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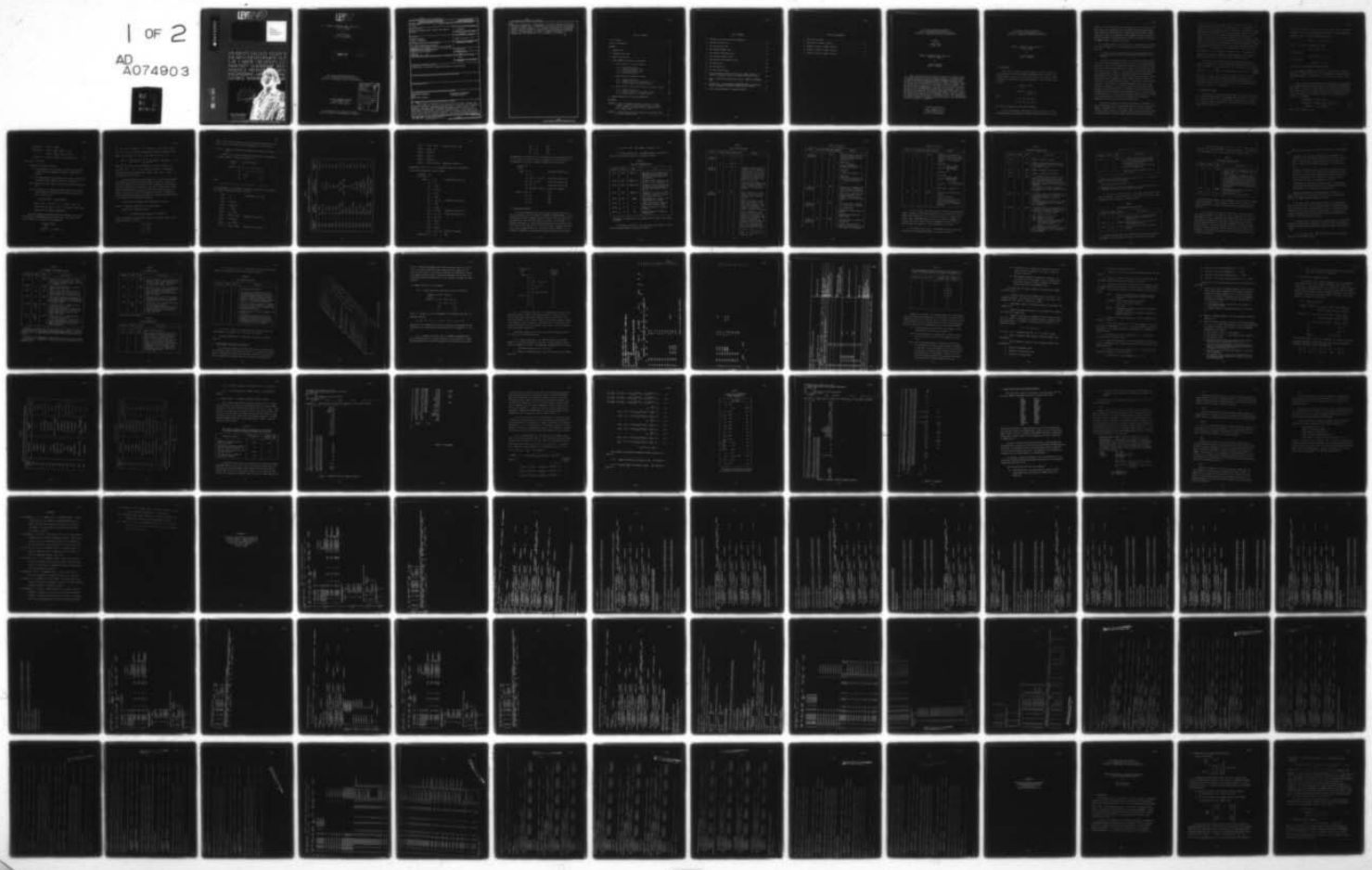

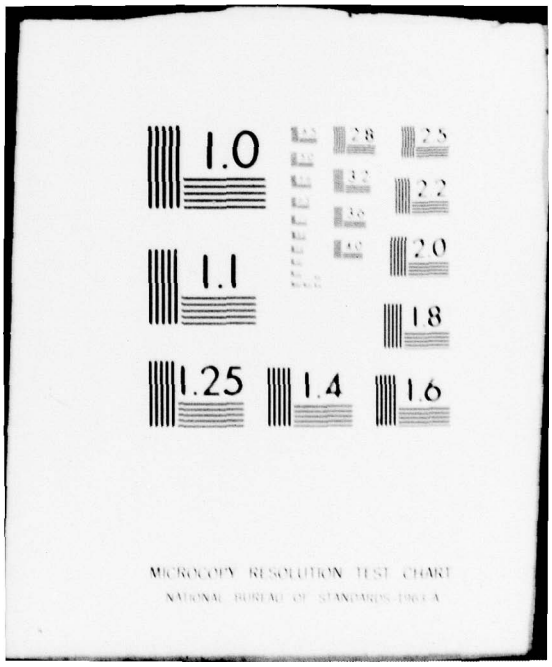
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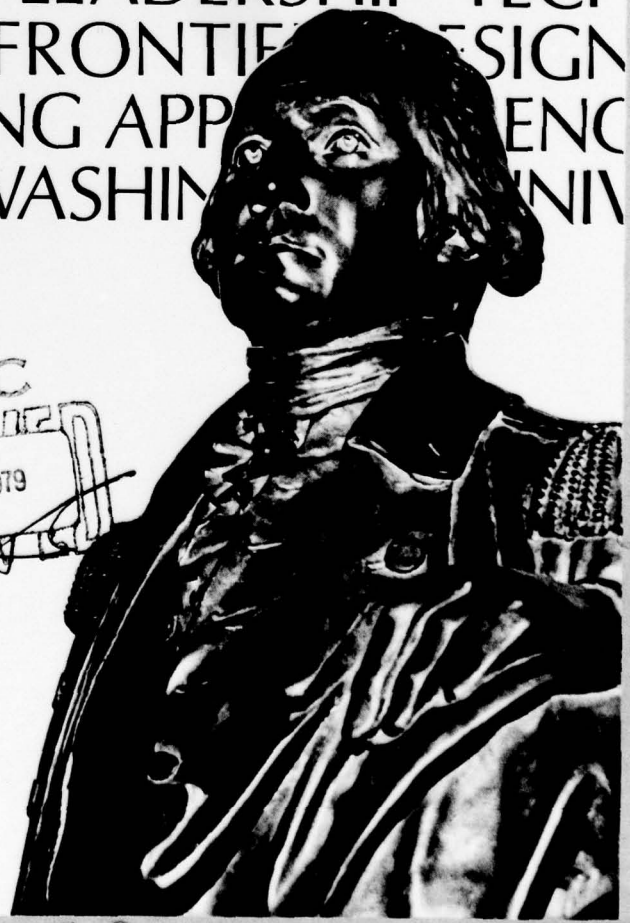

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by

Abolfazl Ghaemi
Garth P. McCormick

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This report is intended to serve as a guide for coding, solving, and conducting sensitivity and elasticity analysis on a solution vector of general parametric nonlinear programming problems using the latest version of the computer programming model of SYMBOLIC FACTORABLE SUMT. The motivation for development of this model, which uses symbolic factorable input, and the process of its evolution is briefly reviewed. The relative merits and advantages of this model over its earlier versions, as well as over the SUMT-Version 4

(Cont'd) over

model [1] is elaborated. The procedure for putting problems into symbolic factorable language that is digestible by the model, and the related model outputs, are fully explored. For illustrative purposes three increasingly difficult sample problems are coded, solved, and discussed in sufficient detail. For completeness, the theory of symbolic factorable sensitivity analysis that was developed in [4] is included as an appendix.

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1. Introduction

This report is intended to guide individuals working in the area of optimization in the use of SYMBOLIC FACTORABLE SUMT, a computer programming model, to code, solve, and conduct sensitivity analysis studies on parametric mathematical optimization problems of the following general structure:

$$\begin{aligned} &\underset{x\in E^n}{\text{minimize}} \quad F(x,y) \\ &\text{subject to} \quad G(x,y) \geq 0 \\ &\qquad\qquad\quad H(x,y) = 0 \ , \end{aligned}$$

where

$$G: \ E^n \times E^K \to E^m \ \text{ and } \ C^2$$

$$H: \ E^n \times E^K \to E^p \ \text{ and } \ C^2$$

and $y\in E^K$ corresponds to the problem parameters.

Since its development, SUMT-Version 4 [1] has undergone a series of modifications and refinements; SYMBOLIC FACTORABLE SUMT is its latest

version. The primary incentive for such modifications has been to relax the coding of the problem functions and their first and second derivatives, required by SUMT-Version 4 as users' subroutines, which in many cases have proved to be cumbersome, time consuming, costly, and prone to various derivational and coding errors. Table 13, provided in Section 4 for sample example 3, supports this claim.

The computer program reported here is an outgrowth of the program developed by McCormick [2]. The rationale for such an attempt was the fact that

> "Almost every nonlinear function can systematically be decomposed to (and hence synthesized from) simple functions of one variable."

Using this fact, which in essence constitutes the backbone structure of factorable programming--the topic of the next section--McCormick developed a series of subroutines which internally calculated the values and the first and second derivatives of single variable functions, many of which are listed in Table 12. Each of these functions was associated with a numeric code, called a transformation code, which aided their identification in the routines. Another subroutine was provided to synthesize any desired combination of these single variable functions to reconstruct the intended problem functions. Interface of these routines with those of SUMT-Version 4 relaxed the provision of the user's subroutines to the model. In fact, the user's task boiled down to decomposition of his problem functions to functions of single variables and earmarking them with appropriate transformation codes. This was a major breakthrough which to a great extent alleviated the user's task in coding and solving optimization problems, and brought about significant savings of the time and cost involved in these steps.

Yet another attempt to facilitate the coding of problems was by deSilva and McCormick [3], through which evolved the original version of SYMBOLIC FACTORABLE SUMT. The essential task in [3] was to associate relevant symbolic names with individual transformation codes which aided the identification of single variable functions in the model software.

Furthermore, this version permitted symbolic input for the problem variables and parameter names of up to four characters. In addition, it simplified the coding of the problems so that users could readily acquaint themselves with the symbolic names of various single variable functions, and could then code their problems without frequent reference to the manual. This was not the case in the earlier version, because the single variable functions had to be associated with the related transformation codes. Later, by exploiting the relative merits of factorable functions, deSilva and McCormick developed new routines to calculate a weighted cumulative sensitivity of the components of a solution vector with respect to problem parameters $\sigma^T V_y x^*$ [Appendix 2, Equation (37)]. The underlying theory for the above developments is fully explained in [4], which is included in Appendix 2 for reference by the interested user.

This latter version has recently been further modified and refined by the authors to calculate sensitivity information for the individual components of the solution vector x^* with respect to the problem parameters y (i.e., $\frac{dx_i^*}{dy_j}$) and corresponding elasticities (i.e., $\frac{dx_i^*/x_i^*}{dy_j^*/y_j^*}$) for $i=1,\ldots,n$; $j=1,\ldots,k$. In order to enhance its scope, a few more functions of a single variable have been added to the model. This latest version of the model is compiled at The George Washington University IBM/370, and can be accessed with the aid of job control cards, as depicted in Figures 2 or 3 of Section 4.

2. Factorable Functions

Most functions of several variables used in nonlinear optimization are complicated compositions of transformed sums and products of functions of a single variable. Once this observation is made formal, an outline of an automatic computer-oriented way of representing nonlinear programming problems becomes clear.

Definition: A function $f(x_1,\ldots,x_n)$ is a *factorable function* of several variables if it can be represented as the last in a finite sequence of functions $f^j(x)$, which are composed as follows:

$$f^j(x) = x_j \ , \quad \text{for} \quad j=1,\ldots,n \ .$$

For $n < j$, $f^j(x)$ is either of the form

$$f^k(x) + f^\ell(x) \ , \quad k,\ell < j \ ,$$

or of the form

$$f^k(x) \cdot f^\ell(x) \ , \quad k,\ell < j \ ,$$

or of the form

$$T[f^k(x)] \ , \quad k < j \ ,$$

where $T(f)$ is a function of a single variable.

It should be emphasized that this is a *natural* way of looking at functions of several variables, since it corresponds to the way they are used when evaluated at a particular set of values of the vector $(x_1,\ldots,x_n)$.

When a function is represented in factorable form, the computation of its gradient vector and Hessian matrix of second derivatives becomes very simple. Consider the general form for the gradient and Hessian matrix of a function which is either the sum of two functions, the product of two functions, or a transformation (function of one variable) of a function. For example,

$$\nabla[f(x)+g(x)] = \nabla f(x) + \nabla g(x)$$

$$\nabla[f(x)\cdot g(x)] = \nabla f(x) \cdot g(x) + \nabla g(x) \cdot f(x) \qquad (1)$$

$$\nabla T[f(x)] = \nabla f(x) \cdot \dot{T}[f(x)] \ , \qquad (2)$$

where $\dot{T}(f) = \partial T(f)/\partial f$; and

$$\nabla^2[f(x)+g(x)] = \nabla^2 f(x) + \nabla^2 g(x)$$

$$\nabla^2[f(x)\cdot g(x)] = \nabla^2 f(x)\cdot g(x) + \nabla^2 g(x)\cdot f(x) + \nabla f(x)\cdot\nabla^T g(x) + \nabla g(x)\cdot\nabla^T f(x) , \quad (3)$$

$$\nabla^2[T[f(x)]] = \nabla^2 f(x)\cdot\dot{T}[f(x)] + \nabla f(x)\ddot{T}[f(x)]\nabla^T f(x) , \quad (4)$$

where $\ddot{T}(f) = \partial^2 T(f)/\partial f^2$.

Several points should be noted.

(i) The computation of (1) involves $f(x), g(x)$, which would already have been computed in order to evaluate their product.

(ii) The computation of (3) involves $\nabla f(x)$, $\nabla g(x)$, which would already have been computed in order to evaluate (1).

(iii) Equation (4) used $\dot{T}[f(x)]$, which was previously needed in (2).

(iv) By induction, it follows that Hessian matrices of factorable functions are naturally given as sums of outer products of vectors, i.e.,

$$\sum u_i(x)\alpha_i(x)v_i^T(x) + v_i(x)\alpha_i(x)u_i^T(x) ,$$

where $u_i(x)$, $v_i(x)$ are $n \times 1$ vectors, $\alpha_i(x)$ are scalars, and the $u_i(x)$, $v_i(x)$ are available, having been required for the gradient computation.

