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PRACTICAL ACCELEROMETER TESTING

by

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SUMMARY

In calibrating modern inertial quality accelerometers which are accurate to a few parts in a million, extreme care must be exercised in measurement technique and interpretation of the results. This Report analyses some of the tests which are performed currently and discusses the effects of errors in the test equipment.

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1 INTRODUCTION

Although an assessment laboratory for testing inertial navigation accelerometers has been in existence for over 20 years at RAE, there has been very little documentation of testing methods and philosophy. The major work entitled "Performance characteristics and methods of testing of force-feedback accelerometers"[1] appeared in 1967 and while this gives a comprehensive account of the underlying physical principles, the test methods described largely relate to the equipment available at the time, with emphasis on manual recording of data and an accuracy of the order of 1 part in 10^4. With the advent of digital data acquisition systems and rapid machine processing of data, many more tests are now performed than hitherto possible and some existing test methods have been improved. Furthermore, accelerometers are now available from a number of different manufacturers who claim accuracies in the order of a few parts in 10^6.

This Report is intended to provide a description and analysis of some of the test methods currently employed at RAE and to indicate where possible the magnitudes and sources of error likely to be encountered. Little will be said about the operating principles of force feedback accelerometers or the characteristics of the servo electronics since these aspects have been discussed in detail elsewhere[1,2]. However, it is assumed that the instruments are pendulous devices with a single degree of freedom, in which the proof mass is captured in a sufficiently tight servo loop so that errors associated with transverse accelerations are small. The pendulum is supported by means of flexure hinges or pivot and jewel arrangements. Non-pendulous devices or those with more than one degree of freedom are not so commonly available, although much of the present analysis may be extended to include these.

There are several tests, for example those which measure the properties of the flexure or pick-off, which indicate that the instrument has been built to the required design standard and is capable of functioning correctly. They do not, however, measure performance under normal operating conditions and are open loop tests requiring access to the pick-off. Many of today's accelerometers are supplied as packaged units comprising integrated electronics and mechanical sensor and no provision is made for the user to obtain access to monitor test points within the unit. Since this Report is concerned mainly with operational performance and such tests have already been described adequately in Ref 1, they will not be considered further.

In most analogue instruments, the output quantity is the current flowing in
149 the restoring coil which is usually measured directly. Some accelerometers,

however, are designed to give a voltage output and this may be influenced by the stability of the built-in load resistor. Digital instruments also exist which employ a pulse torque rebalance loop and the output is then a series of pulses. So far, no instruments of this type have been tested in this laboratory although most of the analysis which follows is still applicable. The output is assumed to be some reasonable function of the input acceleration and will almost certainly be affected by changes in the environmental or operating conditions.

2 PRINCIPLES OF TESTING

It is standard practice to perform a large part of accelerometer assessment using earth's gravity as the input. In spite of the fact that the range of most instruments (typically ±20g) is much greater than the ±1g available from earth's field testing, a large amount of useful information can be gained. Experience in several test laboratories, mainly in the USA, has indicated that extrapolated results from ±1g tests usually compare favourably with those using a precision centrifuge to provide the necessary high-g levels. In any case, it is very difficult to eliminate or correct for all the errors associated with practical centrifuges.

In order to evaluate and compare units from the present generation of precision accelerometers, both output current and input acceleration must be known to a high degree of accuracy. The instrument generally used at RAE to measure output is a digital integrating milliammeter[3] whose resolution can be as good as the equivalent of 0.1 μg for sufficiently long integration periods. Such accuracy is rarely required except for the most precise stability tests, and in most cases resolutions of 1 or 10 μg are accepted so that shorter integration times can be employed. To determine the input, it is usual to place the accelerometer at precisely known angles with respect to the gravity vector by mounting it securely on a fixture attached to the faceplate of an accurate dividing head. The Leitz dividing heads used at RAE and several other test centres have an angular setting accuracy of about ±1 arc second which corresponds to about 5 μg when considered as a deviation from the horizontal plane. The precision with which the dividing head is initially set up and the orthogonality of the mounting fixture are important considerations and the errors associated with these are discussed in section 4.

Account must be taken of any external disturbances which are likely to affect the performance of a test. Temperature is usually the dominant influence and since it is difficult to control the test environment to better than 1°C, the temperature at the accelerometer should be monitored continuously and used to correct the raw data if necessary. In this laboratory, nickel resistance temperature sensors are generally employed in conjunction with digital thermometers having a resolution of 0.01°C in the range −10 to +120°C [4]. It is usually found that there is excellent correlation between temperature and accelerometer output at a fixed attitude and thus corrections can be made with a high degree of confidence.

Tests should be repeated a number of times under the same conditions to enable an estimate to be made of the test uncertainty. An example is the four-point test in which the output is measured at the two 0g positions and the two 1g positions as the accelerometer is rotated about one of its non-sensitive axes (see section 3.2). It is usual practice to perform this test four times with a repeat reading of the +1g output during each revolution. The mean values and standard deviations of the parameters to be measured are calculated together with the in-test rms deviation which estimates the test accuracy. Should this value be unacceptably large, then the results should be rejected if there is any suspicion that the operator or measuring equipment were at fault. It is damaging to the reputation of the laboratory and unfair to the manufacturer if his accelerometer is blamed for imprecise or faulty test methods.

It is also good practice to determine performance parameters using different test procedures. For example, temperature coefficients can be measured by performing tests at fixed temperatures within a range, or by establishing steady conditions at a high temperature and recording the cooling curves at different attitudes. Similar results give added confidence in the values obtained.

The introduction of automatic data acquisition and processing has resulted in a number of significant improvements in accelerometer assessment, which include

(a) less reliance on operator skill and virtual elimination of arithmetical mistakes;

(b) a greatly increased number of tests and hence better estimates of scatter and drift;

(c) more reliable output stability information over long periods of time with automatic correction for temperature;

(d) the ability to analyse test data obtained at a large number of intermediate g-levels and hence more accurate measurements;

(e) the collection of statistical information which enables levels of confidence to be established and predictions to be made about long term behaviour;

149 (f) automatic graph plotting.

