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AN ALGEBRAIC MODEL OF ADAPTIVE OPTICS FOR CONTINUOUS-WAVE THERM--ETC(U)

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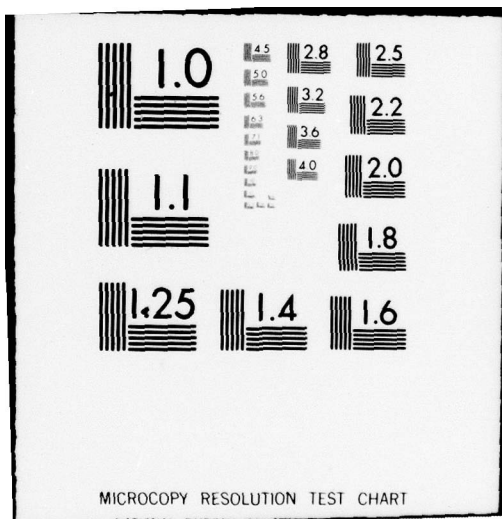
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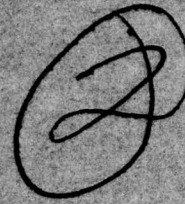
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**AN ALGEBRAIC MODEL OF ADAPTIVE OPTICS  
FOR CONTINUOUS-WAVE THERMAL BLOOMING**

by

**Dr. Cynthia Whitney**

January 1979

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random phase aberrations, compensation is a linear process the net result of which is a simple scaling down. By contrast, the thermal-blooming aberrations are complicated nonlinear functions of the applied compensations. Consequently, for this case, the interaction of applied and natural aberrations must be modeled in detail. The model shows that sometimes conventional adaptive optics concepts can produce counter-intuitive and very disappointing results when applied to thermal blooming. However, the analysis suggests novel remedies that will tend to optimize the corrections made, thus better realizing the full potential of adaptive optics.

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*David G. Hoag* 42379  
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Dr. Kenneth W. Kaiser of Draper Laboratory contributed much to the dynamic analysis reported herein.

Publication of this report does not constitute approval by the U.S. Army of the findings or conclusions contained herein. It is published for the exchange and stimulation of ideas.

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## SECTION 1

### INTRODUCTION

The term adaptive optics generally refers to the use of deformable mirrors to control phase aberrations that are involved in the generation of electromagnetic radiation and its propagation through a medium such as the earth's atmosphere. Such control is important both for passive celestial observation through the atmosphere and for laser-beam propagation, especially high power. The latter application is the main motivation for this report.

There are several physical phenomena that give rise to the need for adaptive optics, including beam quality, jitter, turbulence, and thermal blooming. Of these, all but the last are said to be linear: correcting the phase aberrations they introduce does not in any way feed back to change the phenomena causing the aberrations. Thermal blooming is different: it consists of a lensing effect in the atmosphere caused by the heating induced by the propagating beam, which in turn is modified by any phases introduced at the source.

Much work on theoretical formulation, computer simulation, and experimental demonstration has been undertaken in order to arrive at a workable adaptive optics capability.<sup>(1,2,3)\*</sup> So far, results are rather less than what might have been hoped for. Apparently, diffraction limited performance cannot be guaranteed in the presence of thermal blooming.

Given the suspected disappointing behavior, there has been a pressing need for a physical understanding of the fundamental limitations involved in the adaptive optics concept. Furthermore, the engineering procedures for implementing adaptive optics need to be thoroughly reviewed in the light of this new understanding. This

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\* Superscript numerals refer to similarly numbered items in the List of References.

report attempts to provide the necessary analysis at a level simple enough to facilitate physical understanding, but accurate enough to assist in contriving remedies to the extent possible.

The analysis is based upon an algebraic model for continuous-wave (CW) thermal blooming reported earlier by Breaux, Evers, Sepucha, and Whitney.<sup>(4)</sup> This model was developed for the purpose of providing a standard hand calculation tool for systems analysis exercises, where the numerous parameters are to be explored, making more detailed computer simulation of atmospheric propagation impractical. Its key feature in regard to thermal blooming is characterization of blooming spread in terms of an analytic integral, which represents the accumulation along the range (starting from the center of the aperture) of phase perturbation due to heating in a beam with absorption, scattering, convective clearing, and focusing.

The original algebraic model required several extensions for application to the adaptive optics problem. First, the original model assumed round, nearly Gaussian beams on target, and characterized all beam spreading in terms of a single  $1/e$  radial dispersion. In reality, blooming produces a beam which is not round, and in fact has only one axis of symmetry (a sort of "light-bulb" shape). This means that a number of Zernike modes are involved. The present model has been extended to include appropriate modes up through spherical aberration. Second, the original algebraic model did not include deterministic phase aberrations introduced at the aperture to attempt compensation of the blooming. The present model includes these, plus the resulting change in the blooming spread caused by its nonlinear dependence on what is applied at the aperture. Third, the original model was purely static. The present model allows for temporal variation of the control variables.

Section 2 extends the static model, and Section 3 notes the implications evident at that point. The major implication available is a clear statement of the fundamental limits on adaptive correction of thermal blooming. The numbers are not impressive: a factor of 2.6 for uniform round beams, improvable only to a factor of 8 by manipulating the beam shape. These results mean that any investment in the development of adaptive optics for CW thermal blooming must be motivated not by the hope of correcting that problem, but rather by the hope of reducing its interference with correction of the simpler linear phenomena.

Even though the payoff for optimal thermal blooming is modest, the penalty for doing it conventionally is potentially severe. Section 4 shows that, if applied to thermal blooming, any of the classic adaptive optics concepts commonly considered in the past could be counter productive and end up causing a net degradation. The section also shows how the destructive behavior could be remedied.

The thermal-blooming compensation techniques implied by the static analysis of the phenomenon are, unfortunately, inappropriate for the linear phenomena. The requirements of linear and nonlinear phenomena tend to conflict. The possibility for separating the phenomena so that each may be dealt with appropriately resides in dynamics. Section 5 introduces dynamic models and explores their implications.

Section 6 concludes with recommendations for verifying the findings of this report.

## SECTION 2

### STATIC THERMAL-BLOOMING ANALYSIS

The static analysis constitutes a propagation model, showing the effect of aberrations upon peak intensity and discussing their physical generation.

#### 2.1 Mathematical Model for Aberrated Beams

To reach the heart of the thermal-blooming problem as it relates to adaptive optics, we require a model with reasonable fidelity, facility to vary applied phase, and analytic tractability. Models available to choose from include complex wave-optics computer codes that produce a beam profile in the target plane, more simplified codes, and various analytic approximations, usually dealing only with peak intensity. The requirements are best met by a simple algebraic model along the lines of References 4 and 5. This model represents the results of efforts to fit scaling laws to a large body of data generated by detailed atmospheric propagation simulations done at the Naval Research Laboratory (NRL) using a finite-difference wave-optics code.

Any such wave-optics code provides a detailed intensity profile on target, from which several different summary characteristics can be extracted. These include, for instance, peak intensity, line-of-sight beam dispersion, and beam area. The last of these further admits several definitions, including area to some percentage of total power, and area defined by a ratio of squared integral of intensity to integral of intensity squared (suggested independently by Lincoln Laboratory and Draper Laboratory researchers). If beams on target were Gaussian in shape, all such characteristics would convey equivalent information. The simplified algebraic model assumes that this is nearly the case, and speaks nominally of peak intensity on target.

The original algebraic model approximates peak intensity on target as

$$I_p = PT/\pi Z_t^2 \sigma^2$$

where P is power, T is transmission,  $Z_t$  is target range, and  $\sigma^2$  is the 1/e radial beam spread due to the combination of all dispersive effects, including diffraction and beam quality (D), high-frequency turbulence (T), high-frequency jitter (J), low-frequency wander (W), and blooming (B).

$$\sigma^2 = \sigma_D^2 + \sigma_T^2 + \sigma_J^2 + \sigma_W^2 + \sigma_B^2$$

The above model applies for beams that are nearly round on target. More generally, we can allow an arbitrarily shaped beam with principal axes x and y, so that

$$\sigma^2 \rightarrow 2\sigma_x \sigma_y$$

In addition, the model above contains no deterministic ray-deviating effects other than thermal blooming. More generally, we can allow arbitrary aberrations applied at the aperture by the replacement

$$\sigma_B^2 \rightarrow \sigma_R^2$$

where R refers to the net result of all deterministic ray-bending effects. Since aberrations at the aperture are, literally speaking, "linear" effects, we introduce the notation  $\sigma_L^2(1)$  to refer just to the random linear effects:

$$\sigma_L^2(1) = \sigma_D^2 + \sigma_T^2 + \sigma_J^2 + \sigma_W^2$$

Taking account of the dissimilar principal axes,

$$\sigma_x^2 = \sigma_{Lx}^2(1) + \sigma_{Rx}^2$$

$$\sigma_y^2 = \sigma_{Ly}^2(1) + \sigma_{Ry}^2$$

The dispersions due to ray-deviating effects depend on whatever Zernike mode aberrations are present. A general formulation can be built up, after first considering some special cases, corresponding to each type of aberration that blooming might present (see Table 1). For each case, we shall draw a geometric ray model (see Figure 1), and formulate an algebraic spot model. Then, we shall formulate a general ray model and spot model covering all aberrations and combinations thereof.

Table 1. Zernike optical modes.

i	$P_i(r, \theta)$	Name
0	1	
1	$r \cos \theta$	x-tilt
2	$r \sin \theta$	y-tilt
3	$2r^2 - 1$	defocus
4	$r^2 \cos 2\theta$	astigmatism
5	$r^2 \sin 2\theta$	astigmatism
6	$(3r^3 - 2r) \cos \theta$	primary coma
7	$(3r^3 - 2r) \sin \theta$	primary coma
8	$r^3 \cos 3\theta$	trefoil coma
9	$r^3 \sin 3\theta$	trefoil coma
10	$6r^4 - 6r^2 + 1$	spherical

- Notes: 1. Variable  $r$  is normalized to unity at the aperture radius.
2. Measuring  $\theta$  from the direction of wind, the modes of interest are 3, 4, 6, 10.

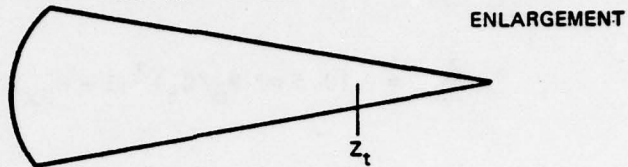
Let us begin with Focus.<sup>(6)</sup> Allowing a constant  $m''$  to account for different beam shapes,<sup>(5)</sup> we have

$$\sigma_{Fx}^2 = \sigma_{Fy}^2 = (0.5 m'' R_m / Z_t)^2 (1 - N_R)^2$$

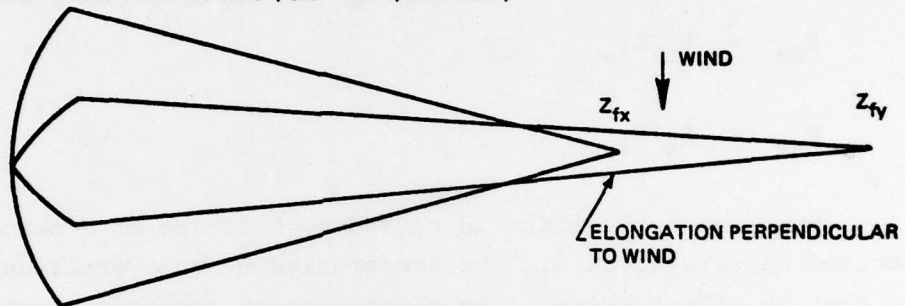
where  $N_R$  is the ratio of target range to focal range

$$N_R = Z_t / Z_f$$

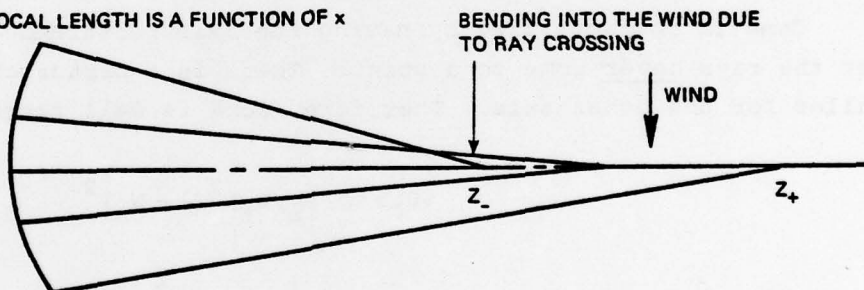
**FOCUS:**  
RAYS CONVERGE TO A POINT OFF TARGET



**ASTIGMATISM:**  
TWO FOCAL LENGTHS EXIST, ONE FOR x, ONE FOR y



**COMA:**  
FOCAL LENGTH IS A FUNCTION OF x



**SPHERICAL ABERRATION:**  
FOCAL LENGTH IS A FUNCTION OF RADIUS

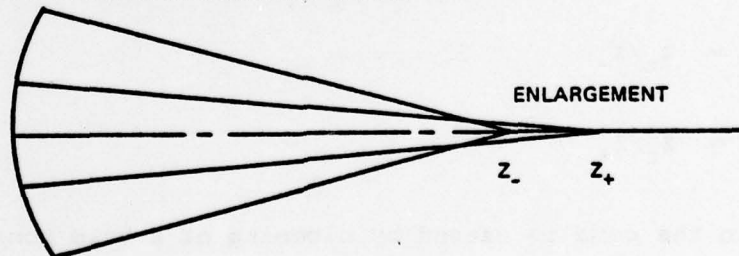


Figure 1. Ray pictures of Zernike mode aberrations.