For those optimization problems which are factorable, the factorable nonlinear programming problem is written in a more compact form (often referred to as canonical factorable form):

$$\begin{aligned} &\text{minimize} \quad X^N(x) \\ &\quad x\in E^n \\ &\text{subject to} \quad L_i \leq X^i(x) \leq U_i , \end{aligned}$$

for $i=1,\ldots,N-1$ (possibly $L_i = -\infty$ and/or $U_i = +\infty$), where $X^i(x) \equiv x_i$ (for $i=1,\ldots,n$), and the remainder are defined recursively as follows: given $X^p(x)$, for $p=1,\ldots,i-1$, then for $i=n+1,\ldots,N$,

$$X^i(x) = \sum_{p=1}^{i-1} T_p^i[X^p(x)] + \sum_{p=1}^{i-1} \sum_{q=1}^{p} V_{q,p}^i[X^p(x)] \cdot U_{p,q}^i[X^q(x)] , \quad (5)$$

where the T's, U's, and V's are functions of a *single* variable. The functions $X^i(x)$, $i=1,\ldots,N$ are usually referred to as concomitant variable functions and abbreviated as CVFs. Also, $X^i(x)$, $i=n+1,\ldots,N$ are referred to as new CVFs.

An important point bears mentioning here. One of the many appealing features of coding and solving problems in factorable form is that they lend themselves in a natural way to the computation of the number of functional evaluations and arithmetic operations involved in the evaluation of problem functions, and in the first and second derivatives required for solution of nonlinear optimization problems. By doing so, a precise quantitative measure is provided for comparing algorithms devised for solving nonlinear optimization problems.

Following are two examples that demonstrate these ideas.

Example 1 [putting a function into factorable form].

Consider the following function:

$$W = \Gamma \cdot B(x_1+x_2)(\ell_t x_4+\ell_m x_5+\ell_b x_6)[x_1 + (x_2^2-x_3^2)^{\frac{1}{2}}]^{-1} ,$$

which corresponds to the objective function of the sample problem 3 provided in Section 4.3, where

$$\begin{aligned} \Gamma \cdot B &= 3736.6 \\ \ell_t &= 495 \\ \ell_m &= 385 \\ \ell_b &= 315 . \end{aligned}$$

Table 1 shows how this function is put into simple factorable form, and depicts the corresponding stepwise reconstruction of the function.

Example ii [putting an optimization problem into canonical factorable form].

Consider the following optimization problem, which corresponds to the sample example 1 provided in Section 4.1:

$$\underset{x_1, x_2}{\text{minimize}} \quad f = (x_1-10)^2 + (x_2-1)^2$$

$$\begin{aligned} \text{subject to} \quad g_1: &\quad C_5\ x_2 - 5 \sin .5x_1 \leq C_1 \\ g_2: &\quad x_2 - C_6\ \ell n\ x_1 \geq C_2 \\ g_3: &\quad x_1 x_2 \geq C_3 \\ g_4: &\quad x_1 \geq C_4 , \end{aligned}$$

where

$$C_1 = C_2 = 0 , \quad C_3 = 3 , \quad C_4 = 1.5 , \quad C_5 = 1 , \quad C_6 = -1.5$$

are the parameters of the problem. Putting the functions of this problem into simple factorable form goes as follows:

$$\begin{aligned} X^1(x) &= x_1 && \text{(functional form of } g_4) \\ X^2(x) &= x_2 \\ X^3(x) &= .5X^1(x) \\ X^4(x) &= \text{Sin}[X^3(x)] \\ X^5(x) &= -5X^4(x) \\ X^6(x) &= C_5 \cdot X^2(x) \\ X^7(x) &= X^6(x) + X^5(x) && \text{(functional form of } g_1) \\ X^8(x) &= \ell n\ [X^1(x)] \\ X^9(x) &= -C_6 \cdot X^8(x) \\ X^{10}(x) &= X^2(x) + X^9(x) && \text{(functional form of } g_2) \end{aligned}$$

TABLE 1

EXAMPLE OF FACTORABLE FUNCTION (EXAMPLE i)

Function in Sequence		Form of Composition	Portion of Original Function	Composition Type
$f^j(x)$	=	x_j , j=1,6		$\Gamma_j(x) = x_j$
$f^7(x)$	=	$f^1(x) + f^2(x)$	$x_1 + x_2$	f + g
$f^8(x)$	=	$\Gamma B \cdot f^7(x)$	$\Gamma B(x_1+x_2)$	T(f)
$f^9(x)$	=	$\ell_t \cdot f^4(x)$	$\ell_t x_4$	T(f)
$f^{10}(x)$	=	$\ell_m \cdot f^5(x)$	$\ell_m x_5$	T(f)
$f^{11}(x)$	=	$\ell_b \cdot f^6(x)$	$\ell_b x_6$	T(f)
$f^{12}(x)$	=	$f^9(x) + f^{10}(x)$	$\ell_t x_4 + \ell_m x_5$	f + g
$f^{13}(x)$	=	$f^{12}(x) + f^{11}(x)$	$\ell_t x_4 + \ell_m x_5 + \ell_b x_6$	f + g
$f^{14}(x)$	=	$[f^2(x)]^2$	x_2^2	T(f)
$f^{15}(x)$	=	$-[f^3(x)]^2$	$-x_3^2$	T(f)
$f^{16}(x)$	=	$f^{14}(x) + f^{15}(x)$	$x_2^2 - x_3^2$	f + g
$f^{17}(x)$	=	$[f^{16}(x)]^{.5}$	$(x_2^2 - x_3^2)^{.5}$	T(f)
$f^{18}(x)$	=	$f^1(x) + f^{17}(x)$	$x_1 + (x_2^2-x_3^2)^{.5}$	f + g
$f^{19}(x)$	=	$[f^{18}(x)]^{-1}$	$[x_1 + (x_2^2-x_3^2)^{.5}]^{-1}$	T(f)
$f^{20}(x)$	=	$f^8(x) \cdot f^{13}(x)$	$\Gamma B(x_1+x_2)(\ell_t x_4+\ell_m x_5+\ell_b x_6)$	f · g
$f^{21}(x)$	=	$f^{20}(x) \cdot f^{19}(x)$	original function	f · g

$$X^{11}(x) = X^1(x) \cdot X^2(x) \qquad \text{(functional form of } g_3\text{)}$$
$$X^{12}(x) = X^1(x) - 10$$
$$X^{13}(x) = [X^{12}(x)]^2$$
$$X^{14}(x) = X^2(x) - 1$$
$$X^{15}(x) = [X^2(x)-1]^2$$
$$X^{16}(x) = X^{13}(x) + X^{15}(x) \qquad \text{(objective function f)} .$$

Representing $X^i(x)$ by X_i for notational simplicity, one canonical form of this problem will read as follows:

$$\underset{X_1,\ldots,X_{15}}{\text{minimize}} \quad X_{16}$$

where
$$X_1 = x_1 \qquad \text{(functional form of } g_4\text{)}$$
$$X_2 = x_2$$
$$X_3 = .5X_1$$
$$X_4 = \text{Sin } X_3$$
$$X_5 = -5X_4$$
$$X_6 = C_5X_2$$
$$X_7 = X_6 + X_5 \qquad \text{(functional form of } g_1\text{)}$$
$$X_8 = \ell n X_1$$
$$X_9 = -C_6X_8$$
$$X_{10} = X_2 + X_9 \qquad \text{(functional form of } g_2\text{)}$$
$$X_{11} = X_1 \cdot X_2 \qquad \text{(functional form of } g_3\text{)}$$
$$X_{12} = X_1 - 10$$
$$X_{13} = (X_{12})^2$$
$$X_{14} = X_2 - 1$$
$$X_{15} = (X_{14})^2$$
$$X_{16} = X_{13} + X_{15} \qquad \text{(objective function)}$$

subject to
$$X_7 \leq C_1 \qquad (g_1)$$

$$x_{10} \geq c_2 \qquad (g_2)$$
$$x_{11} \geq c_3 \qquad (g_3)$$
$$x_1 \geq c_4 \qquad (g_4) .$$

By exploiting the definition of the concomitant variable functions given in (5), many of the above steps may be combined and the problem may be reformulated (as is done in symbolic codings) more concisely as:

$$\begin{aligned}
&\underset{X \in E^6}{\text{minimize}} \quad X^6 \\
&\text{where} \quad X^1 = x_1 && \text{(functional form of } g_4) \\
&\qquad X^2 = x_2 \\
&\qquad X^3 = c_5 X^2 - 5 \sin .5X^1 && \text{(functional form of } g_1) \\
&\qquad X^4 = X^2 - c_6 \ln X^1 && \text{(functional form of } g_2) \\
&\qquad X^5 = X^1 \cdot X^2 && \text{(functional form of } g_3) \\
&\qquad X^6 = (X^1-10)^2 + (X^2-1)^2 && \text{(objective function)} \\
&\text{subject to} \quad X^3 \leq c_1 && (g_1) \\
&\qquad X^4 \geq c_2 && (g_2) \\
&\qquad X^5 \geq c_3 && (g_3) \\
&\qquad X^1 \geq c_4 && (g_4) .
\end{aligned}$$

3. Input Specifications

This section describes how the user must code and supply the required information about his problem to SYMBOLIC FACTORABLE SUMT. In the following, the type of information and its format, name, and description are discussed in the same order that must appear in a coded deck. A schematic depiction of a coded problem deck structure is provided in Figure 1 at the end of this section. For clarification, frequent reference to the code of sample problem 1 of Section 4.1 and to Figures 2 or 3 is advised for beginners as they proceed through the input specifications.

1 JOB and JCL cards. (See Figure 2, Section 4.1.4.)

2 The SUMT parameter card. The SUMT parameter card, which is read in by the subroutine MAIN, is described in Table 2.

TABLE 2

THE SUMT PARAMETER CARD

Columns	Format	Name	Use
01-12	E12.0	EPSI (ε)	Tolerance used to decide if an unconstrained minimum has been achieved for each subproblem (see Option 9)
13-24	E12.0	RHOIN (r_1)	Possible initial value of r (often set at 1.0) (see Option 1)
25-36	E12.0	THETAO (θ_0)	Tolerance used to decide if the solution to the NLP problem (A) has been approximated (see Option 5)
37-48	E12.0	RATIØ (c)	Parameter (> 1) used to compute consecutive values of r; $r_{i+1} = r_i/c$ (often set at 16.0)
49-60	E12.0	TMMAX	Maximum amount of time for solving problem (in seconds)
61-64	I4	M	Number (integer) of inequality constraints, (M+MZ) $\leq$ 200*
65-68	I4	N	Number (integer) of variables, N $\leq$ 100
69-72	I4	MZ	Number (integer) of equality constraints

*The limits on M+MZ and N are governed by the sizes of the arrays in SUMT.

3 The first option card. The first option card, which is read in by the subroutine MAIN, is described in Table 3.

TABLE 3

THE FIRST OPTION CARD

Option	Column	Format	Name	Value	Meaning
1 (normally set to 3)	7	I	NT1	3	
2	14	I	NT2	2	
3 (normally set to 1)	21	I	NT3	1	Standard printout (this includes a call to OUTPUT after the solution of every subproblem). Also the estimates of the "Lagrange multipliers" and first- and second-order solution estimates are printed.
				2	For additional printout (includes standard printout and every intermediate point, gradient of P, and the vector S).
4 (normally set to 1)	28	I	NT4	1	Final convergence is determined on the basis of current solution to the subproblem.
				2	Final convergence is determined on the basis of the first order estimates. The first order estimate of the solution vector must be close to feasible. See below.
				3	Final convergence is determined on the basis of the second order estimates. The second order estimate of the solution vector must be close to feasible before the convergence check is made. If $\bar{x}$ is a solution estimate it is considered close to being feasible if $g_1(x) + \theta_0 \geq 0$, i=1,2,...,m , where θ_0 is defined on the parameter card.

TABLE 3--*continued*

Option	Column	Format	Name	Value	Meaning
5 (normally set to 1)	35	I	NT5		The convergence criterion determining the NLP problem has been solved (only use = 1 when NT4 ≠ 1)
				1	Quit when $\frac{G - f[x(r_k)]}{G[x(r_k),\mu(r_k),\lambda(r_k)]} < \theta_0$
				2	Quit when $\lvert r \sum_{j=1}^{m} \ln g_j[x(r_k)] \rvert < \theta_0$
				3	Quit when first order estimate of $\frac{v_0}{G[x(r_k),\mu(r_k),\lambda(r_k)]} - 1 < \theta_0$
6 (normally set to 1)	42	I	NT6	1	After final convergence the program reads in new data and solves the next problem
				2	After final convergence has been determined a call to PUNCH is made before proceeding on to the next problem
7 (normally set to 1)	49	I	NT7		First move after a minimum to a subproblem is achieved
				1	No extrapolation
				2	Extrapolate through last two minima
				3	Extrapolate through last three minima
8 (normally set to 0)	56	I	NT8	0	No sensitivity or elasticity calculation
				2	Conduct sensitivity and elasticity calculation after each subproblem convergence

TABLE 3--*continued*

Option	Column	Format	Name	Value	Meaning
9	63	I	NT9		Subproblem convergence criterion, or when to stop minimizing P function for fixed value of r (see parameter card)
				1	Quit when $\left\|\nabla_x P^T(x^i,r)\left[\frac{\partial^2 P(x,r)}{\partial x_i \partial x_j}\right]^{-1} \cdot \nabla_x P(x^i,r)\right\| < \varepsilon$
				2	Quit when $\left\|\nabla_x P^T(x^i,r)\left[\frac{\partial^2 P(x,r)}{\partial x_i \partial x_j}\right]^{-1} \cdot \nabla_x P(x^i,r)\right\| < \frac{P(x^{i-1}) - P(x^i)}{5}$
				3	Quit when $\left\|\nabla_x P(x^i,r)\right\| < \varepsilon$
10	70	I	NT10	1	At least one nonlinear constraint
				2	Linear constraints
				3	Linear constraints and linear objective function (i.e., a linear programming problem)

4 The title card. This card, which is read in by the subroutine SYMPUT, contains all pertinent information such as problem title and reference, user's name, date, etc., that the user wishes to appear on the output. This information must be coded in a single card anywhere in columns 1-80, and is read in with the alphanumeric format of 20 A4. A blank card must be supplied if this information is not coded.

5 The SYMPUT parameter card. The SYMPUT parameter card, which is read in by the subroutine SYMPUT, is described in Table 4.