At present, the output from digital instruments (milliammeters, thermometers, etc) is in parallel BCD format and the data from a number of instruments is collected, reformatted and then transferred on to punched paper tape for subsequent computer analysis. Further improvements are envisaged in which the instrumentation would be linked together via a standard interface bus to a digital controller and the paper tape would be replaced by some form of magnetic storage, either cartridge or disc. A more distant aim is to automate the dividing head setting to enable the majority of testing to be performed without manual intervention.

3 DERIVATION OF ACCELEROMETER PARAMETERS

3.1 The model equation

The output of an ideal accelerometer, expressed in units of 'g' can be stated as

$$I = K_1 a_s \qquad (3\text{-}1)$$

where K_1 is the scale factor and a_s is the component of acceleration along the sensitive axis. In practice, the situation is more complex. There will be, in general, a non-zero output when no accelerations are acting (bias); small misalignments between the nominal and true accelerometer axes will be present; the output may not be a strictly linear function of acceleration and there may be a slight sensitivity to accelerations perpendicular to the sensitive axis. There may also be additional errors due to cross coupling. The choice of model equation depends to a certain extent on the type of accelerometer to be tested. Digital instruments employing pulse rebalance electronics sometimes have asymmetrical scale factors and biases because of the separate circuits used to generate the positive and negative pulses. The same may be true of accelerometers which are not symmetrical about their pendulum axis, for example those with only one restoring coil and magnet rather than two. However an equation which has been found to model realistically the output of most accelerometers tested at RAE is

$$I = K_0 + K_1 a_s + K_2 a_s^2 + K_3 a_s^3 + \delta_1 a_p + \delta_2 a_h + K_{sp} a_s a_p + K_{sh} a_s a_h \qquad (3\text{-}2)$$

where K_0 is the bias, K_2 and K_3 are non-linearity coefficients, δ_1 and δ_2 are transverse sensitivity coefficients, and K_{sp}, K_{sh} are the cross-coupling coefficients. The components a_s, a_p and a_h are the accelerations along the true sensitive, pendulum and hinge axes respectively according to an orthogonal system in which $\underline{s} \times \underline{p} = \underline{h}$ (see Fig 1a). The coefficients in equation (3-2) may themselves be functions of the environment or operating conditions. They are therefore likely to change with changes in temperature, amplifier gain, mechanical shock, etc.

In order to determine the coefficients in equation (3-2), the accelerometer is set in various orientations with respect to earth's gravity and the output is measured under constant environmental conditions. This orientation is determined with respect to an orthogonal set of reference (or nominal) axes which are usually defined by the accelerometer structure. Most accelerometers are provided with a lapped mounting surface containing three or four fixing holes, two of which are nominally parallel to the hinge or pendulum axes. In a typical case, we may define the nominal sensitive axis as being perpendicular to the plane of the mounting surface and the nominal hinge axis as being in this plane and joining the centres of the two appropriate fixing holes. The nominal pendulum axis is then perpendicular to both of these. We shall use primed quantities (a'_s, a'_p, a'_n) to denote acceleration components referred to the nominal coordinate frame.

Consider an orthogonal set of axes fixed in space (Fig 1b) such that y_0 is vertically down (parallel to the gravity vector), z_0 is horizontal and in the vertical plane containing the spindle of the dividing head, and x_0 is perpendicular to y_0 and z_0. Then $a_{x_0} = a_{z_0} = 0$ and $a_{y_0} = 1$. For the time being we shall make four assumptions, the validity of which will be tested in the next section:

(a) the dividing head is error-free;

(b) its axis of rotation is accurately level;

(c) the accelerometer is mounted so that one of its nominal axes is parallel to the axis of rotation of the dividing head;

(d) in the reference ($\theta = 0$) position, the other nominal axes are precisely horizontal and vertical.

Then, if x', y' and z' are the nominal accelerometer axes, and the dividing head is rotated through an angle θ, then

$$\begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} . \qquad (3\text{-}3)$$

Note that the dividing head is mechanised so that the direction of increasing θ is as shown in Fig 1, and that accelerometers are conventionally tested by rotating them in the sense +1g through the hinge-up and −1g positions to the hinge-down position. If the accelerometer is mounted so that its nominal hinge axis is parallel to the axis of rotation, which is the most common configuration for testing and referred to here as mounting position 1, then

$$a'_x = a'_s = -\sin\theta \tag{3-4}$$

$$a'_y = a'_p = \cos\theta \tag{3-5}$$

$$a'_z = a'_h = 0 \; . \tag{3-6}$$

In general, the nominal and true axes will not be coincident and an orthogonal transformation can be set up which relates the two frames of reference. Let the transformation from true to nominal axes be described by three sequential rotations; the first through an angle α about the (true) hinge axis; the second through an angle β about the pendulum axis generated by the first rotation and the last through an angle γ about the now nominal sensitive axis. This yields the transformation matrix, $[C]$ =

$$\begin{bmatrix} \cos\alpha\cos\beta & \sin\alpha\cos\beta & -\sin\beta \\ -\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma & \cos\alpha\cos\gamma + \sin\alpha\sin\beta\sin\gamma & \cos\beta\sin\gamma \\ \sin\alpha\sin\gamma + \cos\alpha\sin\beta\cos\gamma & -\cos\alpha\sin\gamma + \sin\alpha\sin\beta\cos\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

in the equation

$$\underline{a}' = [C]\underline{a}$$

where $\underline{a}$ and $\underline{a}'$ are vectors representing the true and nominal axes respectively. Since the matrix is orthogonal, the inverse transformation is

