(I)}_{n+1,i},\ 1 \leq i \leq \tilde{K}\}$ and $\{T^{(A)}_{n+1,i},\ 1 \leq i \leq \tilde{K}\}$ depend on Z_n only through $X_n = (\tilde{S}_n^{(I)}, R_n)$. The sequence $X = \{X_n,\ n \geq 1\}$ is a vector Markov chain over the space $d_{\tilde{K}} \times d$. A flow diagram indicating the transition $X_n \rightarrow X_{n+1}$ is shown in Figure 5.4.

Two possible network control functions which yield irreducible, positive recurrent Markov state sequences are

$$\tilde{U}_{n+1} = I(R_n \geq N_T) \tag{5.30}$$

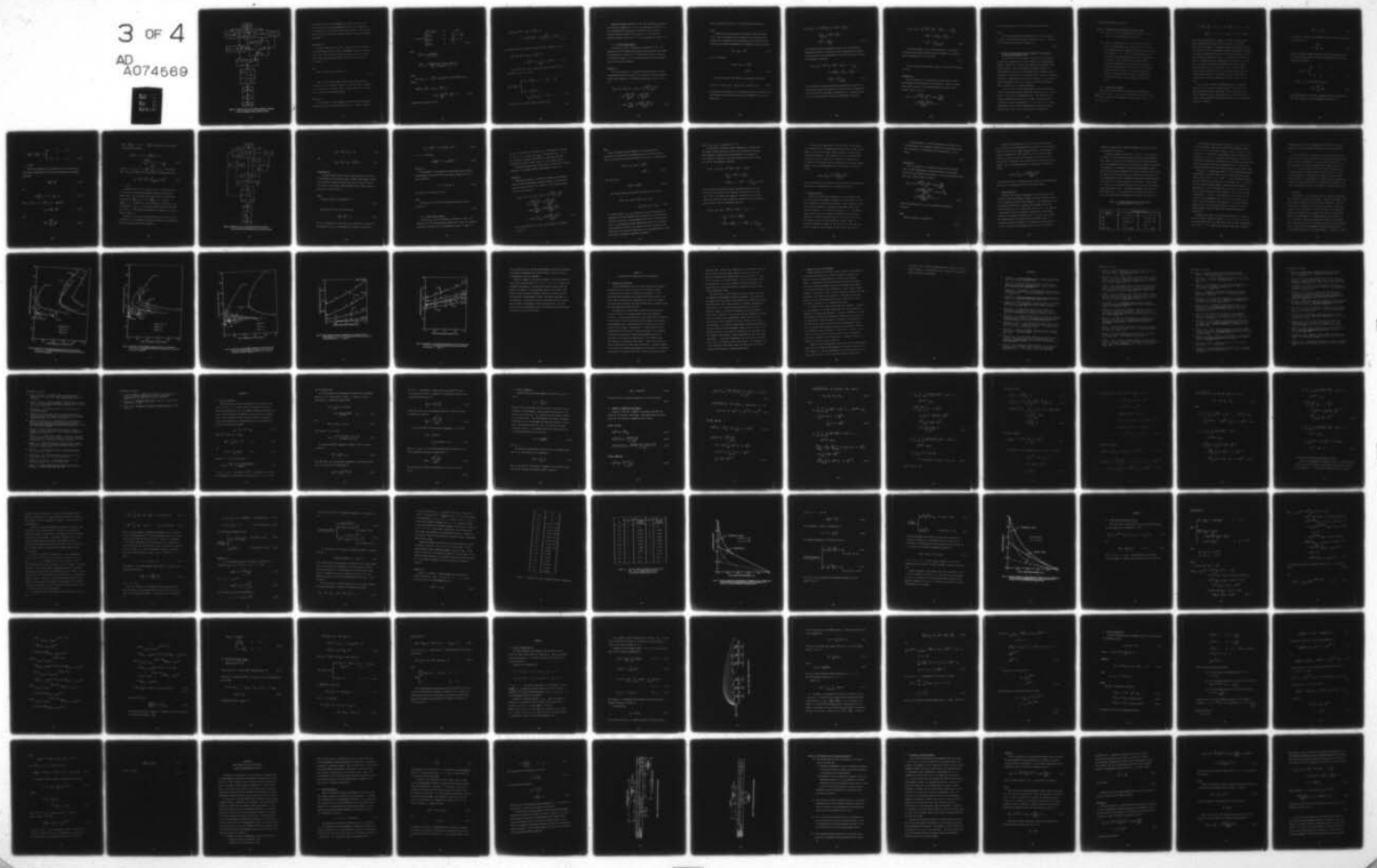
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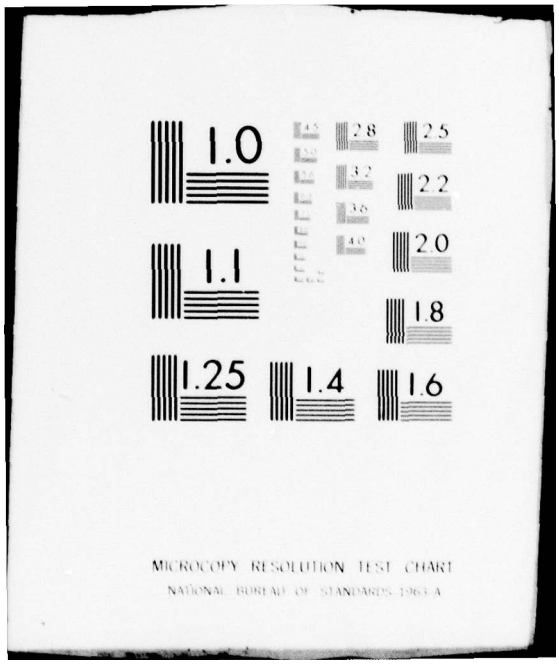
$$\tilde{U}_{n+1} = I(R_n \geq \tilde{K}_{n+1}^{(I)} N_T) \quad . \tag{5.31}$$

Both control functions reject new message arrivals within the (n+1)-st frame if the threshold N_T is exceeded. The control function (5.30) rejects new message arrivals whenever the number of information packet

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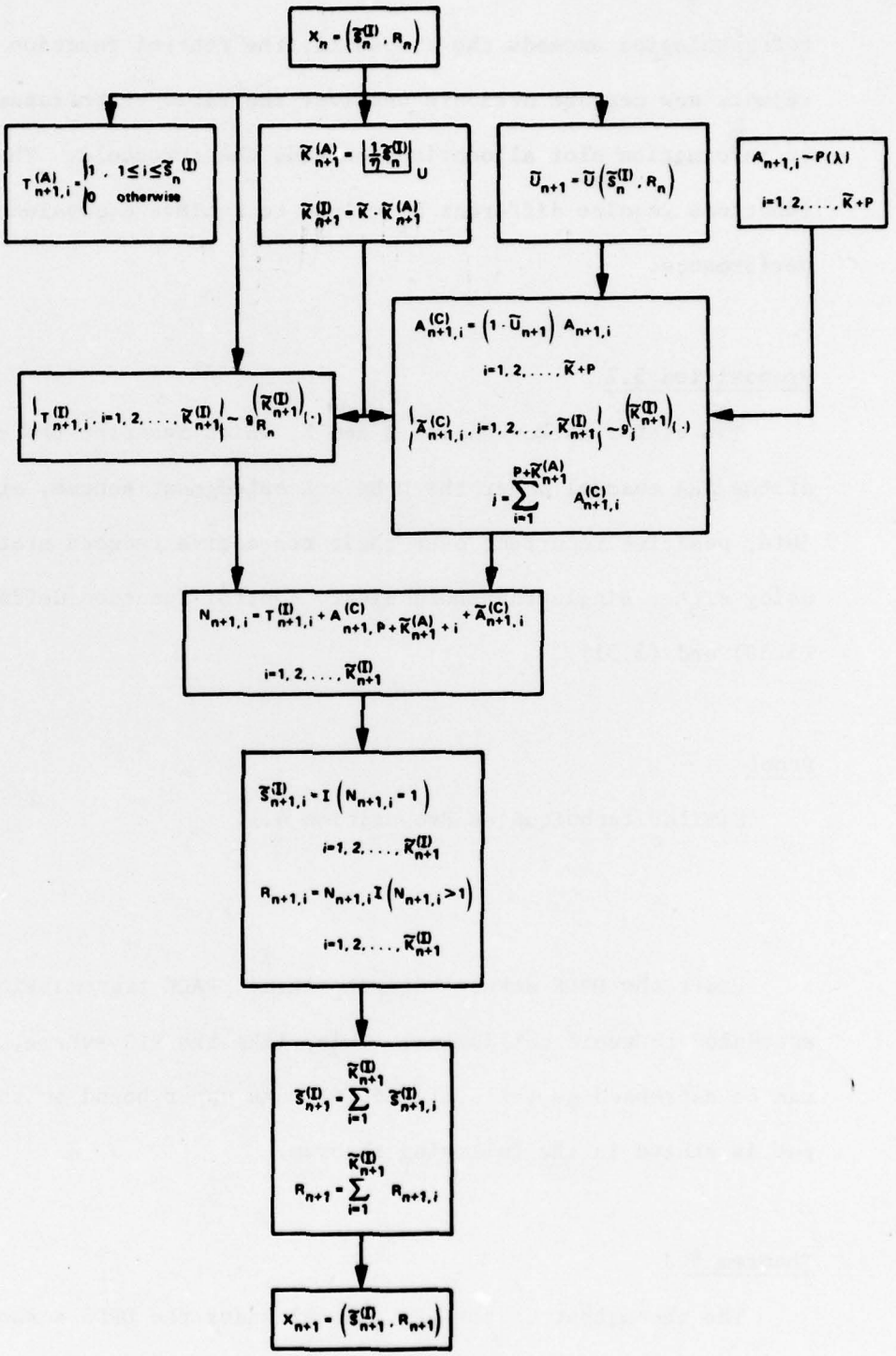


Figure 5.4 Transition $X_n \rightarrow X_{n+1}$ for the Markov State Chain X Associated with the GRA Discipline Under the DPDS ACK Scheme

retransmission exceeds the threshold; the control function (5.31) rejects new message arrivals whenever the ratio of retransmissions to information slot allocations exceeds the threshold. These control functions require different N_T values to achieve equivalent throughput performance.

Proposition 5.2

The vector Markov chains Z and X, which describe the evolution of the GRA channel under the DPDS acknowledgment scheme, are irreducible, positive recurrent over their respective reduced state spaces, using either single-threshold binary control function defined by (5.30) and (5.31).

Proof

Similar technique as Proposition 4.1.

Q.E.D.

Under the DPDS acknowledgment scheme, PACK transmissions are scheduled to avoid collisions. Thus, like the PDS scheme, throughput can be expressed as the limit (5.10). An upper bound on the throughput is stated in the following theorem.

Theorem 5.3

The throughput of the GRA channel under the DPDS acknowledgment scheme specified by the channel structure $(\tilde{K}, \eta)$ is upper bounded by

$$\tilde{\delta} \le H(\tilde{K}, \eta) = \begin{cases} H_o^{(\tilde{K})}\left(\frac{\tilde{K}}{\tilde{K}-2}\right) & \text{if} \quad \eta \le \frac{\tilde{K}}{\tilde{K}-2} \\ H_o^{(\tilde{K})}(\eta) & \text{if} \quad \frac{\tilde{K}}{\tilde{K}-2} < \eta \le \frac{\tilde{K}}{\tilde{K}-3} \\ H_1^{(\tilde{K})}(\eta) & \text{if} \quad \frac{\tilde{K}}{\tilde{K}-3} < \eta \le \tilde{K} \\ H_1^{(\tilde{K})}(\tilde{K}) & \text{if} \quad \tilde{K} < \eta \end{cases} \tag{5.32}$$

where

$$H_o^{(\tilde{K})}(x) = \frac{e^{-1}(\tilde{K}+1)}{\tilde{K}(1+e^{-1}/x)}$$

$$H_1^{(\tilde{K})}(x) = \frac{e^{-1}[\tilde{K}(2\tilde{K}+1)\ (1-1/x) - (4\tilde{K}+1)]}{2\tilde{K}(1+e^{-1}/x)[\tilde{K}(1-1/x) - 2]} .$$

Proof

Since $\{N_{ni},\ 1 \le i \le \tilde{K}_n^{(I)}\}$ is governed by a multinomial distribution,

$$E(\tilde{S}_n^{(I)} | N_n, \tilde{K}_n^{(I)}) = I(N_n = 1, \tilde{K}_n^{(I)} = 1) + N_n\left(1 - \frac{1}{\tilde{K}_n^{(I)}}\right)^{N_n - 1} I(\tilde{K}_n^{(I)} > 1). \tag{5.33}$$

Applying the inequality (4.25),

$$E(\tilde{S}_n^{(I)} | N_n, \tilde{K}_n^{(I)}) \leq I(N_n = 1, \tilde{K}_n^{(I)} = 1)$$

$$+ e^{-1} \tilde{K}_n^{(I)} [(\tilde{K}_n^{(I)} - 1) \ell n(\frac{\tilde{K}_n^{(I)}}{\tilde{K}_n^{(I)} - 1})]^{-1} I(\tilde{K}_n^{(I)} > 1). \quad (5.34)$$

Retaining the first two terms of a Taylor series expansion for $\ell n\ x$,

$$E(\tilde{S}_n^{(I)} | N_n, \tilde{K}_n^{(I)}) \leq I(N_n = 1, \tilde{K}_n^{(I)} = 1)$$

$$+ e^{-1} [\tilde{K}_n^{(I)} + \frac{1}{2} + \frac{1}{2(\tilde{K}_n^{(I)} - 1)}] I(\tilde{K}_n^{(I)} > 1). \quad (5.35)$$

Using (5.19) and (5.20) to upper and lower bound $\tilde{K}_n^{(I)}$ over the appropriate values of η yields

$$E(\tilde{S}_n^{(I)} | \tilde{S}_{n-1}^{(I)}) \leq \begin{cases} e^{-1}[\tilde{K} - \frac{1}{\eta} \tilde{S}_{n-1}^{(I)} + 1] & \text{if } \frac{\tilde{K}}{\tilde{K}-2} \leq \eta \leq \frac{\tilde{K}}{\tilde{K}-3} \\ e^{-1}[\tilde{K} - \frac{1}{\eta} \tilde{S}_{n-1}^{(I)} + \frac{1}{2} & \\ \quad + \frac{1}{2(\tilde{K} - \frac{1}{\eta} \tilde{K} - 2)}] & \text{if } \frac{\tilde{K}}{\tilde{K}-3} \leq \eta \end{cases} \quad (5.36)$$

The bound (5.32) follows using (5.10) and (5.36).

Q.E.D.

Maximum throughput values 0.27, 0.34 and 0.34 packets per service slot have been computed for $\eta = 1, 9, 12$, respectively, with $\tilde{K} = 12$ by a straightforward simulation of the Markov state sequence X. The upper bounds provided by (5.32) are 0.305, 0.370 and 0.373 packets per service slot for $\eta = 1, 9, 12$, respectively, with $\tilde{K} = 12$.

5.3.2 Packet Delay Analysis

The delay D_{S_n} of the n-th message is defined by (5.13). The computation of the limiting average packet delay parallels the technique introduced in Section 4.2.2. Considering the channel structure illustrated in Figure 5.3, the limiting average packet delay $E(D_S)$ is given by the following theorem.

Theorem 5.4

With the operation of a controlled GRA channel under the DPDS acknowledgment scheme described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delay is given by

$$E(D_S) = \frac{3}{2} P + \frac{1}{2} \tilde{K} + \frac{1}{2} (1 + \frac{1}{\eta}) + R + \frac{1}{2} \frac{E[\tilde{K}_n^{(A)}(1 - \tilde{U}_n)]}{(1 - P_R)}$$

$$+ \frac{1}{2\eta} \frac{E(\tilde{S}_n^{(I)} \tilde{S}_n^{(I)})}{E(\tilde{S}_n^{(I)})} - \frac{1}{2} \frac{E(\tilde{S}_n^{(I)} \tilde{K}_n^{(A)})}{E(\tilde{S}_n^{(I)})}$$

$$+ (P+\tilde{K}) \frac{E(R_n)}{E(\tilde{S}_n^{(I)})} + \frac{1}{2} \frac{E[(\tilde{K}_{n+1}^{(A)} - \tilde{K}_n^{(A)})R_n]}{E(\tilde{S}_n^{(I)})} \qquad (5.37)$$

where the expectations are w.r.t. the stationary distribution.

Proof

Consider the two functionals $N(\cdot)$ and $\tilde{W}(\cdot)$ defined in Section 4.2.2. Since new message arrivals admitted by the network control procedure are transmitted and retransmitted until successfully transmitted and since PACK transmissions are scheduled to avoid collisions,

$$N(Z_n, Z_{n+1}) = A_n^{(C)} \tag{5.38}$$

and, in equilibrium,

$$E\{N(Z_n, Z_{n+1})\} = E(A_n^{(C)})$$

$$= E(\tilde{S}_n^{(I)}). \tag{5.39}$$

The system waiting time function is decomposed into the sum

$$\tilde{W}(Z_n, Z_{n+1}) = \tilde{W}_o(Z_n, Z_{n+1}) + \tilde{W}_{11}(Z_n, Z_{n+1}) + \tilde{W}_{12}(Z_n, Z_{n+1}). \tag{5.40}$$

The waiting time components of all new message arrivals within the n-th frame measured from the arrival slot to the transmission slot can be expressed by

$$\tilde{W}_o(Z_n, Z_{n+1}) = (P + \tilde{K}_n^{(A)})A_{n1}^{(C)} + (P + \tilde{K}_n^{(A)} - 1)A_{n2}^{(C)} + \ldots$$

$$+ A^{(C)}_{n,\tilde{K}_n^{(A)}+P} + \tilde{A}_{n2}^{(C)} + 2\tilde{A}_{n3}^{(C)} + \ldots$$

$$+ (\tilde{K}_n^{(I)} - 1)\tilde{A}^{(C)}_{n\tilde{K}_n^{(I)}} \quad . \tag{5.41}$$

For information packet transmissions within the n-th period which experience collision, the interval from the transmission slot within the n-th period to the retransmission slot within the (n+1)-st information subperiod is measured:

$$\tilde{W}_{11}(Z_n, Z_{n+1}) = \tilde{K}_n^{(I)} R_{n1} + (\tilde{K}_n^{(I)} - 1)R_{n2} + \ldots + R_{n\tilde{K}_n^{(I)}}$$

$$+ (P + \tilde{K}_{n+1}^{(A)})R_n + T_{n+1,2}^{(I)} + 2T_{n+1,3}^{(I)} + \ldots$$

$$+ (\tilde{K}_{n+1}^{(I)} - 1)T^{(I)}_{n+1,\tilde{K}_{n+1}^{(I)}} \quad . \tag{5.42}$$

For collision-free information packet transmissions within the n-th period, the waiting time component is measured from the transmission slot within the n-th information subperiod to the transmission mini-slot of its PACK within the (n+1)-st acknowledgment subperiod:

$$\tilde{W}_{12}(Z_n, Z_{n+1}) = \tilde{K}_n^{(I)} \tilde{S}_{n1}^{(I)} + (\tilde{K}_n^{(I)} - 1)\tilde{S}_{n2}^{(I)} + \ldots + \tilde{S}_{n\tilde{K}_n^{(I)}}^{(I)}$$

$$+ P\tilde{S}_n^{(I)} + \frac{1}{\eta}\left[T_{n+1,2}^{(A)} + 2T_{n+1,3}^{(A)} + \ldots\right.$$

$$\left. + (\tilde{S}_n^{(I)} - 1)T_{n+1,\tilde{S}_n^{(I)}}^{(A)}\right] . \qquad (5.43)$$

By applying Lemma 4.1 and by taking advantage of the symmetry provided by the uniform distributed slot allocations for information packet transmissions, (5.37) is proved.