TABLE 4

THE SYMPUT PARAMETER CARD

Columns	Format	Name	Description
01-05	I_5	N	Number of problem variables
06-10	I_5	NN	Number of concomitant variable functions, CVFs (i.e., number of variables in factorable form)
11-15	I_5	MMM	Number of inequality constraints
16-20	I_5	MZ	Number of equality constraints
21-25	I_5	ITEST	• If ITEST=0, at the initial feasible point of the problem the values and first and second derivatives of new CVFs will be evaluated and printed for debugging purposes, without solving the problem • If ITEST=1, the above information will be suppressed and the problem will be solved
26-30	I_5	IPR	• If IPR=1, after each subproblem convergence the values of all CVFs at the subproblem solution point will be printed. • If IPR=0 ($\neq$1), the values of the CVFs will be suppressed after each subproblem convergence
31-35	I_5	ITES	• If ITES=2, after each subproblem convergence the following information will be printed for debugging purposes: (i) Entries of vector $\sigma^{(1)}$ (ii) Entries of vector $\alpha^{(2)}$ (iii) Values of all CVFs (iv) Partial derivatives of new CVFs with respect to problem variables and parameters (v) Entries of vector DELA$^{(3)}$ • If ITES=0 ($\neq$2), the above information will be suppressed after each subproblem convergence

TABLE 4--*continued*

Columns	Format	Name	Description
36-40	I_5	NSWIT	• If NSWIT=1, the modified listing of the coded problem and related tables set up by subroutine SYMPUT will be suppressed in the output • If NSWIT=0 ($\neq$1), the above listing will not be suppressed
41-45	I_5	IID	Number of problem parameters (i.e., P and D cards)

(1) Vector σ is an n-dimensional unit vector e_i, i=1,...,n (n is the number of original problem variables).

(2) See Equation (38) in Appendix 2.

(3) Negative values of partial derivatives of subproblem solution vector components with respect to the problem parameters.

6 *The variable definition cards.* These cards are read in by the subroutine SYMPUT. For each variable of the problem, the user must supply one card with the descriptions as shown in Table 5.

TABLE 5

THE VARIABLE DEFINITION CARDS

Columns	Format	Name	Description
02	A1	ATYPE	Letter "v" must be coded for ATYPE to indicate this card corresponds to a variable card
04-07	A_4	ANAME	Variable name to be specified by the user (right justified)
16-28	F12.5	ARL	Initial value of variable (starting point)

It is obvious that the number of variable definition cards should match the number of the (original) variables of the problem being coded.

7 The problem parameter cards (P and D cards). These cards are read in by the subroutine SYMPUT. For each parameter of the problem, the user must supply one card with a description as shown in Table 6.

TABLE 6

THE PROBLEM PARAMETER CARDS

Columns	Format	Name	Description
02	A_1	ATYPE	• Letter "D" must be coded for ATYPE to indicate this card corresponds to a parameter card • Use of letter "P" instead of letter "D", in conjunction with proper option in the SUMT parameter card, will invoke the sensitivity routines to calculate the sensitivity and elasticities of the solution point with respect to the indicated parameter after each subproblem convergence
04-07	A_4	ANAME	Parameter name to be specified by the user (right justified)
16-28	F12.5	ARG	Numerical value of the parameter

It is obvious that the number of the parameter cards must match the number of problem parameters. Also, parameter cards with "P", on which sensitivity calculation is desired, *must* appear before parameter cards with letter "D".

8 The separable and quadratic cards. These cards are read in by the subroutine SYMPUT. The number and order of appearance of separable and quadratic cards will depend on the problem functions being coded and their canonical formulation by the user. Each separable or quadratic card in essence provides the model with information regarding a single variable function. With this information, the model can systematically synthesize the new concomitant variable functions and subsequently can reconstruct the problem functions and their derivatives for the purposes of various calculations.

While coding these cards, the user must be aware of the following facts:

- Separable cards may appear sequentially in any numbers. As long as they are associated with the same new CVF (CVF in columns 04-07 of Table 7), the single variable functions in each card will be *summed up* recursively by the model to construct the new CVF.

- Quadratic cards must appear as pairs, with any number of pairs permitted. As long as these cards are associated with the same new CVF, the *product* of the related single variable functions in each consecutive pair will be *summed up* recursively (if there are more than one pair) to reconstruct the new CVF.

- When there are separable *and* quadratic cards associated with the same new CVF, the results of the aforementioned calculations will be *summed up* to reconstruct the subject CVF.

Separable and quadratic cards are described in Table 7. For further clarification, the reader is referred to the formulation and coding of the sample problems provided in Section 4.

9 The bound cards. These cards are read in by the subroutine SYMPUT. There are two types of bound cards, upper bound and lower bound. It is natural that these cards must appear after the concomitant variable functions that are being bounded have been defined. Cards relating to equality constraints *must* follow the cards defining the inequality constraints. Table 8 describes bound cards.

10 The flag card. The user must indicate the end of his problem's symbolic factorable code by furnishing a flag card. This card, which is read in by the subroutine SYMPUT, must contain the number 5 in its second column.

11 The tolerance card. This card is read in by the subroutine MAIN, and is described in Table 9.

TABLE 7

THE SEPARABLE AND QUADRATIC CARDS

Columns	Format	Name	Description
02	A1	ATYPE	Letter "S" must be coded for ATYPE if the card is a separable card. Letter "Q" must be coded for ATYPE if the card is a quadratic card
04-07	A4	ANAME	Name of the new CVF must be coded for ANAME (right justified)
09-12	A4	ARG	Name of a previously defined CVF, in terms of which the current new CVF is being defined, must be coded for ARG
14-16	A4	AFUN	Three-letter symbolic name assigned to the type of functional relationship between ANAME and ARG (see Table 12) must be coded for AFUN
17-28	F12.5 or 3A4	AC1†	• The value of C1 in AFUN (see Table 12) must be coded for AC1 • If C1 corresponds to a previously defined problem parameter, then the parameter name must be coded for AC1 (right justified)*
29-40	F12.5 or 3A4	AC2†	• The value of C2 in AFUN (see Table 12) must be coded for AC2 • If C2 corresponds to a previously defined problem parameter, then the parameter name must be coded for AC2 (right justified)*

†Reading of AC1 and/or AC2 with the proper F or A format in the model is made possible by utilizing the subroutine REREAD, which is a built-in subroutine at The George Washington University FORTRAN library (see Section 5).

*Parameters with alphanumeric value *cannot* take negative signs throughout the code (i.e., -ABC does not imply the negative of numeric value of parameter ABC).

TABLE 8

THE BOUND CARDS

Columns	Format	Name	Description
02	A1	ATYPE	Letter "U" must be coded for ATYPE if an upper bound is being imposed, letter "L" for a lower bound. An equality constraint uses either letter.
04-07	A4	ANAME	The name of the first of a sequence of CVFs to which the bound applies must be coded for ANAME (by sequence it is meant the order in which the CVFs appear in the code)
09-12	A4	ARG	The name of the last CVF in the above (uninterrupted) sequence to which the bound applies must be coded for ARG
17-28	F12.5 or 3A4	AC1†	• The value of the bound for the above CVF must be coded for AC1 • If the bound corresponds to a previously defined parameter, then the parameter name must be coded for AC1 (right justified)

†See the footnote of Table 7.

TABLE 9

THE TOLERANCE CARD

Columns	Format	Name	Description
01-12	E12.6	XEP1	Ignored by the program
13-24	E12.6	XEP2	When minimizing the P-function for a given value of r (RHØ) the value of P must decrease by an amount exceeding XEP2 for each iteration after the first. If it does not, then the code prints out the message "apparently roundoff errors prevent a more accurate determination of the minimum of this subproblem," and it is assumed that a minimum has been found. (Usually we set XEP2 equal to 0.)

12 The second option card. This card is read in by the program MAIN. The description of this card is given in Table 10.

TABLE 10

THE SECOND OPTION CARD

Option	Column	Value	Meaning
1	7	1	Ignored in this program
2	14	1	The method for minimizing the unconstrained penalty function is to be the generalized Newton-Raphson method as modified to handle indefinite Hessian matrices. This method requires function values, first and second derivatives
		2	Same as 1, except that when an "orthogonal move" is made because of an indefinite Hessian matrix, -VP is added to the orthogonal move vector
		3	Steepest descent is used to minimize the P-function
		4	The method for minimizing the unconstrained penalty function is the rank one method as reported in the Fiacco-McCormick book [10]. This requires function values and first derivatives

13 End card. This is the usual end card which must follow any coded problem. It contains "//" at the first two columns.

A schematic depiction of a problem deck structure is shown in Figure 1.

4. Coded Examples and Output Illustrations

For illustration purposes the coded input and the resulting output for the sample problem 1 discussed in Section 2 is reviewed in sufficient detail. Although the functional relationships involved in this problem are too simple and the problem itself is too small to illustrate the

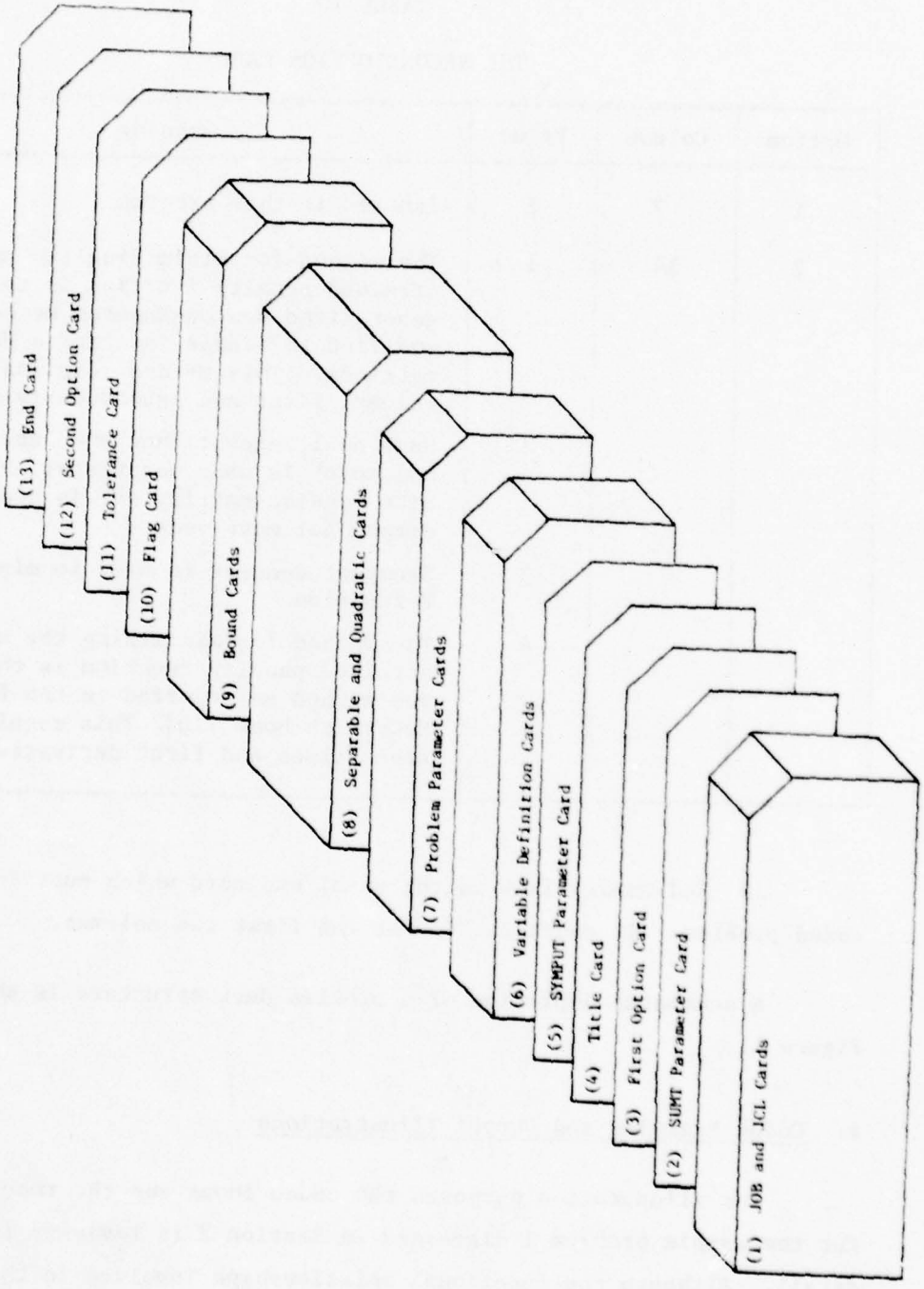

Figure 1.--Data deck structure.

merits of SYMBOLIC FACTORABLE SUMT, we have purposely chosen it so that users can readily follow and become familiar with the model with the least possible effort. For further elucidation the input/output for two more sample problems--Hopkin's problem from Reference [2] and the bulkhead design problem from Reference [5]--are included following the sample problem 1.

4.1 Sample problem 1 (a test problem).

4.1.1. Problem definition (restated for ease of reference):

$$\begin{aligned} \underset{x_1,x_2}{\text{minimize}} \quad & (x_1-10)^2 + (x_2-1)^2 \\ \text{subject to} \quad & C_5\ x_2 - 5 \sin .5x_1 \leq C_1 \\ & x_2 - C_6 \ln x_1 \geq C_2 \\ & x_1 x_2 \geq C_3 \\ & x_1 \geq C_4 , \end{aligned}$$

where C_1 through C_6 are the parameters of the problem, and their respective values are

$$C_1 = C_2 = 0 , \quad C_3 = 3 , \quad C_4 = 1.5 , \quad C_5 = 1 , \quad C_6 = -1.5 .$$

Moreover, let us suppose that we intend to calculate the sensitivity and elasticity of the solution vector with respect to the parameters C_5 and C_6 .

4.1.2. Symbolic factorable form. In SYMBOLIC FACTORABLE SUMT, use of canned routines to evaluate functions of one variable in a single iteration enables us to code the problem in the following concise form:

$$\underset{X \in E^6}{\text{minimize}} \quad X^6$$

		symbolic factorable step
where	$X^1 = x_1$	*(a)*
	$X^2 = x_2$	*(b)*
	$X^3 = C_5 X^2 - 5 \sin .5X^1$	*(c)*
	$X^4 = X^2 - C_6 \ln X^1$	*(d)*
	$X^5 = X^1 \cdot X^2$	*(e)*
	$X^6 = (X^1-10)^2 + (X^2-1)^2$	*(f)*
subject to	$X^3 \leq C_1$	*(g)*
	$X^4 \geq C_2$	*(h)*
	$X^5 \geq C_3$	*(i)*
	$X^1 \geq C_4$.	*(j)*

4.1.3. The keypunch sheet code. The keypunch sheets used to code the above problem in symbolic factorable language is shown in Figure 2. Table 11 shows the correspondence between the steps cited in 4.1.2 and the related symbolic code. Figure 3 depicts an annotated computer listing of the symbolic code for problem 1. The categories of the code are numbered and specified in the left margin of this figure in the same sequence as was illustrated in Section 3.

The symbolic names used in the code are directly taken from Table 12 at the end of the present section.