$$\underline{a} = [C]^{-1}\underline{a}' = [C]^T\underline{a}' \tag{3-7}$$

where $[C]^T$ is the transposed matrix. The transformation has been carried out in this way so that the generally accepted misalignment angles are introduced in their correct sequence; α and β are the misalignments of the true and nominal sensitive axes about the hinge and pendulum axes respectively and γ represents a misalignment about the sensitive axis. For an accelerometer to be of inertial quality the misalignments must be small. If the angles are less than about 3 arc minutes, then an accuracy of 1 part in 10^6 is retained if we take first order approximations. Hence

$$[C]^T = \begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & -\gamma \\ -\beta & \gamma & 1 \end{bmatrix} \qquad (3\text{-}8)$$

and using equations (3-4) and (3-8), we have

$$a_s = -\sin\theta - \alpha\cos\theta \qquad (3\text{-}9)$$

$$a_p = -\alpha\sin\theta + \cos\theta \qquad (3\text{-}10)$$

$$a_h = \beta\sin\theta + \gamma\cos\theta \quad . \qquad (3\text{-}11)$$

Substituting equations (3-9) to (3-11) into the model equation (3-2) and neglecting products of small quantities such as $K_2\alpha$, $K_3\alpha$, $\delta\alpha$, etc; we have (where the additional subscript is used to denote mounting position 1)

$$I_{\theta 1} = K_{01} - K_{11}\sin\theta + K_{21}\sin^2\theta - K_{31}\sin^3\theta$$

$$+ (\delta_1 - K_{11}\alpha)\cos\theta - \tfrac{1}{2}K_{sp}\sin 2\theta \quad . \qquad (3\text{-}12)$$

Thus small misalignments about axes other than that perpendicular to the plane of rotation do not produce any significant errors in the measurement of the coefficients.

A similar relationship holds for mounting position 2 in which the nominal pendulum axis is maintained parallel to the axis of rotation. In this case

$$a'_x = a'_h = -\sin\theta$$

$$a'_y = a'_s = \cos\theta$$

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$$a'_z = a'_p = 0$$

and the components of acceleration along the true axes are,

$$a_s = \cos\theta - \beta \sin\theta$$

$$a_p = \alpha \cos\theta + \gamma \sin\theta$$

$$a_h = -\beta \cos\theta - \sin\theta \ .$$

Equation (3.2) becomes

$$I_{\theta 2} = K_{02} + K_{12} \cos\theta + K_{22} \cos^2\theta + K_{32} \cos^3\theta$$

$$- (\delta_2 + K_{12}\beta) \sin\theta - \tfrac{1}{2}K_{sh} \sin 2\theta \qquad (3\text{-}13)$$

where the extra subscript is used to denote mounting position 2. A third orthogonal mounting position is possible, *ie* that in which the sensitive axis is parallel to the axis of rotation. However, this arrangement is not often used in practice since the results are much more sensitive to errors in the dividing head than they are in the two standard mounting positions (see section 4). Assuming no errors, the output in position 3 is given by

$$I_{\theta 3} = K_{03} - (\delta_1 - K_{13}\alpha) \sin\theta + (\delta_2 + K_{13}\beta) \cos\theta \ . \qquad (3\text{-}14)$$

3.2 Four-point tests

The main performance parameters can be deduced from simple tests in which the accelerometer output is measured at each of four cardinal positions. Assuming the unit is in mounting position 1 and using HU and HD to denote the zero-g hinge-up and hinge-down attitudes respectively, we have the following set of equations.

0g (HU)	$I_0 = K_{01} + (\delta_1 - K_{11}\alpha)$
-1g	$I_{90} = K_{01} - K_{11} + K_{21} - K_{31}$
0g (HD)	$I_{180} = K_{01} - (\delta_1 - K_{11}\alpha)$
+1g	$I_{270} = K_{01} + K_{11} + K_{21} + K_{31}$.

Hence

$$K_{01} = \tfrac{1}{2}(I_0 + I_{180}) = B1_1 \quad \text{(bias at 0g)} \qquad (3\text{-}15)$$

$$K_{01} + K_{21} = \tfrac{1}{2}(I_{90} + I_{270}) = B2_1 \quad \text{(bias at 1g)} \qquad (3\text{-}16)$$

$$K_{21} = B2_1 - B1_1 = BD_1 \quad \text{(bias discrepancy)} \qquad (3\text{-}17)$$

$$\delta_1 - K_{11}\alpha = \tfrac{1}{2}(I_0 - I_{180}) = AE \quad \text{(axis error)} \qquad (3\text{-}18)$$

$$K_{11} + K_{31} = \tfrac{1}{2}(I_{90} - I_{270}) \approx SF_1 \quad \text{(scale factor) .} \qquad (3\text{-}19)$$

There are two points to note here. The first is that it is not possible to separate the effects of any inherent transverse sensitivity from true axis misalignments. What we measure, therefore, is a lumped parameter known as axis error, or more correctly, the component of axis error in the plane containing the sensitive and pendulum axes. The second point is that the difference between the +1g and -1g readings gives the scale factor plus a small additional term due to third order non-linearity. In most cases, this quantity is so small as to be negligible, and the value measured represents scale factor to an adequate degree of accuracy. In mounting position 2, the relationships are

$$K_{02} = \tfrac{1}{2}(I_{90} + I_{270}) = B1_2 \quad \text{(bias at 0g)} \qquad (3\text{-}20)$$

$$K_{02} + K_{22} = \tfrac{1}{2}(I_0 + I_{180}) = B2_2 \quad \text{(bias at 1g)} \qquad (3\text{-}21)$$

$$K_{22} = B2_2 - B1_2 = BD_2 \quad \text{(bias discrepancy)} \qquad (3\text{-}22)$$

$$-\delta_2 - K_{12}\beta = \tfrac{1}{2}(I_{90} - I_{270}) = HAE \quad \text{(hinge axis error)} \qquad (3\text{-}23)$$

$$K_{12} + K_{32} = \tfrac{1}{2}(I_0 - I_{180}) \approx SF_2 \quad \text{(scale factor) .} \qquad (3\text{-}24)$$

Again, equation (3-23) shows that what we measure is a parameter described as the component of axis error in the plane containing the sensitive and hinge axes. Some authorities refer to this as the hinge axis error.