Q.E.D.

The data transfer delay measure, $E(D_R)$, is given by the following corollary.

Corollary 5.2

With the operation of a controlled GRA channel under the DPDS acknowledgment scheme described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delay (data transfer delay) is given by

$$E(D_R) = \frac{P}{2} + \frac{1}{2}\frac{E[\tilde{K}_n^{(A)}(1 - \tilde{U}_n)]}{1 - P_R} + (P+\tilde{K})\frac{E(R_n)}{E(\tilde{S}_n^{(I)})}$$

$$+ \frac{1}{2}\frac{E[(\tilde{K}_{n+1}^{(A)} - \tilde{K}_n^{(A)})R_n]}{E(\tilde{S}_n^{(I)})} + R + 1 \qquad (5.44)$$

where the expectations are w.r.t. the stationary distribution.

Proof

Since PACK transmissions are scheduled to avoid collisions, the difference between $E(D_S)$ and $E(D_R)$ is expressed by (5.16). Therefore, (5.44) follows from the proof of Theorem 5.4.

Q.E.D.

5.4 Random Noise GRA Channel Under the Dynamic Period Division - Scheduled Acknowledgment Scheme

The operation of the GRA channel using ARQ error control procedures has been examined under several random access acknowledgment schemes in Chapter IV and under two scheduled acknowledgment schemes in this chapter. These investigations determine the impact of acknowledgment traffic on the performance (delay, throughput, packet probability of rejection) of a GRA channel with a fixed slot (bandwidth) allocation. Random noise effects are not considered. Specifically, random transmission errors in collision-free PACK and information packet transmissions are assumed negligible.

In this section, the GRA channel under the DPDS acknowledgment scheme is investigated under a stationary transmission error process model. Errors occur as independent events. With probability P_N, random transmission errors occur in a single information packet transmission, and with probability $1 - P_N$, no errors occur. Random transmission errors in PACK transmissions are assumed negligible; however, their effects can be incorporated by extending the

techniques developed in this section.

Protocol: Random Noise GRA Discipline - DPDS ACK Scheme

(1), (2) and (4) See the corresponding steps under the GRA discipline - DPDS ACK scheme in Section 5.3.

(3) A PACK is transmitted in the i-th mini-slot within the (n+1)-st acknowledgment subperiod for the i-th collision-free information packet transmission within the n-th information subperiod, $1 \leq i \leq \tilde{S}_n^{(I)}$, if the information packet is received by the destination station without random transmission errors. Each collision-free information packet transmission within the n-th information subperiod which remains unacknowledged at the end of the two frame acknowledgment time-out interval is retransmitted within the (n+2)-nd information subperiod in a slot determined by a uniform distribution over the $\tilde{K}_{n+2}^{(I)}$ available service slots.

5.4.1 Channel State Process

The evolution of the random noise channel is described by a vector Markov chain $Z = \{Z_n, n \geq 1\}$ over the space $d_{\tilde{K}} \times d_{\tilde{K}}^{\tilde{K}} \times d^{\tilde{K}} \times d_1^{\tilde{K}} \times d^{\tilde{K}+P}$ where d is the set of non-negative integers, $d_{\tilde{K}} = \{0,1,2,\ldots,\tilde{K}\}$, $d_1 = \{0,1\}$, and

$$Z_n = \{\tilde{S}_{n-1}^{(I)}, (T_{ni}^{(F)}, i = 1, 2, \ldots, \tilde{K}), (T_{ni}^{(I)}, i = 1, 2, \ldots, \tilde{K}),$$

$$(T_{ni}^{(A)}, i = 1, 2, \ldots, \tilde{K}), (A_{ni}, i = 1, 2, \ldots, \tilde{K}+P)\}.$$

The variable $\tilde{S}_{n-1}^{(I)}$ denotes the number of collision-free information packet transmissions within the (n-1)-st period. The variable $T_{ni}^{(F)}$ denotes the number of information packet retransmissions in the i-th slot within the n-th information subperiod from among the information packets transmitted within the (n-2)-nd period without collision but with random transmission errors. The variable $T_{ni}^{(I)}$ denotes the number of information packet retransmissions in the i-th slot within the n-th information subperiod from among the information packet collisions within the (n-1)-st period. The 0-1 variable $T_{ni}^{(A)}$ indicates a PACK transmission in the i-th mini-slot within the n-th acknowledgment subperiod. The variable A_{ni} denotes the number of uncontrolled new message arrivals in the i-th slot within the n-th frame.

Let F_n denote the number of information packets transmitted within the n-th information subperiod without collision but with random transmission errors, and let $\tilde{S}_n^{(S)}$ denote the number of successful information packet transmissions (i.e., collision-free and without random transmission noise errors) within the n-th information subperiod. Therefore,

$$\tilde{S}_n^{(I)} = F_n + \tilde{S}_n^{(S)}. \tag{5.45}$$

The number of random noise transmission failures within the n-th period can be expressed by

$$F_n = \sum_{i=1}^{\tilde{S}_n^{(I)}} F_{ni} \tag{5.46}$$

where F_{ni} indicates whether random noise errors are detected in the i-th collision-free information packet transmitted within the n-th information subperiod:

$$P(F_{ni} = j) = \begin{cases} P_N & \text{if} \quad j = 1 \\ 1 - P_N & \text{if} \quad j = 0 \end{cases}, \tag{5.47}$$

$1 \leq i \leq \tilde{S}_n^{(I)}$. The variable $\tilde{S}_n^{(S)}$ is given by

$$\tilde{S}_n^{(S)} = \sum_{i=1}^{\tilde{K}_n^{(I)}} \tilde{S}_{ni}^{(S)} \tag{5.48}$$

where $\tilde{S}_{ni}^{(S)}$ indicates a successful information packet transmission in the i-th slot within the n-th information subperiod with

$$P(\tilde{S}_{ni}^{(S)} = j \mid \tilde{S}_{ni}^{(I)} = 1) = \begin{cases} P_N & \text{if} \quad j = 0 \\ 1 - P_N & \text{if} \quad j = 1 \end{cases}, \tag{5.49}$$

$1 \leq i \leq \tilde{K}_n^{(I)}$.

Since PACK transmissions are scheduled to avoid collision and since random transmission errors in PACK transmissions are assumed negligible,

$$\tilde{S}_{n+1}^{(A)} = \tilde{S}_n^{(S)} \tag{5.50}$$

and

$$P(T_{n+1,i}^{(A)} = j) = 1 - P(F_{ni} = j) \tag{5.51}$$

where $j = \{0,1\}$, $1 \leq i \leq \tilde{S}_n^{(I)}$, $n \geq 1$. Therefore,

$$F_{n-1} = \tilde{S}_{n-1}^{(I)} - \tilde{S}_n^{(A)} \tag{5.52}$$

and

$$\tilde{S}_n^{(A)} = \sum_{i=1}^{\tilde{S}_{n-1}^{(I)}} T_{ni}^{(A)} \quad . \tag{5.53}$$

Hence, $\{T^{(F)}_{n+1,i},\ i = 1, 2, \ldots, \tilde{K}^{(I)}_{n+1}\}$ is governed by the following multinomial distribution:

$$P\{T^{(F)}_{n+1,i} = \alpha_i,\ 1 \leq i \leq \tilde{K}^{(I)}_{n+1} | F_{n-1},\ S_n\}$$

$$= g^{(\tilde{K}^{(I)}_{n+1})}_{F_{n-1}}(\alpha_1, \alpha_2, \ldots, \alpha_{\tilde{K}^{(I)}_{n+1}}) \tag{5.54}$$

where $0 \leq \alpha_i \leq F_{n-1}$, $1 \leq i \leq \tilde{K}^{(I)}_{n+1}$, $\sum_{i=1}^{\tilde{K}^{(I)}_{n+1}} \alpha_i = F_{n-1}$. The variables $\tilde{S}^{(I)}_n$ and R_n are defined by (5.29) and (5.22), respectively, where

$$N_{ni} = T^{(I)}_{ni} + T^{(F)}_{ni} + A^{(C)}_{n,P+\tilde{K}^{(A)}_n+i} + \tilde{A}^{(C)}_{ni}\ , \tag{5.55}$$

$1 \leq i \leq \tilde{K}^{(I)}_n$.

Equations (4.4), (5.19) - (5.23), (5.25) - (5.29) and (5.45) - (5.55) yield the transition probability function for the vector Markov chain Z for the random noise channel under the DPDS scheme. The sequences $\{T^{(I)}_{n+1,i},\ 1 \leq i \leq \tilde{K}\}$, $\{T^{(A)}_{n+1,i},\ 1 \leq i \leq \tilde{K}\}$ and $\{T^{(F)}_{n+1,i},\ 1 \leq i \leq \tilde{K}\}$ depend on Z_n only through $X_n = (\tilde{S}^{(I)}_n, R_n, F_{n-1})$. The sequence $X = \{X_n,\ n \geq 1\}$ is a vector Markov chain over the space $d_{\tilde{K}} \times d \times d_{\tilde{K}}$. A flow diagram indicating the transition $X_n \rightarrow X_{n+1}$ is shown in Figure 5.5.

Analogous to the control functions defined by (5.30) and (5.31) in Section 5.3, the following two control functions yield irreducible, positive recurrent Markov state sequences:

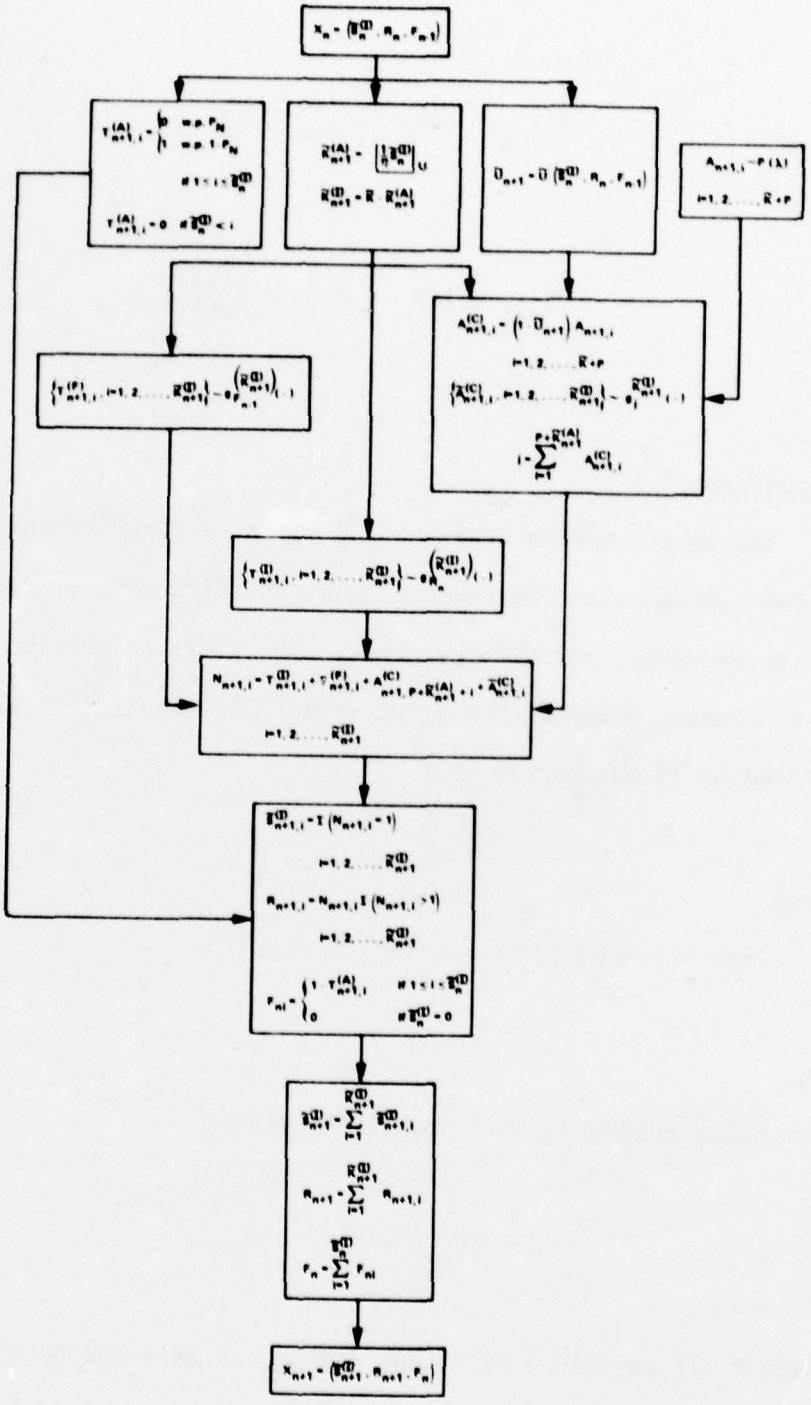


Figure 5.5 Transition $X_n \rightarrow X_{n+1}$ for the Markov State Chain X Associated with the Random Noise GRA Discipline Under the DPDS ACK Scheme

$$\tilde{U}_{n+1} = I(R_n + F_{n-1} \geq N_T) \tag{5.56}$$

and

$$\tilde{U}_{n+1} = I(R_n + F_{n-1} \geq \tilde{K}_{n+1}^{(I)} N_T) \quad . \tag{5.57}$$

Proposition 5.3

The vector Markov chains Z and X, which describe the evolution of the random noise GRA channel under the DPDS acknowledgment scheme, are irreducible, positive recurrent over their respective reduced state spaces, using either single-threshold binary control function defined by (5.56) and (5.57).

Proof

Similar technique as Proposition 4.1.

Q.E.D.

Substituting (5.50) into (5.45) yields

$$\tilde{S}_{n+1}^{(A)} = \tilde{S}_n^{(I)} - F_n \quad . \tag{5.58}$$

Since F_n is expressed by (5.46) and F_{ni} is governed by the Bernoulli distribution (5.47), F_n is governed by the binomial distribution:

$$P(F_n = j | \tilde{S}_n^{(I)} = \ell) = \binom{\ell}{j} P_N^j (1 - P_N)^{\ell - j} \quad , \tag{5.59}$$

$0 \leq j \leq \ell$. Therefore,

$$E(\tilde{S}_{n+1}^{(A)}) = (1 - P_N)\, E(\tilde{S}_n^{(I)}) \quad . \tag{5.60}$$

Theorem 5.5

The throughput of the random noise GRA channel under the *DPDS* acknowledgment scheme specified by the channel structure $(\tilde{K}, \eta)$ is upper bounded by

$$\tilde{\delta} \leq (1 - P_N)\, H(\tilde{K}, \eta) \tag{5.61}$$

where $H(\tilde{K}, \eta)$ is defined by (5.32).

Proof

Substitute (5.60) into (4.16) and apply the upper bound of Theorem 5.3.

Q.E.D.

5.4.2 Packet Delay Analysis

The delay D_{S_n} of the n-th message is defined by (5.13). The computation of the limiting average packet delay follows the technique introduced in Section 4.2.2 by extending the results of Lemma 4.1

so that the functionals $N(\cdot)$ and $\tilde{W}(\cdot)$ are time-homogeneous functions of $\{Y_n, n \geq 1\}$, $Y_n = (Z_n, Z_{n+1}, Z_{n+2})$. The sequence $\{Y_n, n \geq 1\}$ is an irreducible, positive recurrent Markov chain with stationary distribution $\{\pi(\underline{i},\underline{j},\underline{\ell})\}$ where $\pi(\underline{i},\underline{j},\underline{\ell}) = \pi_Z(\underline{i})\, P_Z(\underline{i},\underline{j})\, P_Z(\underline{j},\underline{\ell})$.

The limiting average packet delay $E(D_S)$ is given by the following theorem for the channel structure illustrated in Figure 5.3.

Theorem 5.6

With the operation of a controlled, random noise GRA channel under the DPDS acknowledgment scheme described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delay is given by

$$E(D_S) = \frac{3}{2} P + \frac{1}{2} \tilde{K} + \frac{1}{2}\left(1 + \frac{1}{\eta}\right) + R + \frac{1}{2} \frac{E[\tilde{K}_n^{(A)}(1 - \tilde{U}_n)]}{(1 - P_R)}$$
$$+ \frac{1}{2\eta} \frac{E(\tilde{S}_n^{(S)}\, \tilde{S}_n^{(I)})}{E(\tilde{S}_n^{(A)})} - \frac{1}{2} \frac{E(\tilde{S}_n^{(S)}\, \tilde{K}_n^{(A)})}{E(\tilde{S}_n^{(A)})}$$
$$+ (P+\tilde{K}) \frac{E(R_n)}{E(\tilde{S}_n^{(A)})} + \frac{1}{2} \frac{E[R_n(\tilde{K}_{n+1}^{(A)} - \tilde{K}_n^{(A)})]}{E(\tilde{S}_n^{(A)})}$$
$$+ 2(P+\tilde{K}) \frac{P_N}{1 - P_N} + \frac{1}{2} \frac{E[F_n(\tilde{K}_{n+2}^{(A)} - \tilde{K}_n^{(A)})]}{E(\tilde{S}_n^{(A)})} \qquad (5.62)$$

where the expectations are w.r.t. the stationary distribution $\{\pi(\underline{i},\underline{j},\underline{\ell})\}$.

Proof

Since new message arrivals admitted by the network control procedure are transmitted and retransmitted until successfully transmitted (collision-free and without random noise errors), in equilibrium,

$$E\{N(Z_n, Z_{n+1}, Z_{n+2})\} = E(A_n^{(C)})$$

$$= E(\tilde{S}_n^{(S)}) \quad . \tag{5.63}$$

And from (5.50),

$$E(\tilde{S}_n^{(A)}) = E(\tilde{S}_n^{(S)}) \quad . \tag{5.64}$$

The system waiting time function is decomposed into the sum

$$\tilde{W}(Z_n, Z_{n+1}, z_{n+2}) = \tilde{W}_o(Z_n, Z_{n+1}, Z_{n+2})$$

$$+ \sum_{j=1}^{3} \tilde{W}_{1j}(Z_n, Z_{n+1}, Z_{n+2}) \quad . \tag{5.65}$$

The function $\tilde{W}_o(Z_n, Z_{n+1}, Z_{n+2})$ denotes the sum of the waiting time components of all new message arrivals within the n-th frame measured from the arrival slot to the transmission slot and it is expressed by (5.41). For information packet transmissions within the n-th period which experience collision, the interval from the transmission slot within the n-th information subperiod to the retransmission slot within the (n+1)-st information subperiod is measured;

$\tilde{W}_{11}(Z_n, Z_{n+1}, Z_{n+2})$ is expressed by (5.42).