4.1.4. Output annotations. The computer output for the sample example 1 is depicted in Appendix 1, page 50. Various segments of this output are numbered in circles. Corresponding explanations are as follows:

1 Information contained in the title card

2 Printout of SYMPUT parameter card (the reader may skip reading steps 3-8).

```
//AAAAA JØB (5660,R),ABG
// EXEC FØRG6,DSN='ØR5660.ABG',PRØG=MAIN
//SYSLIB DD
//        DD
//        DD DSN=GWU.FØRTLIB,DISP=SHR
//GØ.FT05F001 DD *
     0.00001          100.0      0.00001          10.0          500.   4   2   0          01
      3       2       1       1       1       1       1       2       1       1          02
        EXAMPLE-1 : SØLUTIØN AND SENSITIVITY ANALYSIS                                     03
   2    6    4    0    1    0    0    0    6                                            04
V  X1                          2.0                                                      05
V  X2                          2.0                                                      06
P  C5                          1.0                                                      07
P  C6                         -1.5                                                      08
D  C1                          0.0                                                      09
D  C2                          0.0                                                      10
D  C3                          3.0                                                      11
D  C4                          1.5                                                      12
S  X3   X1 SIN                 0.5          -5.0                                        13
S  X3   X2 LIN                  C5                                                     14
S  X4   X2 LIN                 1.0                                                      15
```

Figure 2.--Data cards for sample problem 1.

```
S    X4   X1  LNG        1.0          C6
Q    X5   X1  LIN        1.0
Q    X5   X2  LIN        1.0
S    X6   X1  QDR        1.0        -10.0
S    X6   X2  QDR        1.0         -1.0
U    X3   X3              C1
L    X4   X4              C2
L    X5   X5              C3
L    X1   X1              C4
5
          .001                .001
          1                   1
11
```

Figure 2.--continued.

```
//AAAAAA JOB (5660,R),ABG
// EXEC FORG6,DSN='OR5660.ABG',PROG=MAIN
//SYSLIB DD
//        DD
//        DD DSN=GWU.FORTLIB,DISP=SHR
//GO.FT05F001 DD *
    0.00001          100.0      0.00001          10.0        500.   4   2   0
      3      2       1    1     1    1    1    2    1     1
          EXAMPLE-1 : SOLUTION AND SENSITIVITY ANALYSIS
    2   6   4   0   1   0   0   0   6
V   X1                    2.0
V   X2                    2.0
P   C5                    1.0
P   C6                   -1.5
D   C1                    0.0
D   C2                    0.0
D   C3                    3.0
D   C4                    1.5
S   X3   X1 SIN           0.5        -5.0
S   X3   X2 LIN            C5
S   X4   X2 LIN           1.0
S   X4   X1 LNG           1.0          C6
Q   X5   X1 LIN           1.0
Q   X5   X2 LIN           1.0
S   X6   X1 QDR           1.0        10.0
S   X6   X2 QDR           1.0         1.0
U   X3   X3                C1
L   X4   X4                C2
L   X5   X5                C3
L   X1   X1                C4
F
     .001       .001
      1    1
/
```

(1) JOB and JCL cards

(2) SUMT parameter card

(3) First option card

(4) Title card

(5) SYMPUT parameter card

(6) Variable definition cards

(7) Problem parameter cards (sensitivity calculations to be carried on parameters C5 and C6)

(8) Separable and quadratic cards

(9) Bound cards

(10) Flag card

(11) Tolerance card

(12) Second option card

(13) End card

Figure 3.--Computer listing of the sample problem 1 code.

TABLE 11

THE CORRESPONDENCE BETWEEN FUNCTIONS OF SAMPLE PROBLEM 1 IN SYMBOLIC FACTORABLE FORM AND ITS SYMBOLIC FACTORABLE CODE

Symbolic Factorable Step	Statement Serial Number in Keypunch Sheet (Figure 2)
(a)	05
(b)	06
(c)	13 and 14
(d)	15 and 16
(e)	17 and 18
(f)	19 and 20
(g)	21
(h)	22
(i)	23
(j)	24

Segments 3-8 pertain to the categorization of the coded symbolic input by the subroutine SYMPUT which is necessary for synthesis of the original problem from the symbolic code. In order to make these printouts self-explanatory, the reader should notice the following points, which are related to the programming logic of the subroutine SYMPUT:

(i) Separable terms are labeled with a type number "3" and and quadratic terms with a type number "4."

(ii) CVFs are sequentially labeled from 1 to NN (total number of CVFs) in the order of their appearance in the code.

(iii) All the problem parameters and constant coefficients are sequentially labeled by natural sequence of numbers in the order of their appearance in the code and stored in array CC1. Constants having the same numerical value are labeled with the same number. Moreover, in this array the problem parameters are separated from problem constant coefficients with numbers zero and one.

(iv) Constraints of the problem are sequentially labeled by natural sequence of numbers in the order of their appearance in the code.

(v) Each symbolic function, as discussed in Section 1, is associated with a unique number called a transformation code, which serves as a link to juxtapose the symbolic function name with its mathematical code.

With these points in mind we proceed to review briefly segments 3-8 of the output.

3 Modified printout of the symbolic code of the problem (excluding the bounds). For explanation of columns 2, (5,7), and 8 , see comments (i), (ii), and (v), respectively, above.

4 Manipulated printout of problem parameters and constants [see comment (iii)]

5 Bounds on problem constraints. For explanation see comments (iv) and (iii), respectively.

6 Number of times that a concomitant variable function (CVF) is defined in terms of a separable one-variable function or quadratic function (cumulative). For example, concomitant variable function X^4 is defined as follows:

$$X^4 = X^2 - C_2 \ln X^1 ,$$

which has $5-3 = 2$ separable terms and $1-1 = 0$ quadratic terms.

7 Table of separable terms formed by subroutine SYMPUT (self-explanatory).

8 Table of quadratic terms set up by subroutine SYMPUT (self-explanatory).

9 Printout of parameter card.

10 Printout of first option card.

11 Printout of tolerance card.

12 Printout of second option card.

13 Printout of initial vector and corresponding problem function values.

14 Printout of feasible starting vector and corresponding problem function values (initial vector need not be feasible).

15 Objective function value, solution vector, constraint values, and Lagrange multipliers evaluated at solution point of subproblem 1 (at r = 100) . The following information is also evaluated at this solution point:

DOTT	: inverse of Hessian matrix of penalty function pre- and post-multiplied by penalty function gradient
MAGNITUDE:	norm of penalty function gradient
P	: value of the penalty function
G	: dual value
RSIGMA	: residual term related to inequality constraints in penalty function
H	: residual term related to equality constraint in penalty function.

16 Printout of the problem parameters on which sensitivity information is desired.

17 Zero*th* order sensitivity of the first component of the solution vector of subproblem 1 (i.e., x_1) and corresponding elasticities with respect to the problem parameters (repeated for all components of solution vector).

18 Solution to the second subproblem (at r = 10) and first order estimates of solution values.

19 First order sensitivity of the first component of solution vector of subproblem 2 and corresponding elasticities with respect to the problem parameters.

20 Solution to the third subproblem (at r = 1) and first and second order estimates of solution values.

21 Solution to the fourth subproblem (at r = .1) .

22 Solution to the fifth subproblem (at r = .01) .

23 Solution to the sixth subproblem (at r = .001) .

24 Solution to the seventh subproblem (at r = .0001) .

25 Solution to the eighth (final) subproblem (at r = .00001) .

Illustration of a few useful optional outputs seems beneficial at this stage.

- ITEST=0 rather than 1 (in SYMPUT parameter card) would suppress output segments numbered from 15 onwards, and would instead yield output segments A, B, C, and D, as labeled on page 64 of Appendix 1, and which have the following descriptions (all evaluated at the initial feasible point):

  A : the values of CVFs

  B : partial derivatives of the new CVFs with respect to the problem variables

  C : diagonal entries of the Hessian matrix of the new CVFs

  D :

- NSWIT=1 (in SYMPUT parameter card) would suppress the output segments 3-8.

- ITES=2 (in SYMPUT parameter card) would yield output segments E, F, G, H, I, J, and K after solution of each subproblem, which are defined as follows (all evaluated at the corresponding subproblem solution point):

  E : the entries of vector σ (see Table 4)

  F : the entries of vector α [see Equation (38), Appendix 2]

  G : the values of CVFs

  H : partials of the new CVFs with respect to the original problem variables premultiplied by vector α

  I : the joint partial derivatives of the new CVFs with respect to the original problem variables and sensitivity parameters premultiplied by vector α

  J : the partials of the new CVFs with respect to the sensitivity parameters

  K : the negative values of the zero*th* order sensitivity of the original problem variables with respect to the sensitivity parameters.

- NT8=1 (in first option card) would suppress the sensitivity and elasticity analysis calculations.

4.2 Sample problem 2 (Hopkins problem).

This problem has been solved in Reference [2] using FACTORABLE SUMT (earlier version of the SYMBOLIC FACTORABLE SUMT). Here it is chosen as the second sample problem for solution and sensitivity analysis by the SYMBOLIC FACTORABLE SUMT. For brevity, unlike the first sample problem, illustration of this sample problem is limited to the algebraic definition of the problem, computer listing of its code, and selected pages of the computer solution output.

4.2.1. Problem definition.

$$\text{minimize} \quad \left(-100x_1^{.5} - 2x_2 - 25x_3^{.5} - 50x_4^{.5} - 10x_5 + 100(x_6-3)^2 - 30x_1^{.5}x_7^{.5} + 100x_8 - 10x_1/x_3 - 20x_1/x_4\right)$$

$$\text{subject to} \quad g_1 : A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + A_5x_3x_6 + A_6x_4x_6 - A_7x_4x_5x_6 + A_8x_1x_7 - A_9x_1x_8 \leq 25{,}000$$

$$g_2 : 12.25x_1 + x_3x_6 + .7x_4x_6 - .2x_4x_5x_6 \geq 34{,}000$$

$$g_3 : \quad x_5 \leq .5$$

$$g_4 : \quad x_8 \leq .3$$

$$g_5\text{-}g_{14} : \quad x_j \geq 0; \; j=1,8 \; ,$$

where the coefficients of the first constraint, i.e., A_1-A_9 , are chosen as problem parameters on which sensitivity and elasticity analysis is performed. The values of these parameters are:

$A_1 = 24$, $A_2 = 12$, $A_3 = .6$, $A_4 = .4$, $A_5 = .1$

$A_6 = .15$, $A_7 = -.15$, $A_8 = 11$, and $A_9 = 24$.

TABLE 12

LIST OF SINGLE VARIABLE FUNCTIONS USED BY SYMBOLIC FACTORABLE SUMT

Symbolic Name	Description	Algebraic Form	Default Options
CØN	Constant	C1	None
IDN	Identity	Z	None
LIN	Linear	C2 + C1*Z	C2 = 0
EXP	Exponential	C2* EXP(C1*Z)	C2 = 1, C1 = 1
EXC	--	C2* $(C1)^{Z}$	C2 = 1
PØW	Power	C2* $(Z)^{C1}$	C2 = 1
PØL	Polynomial	C2*$(Z)^{C1}$ (C1 integer)	C2 = 1
LNG	Natural Log	C2* LN(Z)	C2 = 1 [no C1]
LØG	Other Log	C2* $LØG_{C1}(Z)$	C2 = 1
SIN	Sine	C2* SIN(C1*Z)	C2 = 1, C1 = 1†
CØS	Cosine	C2* CØS(C1*Z)	C2 = 1, C1 = 1†
TAN	Tangent	C2* TAN(C1*Z)	C2 = 1, C1 = 1†
GAM	Gamma Function	C2* $\Gamma(C_1*Z)$	C2 = 1
NØR	Cumulative Normal	C2* $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z} EXP(\frac{-t^2}{2})dt$	C2 = 1
HEV	Heaviside§	C2* H(Z-C1)	C2 = 1

TABLE 12--*continued*

Symbolic Name	Description	Algebraic Form	Default Options
MIN	Minimum	C2* MIN[Z,C1]	C2 = 1
MAX	Maximum	C2* MAX[Z,C1]	C2 = 1
ARS	Arc Sine	C2* ARCSIN[C1*Z]	C2 = 1
ART	Arc Tangent	C2* ARCTAN[C1*Z]	C2 = 1
QDR	Quadratic	$[C1* Z-C2]^2$	None
ESQ	Exponential Square	C2* EXP[$C1*Z^2$]	C2 = 1
ENT	Entropy Function	C1* Z[C2 + LN(C1*Z)]	None
LGS	Logistic Curve	$[1 + C2* EXP(-C1*Z)]^{-1}$	C2 = 1
EXM	Decay Function	C2*[1 - EXP(-C1*Z)]	C2 = 1
LLN	Log of Linear Function	LN(-C1*Z + C2)	None
DIV	Division	C1/Z + C2	C1 = 1

†Default option C1 = 1 can be used only if the option C2 = 1 is also being exploited, i.e., C2 is not coded.

‡Heaviside function is a step function defined as follows:

$$C2*H(Z-C1) = \begin{cases} 0 , & \text{if } Z < C1 \\ C2 , & \text{if } Z \geq C1 . \end{cases}$$

4.2.2. Computer listing of the symbolic code. (See Figure 4.)

4.2.3. Selected pages from computer output. (See Appendix 1, page 69.)

4.3 Sample problem 3 (corrugated transverse bulkhead design).

This problem, which has been taken from Reference [5], involves more complicated functional relationships than the two previous sample problems. It has been solved by many authors [6], [5], [7], [8]. A comparison of the time, cost, and effort spent to code and solve this problem in [5] and [8]--where the SUMT-Version 4 model was used--with those involved in the present study, shows the enormous advantage of the SYMBOLIC FACTORABLE SUMT model over the SUMT-Version 4 model. Table 13 accounts for this claim.