In the case of astigmatism, there are two focal ranges instead of one, and we have

$$\sigma_{Ax}^2 = (0.5 m'' R_m / z_t)^2 (1 - N_{Rx})^2$$

$$\sigma_{Ay}^2 = (0.5 m'' R_m / z_t)^2 (1 - N_{Ry})^2$$

where

$$N_{Rx} = z_t / z_{fx}$$

$$N_{Ry} = z_t / z_{fy}$$

When the astigmatism is caused by blooming of a beam nominally focused on target, it will be accompanied by an overall defocus as well, so that in effect there is an elongation of the beam perpendicular to the wind, as noted in Figure 1.

Coma is characterized by having one axis perturbed in such a way that the rays never come to a point. There is a perturbation, but much smaller for the other axis. Therefore, coma is well represented by

$$\sigma_{Cx}^2 = (0.5 m'' R_m / z_t)^2 (1 - N_R)^2$$

$$\sigma_{Cy}^2 = (0.5 m'' R_m / z_t)^2 0.5 [(1 - N_{R-})^2 + (1 - N_{R+})^2]$$

where

$$N_{R-} = z_t / z_-$$

$$N_{R+} = z_t / z_+$$

When the coma is caused by blooming of a beam nominally focused on target, it too will be accompanied by an overall defocus. This means that, in the target plane, most ray crossing is displaced slightly into the wind, causing the well known "bending into the wind" associated with blooming (see Figure 1).

Spherical aberration is axially symmetric, but the rays never come to a point so both factors in the dispersion are never zero:

$$\sigma_{Sx}^2 = \sigma_{Sy}^2 = (0.5 m'' R_m / z_t)^2 (0.5) [(1 - N_{R-})^2 + (1 - N_{R+})^2]$$

where

$$N_{R-} = z_t / z_-$$

$$N_{R+} = z_t / z_+$$

Consequently, spherical aberration always enlarges the spot delivered.

The dispersion formulas for the various aberrations previously offered do follow a general pattern. Regardless of the aberration, we can model

$$\sigma_{Rx}^2 = 0.5(\sigma_{R1}^2 + \sigma_{R2}^2)$$

$$\sigma_{Ry}^2 = 0.5(\sigma_{R3}^2 + \sigma_{R4}^2)$$

where

$$\sigma_{Ri}^2 = (0.5 m'' R_m / z_t)^2 (1 - N_{Ri})^2$$

with

$$N_{Ri} = z_t / z_{fi}$$

The particular cases are recovered by the substitutions noted in Table 2.

Normally, the parameterization of an aberrated beam is in terms of Zernike coefficients. The aberration phase  $\Psi$  experienced by a ray originating at the point  $r, \theta$  in the aperture is

$$\Psi(r, \theta) = k \sum_i C_{Zi} P_i(r, \theta)$$

Table 2. Special cases of general ray model.

<p>FOCUS</p> $z_f = z_1 = z_2 = z_3 = z_4$
<p>ASTIGMATISM</p> $(z_1 = z_2) \neq (z_3 = z_4)$
<p>COMA</p> $(z_f = z_1 = z_2) \neq (z_3 \neq z_4)$
<p>SPHERICAL ABERRATION</p> $(z_1 = z_3) \neq (z_2 = z_4)$

where  $k$  is wave number  $2\pi/\lambda$  (with  $\lambda$  being wavelength), and  $C_{zi}$  is the coefficient of the  $i^{\text{th}}$  Zernike mode. The Zernike coefficients of interest in this model are in one-to-one correspondence with linear combinations of the dispersion parameters. The relationships are as follows:

- (1) Because an aberration in focus means all rays are similarly converged off target, the corresponding Zernike coefficient  $C_{z3}$  is proportional to the average

$$(\sigma_{R1} + \sigma_{R2} + \sigma_{R3} + \sigma_{R4})/4$$

- (2) Because astigmatism means a difference between x and y axes, the corresponding Zernike coefficient  $C_{z4}$  is proportional to

$$(\sigma_{R1} + \sigma_{R2} - \sigma_{R3} - \sigma_{R4})/4$$

- (3) Coma implies a dissimilarity between maximum and minimum focal ranges, but only on one axis, not the other. The corresponding Zernike coefficient  $C_{z6}$  is proportional to

$$(\sigma_{R1} - \sigma_{R2} - \sigma_{R3} + \sigma_{R4})/4$$

- (4) Spherical aberration is an axially symmetric dissimilarity between maximum and minimum focal ranges. The corresponding Zernike coefficient  $C_{Z10}$  is proportional to

$$(\sigma_{R1} - \sigma_{R2} + \sigma_{R3} - \sigma_{R4})/4$$

For each of these modes, the actual proportionality constants are functions of beam shape. The simplest example is the uniform beam. For this case, Reference 7 suggests that the proportionality is

$$\alpha_i = \left[ \sigma_D k \left( \int P_i^2 r dr \right)^{1/2} \right]^{-1}$$

Notice the behavior of the model in the limit of axially symmetric random effects and of very small aberrations. For that case, the beam dispersion is equivalently

$$\begin{aligned} \sigma^2 &= \sigma_L^2(1) + 0.5(\sigma_{R1}^2 + \sigma_{R2}^2 + \sigma_{R3}^2 + \sigma_{R4}^2) \\ &= \sigma_L^2(1) + 0.5(\sigma_F^2 + \sigma_A^2 + \sigma_C^2 + \sigma_S^2) \end{aligned}$$

because the various  $\sigma_R$  cross terms in the squared Zernike  $\sigma$ 's cancel each other. That is, whether viewed in terms of dispersions or Zernike coefficients, the model parameters do not cross couple.

## 2.2 Interaction of Ray-Bending Effects

The ray deviation effects represented by the dispersions  $\sigma_{Ri}$  or the Zernike coefficients  $C_{Zi}$  are the net result of aberrations imposed at the aperture (F) and aberrations induced in the atmosphere by thermal blooming (B). The parameters describing the blooming are complicated nonlinear functions of the parameters of the aperture aberrations (and really even each other). But the heart of the adaptive compensation concept is that despite nonlinearities, aberrations applied at the aperture must to some extent cancel those in the atmosphere. In terms of Zernike modes, one might suppose

$$C_{Zi} = C_{Fi} + C_{Bi}$$

The concluding remarks of Section 2.1 show that in the limit of small total aberrations, this would be equivalent to

$$\sigma_{Ri} = \sigma_{Fi} + \sigma_{Bi}$$

In an operational adaptive optics system, the blooming parameters would in some sense be measured (say, in a Zernike mode expansion of a measured phase profile). But for the purposes of this analysis, which intends to anticipate the behavior of such systems, a mathematical model is needed. It should be understood that what such a model lacks in exactness can be remedied by measurement in a real system.

We begin with the scaling law formulations found in References 4 and 5, which model the total beam dispersion in the absence of aperture corrections as

$$\sigma^2 = \sigma_L^2(1) + \sigma_B^2$$

Compare this to the present model evaluated in this limit

$$2\sigma_x \sigma_y = 2[(0.5\sigma_L^2(1) + \sigma_{Bx}^2)(0.5\sigma_L^2(1) + \sigma_{By}^2)]^{1/2}$$

Numerical similarity of the two models requires, approximately

$$\sigma_{Bx}^2 \approx \sigma_{By}^2 = \sigma_B^2/2$$

The above approximate equality between  $\sigma_{Bx}$  and  $\sigma_{By}$  is further confirmed by a ray analysis following Figure 1. Figure 2 shows stylistically the rays at  $x = \pm R_m$ ,  $y = \pm R_m$  as deviated by the blooming. The ray at  $x = -R_m$  is undeviated because of wind clearing. The rays at  $y = \pm R_m$  are somewhat deviated. The ray at  $x = +R_m$  is twice as deviated due to greater heating. The geometry suggests the approximate equality. In addition, it suggests

$$\sigma_{B1} \approx 0$$



Where  $C_B'$  and  $a$  are parameters depending only on beam shape, and  $\psi_h$  is the heating phase defined in Reference 4 as

$$\psi_h = \int_0^1 \psi_h(z) dz$$

where  $z = Z/Z_t$  and

$$\psi_h(z) = \frac{+N_D N_F \exp(-\epsilon Z) \int dt I(x_0(z,t), y_0(z,t))}{N_Q' \left( R_S(z)/R_m \right)^2 \left( 2R_m/U_w \right) \left( P/\pi R_m^2 \right)}$$

with

$$N_D \approx 2.333 \times 10^{-9} P \alpha Z_f^2 / R_m^3 U_w$$

$$N_F = 2\pi R_m^2 / \lambda Z_f$$

$$N_Q' = (\sigma_L^2 - \sigma_W^2)^{1/2} / (\sigma_D/M)$$

and in turn

$P$  = power

$\alpha$  = absorption coefficient

$U_w$  = wind velocity

$M$  = beam quality

The  $\epsilon$  is total extinction, including scattering as well as absorption. The  $x_0$  and  $y_0$  are coordinates in the aperture plane, which depend on time  $t$  as explained in Reference 4. The  $R_s(z)$  represents spot radius at  $z$ , approximated by root sum squaring ray convergence and diffraction effects. For the aberration-free case treated in Reference 4, the ray convergence contribution has the very simple form  $(1-z)^2$ , and

$$R_s(z) = R_m \left[ (1-z)^2 + \left( \frac{2z N_Q}{N_F} \right)^2 \right]^{1/2}$$

where

$$N_Q = \frac{(m^2 \sigma_D^2 + \sigma_T^2 + \sigma_J^2)^{1/2}}{\sigma_{D0} m''}$$

with  $m$  and  $m''$  being constants dependent on beam shape, and

$$\sigma_{D0}^2 = 0.5(\lambda/\pi R_m)^2$$

The first step in generalizing this model to include applied aberrations is to generalize the factor  $\sigma_L^2 - \sigma_W^2$  everywhere it occurs. Allowing for everything, we should have

$$\sigma_L^2 - \sigma_W^2 \rightarrow 2 \left[ (\sigma_{Lx}^2 - \sigma_{Wx}^2) (\sigma_{Ly}^2 - \sigma_{Wy}^2) \right]^{1/2}$$

where

$$\sigma_{Lx}^2 = \sigma_{Lx}^2(1) + 0.5(\sigma_{F1}^2 + \sigma_{F2}^2)$$

$$\sigma_{Ly}^2 = \sigma_{Ly}^2(1) + 0.5(\sigma_{F3}^2 + \sigma_{F4}^2)$$

For scaling laws in both References 4 and 5, the exponent of  $\psi_h$  is not greater than 2. Consequently, the factor in question enters  $\sigma_B^2$  to a power greater than zero. The impact of the generalization is illustrated in Figure 3 for the case where the applied aberration is defocus. The figure shows the  $y$ -axis rays defined by  $\pm(\sigma_{F3} + \sigma_{B3})$  or  $\pm(\sigma_{F4} + \sigma_{B4})$ . These are symmetric because  $y$  is perpendicular to the wind. Because  $\sigma_B$  grows with  $\sigma_{F3}$  and  $\sigma_{F4}$ , the rays have a decreased cone angle beyond the focus. This means the blooming is acting as a diverging lens, which is physically correct.