For successful information packet transmissions (collision-free and without random noise errors) within the n-th period, the waiting time component is measured from the transmission slot within the n-th information subperiod to the PACK transmission mini-slot within the (n+1)-st acknowledgment subperiod:

$$\tilde{W}_{12}(Z_n, Z_{n+1}, Z_{n+2}) = \tilde{K}_n^{(I)} \tilde{S}_{n1}^{(S)} + (\tilde{K}_n^{(I)} - 1) \tilde{S}_{n2}^{(S)} + \ldots$$

$$+ \tilde{S}^{(S)}_{n\tilde{K}_n^{(I)}} + P\tilde{S}_n^{(S)} + \frac{1}{\eta} [T^{(A)}_{n+1,2}$$

$$+ 2T^{(A)}_{n+1,3} + \ldots + (\tilde{S}_n^{(I)} - 1) T^{(A)}_{n+1,\tilde{S}_n^{(I)}}]. \quad (5.66)$$

For collision-free information packet transmissions within the n-th period which are rejected by the destination stations due to random noise errors, the waiting time component is measured from the transmission slot within the n-th information subperiod to the retransmission slot within the (n+2)-nd information subperiod:

$$\tilde{W}_{13}(Z_n, Z_{n+1}, Z_{n+2}) = \tilde{K}_n^{(I)} F_{n1} + (\tilde{K}_n^{(I)} - 1) F_{n2} + \ldots$$

$$+ F_{n\tilde{K}_n^{(I)}} + (2P + \tilde{K} + \tilde{K}_{n+2}^{(A)})F_n$$

$$+ T^{(F)}_{n+2,2} + 2T^{(F)}_{n+2,3} + \ldots + (\tilde{K}_{n+2}^{(I)} - 1) T^{(F)}_{n+2,\tilde{K}_{n+2}^{(I)}} \cdot$$

(5.67)

The packet delay expressions (5.62) and (5.68) for the random noise channel and the corresponding delay expressions for the noiseless channel (5.37) and (5.44) have similar structures. The differences include the appearance of $\tilde{S}_n^{(S)}$ and $\tilde{S}_n^{(A)}$ in (5.62) and (5.68) which accounts for the fact that with the random noise channel not all collision-free information packet transmissions are positively acknowledged. Also the following additional terms appear in (5.62) and (5.68):

$$2(P+\tilde{K})\,\frac{P_N}{1-P_N} + \frac{1}{2}\,\frac{E[F_n(\tilde{K}_{n+2}^{(A)} - \tilde{K}_n^{(A)})]}{E(\tilde{S}_n^{(A)})} \quad .$$

These terms represent the additonal delay experienced when information packets must be retransmitted due to random noise errors.

5.5 Numerical Results

The delay and throughput performance of the GRA channel under the PDS and DPDS acknowledgment schemes has been determined by simulating the appropriate Markov state sequences. These simulations yield the necessary statistics to evaluate the throughput using (4.16) with (5.60) when $P_N \geq 0$ or using (5.10) when $P_N = 0.0$. The limiting average packet delays are computed by substituting the necessary statistics, obtained by simulation, into (5.14) and (5.15) for the PDS scheme and into (5.62) and (5.68) for the DPDS scheme. As previously remarked in Chapter IV, the Markov chain simulations are run for a large but finite number of slots; hence, these results

By applying Lemma 4.1 (with the appropriate extensions) and by taking advantage of the symmetry provided by the uniform distributed slot allocations for information packet transmissions, (5.62) is proved.

Q.E.D.

Corollary 5.3

With the operation of a controlled, random noise GRA channel under the DPDS acknowledgment scheme described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delay (data transfer delay) is given by

$$E(D_R) = \frac{P}{2} + \frac{1}{2}\frac{E[\tilde{K}_n^{(A)}(1 - \tilde{U}_n)]}{(1 - P_R)} + (P+\tilde{K})\frac{E(R_n)}{E(\tilde{S}_n^{(A)})}$$
$$+ \frac{1}{2}\frac{E[R_n(\tilde{K}_{n+1}^{(A)} - \tilde{K}_n^{(A)})]}{E(\tilde{S}_n^{(A)})} + 2(P+\tilde{K})\frac{P_N}{1 - P_N}$$
$$+ \frac{1}{2}\frac{E[F_n(\tilde{K}_{n+2}^{(A)} - \tilde{K}_n^{(A)})]}{E(\tilde{S}_n^{(A)})} + R + 1 \qquad (5.68)$$

where the expectations are w.r.t. the stationary distribution $\{\pi(\underline{i},\underline{j},\underline{\ell})\}$.

Proof

Similar technique as Corollary 5.2.

Q.E.D.

The packet delay expressions (5.62) and (5.68) for the random noise channel and the corresponding delay expressions for the noiseless channel (5.37) and (5.44) have similar structures. The differences include the appearance of $\tilde{S}_n^{(S)}$ and $\tilde{S}_n^{(A)}$ in (5.62) and (5.68) which accounts for the fact that with the random noise channel not all collision-free information packet transmissions are positively acknowledged. Also the following additional terms appear in (5.62) and (5.68):

$$2(P+\tilde{K})\frac{P_N}{1-P_N} + \frac{1}{2}\frac{E[F_n(\tilde{K}_{n+2}^{(A)} - \tilde{K}_n^{(A)})]}{E(\tilde{S}_n^{(A)})} \quad .$$

These terms represent the additonal delay experienced when information packets must be retransmitted due to random noise errors.

5.5 Numerical Results

The delay and throughput performance of the GRA channel under the PDS and DPDS acknowledgment schemes has been determined by simulating the appropriate Markov state sequences. These simulations yield the necessary statistics to evaluate the throughput using (4.16) with (5.60) when $P_N \geq 0$ or using (5.10) when $P_N = 0.0$. The limiting average packet delays are computed by substituting the necessary statistics, obtained by simulation, into (5.14) and (5.15) for the PDS scheme and into (5.62) and (5.68) for the DPDS scheme. As previously remarked in Chapter IV, the Markov chain simulations are run for a large but finite number of slots; hence, these results

represent estimates which indicate performance trends and should not be interpreted as absolute.

Maximum throughput results with $\tilde{K} = 12$, $P_N = 0.0$ are summarized in Table 5.1 for the GRA channel under the PDS and DPDS acknowledgment schemes. Values for the GRA channel under the PDRA (random access) acknowledgment scheme and for the basic GRA channel are included for comparison. As expected the scheduled acknowledgment schemes provide higher maximum throughput values than the random access schemes. In addition, the slot allocations of the DPDS scheme which adapt to the acknowledgment traffic requirement yield higher maximum throughput values than the fixed allocations of the PDS scheme. Larger values of η (the ratio of information packet size to PACK size) yield higher maximum throughput values. The maximum throughput value under the DPDS scheme with $\eta = 9$ is approximately 0.34 packets per service slot which approaches the performance of the basic GRA channel (e^{-1}).

TABLE 5.1. MAXIMUM THROUGHPUT VALUES FOR THE GRA DISCIPLINE WITH $\tilde{K} = 12$, $P_N = 0.0$

Acknowledgment Scheme	Maximum Throughput	
	$\eta = 1$	$\eta = 9$
PDRA	0.14 ($\tilde{K}_I = 6$)	0.26 ($\tilde{K}_I = 10$)
PDS	0.18 ($\tilde{K}_I = 6$)	0.30 ($\tilde{K}_I = 10$)
DPDS	0.27	0.34
Basic GRA	e^{-1} (≈ 0.368)	e^{-1} (≈ 0.368)

Average packet delay D_S versus probability of rejection curves for the GRA channel under the PDS and DPDS acknowledgment schemes with $\tilde{K} = P = R = 12$ are shown in Figure 5.6 with $\eta = 1$ and in Figure 5.7 with $\eta = 9$. Curves for the GRA channel under the DPDS scheme are shown for $P_N = 0.0, 0.1$. In general, larger delays and higher probabilities of rejection are experienced over the random noise GRA channel with non-zero packet noise error probabilities ($P_N > 0$) than over a noiseless GRA channel when both channels operate under the DPDS scheme.

When $P_N = 0.0$, lower delay values for the corresponding rejection probabilities are achieved under the DPDS scheme than under the PDS scheme for all values of $\eta \geq 1$. When $\eta = 1$, the performance advantage with the DPDS scheme is substantial. For example, with $\lambda = 0.1$ packets per slot the DPDS scheme achieves a minimum rejection probability of approximately 0.0 with an average delay of 55 slots. The PDS scheme yields a higher minimum rejection probability ($\simeq 0.11$) with a larger average delay ($\simeq 76$ slots). This difference is due to the 50% slot allocation to acknowledgments (i.e., $\tilde{K}_A = 6$, $\tilde{K} = 12$) under the PDS scheme. Since the number of slots dedicated to each acknowledgment subperiod under the PDS scheme decreases as η increases, the performance margin between the PDS and DPDS schemes decreases with increasing η.

Average packet delay D_R versus packet probability of rejection curves are shown in Figure 5.8 with $\tilde{K} = P = R = 12$, $P_N = 0.0$ for the basic GRA channel and for the GRA channel under the DPDS acknowledgment scheme with $\eta = 1, 9$. These results indicate (analogous to maximum

throughput results) that the performance under the DPDS scheme approaches the basic GRA channel performance with increasing values of η.

Packet probability of rejection versus packet noise error probability curves are shown in Figure 5.9 and average packet delay versus packet noise error probability curves are shown in Figure 5.10 for the GRA channel under the DPDS acknowledgment scheme with $\tilde{K} = P = R = 12$, $\eta = 9$. Constant message arrival rate curves are presented for two network control procedure threshold values ($N_T = \frac{1}{2}, 1$). These results demonstrate the performance loss experienced with random noise GRA channels: larger delays and higher rejection probabilities. In addition, these results indicate a significant sensitivity to the choice of threshold value (N_T) as a function of message arrival rate.

5.6 Conclusions

The operation of the GRA channel under two scheduled acknowledgment schemes was studied in this chapter. Under the PDS scheme, each period was partitioned into two fixed length subperiods. Under the DPDS scheme the subperiod slot allocation was adapted to the acknowledgment traffic requirement. Under both schemes, the information packets were transmitted on a random access basis in one subperiod, while PACK transmissions in the second subperiod were scheduled to avoid collisions. The channel state process under each acknowledgment scheme was described by an irreducible, positive recurrent Markov chain. Upper bounds on the maximum throughput and expressions for the limiting average packet delay were derived. In addition, a random

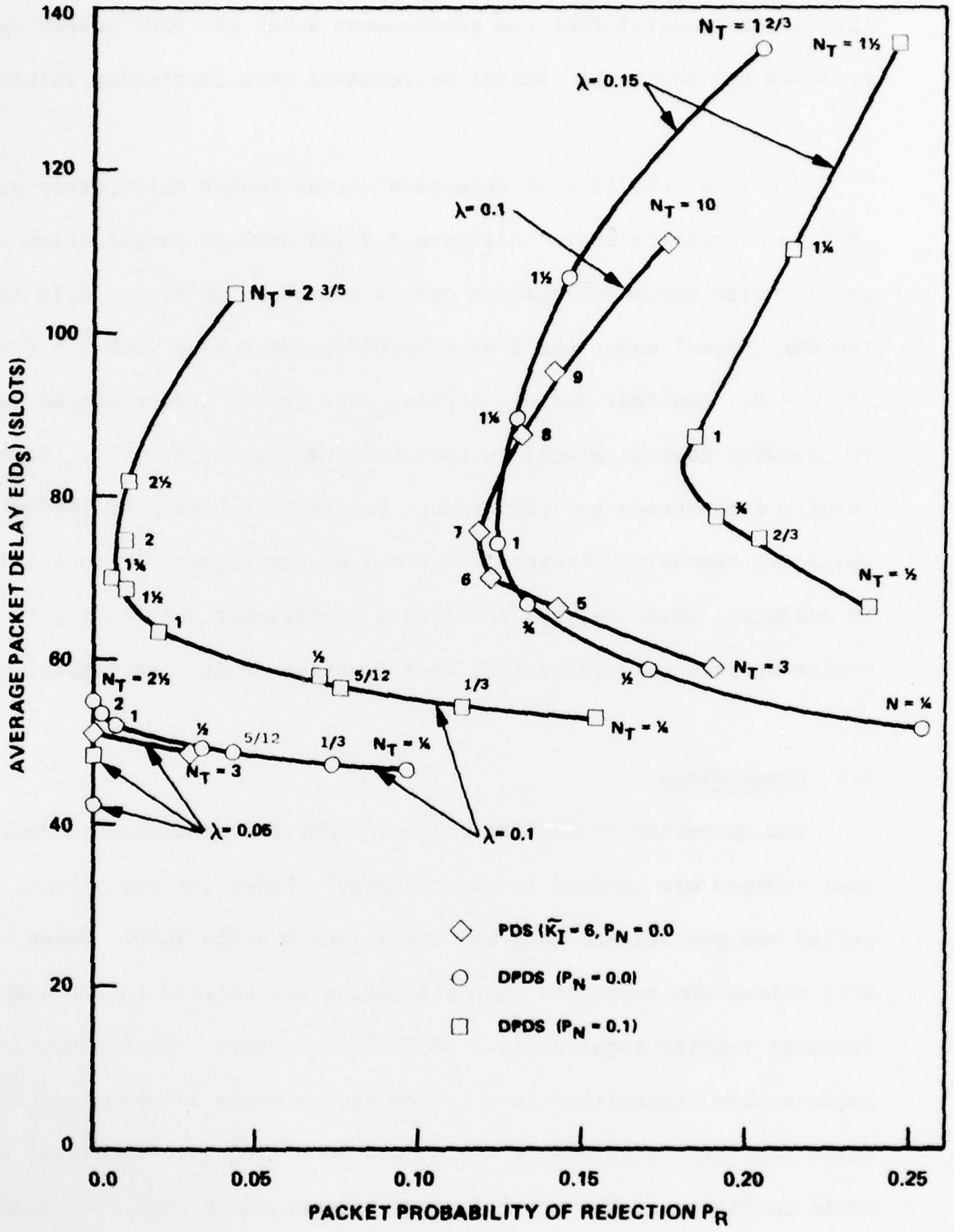


Figure 5.6 Packet Delay vs. Packet Probability of Rejection Curves for a GRA Channel with $\tilde{K} = P = R = 12$, $\eta = 1$ Under the PDS and DPDS ACK Schemes Using Control Functions (5.9) and (5.57), Respectively

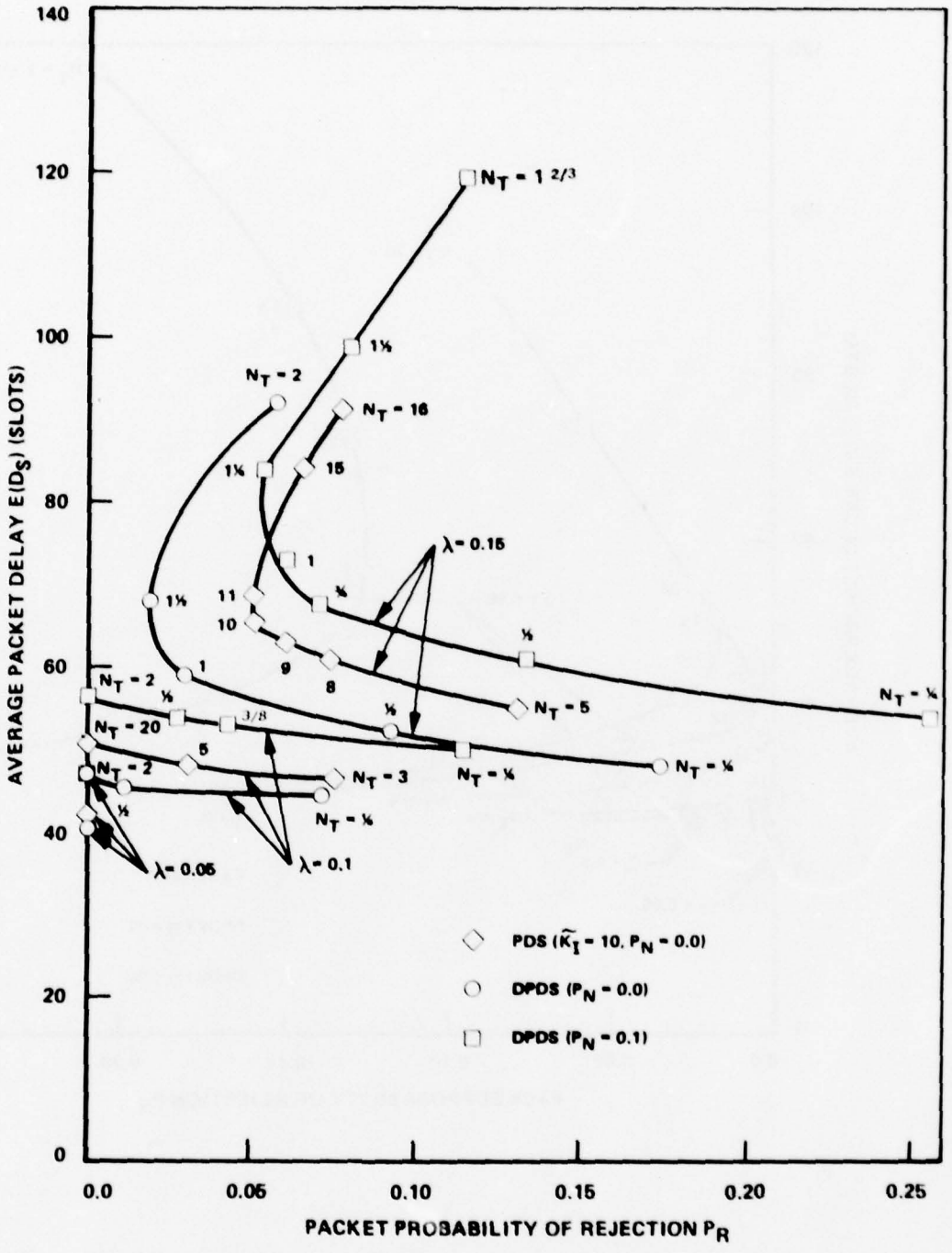


Figure 5.7 Packet Delay vs. Packet Probability of Rejection Curves for a GRA Channel with $\tilde{K} = P = R = 12$, $\eta = 9$ Under the PDS and DPDS ACK Schemes Using Control Functions (5.9) and (5.57), Respectively

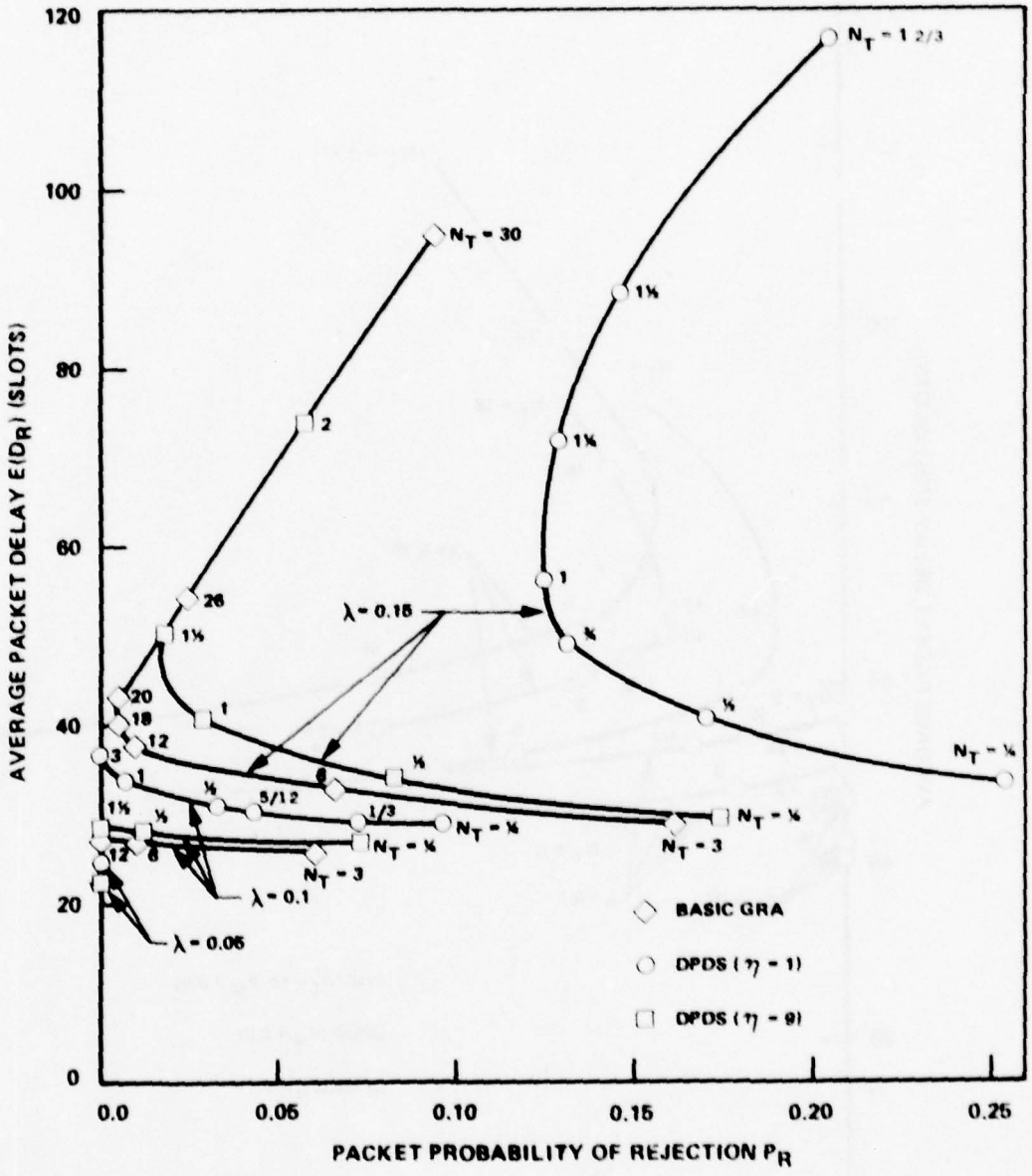


Figure 5.8 Packet Delay vs. Packet Probability of Rejection Curves for the Basic GRA Channel and a GRA Channel Under the DPDS ACK Scheme Using Control Functions (5.9) and (5.57), Respectively, with $\widetilde{K} = P = R = 12$, $P_N = 0.0$