TABLE 13

SUMT-VERSION 4 VERSUS SYMBOLIC FACTORABLE SUMT ON SOLUTION OF OPTIMAL CORRUGATED TRANSVERSE BULKHEAD DESIGN PROBLEM

Comparison Criteria	SUMT-Version 4 [8]	SYMBOLIC SUMT [present study]
Number of statements used to code the problem	300+	100
Approximate time spent formulating and coding the problem, hrs	25+	4
CPU time spent solving the problem/run, seconds	61.63	27.64
Cost incurred in solving the problem/run, dollars	5.08+	2.73

A comparison of the entries in the above table reveals how SYMBOLIC FACTORABLE SUMT facilitates the coding of problems and brings about savings in time and costs involved in the solution of nonlinear problems. As mentioned in Section 1, the primary reason for such reductions, especially in the coding of the problem, stems from the fact that SUMT-Version

```
//AAAAAA JOB (5660,R,3,3),ABG
// EXEC FORG6,DSN='OR5660.ABG',PROG=MAIN
//SYSLIB DD
//        DD
//        DD DSN=GWU.FORTLIB,DISP=SHR
//GO.FT05F001 DD *
     0.00001          100.0          .00001          10.0          300.   12    8    0
       3       2       2       1       1       1       1       2       1       1
EXAMPLE-2 : SOLUTION AND SENSITIVITY ANALYSIS OF HOPKINS PROBLEM
    8   14    12     0     1     0     0     0     9
 V    X1                  180.00
 V    X2                  740.00
 V    X3                 2680.00
 V    X4                14650.00
 V    X5                     .40
 V    X6                    2.30
 V    X7                     .60
 V    X8                     .20
 P    A1                   24.00
 P    A2                   12.00
 P    A3                   0.600
 P    A4                    0.40
 P    A5                    0.10
 P    A6                    0.15
 P    A7                   -0.15
 P    A8                   11.00
 P    A9                  -24.00
 Q    X9    X4  IDN
 Q    X9    X6  IDN
 Q   X10    X3  IDN
 Q   X10    X6  IDN
 S   X11    X1  LIN           A1
 S   X11    X2  LIN           A2
 S   X11    X3  LIN           A3
 S   X11    X4  LIN           A4
 S   X11   X10  LIN           A5
 S   X11    X9  LIN           A6
 Q   X11    X9  LIN           A7
 Q   X11    X5  IDN
 Q   X11    X1  LIN           A8
 Q   X11    X7  IDN
 Q   X11    X1  LIN           A9
 Q   X11    X8  IDN
 S   X12    X1  LIN        12.25
 S   X12   X10  IDN
 S   X12    X9  LIN         0.70
 Q   X12    X9  LIN        -0.20
 Q   X12    X5  IDN
```

Figure 4.--Computer listing of sample problem 2.

```
S   X13    X6 QDR          1.00              3.
S   X14    X1 POW          0.50           -100.
S   X14    X2 LIN         -2.00
S   X14    X3 POW          0.50            -25.
S   X14    X4 POW          0.50            -50.
S   X14    X5 LIN        -10.00
S   X14   X13 LIN        100.00
Q   X14    X1 POW          0.50            -30.
Q   X14    X7 POW          0.50
S   X14    X8 LIN        100.00
Q   X14    X1 LIN        -10.00
Q   X14    X3 POW         -1.
Q   X14    X1 LIN        -20.00
Q   X14    X4 POW         -1.
U   X11   X11           25000.00
L   X12   X12           34000.00
U    X5    X5               .50
U    X8    X8               .30
L    X1    X8              0.00
5
     0.001          .001
      1              1
//
```

Figure 4.--continued

4 requires explicit derivations and coding of the problem function gradients and Hessians (user's subroutine), while in SYMBOLIC FACTORABLE SUMT these requirements are internally calculated and so are relaxed. The derivation and coding of these entries is sometimes extremely tedious and time consuming, as is the case for the present problem, and is prone to mathematical errors whose debugging is another factor in the user's time consumption. It was this very fact that motivated the development of SYMBOLIC FACTORABLE SUMT, and the present example clearly demonstrates that this goal has been very well achieved.

It is not the intention of the present work, or within its scope, to discuss the engineering aspects of this optimal design problem and the significance of its constraints. Readers interested in such aspects should refer to [5]. Instead, similar to the sample problem 2, its discussion is limited to the algebraic definition of the problem, a computer listing of its code, and selected pages from computer solution output.

4.3.1. Problem definition. Following is the algebraic definition of the problem, borrowed from [5]. For sensitivity analysis considerations, the coefficients of K_2 and $t_{\bullet}^{min}$ in constraints 8-16 have been normalized to unity in the code of the problem (i.e., constraints 8, 11, and 14 have been scaled by -1, and constraints 9, 10, 11, 12, 13, 15, and 16 by $-\frac{1}{10}$). The problem is

$$\text{minimize} \quad W = \Gamma B(x_1+x_2)(\ell_t x_4+\ell_m x_5+\ell_b x_6)[x_1 + (x_2^2-x_3^2)^{\frac{1}{2}}]^{-1}$$

subject to

constraint number

$$x_2 - x_3 \geq 0 . \qquad (1)$$

$$\frac{1}{6} x_2x_3x_4 + \frac{e}{2} x_1x_3x_4 - K_1h_t\ell_t^2[x_1 + (x_2^2-x_3^2)^{\frac{1}{2}}] \geq 0 , \qquad (2)$$

$$\frac{1}{6} x_2x_3x_5 + \frac{e}{2} x_1x_3x_5 - K_1h_m\ell_m^2[x_1 + (x_2^2-x_3^2)^{\frac{1}{2}}] \geq 0 , \qquad (3)$$

$$\frac{1}{6} x_2x_3x_6 + \frac{e}{2} x_1x_3x_6 - K_1h_b\ell_b^2[x_1 + (x_2^2-x_3^2)^{\frac{1}{2}}] \geq 0 . \qquad (4)$$

$$\frac{1}{12}\,x_2x_3^2x_4 + \frac{e}{4}\,x_1x_3^2x_4 - 2.2(K_1h_t\ell_t^2)^{4/3}[x_1 + (x_2^2-x_3^2)^{1/2}]^{4/3} \geq 0\ , \qquad (5)$$

$$\frac{1}{12}\,x_2x_3^2x_5 + \frac{e}{4}\,x_1x_3^2x_5 - 2.2(K_1h_m\ell_m^2)^{4/3}[x_1 + (x_2^2-x_3^2)^{1/2}]^{4/3} \geq 0\ , \qquad (6)$$

$$\frac{1}{12}\,x_2x_3^2x_6 + \frac{e}{4}\,x_1x_3^2x_5 - 2.2(K_1h_b\ell_b^2)^{4/3}[x_1 + (x_2^2-x_3^2)^{1/2}]^{4/3} \geq 0\ . \qquad (7)$$

$$x_4 - t_t^{min} \geq 0\ , \qquad (8)$$

$$10x_4 - [3.9 \cdot 1.05(.01h_{1t})^{1/2}(.01x_1) + 10K_2] \geq 0\ , \qquad (9)$$

$$10x_4 - [3.9 \cdot 1.05(.01h_{1t})^{1/2}(.01x_2) + 10K_2] \geq 0\ , \qquad (10)$$

$$x_5 - t_m^{min} \geq 0\ , \qquad (11)$$

$$10x_5 - [3.9 \cdot 1.05(.01h_{1m})^{1/2}(.01x_1) + 10K_2] \geq 0\ , \qquad (12)$$

$$10x_5 - [3.9 \cdot 1.05(.01h_{1m})^{1/2}(.01x_2) + 10K_2] \geq 0\ , \qquad (13)$$

$$x_6 - t_b^{min} \geq 0\ , \qquad (14)$$

$$10x_6 - [3.9 \cdot 1.05(.01h_{1b})^{1/2}(.01x_1) + 10K_2] \geq 0\ , \qquad (15)$$

$$10x_6 - [3.9 \cdot 1.05(.01h_{1b})^{1/2}(.01x_2) + 10K_2] \geq 0\ . \qquad (16)$$

$$x_i \geq 0;\ i=1,2,\text{ and } 3\ .$$

The listing of the problem parameters and their values is in Table 14.

4.3.2. Computer listing of the symbolic code. (See Figure 5.)

4.3.3. Selected pages from computer output. (See Appendix 1, page 78.)

TABLE 14

PROBLEM DESIGN PARAMETERS
(SAMPLE PROBLEM 3)

Input Data		
	B = 476	cm
	Γ = 7.85	ton/cm
*	ℓ_t = 495	cm
*	ℓ_m = 385	cm
*	ℓ_b = 315	cm
	h_t = 498	cm
	h_m = 938	cm
	h_b = 1288	cm
	h_{1t} = 745	cm
	h_{1m} = 1130	cm
	h_{1b} = 1445	cm
*	t_t^{min} = 1.05	cm
*	t_m^{min} = 1.05	cm
*	t_b^{min} = 1.05	cm
	e = 0.8	cm
	k_1 = 6.94×10^{-8}	cm^{-1}
*	k_2 = 0.15	cm
	h_a = 250	cm

*Parameters marked with an asterisk are selected for sensitivity analysis.

```
//AAAAAA JOB (5660,R,4,4),ABG
// EXEC FORG6,DSN='0R5660.ABG',PROG=MAIN
//SYSLIB DD
//        DD
//        DD DSN=GWU.FORTLIB,DISP=SHR
//GO.FT05F001 DD *
   0.00001            100.    .000001            4.0          800.  19   6   0
      3       2       2       1       1       1       1       2       1       1
EXAMPLE-3 : SOLUTION AND SESITIVITY ANALYSIS OF CORRUGATED BULKHEAD
    6   45   19    0    1                          15
 V    X1                    45.8
 V    X2                    43.2
 V    X3                    30.5
 V    X4                     1.2
 V    X5                     1.2
 V    X6                     1.3
 P    LT                     495.
 P    LM                     385.
 P    LB                     315.
 P   TTM                     1.05
 P   TMM                     1.05
 P   TBM                     1.05
 P    K2                    0.15
 D     E                      .8
 D    GB                  3736.6
 D   KHT            0.00003456
 D   KHM            0.00006510
 D   KHB            0.00008938
 D    CT               .011177
 D    CM               .013765
 D    CB               .015566
 S    Z1    X2 POW           2.
 S    Z1    X3 POW           2.          -1.
 S    Z2    X1 IDN
 S    Z2    Z1 POW           .5
 Q    Z3    X3 LIN           .5
 Q    Z3    X4 IDN
 Q    Z4    X3 LIN           .5
 Q    Z4    X5 IDN
 Q    Z5    X3 LIN           .5
 Q    Z5    X6 IDN
 S    Z6    X2 LIN  0.33333333
 S    Z6    X1 LIN            E
 Q    Z7    Z3 IDN
 Q    Z7    Z6 IDN
 Q    Z8    Z4 IDN
 Q    Z8    Z6 IDN
 Q    Z9    Z5 IDN
 Q    Z9    Z6 IDN
 S   Z10    Z2 LIN          KHT
 S   Z11    Z2 LIN          KHM
 S   Z12    Z2 LIN          KHB
 S   Z13   Z10 LIN           LT
 S   Z14   Z11 LIN           LM
 S   Z15   Z12 LIN           LB
```

Figure 5.--Computer listing of sample problem 3.

```
S   Z16   Z13 LIN            LT
S   Z17   Z14 LIN            LM
S   Z18   Z15 LIN            LB
S    G1    X2 IDN
S    G1    X3 LIN           -1.
S    G2    Z7 IDN
S    G2   Z16 LIN           -1.
S    G3    Z8 IDN
S    G3   Z17 LIN           -1.
S    G4    Z9 IDN
S    G4   Z18 LIN           -1.
Q    G5    Z7 IDN
Q    G5    X3 LIN            .5
S    G5   Z16 POW  1.33333333          -2.2
Q    G6    Z8 IDN
Q    G6    X3 LIN            .5
S    G6   Z17 POW  1.33333333          -2.2
Q    G7    Z9 IDN
Q    G7    X3 LIN            .5
S    G7   Z18 POW  1.33333333          -2.2
S    G8    X4 LIN           -1.          TTM
S    G9    X4 LIN           -1.
S    G9    X1 LIN            CT          K2
S   G10    X4 LIN           -1.
S   G10    X2 LIN            CT          K2
S   G11    X5 LIN           -1.          TMM
S   G12    X5 LIN           -1.
S   G12    X1 LIN            CM          K2
S   G13    X5 LIN           -1.
S   G13    X2 LIN            CM          K2
S   G14    X6 LIN           -1.          TBM
S   G15    X6 LIN           -1.
S   G15    X1 LIN            CB          K2
S   G16    X6 LIN           -1.
S   G16    X2 LIN            CB          K2
S   Z19    X1 IDN
S   Z19    X2 IDN
S   Z20   Z19 LIN            GB
S   Z21    X4 LIN            LT
S   Z21    X5 LIN            LM
S   Z21    X6 LIN            LB
Q   Z22   Z20 IDN
Q   Z22   Z21 IDN
Q   OBJ   Z22 IDN
Q   OBJ    Z2 POW           -1.
L    X1    X3                0.
L    G1    G7                0.
U    G8   G16                0.
5
        .001                    .001
         1                       1
//
```

Figure 5.--continued

5. General Description of the New Subroutines

The following is a list of the names of the subroutines that comprise the SYMBOLIC FACTORABLE SUMT model in alphabetical order:

BODY	OUTPUT
CHCKER	PARSEN*
CONVRG	PEVALU
DLPEN*	PUNCH
ESTIM	REJECT
EVALU	RESTNT**
EXDELP*	RHOCOM
FEAS	SECORD
FINAL	SENS*
GRAD	STORE
GRAD1**	SYMPUT*
GVALU*	TCHECK
INVERS	TIMEC
MAIN	VALU*
MATRIX**	XMOVE
OPT	

Subroutines marked with a single asterisk (*) are the new subroutines that have been added to the SUMT-Version 4 model in its various developmental stages to result in SYMBOLIC SENSITIVITY SUMT model. Subroutines marked with a double asterisk (**) correspond to the user's subroutine in SUMT-Version 4.

In order to provide problem function values, their gradients and Hessian matrices, users had to code the latter subroutines in SUMT-Version 4, while here, in conjunction with the new subroutines, they are modified to serve the same purpose internally. The remainder of the subroutines virtually serve the same purposes as in SUMT-Version 4.

For brevity, a general description of the new subroutines is provided here. Readers interested in general descriptions of the remaining subroutines must refer to [1].

The new subroutines fall into two categories,

(i) those developed to relax the user from coding old users' subroutines--these subroutines are SYMPUT, VALU, and GVALU; and

(ii) those developed to perform sensitivity and elasticity calculations--these subroutines are PARSEN, SENS, EXDELP, and DLPEN.

Following is a brief and general description of the role of each of these subroutines.

SYMPUT

Reads in and stores in arrays the coded problem in symbolic factorable format such that problem functions can be synthesized with the aid of these arrays. The title, SYMPUT parameter, and flag cards are also read in by this subroutine. It prints out a modified listing of the coded problem if desired. As described in Section 3 (Tables 7 and 8), this subroutine selects F or A format internally, while reading the coded problem, for numeric and alphanumeric inputs, respectively. This is made possible with the aid of subroutine REREAD, which is a special purpose George Washington University FORTRAN Library routine. Following is a brief description of this subroutine.

"Subroutine Name: REREAD - permits reformating of a record.

Description: REREAD patches the FORTRAN I/O routine so that upon execution it examines every read for a data set reference number (device code) of 99. If 99 is not found normal reading takes place. If 99 is found reading does not take place and the record read by the last normal read is formated according to the new format. The call needs to be issued only once for the program.

Usage: CALL REREAD

Example:

```
      DIMENSION A(10),B(20)
      CALL REREAD
      ..........
      ..........
      READ (5,100) A  This will read 10 variables
                      on a card.
      READ (99,101) B  This will reformat the card
                      into 20A1.
      ..........
      ..........
100   FORMAT(10F2.0)
101   FORMAT(20A1)."
```

VALU

Evaluates the functional values, first and second derivatives of single variable functions as required by GVALU. These results are transferred to subroutine GVALU via common blocks to reconstruct the problem function values, gradients, and Hessian matrices.

