UNIVERSITY OF MARYLAND
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(14) CSC-TR-702
(15) DAAG-53-76C-0138
DARPA Order-3206

(11) Oct[redacted]1978

(12) 43

(6) POINT PATTERN MATCHING BY RELAXATION

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ABSTRACT

Let $P=P_1,...,P_m$ and $Q=Q_1,...,Q_n$ be two patterns of points. Each pairing (P_i,Q_j) of a point of P with a point of Q defines a relative displacement $\underline{\delta}_{ij}$ of the two patterns. We can define a figure of merit for $\underline{\delta}_{ij}$ according to how closely other point pairs coincide under $\underline{\delta}_{ij}$. If there exists a displacement $\underline{\delta}_o$ for which P and Q match reasonably well, the pairings for which $\underline{\delta}_{ij} \approx \underline{\delta}_o$ will have high merit scores, while other pairings will not. The scores can then be recomputed, giving weights to the other point pairs based on their own scores; and this process can be iterated. When this is done, the scores of pairs that correspond under $\underline{\delta}_o$ remain relatively high, while those of other pairs become low. Examples of this method of point pattern matching are given, and its possible advantages relative to other methods are discussed.

The support of the Defense Advanced Research Projects Agency and U.S. Army Night Vision Laboratory under Contract DAAG-53-76C-0138 (DARPA order 3206) is gratefully acknowledged, as is the help of Kathryn Riley in preparing this paper.

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1. Introduction

In [1] some experiments in point pattern matching are described. Given two point patterns $P=P_1,\ldots,P_m$ and $Q=Q_1,\ldots,Q_n$, we count, for each displacement $\underline{\delta}$ of P relative to Q, how many pairs (P_i,Q_j) lie closer together than some threshold t. If P and Q have many points in common, this process will yield a large match peak for that $\underline{\delta}$ that maps these points into themselves, while others $\underline{\delta}$'s will yield at best low match values where a few pairs happen to coincide.

The matching process just described is essentially equivalent to cross-correlating P with a "blurred" version of Q in which each point has been expanded into a disk of radius t. (Actually, for computational simplicity, only displacements that map some $P_i \in P$ into some $Q_j \in Q$ were considered.) It will tolerate local distortions, as long as they do not generally give rise to relative displacements that exceed t. On the other hand, t must be substantially smaller than the average interpoint distance in P and Q, since otherwise many false matches will be detected.

This paper discusses an alternative approach to point pattern matching in which a figure of merit is assigned to each pair (P_i,Q_j), according to how closely other pairs (P_h,Q_k) match when P_i is mapped into Q_j. Let $\underline{\delta}_{ij}$ be the displacement that maps P_i into Q_j; if there exists a displacement $\underline{\delta}_o$ for which P and Q match reasonably well, and

$\underline{\delta}_{ij}$ is close to $\underline{\delta}_o$, then (P_i, Q_j) will have a high merit score, but otherwise it will not. The scores can then be recomputed, giving weights to the other point pairs (P_h, Q_k) based on their own scores; and this process can be iterated. When this is done, the scores of pairs that correspond under $\underline{\delta}_o$ remain relatively high, while those of other pairs become low.

The merit score computation is described in greater detail in Section 2, and some examples of point pattern matching by this method are given in Section 3. For earlier work on the use of iterative ("relaxation") methods in image matching see [2].

2. Match merit computation

When we pair P_i with Q_j, we would like to find Q's in the same positions relative to Q_j that the P's have relative to P_i. Let P_h and Q_k be any P and Q other than P_i and P_j, and let $\underline{\delta}_{ij}(h,k)$ be the position difference of P_h and Q_k when P_i is mapped into Q_j. If $|\underline{\delta}_{ij}(h,k)|$ is zero, Q_k is exactly in the same position relative to Q_j that P_h is relative to P_i, so that the pair (P_h,Q_k) should give the pair (P_i,Q_j) maximal support; while as $|\underline{\delta}_{ij}(h,k)|$ increases, this support should decline. Let the support given to (P_i,Q_j) by (P_h,Q_k) be denoted by $\varphi(|\delta_{ij}(h,k)|)$. In the experiments described in the next section, we used $\varphi(x)=\frac{1}{1+x^2}$; many other functions could have been used instead.

For each P_h, there may be several Q_k's that lie close to it; but we only want one Q_k to correspond to P_h when P_i is paired with Q_j. Thus it is reasonable to define the support to (P_i,Q_j) associated with P_h as

$$\max_{k\neq j}[\varphi(|\delta_{ij}(h,k)|)] \tag{1}$$

In this way, we can compute the support provided to (P_i,Q_j) by each P_h. To obtain the total support for (P_i,Q_j), we average the contributions of all the P_h's:

$$\frac{1}{m-1}\sum_{h\neq i}\{\max_{k\neq j}[\varphi(|\delta_{ij}(h,k)|)]\} \tag{2}$$

We shall denote this quantity by $s^{(0)}(P_i,Q_j)$.

In computing $s^{(0)}$, we treated all pairs (P_h,Q_k) equally, since a priori any two points can correspond. Now, however, we can recompute the support, taking into account the fact that each pair has a figure of merit $s^{(0)}(P_h,Q_k)$. Specifically, the support given to (P_i,Q_j) by (P_h,Q_k) should depend not only on the position difference between P_h and Q_k, but also on their $s^{(0)}$ value. These two factors can be combined in various ways; we have chosen to use their minimum, i.e., to define the support for (P_i,Q_j) given by (P_h,Q_k) as

$$\min[\varphi(|\delta_{ij}(h,k)|),\ s^{(0)}(P_h,Q_k)] \tag{3}$$

We can then compute the support for (P_i,Q_j) associated with P_h as

$$\max_{k\neq j}(\min[\varphi(|\delta_{ij}(h,k)|),\ s^{(0)}(P_h,Q_k)]) \tag{1'}$$

and the total support for (P_i,Q_j) from all the P_h's as

$$s^{(1)}(P_i,Q_j)=\frac{1}{m-1}\sum_{h\neq i}\{\max_{k\neq j}(\min[\varphi(|\delta_{ij}(h,k)|),s^{(0)}(P_h,Q_k)])\} \tag{2'}$$

This process can then be iterated; at the rth step we have

$$s^{(r)}(P_i,Q_j)=\frac{1}{m-1}\sum_{h\neq i}\{\max_{k\neq j}(\min[\varphi(|\delta_{ij}(h,k)|),s^{(r-1)}(P_h,Q_k)]) \tag{2*}$$

for r=1,2,3,... .

3. Experiments

The process described in Section 2 was applied to the same data sets used in [1]. In each of the following figures, parts (a-b) show the two point patterns P and Q. Part (c) tabulates, for each $P_i \in P$, the two points $Q_j \in Q$ for which the match merit of the pair (P_i, Q_j) is highest. Part (d) shows the total match merit associated with each relative displacement of P and Q, multiplied by 100.

The point patterns in Figures 1 and 2 represent two sets of local features extracted from a picture of a tank and from a road map of the Washington, D.C. area, respectively. In each case, there is a strong match peak corresponding to the proper relative displacement of the two patterns. Note that the merit of the second best Q_j is not always very much lower than that of the best Q_j, but it is often at least twice as low.

Figures 3-5 show results for sets of edge points extracted from a succession of FLIR images of a tank. The comparisons are for frames 1 and 10, 1 and 5, and 5 and 10, respectively. The smearing of the peaks indicates the motion of the tank from frame to frame. Figures 6-7 show results using two different edge detectors on an image of a tank and of an APC.

Figures 8-11 show the results of applying various amounts of "random walk noise" to the second set of map feature points used in Figure 2. The random jumps had uniformly distributed directions; their magnitudes were normally distributed with mean 0 and standard deviation 3%, $4\frac{1}{2}$%, 6%, and 9% (of the picture diameter) in Figures 8-11, respectively. Good match peaks are obtained for the first three cases.

Figures 12-13 show the results of rotating one of the map feature point sets by 5° and 10° relative to the other. Good match peaks are obtained in both cases; in [1], on the other hand, the 10° rotation did not yield a good peak.

In all of these examples, the results shown are for the fourth iteration (i.e., $s^{(4)}(P_i,Q_j)$). Figure 14 shows the results of iterations 0,1,2,3 for the case in Figure 1. Note that even for $s^{(0)}$, the peak is four times higher than the largest non-peak values, and that this ratio is (at least) maintained while the number of non-peak values drops substantially. Note also that in this case the $s^{(3)}$ values are the same as the $s^{(4)}$ values in Figure 1d--i.e., the process has stabilized after three iterations.

4. Discussion

The relaxation scheme for point pattern matching described in this paper is comparable in computational cost to the simpler scheme of [1]. Both schemes consider only the mn displacements that pair off all possible points of P with points of Q; and for each such displacement, they compare all other points of P with all other points of Q. Thus the total amount of computation is $O(m^2n^2)$. The individual computations involved in the relaxation scheme are somewhat more complex, and must be iterated several times; but this additional complexity represents only a constant factor which does not grow with m and n.

On the other hand, the relaxation approach appears to be more tolerant to global distortion than the method of [1]. For example, consider the case where one pattern is rotated relative to the other. This rotation shifts different parts of the pattern in different directions; hence when we compute a match score for any given relative shift, as in [1], it will be relatively low. On the other hand, for any given point P, there will be neighboring points that are shifted in approximately the same way as P, so that in the relaxation scheme these points will provide some support for pairing P with an appropriate Q. Thus the relaxation method should give better results under rotation than the method of [1], and this is indeed the case (cf. Figures 12-13).