The next step in generalizing the model is to generalize the phase integral  $\psi_h$ . The analytic integration procedure given in Reference 5 applies for beams focused on target, and readily generalizes to beams focused off target ( $N_R \neq 1$ ). The result is

$$\psi_h \approx \left[ N_D N_F N_R / N_Q^2 A \right] \exp(-\epsilon Z_t/2) \ln \left| \frac{(N_S + N_R + A)(N_S + 1)}{-BN_R + N_S + N_R + AC} \right|$$

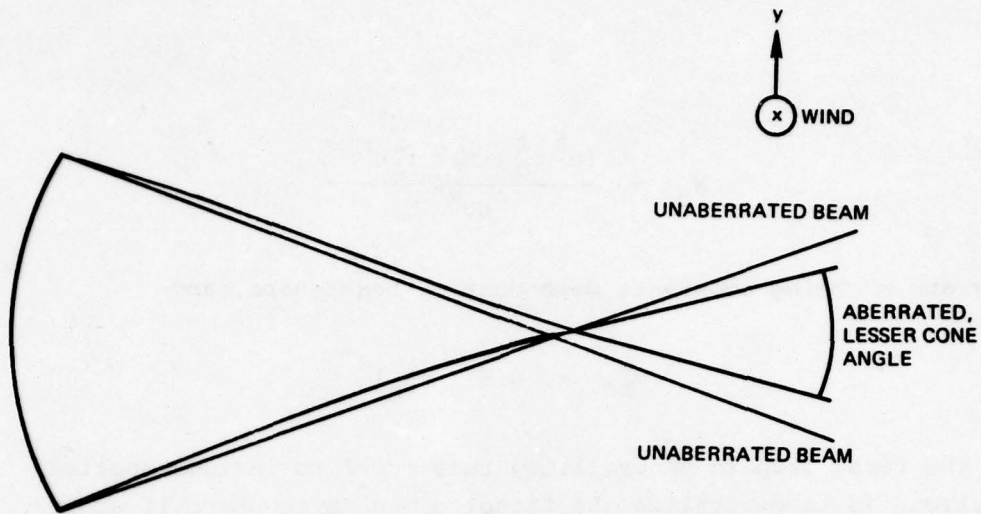


Figure 3. Gross behavior of  $\sigma_B$ .

where

$$A = \left[ (N_S + N_R)^2 + (2N_R N_Q / N_F)^2 \right]^{1/2}$$

$$B = N_S + N_R + (2N_Q / N_F)^2 N_R$$

$$C = \left[ (1 - N_R)^2 + (2N_R N_Q / N_F)^2 \right]^{1/2}$$

In principle, it is possible to generalize the expression for  $\Psi_h$  even further by using a more correct spot model  $R_S(z)$ . What is required is that  $(1-z)^2$  be replaced by

$$a(z)^2 = 2a_x(z) a_y(z)$$

where

$$a_x^2(z) = 0.25 \left( (1-z_1)^2 + (1-z_2)^2 \right)$$

$$a_y^2(z) = 0.25 \left( (1-z_3)^2 + (1-z_4)^2 \right)$$

with

$$z_i = z_f N_{Ri}$$

Such a substitution can always be handled by numerical integration. However, it does not appear likely that analytic integration would remain possible without further approximations.

In order to retain an essentially analytical model we shall use the functional form of  $\Psi_h(N_R)$ , and with linear  $N_R$  replaced by the average  $N_{Ri}$ , and quadratic  $(1 - N_R)^2$  replaced by the area excess caused by the  $N_{Ri}$ :

$$(1 - N_R)^2 = 0.5 \left[ (1 - N_{R1})^2 + (1 - N_{R2})^2 \right]^{1/2} \\ \times \left[ (1 - N_{R3})^2 + (1 - N_{R4})^2 \right]^{1/2}$$

### SECTION 3

#### IMPLICATIONS OF STATIC THERMAL-BLOOMING ANALYSIS

The algebraic model developed in the previous section can be used to show that peak intensity on target is fundamentally limited, but that presently conceived schemes for accomplishing adaptive optics still ought to be reviewed and changed, not so much to provide impressive correction, as to prevent serious degradation.

#### 3.1 Optimization of Peak Intensity

All other things being equal, optimization of peak intensity means minimization of the dispersion product  $\sigma_x \sigma_y$ , where

$$\sigma_{\begin{matrix} x \\ y \end{matrix}}^2 = \sigma_{\begin{matrix} L \\ Y \end{matrix}}^2 \begin{matrix} (1) \\ (Y) \end{matrix} + \sigma_{\begin{matrix} R \\ Y \end{matrix}}^2 \begin{matrix} (x) \\ (Y) \end{matrix}$$

with

$$\sigma_{\begin{matrix} R \\ Y \end{matrix}}^2 \begin{matrix} (x) \\ (Y) \end{matrix} = 0.5 \left( \sigma_{\begin{matrix} R \\ (3) \end{matrix}}^2 \begin{matrix} (1) \\ (3) \end{matrix} + \sigma_{\begin{matrix} R \\ (4) \end{matrix}}^2 \begin{matrix} (2) \\ (4) \end{matrix} \right)$$

The nonlinear physical relationship between the  $\sigma_{Bi}$  and  $\sigma_{Fi}$  can be expected to prevent the set of  $\sigma_{Ri}^2$  from ever going to zero. The various  $\sigma_{Ri}^2$  can at best be minimized. For an individual  $\sigma_{Ri}^2$ , consider the variation of  $\sigma_{Fi}$  to achieve a minimum. The minimum of

$$\sigma_{Ri}^2 = (\sigma_{Fi} + \sigma_{Bi})^2$$

must occur when its derivative is zero:

$$0 = (\sigma_{Fi} + \sigma_{Bi}) \left[ \frac{\partial \sigma_{Fi}}{\partial N_{Ri}} + \left( \frac{\partial \sigma_{Bi}}{\partial \sigma_{Fi}} \right) \left( \frac{\partial \sigma_{Fi}}{\partial N_{Ri}} \right) + \left( \frac{\partial \sigma_{Bi}}{\partial \Psi_h} \right) \left( \frac{\partial \Psi_h}{\partial N_{Ri}} \right) \right]$$

To solve this equation formally for the optimal  $\sigma_{Fi}$ , let us expand to first order in  $\sigma_{Fi}$ . Note that the dependence of  $\sigma_{Bi}$  on  $\sigma_{Fi}$  is such that  $\partial\sigma_{Bi}/\partial\sigma_{Fi}$  is proportional to  $\sigma_{Fi}$ :

$$\begin{aligned} \frac{\partial\sigma_{Bi}}{\partial\sigma_{Fi}} &= \frac{\sigma_{Fi} \partial^2\sigma_{Bi}}{\sigma_{Bi} \partial\sigma_{Fi}^2} \\ &= \frac{\sigma_{Fi} \sigma_{Bi}}{2(\sigma_L^2(1) + \sigma_{Fi}^2 + \sigma_{Fj}^2 - \sigma_W^2)} \end{aligned}$$

where  $j = i+1$  for  $i$  odd,  $i-1$  for  $i$  even. The consequence of this dependence is that the optimal  $\sigma_{Fi}$  has the form

$$\sigma_{Fi}^0 = -\sigma_{Bi} \frac{\left[ 1 + \frac{\partial\sigma_{Bi}}{\partial\psi_h} \frac{\partial\psi_h}{\partial N_{Ri}} \bigg/ \frac{\partial\sigma_{Fi}}{\partial N_{Ri}} \right]}{\left[ 1 + \frac{\sigma_{Bi}}{\sigma_{Fi}} \frac{\partial\sigma_{Bi}}{\partial\sigma_{Fi}} + \frac{\partial\sigma_{Bi}}{\partial\psi_h} \frac{\partial\psi_h}{\partial N_{Ri}} \bigg/ \frac{\partial\sigma_{Fi}}{\partial N_{Ri}} \right]}$$

This expression for the optimal  $\sigma_{Fi}^0$  makes it clear that the compensation usually cannot be perfect. In addition, it is interesting to note, that with  $\partial\sigma_{Fi}/\partial N_{Ri}$  being a negative number and everything else being positive, it is entirely possible for the other parameters to be severe enough that  $\sigma_{Fi}^0$  turns out the same sign as  $\sigma_{Bi}$ , rather than opposite. This is a most startling observation.

Now consider optimization of  $\sigma_x \sigma_y$  as a whole, rather than the  $\sigma_{Ri}$  individually. Because the formula for  $\sigma_x \sigma_y$  is entirely symmetric in the various  $\sigma_{Ri}$ , it seems most likely that optimal overall correction ends up with the various  $\sigma_{Ri}$  all equal, though not necessarily individually minimum with respect to their corresponding  $\sigma_{Fi}$ . In terms of Zernike modes, such a solution would have all the high-order modes corrected exactly, and focus corrected optimally. The beam would be round, and the rays would be converged on or near target.

Even if not strictly optimal in terms of peak intensity, such a solution would be optimum in more general terms, such as its interaction with a hot-spot tracking scheme. Consider, for instance,

Figure 1, which shows how the combination of coma with defocus causes a displacement of the beam bright spot. The reverse will also be true: the image of the hot spot will also be displaced in the same direction. Thus, coma causes the hot spot to be missensed and the beam to be misplaced with a net double error. This type of error is not a wave-front tilt, but does affect the system like a tilt, and can be referred to as tilt aliasing. Tilt aliasing is cured in the optimization described previously.

For this type of solution, the optimum focus would occur for minimum

$$C_{Z3}^2 = (C_{F3} + C_{B3})^2$$

The problem of minimizing  $C_{Z3}^2$  with all the other modes perfectly corrected is formally similar to that of minimizing  $\sigma_{Ri}^2$  with only  $\sigma_{Fi}$  to vary. The equation for zero derivative is

$$0 = (C_{F3} + C_{B3}) \left[ \frac{\partial C_{F3}}{\partial N_R} + \frac{\partial C_{B3}}{\partial C_{F3}} \frac{\partial C_{F3}}{\partial N_R} + \frac{\partial C_{F3}}{\partial \psi_h} \frac{\partial \psi_h}{\partial N_R} \right]$$

The term  $\partial C_{B3} / \partial C_{F3}$  can be expressed as

$$\frac{\partial C_{B3}}{\partial C_{F3}} = C \frac{\partial \sigma_B}{\partial \sigma_F}$$

Where C is the ratio of the blooming focus dispersion to the total blooming dispersion. Carrying out the differentiation, we have

$$\frac{\partial C_{B3}}{\partial C_{F3}} = \frac{C \sigma_F \sigma_B}{\sigma_L^2 - \sigma_W^2} = \frac{(4/\alpha_3)^2 C_{F3} C_{B3}}{\sigma_L^2 - \sigma_W^2}$$

Which, being linear in  $C_{F3}$ , leads to the analog for the optimization of  $\sigma_{Fi}$ :

$$C_{F3}^o = -C_{B3} \frac{\left[ 1 + \frac{\partial C_{B3}}{\partial \psi_h} \frac{\partial \psi_h}{\partial N_R} / \frac{\partial C_{F3}}{\partial N_R} \right]}{\left[ 1 + \frac{C_{B3}}{C_{F3}} \frac{\partial C_{B3}}{\partial C_{F3}} + \frac{\partial C_{B3}}{\partial \psi_h} \frac{\partial \psi_h}{\partial N_R} / \frac{\partial C_{F3}}{\partial N_R} \right]}$$

Again, we see the compensation cannot be perfect, and we see for strong enough blooming, the optimum compensation may even be in the same sense as the blooming itself.