GVALU

Using the information stored in subroutine SYMPUT regarding the coded problem, via common blocks, GVALU reconstructs the problem function values, gradients, and Hessian matrices with the aid of subroutine VALU.

PARSEN

Acts as a coordinating routine to conduct sensitivity analysis in the solution vector of each subproblem. It also calculates the first order sensitivity information, elasticities of solution vector with respect to the problem parameters, and prints them out.

SENS

Evaluates the functional values, first and second derivatives of single variable functions, partial derivatives of these functions with respect to function parameters (C1 and C2), and joint partials with respect to parameters and function variables as required by subroutine EXDELP. This information is transferred to subroutine EXDELP for reconstruction of functional values, gradients, Hessian matrices of the problem functions, and related partial derivatives and cross partial matrices required for sensitivity analysis.

EXDELP

Using the information stored in the subroutine SYMPUT about the coded problem, and utilizing the information calculated in SENS, via common blocks, reconstructs the functional values, gradients, Hessian matrices of the problem functions, their partials with respect to the problem parameters, and joint partials with respect to the problem variables and parameters.

DLPEN

Using the information calculated in EXDELP via common blocks, uses the sensitivity formulas given in Appendix 2, to calculate the zero*th* order sensitivity of the solution vector with respect to the problem parameters.

An important point worth noting here is that the subroutines SYMPUT, VALU, GVALU, SENS, and EXDELP are quite general subroutines that can be interfaced with any nonlinear programming algorithm which uses the first or the first and second derivative methods.

Subroutines comprising SYMBOLIC FACTORABLE SUMT are currently dimensioned to handle problems of the following maximum scale:

- total number of CVFs $\leq$ 100
- number of original variables $\leq$ 30
- total number of parameters $\leq$ 75
- number of sensitivity parameters $\leq$ 15
- total number of original problem constraints $\leq$ 200 .

However, if computer capacity permits, they can readily be redimensioned to solve problems of higher dimensions. All of these subroutines are separately filed in the TED facility [9] at The George Washington University under their corresponding names. They may be accessed from the above source using account number 55599 and the proper password.

REFERENCES

[1] MYLANDER, W. C., R. L. HOLMES, and G. P. McCORMICK (1971). A guide to SUMT-Version 4: The computer program implementing the sequential unconstrained minimization technique for nonlinear programming. RAC Paper RAC-P-63, Research Analysis Corporation, McLean, Virginia.

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[3] deSILVA, A. H. and G. P. McCORMICK (1977). An interim manual for using the SUMT algorithm in conjunction with the symbolic factorable programming language. Draft paper.

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[9] TED FACILITY USER'S GUIDE, V2M0 (1977). Center for Academic and Administrative Computing, George Washington University.

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APPENDIX 1

Annotated Computer Output for Solution of Sample Problem 1 (pages 50-68) and Selected Pages of Computer Solution Output for Sample Problems 2 (pages 69-77) and 3 (pages 78-84)

(1) EXAMPLE-1 : SOLUTION AND SENSITIVITY ANALYSIS

(2) SYMPUT PARAMETER CARD
N= 2 NN= 6 M= 4 MZ= 0 ITEST= 1 IPR= 0 ITES= 0 NSWIT= 0 IIC= 6

TYPE NAME	TYPE #	TYPE DESCRIP.	VAR. NAME	NUMBR. ASSND.	ARG. VARIAB. NAME	NUMBR.	FUNCT. TYPE	NUMBR. ASSND.	CONSTANT C1	CONSTANT C2
V		ORIG. VARIABLE	X1	1	0.20000D 01					
V		ORIG. VARIABLE	X2	2	0.20000D 01					
P		SENSITIVITY PAR	C5					1	0.10000D 01	
P		SENSITIVITY PAR	C6					2	-0.15000D 01	
D		DATA DEFINITION	C1						0.0	
D		DATA DEFINITION	C2						0.0	
(3) D		DATA DEFINITION	C3						0.30000000D 01	
D		DATA DEFINITION	C4						0.15000000D 01	
S	3	SEPARABL	X3	3	X1	1	SIN	20	0.50000000D 00	-.50000000D 01
S	3	SEPARABL	X3	3	X2	2	LIN	3	0.10000000D 01 C5	0.0
S	3	SEPARABL	X4	4	X2	2	LIN	3	0.10000000D 01	0.0
S	3	SEPARABL	X4	4	X1	1	LNG	15	0.10000000D 01	-.15000000D 01 C6
Q	4	QUADRATC	X5	5	X1	1	LIN	3	0.10000000D 01	0.0
Q	4	QUADRATC	X5	5	X2	2	LIN	3	0.10000000D 01	0.0
S	3	SEPARABL	X6	6	X1	1	QDR	33	0.10000000D 01	0.10000000D 02
S	3	SEPARABL	X6	6	X2	2	QDR	33	0.10000000D 01	0.10000000D 01

ARRAY OF CONSTANTS CC1(.)
****

(4)

LOCATION	VALUE
1	0.10000D 01
2	-0.15000D 01
3	0.0
4	0.0
5	0.30000D 01
6	0.15000D 01
7	0.0
8	0.10000D 01
9	0.50000D 00
10	-0.50000D 01
11	0.10000D 02

BOUNDS:NCONC(.,.) : LOWER BOUND HAS MINUS SIGN

(5)

	CVF	CVF#	VALUE OF BOUND	LOCATION IN CC1(.)
1	X3	3	0.0	3
2	X4	4	0.0	-4
3	X5	5	0.3000000 01	-5
4	X1	1	0.1500000 01	-6

RELATIVE LOCATION:ITAB(.,.)

(6)

CVF#	SEPARABLE	QUADRATIC
3	1	1
4	3	1
5	5	1
6	5	2
7	7	2

T-402

	ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN CC1	C2	LOC.IN CC1
1	X1	1	20	SIN	0.5000D 00	9	-.500D 01	10
2	X2	2	3	LIN	0.1000D 01	1	0.0	7
3	X2	2	3	LIN	0.1000D 01	8	0.0	7
4	X1	1	15	LNG	0.1000D 01	8	-.150D 01	2
5	X1	1	33	QDR	0.1000D 01	8	0.100D 02	11
6	X2	2	33	QDR	0.1000D 01	8	0.100D 01	8

QUADRATIC DEFINITIONS : IUADTB(.,.) : EACH LINE REFERS TO A PAIR OF ARGUMENTS

	FIRST ARGUMENT									SECOND ARGUMENT								
	ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN CC1	C2	LOC.IN CC1		ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN CC1	C2	LOC.IN CC1	
1	X1	1	3	LIN	0.1000D 01	8	0.0	7	*	X2	2	3	LIN	0.1000D 01	8	0.0	7	

ROUTINE-SUNT VERSION 4 08/10/71

(9) N= 2 M= 4 MZ= 0

MAX. TIME= 0.50000000 03 R= 0.10000000 03 RATIO= 0.10000000 02 EPSILON= 0.10000000-04 THETA= 0.10000000-04

(10) OPTIONS SELECTED
3 2 1 1 1 1 1 2 1 1

(11) TOLERANCES
0.10000000-02 0.10000000-02

(12) SECOND SET OF OPTIONS
1 1
F= 0.65000000 02 P= 0.0 G= 0.0 RSIGMA= 0.0 H= 0.0
THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

(13) X1= 0.20000000 01 X2= 0.20000000 01
THE CONSTRAINT VALUES
0.22073550 01 0.96027920 00 0.10000000 01 0.50000000 00

*****THE FEASIBLE STARTING POINT TO BE USED IS ...
F= 0.65000000 02 P= 0.0 G= 0.0 RSIGMA= 0.0 H= 0.0
(14) THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.20000000 01 X2= 0.20000000 01
THE CONSTRAINT VALUES
0.22073550 01 0.96027920 00 0.10000000 01 0.50000000 00

****
POINT= 3 DOTT= 0.95411290-05 RHO= 0.10000000 03 MAGNITUDE= 0.31566160-01 PHASE= 2
F= 0.46097530 02 P= -0.33441220 03 G= -0.35390250 03 RSIGMA= -0.38050970 03 H= 0.0
THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.37505537 01 X2= 0.36536666 01
THE CONSTRAINT VALUES
(15) 0.11163480 01 0.16708110 01 0.10703270 02 0.22505540 01

LAGRANGE MULTIPLIERS
F= 0.46097530 02 P= -0.33441220 03 G= -0.35390250 03 RSIGMA= -0.38050970 03 H= 0.0
THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.37505537 01 X2= 0.36536666 01
THE CONSTRAINT VALUES
0.89577840 02 0.59851160 02 0.93429370 01 0.44433510 02

SENSITIVITY OF SOLUTION VECTOR WRT PROBLEM PARAMETERS
****

PROBLEM PARAMETERS ARE
==========

(16) C5 C6

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X1 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.46514D 00 0.31499D 00

RESPECTIVE ELASTICITIES ARE :

```
 -0.12402D 00 -0.12598D 00

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X2  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-----------------------------------------------------------------------------

-0.27637D 01 -0.48034D 00

RESPECTIVE ELASTICITIES ARE :
*****************************

 -0.75642D 00  0.19720D 00
APPARENTLY ROUNDOFF ERRORS PREVENT A MORE ACCURATE DETERMINATION OF THE MINIMUM OF THIS SUBPROBLEM.

*****************************************
        POINT=   6        DOTT=  0.14712310-04       RHO=  0.1000000D 02       MAGNITUDE=  0.20370570-01       PHASE= 2
(18)  F=  0.3680283D 02     P=  0.32881460 01     G= -0.31971710 01     RSIGMA= -0.3351468D 02     H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------
   X1=  0.43303384D 01    X2=  0.31581859D 01
     THE CONSTRAINT VALUES
      0.98431330 00        0.95971740 00        0.10676010 02        0.28303380 01


    1ST  ORDER ESTIMATES
      F=  0.3584189D 02     P=  0.40810410 02     G=  0.35770090 02     RSIGMA=  0.0               H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------
   X1=  0.43947589D 01    X2=  0.31031325D 01
     THE CONSTRAINT VALUES
      0.94704650 00        0.88251350 00        0.10637520 02        0.28947590 01


  LAGRANGE MULTIPLIERS
      F=  0.3584189D 02     P=  0.40810410 02     G=  0.35770090 02     RSIGMA=  0.0               H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------
   X1=  0.43947589D 01    X2=  0.31031325D 01
     THE CONSTRAINT VALUES
      0.10159370 02        0.10419730 02        0.93667920 00        0.35331460 01


SENSITIVITY OF SOLUTION VECTOR WRT PROBLEM PARAMETERS
*****************************************************

PROBLEM PARAMETERS ARE
**********************

  C5          C6

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X1  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-----------------------------------------------------------------------------

-0.87601D 00  0.48925D 00

THE ESTIMATES OF FIRST  ORDER SENSITIVITY OF  X1  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-----------------------------------------------------------------------------
(19)
-0.92166D 00  0.50861D 00

RESPECTIVE ELASTICITIES ARE :
*****************************
```

```
THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X2  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-----------------------------------------------------------------------------------------

-0.13339D 01 -0.85640D 00

THE ESTIMATES OF FIRST  ORDER SENSITIVITY OF  X2  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-----------------------------------------------------------------------------------------

-0.11750D 01 -0.89818D 00

RESPECTIVE ELASTICITIES ARE :
*****************************

 -0.37206D 00  0.42660D 00

*******************************************
(20)      POINT=  10       DOTT=  0.5886959D-05      RHO=  0.1000000D 01      MAGNITUDE=  0.1815338D-01      PHASE= 2
     F=  0.2673064D 02      P=  0.2645202D 02      G=  0.2273064D 02      RSIGMA= -0.2786173D 00      H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------------------------
  X1=  0.507908120 01     X2=  0.258593750 01
    THE CONSTRAINT VALUES
     0.24573700 00        0.14824190 00        0.10134190 02        0.35790810 01


   2ND  ORDER ESTIMATES
     F=  0.2562837D 02      P=  0.2890675D 02      G=  0.2561151D 02      RSIGMA=  0.0              H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------------------------
  X1=  0.517002760 01     X2=  0.251648790 01
    THE CONSTRAINT VALUES
     0.12493470 00        0.52170830-01        0.10010310 02        0.36700280 01


   1ST  ORDER ESTIMATES
     F=  0.2572115D 02      P=  0.2902579D 02      G=  0.2561151D 02      RSIGMA=  0.0              H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------------------------
  X1=  0.516227490 01     X2=  0.252235430 01
    THE CONSTRAINT VALUES
     0.13550480 00        0.60288280-01        0.10021090 02        0.36622750 01


 LAGRANGE MULTIPLIERS
     F=  0.2572115D 02      P=  0.2902579D 02      G=  0.2561151D 02      RSIGMA=  0.0              H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------------------------
  X1=  0.516227490 01     X2=  0.252235430 01
    THE CONSTRAINT VALUES
     0.40693910 01        0.67457310 01        0.98675900-01        0.27940130 00


SENSITIVITY OF SOLUTION VECTOR WRT PROBLEM PARAMETERS
*****************************************************

PROBLEM PARAMETERS ARE
**********************

  C5          C6

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X1  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-----------------------------------------------------------------------------------------
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```
-0.10112D 01  0.65410D 00

THE ESTIMATES OF FIRST  ORDER SENSITIVITY OF  X1  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-----------------------------------------------------------------------------------------

-0.10262D 01  0.67242D 00

RESPECTIVE ELASTICITIES ARE :
*****************************

 -0.20204D 00 -0.19858D 00


THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X2  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-----------------------------------------------------------------------------------------

-0.40327D 00 -0.13672D 01

THE ESTIMATES OF FIRST  ORDER SENSITIVITY OF  X2  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-----------------------------------------------------------------------------------------

-0.29986D 00 -0.14239D 01

RESPECTIVE ELASTICITIES ARE :
*****************************

 -0.11596D 00  0.82595D 00
APPARENTLY ROUNDOFF ERRORS PREVENT A MORE ACCURATE DETERMINATION OF THE MINIMUM OF THIS SUBPROBLEM.

*********************************************
        POINT=  13        DOTT=  0.6592370D-04       RHO=  0.1000000D 00       MAGNITUDE=  0.2027132D 00       PHASE= 2
(21)      F=  0.2503643D 02      P=  0.2544952D 02      G=  0.2463643D 02      RSIGMA=  0.4130901D 00      H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------
   X1=  0.5225095AD 01     X2=  0.2495565ED 01
     THE CONSTRAINT VALUES
      0.2798036D-01         0.1535560D-01         0.1003957D 02         0.3725095D 01


    2ND  ORDER ESTIMATES
       F=  0.2484312D 02      P=  0.2530088D 02      G=  0.2484818D 02      RSIGMA=  0.0               H=  0.0
     THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------
   X1=  0.5242117AD 01     X2=  0.2481518D 01
     THE CONSTRAINT VALUES
      0.1565042D-02         0.6348279D-04         0.1002746D 02         0.3742118D 01


    1ST  ORDER ESTIMATES
       F=  0.2485182D 02      P=  0.2533813D 02      G=  0.2484818D 02      RSIGMA=  0.0               H=  0.0
     THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------
   X1=  0.5241319ED 01     X2=  0.2485523BD 01
     THE CONSTRAINT VALUES
      0.2924514D-02         0.6639904D-03         0.1002742D 02         0.3741319D 01


  LAGRANGE MULTIPLIERS
       F=  0.2485182D 02      P=  0.2533813D 02      G=  0.2484818D 02      RSIGMA=  0.0               H=  0.0
     THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------
   X1=  0.5241319ED 01     X2=  0.2485523BD 01
     THE CONSTRAINT VALUES
      0.3573936D 01         0.6512281D 01         0.9960593D-02         0.2684495D-01
```