The values of the other parameters should be the same whether measured in mounting positions 1 or 2. In practice, because of dividing head errors and other small misalignments considered negligible in the above analysis, small differences may be found, but should be insignificant in any reasonable application. It should also be noted that the cross-coupling coefficients do not appear in the analysis of four-point tests.

3.3 Multiposition tests

The four-point test only provides information at the zero-g and two extreme 1g levels. While this is not usually a serious limitation for routine day-to-day assessment, better estimates of the performance parameters are obtained if they are derived from data taken at a large number of intermediate g-levels as well as the extreme values. In a multiposition test, the output is recorded at different angles and regression analysis is used to fit the data to the chosen model equation. An additional advantage is that the validity of the model can be tested by examining a plot of the residuals, and if necessary the data can be fitted to a different model, for example one using asymmetrical scale factors.

In the following analysis we shall assume that the model equation (3-2) is appropriate and that the accelerometer is in mounting position 1. Then if $\bar{I}_k$ denotes the best fit value of output current determined from the theoretical curve at position k, equation (3-12) becomes

$$\bar{I}_k = K_{01} - K_{11}(-\sin\theta_k + \alpha\cos\theta_k) + K_{21}\sin^2\theta_k - K_{31}\sin^3\theta_k + \delta_1\cos\theta_k - \tfrac{1}{2}K_{sp}\sin 2\theta_k \quad . \qquad (3\text{-}25)$$

If I_k denotes the measured output at position k, then we use the method of least mean squares to minimise the sum of the squares of the residuals with respect to each unknown coefficient we wish to determine; *ie* we need to minimise

$$\sum_{\text{all } k} r_k^2 = \sum (I_k - \bar{I}_k)^2$$

$$= \sum \left[I_k - K_{01} + K_{11}(\sin\theta_k + \alpha\cos\theta_k) - \ldots\ldots + \tfrac{1}{2}K_{sp}\sin 2\theta_k \right]^2 \qquad (3\text{-}26)$$

with respect to K_{01}, K_{11}, K_{21}, K_{31}, δ, K_{sp} and α. We therefore form the partial derivatives of equation (3-26) with respect to each unknown and set these equal to zero. It might be argued, with some justification, that the θ_k should be chosen to produce approximately equal increments of applied acceleration. If this scheme is employed, however, the computations are extremely involved, requiring the inversion of an $m \times m$ matrix where m is the number of unknown 14 149 coefficients. To simplify the data reduction procedure, we shall assume that $\theta_k = k\pi/2n$ where $n = 1, 2, 3, \ldots\ldots,$ and $k = 0, 1, 2 \ldots\ldots (4n - 1)$. This means that we perform an N-position test, where N is a multiple of 4, and we use equal angular increments.

Differentiating equation (3-26) with respect to K_{01}, we have

$$\frac{\partial}{\partial K_{01}} \sum r_k^2 = 2 \sum \left[(I_k - \bar{I}_k) \frac{\partial}{\partial K_{01}} (I_k - \bar{I}_k) \right] = 0$$

$$= 2 \sum \left[I_k - K_{01} + K_{11} \sin\left(\frac{k\pi}{2n}\right) + \ldots\ldots + \tfrac{1}{2} K_{sp} \sin\left(\frac{k\pi}{n}\right) \right] (-1) ,$$

where all summations are taken from $k = 0$ to $k = 4n - 1$.

Therefore
$$\sum I_k = \sum K_{01} - K_{11} \sum \sin\left(\frac{k\pi}{2n}\right) - \alpha K_{11} \sum \cos\left(\frac{k\pi}{2n}\right)$$
$$+ K_{21} \sum \sin^2\left(\frac{k\pi}{2n}\right) - K_{31} \sum \sin^3\left(\frac{k\pi}{2n}\right)$$
$$+ \delta_1 \sum \cos\left(\frac{k\pi}{2n}\right) - \tfrac{1}{2} K_{sp} \sum \sin\left(\frac{k\pi}{n}\right) .$$

This simplifies to

$$\sum I_k = 4nK_{01} + 2nK_{21} . \qquad (3\text{-}27)$$

When we differentiate equation (3-26) with respect to all the other unknown quantities and simplify, we obtain the following expressions;

$$\sum I_k \sin\left(\frac{k\pi}{2n}\right) + \alpha \sum I_k \cos\left(\frac{k\pi}{2n}\right) = -2nK_{11} - \frac{3n}{2} K_{31} \qquad (3\text{-}28)$$

$$\sum I_k \sin^2\left(\frac{k\pi}{2n}\right) = 2nK_{01} + \frac{3n}{2} K_{21} \qquad (3\text{-}29)$$

$$\sum I_k \sin^3\left(\frac{k\pi}{2n}\right) = -\frac{3n}{2} K_{11} - \frac{5n}{4} K_{31} \qquad (3\text{-}30)$$

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$$\sum I_k \cos\left(\frac{k\pi}{2n}\right) = 2n(\delta_1 - K_{11}\alpha) \tag{3-31}$$

$$\sum I_k \sin\left(\frac{k\pi}{n}\right) = -nK_{sp} \quad . \tag{3-32}$$

These expressions, with the exception of equation (3-31), are not valid for $n = 1$, *ie* a four-point test. Note that equation (3-31) is obtained when we differentiate with respect to either δ_1 or α , again showing that it is not possible to separate these coefficients. Equations (3-28) to (3-32) reduce to

$$K_{01} = \frac{1}{4n} \sum I_k \left\{1 + 2\cos\left(\frac{k\pi}{n}\right)\right\} \tag{3-33}$$

$$K_{11} = -\frac{1}{n} \sum I_k \sin\left(\frac{k\pi}{2n}\right) \left\{2 + 3\cos\left(\frac{k\pi}{n}\right)\right\} \tag{3-34}$$

$$K_{21} = -\frac{1}{n} \sum I_k \cos\left(\frac{k\pi}{n}\right) \tag{3-35}$$

$$K_{31} = \frac{2}{n} \sum I_k \sin\left(\frac{k\pi}{2n}\right) \left\{1 + 2\cos\left(\frac{k\pi}{n}\right)\right\} \tag{3-36}$$

$$K_{sp} = -\frac{1}{n} \sum I_k \sin\left(\frac{k\pi}{n}\right) \tag{3-37}$$

$$\delta_1 - K_{11}\alpha = \frac{1}{2n} \sum I_k \cos\left(\frac{k\pi}{2n}\right) \quad . \tag{3-38}$$