For future reference, note that the optimum  $C_{F3}^o$  corresponds to an optimum focus dispersion  $\sigma_F^o$ , an optimum range ratio  $N_R^o$ , or an optimum focal range  $Z_f^o$ :

$$\sigma_F^o = C_{F3}^o \sqrt{2}/\alpha_3$$

$$N_R^o = 1 - \sigma_F^o Z_t / \sqrt{0.5} m'' R_m$$

$$Z_f^o = Z_t / N_R^o$$

The optimum peak intensity corresponding to  $C_{F3}^o$  can now be computed. The resulting formula is important because it represents a fundamental limit and so allows a prior judgment of the value of adaptive optics, without regard to particular implementations. We begin by evaluating

$$C_{F3}^o + C_{B3} = \frac{C_{B3} \frac{\partial C_{B3}}{\partial C_{F3}}}{1 + \frac{C_{B3}}{C_{F3}} \frac{\partial C_{B3}}{\partial C_{F3}} + \frac{\partial C_{B3}}{\partial \psi_h} \frac{\partial \psi_h}{\partial N_R} / \frac{\partial C_{F3}}{\partial N_R}}$$

Then, we form

$$\sigma_{Rx}^2 = \sigma_{Ry}^2 = (C_{F3}^o + C_{B3})^2 / \alpha_3^2$$

and

$$\sigma_R^2 = 2 \sigma_{Rx} \sigma_{Ry}$$

and evaluate

$$I_p = \frac{PT}{\pi z_t^2 (\sigma_L^2(1) + \sigma_R^2)}$$

### 3.2 Limits of Thermal-Blooming Compensation

Section 3.1 developed the fundamental equations describing the limits on improvement to peak intensity by phase compensation of thermal blooming, and this section explores the numerical consequences of those equations.

We begin by assuming the modes higher than focus are fully corrected. As an example of what this step by itself may accomplish, consider the blooming components  $\sigma_{Bi}$  cited in Section 2 as typical for a uniform round beam:

$$\sigma_{B1} = 0$$

$$\sigma_{B2} = \sigma_B$$

$$\sigma_{B3} = \sigma_B/\sqrt{2}$$

$$\sigma_{B4} = \sigma_B/\sqrt{2}$$

If compensations  $\sigma_{Fi}$  are applied such that there is no net refocus

$$\sum_i \sigma_{Fi} = 0$$

and such that all modes higher than focus are compensated,

$$\sigma_{Ri} = \sigma_{Fi} + \sigma_{Bi} = \text{Constant}$$

then, the constant must be

$$\left(\frac{1}{4}\right) \sum_i \sigma_{Bi} = \left[\frac{1}{4} + \frac{1}{2\sqrt{2}}\right] \sigma_B$$

and the compensations must be

$$\sigma_{F1} = \left(\frac{1}{4} + \frac{1}{2\sqrt{2}}\right) \sigma_B$$

$$\sigma_{F2} = \left(-\frac{3}{4} + \frac{1}{2\sqrt{2}}\right) \sigma_B$$

$$\sigma_{F3} = \left(\frac{1}{4} - \frac{1}{2\sqrt{2}}\right) \sigma_B$$

$$\sigma_{F4} = \left(\frac{1}{4} - \frac{1}{2\sqrt{2}}\right) \sigma_B$$

The individual  $\sigma_{Ri}$  remaining are equivalent to an overall

$$\sigma_R = \sqrt{2} \left(\frac{1}{4} + \frac{1}{2\sqrt{2}}\right) \sigma_B$$

$$\approx 0.85 \sigma_B$$

This is the same thing as the focus component of the original set of  $\sigma_{Bi}$ 's. The factor of 0.85 may seem surprisingly large for a focus content of thermal blooming, due to the fact that it has been so customary to think in terms of Zernike coefficients rather than actual dispersions. Indeed, even though the focus Zernike coefficient may be unimpressive compared to other modes, the relationship

$$\sigma_{Zi}^2 = C_{Zi}^2 / \alpha_i^2$$

and the numerical values of the  $\alpha_i$  on the order of

$$\alpha_i = \left[ \sigma_D k \left( \int P_i^2 r dr \right)^{1/2} \right]^{-1}$$

make focus the important contributor in terms of dispersion. Its fractional contribution to the overall blooming area  $\sigma_B^2$  is  $(0.85)^2 = 0.73$ . Consequently, correction of modes higher than focus reduces the deterministic ray-bending dispersion of the beam by a factor of only 0.73.

Now, let us proceed to apply refocus to achieve whatever further correction may be possible. The preceding section provides a formula for the optimal combination of applied focus and blooming focus Zernike coefficient, which when taken to first order in  $C_{B3}$  leads to the implication that optimal focus correction will reduce the residual beam dispersion due to deterministic ray bending by roughly a factor of

$$\left( \frac{C_{B3}}{C_{F3}} \frac{\partial C_{B3}}{\partial C_{F3}} \right)^2$$

This can be recast simply in terms of dispersions as

$$\left( 0.73 \frac{\partial \sigma_B^2}{\partial \sigma_F^2} \right)^2$$

and is approximately equal to

$$\left( \frac{0.73 \sigma_B^2}{\sigma_L^2 - \sigma_W^2} \right)^2$$

The overall reduction in  $\sigma_B^2$  due to correction of higher modes, and due to optimal correction of focus, is thus approximately

$$\frac{0.73 (0.73 \sigma_B^2)^2}{(\sigma_L^2 - \sigma_W^2)^2}$$

In terms of peak intensity, the overall improvement factor is

$$\frac{\sigma_L^2(1) + \sigma_B^2}{\sigma_L^2(1) + \frac{(0.73 \sigma_B^2)^3}{(\sigma_L^2 - \sigma_W^2)^2}}$$

or

$$\frac{(\sigma_L^2(1) + \sigma_B^2)(\sigma_L^2 - \sigma_W^2)^2}{\sigma_L^2(1)(\sigma_L^2 - \sigma_W^2)^2 + (0.73 \sigma_B^2)^3}$$

If the blooming is small in comparison to diffraction, improvement is neither possible nor necessary. But if blooming dominates, the improvement ratio is very definitely limited. Even if we allow that the  $\sigma_L^2$  includes not only the random effects represented by  $\sigma_L^2(1)$ , but also the deterministic effects of the applied  $\sigma_{Fi}$ ; and even if the latter are assumed to roughly equal minus the  $\sigma_{Bi}$ , the improvement ratio cannot exceed  $1/(0.73)^3 \approx 2.6$ . This result confirms the limitation that other researchers have previously encountered in simulations and tried, more or less fruitlessly, to improve upon.

One technique to improve the result a little bit is to change the problem geometry in a way that will change the spreading in the y dimension from the nearly uniform situation represented by  $\sigma_{B3} \approx \sigma_{B4} \approx \sigma_B/\sqrt{2}$  to one with more disparity—say, by introducing a nonuniform or noncircular beam. If such manipulation could get to the limit  $\sigma_{B3} \approx 0$ ,  $\sigma_{B4} \approx \sigma_B$ , then correction of modes higher than focus would result in

$$\sigma_R = \frac{\sqrt{2}}{4} \sum \sigma_{Bi} = \frac{\sigma_B}{\sqrt{2}}$$

Accordingly, the factor of 0.73 changes to 0.5, and the upper-bound correction ratio changes to  $(1/0.5)^3 = 8$ . The factor of 8 seems much more impressive, except when viewed in the perspective that it would occur only in situations that were catastrophically bloomed in the first place.

We can say in general, that if C is the ratio of blooming focus dispersion to total blooming dispersion, then the upper bound of blooming compensation is  $1/C^3$ . This result constitutes a very strong statement because it is not at all scenario or hardware implementation dependent. The only ingredients of the analysis are the basic form of the algebraic model and the basic geometry of the ray-propagation process.

### 3.3 Implications to Linear Compensation

It seems intuitively obvious that maximum peak intensity ought to result if beam spread due to turbulence and jitter is made as small as possible. This supposition turns out to be naive, however, given the limitations on thermal-blooming compensation cited in the last section. The problem is that the total beam degradation includes both the residual "high-frequency" beam spread

$$\sigma_H^2 = \sigma_T^2 + \sigma_J^2$$

and the optimal ray-bending dispersion

$$\sigma_{R0}^2 = \frac{(C \sigma_B^2)^3}{(\sigma_L^2 - \sigma_W^2)^2}$$

which depends upon  $\sigma_H^2$  through  $\sigma_L^2 - \sigma_W^2$  and  $\sigma_B^2$ . The  $\sigma_H^2$  is a linear contributor to  $\sigma_L^2 - \sigma_W^2$ ,

$$\sigma_L^2 - \sigma_W^2 \approx \sigma_D^2 + \sigma_H^2 + \sigma_B^2$$

Here  $\sigma_D^2$  is the diffraction term, and  $\sigma_B^2$  approximates the aberration compensations forced at the aperture. Then, the  $\sigma_H^2$  also enters  $\sigma_B^2$  through the definition

$$\sigma_B^2 = C'_B (\sigma_L^2 - \sigma_W^2) \psi_h^a$$

Not only is it there explicitly in  $\sigma_L^2 - \sigma_W^2$ , but also in the  $\psi_h$  through the parameters  $N_Q$  and  $N'_Q$  noted in Section 2.2.

Optimal linear compensation has to mean minimization of the combination:

$$\sigma_H^2 + \sigma_{R0}^2$$

This may occur for nonzero  $\sigma_H^2$ , if the derivative

$$1 + \frac{\partial \sigma_{R0}^2}{\partial \sigma_H^2}$$

is zero for a nonzero  $\sigma_H^2$  (or negative for zero  $\sigma_H^2$ ). The expression for  $\sigma_{R0}^2$  can be simplified to

$$\sigma_{R0}^2 = (C C_B' \psi_h^a)^3 (\sigma_L^2 - \sigma_W^2)$$

so the required derivative is

$$1 + \sigma_{R0}^2 \left[ \frac{3a}{\psi_h} \frac{\partial \psi_h}{\partial \sigma_H^2} + \frac{1}{\sigma_L^2 - \sigma_W^2} \right]$$

where

$$\frac{\partial \psi_h}{\partial \sigma_H^2} = \frac{\partial \psi_h}{\partial N_Q} \frac{\partial N_Q}{\partial \sigma_H^2} + \frac{\partial \psi_h}{\partial N_Q'} \frac{\partial N_Q'}{\partial \sigma_H^2}$$

will be a negative number, possibly allowing the required derivative to be zero for nonzero  $\sigma_H^2$ .

To examine the situation more quantitatively, we required the derivatives

$$\frac{\partial N_Q}{\partial \sigma_H^2} = \frac{N_Q}{2(m^2 \sigma_D^2 + \sigma_H^2)}$$

$$\frac{\partial N_Q'}{\partial \sigma_H^2} = \frac{N_Q'}{2(\sigma_L^2 - \sigma_W^2)}$$

and the corresponding derivatives of  $\psi_h$ . These are difficult to evaluate unless we introduce some approximations. If the Fresnel number  $N_F$  is large and the range ratio  $N_R$  is close to 1, we have

$$\psi_h \approx \frac{N_D N_F}{N_Q (N_S + 1)} \exp\left(\frac{-\epsilon Z_t}{2}\right) \ln\left(\frac{(N_S + 1) N_F}{N_Q}\right)$$

and

$$\frac{\partial \psi_h}{\partial N_Q} = \frac{-\psi_h / N_Q}{\ln\left(\frac{(N_S + 1) N_F}{N_Q}\right)}$$

$$\frac{\partial \psi_h}{\partial N_Q^2} = \frac{-\psi_h}{N_Q^2}$$

The required derivative can be evaluated in terms of the above, and for large  $N_F$ , reduces approximately to

$$1 + (C C_B \psi_h^a)^3 \left[\frac{-3a}{2} + 1\right]$$

In any situation where the above is negative, perfect linear correction is not optimal. The existence of such situations is a surprising result. But it seems to suggest an explanation for what has previously been observed in simulations:<sup>(2)</sup> that the presence of even a small amount of thermal blooming can make turbulence correction unstable. The above formula depends only on heating phase  $\psi_h$  and not on actual blooming dispersion  $\sigma_B^2$ . Even if the latter seems negligibly small, the former may be large enough to make full linear correction suboptimal.