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SENSITIVITY OF SOLUTION VECTOR WRT PROBLEM PARAMETERS
*****************************************************

PROBLEM PARAMETERS ARE
**********************

  C5            C6

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X1  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.96309D 00  0.65477D 00

THE ESTIMATES OF FIRST  ORDER SENSITIVITY OF  X1  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.95775D 00  0.65484D 00

RESPECTIVE ELASTICITIES ARE :
*****************************

 -0.18330D 00 -0.18799D 00

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X2  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.28617D 00 -0.14148D 01

THE ESTIMATES OF FIRST  ORDER SENSITIVITY OF  X2  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.27316D 00 -0.14201D 01

RESPECTIVE ELASTICITIES ARE :
*****************************

 -0.10946D 00  0.85359D 00

*********************************************
        POINT=  15        DOTT=  0.56553240-06       RHO=  0.1000000D-01       MAGNITUDE=  0.64527180-01       PHASE= 2
(22)   F=  0.24857180 02      P=  0.24944370 02      G=  0.24817180 02      RSIGMA=  0.87188820-01      H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------
   X1=  0.52410000D 01     X2=  0.248630490 01
     THE CONSTRAINT VALUES
      0.28353040-02        0.15364480-02        0.10030720 02        0.37410000 01


    2ND  ORDER ESTIMATES
       F=  0.24837160 02      P=  0.24883700 02      G=  0.24837270 02      RSIGMA=  0.0               H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------
   X1=  0.524278190 01     X2=  0.248527350 01
     THE CONSTRAINT VALUES
      0.24252320-05       -0.48471950-05        0.10029750 02        0.37427820 01


    1ST  ORDER ESTIMATES
       F=  0.24837310 02      P=  0.24888240 02      G=  0.24837270 02      RSIGMA=  0.0               H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------------
   X1=  0.524276720 01     X2=  0.248527600 01
     THE CONSTRAINT VALUES
```

```
 LAGRANGE MULTIPLIERS
     F=  0.24837310 02      P=  0.24888240 02      G=  0.24837270 02      RSIGMA=  0.0                H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------
   X1=  0.52427672D 01     X2=  0.24852760D 01
    THE CONSTRAINT VALUES
     0.35269580 01         0.65085180 01         0.99693700-03         0.26730820-02

SENSITIVITY OF SOLUTION VECTOR WRT PROBLEM PARAMETERS
*****************************************************

PROBLEM PARAMETERS ARE
**********************

  C5            C6

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X1  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.99476D 00  0.68074D 00

THE ESTIMATES OF FIRST  ORDER SENSITIVITY OF  X1  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.99828D 00  0.68363D 00

RESPECTIVE ELASTICITIES ARE :
*****************************

 -0.19047D 00 -0.19566D 00

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X2  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.28570D 00 -0.14761D 01

THE ESTIMATES OF FIRST  ORDER SENSITIVITY OF  X2  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.28565D 00 -0.14830D 01

RESPECTIVE ELASTICITIES ARE :
*****************************

 -0.11489D 00  0.89468D 00

*********************************************
(23)     POINT=  17       DOTT=  0.43389810-06      RHO=  0.10000000-02      MAGNITUDE=  0.15291740 00      PHASE= 2
     F=  0.24839180 02      P=  0.24852510 02      G=  0.24835180 02      RSIGMA=  0.13327140-01      H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------
   X1=  0.52426029D 01     X2=  0.24853800D 01
    THE CONSTRAINT VALUES
     0.28400250-03         0.15286360-03         0.10029860 02         0.37426030 01


    2ND  ORDER ESTIMATES
     F=  0.24837180 02      P=  0.24841840 02      G=  0.24837180 02      RSIGMA=  0.0                H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
```

```
THE CONSTRAINT VALUES
 0.1104730D-06       -0.8866205D-06       0.10029770 02       0.3742781D 01


 1ST  ORDER ESTIMATES
   F=  0.2483718D 02     P=  0.2484230D 02     G=  0.2483718D 02     RSIGMA=  0.0              M=  0.0
 THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------------
 X1=  0.5242781DD 01    X2=  0.248527730 01
 THE CONSTRAINT VALUES
 0.4258959D-06       -0.8593484D-06       0.10029760 02       0.3742781D 01


 LAGRANGE MULTIPLIERS
   F=  0.2483718D 02     P=  0.2484230D 02     G=  0.2483718D 02     RSIGMA=  0.0              M=  0.0
 THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------------
 X1=  0.5242781DD 01    X2=  0.248527730 01
 THE CONSTRAINT VALUES
 0.3521096D 01       0.6541779D 01       0.9970228D-04       0.2671937D-03


SENSITIVITY OF SOLUTION VECTOR WRT PROBLEM PARAMETERS
*****************************************************

PROBLEM PARAMETERS ARE
======================

  C5          C6

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X1  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.99185D 00  0.71022D 00

THE ESTIMATES OF FIRST  ORDER SENSITIVITY OF  X1  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.99153D 00  0.71350D 00

RESPECTIVE ELASTICITIES ARE :
=============================

 -0.189130 00 -0.204140 00

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X2  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.283890 00 -0.154060 01

THE ESTIMATES OF FIRST  ORDER SENSITIVITY OF  X2  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.283690 00 -0.154780 01

RESPECTIVE ELASTICITIES ARE :
=============================

 -0.114140 00  0.934130 00

*****************************************
(24)     POINT=  19      DOTI=  0.8083987D-06      RHO=  0.1000000D-03      MAGNITUDE=  0.8201477D 00      PHASE= 2
```

THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.52427628D 01 X2= 0.24852884D 01

THE CONSTRAINT VALUES

0.2882452D-04 0.1542979D-04 0.1002978D 02 0.3742763D 01

2ND ORDER ESTIMATES

F= 0.2483719D 02 P= 0.2483765D 02 G= 0.2483719D 02 RSIGMA= 0.0 H= 0.0

THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.52427806D 01 X2= 0.24852782D 01

THE CONSTRAINT VALUES

0.4708845D-06 0.1697422D-06 0.1002977D 02 0.3742781D 01

1ST ORDER ESTIMATES

F= 0.2483719D 02 P= 0.2483770D 02 G= 0.2483719D 02 RSIGMA= 0.0 H= 0.0

THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.52427806D 01 X2= 0.24852782D 01

THE CONSTRAINT VALUES

0.4704346D-06 0.1594513D-06 0.1002977D 02 0.3742781D 01

LAGRANGE MULTIPLIERS

F= 0.2483719D 02 P= 0.2483770D 02 G= 0.2483719D 02 RSIGMA= 0.0 H= 0.0

THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.52427806D 01 X2= 0.24852782D 01

THE CONSTRAINT VALUES

0.3469268D 01 0.6480970D 01 0.9970311D-05 0.2671823D-04

SENSITIVITY OF SOLUTION VECTOR WRT PROBLEM PARAMETERS

****

PROBLEM PARAMETERS ARE

==========

C5 C6

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X1 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.8119D 00 0.6582D 00

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X1 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.7911D 00 0.6524D 00

RESPECTIVE ELASTICITIES ARE :

==========

-0.1509D 00 -0.1866D 00

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X2 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.2321D 00 -0.1427D 01

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X2 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.22634D 00 -0.14153D 01

RESPECTIVE ELASTICITIES ARE :
****

-0.91074D-01 0.85420D 00

****
(25) POINT= 21 DOTT= 0.63905740-06 RHO= 0.10000000-04 MAGNITUDE= 0.25509340 01 PHASE= 2
F= 0.24837210 02 P= 0.24837430 02 G= 0.24837170 02 RSIGMA= 0.22514500-03 H= 0.0
THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.52427791D 01 X2= 0.24852791D 01
THE CONSTRAINT VALUES
0.2836728D-05 0.1565929D-05 0.1002977D 02 0.3742779D 01

2ND ORDER ESTIMATES
F= 0.2483718D 02 P= 0.2483723D 02 G= 0.2483718D 02 RSIGMA= 0.0 H= 0.0
THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.52427809D 01 X2= 0.24852781D 01
THE CONSTRAINT VALUES
-0.5607949D-07 0.2414813D-07 0.1002977D 02 0.3742781D 01

1ST ORDER ESTIMATES
F= 0.2483718D 02 P= 0.2483724D 02 G= 0.2483718D 02 RSIGMA= 0.0 H= 0.0
THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.52427809D 01 X2= 0.24852781D 01
THE CONSTRAINT VALUES
-0.5081435D-07 0.2550116D-07 0.1002977D 02 0.3742781D 01

LAGRANGE MULTIPLIERS
F= 0.2483718D 02 P= 0.2483724D 02 G= 0.2483718D 02 RSIGMA= 0.0 H= 0.0
THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.52427809D 01 X2= 0.24852781D 01
THE CONSTRAINT VALUES
0.3525188D 01 0.6385985D 01 0.9970319D-06 0.2671811D-05

SENSITIVITY OF SOLUTION VECTOR WRT PROBLEM PARAMETERS
****

PROBLEM PARAMETERS ARE
****

C5 C6

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X1 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.5946D 00 0.5292D 00

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X1 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.5705D 00 0.5148D 00

RESPECTIVE ELASTICITIES ARE :

-0.108830 00 -0.147310 00

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X2 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.170130 00 -0.114800 01

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X2 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.163250 00 -0.111690 01

RESPECTIVE ELASTICITIES ARE :
==========

-0.656870-01 0.674120 00

EXAMPLE-1 : SOLUTION AND SENSITIVITY ANALYSIS

SYMPUT PARAMETER CARD
N= 2 NN= 6 M= 4 MZ= 0 ITEST= 0 IPR= 0 ITES= 0 NSWIT= 0 IID= 6

TYPE NAME	TYPE #	TYPE DESCRIP.	VAR. NAME	NUMBR. ASSND.	ARG. VARIAB. NAME	NUMBR.	FUNCT. TYPE	NUMBR. ASSND.	CONSTANT C1	CONSTANT C2
V		ORIG. VARIABLE	X1	1	0.20000D 01					
V		ORIG. VARIABLE	X2	2	0.20000D 01					
P		SENSITIVITY PAR	C5					1	0.10000D 01	
P		SENSITIVITY PAR	C6					2	-0.15000D 01	
D		DATA DEFINITION	C1						0.0	
D		DATA DEFINITION	C2						0.0	
D		DATA DEFINITION	C3						0.300000000 01	
D		DATA DEFINITION	C4						0.150000000 01	
S	3	SEPARABL	X3	3	X1	1	SIN	20	0.500000000 00	-.500000000 01
S	3	SEPARABL	X3	3	X2	2	LIN	3	0.100000000 01	0.0
									C5	
S	3	SEPARABL	X4	4	X2	2	LIN	3	0.100000000 01	0.0
S	3	SEPARABL	X4	4	X1	1	LNG	15	0.100000000 01	-.150000000 01
										C6
Q	4	QUADRATC	X5	5	X1	1	LIN	3	0.100000000 01	0.0
Q	4	QUADRATC	X5	5	X2	2	LIN	3	0.100000000 01	0.0
S	3	SEPARABL	X6	6	X1	1	QDR	33	0.100000000 01	0.100000000 02
S	3	SEPARABL	X6	6	X2	2	QDR	33	0.100000000 01	0.100000000 01

ARRAY OF CONSTANTS CC1(.)
****

LOCATION	VALUE
1	0.10000D 01
2	-0.15000D 01
3	0.0
4	0.0
5	0.30000D 01
6	0.15000D 01
7	0.0
8	0.10000D 01
9	0.50000D 00
10	-0.50000D 01
11	0.10000D 02

BOUNDS:NCONC(.,.) : LOWER BOUND HAS MINUS SIGN

	CVF	CVF#	VALUE OF BOUND	LOCATION IN CC1(.)
1	X3	3	0.0	3
2	X4	4	0.0	-4
3	X5	5	0.3000000 01	-5
4	X1	1	0.1500000 01	-6

RELATIVE LOCATION:ITAB(.,.)

CVF#	SEPARABLE	QUADRATIC
3	1	1
4	3	1
5	5	1
6	5	2
7	7	2

SEPARABLE DEFINITIONS : IEPTAB(...)

	ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN CC1	C2	LOC.IN CC1
1	X1	1	20	SIN	0.5000D 00	9	-.500D 01	10
2	X2	2	3	LIN	0.1000D 01	1	0.0	7
3	X2	2	3	LIN	0.1000D 01	8	0.0	7
4	X1	1	15	LNG	0.1000D 01	8	-.150D 01	2
5	X1	1	33	QDR	0.1000D 01	8	0.1000 02	11
6	X2	2	33	QDR	0.1000D 01	8	0.100D 01	8

QUADRATIC DEFINITIONS : IUADTB(...) : EACH LINE REFERS TO A PAIR OF ARGUMENTS

	********** FIRST ARGUMENT **********									********** SECOND ARGUMENT **********							
	ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN CC1	C2	LOC.IN CC1		ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN CC1	C2	LOC.IN CC1
1	X1	1	3	LIN	0.1000D 01	8	0.0	7	*	X2	2	3	LIN	0.1000D 01	8	0.0	7

```
NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4 08/10/71

    N=  2      M=  4      MZ=  0

     MAX. TIME= 0.50000000D 03    R= 0.10000000D 03    RATIO= 0.10000000D 02      EPSILON= 0.10000000D-04    THETA= 0.10000000D-04

OPTIONS SELECTED
    3      2      1      1      1      1      1      2      1      1

TOLERANCES
     0.10000000D-02       0.10000000D-02

SECOND SET OF OPTIONS
    1      1
      F=  0.65000000 02     P=  0.0               G=  0.0              RSIGMA=  0.0              H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------------
   X1=  0.20000000D 01    X2=  0.20000000D 01
    THE CONSTRAINT VALUES
     0.2207355D 01       0.9602792D 00       0.10000000D 01       0.50000000D 00

