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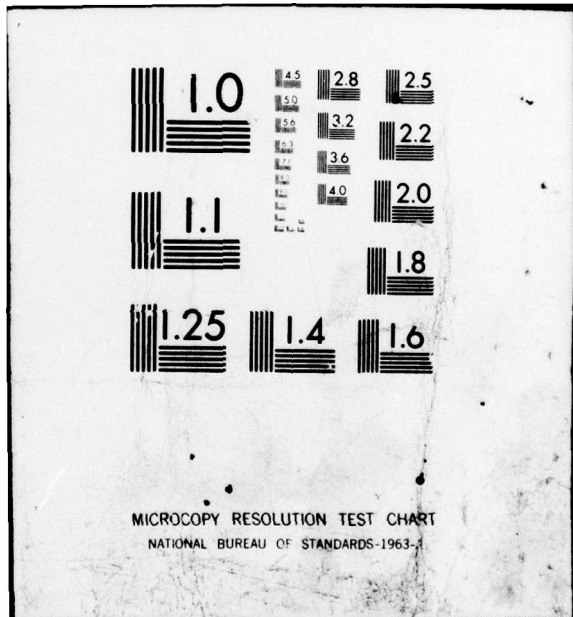
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**DETERMINATION OF  
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by

Yu. N. Bunakov

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DETERMINATION OF BENDING STRESSES IN A SPUR GEAR TOOTH

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TRANSLATOR'S SUMMARY

The bending stress of spur gear teeth is estimated assuming it approximates to a cantilever of varying thickness and applying Ritz's variational method for point loading. The stress is determined for a particular case showing the increase near the end of the teeth.

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Increasing the reliability of operation of gears in aircraft engines depends on the correct assessment of the stresses in the gear teeth. The strength of a spur gear tooth is calculated<sup>1,2</sup> assuming that the bending stresses are identical at all points over the length of the tooth. By considering the tooth as a rectangular cantilever plate of varying thickness, the values for the nominal stresses can be more accurately determined than those obtained in paper<sup>2</sup> using the Ritz's variational method.

The bending function should satisfy only geometrical limiting conditions. The static limiting conditions will be approximately satisfied. The bending function is determined in the form of a binary series:-

$$\omega = \sum_{i=1}^m \sum_{j=1}^n a_{ij} f_i(x) f_j(y) \quad (1)$$

where  $a_{ij}$  are the unknown parameters,  
 $f_i(x), f_j(y)$  are the known coordinate functions which satisfy the geometrical limiting conditions, the linear independence and the condition for completeness of energy,

$$f_i(x) = \left(\frac{x}{L}\right)^{i-1}, \quad f_j(y) = \left(\frac{y}{H}\right)^{j+1}$$

$$i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n.$$

The potential energy of deformation  $E$  of the plate and the work of the external forces  $A$  are determined from the formulae

$$E = \int_x \int_y \frac{D}{2} \left\{ \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right)^2 - 2(1 - \nu) \left[ \frac{\partial^2 \omega}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} - \left( \frac{\partial^2 \omega}{\partial x \partial y} \right)^2 \right] \right\} dx dy, \quad (2)$$

$$A = \int_x \int_y p \omega dx dy,$$

where  $D$  is the cylindrical rigidity of the plate;

$\nu$  is Poisson's ratio;

$p$  is the specific transverse load.

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We obtain the parameters  $a_{ij}$  for the solution of the linear algebraic system of equations, on the basis of the expression

$$\frac{\partial}{\partial a_{ij}} \left[ \iint_{x,y} \frac{D}{2} \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) - 2(1 - \nu) \left[ \frac{\partial^2 \omega}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} - \left( \frac{\partial^2 \omega}{\partial x \partial y} \right)^2 \right] - p\omega \right] dx dy = 0 \quad \dots (3)$$