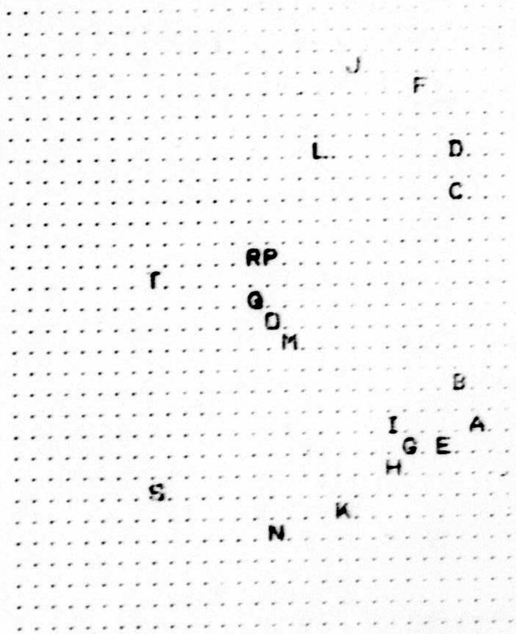
In summary, the relaxation approach deserves consideration as a method of evaluating point pattern matches under distortion.

References

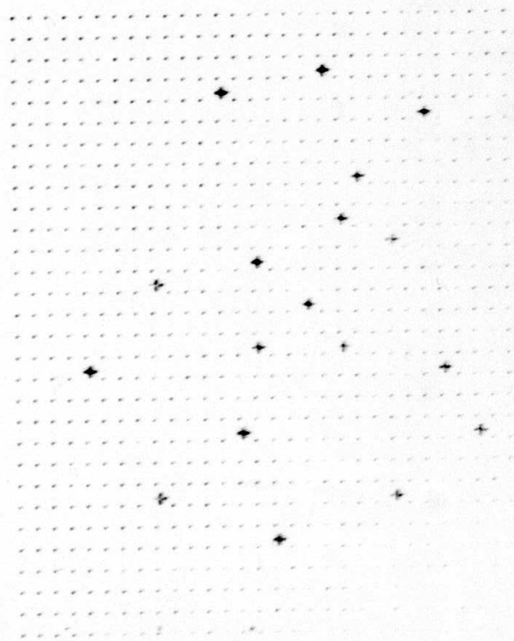
1. D.J. Kahl, A. Rosenfeld, and A. Danker, Some experiments in point pattern matching, Univ. of Md. Computer Science Ctr. TR-690, September 1978.

2. L.S. Davis and A. Rosenfeld, An application of relaxation labelling to spring-loaded template matching, Proc. 3IJCPR, November 1976, 591-597.

a)



b)



c)

ipoint			match		confidence			next best
A	10	28	10	28	0.12	18	31	0.03
B	12	27	13	26	0.10	10	28	0.10
C	21	27	19	23	0.05	18	31	0.05
D	23	27	25	25	0.08	20	20	0.03
E	9	26	10	28	0.09	18	31	0.03
F	26	25	25	25	0.10	20	20	0.02
G	9	24	7	23	0.11	10	28	0.06
H	8	23	7	23	0.10	10	28	0.05
I	10	23	7	23	0.08	10	28	0.05
J	27	21	27	19	0.1	25	25	0.05
K	6	20	7	23	0.07	13	26	0.03
L	23	19	22	21	0.1	20	20	0.08
M	14	17	14	15	0.09	16	18	0.09
N	5	16	5	16	0.10	7	23	0.03
O	15	16	14	15	0.10	16	18	0.08
P	18	16	18	15	0.1	16	18	0.09
Q	16	15	18	15	0.09	14	15	0.09
R	18	15	18	15	0.09	16	18	0.06
S	7	9	7	9	0.09	10	14	0.03
T	17	9	17	9	0.09	18	15	0.03

Figure 2 (a-c)

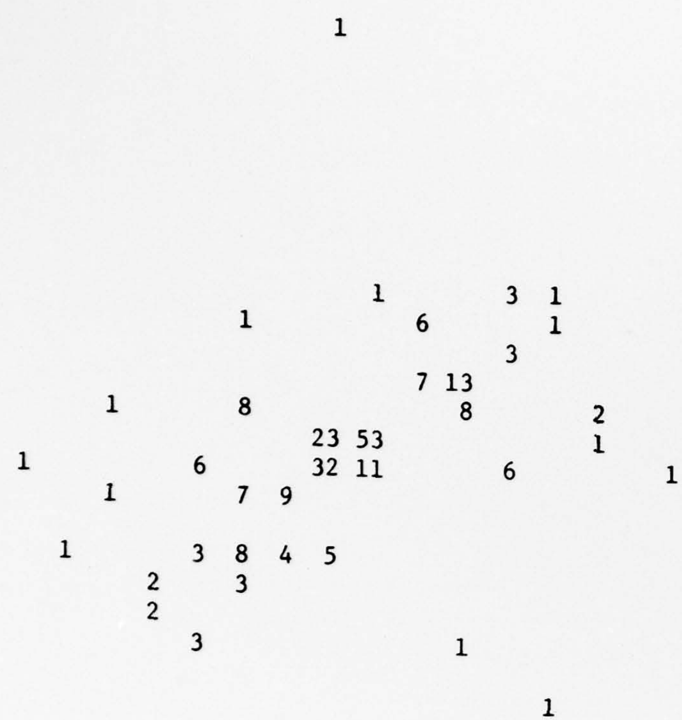
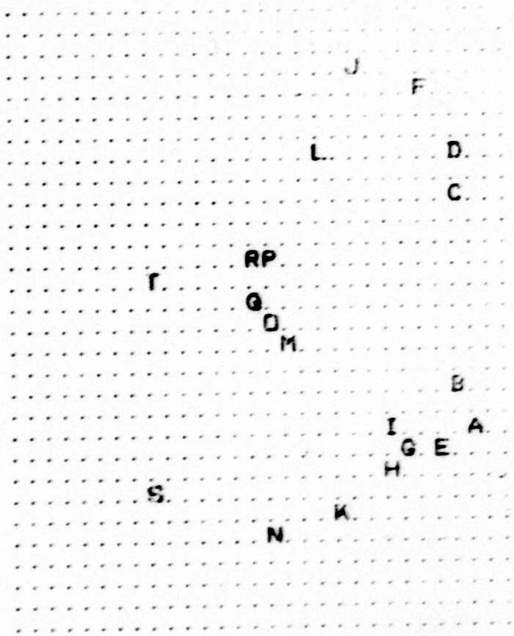


Figure 1d

a)



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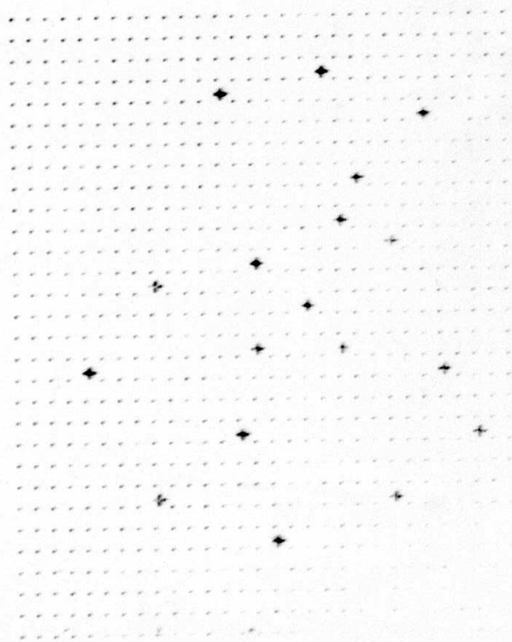


c)

ipoint			match		confidence			next best
A	10	28	10	28	0.12	18	31	0.03
B	12	27	13	26	0.10	10	28	0.10
C	21	27	19	23	0.05	18	31	0.05
D	23	27	25	25	0.08	20	20	0.03
E	9	26	10	28	0.09	18	31	0.03
F	26	25	25	25	0.10	20	20	0.02
G	9	24	7	23	0.11	10	28	0.06
H	8	23	7	23	0.10	10	28	0.05
I	10	23	7	23	0.08	10	28	0.05
J	27	21	27	19	0.1	25	25	0.05
K	6	20	7	23	0.07	13	26	0.03
L	23	19	22	21	0.1	20	20	0.08
M	14	17	14	15	0.09	16	18	0.09
N	5	16	5	16	0.10	7	23	0.03
O	15	16	14	15	0.10	16	18	0.08
P	18	16	18	15	0.1	16	18	0.09
Q	16	15	18	15	0.09	14	15	0.09
R	18	15	18	15	0.09	16	18	0.06
S	7	9	7	9	0.09	10	14	0.03
T	17	9	17	9	0.09	18	15	0.03

Figure 2 (a-c)