Naturally, one wants a generally applicable rule for defining optimal turbulence and jitter correction, assuming optimal thermal-blooming compensation is available. The rule follows from the fact that we would like to end up with the heating phase no larger than

$$\psi_{h0} = \left[ \frac{(\frac{3a}{2} - 1)^{1/3}}{C C_B} \right]^{1/a}$$

after driving the high-frequency beam spread to an optimal value  $\sigma_{H0}^2$ , and applying the optimal blooming correction. Let us suppose that the sensitivity of heating phase is mainly in the  $N_Q'$  factor, and that  $N_Q'$  without corrections is proportional to

$$\sigma_D^2 + \sigma_H^2$$

and at optimal correction to

$$\sigma_D^2 + \sigma_{H0}^2 + \sigma_B^2$$

where  $\sigma_B^2$  approximates the aperture corrections for blooming. This is not explicitly known until  $\sigma_{H0}^2$  is already specified, so we proceed by further approximating the  $\sigma_B^2$  in terms of the algebraic model.<sup>(4)</sup> This makes  $N_Q'$  corrected proportional to

$$m^2 \sigma_D^2 + \sigma_{H0}^2$$

This means high-frequency beam spread should be made no smaller than

$$\sigma_{H0}^2 = (\sigma_D^2 + \sigma_H^2) \frac{\Psi_h}{\Psi_{h0}} - m^2 \sigma_D^2$$

if that is a positive number.

### 3.4 Implications for Power Throttling

The concept of varying laser power for optimization has been discussed elsewhere<sup>(4)</sup> and shown to be exceedingly sensible. It seems to have a clear application in regard to adaptive optics. In Sections 3.2 and 3.3, we have optimized the ray-bending dispersion  $\sigma_R^2$  for fixed high-frequency beam spread  $\sigma_H^2$  with P held fixed, and now finally we can consider optimizing P.

The ultimate objective is to optimize the peak intensity

$$I_p = \frac{P}{\pi z_t^2 (\sigma_D^2 + \sigma_W^2 + \sigma_{H0}^2 + \sigma_{R0}^2)}$$

given the functional dependence of  $\sigma_{R0}^2$  and  $\sigma_{H0}^2$  upon  $P$ . Using the algebraic model (4) parameter  $m$  to estimate the  $\sigma_B^2$  that would not be known prior to specifying  $P$ , we have

$$\sigma_{R0}^2 = (C C_B' \psi_{h0}^a)^3 (m^2 \sigma_D^2 + \sigma_{H0}^2)$$

where

$$\sigma_{H0}^2 = (\sigma_D^2 + \sigma_H^2) \frac{\psi_h}{\psi_{h0}} - m^2 \sigma_D^2$$

if that is positive, zero otherwise, and where

$$\psi_{h0} = \left[ \frac{\left( \frac{3a}{2} - 1 \right)^{1/3}}{C C_B'} \right]^{1/a}$$

Since  $\psi_h$  is linear in  $P$ ,  $I_p$  essentially has a denominator which is a polynomial in  $P$ , possibly going up to fourth order. Analytic optimization of the full expression looks intractable, but a quick one-dimensional computer search should suffice to find a suitable operating point.

## SECTION 4

### CONVENTIONAL ADAPTIVE OPTICS SCHEMES

The fact is that no existing adaptive optics scheme fully realizes even the modest corrections available for thermal blooming, and each may actually go seriously wrong. This section shows what goes wrong and how it may be remedied.

#### 4.1 What Goes Wrong

The classic techniques for achieving adaptive optics include open-loop, predictive compensation, and at least three versions of closed-loop compensation. The first of these is referred to as "phase conjugate". A point source at the receiver gives rise to a phase profile at the aperture, which is exactly compensated by the adaptive mirror so that, by reciprocity, the output beam will propagate to that point. The next technique is referred to as "multidither". The adaptive mirror is dithered, either by zones or by Zernike modes, and the radiation reflected from the receiver is monitored to determine adjustments in the mirror. Another technique is referred to as "image sharpening". It is similar to multidither, except that the sharpness of the received image, rather than the intensity of reflected radiation, is monitored. The problems with each of the techniques are discussed in the following paragraphs.

##### 4.1.1 Predictive Compensation

Given all the relevant variables, it is possible to compute the entire heating-phase profile to be expected from an uncorrected beam. Normally, what is done with this is to set the applied phase  $\Psi_{app}$  equal to that for a spherical wave converging on target,  $\Psi_{sph}(Z_t)$ , plus the conjugate of the computed heating-phase profile,  $\Psi_{hpp}$ :

$$\Psi_{app} = \Psi_{sph}(Z_t) + \Psi_{hpp}$$

The static thermal-blooming model indicates that one should not do this. Instead, one should go through the following steps:

- (1) Compute optimum focus, and assume the optimum focus is applied.
- (2) Compute the phase that would then result, and assume modes higher than focus are fully corrected.

The difference between these two strategies is that the conventional one always applies more focus than  $\Psi_{\text{sph}}(Z_t)$ , whereas the newly suggested one can apply less focus than  $\Psi_{\text{sph}}(Z_t)$  in case of heavy blooming.

It is noteworthy that one of the major motivations for closed-loop adaptive optics has been the lack of any proof that open-loop correction is optimal. The previous argument indicates that the conventional open-loop correction is indeed not optimal, but the analysis also suggests how to optimize it without having to resort to a closed-loop fix—such as dithering the amplitude of  $\Psi_{\text{app}}$ .

#### 4.1.2 Phase Conjugate Adaptive Optics

Phase conjugate adaptive optics is normally implemented simply by driving  $\Psi_{\text{app}}$  to the conjugate sensed phase  $\Psi_{\text{sen}}$ . But the static thermal-blooming analysis provides some additional information that can be used. We know the optimal focus correction is  $\Psi_{\text{sph}}(Z_f^0)$ , and it is clear that if the optimal correction were applied, the conjugate sensed phase would be  $\Psi_{\text{sen}} = \Psi_{\text{sph}}(Z_t) + \Psi_{\text{hpf}}(N_R^0)$ . Let  $\Psi_{\text{hpf}}$  be the focus part of  $\Psi_{\text{hpp}}$ . We can define the ratio of focus phases

$$N_R^0 = \frac{\Psi_{\text{sph}}(Z_f^0)}{\Psi_{\text{sph}}(Z_t) + \Psi_{\text{hpf}}(N_R^0)}$$

Apparently, optimal correction cannot be found by driving  $\Psi_{\text{app}}$  to  $\Psi_{\text{sen}}$ . Instead it requires driving only the higher modes to such an equality. For focus parts, one should drive  $\Psi_{\text{app}}$  to  $N_R^0 \Psi_{\text{sen}}$ .

The previous observations can help to explain and remedy some mysterious results that have been thought to occur with conventional phase conjugate adaptive optics. Figure 4 shows three major categories of what a phase conjugate system can do, according to the nonlinear

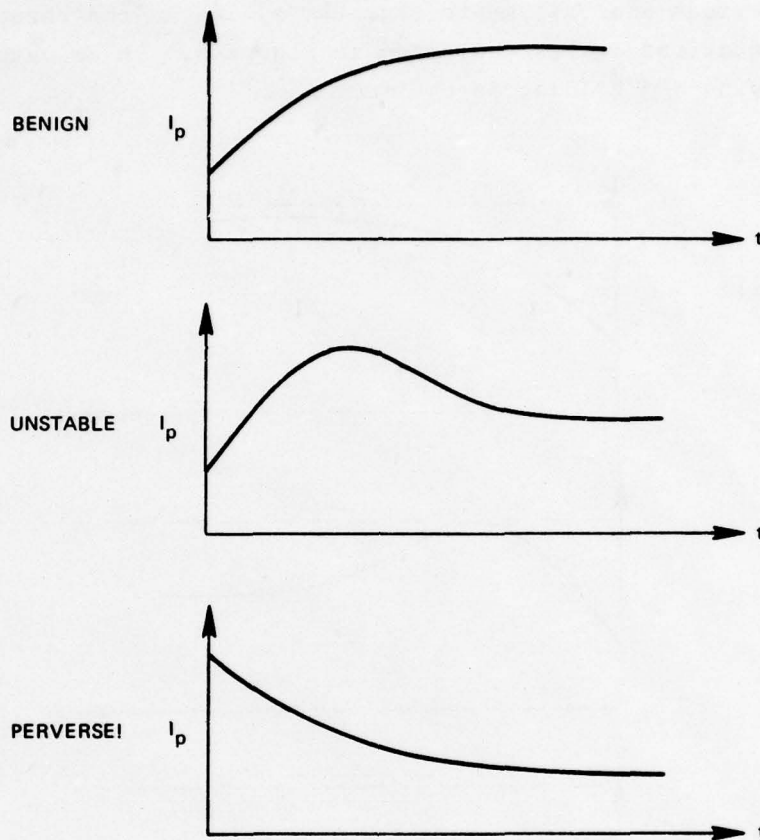


Figure 4. Conventional phase conjugate results.

lens models of Hermann<sup>(1)</sup> and the computer simulation of Brown, et al.<sup>(2)</sup> These three possible phase conjugate results can be identified with three possible circumstances that can occur according to the static thermal-blooming analysis:

- (1)  $N_R^0 > 1$ ,  $N_{R,}^0 > 1$ ; the optimum focus  $\Psi_{app} > \Psi_{sen}$  is almost reached, but not quite. This situation can be called light blooming.
- (2)  $N_R^0 > 1$ ,  $N_{R,}^0 < 1$ ; the system walks through the optimum focus correction, which occurs for  $\Psi_{app} < \Psi_{sen}$ . This situation can be called moderate blooming.
- (3)  $N_R^0 < 1$ ,  $N_{R,}^0 < 1$ ; the system goes wrong way, focusing in when the optimum is behind the target. This situation constitutes heavy blooming.

The previous analysis means that the optima in the three phase conjugate situations are as indicated in Figure 5. In no case is the system achieving and holding an optimum!

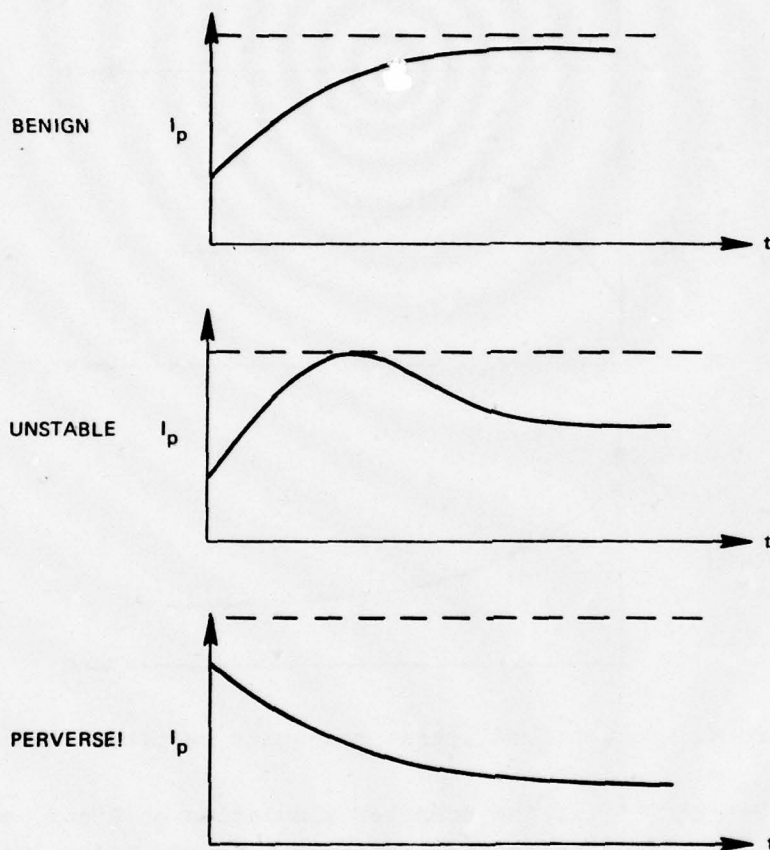


Figure 5. Conventional results in relation to optimum.

It is noteworthy that the terms light, moderate, and heavy are defined by the values of  $N_R^O$  and  $N_{R'}^O$ , and not by the Strehl ratio  $I_p/I_{pu}$ , where u denotes unbloomed. Because  $N_R^O$  and  $N_{R'}^O$  depend on a single phase, whereas  $I_p/I_{pu}$  depends on a whole phase profile, it is possible for heavy blooming to have a large Strehl ratio  $I_p/I_{pu}$ .

The behavior already described has not previously been subject to remedy. Instead, the poor behavior has been attributed to uncorrectability of blooming far from the aperture. Indeed such blooming is somewhat uncorrectable, but this model indicates that the uncorrectability only determines what the optimum peak intensity is, and not

why it is not achieved and held. Also, poor behavior has previously been attributed to the use of a linear control law in a nonlinear system. Indeed, the linear control law  $\psi_{app} = \psi_{sen}$  does not work, but only because it has been applied to all modes equally.