The above equations represent the best estimates of the coefficients in equation (3-12) when measured from an N-position test, where $N = 8, 12, 16, 20, \ldots,$. Since K_{31} and K_{sp} should be very small, it is unlikely that any test over the narrow ±1g range would produce meaningful values. Should they in fact be significantly greater than the test uncertainty as measured by the standard deviation of the residuals, then the test would almost certainly indicate a poor or faulty accelerometer.

An identical analysis holds for mounting position 2 which allows measurements to be made of the hinge axis error and of the sensitive/hinge axes cross coupling coefficient.

4 DIVIDING HEAD AND LEVELLING ERRORS

It has been assumed so far that there have been no errors associated with the dividing head and mounting fixture. In practice, of course, this is not the case and in this section, an attempt is made to identify those errors which exist and to determine the magnitudes that can be tolerated while still maintaining the required measurement accuracy.

Let us assume that, instead of lying along the z_0 (horizontal) axis, the axis of rotation of the dividing head is inclined at a small angle ε_1 to the horizontal (see Fig 2). Then the acceleration components along the x_1 y_1 z_1 axes are

$$\begin{bmatrix} a_{x_1} \\ a_{y_1} \\ a_{z_1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon_1 & \sin \varepsilon_1 \\ 0 & -\sin \varepsilon_1 & \cos \varepsilon_1 \end{bmatrix} \begin{bmatrix} a_{x_0} \\ a_{y_0} \\ a_{z_0} \end{bmatrix} \qquad (4\text{-}1)$$

where $\left\{a_{x_0}, a_{y_0}, a_{z_0}\right\} = \{0, 1, 0\}$. If the $\theta = 0$ position is not precisely horizontal, but misaligned by a small angle ε_2, the new acceleration components are given by

$$\begin{bmatrix} a_{x_2} \\ a_{y_2} \\ a_{z_2} \end{bmatrix} = \begin{bmatrix} \cos \varepsilon_2 & -\sin \varepsilon_2 & 0 \\ \sin \varepsilon_2 & \cos \varepsilon_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{x_1} \\ a_{y_1} \\ a_{z_1} \end{bmatrix} . \qquad (4\text{-}2)$$

This is the situation when the accelerometer has been nominally levelled ($\theta = 0^\circ$), where $\left\{a_{x_2}, a_{y_2}, a_{z_2}\right\}$ are the acceleration components along the 3 nominal axes. As the dividing head spindle is rotated through any angle θ, the components are:

$$\begin{bmatrix} a_{x_3} \\ a_{y_3} \\ a_{z_3} \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{x_2} \\ a_{y_2} \\ a_{z_2} \end{bmatrix} \qquad (4\text{-}3)$$

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and combining equations (4-1), (4-2) and (4-3) we have

$$\begin{bmatrix} a_{x_3} \\ a_{y_3} \\ a_{z_3} \end{bmatrix} = \begin{bmatrix} \cos(\theta + \varepsilon_2) & -\sin(\theta + \varepsilon_2)\cos\varepsilon_1 & -\sin(\theta + \varepsilon_2)\sin\varepsilon_1 \\ \sin(\theta + \varepsilon_2) & \cos(\theta + \varepsilon_2)\cos\varepsilon_1 & \cos(\theta + \varepsilon_1)\sin\varepsilon_1 \\ 0 & -\sin\varepsilon_1 & \cos\varepsilon_1 \end{bmatrix} \begin{bmatrix} a_{x_0} \\ a_{y_0} \\ a_{z_0} \end{bmatrix} .$$

Hence

$$a_{x_3} = -\sin(\theta + \varepsilon_2)\cos\varepsilon_1 \qquad \text{(A-4)}$$

$$a_{y_3} = \cos(\theta + \varepsilon_2)\cos\varepsilon_1 \qquad \text{(4-5)}$$

$$a_{z_3} = -\sin\varepsilon_1 \ . \qquad \text{(4-6)}$$

This is usually as far as the analysis is taken and it shows that, provided the accelerometer is mounted in a position such that its nominal sensitive axis rotates in the xy plane, only the misalignment ε_2 contributes significantly to errors in measuring the performance parameters, specifically the axis error. In practice, care is taken to reduce ε_2 to within about 2 arc seconds, whereas an angular misalignment of a few arc minutes in ε_1 can be tolerated before noticeable measurement errors are introduced. When the sensitive axis, however, is aligned along the z-direction, dividing head and mounting block inaccuracies are significant as we shall see below.

Two further misalignments can be introduced as follows. An accelerometer mounting fixture is usually designed to allow one of the nominal axes to be aligned parallel to the axis of rotation of the dividing head. As a result of machining limitations or setting up inaccuracies, there can be a misalignment ε_3 between this accelerometer nominal axis and the dividing head spindle about the x-direction and an additional misalignment ε_4 about the y-direction (see Fig 2). The way in which these errors arise depends upon the detailed fixing arrangements, but in a typical case a plane lapped surface is provided with a pattern of fixing holes on to which the accelerometer is bolted. A skew error ε_3 will be present if this plane is not precisely perpendicular to the dividing head faceplate; or indeed if the faceplate is not perpendicular to the spindle axis. If, in addition, the mounting hole configuration does not permit precise alignment between the 149
required accelerometer nominal axis and the axis of rotation, then a skew error

ε_4 will arise. In some circumstances, ε_4 can be relatively large (~ 20 arc minutes) since there is usually a certain slackness in the fixing holes and the adjustment is done by eye. It is worth noting also that significant errors may still be present in a system which appears to be perfectly aligned. For example, the accelerometer mounting surface may be adjusted to be precisely horizontal at one particular dividing head setting, say $\theta = 0^{\circ}$, and it would be tempting to believe that all errors had been reduced to zero. However, the same result would have been obtained if the axis of rotation had not been level and the mounting surface had been skewed by an equal but opposite amount, *ie* $\varepsilon_1 = -\varepsilon_3$. In this case gross errors might result in the measurements of the performance parameters. It is therefore important to ensure that the dividing head is set up to a sufficient degree of accuracy before attaching any mounting fixtures, and that reasonably tight tolerances are applied in the manufacture of such fixtures.