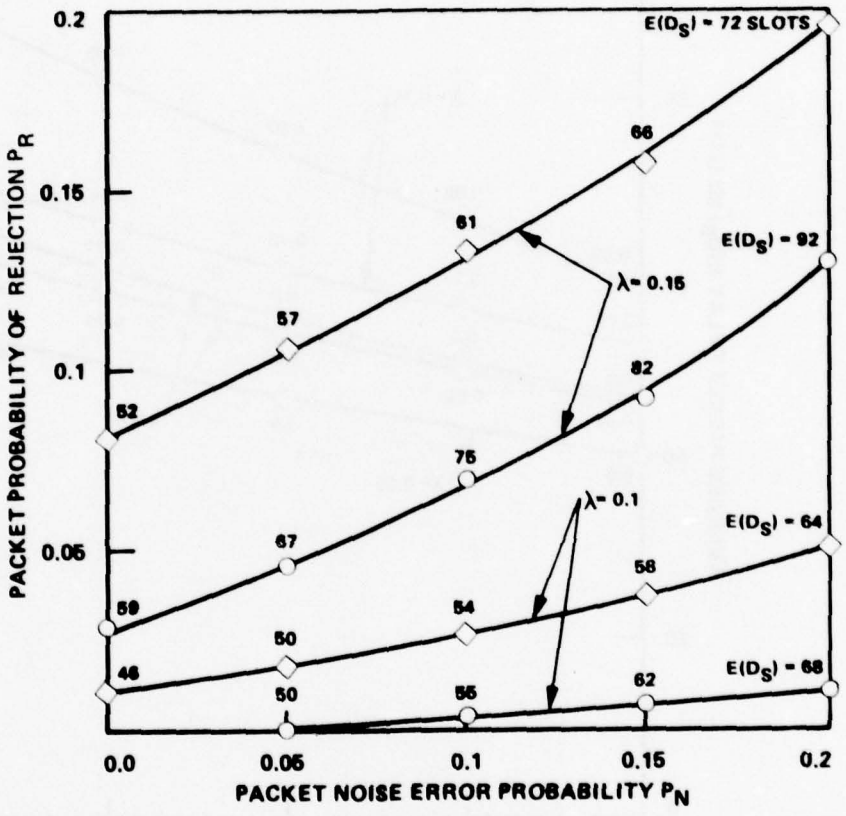


Figure 5.9 Packet Probability of Rejection vs. Packet Noise Error Probability Curves for a GRA Channel with $\bar{K}$ = P = R = 12, η =9 Under the DPDS ACK Scheme Using Control Function (5.57) with N_T = ½ ◇ and N_T = 1 ○

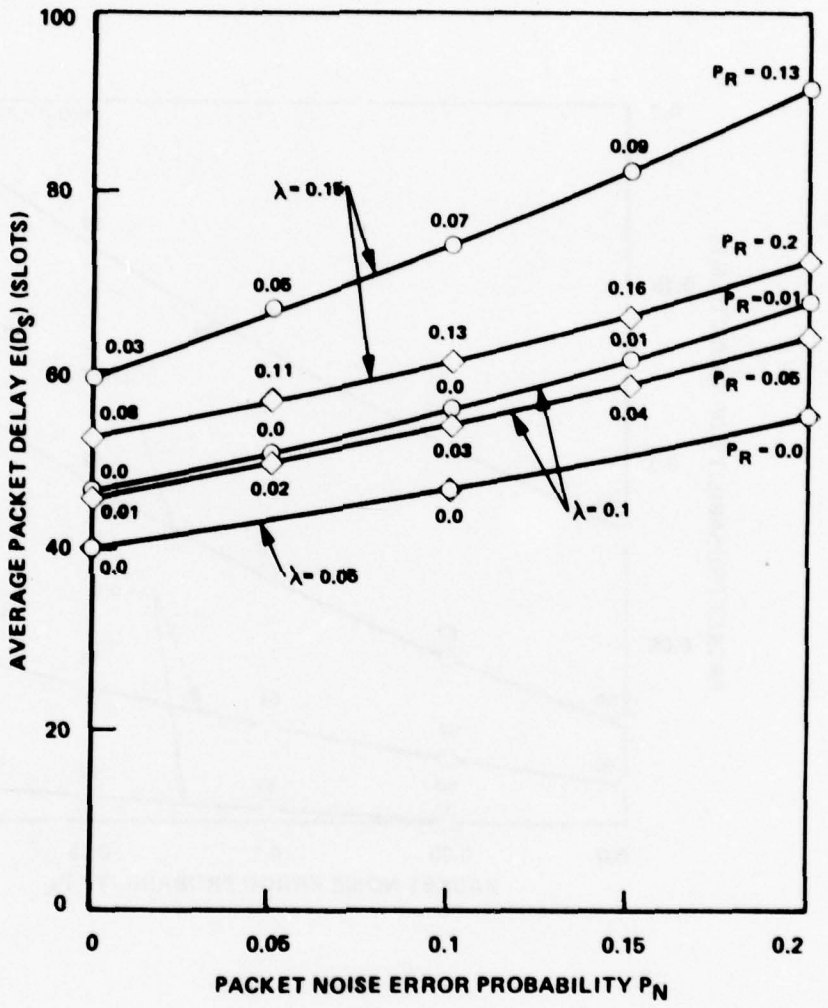


Figure 5.10 Packet Delay vs. Packet Noise Error Probability Curves for a GRA Channel with $\widetilde{K} = P = R = 12, \eta = 9$ Under the DPDS ACK Scheme Using Control Function (5.57) with $N_T = \frac{1}{2}$ ◇ and $N_T = 1$ ○

noise GRA channel under the DPDS acknowledgment scheme was investigated. A stationary transmission error process model in which errors occur as independent events was examined.

Numerical examples of the delay-throughput (rejection probability) function were presented with $\tilde{K} = P = R = 12$, $\eta = 1, 9$. The delay and throughput performance of the GRA channel under the DPDS scheme was shown to be superior to the performance under the PDS scheme. Both schemes provided reduced performance compared to the basic GRA discipline without acknowledgment; however, the margin diminished as η was increased. The performance loss of the random noise GRA channel under the DPDS acknowledgment scheme was demonstrated. Non-zero packet noise error probabilities ($P_N > 0$) yielded larger delay values and higher rejection probabilities.

CHAPTER VI

CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

6.1 Summary and Conclusions

In this dissertation, ARQ error control systems were applied to communication over a multi-user multi-access broadcast channel. Two channel access-control disciplines were examined: a fixed scheduling TDMA discipline and a random access GRA discipline. ARQ error control procedures were integrated with these access-control disciplines and the normal performance indices such as channel throughput and message delay were evaluated.

The operation of a TDMA channel using SW and Block ARQ error recovery procedures was examined under several acknowledgment mechanisms. The generating function of the message delay distribution at steady state was derived for multi-packet messages under random acknowledgment delays. Expressions for the message delay mean and variance were obtained. The operation of a TDMA channel was also examined under the SR ARQ error recovery procedure. By considering single-packet messages and constant acknowledgment delays, the evolution of the channel was described by a vector Markov chain. The conditions for ergodicity were stated. Upper and lower bounds on the average delay at steady state were derived. Numerical examples of the delay and throughput performance were presented for a stationary transmission error process model with a Poisson distributed message

arrival process. These results demonstrated the performance advantage possible with the SR ARQ scheme compared to the SW or Block ARQ schemes for communication channels with non-zero error rates. These results, derived for the TDMA channel, are with obvious modifications also appropriate for a slotted Frequency Division Multiple Access channel and for a slotted statistical concentrator used with single-access communication links.

The operation of the GRA channel under three random access and two scheduled acknowledgment schemes was examined. The evolution of the channel under each scheme was described by an irreducible, positive recurrent Markov chain. Upper bounds on the maximum throughput and expressions for the limiting average packet delay were derived. Numerical examples of the delay-throughput (rejection probability) function were presented. Both the delay and the throughput performance under the random access acknowledgment schemes were found to be substantially reduced compared to the basic GRA discipline without acknowledgment. The scheduled acknowledgment schemes performed better than the random access acknowledgment schemes and as the ratio of the information packet size-to-PACK size increased, the performance under the scheduled acknowledgment schemes approached the basic GRA channel performance. A stationary transmission error process model in which errors occur as independent events was examined. The performance loss of the random noise GRA channel under the Dynamic Period Division - Scheduled acknowledgment scheme was demonstrated.

6.2 Suggestions for Future Research

Several possible directions for future work will now be indicated. In this dissertation ARQ error control systems were analyzed for a network of stations communicating over a multi-access broadcast channel. Forward error correction systems and hybrid systems which use both FEC and ARQ techniques should be investigated and compared to the results obtained in this dissertation. These results could be extended to include the effects of various modulation schemes and coding procedures. Such studies could relate the information rate of the channel at prescribed message delay and error probability values to the various components of the communication system.

Two channel access control disciplines were examined. In particular, the operation of the TDMA channel under the SR ARQ system was examined for single-packet messages. It would be worthwhile to extend these results to multi-packet messages and obtain both mean and variance expressions for message delay. Furthermore, investigations of the TDMA and GRA channels under alternative acknowledgment systems such as negative acknowledgment or positive acknowledgment with time-out thresholds could be conducted. In addition, a stationary transmission error process model in which errors occur as independent events was considered in this dissertation. Developing results for a Markovian noise model would be important.

Finally, in this dissertation, various error control implementation protocols for the TDMA and GRA channels were proposed, analyzed, and then compared. To develop a methodology for synthesizing not only the error control system but also the basic channel access-control

discipline to meet objectives under prescribed constraints would be highly rewarding. The synthesis of such a communication system would answer the question of optimality.

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APPENDIX A

A.1 Proof of Lemma 2.1

The group arrival process is described by the stochastic jump process $\{(\tilde{t}_n, G_n),\ n \geq 1\}$ and the service requirement of the n-th group S_n is governed by (2.3). The number of groups buffered at the source station following the departure of the n-th group is denoted by L_n. With the number of group arrivals during the n-th group's service time denoted by M_n, the group queue size is governed by the following recursive relationship:

$$L_{n+1} = [L_n - 1]^+ + M_{n+1} \tag{A.1}$$

where $[x]^+ = \max(0,x)$. Define

$$L_n^*(z) = \sum_{j=0}^{\infty} P(L_n = j) z^j \qquad |z| < 1 \tag{A.2}$$

and

$$L^*(z,w) = \sum_{n=0}^{\infty} L_n^*(z) w^n \qquad |w| < 1 \ . \tag{A.3}$$

From (A.1) - (A.3), it follows that

$$L^*(z,w) = \frac{zL_o^*(z) + (z-1)\, L^*(0,w)\, M^*(z)w}{z - M^*(z)w} \tag{A.4}$$

for $|z| < 1$, $|w| < 1$ where $M^*(z) = E\{z^{M_n}\}$. The boundary term $L^*(0,w)$ is determined by the analyticity of $L^*(z,w)$ in $|z| < 1$, $|w| < 1$ (see

Chu and Konheim [57]).

It is well known that the stationary distribution of the Markov chain $\{L_n, n \geq 0\}$ exists when $\rho = E(M_n) < 1$. With $L_0 = 0$, the application of a Tauberian theorem yields

$$L^*(z) = \lim_{w \uparrow 1} (1 - w)\, L^*(z,w)$$

$$= \frac{(1 - \rho)(z - 1)\, M^*(z)}{z - M^*(z)} \qquad |z| < 1 \tag{A.5}$$

But

$$M^*(z) = S^*(z(1 - \eta_o) + \eta_o) \tag{A.6}$$

and, therefore, (A.5) becomes

$$L^*(z) = \frac{(1-\rho)\ (z-1)\ S^*(z(1 - \eta_o) + \eta_o)}{z - S^*(z(1 - \eta_o) + \eta_o)} \tag{A.7}$$

The system delay $D_n^{(S)}$ (measured in frames) of the n-th group is given by

$$D_n^{(S)} = \frac{1}{M} W_n^{(L)} + S_n \tag{A.8}$$

Since $W_n^{(L)}$ and S_n are statistically independent, the generating function of the system delay distribution is

$$D_{S_n}^*(z) = W_{L_n}^*(z^{1/M})\ S^*(z) \tag{A.9}$$

for $|z| < 1$. The number of groups arriving during $D_n^{(S)}$ is equal to the group queue size L_n; therefore, the generating function of the system delay distribution is

$$D^*_{S_n}(z) = L^*_n\left(\frac{z - \eta_o}{1 - \eta_o}\right) \tag{A.10}$$

Using (A.9) and (A.10), the generating function of the waiting time distribution is given by

$$W^*_{L_n}(z) = \frac{L^*_n\left(\frac{z^M - \eta_o}{1 - \eta_o}\right)}{S^*(z^M)} \tag{A.11}$$

Since G_n and $S_n^{(i)}$ are statistically independent, (2.3) yields

$$S^*(z) = G^*(s^*(z))$$

$$= \frac{1}{1 - \eta_o}\left[N^*(s^*(z)) - \eta_o\right] \tag{A.12}$$

Thus for $\rho = s_1 n_1 < 1$, the limiting waiting time distribution exists and the generating function at steady state is

$$W^*_L(z) = \frac{L^*\left(\frac{z^M - \eta_o}{1 - \eta_o}\right)}{S^*(z^M)} \tag{A.13}$$

The desired result follows by substituting (A.7) and (A.12) into (A.13).

Q.E.D.

A.2 Proof of Lemma 2.2

Consider the discrete-time renewal point process $\{e_m, m \geq 1\}$ where

$$e_m = 1 + \sum_{j=1}^{m-1} G_j \quad .$$

Messages form the time index for this process, while events are the occurrence of group leaders. Thus e_m marks the occurrence of the leader of the m-th group. Let γ_n denote the time between n and the last occurring event. This time represents the number of messages served ahead of the n-th message from among the messages of its own group. From renewal theory [58, 59], γ_n is the age or backward recurrence time at n. Its limiting distribution always exists and its generating function $\gamma^*(z)$ is given by

$$\gamma^*(z) = \frac{1 - G^*(z)}{g_1(1-z)} \tag{A.14}$$

for $|z| < 1$.

The waiting time (measured in slots) of the n-th message beyond that of its group leader can be expressed as

$$W_n^{(G)} = M \sum_{i=1}^{\gamma_n} S_m^{(i)} \tag{A.15}$$

Since γ_n and $S_m^{(i)}$ are statistically independent, the generating function for the limiting distribution of $W_n^{(G)}$ is given by

$$W_G^*(z) = \gamma^*(s^*(z^M)) \tag{A.16}$$

The desired result follows by using (A.12), (A.14) and (A.16).

Q.E.D.

A.3 Summary of Completion Time Moments

Results for the first 3 moments of completion time $S_n^{(i)}$ are stated for the six ARQ - ACK schemes. These expressions are appropriate for the stationary transmission error process.

SW ARQ - PP ACK

$$E(S_n^{(i)}|B_\ell) = \frac{k_1 B_\ell}{1 - P_N} \tag{A.17}$$

$$E(S_n^{(i)} S_n^{(i)}|B_\ell) = \frac{k_2 B_\ell (B_\ell + P_N)}{(1 - P_N)^2} \tag{A.18}$$

$$E(S_n^{(i)} S_n^{(i)} S_n^{(i)}|B_\ell) = \frac{k_3 B_\ell [P_N(1 + P_N) + 3P_N B_\ell + B_\ell^2]}{(1 - P_N)^3} \tag{A.19}$$

SW ARQ - MMNS ACK

$$E(S_n^{(i)}|B_\ell) = \frac{B_\ell + k_1 - 1}{(1 - P_N)^{B_\ell}} \tag{A.20}$$

$$E(S_n^{(i)} S_n^{(i)} | B_\ell) = \frac{[B_\ell^2 + 2B_\ell(k_1-1) + k_2 - 2k_1+1][2 - (1 - P_N)^{B_\ell}]}{(1 - P_N)^{2B_\ell}} \tag{A.21}$$

$$E(S_n^{(i)} S_n^{(i)} S_n^{(i)} | B_\ell) = [B_\ell^3 + 3B_\ell^2(k_1-1) + 3B_\ell(k_2-2k_1+1) + k_3 - 3k_2 + 3k_1-1][6 - 6(1 - P_N)^{B_\ell} + (1 - P_N)^{2B_\ell}] (1 - P_N)^{-3B_\ell} \tag{A.22}$$

SW ARQ - MMS ACK

$$E(S_n^{(i)} | B_\ell) = \frac{B_\ell}{1 - P_N} + (k_1-1) \sum_{m=0}^{\infty} [1 - (1 - P_N^m)^{B_\ell}] \tag{A.23}$$

$$E(S_n^{(i)} S_n^{(i)} | B_\ell) = \frac{B_\ell(B_\ell + P_N)}{(1 - P_N)^2} + (k_2 - 2k_1+1) \sum_{m=0}^{\infty} (2m+1) [1 - (1 - P_N^m)^{B_\ell}] + 2B_\ell(k_1-1) \{\frac{1}{1 - P_N} \sum_{m=0}^{\infty} [1 - (1 - P_N^m)^{B_\ell}] + \sum_{m=1}^{\infty} m P_N^m (1 - P_N^m)^{B_\ell - 1}\} \tag{A.24}$$

$$E(S_n^{(i)} S_n^{(i)} S_n^{(i)} | B_\ell) = \xi_3 + 3(k_1 - 1)\xi_2 + 3(k_2 - 2k_1 + 1)\xi_1$$

$$+ (k_3 - 3k_2 + 3k_1 - 1)\xi_0 \qquad \text{(A.25)}$$

where

$$\xi_0 = \sum_{j=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P(B_\ell^{(0)} > 0, E_\ell^{(1)} > 0, \ldots, B_\ell^{(j \wedge n \wedge m)} > 0 | B_\ell)$$

$$= 3 \sum_{m=1}^{\infty} (m^2 + m) \left[1 - (1 - P_N^m)^{B_\ell}\right]$$

$$+ \sum_{m=0}^{\infty} \left[1 - (1 - P_N^m)^{B_\ell}\right] \qquad \text{(A.26)}$$

$$\xi_1 = \sum_{j=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} E\{B_\ell^{(j)} I(B_\ell^{(0)} > 0, B_\ell^{(1)} > 0, \ldots,$$

$$B_\ell^{(j \wedge n \wedge m)} > 0) | B_\ell\}$$

$$= \frac{B_\ell P_N}{(1-P_N)^3} + \frac{B_\ell}{1-P_N} + \frac{B_\ell}{1-P_N} \sum_{m=1}^{\infty} [(2 - P_N)m + 1][1 - (1 - P_N^m)^{B_\ell}]$$

$$+ 2B_\ell \sum_{m=1}^{\infty} m^2 P_N^m (1 - P_N^m)^{B_\ell - 1}$$

$$+ \frac{B_\ell P_N}{1 - P_N} \sum_{m=2}^{\infty} m(1 - P_N^{m-1}) \left[1 - (1 - P_N^m)^{B_\ell - 1}\right] \qquad \text{(A.27)}$$

$$\xi_2 = \sum_{j=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} E\{B_\ell^{(n)} B_\ell^{(m)} \; I(B_\ell^{(0)} > 0,\; B_\ell^{(1)} > 0,\; \ldots,$$

$$B_\ell^{(n \wedge j \wedge m)} > 0) \,|\, B_\ell\}$$

$$= \frac{B_\ell (B_\ell + P_N)}{(1 - P_N)^2} \sum_{m=1}^{\infty} [1 - (1 - P_N^m)^{B_\ell}]$$

$$+ B_\ell \sum_{m=1}^{\infty} m^2 (1 - B_\ell P_N^m)\, P_N^m (1 - P_N^m)^{B_\ell - 2}$$

$$+ \frac{2B_\ell^2}{1 - P_N} \sum_{m=1}^{\infty} m\, P_N^m (1 - P_N^m)^{B_\ell - 1}$$

$$+ \frac{B_\ell (1 + P_N) + B_\ell (B_\ell - 1)}{(1 - P_N)^2} \qquad \text{(A.28)}$$

$$\xi_3 = \sum_{j=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} E\{B_\ell^{(j)} B_\ell^{(n)} B_\ell^{(m)} \; I(B_\ell^{(0)} > 0,\; B_\ell^{(1)} > 0,\; \ldots,$$

$$B_\ell^{(j \wedge n \wedge m)} > 0) \,|\, B_\ell\}$$

$$= \frac{1}{(1 - P_N)^3} [B_\ell (1 + 4P_N + P_N^2)$$

$$+ 3(1 + P_N)\, B_\ell (B_\ell - 1) + B_\ell (B_\ell - 1)\, (B_\ell - 2)] \qquad \text{(A.29)}$$

$j \wedge n \wedge m = \max(j, n, m)$.