#### 4.1.3 Multidither Adaptive Optics

The intent of the multidither scheme is to drive  $N_R$  to  $N_R^O$  by observing the time average value of the product of commanded phase deviation and observed intensity deviation. In principle, the system should work; but there are several practical problems. The system tends to have difficulty if started very far from the solution it seeks, and tends to provide inadequate information concerning the size of steps required. There are other problems as well, but those mentioned are potentially reducible on the basis of knowledge gained from the static thermal-blooming analysis. Figure 6 illustrates the problems.

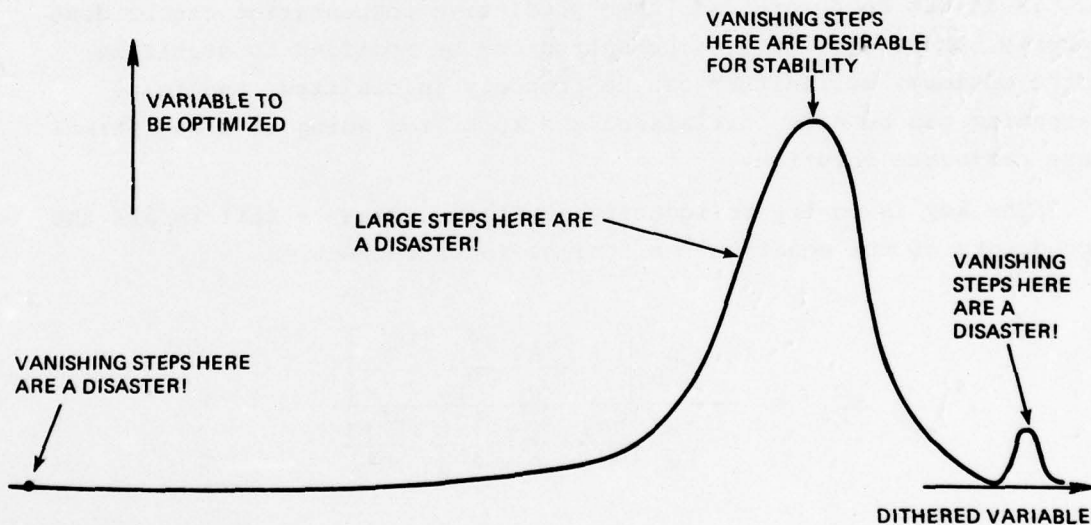


Figure 6. Illustration of some multidither problems.

The analysis is helpful because knowledge of  $N_R^O$  allows the system to be initialized predictively near the optimum, and having a model for  $I_p(N_R)$  means step size can be properly chosen. To control the step

size, it is useful to measure the curvature as well as the slope of  $I_p$  versus  $\psi_{com}$  by using the harmonics of the return signal. Then, the slope and curvature can be used to define a parabola. If the parabola has a peak, the step should go directly to it. If not, the ratio of the first and second derivatives may provide some guidance.

#### 4.1.4 Image-Sharpening Adaptive Optics

Finally, let us consider adaptive optics by the image-sharpening technique. This technique is subject to the same suboptimal equilibrium as the phase conjugate technique, plus the same stalls and overshoots as the multidither technique. All of the same remedies are required.

#### 4.2 Remedy

The hope for remedying any of the conventional adaptive optics approaches in regard to thermal blooming resides in techniques for determining, from available data, what the optimal focus correction really is. If it can be determined, then predictive compensation can be done properly, phase conjugate compensation can be modified to stabilize at the optimum, multidither can be properly initialized, and image sharpening can be both initialized and kept from going to a suboptimal phase conjugate solution.

The key is to try to identify sufficient data to fill in all the ingredients of the equation for optimal focus correction

$$C_{F3}^o = \frac{-C_{B3} \left[ 1 + \frac{\partial C_{B3}}{\partial \psi_h} \frac{\partial \psi_h}{\partial N_R} / \frac{\partial C_{F3}}{\partial N_R} \right]}{1 + \frac{C_{B3}}{C_{F3}} \frac{\partial C_{B3}}{\partial C_{F3}} + \frac{\partial C_{B3}}{\partial \psi_h} \frac{\partial \psi_h}{\partial N_R} / \frac{\partial C_{F3}}{\partial N_R}}$$

Let us suppose a thermal-blooming phase profile is available, either predicted or extracted from a return wave by an interferometer or an image-sharpening mirror. That is, let us assume we have access to characteristics like blooming Zernike coefficients and phase perturbation at the center of the aperture.

Let us suppose also that some basic system parameters are available, including:

$$\begin{aligned}
R_m &= \text{aperture radius} \\
\lambda &= \text{wavelength} \\
z_t &= \text{target range} \\
M &= \text{beam quality} \\
\sigma_T^2 + \sigma_J^2 &= \text{high-frequency tilt statistics} \\
\sigma_W^2 &= \text{low-frequency tilt statistics}
\end{aligned}$$

The algebraic propagation model<sup>(4)</sup> provides some key numbers

	$C'_B$	a	m	m'	m''
Infinite Gaussian	0.010590	1.6419	2.0	0.6366	1
Truncated Gaussian	0.028727	1.3715	4.0	0.9166	0.8893
Uniform Beam	0.014264	1.1777	1.0	0.9202	1.124

and some key equations

$$\begin{aligned}
N_F &= 2\pi R_m^2 / \lambda z_t \\
\sigma_{D0}^2 &= 0.5 (\lambda / \pi R_m)^2 \\
\sigma_D^2 &= 0.5 (m' M \lambda / 2 R_m)^2 \\
k &= 2\pi / \lambda \\
\sigma_L^2 &= \sigma_D^2 + \sigma_T^2 + \sigma_J^2 + \sigma_W^2 + \sigma_B^2 \quad (\sigma_B^2 \approx \text{corrections}) \\
N_Q &= (m^2 \sigma_D^2 + \sigma_T^2 + \sigma_J^2)^{1/2} / \sigma_{D0} m'' \\
N'_Q &= (\sigma_L^2 - \sigma_W^2)^{1/2} / (\sigma_D / M)
\end{aligned}$$

Section 2 of this report provides the definition of conversion factors

$$\alpha_i = \left[ \sigma_D k \left( \int P_i^2 r dr \right)^{1/2} \right]^{-1}$$

which allow evaluation of the blooming dispersions

$$\sigma_B^2 = \sigma_D^2 \sum_i (C_{Bi}/\alpha_i)^2$$

and, consequently, the fraction of total blooming area due to focus aberration

$$c = \frac{\sigma_D^2 (C_{B3}/\alpha_3)^2}{\sigma_B^2}$$

(which we already know from Section 3 to be on the order of 0.73).

The algebraic propagation model<sup>(4)</sup> depends upon a heating phase which can be defined from the data as

$$\Psi_h = \left[ \sigma_B^2 / C_B' (\sigma_L^2 - \sigma_W^2) \right]^{1/a}$$

From the definition of  $\Psi_h$  shown in Section 2, it must also be rather similar to  $(N_Q')^{-1}$  times the phase perturbation existing at the center of the aperture, so that pieces of data may provide an alternative basis for evaluation.

To fill in the equation for the optimal focus correction, we also need a derivative of this phase:  $\partial \Psi_h / \partial N_R$ . Using any of the analytic approximations for  $\Psi_h$ ,  $\partial \Psi_h / \partial N_R$  can be approximated analytically. For example, suppose we consider  $\Psi_h$  in the limit of large Fresnel number

$$\Psi_h \approx [N_D N_F N_R / N_Q' (N_S + N_R)] \exp\left(\frac{-\epsilon Z_t}{2}\right) \ln \left| \frac{2(N_S + 1)}{(1 - N_R) + \left[ (1 - N_R)^2 + (2N_R N_Q' / N_F)^2 \right]^{1/2}} \right|$$

Then,  $\partial \Psi_h / \partial N_R$  is obtained as follows. First, we note that the factors  $N_D$ ,  $N_F$ , and  $N_R$  are each individual functions of  $N_R$  for a given  $Z_t$ , but the dependencies cancel leaving the product  $N_D N_F N_R$  independent of  $N_R$ . Thus,  $\partial \Psi_h / \partial N_R$  has only three contributors

$$\frac{\partial \Psi_h}{\partial N_R} = \frac{-\Psi_h}{N_S + N_R} + \frac{-\Psi_h}{N_Q'} \frac{\partial N_Q'}{\partial N_R} + \frac{\Psi_h}{\ln} \frac{\partial \ln}{\partial N_R}$$

where

$$N'_Q = \frac{(\sigma_L^2(1) + \sigma_F^2 - \sigma_W^2)^{1/2}}{\sigma_D/M}$$

so that

$$\frac{\partial N'_Q}{\partial N_R} = \frac{-0.5(R_m/z_t)^2(1-N_R)}{N'_Q(\sigma_D/M)^2}$$

and where  $\partial \ln / \partial N_R$  can be approximated simply by neglecting terms of order  $1/N_F$ . The result is

$$\frac{\partial \ln}{\partial N_R} = \left[ (1-N_R)^2 + (2N_R N'_Q / N_F)^2 \right]^{-1/2}$$

The equations for  $\partial N'_Q / \partial N_R$  and  $\partial \ln / \partial N_R$  can be evaluated from the data using the approximation that, to first order, the correction  $\sigma_F$  simply compensates  $\sigma_B$ :

$$1 - N_R \approx \frac{-\sigma_B}{\sqrt{0.5} (R_m/z_t)}$$

Given all the aforementioned, we can evaluate all the required derivatives

$$\frac{C_{B3}}{C_{F3}} \frac{\partial C_{B3}}{\partial C_{F3}} = C C'_B \psi_h^a$$

$$\frac{\partial C_{F3}}{\partial N_R} = \frac{-\alpha_3 R_m}{2z_t}$$

$$\frac{\partial C_{B3}}{\partial \psi_h} = \frac{a}{2} \frac{C_{B3}}{\psi_h}$$

$$\frac{\partial \psi_h}{\partial N_R} \approx \frac{-\psi_h}{N_S + N_R} + \frac{-\psi_h}{N'_Q} \frac{\partial N'_Q}{\partial N_R} + \frac{\psi_h}{\ln} \frac{\partial \ln}{\partial N_R}$$

and thereby determine the optimal focus.

One may thus conclude that:

- (1) Predictive compensation is readily achievable if the blooming phase profile can be adequately predicted.
- (2) Phase conjugate compensation can be suitably modified if blooming can be separated from turbulence.
- (3) Multidither compensation can be helpfully initialized if the blooming phase profile can be predicted.
- (4) Image-sharpening compensation can be helpfully initialized and suitably modified if blooming can be predicted and then separated from turbulence.

So far as closed-loop adaptive optics is concerned, the key problem remaining is separation of blooming from turbulence, which has to be simultaneously sensed. Dynamic behavior is the only basis for such a separation.

## SECTION 5

### DYNAMICS

The existence of temporal dynamics is both an assistance in the design of an operational adaptive optics system and a confusing factor in the prediction of its performance. This section discusses both aspects of the situation.

#### 5.1 Discrimination Between Phenomena

Section 4 presented a complete algorithm for optimal thermal blooming correction, leaving as a key requirement the discrimination between turbulence and thermal blooming. Dynamic behavior provides the basis for such discrimination.

Taking a first cut at the discrimination problem, one may note that there tends to be a difference in frequency content between the two phenomena. Each can be thought of as a set of phase screens moving past the aperture due to wind and slow convection, while also evolving slowly due to conduction as they go. The blooming phase screens are zero everywhere except where deposited heat has caused density perturbation; whereas, the turbulence phase screens continue indefinitely on a spatial scale of  $\rho$ , the coherence length. This means that, in a condition of high slow and/or small coherence length, changes in turbulence profile will be on a shorter time scale than those in the blooming profile.

Now, suppose we have to face the more difficult situation where the time scales for blooming and turbulence indicate temporal overlapping of the two phenomena. There is still a basis for discrimination in the spatial peculiarity of the blooming profile. The main peculiarity is that, either by convection or buoyancy, the blooming tends to move across the aperture, being zero on one side and nonzero on the other. Normally, the zero edge is leading into the wind, and in any event it can be located on the basis of the time average squared phase amplitude at the edge.