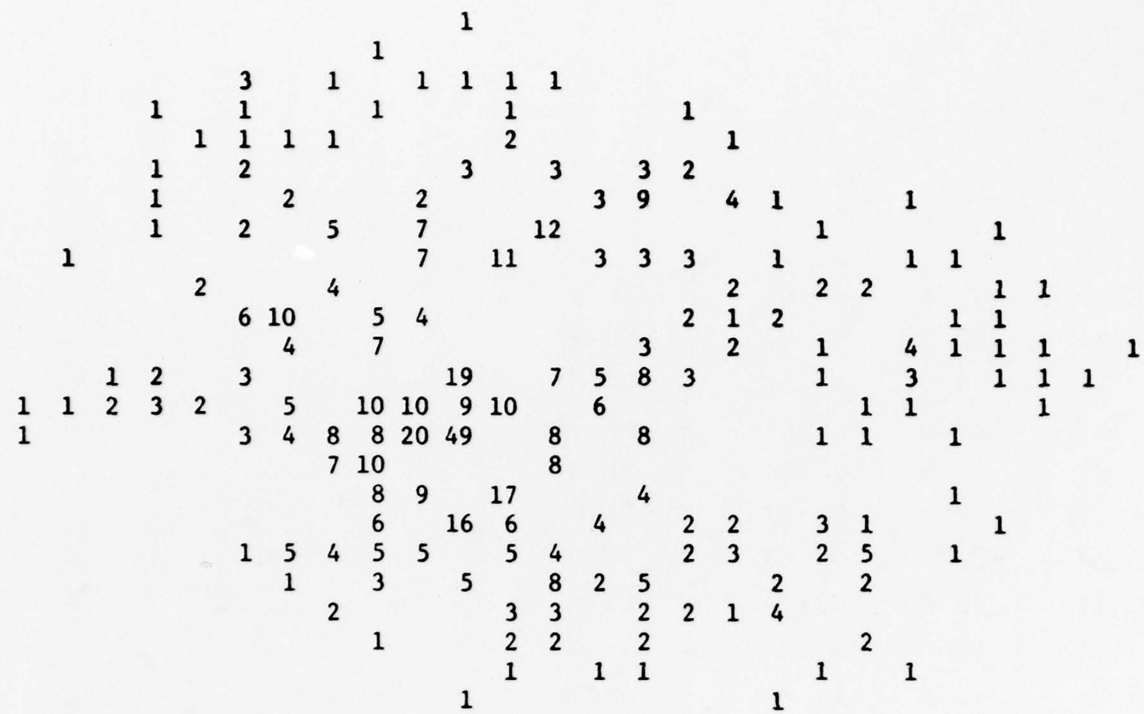
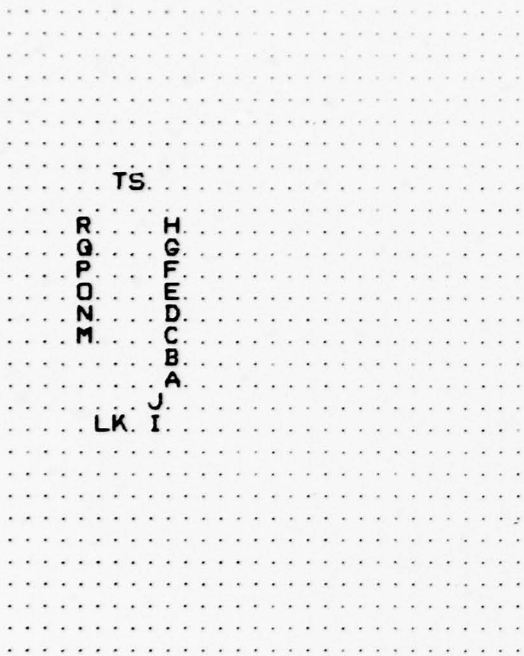
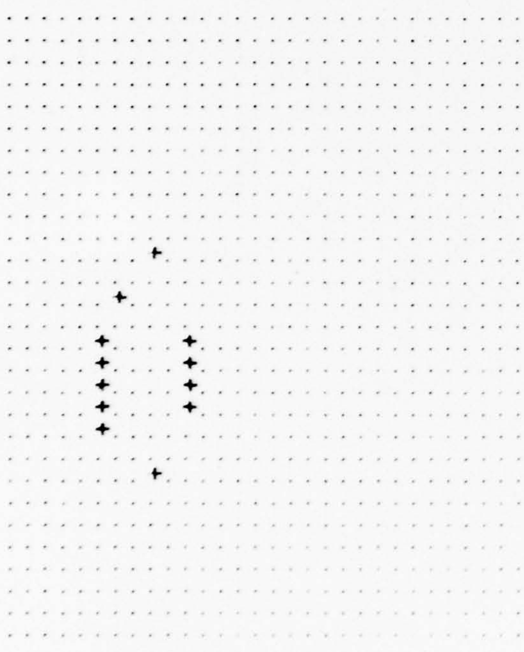


Figure 2d

a)



b)



c)

ipoint		match		confidence	next best		
A 13	10	12	11	0.26			
B 14	10	12	11	0.29			
C 15	10	12	11	0.29			
D 16	10	13	11	0.29	12	11	0.27
E 17	10	14	11	0.29	13	11	0.27
F 18	10	15	11	0.29	14	11	0.27
G 19	10	15	11	0.26	14	11	0.22
H 20	10	15	11	0.22	14	11	0.17
I 11	9	9	9	0.25	12	11	0.13
J 12	9	9	9	0.26	12	11	0.18
K 11	7	9	9	0.26	12	11	0.07
L 11	6	9	9	0.19	12	11	0.04
M 15	5	12	6	0.27	11	6	0.26
N 16	5	13	6	0.27	12	6	0.26
O 17	5	14	6	0.27	13	6	0.25
P 18	5	15	6	0.27	14	6	0.25
Q 19	5	17	7	0.25	19	9	0.09
R 20	5	17	7	0.25	19	9	0.10
S 22	8	19	9	0.26	15	11	0.08
T 22	7	19	9	0.25	15	11	0.05

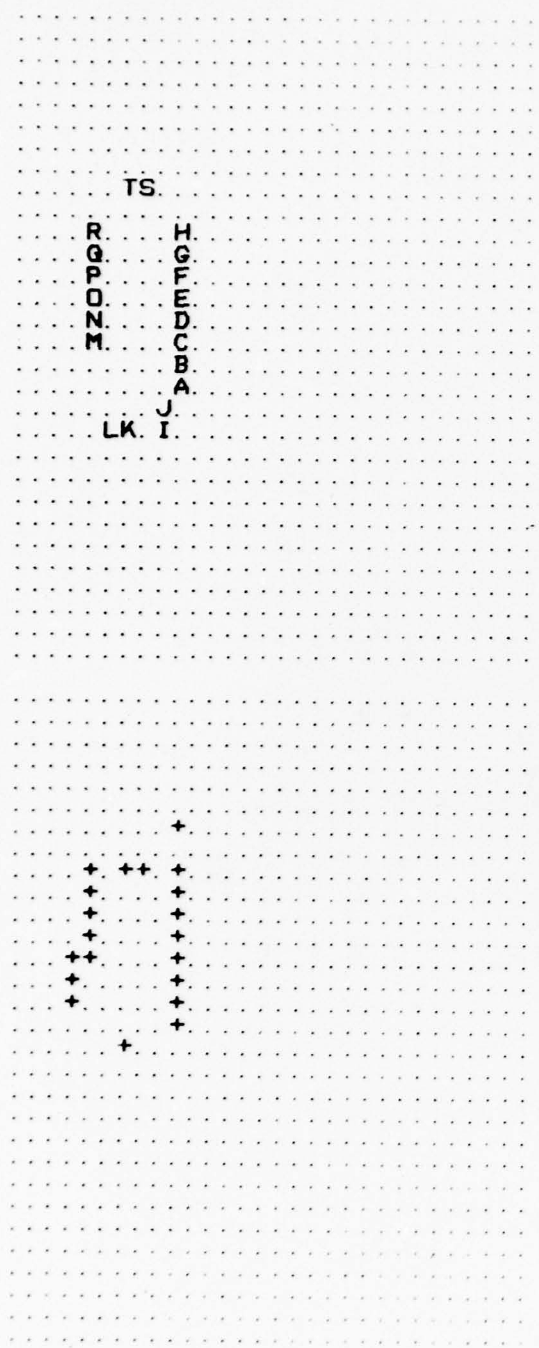
Figure 3 (a-c)

		1	4	8	12	14	14	12	13	13	11	6	2						
								4	4	9	13	13	11	9	5				
1	2	4	5	7												2	1		
3	7	10	15	22	14	32	17			14	23	19	15	11	7	2	1		
						19		25	25		16	12	9	6	4		1	1	
		6	27	63	116	191	233	250	196	147	74	27	10						1
								50	51	22	35	38	29	13	4				
	2	4	5	7					19										
	3	5	7	9	5					10	8	14	11	8	6				
												4	4	3	2				
			1	3	4	4	3	6	5	5	2								

Figure 3d

a)

b)



c)

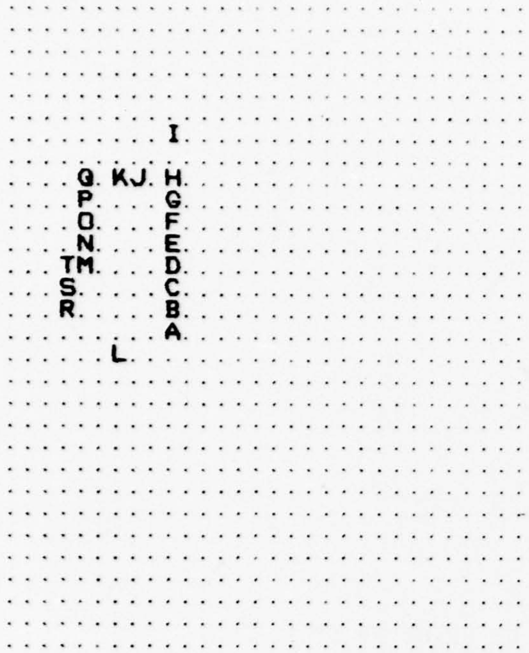
ipoint			match		confidence	next best		
A	13	10	16	10	0.39	15	10	0.38
B	14	10	17	10	0.39	16	10	0.38
C	15	10	18	10	0.38	17	10	0.37
D	16	10	19	10	0.38	18	10	0.37
E	17	10	20	10	0.38	19	10	0.37
F	18	10	21	10	0.37	20	10	0.36
G	19	10	22	10	0.37	21	10	0.36
H	20	10	22	10	0.36	21	10	0.33
I	11	9	15	10	0.33			
J	12	9	15	10	0.36			
K	11	7	14	7	0.36	15	10	0.14
L	11	6	14	7	0.36	15	10	0.08
M	15	5	18	5	0.35	14	7	0.13
N	16	5	19	5	0.35	18	5	0.35
O	17	5	20	5	0.35	19	5	0.34
P	18	5	21	5	0.35	20	5	0.34
Q	19	5	22	5	0.34	21	5	0.34
R	20	5	22	5	0.34	21	5	0.31
S	22	8	22	8	0.27	24	10	0.21
T	22	7	22	7	0.26	22	8	0.25

Figure 4 (a-c)