Block ARQ - PP ACK

$$E(S_n^{(i)} | B_\ell) = B_\ell \left[\frac{k_1 P_N}{1 - P_N} + 1\right] \tag{A.30}$$

$$E(S_n^{(i)} S_n^{(i)} | B_\ell) = \frac{k_2 B_\ell P_N (1 + B_\ell P_N)}{(1 - P_N)^2} + \frac{2B_\ell^2 k_1 P_N}{1 - P_N} + B_\ell^2 \tag{A.31}$$

$$E(S_n^{(i)} S_n^{(i)} S_n^{(i)} | B_\ell) = \frac{k_3}{(1 - P_N)^3} [B_\ell P_N (1 + P_N) + 3B_\ell^2 P_N^2 + B_\ell^3 P_N^3]$$
$$+ \frac{3k_2 P_N B_\ell^2 (1 + B_\ell P_N)}{(1 - P_N)^2}$$
$$+ \frac{3k_1 B_\ell^3 P_N}{1 - P_N} + B_\ell^3 \tag{A.32}$$

Block ARQ - MMNS ACK

$$E(S_n^{(i)} | B_\ell) = \frac{B_\ell + (k_1 - 1)[1 - (1 - P_N)^{B_\ell}]}{(1 - P_N)^{B_\ell}} \tag{A.33}$$

$$E(S_n^{(i)} S_n^{(i)} | B_\ell) = [B_\ell^2 + 2B_\ell (k_1 - 1) + k_2 - 2k_1 + 1][2 - (1 - P_N)^{B_\ell}]$$
$$\cdot (1 - P_N)^{-2B_\ell} - [2(k_2 - 2k_1 + 1)$$
$$+ 2(k_1 - 1)B_\ell] (1 - P_N)^{-B_\ell}$$
$$+ k_2 - 2k_1 + 1 \tag{A.34}$$

$$E(S_n^{(i)} S_n^{(i)} S_n^{(i)} | B_\ell) = [B_\ell^3 + 3B_\ell^2(k_1-1) + 3B_\ell(k_2 - 2k_1 + 1)$$

$$+ k_3 - 3k_2 + 3k_1 - 1][6 - 6(1 - P_N)^{B_\ell}$$

$$+ (1 - P_N)^{2B_\ell}] \cdot (1 - P_N)^{-3B_\ell}$$

$$- 3[B_\ell^2(k_1-1) + 2B_\ell(k_2 - 2k_1 + 1)$$

$$+ k_3 - 3k_2 + 3k_1 - 1][2 - (1 - P_N)^{B_\ell}](1 - P_N)^{-2B_\ell}$$

$$+ 3[B_\ell(k_2 - 2k_1 + 1) + k_3 - 3k_2 + 3k_1 - 1]$$

$$\cdot (1 - P_N)^{-B_\ell} - (k_3 - 3k_2 + 3k_1 - 1) \qquad \text{(A.35)}$$

Block ARQ - MMS ACK

$$E(S_n^{(i)} | B_\ell) = \frac{B_\ell}{1 - P_N} + (k_1-1) \sum_{m=1}^{\infty} [1 - (1 - P_N^m)^{B_\ell}] \qquad \text{(A.36)}$$

$$E(S_n^{(i)} S_n^{(i)} | B_\ell) = \frac{B_\ell(B_\ell + P_N)}{(1 - P_N)^2} + (k_2 - 2k_1 + 1) \sum_{m=1}^{\infty} (2m-1)[1 - (1-P_N^m)^{B_\ell}]$$

$$+ 2(k_1-1)\, B_\ell \{\frac{1}{1-P_N} \sum_{m=1}^{\infty} [1 - (1-P_N^m)^{B_\ell}] + \sum_{m=1}^{\infty} m\, P_N^m (1-P_N^m)^{B_\ell - 1}\} \qquad \text{(A.37)}$$

$$E(S_n^{(i)} S_n^{(i)} S_n^{(i)} | B_\ell) = \psi_3 + 3(k_1 - 1)\psi_2 + 3(k_2 - 2k_1 + 1)\psi_1$$

$$+ (k_3 - 3k_2 + 3k_1 - 1)\psi_0 \qquad \text{(A.38)}$$

where

$$\psi_0 = \sum_{j=1}^{\infty} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} P(B_\ell^{(0)} > 0,\ B_\ell^{(1)} > 0,\ \ldots,\ B_\ell^{(j \wedge n \wedge m)} > 0 | B_\ell)$$

$$= 3 \sum_{m=1}^{\infty} (m^2 - m)\ [1 - (1 - P_N^m)^{B_\ell}] + \sum_{m=1}^{\infty} [1 - (1 - P_N^m)^{B_\ell}] \qquad \text{(A.39)}$$

$$\psi_1 = \sum_{j=0}^{\infty} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} E\{B_\ell^{(j)}\ I(B_\ell^{(0)} > 0,\ B_\ell^{(1)} > 0,\ \ldots,$$

$$B_\ell^{(j \wedge n \wedge m)} > 0) | B_\ell\}$$

$$= \frac{B_\ell P_N}{(1 - P_N)^3} + 2B_\ell \sum_{m=1}^{\infty} (m^2 - m)\ P_N^m (1 - P_N^m)^{B_\ell - 1}$$

$$+ \frac{B_\ell P_N}{1 - P_N} \sum_{m=2}^{\infty} m(1 - P_N^{m-1})\ [1 - (1 - P_N^m)^{B_\ell - 1}]$$

$$+ \frac{B_\ell}{1 - P_N} \sum_{m=1}^{\infty} [(2 - P_N)\ell - 1][1 - (1 - P_N^m)^{B_\ell}] \qquad \text{(A.40)}$$

$$\psi_2 = \sum_{j=1}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} E\{B_\ell^{(n)} B_\ell^{(m)} \; I(B_\ell^{(0)} > 0, \; B_\ell^{(1)} > 0, \; \ldots,$$

$$B_\ell^{(n \wedge j \wedge m)} > 0) \,|\, B_\ell\}$$

$$= \frac{B_\ell (B_\ell + P_N)}{(1 - P_N)^2} \sum_{m=1}^{\infty} [1 - (1 - P_N^m)^{B_\ell}]$$

$$+ B_\ell \sum_{m=1}^{\infty} m^2 (1 - B_\ell \, P_N^m) \, P_N^m (1 - P_N^m)^{B_\ell - 2}$$

$$+ \frac{2B_\ell^2}{1 - P_N} \sum_{m=1}^{\infty} m \, P_N^m (1 - P_N^\ell)^{B_\ell - 1} \qquad \text{(A.41)}$$

$$\psi_3 = \sum_{j=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} E\{B_\ell^{(j)} B_\ell^{(n)} B_\ell^{(m)} \; I(B_\ell^{(0)} > 0, \; B_\ell^{(1)} > 0, \; \ldots,$$

$$B_\ell^{(j \wedge n \wedge m)} > 0) \,|\, B_\ell\}$$

$$= \frac{1}{(1 - P_N)^3} [B_\ell (1 + 4P_N + P_N^2) + 3(1 + P_N) \, B_\ell (B_\ell - 1)$$

$$+ B_\ell (B_\ell - 1)(B_\ell - 2)] \qquad \text{(A.42)}$$

A.4 Positive Acknowledgment with Time-Out Periods

Positive acknowledgment with time-out requires a PACK be returned to the source station within a predetermined time interval following data transmission. If a PACK is not returned within this time-out

interval, a NACK is understood. In actual system implementations, time-out mechanisms are also used with positive-negative acknowledgment schemes. Time-out thresholds are required to recover from lost acknowledgments.

In Chapter II, ideal acknowledgment mechanisms are assumed; PACKs and NACKs are never misinterpreted. A method for extending these results to include the possibility of exceeding the time-out threshold is outlined in this appendix. The results of Theorem 2.1 are applicable with appropriate modifications to the completion time. In particular, the completion times for the SW ARQ - PP ACK and the Block ARQ - PP ACK schemes are developed and maximum throughput results are discussed.

The time-out threshold is denoted by K_T. The acknowledgment delay of the j-th packet of the ℓ-th message is denoted by $K_{\ell j}$ and $\{K_{\ell j}, \ell \geq 1, j \geq 1\}$ is assumed to be an i.i.d. sequence. Thus, if the acknowledgment delay $K_{\ell j}$ exceeds K_T, a NACK is assumed. To simplify analysis, a late arriving PACK is disregarded; delay and throughput performance calculated under this assumption serve as bounds to systems which accept the late PACK and halt the retransmission - acknowledgment sequence. Hence, a network of stations with random response times is modeled. The stations have incomplete acknowledgment delay information which necessitates the use of a single constant time-out threshold for all source-destination station pairs.

The completion times using positive acknowledgment with time-out are

$$S_n^{(i)} = \sum_{j=1}^{B_\ell} [(R_{\ell j} - 1)K_T + K_{\ell j}^{(L)}] \qquad \text{for SW ARQ-PP ACK} \qquad (A.43)$$

$$S_n^{(i)} = \sum_{j=1}^{B_\ell} [(R_{\ell j} - 1)K_T + 1] \qquad \text{for Block ARQ-PP ACK} \qquad (A.44)$$

where $R_{\ell j}$ is the number of transmissions required by the j-th packet of the ℓ-th message and $K_{\ell j}^{(L)}$ is the acknowledgment delay for the PACK which arrives within the time-out period. A packet is retransmitted if (1) random transmission errors are detected or (2) random transmission errors are not detected but the time-out threshold is exceeded. Thus the variables $\{R_{\ell j}\}$ form an independent sequence of geometric distributed random variables, $\{P(R_{\ell j} = m) = (1 - p)p^{m-1}, m = 1, 2, \ldots\}$ where

$$p = 1 - (1 - P_N)\, P(K_{\ell j} \leq K_T) \qquad (A.45)$$

The sequence of last acknowledgment delays $\{K_{\ell j}^{(L)}\}$ is governed by the following distribution:

$$P(K_{\ell j}^{(L)} \leq m) = \frac{P(K_{\ell j} \leq m)}{P(K_{\ell j} \leq K_T)} \qquad (A.46)$$

$m = 1, 2, \ldots, K_T$.

The first 3 moments of the completion time necessary for the mean and variance of the message delay at steady state can be calculated in a straightforward manner. In particular, the first moments are

$$s_1 = \frac{b}{1 - p} \left[K_T p + (1 - p)\, E(K_{\ell j}^{(L)})\right] \quad \text{for SW ARQ-PP ACK} \qquad (A.47)$$

$$s_1 = \frac{b}{1 - p} \left[K_T p + 1 - p\right] \quad \text{for Block ARQ-PP ACK} \qquad (A.48)$$

and the maximum throughputs become

$$\text{maximum throughput} = \begin{cases} \dfrac{1 - p}{K_T p + (1-p)\, E(K_{\ell j}^{(L)})} & \text{for SW ARQ - PP ACK} \qquad (A.49) \\ \dfrac{1 - p}{K_T p + 1 - p} & \text{for Block ARQ - PP ACK} \qquad (A.50) \end{cases}$$

Example A.4.1

If the acknowledgment delay is governed by a geometric distribution $\{P(K_{\ell j} = m) = a^{m-1}(1-a),\ m = 1, 2, \ldots\}$, then

$$P(K_{\ell j}^{(L)} = m) = \frac{a^{m-1}(1 - a)}{1 - a^{K_T}} \qquad (A.51)$$

for $m = 1, 2, \ldots, K_T$ and

$$E(K_{\ell j}^{(L)}) = \frac{1}{1 - a} - \frac{K_T\, a^{K_T}}{1 - a^{K_T}} \qquad (A.52)$$

The probability of packet retransmission is

$$p = 1 - (1 - P_N)\,(1 - a^{K_T}) \qquad (A.53)$$

Using (A.49) and (A.50), the maximum throughputs as a function of K_T are

$$\text{maximum throughput as a function of } K_T = \begin{cases} \dfrac{(1-P_N)(1-a^{K_T})(1-a)}{K_T(1-a)P_N + (1-P_N)(1-a^{K_T})} & \text{for SW ARQ - PP ACK} \quad (A.54) \\ \dfrac{(1-P_N)(1-a^{K_T})}{K_T a^{K_T} + [P_N(K_T-1)+1](1-a^{K_T})} & \text{for Block ARQ - PP ACK} \quad (A.55) \end{cases}$$

For the SW ARQ - PP ACK scheme the maximum throughput is maximum with $K_T = 1$:

$$\text{maximum throughput} = (1 - P_N)(1 - a) \qquad (A.56)$$

Under SW ARQ, packet transmissions are halted until the current transmission is positively acknowledged or the time-out period elapses. Since the probability of a PACK reception in each successive frame (given no transmission errors are detected) is equal to 1-a, the optimal strategy is to wait the minimum time-out period ($K_T = 1$) and retransmit.

For the Block ARQ - PP ACK scheme the value of K_T which maximizes the maximum throughput satisfies the following relationship:

$$a^{K_T}(1 - a^{K_T}) + P_N(1 - a^{K_T})^2 + a^{K_T} \ln(a^{K_T}) = 0 \qquad (A.57)$$

with K_T an integer, $K_T \geq 1$. Solutions to (A.57) are presented in Table A.1 in terms of a^{K_T}. Solutions for $P_N > 0.5$ do not exist under the requirement $K_T \geq 1$ (i.e., $a^{K_T} < 1$). Hence, for $P_N > 0.5$, $K_T = 1$ provides the maximum throughput for $0 \leq a < 1$. Time-out threshold values which maximize the maximum throughput are presented in Table A.2 for $a = 0.5, 0.8$. Since packet transmissions are continuous under Block ARQ, long thresholds are desirable with small P_N. However, as P_N increases, shorter thresholds which anticipate negative acknowledgments become optimal.

Maximum throughput versus packet noise error probability curves are shown in Figure A.1 for $a = 0.5, 0.8$. The Block ARQ - PP ACK scheme exhibits larger maximum throughputs than the SW ARQ - PP ACK scheme for small P_N. The maximum throughput of the Block ARQ - PP ACK scheme decreases rapidly as P_N increases. When the optimum threshold is 1, the maximum throughputs of the SW and Block ARQ schemes are identical.