The phase profile history along the edge where blooming should be zero can provide a good estimate of turbulence everywhere later on. The trick is to break the observed edge profile into separate phase sheets corresponding to different transverse velocities and reassemble them with appropriate time shifts for other points in the aperture. For convenience, suppose this edge is leading into the wind and the wind is along  $x$ . Let  $-x_m$  be the  $x$  coordinate of a point on the edge. The phase here is  $\Psi(-x_m, y, t)$ . The differential phase screen moving at velocity  $v$  can be identified as the frequency component of  $\Psi$  at frequency  $f = v/\rho$ . The turbulence phase at  $x$  can be estimated by multiplying the frequency component of  $\Psi(-x_m, y, t)$  at frequency  $f$  by the phase shift factor  $e^{i2\pi f\tau}$ , where  $\tau = (-x_m + x)/v$ , and resynthesizing.

## 5.2 Modeling Expected Behavior

All of the conventional adaptive optics algorithms and the new ones here, suggested for optimal thermal-blooming correction and for turbulence-blooming discrimination, are apt to exhibit complex behavior in a dynamic situation. In order to anticipate what may happen, it is necessary to have some models.

Let us begin with the dynamics of thermal blooming. Appropriate models can be based upon the algebraic model developed so far, but the temporal variation requires some modification to the quantities involved. An extended model incorporating dynamics must take account of the fact that the atmosphere has some memory due to the finite time required for heated air to pass out of the propagation path, and it must take account of the fact that the corrections actually applied may lag commands somewhat because they are produced by some type of servo mechanism.

Atmospheric memory will appear in an effective heating phase representing some type of time average over the recent past. This effective phase must impact the instantaneous peak intensity  $I_p(t)$  directly, and also indirectly through the instantaneous thermal-blooming beam spread parameter  $m$ . Figure 7 is a functional block diagram of the relationships in the dynamic system.

The actual computations involved in most of the blocks have already been described in Section 2. Only  $\bar{\Psi}_h$  and  $m(t)$  remain to be

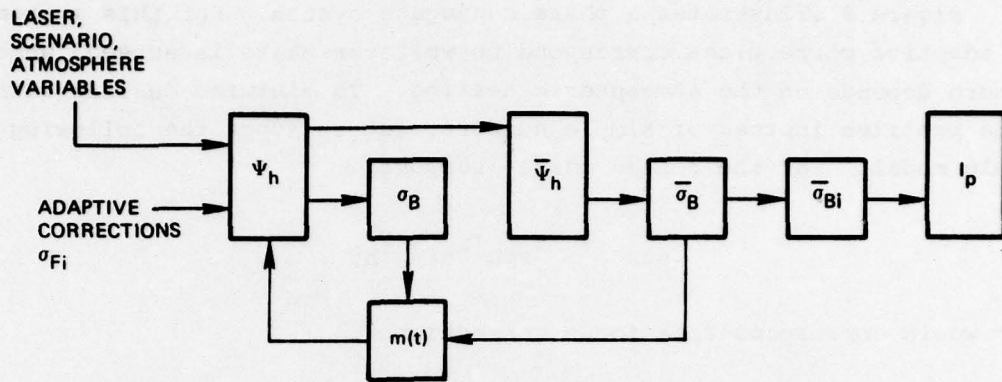


Figure 7. Dynamic model of propagation process.

defined. Let the  $\bar{\psi}_h$  be the time average over the recent past, with the beam profile at the aperture as a weighting function:

$$\bar{\psi}_h = \frac{\int_0^R I(r, t - \frac{r}{U_w}) \psi_h(t - \frac{r}{U_w}) dr}{\int_0^R I(r, t - \frac{r}{U_w}) dr}$$

Let the  $m(t)$  be contrived to have reasonable limiting values:

- $m(t) = 1$  before blooming develops
- $m(t) = m$  at steady state
- $m(t) > m$  if blooming has been heavier in the past.

A simple form for  $m(t)$  is

$$m(t) = 1 + (m - 1) \bar{\sigma}_B / \sigma_B$$

Consider now the imposition of any of the standard adaptive optics algorithms on the system described by Figure 7. In the case of predictive compensation, the  $\sigma_{Fi}$  simply correspond to constants equal to minus the values the  $\bar{\sigma}_{Bi}$  would have in uncorrected steady state. Imposition of any of the closed-loop schemes, however, is more complicated, requiring an additional loop in the system.

Figure 8 illustrates a phase conjugate system. For this system, the adaptive corrections correspond to whatever phase is sensed, which in turn depends on the atmospheric heating. To minimize dealing with whole profiles instead of simple numbers, let us adopt the following simple model. For the sensed phase, suppose

$$\psi_{\text{sen}} = \psi_{\text{sph}}(z_t) + \bar{\psi}_h$$

This would correspond to a focus correction

$$\sigma_F = -\sqrt{0.5} \frac{R_m}{z_t} \frac{\bar{\psi}_h}{\psi_{\text{sph}}(z_t)}$$

Partitioning into individual  $\sigma_{Fi}$  in the usual way,

$$\sigma_{F1} \approx 0$$

$$\sigma_{F2} \approx \sigma_F$$

$$\sigma_{F3} \approx \sigma_{F4} \approx \sigma_F / \sqrt{2}$$

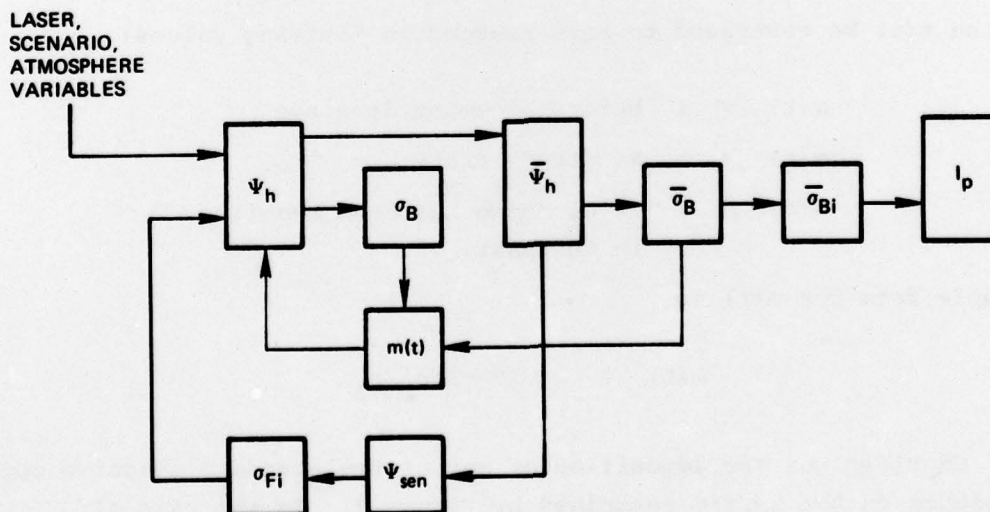


Figure 8. Dynamics of phase conjugate compensation.

Figure 9 shows a multidither system. For this system, oscillating perturbation corrections  $\Delta\sigma_{Fi}$  are observed via a glint return, and some update to  $\sigma_{Fi}$  is rendered on the basis of how they compare. The equations can be modeled as

$$\frac{d\sigma_{Fi}}{dt} = c \langle \Delta\sigma_{Fi} \Delta I_p \rangle$$

$$\Delta I_p = I_p - \bar{I}_p$$

$$\bar{I}_p = \int_{\tau=0}^{\infty} I_p \epsilon^{\epsilon(t-\tau)} \epsilon d\tau$$

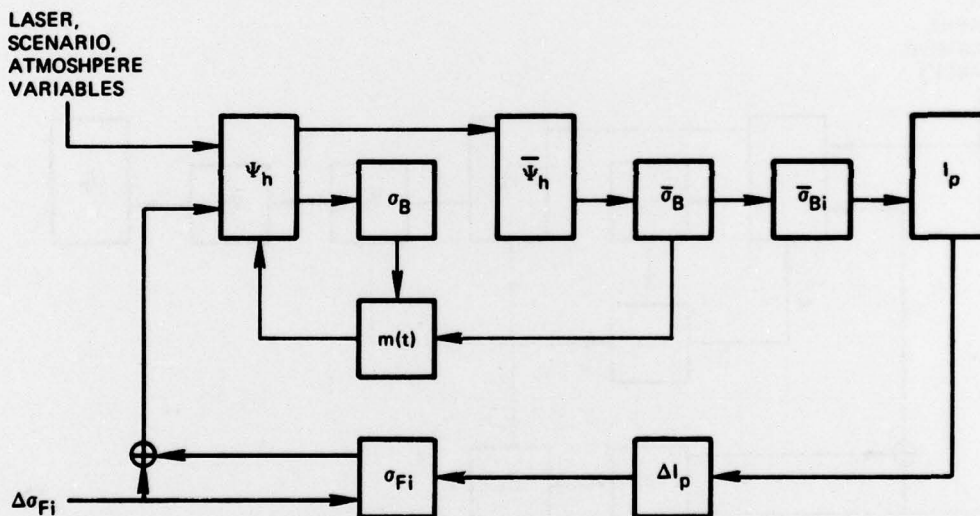


Figure 9. Dynamics of multidither system.

Figure 10 shows an image-sharpening system. Here, oscillating perturbation corrections  $\Delta\sigma_{Fi}$  are entered, and the resulting target image is monitored for oscillating sharpness  $S$ , with some update to  $\sigma_{Fi}$  being based upon the comparison of the two. (Monitoring sharpness is essentially equivalent to monitoring phase.) The equations are

$$\frac{d\sigma_{Fi}}{dt} = c \langle \Delta\sigma_{Fi} \Delta s \rangle$$

$$\Delta s = s - \bar{s}$$

$$\bar{s} = \int_{\tau=0}^{\infty} s e^{\epsilon(t-\tau)} \epsilon d\tau$$

$$s = \int I^2 dx dy / \left[ \int I dx dy \right]^2$$

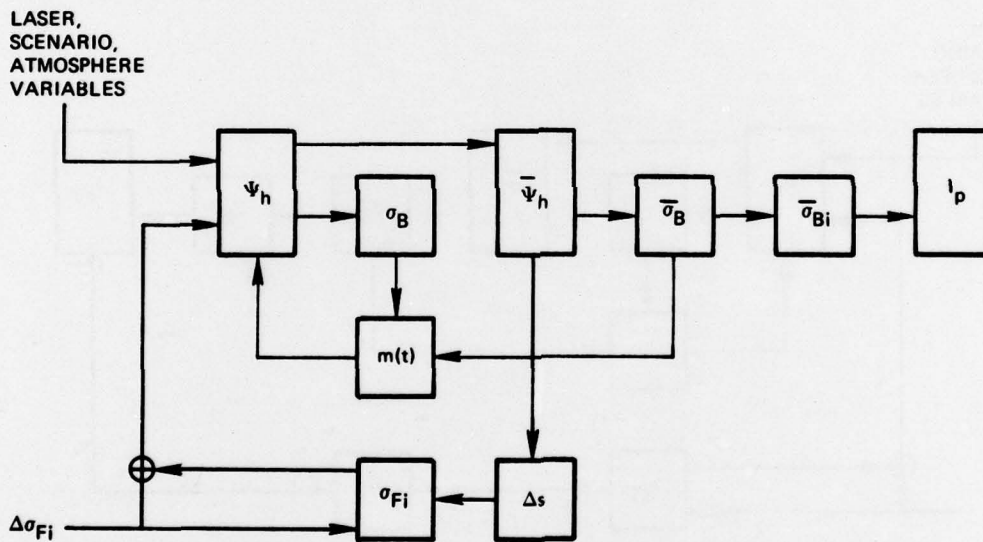


Figure 10. Dynamics of image-sharpening system.

Now, consider the implementation of the new algorithm suggested for optimal blooming correction. This does not affect the multidither system, and only affects the others to the extent of changing the equations for the  $\sigma_{Fi}$ , leaving the drawings unchanged.

Next, consider the impact of turbulence on these systems. Any turbulence not adequately discriminated and compensated enters in two places: as a noise corrupting  $\bar{\psi}_h$ , and as an additional degradation entering  $I_p$ .