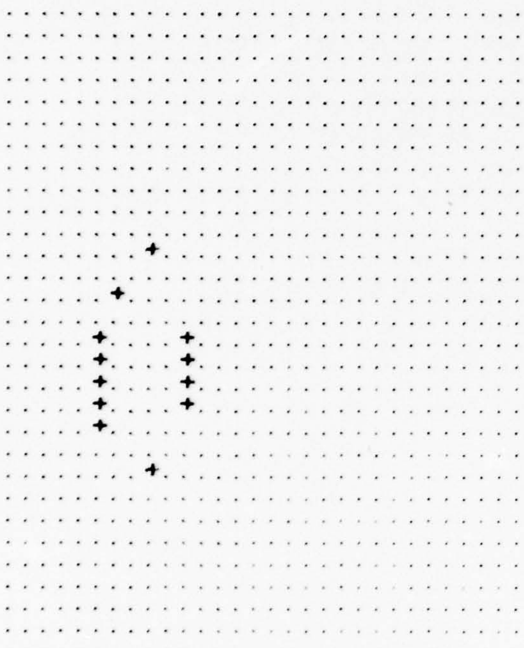
			1	4	8	9	10	10	10	10	7	3								
					4	9	15	22	28	27	31	33	26	15	6	2				
	1	2	4										5	9	7	6	4	1		
	4	8	16	15	19	23	26	14	14	14	11	19	15	12	4	3				
			7	9	13	16	19		43	40	17	29	25	30	15	11	7	5		
			7	20	44	64	107	91	65	32				13	10	7	10	6		
		5	16	34	77	151	305	361	464	475	379	257	148	76	35	17	5	7		
							24			72	65	53	41	28	21	15	10	10	2	1
1	5	8	11	16	20	26	15		41	22	21	19	15	11			2			
1	2	3	4	5	6	7	8		26	14	28	25	20	17	6	4	3	1		
											7	6	5	5	3	3	2	1		
		1	3	7	11	14	19	23	29	27	25	17	11	6	2	1				

Figure 4d

a)



b)



c)

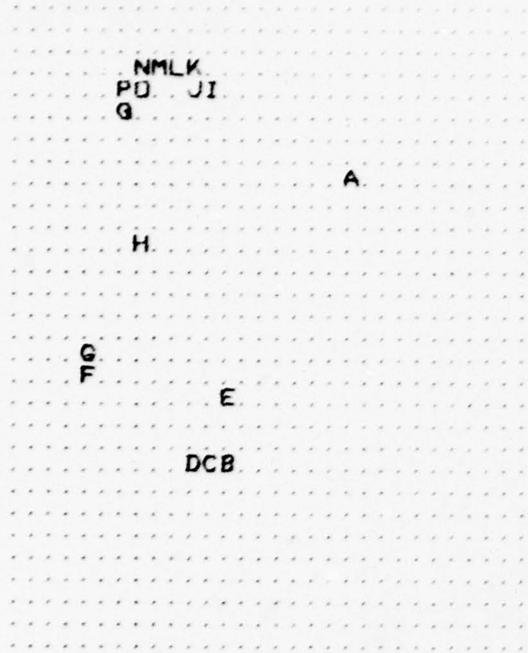
ipoint		match	confidence	next best	
A	15 10	12 11	0.25		
B	16 10	12 11	0.28		
C	17 10	12 11	0.28		
D	18 10	13 11	0.28	12 11	0.27
E	19 10	14 11	0.27	13 11	0.27
F	20 10	15 11	0.27	14 11	0.27
G	21 10	15 11	0.26	14 11	0.24
H	22 10	15 11	0.23	14 11	0.20
I	24 10	19 9	0.17	15 11	0.15
J	22 8	19 9	0.24	15 11	0.17
K	22 7	17 7	0.24	19 9	0.23
L	14 7	9 9	0.25	12 11	0.10
M	18 5	13 6	0.26	12 6	0.26
N	19 5	14 6	0.26	13 6	0.25
O	20 5	15 6	0.26	14 6	0.25
P	21 5	15 6	0.25	17 7	0.24
Q	22 5	17 7	0.24	19 9	0.11
R	16 4	12 6	0.24	11 6	0.24
S	17 4	13 6	0.24	12 6	0.24
T	18 4	14 6	0.24	13 6	0.24

Figure 5 (a-c)

	1	4	8	13	14	19	19	15	17	11	8	4	1			
						5		5	5	4	3	3	2	1	1	
		4	6	8	0	0										
2	4	12	18	23	28	31	17	33		28	23	16	11	6	2	1
																1
3	5	15	45	87	136	207	207	187	162	147	61	30	10			
						21	45	120	96	110	55	29				
			7	10	14	16						11	8	5		
1	2	3	10	12	7	9				11	20	16	12	8		
				2	3	3								3	2	1
			2	5	7	9	8	6	5	2						
							1	2	4	6	5	2				

Figure 5d

a)



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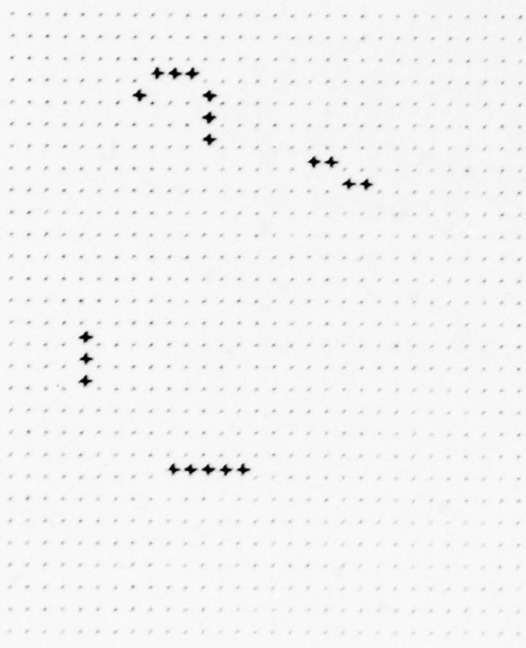


c)

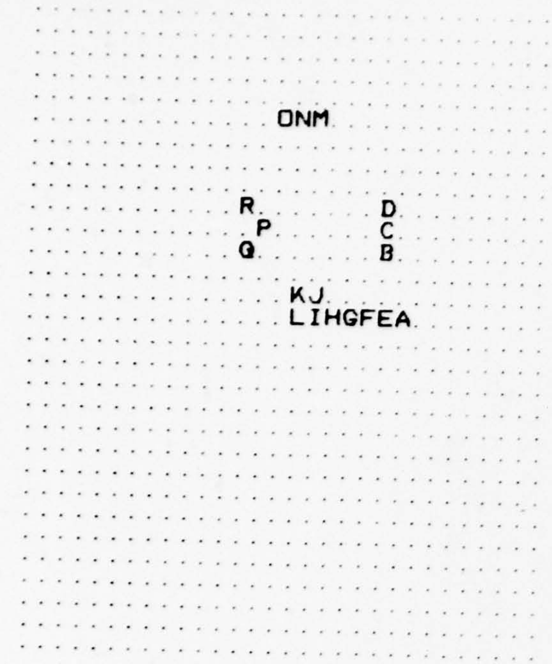
ipoint	match	confidence	next best	
A 22 20	22 21	0.34	22 20	0.34
B 9 13	9 14	0.34	9 13	0.34
22 20	0.00			
C 9 12	9 13	0.33	9 14	0.29
D 9 11	9 12	0.33	9 13	0.29
E 12 13	9 13	0.13	9 14	0.13
F 13 5	13 5	0.32	9 10	0.05
G 14 5	14 5	0.32	13 5	0.31
H 19 8	15 5	0.02	14 5	0.00
I 26 12	26 12	0.31	23 18	0.05
J 26 11	26 12	0.31	23 18	0.04
K 27 11	27 11	0.30	26 12	0.30
L 27 10	27 11	0.30	26 12	0.25
M 27 9	27 10	0.30	27 11	0.27
N 27 8	27 9	0.29	27 10	0.27
O 26 8	26 8	0.29	27 9	0.28
P 26 7	26 8	0.29	27 9	0.25
Q 25 7	26 8	0.28	27 9	0.17

Figure 6 (a-c)

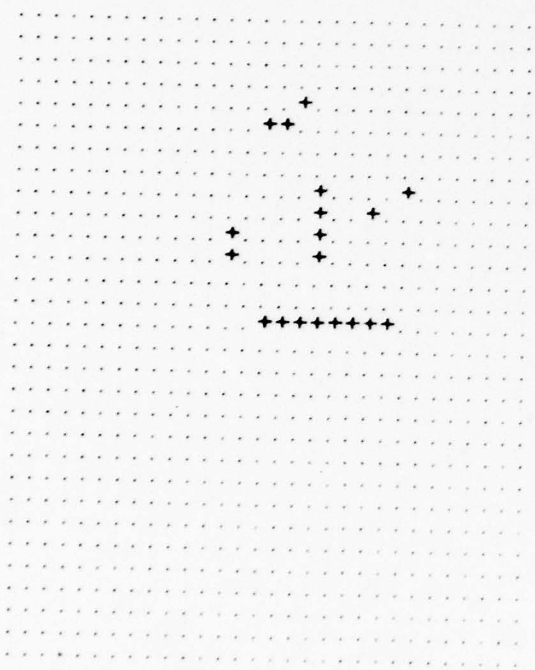
		1									
		2	1								
									8		
					1	5		13	27	12	
	1					8		19	66	61	
1	2					11		26	147	80	
2	3					13	22	88	347	91	22
3	3					25	42	59	281	55	
4	4					11	17	25	112	49	17
4	3					9	13	18	38	34	11
3	1					7	17	21	10	9	6
2	1			4	5		5	13	14	5	
	1			3	4	4					
				2	6	8					
				3	11	3					
				6	15						
				5	7	2					
				5	2	4	1				
				4	2	1	1				
				1	2	1					

Figure 6d

a)



b)



c)