We find the moments and stresses from the formulae:

$$\begin{aligned} M_x &= -D \left( \frac{\partial^2 \omega}{\partial y^2} + \nu \frac{\partial^2 \omega}{\partial x^2} \right), & \sigma_y &= \frac{12M_x z}{h^3} \\ M_y &= -D \left( \frac{\partial^2 \omega}{\partial x^2} + \nu \frac{\partial^2 \omega}{\partial y^2} \right), & \sigma_x &= \frac{12M_y z}{h^3} \\ M_{xy} &= -D(1 - \nu) \frac{\partial^2 \omega}{\partial x \partial y}, & \tau_{xy} &= \frac{12M_{xy} z}{h^3} \end{aligned} \quad (4)$$

The stresses were determined in a gear tooth with the tooth number  $z \geq 100$ , angle  $\alpha = 20^\circ$ , modulus  $m = 5$  mm, tooth length  $L = 50$  mm, tooth height  $H = 11.25$  mm, the tooth base thickness  $h_0 = 12.4$  mm. We shall disregard the fillet radius. The plane of symmetry of the tooth was divided along the  $x$  axis into twelve equal parts, and along the  $y$  axis into eight equal parts (Fig 1). The total load acting on the surface of the tooth was replaced by 13 equal concentrated forces  $q$  applied along their lines of action at the plane of symmetry of the tooth; ie, along the line C-C at the nodes of the coordinate network in Fig 1. The stresses are obtained from the transverse components  $p$  of the forces. The bending functions<sup>1</sup> were determined assuming that:  $m = 4$ ,  $n = 5$ .

Calculations performed on a Ural 2 electronic computer gave the following values:  $\omega$  for the nodes of the coordinate network;  $\sigma_x$ ,  $\sigma_y$ ,  $\tau_{xy}$  for the 117 points on the tooth flank; and the 20 parameters  $a_{ij}$ .

The graph (Fig 2) shows the change  $\frac{\sigma_y^*}{P}$  along the line K - K and the value  $\frac{\sigma_y^*}{P}$  calculated from the formula

$$\frac{\sigma_y^*}{P} = \frac{6l}{Lh_0^2} \quad (5)$$

where  $P$  is the total transverse load.

The magnitudes  $\sigma_x$  and  $\tau_{xy}$  account for not more than 35% of  $\sigma_y$ . The bending stresses  $\sigma_y$  may be obtained using the derived stress concentrations and the coefficients given in papers <sup>1,3</sup>.

#### CONCLUSIONS

- (1) The results obtained show a picture of the change of  $\sigma_y$  along a spur gear tooth.
- (2) The maximum value of  $\sigma_y$  exceeds  $\sigma_y^*$  given by (5) by 19%.
- (3) The good agreement of the mean  $\sigma_y$  and  $\sigma_y^*$  (1%) shows that Ritz's method using functions of the form (1) is fully appropriate for determining the bending stresses in gear teeth with random loading.

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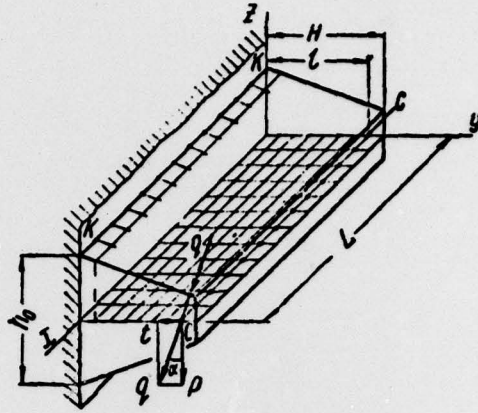


Fig 1 Diagram of the load on a tooth

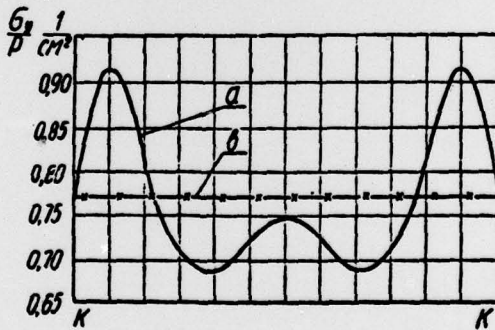


Fig 2 Bending stresses in a tooth along the line K-K

- (a) according to formula (4)
- (b) according to formula (5)

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