Finally, consider the impact of servo lag on all the systems. The  $\sigma_{Fi}$  actually applied are a result from a servo mechanism trying to implement the commands. Therefore, residual discrepancies will always exist, usually in the form of lags. An example is illustrated in Figure 11: a type zero servo drives  $\sigma_{Fi\ app}$  to  $\sigma_{Fi\ com}$ . For this example, the frequency domain equations are

$$\sigma_{Fi\ app}(s) = \left[ \sigma_{Fi\ com}(s) - \sigma_{Fi\ app}(s) \right] \frac{\gamma}{s + \gamma}$$

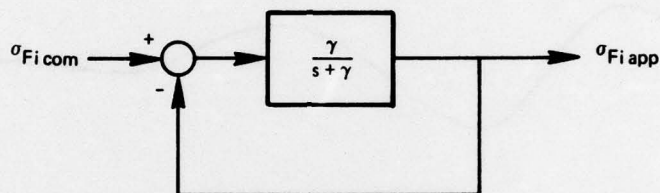


Figure 11. Example: type-zero servo driving  $\sigma_{Fi\ com}$  to  $\sigma_{Fi\ app}$ .

and the time-domain equations are

$$\sigma_{Fi\ app}(t) = \sigma_{Fi\ app}(0) + \int_0^t (\sigma_{Fi\ com}(\tau) - \sigma_{Fi\ app}(\tau)) e^{-\gamma(t-\tau)} \gamma d\tau$$

The equations reveal the lag: whatever  $\sigma_{Fi}$  is commanded after  $t=0$  is not fully applied for a while. More generally, the main point of any such model is that  $\sigma_{Fi\ app}(t)$  lags  $\sigma_{Fi\ com}(t)$  because an integration is involved.

### 5.3 Implications

Inclusion of temporal variation adds some nuances to the analyses of what each of the adaptive optics systems will do, and how to improve the behavior. Generally, the dynamics will cause overshoots and oscillations, and sometimes even wrong commands.

Consider the case of predictive compensation. When the system is first turned on, peak intensity is lower than it will become, so blooming is less than it will become. Therefore, the peak intensity will jump initially (on the time scale of the servo) to some value, and then it will either climb or decline slowly (with the atmospheric time scale) toward equilibrium. In either case, it might overshoot and oscillate, as shown in Figure 12.

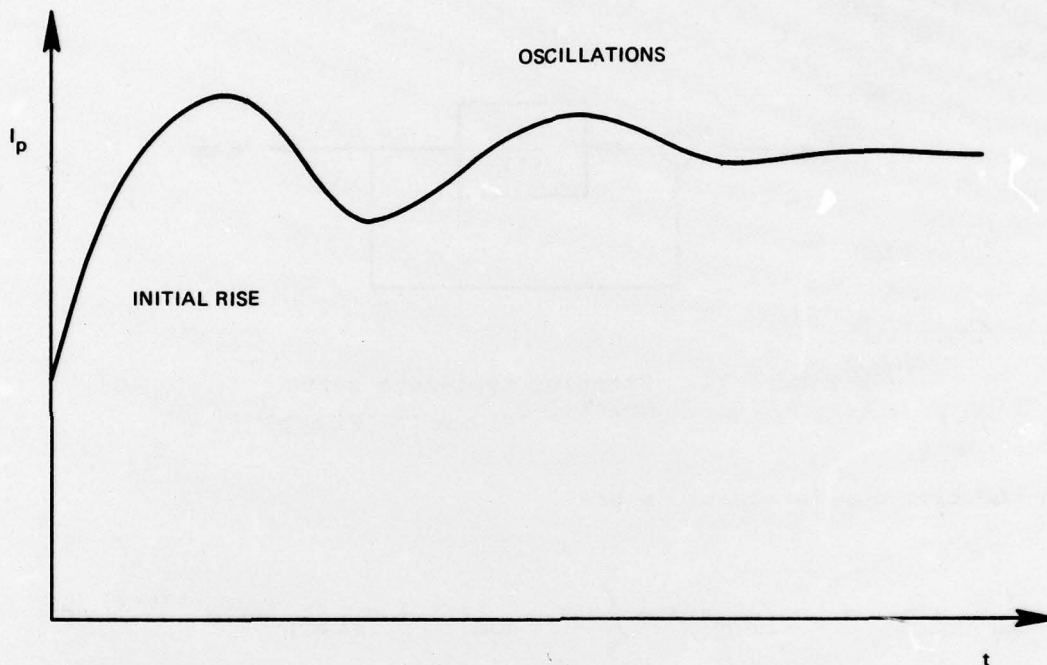


Figure 12. Oscillations superposed on basic predictive behavior.

Next, consider the case of phase conjugate adaptive optics. For moderate and heavy blooming,  $I_p(t)$  is declining as it approaches its equilibrium value. Because  $\psi_{sen}$  lags  $I_p$ ,  $\psi_{sen}$  is then larger than at equilibrium; and because  $\psi_{app}$  lags  $\psi_{sen}$ , it is then even larger than the equilibrium  $\psi_{sen}$ . Consequently, the system overshoots, and oscillations follow, as illustrated in Figure 13.

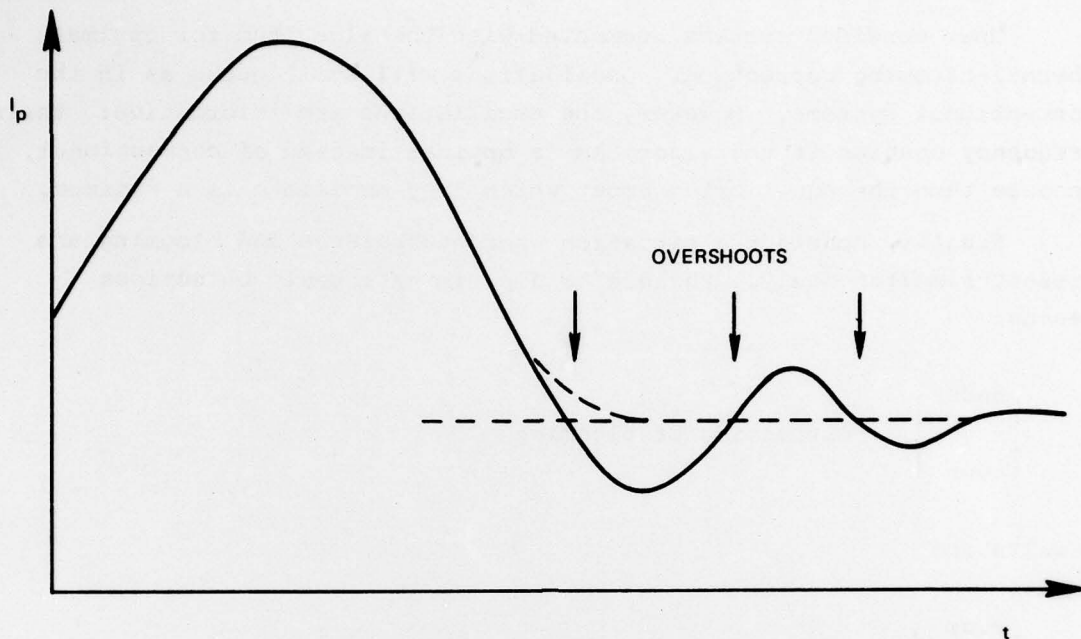


Figure 13. Oscillations superposed on basic phase conjugate behavior.

Now, consider multidither adaptive optics. The scheme uses the expected value  $\langle \Delta I_p \Delta \Psi_{com} \rangle$  to drive  $N_R$  to  $N_R^0$ . But servo lag and atmospheric integration time can significantly change the phase between  $\Delta I_p$  and  $\Delta \Psi_{com}$ , possibly giving the wrong sign! This will at least delay, and may even prevent, convergence.

To encourage proper functioning of a multidither scheme, it is helpful to try to overcome the lags. One approach is to dither as slowly as practical. This approach is of limited value because it compromises the bandwidth capability of the system. Another approach is to attempt to measure the phase of  $\Delta I_p$  with respect to  $\Delta \Psi_{com}$ . This can be difficult, and  $2\pi$  ambiguities are a problem. Further assistance can be gained by using at least two dither frequencies per mode and extrapolating to zero dither frequency.

Next, consider image-sharpening adaptive optics. It has the same problems due to phasing between perturbation introduced and perturbation observed as does multidither, and perhaps worse, because it dithers a low-power mirror which has better bandwidth capability than the high-power mirror used by multidither. But the problems shared with multidither can be overcome similarly.

Now, consider systems augmented with the algorithm for optimal thermal-blooming correction. Oscillations will still occur as in the conventional systems. However, the oscillations are informative: their frequency doubles if the algorithm is optimal instead of conventional, because then the equilibrium about which they oscillate is a maximum.

Finally, consider a situation where turbulence and blooming are present simultaneously. Failure to discriminate could be serious because

$\left. \begin{array}{c} \text{under} \\ \phantom{\text{under}} \\ \text{over} \end{array} \right\}$  estimation of blooming

results in

$\left. \begin{array}{c} \text{over} \\ \phantom{\text{over}} \\ \text{under} \end{array} \right\}$  correction of both phenomena

and hence suboptimal results.

## SECTION 6

### CONCLUSIONS AND RECOMMENDATIONS

This report argues that the nonlinear phenomenon of thermal blooming presents a serious problem for continuous-wave adaptive optics. The potential for blooming correction is fundamentally limited. If adaptive optics is to be used at all for this type of system, the motivation must lie in the potential for correction of simpler linear effects. But if adaptive optics is used for those purposes, blooming has to be estimated and treated intelligently, because naive approaches will result in catastrophic failure to do even the simple task of linear correction, just because the blooming is present. If this is true, then it is obvious that adaptive optics has not yet been implemented with a proper understanding. Forging ahead without that understanding would be wasteful.

Verifying the analyses presented here requires that a number of key assertions and conclusions be confirmed by further analysis, by simulation, or by experiment. It is important to break the verification problem into as many independent questions as possible, in order that each one be as simple and straightforward as possible. The following partitioning of the problem is recommended:

- (1) In an unbloomed beam with aberrations of focus, astigmatism, primary coma, and spherical aberration, the Zernike coefficients imply beam-spread parameters that accurately predict peak-intensity Strehl ratio below beam breakup.
- (2) In an uncorrected bloomed beam below  $P_{\text{crit}}$ , the ratio of defocused beam spread to total beam spread is a function of beam shape only.
- (3) For a uniform round beam, the ratio of defocused beam spread to total beam spread is approximately

$$\frac{\sigma_{B3}^2}{\sigma_B^2} \approx 0.73$$

- (4) Application of any reasonable blooming correction strategy has the effect of increasing the value of  $P_{crit}$ , as described in Reference 4.
- (5) If arbitrary Zernike coefficients  $C_{Fi}$  are applied at the aperture and resulting blooming coefficients  $C_{Bi}$  are sensed, the sum of Zernike coefficients

$$C_{Zi} = C_{Fi} + C_{Bi}$$

determine a value of beam spread on target that accurately predicts the peak-intensity Strehl ratio so long as  $P < P_{crit}$ .

- (6) The combination of uncorrected focus and coma aberrations can result in tilt aliasing that would in principle affect hot-spot tracking.
- (7) The optimum thermal-blooming correction strategy is to fully correct modes other than focus, and to optimize the focus.
- (8) The optimum focus correction  $C_{F3}$  is not generally equal to  $-C_{B3}$ .
- (9) It is possible for the optimum  $C_{F3}$  to have the same sign as  $C_{B3}$ .
- (10) The numerical value of the optimum  $C_{F3}$  can be estimated from the algorithm cited in this report.
- (11) The limits on blooming correction are a function of beam shape only, and amount to a factor of

$$\left( \frac{\sigma_B^2}{\sigma_{B3}^2} \right)^3$$

- (12) If there are correctable linear dispersions (jitter or turbulence) as well as blooming, in some cases, maximum peak intensity requires leaving some of the linear spread uncorrected.

- (13) The optimum linear spread to leave uncorrected can be estimated with the algorithm cited in this report.
- (14) If blooming and linear effects are all to be optimally corrected, there is then an optimum power to operate at, as described in this report.
- (15) Discrimination between blooming and turbulence can be accomplished on the basis of temporal frequency content when slew is high or coherence length is small.
- (16) Even if blooming and turbulence frequency spectra overlap, discrimination can be accomplished in the manner described in this report.

The analyses, simulations, and experiments aimed at verifying these items need to be designed in the least burdensome way possible. It should be possible to concentrate on geometric analyses, limited wave-optics simulations, and small-scale thermal-blooming cell-laboratory-type experiments.

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