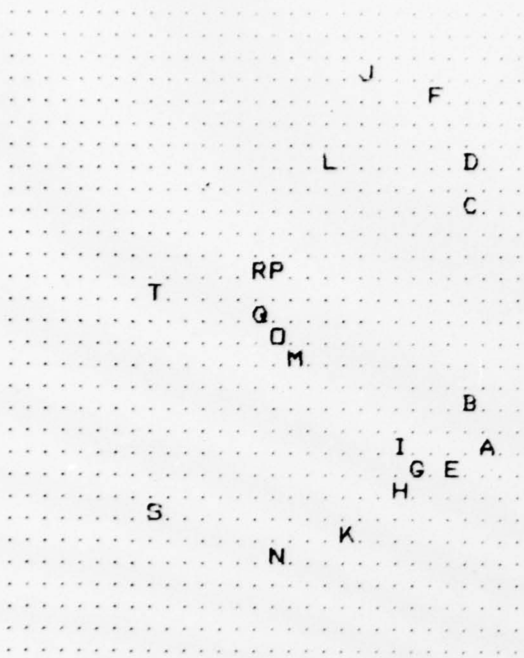
ipoint	match	confidence	next best	
A 16 22	16 22	0.45		
B 19 21	19 18	0.27	21 21	0.24
C 20 21	21 21	0.40	16 21	0.07
D 21 21	21 21	0.45	16 22	0.05
E 16 21	16 21	0.44	16 22	0.40
F 16 20	16 20	0.44	16 21	0.40
G 16 19	16 19	0.44	16 20	0.40
H 16 18	16 18	0.43	16 19	0.39
I 16 17	16 17	0.43	16 18	0.39
J 17 17	16 17	0.41	16 18	0.34
K 17 16	16 16	0.40	16 17	0.34
L 16 16	16 16	0.43	16 17	0.39
M 25 17	25 16	0.41	26 17	0.38
N 25 16	25 16	0.42	26 17	0.33
O 25 15	25 15	0.42	25 16	0.38
P 20 14	20 13	0.41	19 13	0.38
Q 19 13	19 13	0.41	16 15	0.11
R 21 13	20 13	0.39	19 13	0.23

Figure 7 (a-c)

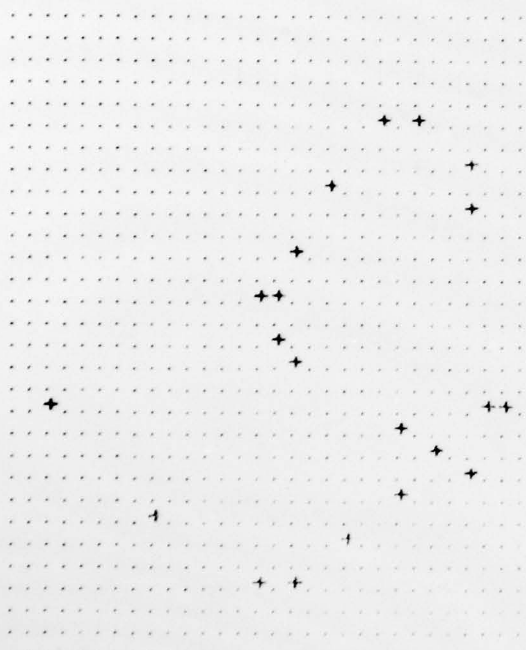
										1	1						
					1	3	4	2		2	2						
							2			3	3					1	
			1	2			13			6	10	4	3			2	
		1	1	4			34			7	12	6	5			4	1
	1	3	2	6			71		12	24	15	13	12	5		5	1
	1	3	3	9	17	50	216	69	45	38	15	6	6			6	2
	1	5	5	11		31	251			11	8	7	6		3	5	1
		3	6	13		115	419			13	10	19	7		8	4	1
		4	6	12	23	120	476	116	24	16	12	19	7		5	5	1
1	3	9	19	24		67	275	33	23	32	26	41	18		3	1	
1	3	8	11	21		49	147	50	39	46	37	40	26				
	3	8	11	19		37	81				11	9	24				
		3	11	8		37	53	14	12		8	15	7	6			
		2	8	6	7	28	40	18	7	6	6	5	6				
		2	6	10		6	6					4	4				
		1	2	7				4				3	3				
		1	1	4			3		1								
				1													

Figure 7d

a)



b)



c)

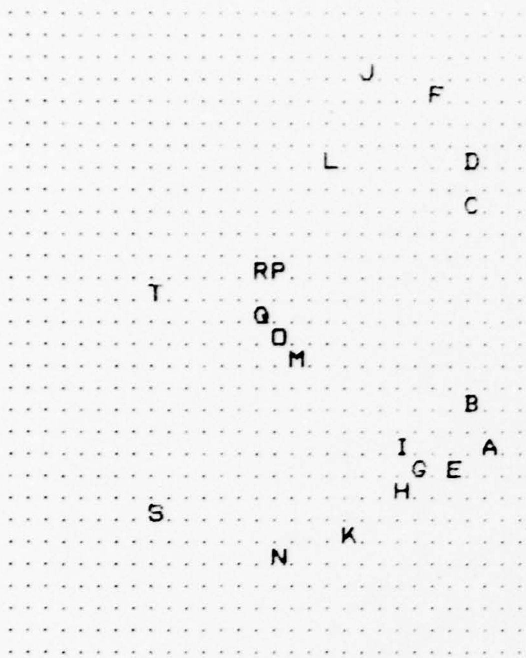
ipoint	match	confidence		next best
A 10 28	9 27	0.25	12 28	0.17
B 12 27	12 28	0.25	12 29	0.21
C 21 27	21 27	0.26	12 28	0.00
D 23 27	23 27	0.26	21 27	0.23
E 9 26	9 27	0.25	12 28	0.10
F 26 25	25 24	0.24	23 27	0.1
G 9 24	8 23	0.24	10 25	0.22
H 8 23	8 23	0.25	10 25	0.14
I 10 23	8 23	0.22	10 25	0.20
J 27 21	25 22	0.20	25 24	0.1
K 6 20	6 20	0.25	8 23	0.09
L 23 19	22 19	0.24	25 22	0.09
M 14 17	14 17	0.24	11 23	0.01
N 5 16	4 17	0.23	4 15	0.23
O 15 16	15 16	0.24	14 17	0.23
P 18 16	17 16	0.24	19 17	0.21
Q 16 15	15 16	0.23	14 17	0.16
R 18 15	17 15	0.23	17 16	0.23
S 7 9	7 9	0.23	4 15	0.01
T 17 9	17 15	0.02	17 16	0.02

Figure 8 (a-c)

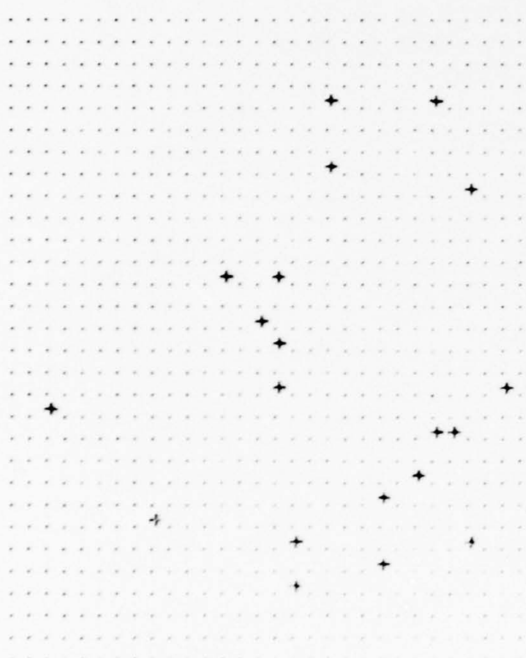
	2												
		1											
1			1										
	1												
			2	2									
					7			2	2		1		
	1		3	4		5							
	1		4		16	22	11		13		2		
		2	6		16					6			
						118		40	29	9			
				30	44	71	172	42	50	11		4	
			7	14	20	91	50	63	15		7		
		3	6	9	15		41	16	13	19			
					9		13		18	7	5	3	
						7		15		5	4		1
									9	4	7		
				2			2		3		2		
							1			3			
									1				

Figure 8d

a)



b)



c)

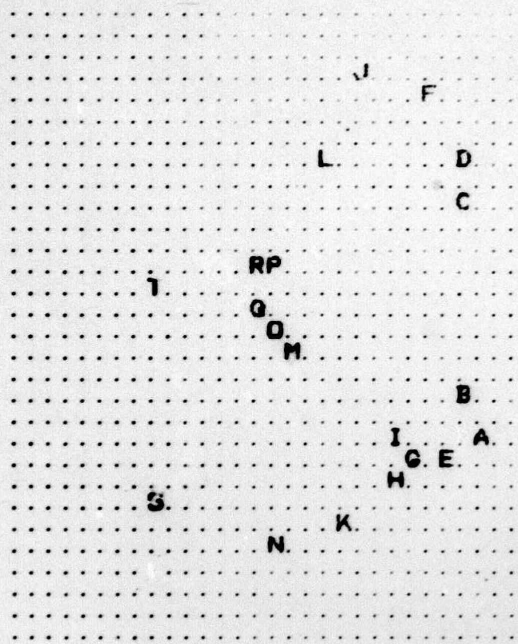
ipoint	match	confidence	next best	
A 10 28	11 26	0.16	13 29	0.12
B 12 27	11 26	0.19	13 29	0.15
C 21 27	22 27	0.18	22 27	0.18
D 23 27	22 27	0.19	22 27	0.19
E 9 26	9 24	0.17	11 26	0.16
F 26 25	26 25	0.19	22 27	0.06
22 27	0.06			
G 9 24	9 24	0.18	11 26	0.13
H 8 23	8 22	0.18	9 24	0.17
I 10 23	9 24	0.16	11 26	0.11
J 27 21	26 19	0.16	26 25	0.04
K 6 20	5 22	0.14	8 22	0.12
L 23 19	23 19	0.18	26 19	0.12
M 14 17	13 16	0.17	22 27	0.00
22 27	0.00			
N 5 16	6 17	0.16	8 22	0.02
O 15 16	15 16	0.17	13 16	0.16
P 18 16	18 16	0.17	15 16	0.12
Q 16 15	16 15	0.17	15 16	0.15
R 18 15	18 16	0.16	15 16	0.11
S 7 9	7 9	0.16	13 16	0.00
T 17 9	18 13	0.06	16 15	0.00