If x_4, y_4, z_4 and x_5, y_5, z_5 are the axes after rotation through ε_3 and ε_4, we have for the acceleration components:

$$\begin{bmatrix} a_{x_4} \\ a_{y_4} \\ a_{z_4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon_3 & \sin\varepsilon_3 \\ 0 & -\sin\varepsilon_3 & \cos\varepsilon_3 \end{bmatrix} \begin{bmatrix} a_{x_3} \\ a_{y_3} \\ a_{z_3} \end{bmatrix} \qquad (4\text{-}7)$$

and

$$\begin{bmatrix} a_{x_5} \\ a_{y_5} \\ a_{z_5} \end{bmatrix} = \begin{bmatrix} \cos\varepsilon_4 & 0 & -\sin\varepsilon_4 \\ 0 & 1 & 0 \\ \sin\varepsilon_4 & 0 & \cos\varepsilon_4 \end{bmatrix} \begin{bmatrix} a_{x_4} \\ a_{y_4} \\ a_{z_4} \end{bmatrix} . \qquad (4\text{-}8)$$

This final transformation gives the components of acceleration along the accelerometer nominal axes after taking into account all the levelling and mounting misalignments which should remain constant during a test in which the instrument is not removed from its fixture. No account has been taken of test plinth instability caused by random vibrations or temperature changes, but these effects are largely unpredictable and should be negligibly small in any worthwhile test facility. It has also been assumed that the dividing head is of such precision that backlash in the gearing can be ignored.

Since primed quantities are being used to denote accelerations along the nominal axes, we have $\left\{a_{x_5}, a_{y_5}, a_{z_5}\right\} = \left\{a'_x, a'_y, a'_z\right\}$ and equations (4-1) to (4-8) can be combined to give

$$a'_x = -\sin(\theta+\varepsilon_2)\cos\varepsilon_1\cos\varepsilon_4 + \cos(\theta+\varepsilon_2)\cos\varepsilon_1\sin\varepsilon_3\sin\varepsilon_4 + \sin\varepsilon_1\cos\varepsilon_3\sin\varepsilon_4 \qquad (4\text{-}9)$$

$$a'_y = \cos(\theta+\varepsilon_2)\cos\varepsilon_1\cos\varepsilon_3 - \sin\varepsilon_1\sin\varepsilon_3 \qquad (4\text{-}10)$$

$$a'_z = -\sin(\theta+\varepsilon_2)\cos\varepsilon_1\sin\varepsilon_4 - \cos(\theta+\varepsilon_2)\cos\varepsilon_1\sin\varepsilon_3\cos\varepsilon_4 - \sin\varepsilon_1\cos\varepsilon_3\cos\varepsilon_4 \ . \qquad (4\text{-}11)$$

Note that if there are no errors, these reduce to equation (3-3).

To retain an accuracy of 1 part in 10^6, we may take first order angle approximations only if the angles are less than about 3 arc minutes. Although this is true of ε_2 (which we always take care to reduce to near-zero) it is not necessarily the case with ε_1, ε_3 and ε_4. However, these angles will almost certainly be less than $\frac{1}{2}^{\circ}$ and there is no loss of accuracy if we make the approximations

$$\sin\varepsilon_i \sim \varepsilon_i \quad ; \quad \cos\varepsilon_i \simeq 1 - \varepsilon_i^2/2 \quad (i = 1, 3, 4) \ .$$

Equations (4-9), (4-10) and (4-11) then become

$$a'_x = -\left(1 - \frac{\varepsilon_1^2}{2} - \frac{\varepsilon_4^2}{2}\right)\sin\theta - (\varepsilon_2 - \varepsilon_3\varepsilon_4)\cos\theta + \varepsilon_1\varepsilon_4 \qquad (4\text{-}12)$$

$$a'_y = -\varepsilon_2\sin\theta + \left(1 - \frac{\varepsilon_1^2}{2} - \frac{\varepsilon_2^2}{2}\right)\cos\theta - \varepsilon_1\varepsilon_3 \qquad (4\text{-}13)$$

$$a'_z = -\varepsilon_4\sin\theta - \varepsilon_3\cos\theta - \varepsilon_1 \ . \qquad (4\text{-}14)$$

To determine the acceleration components along the true axes, we use the transformation (3-8). Assuming first that the accelerometer is mounted in position 1, *ie* its hinge axis is nominally parallel to the axis of rotation, we have, after making the appropriate small angle approximations:

$$a_s = -\left(1 - \frac{\varepsilon_1^2}{2} - \frac{\varepsilon_4^2}{2}\right) \sin\theta - \left(\alpha + \varepsilon_2 - \varepsilon_3\varepsilon_4\right) \cos\theta + \varepsilon_1\varepsilon_4 \qquad (4\text{-}15)$$

$$a_p = -\left(\alpha + \varepsilon_2\right) \sin\theta + \left(1 - \frac{\varepsilon_1^2}{2} - \frac{\varepsilon_3^2}{2}\right) \cos\theta + \varepsilon_1\varepsilon_3 \qquad (4\text{-}16)$$

$$a_n = \left(\beta - \varepsilon_4\right) \sin\theta + \left(\gamma - \varepsilon_3\right) \cos\theta - \varepsilon_1 \quad . \qquad (4\text{-}17)$$