Example A.4.2

Consider the sequence of acknowledgment delays governed by a uniform distribution, $\{P(K_{\ell j} = m) = \frac{1}{N}, m = 1, 2, \ldots, N\}$. The distribution of the last acknowledgment delay is

$$P(K_{\ell j}^{(L)} = m) = \frac{1}{K_T} \tag{A.58}$$

P_N	K_{T_a}
0.0	0.0
10^{-6}	6.4266×10^{-8}
10^{-5}	7.6428×10^{-7}
10^{-4}	9.4629×10^{-6}
10^{-3}	1.2519×10^{-4}
10^{-2}	1.8891×10^{-3}
0.05	1.5125×10^{-2}
0.1	4.1222×10^{-2}
0.15	7.8456×10^{-2}
0.2	0.12885
0.25	0.19548
0.3	0.28263
0.35	0.39627
0.4	0.544781
0.45	0.74015
0.5	0.99998

TABLE A.1: SOLUTIONS TO (A.57), NORMALIZED TIME-OUT THRESHOLDS

P_N	a = 0.5		a = 0.8	
	K_T	Maximum Throughput	K_T	Maximum Throughput
0.0	∞	1.0	∞	1.0
0.05	6	0.7062	19	0.4362
0.1	5	0.5764	14	0.3057
0.15	4	0.4952	11	0.2405
0.2	3	0.4375	9	0.2002
0.25	2	0.3913	7	0.1721
0.3	2	0.3559	6	0.1511
0.35	1	0.3250	4	0.1347
0.4	1	0.3	3	0.1213
0.45	1	0.275	1	0.11
0.5	1	0.25	1	0.1

TABLE A.2: OPTIMUM TIME-OUT THRESHOLDS FOR THE BLOCK ARQ - PP ACK SCHEME UNDER GEOMETRIC DISTRIBUTED ACKNOWLEDGMENT DELAYS

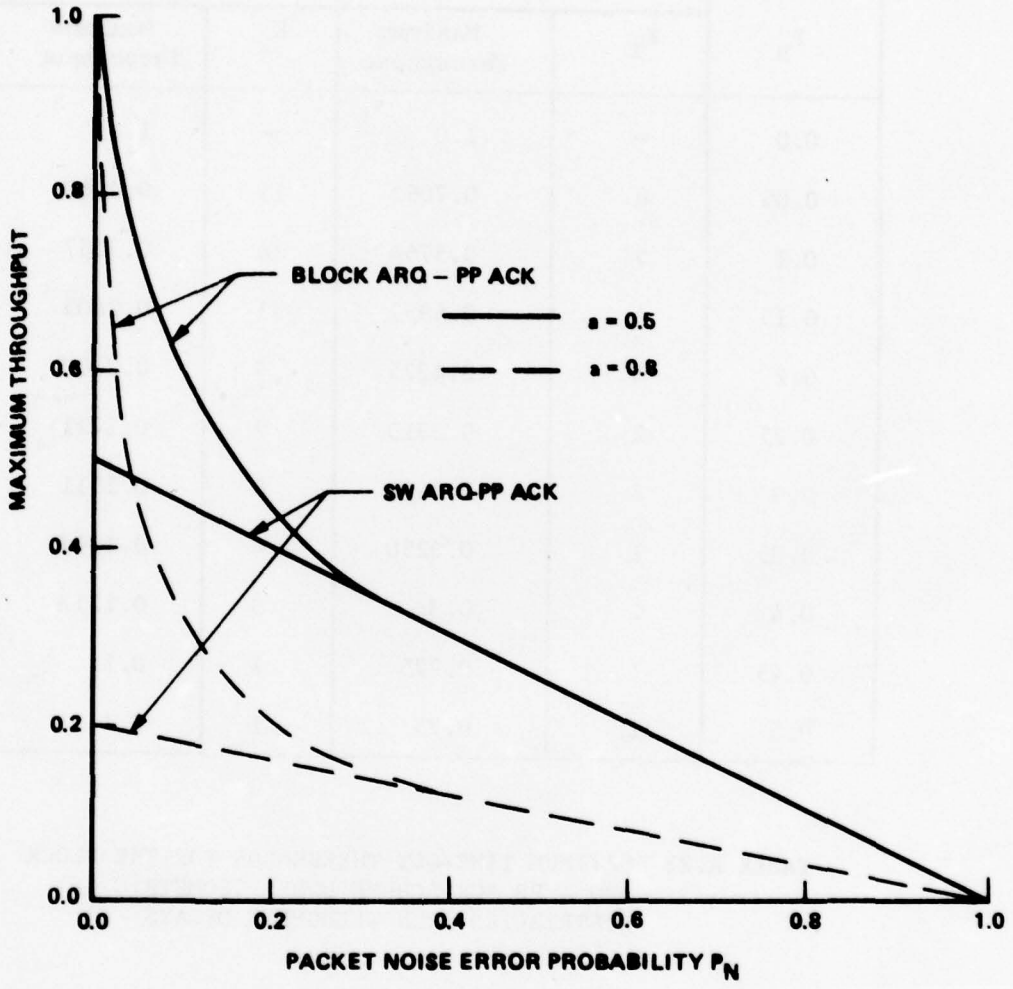


Figure A.1. Maximum Throughput versus Packet Noise Error Probability Curves for a TDMA Channel Using SW ARQ-PP ACK and Block ARQ-PP ACK, Positive Acknowledgment with Time-Out, with Geometric Distributed Acknowledgment Delays

for $m = 1, 2, \ldots, K_T$ and

$$E(K_{\ell j}^{(L)}) = \frac{K_T + 1}{2} \tag{A.59}$$

The probability of packet retransmission is

$$p = 1 - (1 - P_N)^{\frac{K_T}{N}} \tag{A.60}$$

The maximum throughputs as a function of K_T are

$$\text{maximum throughput as a function of } K_T = \begin{cases} \dfrac{2(1 - P_N)}{2N - K_T(1 - P_N) + 1 - P_N} & \text{for SW ARQ - PP ACK} \quad \text{(A.61)} \\ \dfrac{1 - P_N}{N - K_T(1 - P_N) + 1 - P_N} & \text{for Block ARQ - PP ACK} \quad \text{(A.62)} \end{cases}$$

The value of K_T which maximizes the maximum throughput for both schemes is $K_T = N$:

$$\text{maximum throughput} = \begin{cases} \dfrac{2(1 - P_N)}{N(1 + P_N) + 1 - P_N} & \text{for SW ARQ - PP ACK} \quad \text{(A.63)} \\ \dfrac{1 - P_N}{(N-1)P_N + 1} & \text{for Block ARQ - PP ACK} \quad \text{(A.64)} \end{cases}$$

With acknowledgment delays which are uniformly distributed over N frames, (given that no transmission errors are detected) the probability of PACK reception in each successive frame increases to 1 as shown by the following conditional probabilities:

$$P(K_{\ell j} = m \mid K_{\ell j} \geq m) = \frac{1}{N - m + 1} \quad \text{(A.65)}$$

for $m = 1, 2, \ldots, N$. Thus the optimal strategy is to wait the maximum time-out period $K_T = N$ before assuming a negative acknowledgment.

Maximum throughput versus packet noise error probability curves are shown in Figure A.2 for N = 3, 9. Since the time-out thresholds are set to N (optimum value), the Block ARQ scheme provides larger maximum throughputs for all $P_N < 1$ (unlike the geometric distributed acknowledgment delay example).

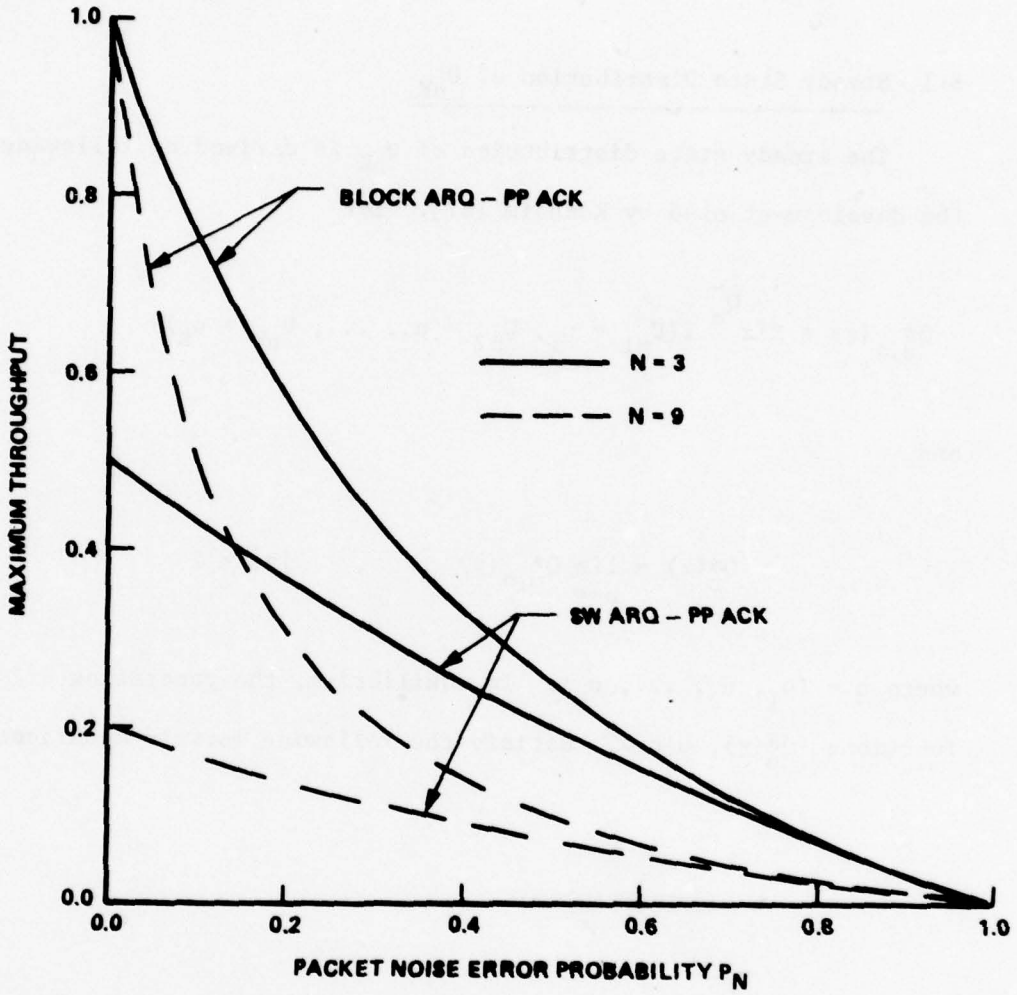


Figure A.2. Maximum Throughput versus Packet Noise Error Probability Curves for a TDMA Channel Using SW ARQ-PP ACK and Block ARQ-PP ACK, Positive Acknowledgment with Time-Out, with Uniform Distributed Acknowledgment Delays

APPENDIX B

B.1 Steady State Distribution of U_{nK}

The steady state distribution of U_{nK} is derived by following the development used by Konheim [47]. Let

$$Q^*_{\underline{u},n}(z) = E\{z^{Q_n^-} I(U_{n1} = u_1, U_{n2} = u_2, \ldots, U_{nK} = u_K)\} \tag{B.1}$$

and

$$Q^*_{\underline{u}}(z) = \lim_{n\to\infty} Q^*_{\underline{u},n}(z) \qquad , \qquad |z| < 1 \tag{B.2}$$

where $\underline{u} = (u_1, u_2, \ldots, u_K)$. In equilibrium, the generating functions $\{Q^*_{\underline{u}}(z), \underline{u} \in d_1^K\}$ satisfy the following forward equations.

Proposition B.1

$$Q^*_{\underline{u}}(z) = \begin{cases} N^*(z)\,[Q^*_{\underline{u}_0}(0) + (1-P_N)\,Q^*_{\underline{u}_0}(0)] & \text{if } u_1 = 0 \\ \dfrac{N^*(z)}{z}\,[Q^*_{\underline{u}_0}(z) - Q^*_{\underline{u}_0}(0)] & \\ \quad + (1-P_N)\,\dfrac{N^*(z)}{z}\,[Q^*_{\underline{u}_1}(z) - Q^*_{\underline{u}_1}(0)] & \\ \quad + P_N\,N^*(z)\,Q^*_{\underline{u}_1}(z) & \text{if } u_1 = 1 \end{cases} \quad (B.3)$$

where

$$\underline{u}_0 = (u_2, u_3, \ldots, u_K, 0)$$

$$\underline{u}_1 = (u_2, u_3, \ldots, u_K, 1).$$

Proof

Let $\underline{U}_n = (U_{n1}, U_{n2}, \ldots, U_{nK})$

$$Q^*_{(0,u_2,\ldots,u_K),n}(z) = E\{z^{Q_n^-}\, I(\underline{U}_n = (0, u_2, \ldots, u_K))\}$$

$$= E\{E\{z^{Q_n^-}\, I(\underline{U}_n = (0,u_2,\ldots,u_K)) \mid U_{n-1,K}, \text{ACK}_{n-1,K}\}\}$$

$$= E\{z^{N_n}\, I(\underline{U}_{n-1} = (u_2,u_3,\ldots,u_K,1))\}\,(1 - P_N)$$

$$+ E\{z^{N_n}\, I(\underline{U}_{n-1} = (u_2,u_3,\ldots,u_K,0))\}$$

$$= N^*(z)[(1-P_N)\,P(\underline{U}_{n-1} = (u_2,u_3,\ldots,u_K,1))$$

$$+ P(\underline{U}_{n-1} = (u_2,u_3,\ldots,u_K,0))] \quad (B.4)$$

$$Q^*_{(1,u_2,\ldots,u_K),n}(z) = E\{z^{Q_n^-} I(\underline{U}_n = (1,u_2,\ldots,u_K))\}$$

$$= E\{E\{z^{Q_n^-} I(\underline{U}_n = (1,u_2,\ldots,u_K)) | U_{n-1,K}, ACK_{n-1,K}\}\}$$

$$= E\{z^{Q_{n-1}^- - 1 + N_n} I(\underline{U}_{n-1} = (u_2,u_3,\ldots,u_K,0), U_{n1} = 1)\}$$

$$+ (1-P_N)\, E\{z^{Q_{n-1}^- - 1 + N_n} I(\underline{U}_{n-1} = (u_2,u_3,\ldots,u_K,1),$$

$$U_{n1} = 1)\}$$

$$+ P_N\, E\{z^{Q_{n-1}^- + N_n} I(\underline{U}_{n-1} = (u_2,u_3,\ldots,u_K,1))\}$$

$$= \frac{N^*(z)}{z} [Q^*_{(u_2,u_3,\ldots,u_K,0),n-1}(z) - P(Q_{n-1}^- = 0)]$$

$$+ (1-P_N) \frac{N^*(z)}{z} [Q^*_{(u_2,u_3,\ldots,u_K,1),n-1}(z)$$

$$- P(Q_{n-1}^- = 0)]$$

$$+ P_N\, N^*(z)\, Q^*_{(u_2,u_3,\ldots,u_K,1),n-1}(z) \qquad (B.5)$$

The desired results follow by taking the limits as $n \to \infty$ in (B.4) and (B.5) Q.E.D.

Let

$$U^*_{(i)}(z) = \sum_{\{u:u_K=i\}} Q^*_{\underline{u}}(z)\, z^{u_1+u_2+\ldots+u_K}, \quad i = 0, 1 \qquad (B.6)$$

$$+ N^*(z) \sum_{\{(u_2,u_3,\ldots,u_{K-1})\}} zz^{1+u_2+\ldots+u_{K-1}}$$

$$\cdot \left[Q^*_{(u_2,u_3,\ldots,u_{K-1},1,0)}(z)\right.$$

$$\left. + (1-P_N + P_N z)Q^*_{(u_2,u_3,\ldots,u_{K-1},1,1)}(z)\right]$$

$$= N^*(z) \sum_{\{(u_2,u_3,\ldots,u_{K-1},0)\}} z^{1+u_2+\ldots+u_{K-1}}$$

$$\cdot \left[Q^*_{(u_2,u_3,\ldots,u_K,0)}(z) + (1-P_N + P_N z)Q^*_{(u_2,u_3,\ldots,u_K,1)}(z)\right]$$

$$+ N^*(z) \sum_{\{(u_2,u_3,\ldots,u_{K-1},1)\}} zz^{1+u_2+\ldots+u_{K-1}}$$

$$\cdot \left[Q^*_{(u_2,u_3,\ldots,u_K,0)}(z) + (1-P_N + P_N z)Q^*_{(u_2,u_3,\ldots,u_K,1)}(z)\right]$$

$$= N^*(z) \sum_{\{(u_2,u_3,\ldots,u_K)\}} z^{1+u_2+\ldots+u_K} \left[Q^*_{(u_2,u_3,\ldots,u_K,0)}(z)\right.$$

$$\left. + (1-P_N + P_N z)Q^*_{(u_2,u_3,\ldots,u_K,1)}(z)\right]$$

$$= N^*(z) \sum_{\{(u_3,u_4,\ldots,u_K)\}} z^{1+u_3+\ldots+u_K} \left[Q^*_{(0,u_3,u_4,\ldots,u_K,0)}(z)\right.$$

$$\left. + (1-P_N + P_N z)Q^*_{(0,u_3,u_4,\ldots,u_K,1)}(z)\right]$$

$$+ N^*(z) \sum_{\{(u_3,u_4,\ldots,u_K)\}} z^{1+u_3+\ldots+u_K}$$

$$\cdot \left[Q^*_{(1,u_3,\ldots,u_K,0)}(z) + (1-P_N + P_N z)Q^*_{(1,u_3,\ldots,u_K,1)}(z)\right]$$

$$= N^*(z) \sum_{\{(u_3,u_4,\ldots,u_K)\}} z^{1+u_3+\ldots+u_K} \left[Q^*_{(0,u_3,\ldots,u_K,0)}(z)\right.$$

$$\left. + z\, Q^*_{(1,u_3,\ldots,u_K,0)}(z)\right] + N^*(z)\,(1 - P_N + P_N z)$$

$$\cdot \sum_{\{(u_3,u_4,\ldots,u_K)\}} z^{1+u_3+\ldots+u_K} \left[Q^*_{(0,u_3,\ldots,u_K,1)}(z)\right.$$

$$\left. + z\, Q^*_{(1,u_3,\ldots,u_K,1)}(z)\right]$$

$$= z\, N^*(z)\, U^*_{(1)}(z) + N^*(z)\,(1 - P_N + P_N z)\, U^*_{(1)}(z) \qquad \text{(B.10)}$$

Q.E.D.

Equation (B.7) yields

$$\frac{U^*_{(1)}(1)}{U^*_{(0)}(1)} = \frac{n_1}{1 - n_1 - P_N} \qquad \text{(B.11)}$$

The normalization $U^*_{(1)}(1) + U^*_{(0)}(1) = 1$ together with (B.11) determines the limiting distribution of U_{nK}:

$$P(U_{\infty K} = j) = Q^*_{(j)}(1)$$

$$= \begin{cases} \dfrac{n_1}{1 - P_N} & \text{if} \quad j = 1 \\ 1 - \dfrac{n_1}{1 - P_N} & \text{if} \quad j = 0 \end{cases} \tag{B.12}$$

B.2 Derivations of W_{LK} and W_{UK}

Equation (3.17) yields

$$E\{Q_n^- \, I(\varepsilon_n, Q_n^- > 0)\} = (1-P_N)\, E(Q_n^-) + P_N \, E\{Q_n^- \, I(U_{nK} = 0)\} \tag{B.13}$$

Solving (3.4) recursively with $I_j = I(\varepsilon_j, Q_j^- > 0)$, the transmit queue size becomes

$$Q_n^- = N_n + N_{n-1} + \ldots + N_{n-K+1} - [I_{n-1} + I_{n-2} + \ldots + I_{n-K+1}]$$

$$+ Q_{n-K}^- - I_{n-K} \tag{B.14}$$

Multiplying (B.14) by $I(U_{nK} = 0)$,

$$E\{Q_n^- \, I(U_{nK} = 0)\} = Kn_1 \, P(U_{nK} = 0)$$

$$- E\{[I_{n-1} + I_{n-2} + \ldots + I_{n-K+1}] \, I(U_{nK} = 0)\}$$

$$+ E\{[Q_{n-K}^- - I_{n-K}] \, I(U_{nK} = 0)\} \qquad \text{(B.15)}$$

But $U_{nK} = 0$ implies that $Q_{n-K}^- = 0$ and, therefore,

$$E\{Q_n^- \, I(U_{nK} = 0)\} = \begin{cases} Kn_1 \, P(U_{nK} = 0) - E\{[I_{n-1} + I_{n-2} + \ldots + I_{n-K+1}] \\ \quad \cdot I(U_{nK} = 0)\} & \text{if} \quad K > 1 \\ n_1 \, P(U_{n1} = 0) & \text{if} \quad K = 1 \end{cases} \qquad \text{(B.16)}$$

It immediately follows that

$$E\{Q_n^- \, I(U_{nK} = 0)\} \leq Kn_1 \, P(U_{nK} = 0) \qquad \text{(B.17)}$$

for $K \geq 1$. In addition,

$$E\{I_i \, I(U_{nK} = 0)\} = P(\varepsilon_i, \, Q_i^- > 0, \, U_{nK} = 0)$$

$$\leq P(Q_i^- > 0 \,|\, U_{nK} = 0) \, P(U_{nK} = 0) \qquad \text{(B.18)}$$

and using (B.14)

$$P(Q_i^- > 0 | U_{nK} = 0) \leq P(N_i + N_{i-1} + \ldots + N_{n-K+1} > 0) \qquad (B.19)$$

for $i = n-1, n-2, \ldots, n-K+1$ and $K > 1$. Substituting (B.18) and (B.19) into (B.16),

$$E\{Q_n^- \, I(U_{nK} = 0)\} \geq (Kn_1 - N_K) \, P(U_{nK} = 0) \qquad (B.20)$$

where

$$N_K = \begin{cases} \sum_{i=1}^{K-1} P(N_1 + N_2 + \ldots + N_i > 0) & \text{if} \quad K > 1 \\ 0 & \text{if} \quad K = 1 \end{cases}$$

Thus, substituting the inequalities (B.17) and (B.20) into (B.13) yields the relationships necessary to evaluate (3.16). Bounds on the average steady state queue size result, and the desired bounds (W_{LK}, W_{UK}) on the average waiting time E(W) at steady state follow from (3.14) and (3.15).