Figure 9 (a-c)

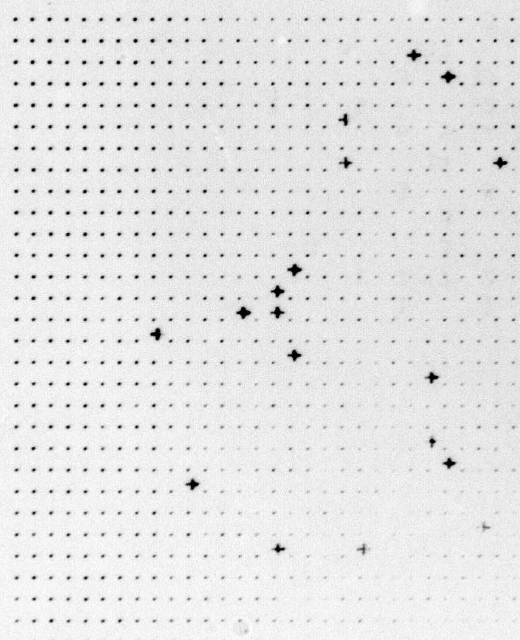
			1		1											
		1	2	2	2	4		2								
	1			3												
				3			15					2				
			3		8	9		24	12		5					
				14			47	33	15	23	7		3			
			6	9	12	30	36	17	33	14		5				
1		5	6		12	31	37	122	36	16	23					
			5		35		47	16	33	29	12					
				12			14		30	25	9					
					5				10		7	6	2			
				2		4	4		5		5		1		1	
												2	1			
								1			5		3			
								1								
																1

Figure 9d

a)



b)



c)

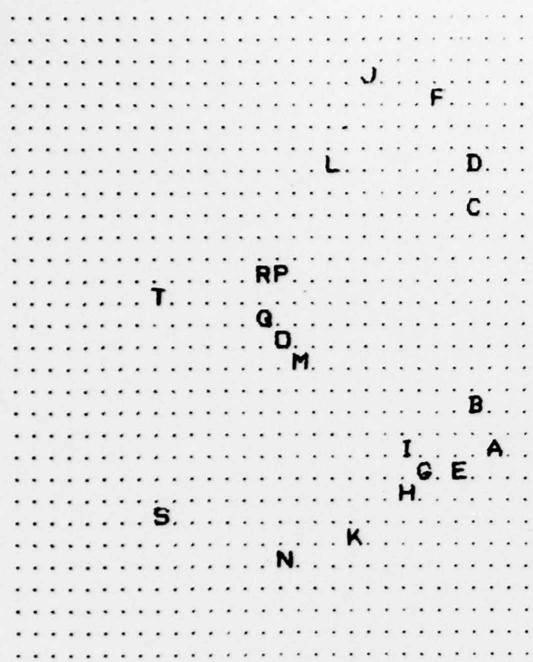
ipoint			match		confidence	next best		
A	10	28	9	26	0.09	9	26	0.09
B	12	27	13	25	0.08	10	25	0.08
C	21	27	23	29	0.13	10	25	0.00
D	23	27	23	29	0.15	10	25	0.00
E	9	26	9	26	0.15	9	26	0.15
F	26	25	27	26	0.15	23	29	0.04
G	9	24	10	25	0.15	6	28	0.04
H	8	23	10	25	0.12	6	28	0.03
I	10	23	10	25	0.14	6	28	0.02
J	27	21	28	24	0.12	27	26	0.05
K	6	20	5	21	0.14	6	28	0.01
L	23	19	23	20	0.14	25	20	0.13
M	14	17	14	17	0.14	13	25	0.01
N	5	16	5	16	0.14	5	21	0.05
O	15	16	16	16	0.14	14	17	0.14
P	18	16	18	17	0.14	16	16	0.11
Q	16	15	16	16	0.14	14	17	0.11
R	18	15	17	16	0.13	18	17	0.12
S	7	9	8	11	0.12	5	16	0.02
T	17	9	15	9	0.11	16	14	0.04

Figure 10 (a–c)

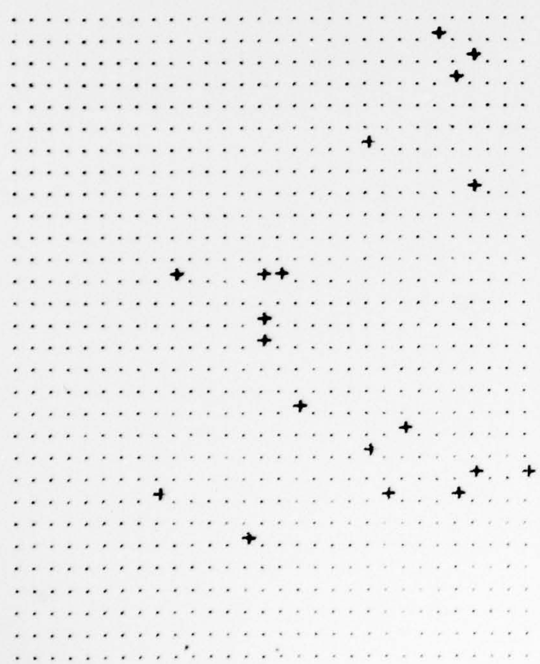
			1										
		3		2			2		1				
1	1			1		1							
1			2	2		2							
						4							
	1		3				6		4	3		1	
		5		4	14	27		16					
			4	22	18		11	13	19	6	9		1
			6		21	12	73	13	11		5		
			6		11	41	41	42	13	9	6		
			5	8	11		84	11	36	9		4	
						36		47					
				8									
			1		3	4	9						
										2			
					1	1	1						
			1			1	1						
		1											

Figure 10d

a)



b)



c)

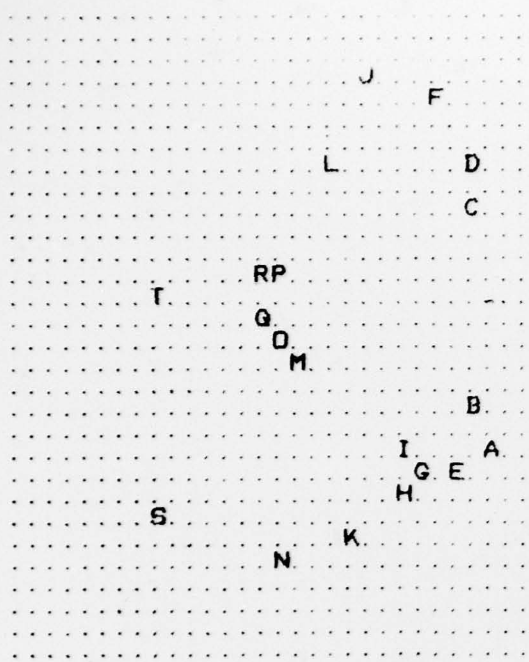
ipoint	match	confidence		next best
A 10 28	9 27	0.13	9 30	0.12
B 12 27	9 27	0.08	11 23	0.07
C 21 27	22 27	0.13	11 23	0.00
D 23 27	22 27	0.13	11 23	0.00
E 9 26	9 27	0.14	8 26	0.13
F 26 25	27 26	0.13	28 27	0.1
G 9 24	11 23	0.12	8 26	0.12
H 8 23	8 22	0.13	11 23	0.10
I 10 23	11 23	0.12	8 26	0.07
J 27 21	24 21	0.08	29 25	0.05
K 6 20	8 22	0.09	10 21	0.06
10 21	0.06			
L 23 19	24 21	0.10	22 27	0.01
M 14 17	15 15	0.11	12 17	0.10
N 5 16	6 14	0.11	8 22	0.01
O 15 16	15 15	0.12	12 17	0.07
P 18 16	18 16	0.12	15 15	0.07
Q 16 15	16 15	0.12	15 15	0.12
R 18 15	18 15	0.12	18 16	0.12
S 7 9	8 9	0.11	6 14	0.03
T 17 9	18 10	0.11	16 15	0.02

Figure 11 (a-c)

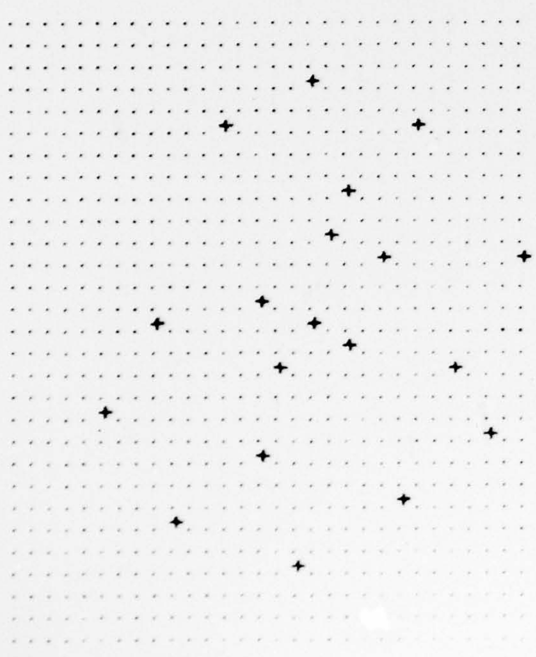
									1				
											1		
						7			5				
				7		7	4	3	6	2			
		1				4	16	4					
				3	12								
	1						17	8					
				9	10	21	21	28		4		1	
		6	7	19	12	36	11	11	8	13		2	
			22	19	37	36	36	10	30		4		1
2			6			25	23	10		12			
1		9			23		10	19					
			5	6		18					1		
		4			4	6	6	4					
	2				3	3					1		
			1	1	2	2	4	1	1				
					1		3						
					1								