Hence, substituting equations (4-15), (4-16) and (4-17) into the model equation (3-2),

$$I_\theta = K_0 + K_1\varepsilon_1\varepsilon_4 + K_1\left(1 - \frac{\varepsilon_1^2}{2} - \frac{\varepsilon_4^2}{2}\right) \sin\theta + K_2 \sin^2\theta - K_3 \sin^3\theta$$

$$+ \left(\delta_1 - K_1\alpha - K_1\varepsilon_2 + K_1\varepsilon_3\varepsilon_4\right) \cos\theta - \tfrac{1}{2}K_{sp} \sin\theta \cos\theta \ . \qquad (4\text{-}18)$$

Comparing equation (4-18) with (3-12), it can be seen that the measured bias differs from the true bias by an amount $K_1\varepsilon_1\varepsilon_4$, the difference in the scale factors is $K_1\left(\varepsilon_1^2/2 + \varepsilon_4^2/2\right)$ and the axis errors differ by $K_1\left(\varepsilon_2 - \varepsilon_3\varepsilon_4\right)$. The first two quantities should be very small, depending as they do on products of small angles. If we assume $\varepsilon_1 \sim \varepsilon_4 \sim 10$ arc minutes (3×10^{-3} radians) then the bias will be in error by about 9 μg and the scale factor by about 9 ppm. These may be considered as acceptably small errors in normal testing; furthermore, ε_1 is more likely to be less than 2 arc minutes (6×10^{-4} radians), reducing the bias and scale factor errors to < 2 μg and < 5 ppm respectively. The dominant contributor to the axis error uncertainty is the $K_1\varepsilon_2$ term which shows that great care must be taken to level the mounting surface in the plane of rotation. An error $\varepsilon_2 = 2$ arc seconds is equivalent to an axis error change of 10 μg/g .

A similar analysis holds for mounting position 2 in which the pendulum axis is kept nominally parallel to the axis of rotation. Again the dominant error
.49 angle is ε_2 which determines the accuracy with which the appropriate axis error

is measured. Provided that the other dividing head and mounting fixture errors are kept below a few arc minutes, they do not affect the precision with which accelerometer parameters are determined.

The same argument, however, is not true for the third orthogonal mounting position; that in which the accelerometer is rotated about its sensitive axis. In this case, the acceleration components along the true axes are

$$a_s = (\alpha - \varepsilon_4)\sin\theta + (\beta - \varepsilon_3)\cos\theta - \varepsilon_1 \qquad (4\text{-}19)$$

$$a_p = -\left(1 - \frac{\varepsilon_1^2}{2} - \frac{\varepsilon_4^2}{2}\right)\sin\theta - (\gamma + \varepsilon_2 - \varepsilon_3\varepsilon_4)\cos\theta + \varepsilon_1\varepsilon_4 \qquad (4\text{-}20)$$

$$a_n = -(\gamma + \varepsilon_2)\sin\theta + \left(1 - \frac{\varepsilon_1^2}{2} - \frac{\varepsilon_3^2}{2}\right)\cos\theta - \varepsilon_1\varepsilon_3 \qquad (4\text{-}21)$$

and the model equation (3-2) becomes

$$I = K_0 - K_1\varepsilon_1 - \left[(\delta_1 - K_1\alpha) + K_1\varepsilon_4\right]\sin\theta + \left[(\delta_2 + K_1\beta) - K_1\varepsilon_3\right]\cos\theta \ . \qquad (4\text{-}22)$$

The difference between the true and measured bias is $K_1\varepsilon_1$ which is likely to be significant. For example, if $\varepsilon_1 \sim 5$ arc minutes, then the bias error is ~ 1500 μg, which in a good accelerometer is much larger than K_0 itself! This, however, suggests a method for levelling the axis of rotation, for if the true bias is known from tests in one of the other mounting positions, then the axis orientation can be adjusted until the two values agree, and hence $\varepsilon_1 = 0$.

Measurements of the components of axis error are also unlikely to agree with those obtained in the other mounting positions because of the terms $K_1\varepsilon_4$ and $K_1\varepsilon_3$, both of which can be large. It is not unusual for ε_4 to be of the order of 10 arc minutes, thus contributing an additional 3000 μg/g to the component of axis error. This is a relatively enormous quantity and it is difficult to envisage how ε_3 and ε_4 could be reduced to the few arc seconds necessary to make measurements in this mounting position meaningful.

5 CONCLUSIONS

In this Report, a mathematical modelling approach has been used to explore methods for calibrating precision accelerometers. While the conventional four-point test remains an important means of monitoring instrument performance, multiposition tests are considered superior when measurements of the highest accuracy are required. Because such tests allow the accelerometer characteristics to be examined in much finer detail, it is possible to isolate any anomalous effects and to decide whether the chosen model equation is appropriate. If necessary, different models may be tested to determine the best fit to the experimental data. An additional advantage of multiposition testing is that bad data points, caused by operator mistakes or spurious signals, are immediately apparent from the plot of residuals.

With the present emphasis on measurement precision, errors associated with the test facility must be carefully considered. Assuming the accelerometer to be mounted on the dividing head so that it rotates about one of its non-sensitive axes, then it has been shown that only levelling in the plane of rotation is of prime importance. Other angular errors of the order of a few arc minutes can be tolerated without significant loss of accuracy. However, when the accelerometer is rotated about its sensitive axis, then these errors have a profound effect on the results.