APPENDIX C

C.1 Proof of Proposition 4.1

The state sequences are irreducible and aperiodic on their respective reduced state spaces by construction. Since Z_{n+1} depends on Z_n only through X_n, Z is positive recurrent if and only if X is positive recurrent.

Construct the following sets:

$$C_o = \{\underline{i} = (i_s, i_r): \; i_s + i_r < N_T, \; i_s \in \tilde{d_K}, \; i_r \in d\}$$

$$C_\ell = \{\underline{i} = (i_s, i_r): \; i_s + i_r = N_T + \ell - 1, \; i_s \in \tilde{d_K}, \; i_r \in d\}$$

$\ell = 1, 2, \ldots$. The reduced state space of X is denoted by C_X and $C_X \subseteq \bigcup_{\ell=0}^{\infty} C_\ell$. New message arrivals within the (n+1)-st frame are admitted by the network control procedure if $X_n = \underline{i} \in C_o$; if $X_n = \underline{i} \notin C_o$, new message arrivals are rejected.

Suppose X is transient and $X_n = \underline{i} \in C_o$. Since the number of elements in C_o (denoted by $|C_o|$) is finite, there exists an $m > n$ such that $X_m = \underline{j} \notin C_o$. But the number of states in $C_o \cup \{\underline{k}: \; k_s + k_r \leq j_s + j_r\}$ is finite; and, therefore, there exists an $\ell > m$ such that $X_\ell = \underline{k} \in C_o$. Thus X visits C_o infinitely often, a contradiction. Hence, X is a recurrent Markov chain.

Let $m_{\underline{i}}$ denote the mean recurrence time of state $\underline{i} \in C_X$. To prove that X is positive recurrent, it is sufficient to show that $m_{\underline{i}}$ is finite for at least one state $\underline{i} \in C_o$.

Consider the scalar Markov chain $y = \{y_n, n \geq 1\}$ over the state space d with transition probabilities

$$P_y(0,\ell) = \max_{\underline{i} \in C_o} \{ \sum_{\underline{j} \in C_\ell} P_X(\underline{i},\underline{j}) \} \qquad \text{if } \ell > 0 \qquad (C.1)$$

$$P_y(0,0) = 1 - \sum_{\ell=1}^{\infty} P_y(0,\ell) \qquad (C.2)$$

$$P_y(\ell,\ell-1) = \min_{\underline{i} \in C_\ell} \{ \sum_{\underline{j} \in C_{\ell-1}} P_X(\underline{i},\underline{j}) : \sum_{j \in C_{\ell-1}} P_X(\underline{i},\underline{j}) > 0 \}$$

$$\text{if } \ell \geq 1 \qquad (C.3)$$

$$P_y(\ell,\ell) = 1 - P_y(\ell,\ell-1) \qquad \text{if } \ell \geq 1. \qquad (C.4)$$

The sequence y is irreducible and recurrent with state transition diagram illustrated in Figure C.1.

By construction,

$$m_{\underline{i}} < |C_o| \, \tilde{m}_o \qquad (C.5)$$

for at least one state $\underline{i} \in C_o$ where $\tilde{m}_o$ denotes the mean recurrence

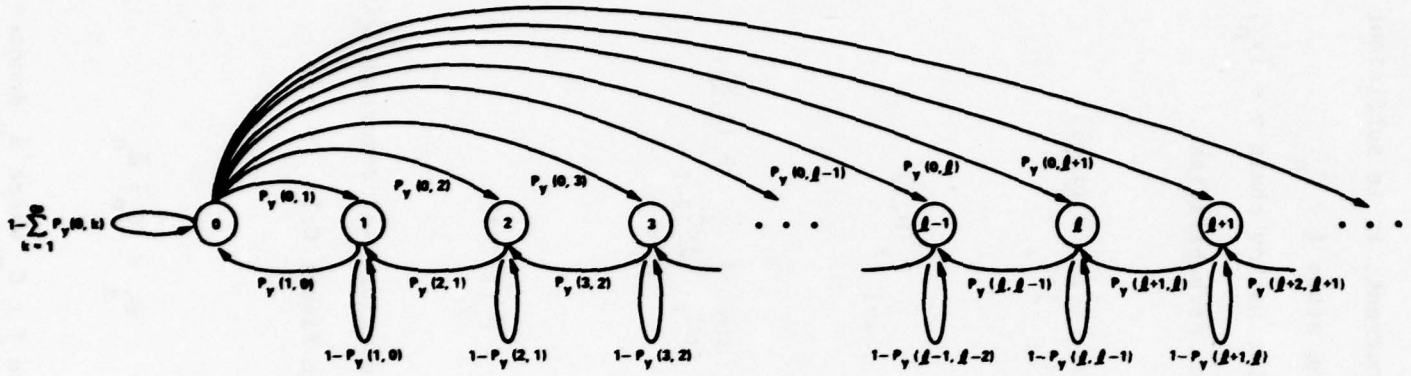


Figure C.1 State Transition Diagram for the Markov Chain y

time of the state 0 for the Markov chain y. The mean recurrence time can be expressed by

$$\tilde{m}_o = 1 + \sum_{\ell=1}^{\infty} P_y(0,\ell)\, \tilde{m}_{\ell o} \tag{C.6}$$

where $\tilde{m}_{\ell o}$ is the mean first passage time from ℓ to 0 for the Markov chain y. For $\ell > 0$

$$\tilde{m}_{\ell o} = \sum_{k=1}^{\ell} \tilde{m}_{k,k-1} \tag{C.7}$$

where

$$\tilde{m}_{k,k-1} = \frac{1}{P_y(k,k-1)} \quad . \tag{C.8}$$

Thus $\tilde{m}_o$ is upper bounded by upper bounding $P_y(0,\ell)$, $\ell = 1, 2, \ldots$ and lower bounding $P_y(k,k-1)$, $k = 1, 2, \ldots$.

Using (C.1),

$$P_y(0,\ell) \leq \sum_{\underline{i} \in C_o} \sum_{\underline{j} \in C_\ell} P_X(\underline{i},\underline{j}) \quad , \tag{C.9}$$

$\ell > 0$. The number of transmissions in the (n+1)-st period is equal to the number of collisions plus collision-free information and PACK transmissions (i.e., $R_{n+1} + \tilde{S}_{n+1}^{(I)} + \tilde{S}_{n+1}^{(A)}$). It is also equal to the number of collision-free information packet transmissions plus the number of collisions in the n-th period plus the number of new message arrivals in the (n+1)-st frame (i.e., $R_n + \tilde{S}_n^{(I)} + A_{n+1}^{(C)}$). Therefore,

$$\tilde{S}_{n+1}^{(I)} + R_{n+1} = R_n + \tilde{S}_n^{(I)} + A_{n+1}^{(C)} - \tilde{S}_{n+1}^{(A)} \tag{C.10}$$

and

$$\sum_{\underline{j} \in C_\ell} P_X(\underline{i},\underline{j}) = \sum_{(j_s,j_r) \in C_\ell} P\{\tilde{S}_{n+1}^{(I)} = j_s,\ R_{n+1} = j_r \Big| \tilde{S}_n^{(I)} = i_s,\ R_n = i_r,$$

$$i_s + i_r < N_T\}$$

$$\leq \sum_{k=0}^{i_s} P\{A_{n+1}^{(C)} = N_T + \ell - 1 - (i_s + i_r) + k \Big| \tilde{S}_n^{(I)} = i_s,$$

$$R_n = i_r,\ i_s + i_r < N_T,\ \tilde{S}_{n+1}^{(A)} = k\} \tag{C.11}$$

for $\underline{i} \in C_o$, $\ell > 0$. Substituting (C.11) into (C.9) yields

$$P_y(0,\ell) \leq \sum_{t=0}^{N_T-1} \sum_{k=0}^{t} P(A_{n+1} = N_T - 1 + \ell - t + k)$$

$$\leq \theta_o e^{-\lambda_o} \frac{\lambda_o^\ell}{\ell!} \tag{C.12}$$

where θ_o is a (positive) finite constant and $\lambda_o = \lambda(\tilde{K}+P)$. From (C.3)

$$P_y(\ell,\ell-1) = \sum_{(j_s,j_r)\in C_{\ell-1}} P\{\tilde{S}_{n+1}^{(I)} = j_s,\ R_{n+1} = j_r \Big| \tilde{S}_n^{(I)} = 1,$$

$$R_n = N_T + \ell - 2\}$$

$$\geq P\{\tilde{S}_{n+1}^{(I)} = 0,\ R_{n+1} = N_T + \ell - 2 \Big| \tilde{S}_n^{(I)} = 1,\ R_n = N_T + \ell - 2\}$$

$$= \left(\frac{1}{\tilde{K}-1}\right)^{N_T+\ell-1}$$

$$> \left(\frac{1}{\tilde{K}}\right)^{N_T+\ell-1}, \qquad (C.13)$$

$\ell > 0$.

Using (C.7), (C.8) and (C.13),

$$\tilde{m}_{\ell o} < \sum_{k=1}^{\ell} \tilde{K}^{N_T+k-1}$$

$$< \ell\, \tilde{K}^{N_T+\ell-1}. \qquad (C.14)$$

Substituting (C.12) and (C.14) into (C.6) yields

$$\tilde{m}_o < 1 + \sum_{\ell=1}^{\infty} \theta_o\, e^{-\lambda_o} \frac{\lambda_o^{\ell}}{\ell!}\, \ell\, \tilde{K}^{N_T-1+\ell}$$

$$= 1 + \theta_o\, \lambda_o\, \tilde{K}^{N_T}\, e^{\lambda_o(\tilde{K}-1)}$$

$$< \infty. \qquad (C.15)$$

Q.E.D.

C.2 Proof of Theorem 4.6

To derive the upper bound on throughput given by (4.105) consider the following function:

$$f(x) = x(1 - a^{x-1}) \tag{C.16}$$

where $a < 1$ with $f^{(i)}(x) = \frac{d^i}{dx^i} f(x)$, $i \geq 1$.

Lemma C.1

$$f(x) \geq f^{(1)}(x_o)\ (x - x_o) + f(x_o) \quad , \quad x \geq 0 \tag{C.17}$$

where

$$x_o \in \{x:\ 0 < f^{(1)}(x) \leq 1\}.$$

Proof

The first 3 derivatives of $f(x)$ are

$$f^{(1)}(x) = 1 - a^{x-1} - xa^{x-1}\ \ell n(a) \tag{C.18}$$

$$f^{(2)}(x) = a^{x-1}[2 + x\ \ell n(a)]\ \ell n(a^{-1}) \tag{C.19}$$

$$f^{(3)}(x) = -a^{x-1}[3 + x\ \ell n(a)]\ [\ell n(a)]^2 \quad . \tag{C.20}$$

The function $f(x)$ has the following properties:

- $f^{(2)}(x) > 0$ if $x < \frac{2}{\ell n(a^{-1})}$

- $f^{(1)}(x) \leq 1$ if $x \leq \frac{1}{\ell n(a^{-1})}$

- $f^{(1)}(x) > 0$ if $x > x_1$ where x_1 satisfies $f^{(1)}(x_1) = 0$, $0 < x_1 < 1$.

- $\lim_{x \to \infty} f^{(1)}(x) = 1$.

Thus f(x) can be described as follows:

- f(0) = 0 and f(x) is a decreasing function of x if $x \in [0, x_1)$.

- f(x) is an increasing function of x with strictly increasing slope, $f^{(1)}(x)$, if $x_1 \leq x \leq \frac{2}{\ell n(a^{-1})}$.

- f(x) is an increasing function of x with slope decreasing to 1 if $x \geq \frac{2}{\ell n(a^{-1})}$.

Therefore, f(x) can be lower bounded by choosing a line tangent at x_o where $x_o \in \{x: 0 < f^{(1)}(x) \leq 1\}$ or, equivalently, $x_1 < x_o \leq \frac{1}{\ell n(a^{-1})}$.

Q.E.D.

Proof (Theorem 4.6)

From (4.110)

$$E(\tilde{S}_{n+1}^{(A)} | \tilde{S}_n^{(I)}, N_n) = \tilde{S}_n^{(I)} - E(R_{n+1}^{(A)} | \tilde{S}_n^{(I)}, N_n) \quad . \tag{C.21}$$

Substituting (4.98) into (C.21) and rearranging terms,

$$E(R_{n+1}^{(A)} | \tilde{S}_n^{(I)}, N_n) = \tilde{S}_n^{(I)} [1 - (1 - \frac{1}{\tilde{K}_\eta})^{\tilde{S}_n^{(I)} - 1}] \tag{C.22}$$

and

$$E(\tilde{S}_{n+1}^{(A)} | N_n) = E(\tilde{S}_n^{(I)} | N_n) - E\{\tilde{S}_n^{(I)} [1 - (1 - \frac{1}{\tilde{K}_\eta})^{\tilde{S}_n^{(I)} - 1}] | N_n\}. \tag{C.23}$$

Applying Lemma C.1 to the second term on the right hand side of (C.23) with $a = (1 - \frac{1}{\tilde{K}_\eta})$,

$$E\{\tilde{S}_n^{(I)} [1 - (1 - \frac{1}{\tilde{K}_\eta})^{\tilde{S}_n^{(I)} - 1}] | N_n\} \geq f^{(1)}(x_o)[E(\tilde{S}_n^{(I)} | N_n) - x_o] + f(x_o). \tag{C.24}$$

Substituting (C.24) into (C.23) yields

$$E(\tilde{S}_{n+1}^{(A)} | N_n) \leq E(\tilde{S}_n^{(I)} | N_n) \; [1 - f^{(1)}(x_o)] + x_o \; f^{(1)}(x_o) - f(x_o). \tag{C.25}$$

In addition,

$$E(S_n^{(I)} | N_n) = N_n (1 - \frac{1}{\tilde{K}_I})^{N_n - 1}$$

$$\leq V_o(\tilde{K}_I) \tag{C.26}$$

where

$$V_o(\tilde{K}_I) = e^{-1}\,\tilde{K}_I\{(\tilde{K}_I - 1)\,\ell n[1 + (\tilde{K}_I - 1)^{-1}]\}^{-1}.$$

Since $f^{(1)}(x_o) \leq 1$, (C.25) and (C.26) yield

$$E(\tilde{S}_{n+1}^{(A)}) \leq f^{(1)}(x_o)[x_o - V_o(\tilde{K}_I)] + V_o(\tilde{K}_I) - f(x_o) \triangleq F(x_o). \qquad (C.27)$$

To choose an x_o which minimizes $F(x_o)$ under the constraint

$$x_1 < x_o \leq \frac{1}{\ell n[(1 - \frac{1}{\tilde{K}_\eta})^{-1}]} \triangleq V_2(\tilde{K}_\eta)\ , \qquad (C.28)$$

consider

$$F^{(1)}(x_o) = \frac{d}{dx_o} F(x_o)$$

$$= f^{(2)}(x_o)[x_o - V_o(\tilde{K}_I)]\ . \qquad (C.29)$$

Under (C.28), $F^{(1)}(x_o)$ is equal to 0 with $x_o = V_o(\tilde{K}_I)$ if $V_o(\tilde{K}_I) \leq V_2(\tilde{K}_\eta)$. Therefore,

$$E(\tilde{S}_{n+1}^{(A)}) \leq V_o(\tilde{K}_I)\ (1 - \frac{1}{\tilde{K}_\eta})^{V_o(\tilde{K}_I)-1} \qquad (C.30)$$

if $V_o(\tilde{K}_I) \leq V_2(\tilde{K}_\eta)$. On the otherhand, if $V_o(\tilde{K}_I) > V_2(\tilde{K}_\eta)$, then $F^{(1)}(x_o) < 0$; and, hence, $F(x_o)$ is minimum with $x_o = V_2(\tilde{K}_\eta)$ (i.e., the largest value of x_o allowable under (C.28)). Therefore,

$$E(S_{n+1}^{(A)}) \leq V_o(\tilde{K}_\eta) \tag{C.31}$$

if $V_o(\tilde{K}_I) > V_2(\tilde{K}_\eta)$.

Q.E.D.

APPENDIX D

GROUP RANDOM ACCESS USING FREQUENCY DIVISION ACKNOWLEDGMENT PROTOCOLS

In Chapter V the operation of the GRA channel is examined under two acknowledgment schemes which schedule PACK transmissions to avoid collisions. PACK transmissions and information packet transmissions share the available channel capacity (bandwidth) on a time division basis. Each channel access period is partitioned into two distinct subperiods; information packets and PACKs are transmitted in separate subperiods utilizing the entire channel capacity (C bits per second).

The operation of the GRA channel under frequency division acknowledgment schemes is examined in this appendix. The available channel capacity is divided into two smaller capacity subchannels: an information subchannel and an acknowledgment subchannel. Information packets are transmitted over the information subchannel on a random access basis within the channel access periods. PACK transmissions over the acknowledgment subchannel are scheduled to avoid collision. Like the time division PDS acknowledgment scheme in which the subperiod slot allocations are fixed, the subchannel capacity assignments under the frequency division acknowledgment schemes considered in this appendix are fixed (not adaptive).

Two frequency division acknowledgment schemes are considered:

- Frequency Division Asynchronous (FDA)
- Frequency Division Synchronous (FDS).

Under the FDA scheme, the destination stations transmit PACKs over the acknowledgment subchannel immediately following the collision-free reception of information packets. Under the FDS scheme, the information and acknowledgment subchannels are structured so that the start of their channel access periods are coincident in time. Hence, PACK transmissions are not necessarily made immediately following the collision-free reception of an information packet; in general, they must wait for a channel access period.

D.1 System Description

Under the frequency division acknowledgment schemes, the available channel bandwidth is divided into two subchannels: an information subchannel and an acknowledgment subchannel. It is assumed that the transmission rate per unit bandwidth is the same whether single channel or two channel operation is considered; therefore, the information and acknowledgment subchannels have capacities C_I and C_A, respectively, such that

$$C = C_I + C_A \quad \text{bits/second.} \tag{D.1}$$

The information subchannel is divided into slots of duration $\tau_I = b/C_I$ seconds and the acknowledgment subchannel is divided into slots of duration $\tau_A = b/\eta C_A$ seconds where b denotes the information packet size in bits and η is the ratio of the information packet size to PACK size. Let ζ denote the ratio of transmission times

$$\zeta = \frac{\tau_A}{\tau_I} \quad . \tag{D.2}$$

The parameter ζ is restricted to $\zeta \leq 1$ to simplify the acknowledgment protocols. Ratios greater than 1 ($\zeta > 1$) could be handled but with more complex protocol algorithms.

In Chapter V the operation of the GRA channel under the time division acknowledgment schemes is examined with the channel divided into fixed length slots of duration $\tau = b/C$ seconds. The slots are organized into frames with $\tilde{K}+P$ slots per frame and $\tilde{K}$ service slots per channel access period. For a fair comparison between the time and frequency division schemes, the information subchannel of the frequency division schemes is organized into frames with $\hat{P} + \hat{K} + \hat{K}_\Delta$ slots of duration τ_I seconds such that

$$(P+\tilde{K})\tau = (\hat{P} + \hat{K} + \hat{K}_\Delta)\tau_I \tag{D.3}$$

and

$$\tilde{K}\tau \geq \hat{K}\,\tau_I \quad . \tag{D.4}$$

To minimize the number of information packet collisions per channel access period, the number of slots in the information period should be maximized under the restriction $\zeta \leq 1$. The maximum number is given by

$$\hat{R} = \left\lfloor \frac{\tilde{K}\eta}{1 + \eta} \right\rfloor \qquad \text{if} \quad \zeta_o \leq \zeta \leq 1 \tag{D.5}$$

where $\lfloor x \rfloor$ denotes the integer part of x and

$$\zeta_o = \frac{\hat{K}}{(\tilde{K} - \hat{K})\eta} \quad . \tag{D.6}$$

The fractional slot given by

$$\hat{K}_\Delta = \frac{\tilde{K}\tau - \hat{K}\tau_I}{\tau_I}$$

$$= \frac{\tilde{K}\zeta\eta}{1 + \zeta\eta} - \hat{K} \tag{D.7}$$

cannot be used for information packet transmissions. It is (arbitrarily) placed at the end of each channel access period.