Figure 11d

a)



b)



c)

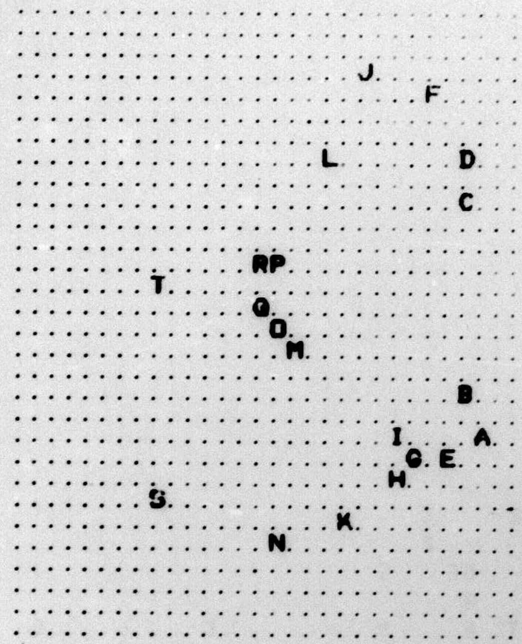
ipoint			match		confidence	next best		
A	10	28	11	28	0.12	19	30	0.03
B	12	27	11	28	0.11	19	30	0.03
C	21	27	19	30	0.07			
D	23	27	25	24	0.05	20	19	0.03
E	9	26	8	23	0.08	11	28	0.07
F	26	25	25	24	0.11	20	19	0.02
G	9	24	8	23	0.11	11	28	0.05
H	8	23	8	23	0.11	11	28	0.04
I	10	23	8	23	0.11	11	28	0.04
J	27	21	27	18	0.07	25	24	0.07
K	6	20	5	17	0.07	8	23	0.05
L	23	19	22	20	0.10	20	19	0.09
M	14	17	14	16	0.11	15	20	0.08
N	5	16	5	17	0.10	8	23	0.03
O	15	16	14	16	0.10	16	18	0.08
P	18	16	17	15	0.1	16	18	0.08
Q	16	15	17	15	0.1	14	16	0.09
R	18	15	17	15	0.1	16	18	0.06
S	7	9	7	10	0.09	10	15	0.02
T	17	9	16	9	0.09	17	15	0.03

Figure 12 (a-c)

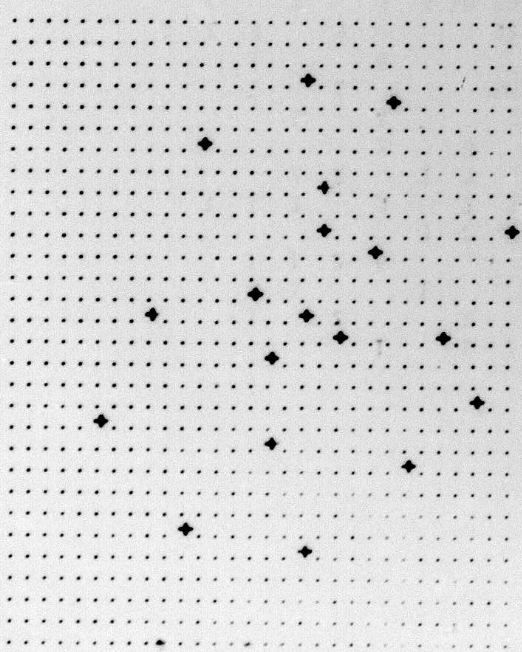
												1													
								1		1	1														
					1		2			1			1	1											
					3	1			1				1				1	1							
					1	1	2	4					3			3		1							
									3	2	4	3		5			3		1	1					
				1	1	2		3		2	3		11		3	2		1				1	1		
						2	2		2	3		3		5				1	1		1				
								5		8						2		1	1	1			2		
						1		4										2		1		3	1	1	1
					1		1				15	7		5		4	5		4			1		1	1
					1			4							4	4			2		1	3			
			1	1	3		5				32	10		17		4	3				1		1		
				1		2		5	8	10	29	10	21				3				4				
1		2		2				4		9	21	19		7											
										8			8	7			3	2			4		1	1	
								5		19		7	7	10		3	2	2	3	2	1		1		
									2			6		8		6			2	1		4			
									2		3		4	4	4	2	2	1							
												2	3		2	2				2		3			
													2		4				1		1		2		
																	1					1			
																					1				

Figure 12d

a)



b)



c)

ipoint	match	confidence		next best
A 10 28	12 28	0.08	20 30	0.03
B 12 27	12 28	0 11	20 30	0.03
C 21 27	20 30	0.08		
D 23 27	20 30	0.05		
E 9 26	9 24	0.09	12 28	0.06
F 26 25	26 23	0.09	20 19	0.02
G 9 24	9 24	0.11	12 28	0.04
H 8 23	9 24	0.10	12 28	0.03
I 10 23	9 24	0.11	12 28	0.04
J 27 21	26 23	0.08	20 19	0.01
K 6 20	5 18	0.08	9 24	0.04
L 23 19	22 19	0.10	20 19	0.07
M 14 17	14 16	0.09	16 18	0.08
N 5 16	5 18	0.09	15 20	0.02
O 15 16	14 16	0.1	16 18	0.08
P 18 16	17 15	0.09	16 18	0.08
Q 16 15	17 15	0.09	14 16	0.08
R 18 15	17 15	0.09	16 18	0.06
S 7 9	6 11	0.08	10 16	0.02
T 17 9	16 9	0.09	17 15	0.02

Figure 13 (a-c)

								1															
					1		3		1		1												
				1				1	2				2										
				2		2	1					3				1	1						
				5		4		2			3				4		1						
							3	3		9		9	3			3		2	1				
			1		2				3		4	12	4	4	3		1					1	
					2		5		3		4						2	2		2			
						2	3	2	4			4					2	3	2		1	2	
					1					5				4	3	3	5		2	1	3		1
				1	1			5			7				3	3	3	2	2				
				1		2				8	18			5		5					3		
			1	1	2		3			9	9		7	5	4					2		1	
						2	4	7		37	10	8	8			3	3			5			
2		2	2		1		4		8	10	11	10	7										
									7	15	8	8		5			5		2	2	3	1	1
							3	5	6	8	7	7	6		3	6		6	3	1		1	
								3			6		4	11	5		2				2	2	
								2		4			3		3	2							
											2	2			2					1	1	1	1
												1		4							1	1	1
															1		1						
												1											