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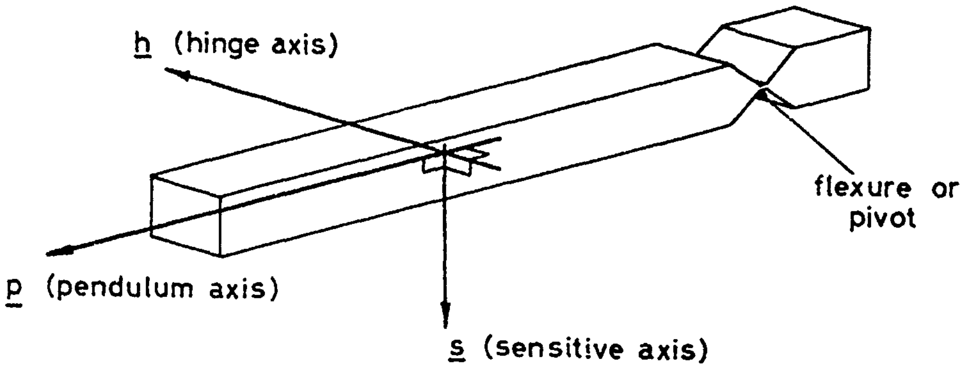

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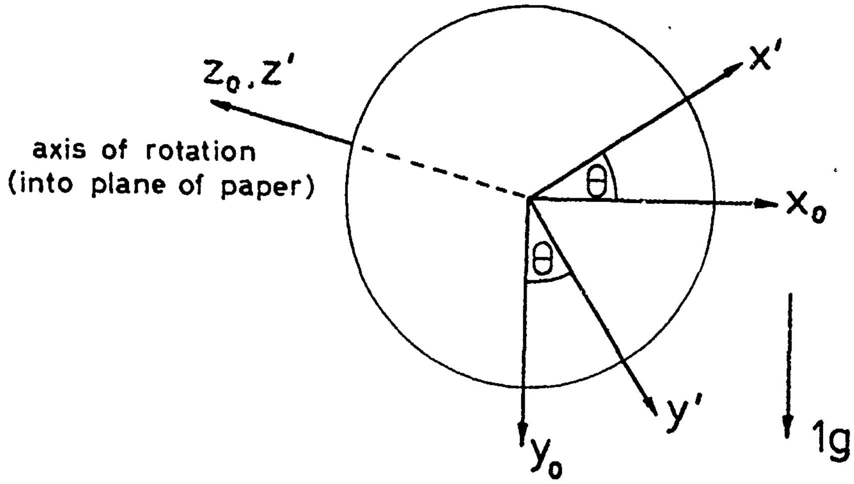

Position 1 :	x' = s'	y' = p'	z' = h'
" 2 :	x' = h'	y' = s'	z' = p'
" 3 :	x' = p'	y' = h'	z' = s'

b

Fig 1a&b Accelerometer axes and coordinate system

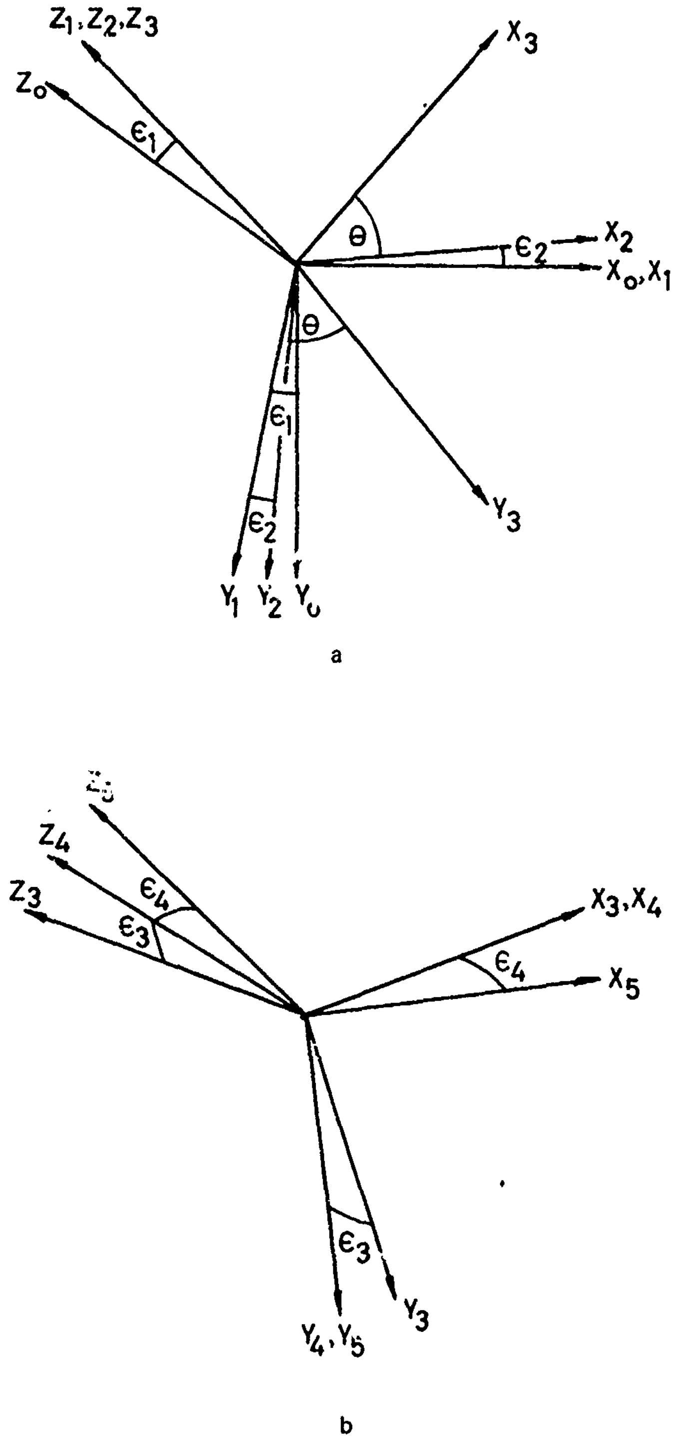

Fig 2a&b Dividing head error angles

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17. Abstract

In calibrating modern inertial quality accelerometers which are accurate to a few parts in a million, extreme care must be exercised in measurement technique and interpretation of the results. This Report analyses some of the tests which are performed currently and discusses the effects of errors in the test equipment.

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