The GRA channel structures under the FDA and FDS acknowledgment schemes are illustrated in Figures D.1 and D.2, respectively. Under the FDA scheme, the (n+1)-st acknowledgment period begins $(\hat{R}+1)\tau_I$ seconds following the start of the n-th information period, since PACKs are transmitted immediately following collision-free information packet receptions. Under the FDS scheme, the start of the information and acknowledgment periods coincides.

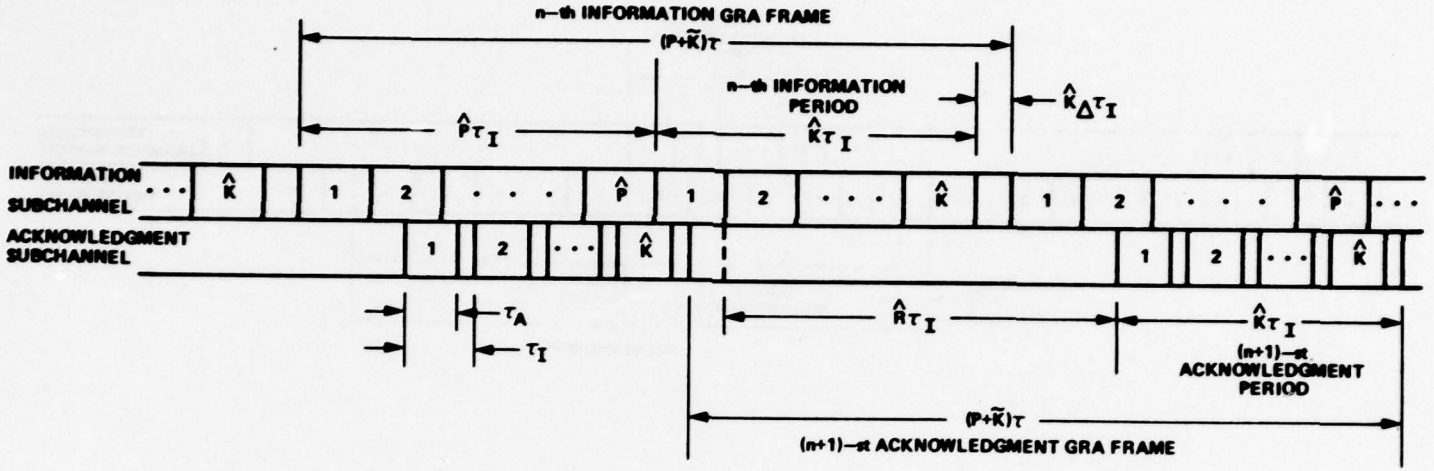


Figure D.1 GRA Channel Structure Under the FDA ACK Scheme

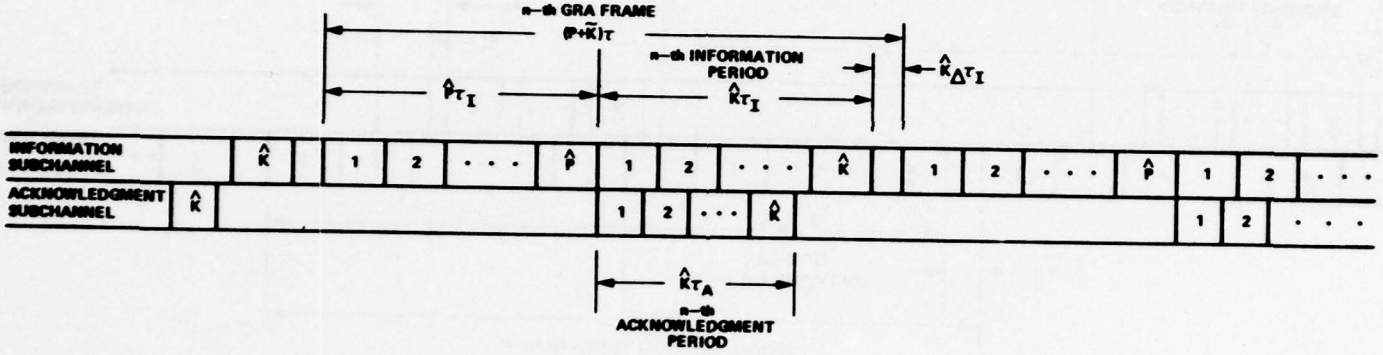


Figure D.2 GRA Channel Structure Under the FDS ACK Scheme

Protocol: GRA Discipline - FDA and FDS ACK Scheme

(1) New message arrivals which are admitted by the network control procedure are

i) transmitted immediately over the information subchannel, if they arrive during a channel access period allocated to information packet transmissions

ii) transmitted over the information subchannel in the next information period in a slot determined by a uniform distribution over the $\hat{K}$ available service slots, if they arrive in the interval between information periods.

Messages which are not admitted by the network control procedure are rejected.

(2) Each information packet colliding within the n-th period is retransmitted over the information subchannel within the (n+1)-st period in a slot determined by a uniform distribution over the $\hat{K}$ available service slots.

(3) For a collision-free information packet transmission in the i-th slot within the n-th information period, a PACK is transmitted over the acknowledgment subchannel in the i-th slot within the (n+1)-st acknowledgment period.

(4) Each information packet admitted by the network control procedure is transmitted and retransmitted until successful.

D.2 Throughput and Delay Analysis

Under the PDS time division acknowledgment scheme, the number of slots ($\tilde{K}_I$) in each period allocated to information packet transmissions is determined by (5.2); under the frequency division acknowledgment schemes, the number of slots ($\hat{K}$) in each period allocated to information packet transmissions is determined by (D.5). By comparing (5.2) with (D.5), $\tilde{K}_I = \hat{K}$. Although the slot durations (τ_I, τ) are different under the frequency division and time division acknowledgment schemes, the operating statistics for these three schemes are identical, since the same group of network stations considered in Chapters IV and V with overall message arrival stream described by a Poisson point process is considered. Hence, the evolution of the GRA channel under the FDA and FDS acknowledgment schemes is described by a Markov state sequence Z whose structure is identical to the structure of the state sequence developed in Section 5.2.1 for the GRA channel under the PDS scheme. Therefore, the throughput of the GRA channel under the frequency division schemes is equal to the throughput under the PDS scheme (throughput measured in packets per frame).

Although the statistical behavior of the GRA channel under the FDA, FDS and PDS schemes is equivalent, the limiting average packet delay expressions are different. The channel structures illustrated in Figures D.1, D.2, and 5.1 are dissimilar. The data transfer delay for the GRA channel under the frequency division acknowledgment schemes is given by the following lemma.

Lemma D.1

With the operation of a controlled GRA channel under the FDA and FDS acknowledgment schemes described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delay (measured in slots of duration τ seconds) is given by

$$E(D_R) = \frac{P}{2} + \frac{(\hat{K}_\Delta + 2)(1 + \zeta\eta) + 1}{2\zeta\eta} + (\tilde{K}+P)\frac{E(R_n)}{E(\tilde{S}_n^{(I)})} + R \qquad (D.8)$$

where the expectations are w.r.t. the stationary distribution.

Proof

Under the FDA and FDS acknowledgment schemes, PACK transmissions are scheduled to avoid collision. Hence, the operation of the information subchannel is identical to the basic GRA channel described in Section 4.1.2. Therefore, the limiting average packet delay $E(\hat{D}_R)$ measured in slots of duration τ_I seconds is given by (4.3) with the GRA structure parameters $(\tilde{K}, P, R)$ replaced with $(\hat{K}, \hat{P}+\hat{K}_\Delta, \hat{R})$:

$$E(\hat{D}_R) = \frac{\hat{P} + \hat{K}_\Delta}{2} + (\hat{P} + \hat{K}_\Delta + \hat{K})\frac{E(R_n)}{E(\tilde{S}_n^{(I)})} + \hat{R} + 1 \quad . \qquad (D.9)$$

Conversion to slots of duration τ seconds is accomplished by multiplying the right hand side of (D.9) by

$$\frac{\tau_I}{\tau} = \frac{1 + \zeta\eta}{\zeta\eta}$$

and adding $1/2\zeta\eta$. Although new message arrivals at the source stations are recorded only at the start of slots, for a Poisson arrival stream, message arrivals within a slot experience an average $\frac{1}{2}$ slot delay. But the slot durations under the time and frequency division acknowledgment schemes differ by $\tau_I - \tau$ seconds. Hence,

$$\frac{\tau_I - \tau}{2\tau} \approx \frac{1}{2\zeta\eta} \tag{D.10}$$

must be added.

Q.E.D.

Expressions for the average holding times in the source station's buffer are stated in the following theorem.

Theorem D.1

With the operation of a controlled GRA channel under the frequency division acknowledgment schemes described by an irreducible, positive recurrent Markov state sequence Z, the limiting average packet delays (measured in slots of duration τ seconds) are given by

$$E(D_S) = \frac{P}{2} + \frac{(\hat{K}_\Delta + 2)(1 + \zeta\eta) + 1}{2\zeta\eta} + (\tilde{K}+P)\frac{E(R_n)}{E(\tilde{S}_n^{(I)})}$$

$$+ 2R + \frac{1 + \zeta\eta}{\eta} \tag{D.11}$$

under the FDA scheme and

$$E(D_S) = \frac{3}{2} P + \frac{3\hat{K}_\Delta(1 + \zeta\eta) + 1}{2\zeta\eta} + (\tilde{K}+P) \frac{E(R_n)}{E(\tilde{S}_n^{(I)})} + R + \frac{\tilde{K}(1 + \zeta)}{2}$$

$$+ \frac{(1 - \hat{K}_\Delta)(1+\zeta)(1 + \zeta\eta)}{2\zeta\eta} \qquad (D.12)$$

under the FDS scheme where the expectations are w.r.t. the stationary distribution.

Proof

Under the FDA scheme, a PACK is trasnmitted immediately following a collision-free information packet reception. Therefore,

$$E(D_S) = E(D_R) + \frac{\tau_A}{\tau} + R \qquad (D.13)$$

and (D.11) follows by substituting (D.8) into (D.13) with

$$\frac{\tau_A}{\tau} = \frac{1 + \zeta\eta}{\eta} .$$

Under the FDS scheme, the limiting average packet delay $E(\hat{D}_S)$, measured in slots of duration τ_I seconds, can be expressed as

$$E(\hat{D}_S) = E(\hat{D}_R) - 1 + \frac{E\{\hat{W}_{12}(Z_n, Z_{n+1})\}}{E(\tilde{S}_n^{(I)})} + \zeta \qquad (D.14)$$

where $\hat{W}_{12}(Z_n, Z_{n+1})$ is the sum of the waiting time components of all collision-free information packets transmitted within the n-th period. This waiting time is measured from the transmission slot within the n-th period to the PACK transmission slot within the (n+1)-st period:

$$\hat{W}_{12}(Z_n, Z_{n+1}) = \hat{K}\,\tilde{S}_{n1}^{(I)} + (\hat{K}-1)\,\tilde{S}_{n2}^{(I)} + \ldots + \tilde{S}_{n\hat{K}}^{(I)}$$

$$+ (\hat{P} + \hat{K}_\Delta)\,\tilde{S}_n^{(I)} + \zeta[T_{n+1,2}^{(A)} + 2T_{n+1,3}^{(A)} + \ldots$$

$$+ (\hat{K} - 1)\,T_{n+1,\hat{K}}^{(A)}] \,. \qquad \text{(D.15)}$$

Applying Lemma 4.1 to (D.15) with $N(Z_n, Z_{n+1}) = \tilde{S}_n^{(I)}$,

$$\frac{E\{\hat{W}(Z_n, Z_{n+1})\}}{E(\tilde{S}_n^{(I)})} = \hat{P} + \hat{K}_\Delta + \frac{\hat{K}(1+\zeta)}{2} + \frac{1-\zeta}{2} \,. \qquad \text{(D.16)}$$

Substituting (D.16) into (D.14), multiplying the result by τ_I/τ and adding $1/2\zeta\eta$ yields (D.12).

Q.E.D.

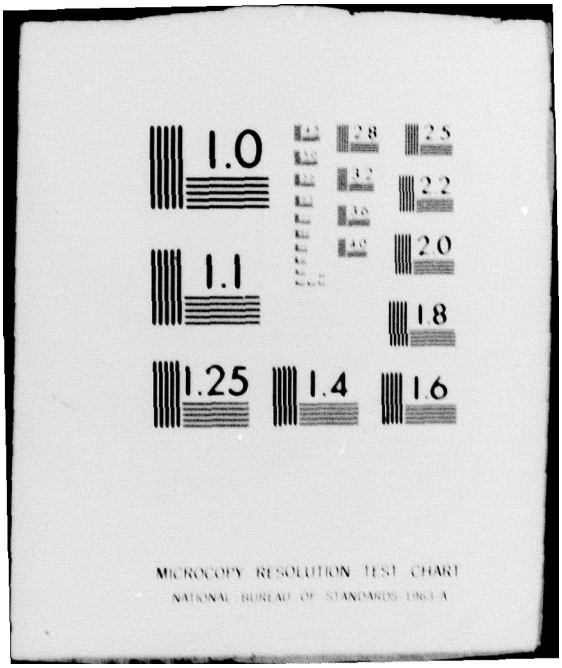
Since the operating statistics of the GRA channel under the FDA, FDS and PDS acknowledgment schemes are identical, the average number of information packet collisions per admitted message, $E(R_n)/E(\tilde{S}_n^{(I)})$, is the same for all three schemes. Thus the differences among the delay expressions can be evaluated in terms of the GRA structure parameters. The difference between the limiting average

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packet delay (data transfer delay) is

$$E(D_R)_{PDS} - E(D_R)_{\substack{FDA \\ FDS}} = \frac{\tilde{K}_I - 3}{2\zeta\eta} \quad . \tag{D.17}$$

Therefore, in general (i.e., $\tilde{K}_I \geq 3$),

$$E(D_R)_{FDA} = E(D_R)_{FDS} \leq E(D_R)_{PDS} \quad . \tag{D.18}$$

The average delay under the PDS acknowledgment scheme is larger, since new message arrivals during the acknowledgment subperiod experience an additional delay.

The differences among the limiting average packet delays (holding times) are given by the following expressions:

$$E(D_S)_{FDS} - E(D_S)_{FDA} = \tilde{K} + P - R - \frac{(1 + \zeta\eta)[(1 - \zeta)\hat{K} + (1 + \zeta)]}{2\zeta\eta} \tag{D.19}$$

$$E(D_S)_{PDS} - E(D_S)_{FDA} = P - R + \tilde{K}_\Delta + \frac{\tilde{K}_I(\zeta\eta + \zeta + 1)}{2\zeta\eta} - \frac{\zeta\eta(1 + 2\zeta) + \zeta + 3}{2\zeta\eta} \tag{D.20}$$

$$E(D_S)_{FDS} - E(D_S)_{PDS} = \hat{K} - \tilde{K}_\Delta + \tilde{K}_I\left(\frac{\zeta}{2} - \frac{1}{\zeta\eta} - 1\right) + \frac{2 + \zeta^2\eta}{2\zeta\eta} \quad . \tag{D.21}$$

Therefore,

$$E(D_S)_{FDA} \leq E(D_S)_{PDS} \leq E(D_S)_{FDS} \quad . \tag{D.22}$$

The FDA scheme yields the smallest average delay, since PACKs are transmitted immediately following the reception of collision-free information packet transmissions. Since the acknowledgment subperiod precedes the information subperiod under the PDS scheme and since $\tau/\eta \leq \tau_A$, the PDS scheme yields a smaller average delay than the FDS scheme.

D.3 Numerical Results

The average delay relationships expressed by (D.18) and (D.22) are valid under the restriction $\zeta_o \leq \zeta \leq 1$ (or $\tilde{K}_I = \hat{K}$). With $\tilde{K} = P = R = 12$, for example, $\zeta_o = 1$ when $\eta = 1$ and $\zeta_o = 5/9$ when $\eta = 9$. Both $E(D_S)$ and $E(D_R)$ increase with increasing values of ζ. The differences between the limiting average delays evaluated with $\zeta = 5/9$ and $\zeta = 1$, $\eta = 9$ are

$$E(D_R)_{\zeta=1} - E(D_R)_{\zeta=5/9} = 0.31 \qquad \text{under FDA or FDS}$$

$$E(D_S)_{\zeta=1} - E(D_S)_{\zeta=5/9} = \begin{cases} 0.75 & \text{under FDA} \\ 3.24 & \text{under FDS} \; . \end{cases}$$

Numerical data comparing the delay performance under the FDA, FDS

and PDS schemes are presented in Table D.1. These results agree with (D.18) and (D.22).

Average packet delay $E(D_S)$ versus probability of rejection P_R curves for the GRA channel under the FDA and DPDS acknowledgment schemes with $\tilde{K} = P = R = 12$ are shown in Figure D.3 with $\eta = 1$ and in Figure D.4 with $\eta = 9$. When $\eta = 1$, the performance under the DPDS scheme is superior to the performance under the FDA scheme for all new message arrival rates (λ). For example with $\lambda = 0.1$, the minimum rejection probability is ≈ 0.0 with a corresponding average delay of ≈ 55 slots under the DPDS scheme; under the FDA scheme, only a minimum rejection probability of ≈ 0.12 with a corresponding average delay of ≈ 64 slots can be achieved.

When $\eta = 9$, the performance under the FDA scheme is better than the performance under the DPDS scheme for small values of λ (i.e., $\lambda = 0.05, 0.1$). For these small new message arrival rates, the average number of information packet collisions per admitted message is small. Thus, the FDA protocol which allows immediate acknowledgment over the acknowledgment subchannel yields the lower average delays. For larger values of λ (i.e., $\lambda = 0.15$), the average number of information packet collisions per admitted message increases and it becomes the dominant term in the delay expressions. The adaptive acknowledgment slot allocations under the DPDS scheme yield the smaller number of information packet collisions per admitted message and, thus, the smaller delay.

TABLE D.1. DELAY PERFORMANCE COMPARISONS OF THE GRA CHANNEL UNDER THE FDA, FDS AND PDS ACKNOWLEDGMENT SCHEMES
$\tilde{K} = P = R = 12$

Acknowledgment Schemes	Delay Comparisons*		
	$\eta = 1$	$\eta = 9$	
	$\zeta = 1$	$\zeta = \zeta_o = 5/9$	$\zeta = 1$
$E(D_R)_{FDA} - E(D_R)_{FDS}$	0.0	0.0	0.0
$E(D_R)_{PDS} - E(D_R)_{FDA}$	1.5	0.7	0.39
$E(D_S)_{FDS} - E(D_S)_{FDA}$	10.0	8.4	10.89
$E(D_S)_{PDS} - E(D_S)_{FDA}$	5.5	6.03	5.28
$E(D_S)_{FDS} - E(D_S)_{PDS}$	4.5	2.37	5.61
$E(D_S - D_R)_{FDA}$	14.0	12.67	13.11
$E(D_S - D_R)_{FDS}$	24.0	21.07	24.0
$E(D_S - D_R)_{PDS}$	18.0	18.0	18.0

*Measured in slots of duration τ seconds.