Figure 13d

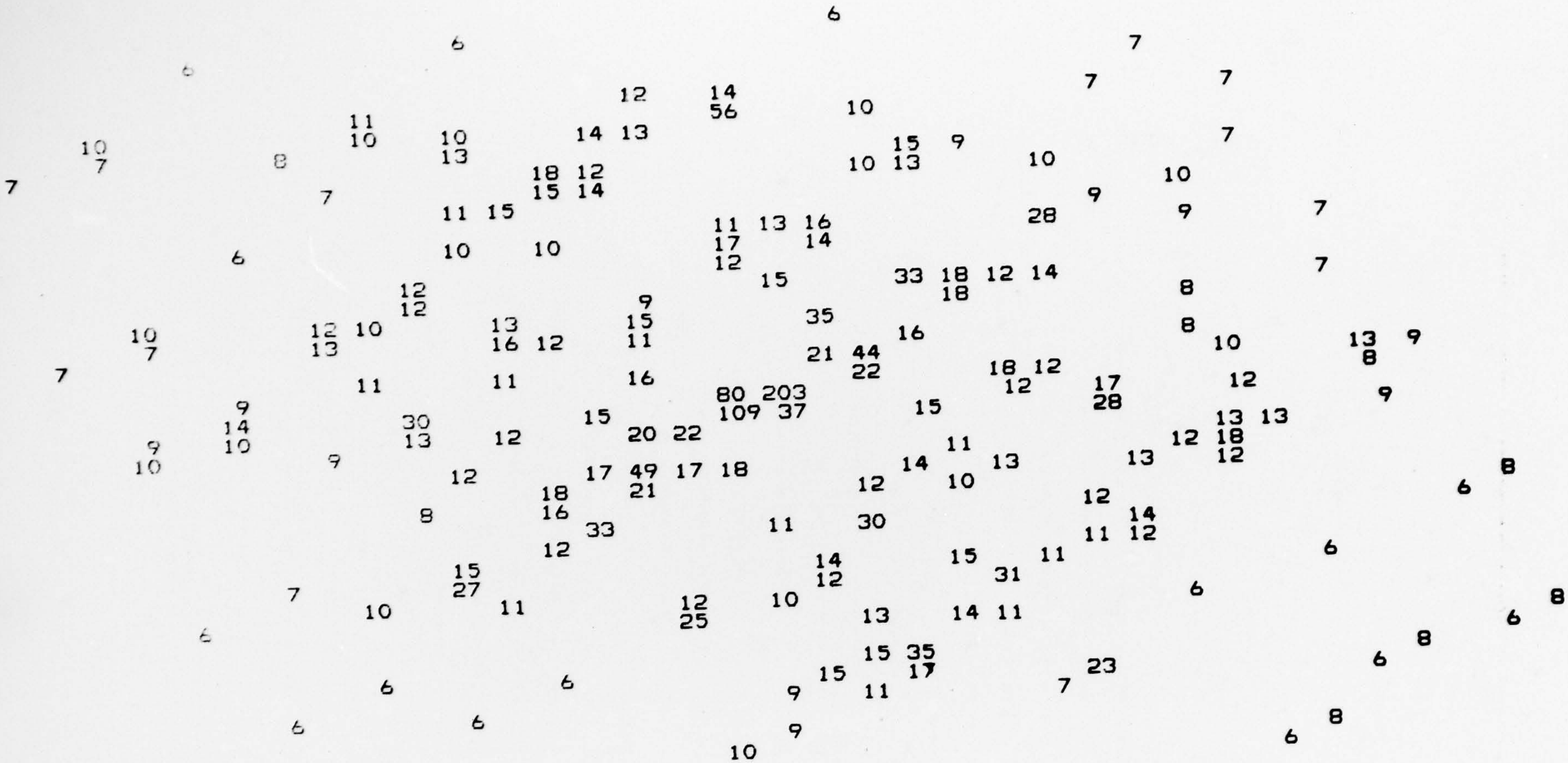


Figure 14a

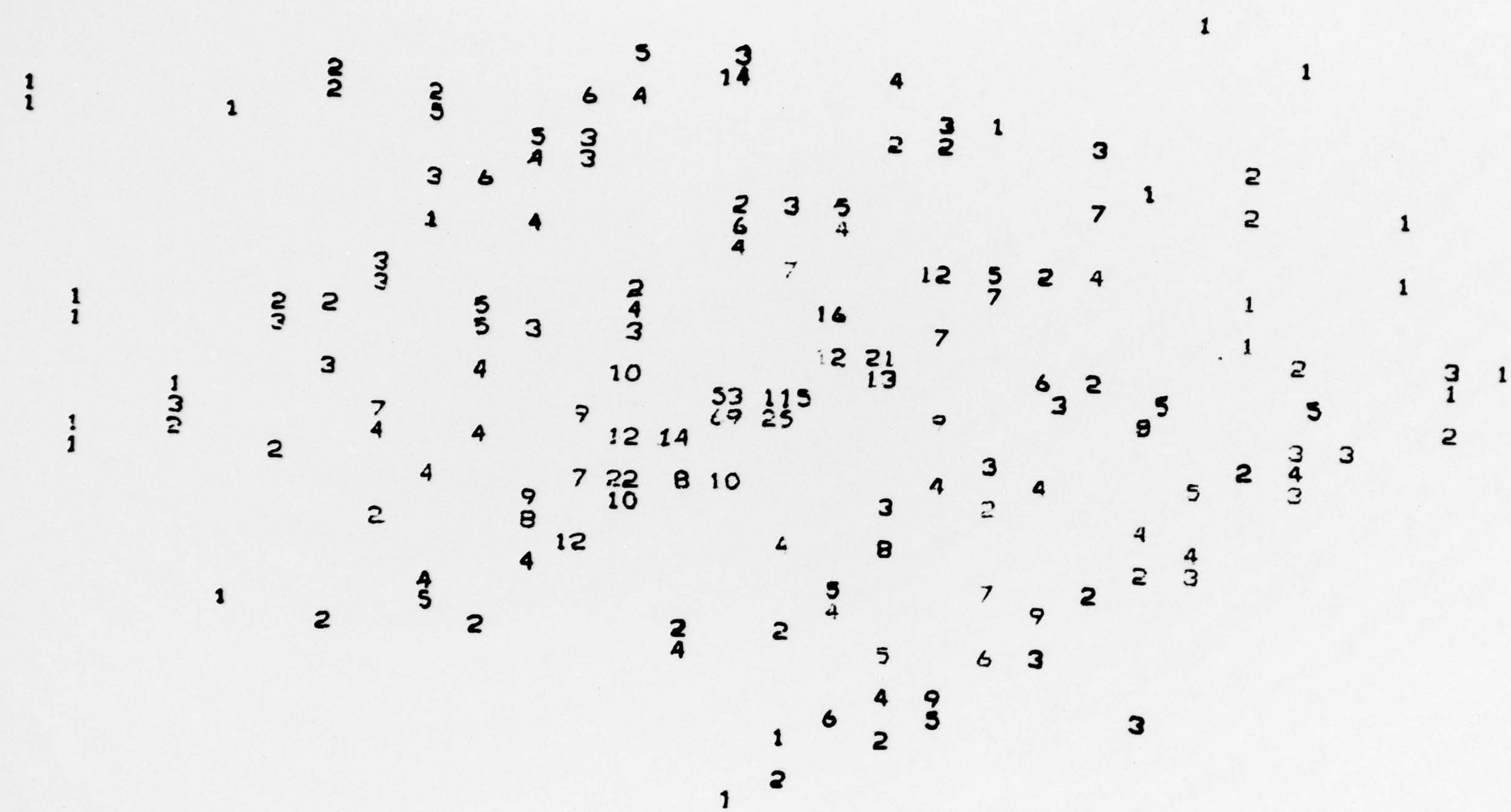


Figure 14b

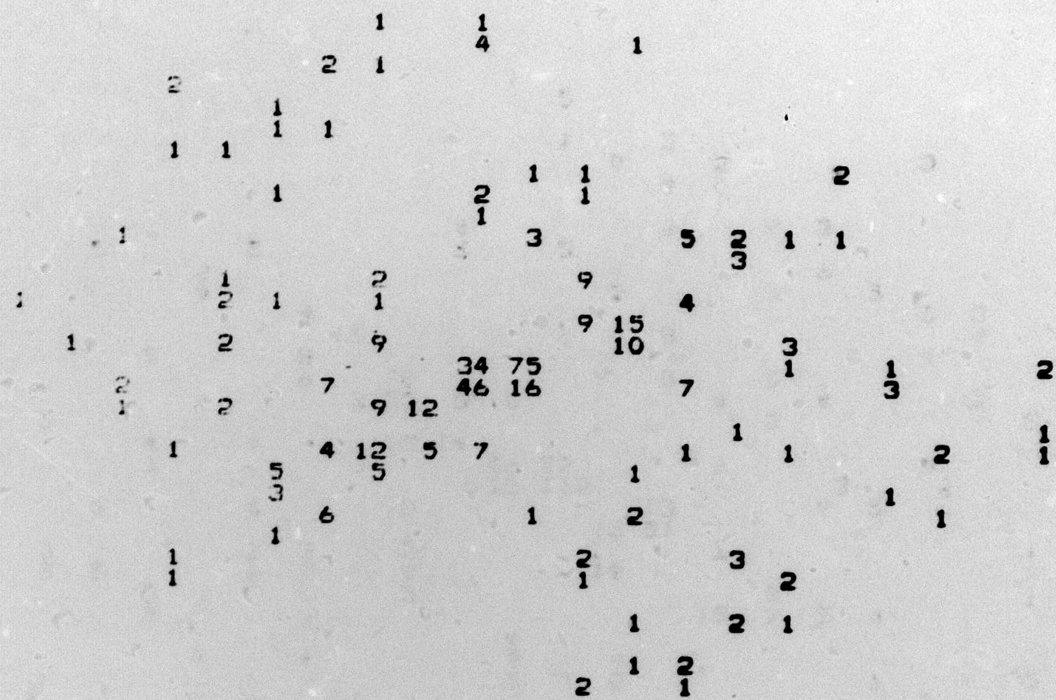


Figure 14c

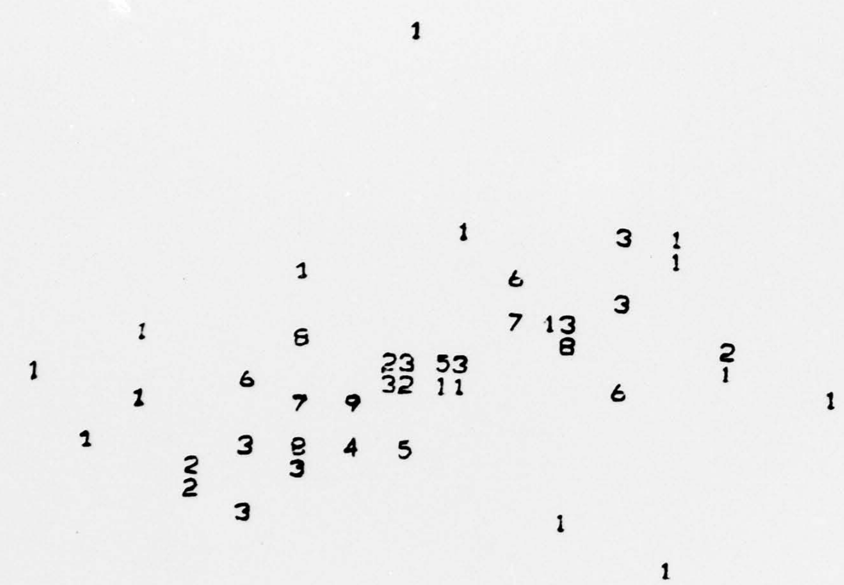


Figure 14d

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	6. PERFORMING ORG. REPORT NUMBER TR-702
7. AUTHOR(s) Sanjay Ranade, Azriel Rosenfeld	8. CONTRACT OR GRANT NUMBER(s) DAAG-53-76C-0138
9. PERFORMING ORGANIZATION NAME AND ADDRESS Computer Science Center, University of Maryland, College Park, MD 20742	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Night Vision Laboratory, Ft. Belvoir, VA 20060	12. REPORT DATE October 1978
	13. NUMBER OF PAGES 40
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
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Relaxation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Let $P=P_1,\ldots,P_m$ and $Q=Q_1,\ldots,Q_n$ be two patterns of points. Each pairing (P_i,Q_j) of a point of P with a point of Q defines a relative displacement δ_{ij} of the two patterns. We can define a figure of merit for δ_{ij} according to how closely other point pairs coincide under δ_{ij}. If there exists a displacement δ_o for which P and Q match reasonably well, the pairings for which $\delta_{ij} \approx \delta_o$ will have high merit scores, while other pairings will not. The scores can then be recomputed, giving weights to the other point pairs

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based on their own scores; and this process can be iterated. When this is done, the scores of pairs that correspond under δ_0 remain relatively high, while those of other pairs become low. Examples of this method of point pattern matching are given, and its possible advantages relative to other methods are discussed.