*****THE FEASIBLE STARTING POINT TO BE USED IS ...
      F=  0.65000000D 02     P=  0.0               G=  0.0              RSIGMA=  0.0              H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------------
   X1=  0.20000000D 01    X2=  0.20000000D 01
    THE CONSTRAINT VALUES
     0.2207355D 01       0.9602792D 00       0.10000000D 01       0.50000000D 00
     1                0.20000000D 01
     2                0.20000000D 01
     3               -0.22073549D 01
     4                0.96027923D 00
     5                0.40000000D 01
     6                0.65000000D 02
 1
    -0.1350760 01         0.1000000 01
 2
    -0.7500000 00         0.1000000 01
 3
     0.2000000 01         0.2000000 01
 4
    -0.1600000 02         0.2000000 01
 1
     0.1051849 01         0.0
 2
     0.3750000 00         0.0
 3
     0.0                  0.0
 4
     0.2000000 01         0.2000000 01
 1    1
 2    1
 3    1
 1    2         0.1000000 01
 4    2
```

EXAMPLE-1 : SOLUTION AND SENSITIVITY ANALYSTS

SYMPUT PARAMETER CARD

N= 2 NN= 6 M= 4 MZ= 0 ITEST= 1 IPR= 0 ITES= 2 NSWIT= 0 IIC= 6

TYPE NAME	TYPE #	TYPE DESCRIP.	VAR. NAME	NUMBR. ASSND.	ARG. VARIAB. NAME	NUMBR.	FUNCT. TYPE	NUMBR. ASSND.	CONSTANT C1	CONSTANT C2
V		ORIG. VARIABLE	X1	1	0.200000D 01					
V		ORIG. VARIABLE	X2	2	0.200000D 01					
P		SENSITIVITY PAR	C5					1	0.100000D 01	
P		SENSITIVITY PAR	C6					2	-0.150000D 01	
D		DATA DEFINITION	C1						0.0	
D		DATA DEFINITION	C2						0.0	
D		DATA DEFINITION	C3						0.300000000 01	
D		DATA DEFINITION	C4						0.150000000 01	
S	3	SEPARABL	X3	3	X1	1	SIN	20	0.500000000 00	-.500000000 01
S	3	SEPARABL	X3	3	X2	2	LIN	3	0.100000000 01 C5	0.0
S	3	SEPARABL	X4	4	X2	2	LIN	3	0.100000000 01	0.0
S	3	SEPARABL	X4	4	X1	1	LNG	15	0.100000000 01	-.150000000 01 C6
Q	4	QUADRATC	X5	5	X1	1	LIN	3	0.100000000 01	0.0
Q	4	QUADRATC	X5	5	X2	2	LIN	3	0.100000000 01	0.0
S	3	SEPARABL	X6	6	X1	1	QDR	33	0.100000000 01	0.100000000 02
S	3	SEPARABL	X6	6	X2	2	QDR	33	0.100000000 01	0.100000000 01

ARRAY OF CONSTANTS CC1(.)

LOCATION	VALUE
1	0.100000D 01
2	-0.150000D 01
3	0.0
4	0.0
5	0.300000D 01
6	0.150000 01
7	0.0
8	0.100000D 01
9	0.500000 00
10	-0.500000 01
11	0.100000D 02

BOUNDS:NCONC(...) : LOWER BOUND HAS MINUS SIGN

	CVF	CVF#	VALUE OF BOUND	LOCATION IN CC1(.)
1	X3	3	0.0	3
2	X4	4	0.0	-4
3	X5	5	0.3000000 01	-5
4	X1	1	0.1500000 01	-6

RELATIVE LOCATION:ITAB(...)

CVF#	SEPARABLE	QUADRATIC
3	1	1
4	3	1
5	5	1
6	5	2
7	7	2

SEPARABLE DEFINITIONS : IEPTAB(.,.)

	ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN CC1	C2	LOC.IN CC1
1	X1	1	20	SIN	0.5000D 00	9	-.5000 01	10
2	X2	2	3	LIN	0.1000D 01	1	0.0	7
3	X2	2	3	LIN	0.1000D 01	8	0.0	7
4	X1	1	15	LNG	0.1000D 01	8	-.150D 01	2
5	X1	1	33	QDR	0.1000D 01	8	0.100D 02	11
6	X2	2	33	QDR	0.1000D 01	8	0.100D 01	8

QUADRATIC DEFINITIONS : IJADTB(.,.) : EACH LINE REFERS TO A PAIR OF ARGUMENTS

	********** FIRST ARGUMENT **********									********** SECOND ARGUMENT **********							
	ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN CC1	C2	LOC.IN CC1		ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN CC1	C2	LOC.IN CC1
1	X1	1	3	LIN	0.1000D 01	8	0.0	7	*	X2	2	3	LIN	0.1000D 01	8	0.0	7

```
NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4 08/10/71

    N=  2      M=  4      MZ=  0

      MAX. TIME= 0.5000000D 03    R= 0.1000000D 03    RATIO= 0.10000000 02      EPSILON= 0.1000000D-04    THETA= 0.1000000D-04

OPTIONS SELECTED
    3      2      1      1      1      1      1      2      1      1

TOLERANCES
     0.10000000-02       0.1000000D-02

SECOND SET OF OPTIONS
    1      1
      F=  0.6500000D 02     P=  0.0                G=  0.0              RSIGMA=  0.0              H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------------------
   X1=  0.20000000D 01    X2=  0.20000000D 01
    THE CONSTRAINT VALUES
     0.2207355D 01       0.9602792D 00       0.1000000D 01       0.50000000 00

*****THE FEASIBLE STARTING POINT TO BE USED IS ...
      F=  0.6500000D 02     P=  0.0                G=  0.0              RSIGMA=  0.0              H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------------------
   X1=  0.20000000D 01    X2=  0.20000000D 01
    THE CONSTRAINT VALUES
     0.2207355D 01       0.9602792D 00       0.1000000D 01       0.50000000 00

**********************************************
       POINT=   3       DOTT=  0.9541129D-05      RHO=  0.1000000D 03      MAGNITUDE=  0.3156616D-01      PHASE= 2
      F=  0.4609753D 02     P= -0.3344122D 03     G= -0.3539025D 03     RSIGMA= -0.3805097D 03     H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------------------
   X1=  0.37505537D 01    X2=  0.36536666D 01
    THE CONSTRAINT VALUES
     0.1116348D 01       0.1670811D 01       0.1070327D 02       0.2250554D 01


  LAGRANGE MULTIPLIERS
      F=  0.4609753D 02     P= -0.3344122D 03     G= -0.3539025D 03     RSIGMA= -0.3805097D 03     H=  0.0
    THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------------------
   X1=  0.37505537D 01    X2=  0.36536666D 01
    THE CONSTRAINT VALUES
     0.8957784D 02       0.5985116D 02       0.9342937D 01       0.4443351D 02


SENSITIVITY OF SOLUTION VECTOR WRT PROBLEM PARAMETERS
******************************************************

PROBLEM PARAMETERS ARE
**********************

  C5           C6

(E) THE VALUES OF SIG(J) IN DLPEN ARE:
0.100000000 01  0.0

(F) THE VALUES OF ALF RETURNED BY INVERS ARE :
0.60008081D-02 -0.222974260-02

 IN EXPELR . THE VALS. OF CVEC XX(I) ARE :
```

```
 6  0.460975260 02

 THE VALS. OF DM(J)      : I.E. PARTIALS OF NEW CVFS W.R.T. ORIGINAL PROB. VARS. (N BY 1) PREMULT. BY ALF ;

0.226782740-02 -0.462971150-02  0.135621830-01 -0.108014550 00

 THE VALS. OF DIAGA (I,J) I.E. SECOND PARTIALS OF NEW CVFS W.R.T. Y    & X PREMULT. BY ALF;

 3 -0.222974260-02  0.0
 4  0.0              0.159997930-02
 5  0.0              0.0
 6  0.0              0.0

THE VALS OF DELEX(I,J) ;PARTIALS OF NEW CVF WRT SENS VAR. Y;

 3  0.365366660 01  0.0
 4  0.0              0.132190350 01
 5  0.0              0.0
 6  0.0              0.0

 THE VALUES OF DELA(J) ARE:
0.465139590 00 -0.314990030 00


THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF  X1  WRT PROBLEM PARAMETERS ARE RESPECTIVELY :
-------------------------------------------------------------------------------------------

-0.46514D 00  0.31499D 00

RESPECTIVE ELASTICITIES ARE :
*****************************

 -0.124020 00 -0.125980 00

 THE VALUES OF SIG(J) IN DLPEN ARE:
 0.0              0.100000000 01

 THE VALUES OF ALF RETURNED BY INVERS ARE :
-0.222974260-02  0.850058150-02

 IN EXDELP : THE VALS. OF CVFS XX(J) ARE ;
  1  0.375055370 01       2  0.365366660 01       3  -.111634750 01       4  0.167081140 01       5  0.137032730 02
  6  0.460975260 02

 THE VALS. OF DM(J)      : I.E. PARTIALS OF NEW CVFS W.R.T. ORIGINAL PROB. VARS. (N BY 1) PREMULT. BY ALF ;

 0.682940270-02  0.939234690-02  0.237351510-01  0.401353670-01

 THE VALS. OF DIAGA (I,J) I.E. SECOND PARTIALS OF NEW CVFS W.R.T. Y    & X PREMULT. BY ALF;

 3  0.850058150-02  0.0
 4  0.0             -0.594510250-03
 5  0.0              0.0
 6  0.0              0.0

THE VALS OF DELEX(I,J) ;PARTIALS OF NEW CVF WRT SENS VAR. Y;

 3  0.365366660 01  0.0
 4  0.0              0.132190350 01
 5  0.0              0.0
 6  0.0              0.0

  THE VALUES OF DELA(J) ARE:
0.276368840 01  0.480335270 00
```

EXAMPLE-2 : SOLUTION AND SENSITIVITY ANALYSIS OF HOPKINS PROBLEM

SYMPUT PARAMETER CARD
N= 8 NN= 14 M= 12 MZ= 0 ITEST= 1 IPR= 0 ITES= 0 NSWIT= 0 IIC= 9

TYPE NAME	TYPE #	TYPE DESCRIP.	VAR. NAME	NUMBR. ASSND.	ARG. VARIAB. NAME	NUMBR.	FUNCT. TYPE	NUMBR. ASSND.	CONSTANT C1	CONSTANT C2
V		ORIG. VARIABLE	X1	1	0.18000D 03					
V		ORIG. VARIABLE	X2	2	0.74000D 03					
V		ORIG. VARIABLE	X3	3	0.26800D 04					
V		ORIG. VARIABLE	X4	4	0.14650D 05					
V		ORIG. VARIABLE	X5	5	0.40000D 00					
V		ORIG. VARIABLE	X6	6	0.23000D 01					
V		ORIG. VARIABLE	X7	7	0.60000D 00					
V		ORIG. VARIABLE	X8	8	0.20000D 00					
P		SENSITIVITY PAR	A1					1	0.24000D 02	
P		SENSITIVITY PAR	A2					2	0.12000D 02	
P		SENSITIVITY PAR	A3					3	0.60000D 00	
P		SENSITIVITY PAR	A4					4	0.40000D 00	
P		SENSITIVITY PAR	A5					5	0.10000D 00	
P		SENSITIVITY PAR	A6					6	0.15000D 00	
P		SENSITIVITY PAR	A7					7	-0.15000D 00	
P		SENSITIVITY PAR	A8					8	0.11000D 02	
P		SENSITIVITY PAR	A9					9	-0.24000D 02	
Q	4	QUADRATC	X9	9	X4	4	IDN	2	0.0	0.0
Q	4	QUADRATC	X9	9	X6	6	IDN	2	0.0	0.0
Q	4	QUADRATC	X10	10	X3	3	IDN	2	0.0	0.0
Q	4	QUADRATC	X10	10	X6	6	IDN	2	0.0	0.0
S	3	SEPARABL	X11	11	X1	1	LIN	3	0.240000000D 02 A1	0.0
S	3	SEPARABL	X11	11	X2	2	LIN	3	0.120000000D 02 A2	0.0
S	3	SEPARABL	X11	11	X3	3	LIN	3	0.600000000D 00 A3	0.0
S	3	SEPARABL	X11	11	X4	4	LIN	3	0.400000000D 00 A4	0.0
S	3	SEPARABL	X11	11	X10	10	LIN	3	0.100000000D 00 A5	0.0
S	3	SEPARABL	X11	11	X9	9	LIN	3	0.150000000D 00 A6	0.0
Q	4	QUADRATC	X11	11	X9	9	LIN	3	-.150000000D 00 A7	0.0
Q	4	QUADRATC	X11	11	X5	5	IDN	2	0.0	0.0
Q	4	QUADRATC	X11	11	X1	1	LIN	3	0.110000000D 02 A8	0.0
Q	4	QUADRATC	X11	11	X7	7	IDN	2	0.0	0.0
Q	4	QUADRATC	X11	11	X1	1	LIN	3	-.240000000D 02 A9	0.0
Q	4	QUADRATC	X11	11	X8	8	IDN	2	0.0	0.0
S	3	SEPARABL	X12	12	X1	1	LIN	3	0.122500000D 02	0.0
S	3	SEPARABL	X12	12	X10	10	IDN	2	0.0	0.0
S	3	SEPARABL	X12	12	X9	9	LIN	3	0.700000000D 00	0.0

4	4	QUADRATIC	X12	12	X5	5	[illegible]	2	0.0	0.0
5	3	SEPARABL	X13	13	X6	6	[illegible]	33	[illegible]	[illegible]
5	3	SEPARABL	X14	14	X1	1	[illegible]	11	[illegible]	-.1[illegible]
5	3	SEPARABL	X14	14	X2	2	[illegible]	2	-.2[illegible]	0.0
5	3	SEPARABL	X14	14	X3	3	[illegible]	11	[illegible]	-.2[illegible]
5	3	SEPARABL	X14	14	X4	4	[illegible]	11	[illegible]	-.5[illegible]
5	3	SEPARABL	X14	14	X5	5	[illegible]	2	-.1[illegible]	0.0
5	3	SEPARABL	X14	14	X13	13	[illegible]	2	[illegible]	0.0
6	4	QUADRATIC	X14	14	X1	1	[illegible]	11	[illegible]	-.3[illegible]
6	4	QUADRATIC	X14	14	X7	7	[illegible]	10	[illegible]	0.0
6	3	SEPARABL	X14	14	X8	8	[illegible]	2	[illegible]	0.0
6	4	QUADRATIC	X14	14	X1	1	[illegible]	2	-.1[illegible]	0.0
6	4	QUADRATIC	X14	14	X3	3	[illegible]	10	-.1[illegible]	0.0
6	4	QUADRATIC	X14	14	X1	1	[illegible]	2	-.2[illegible]	0.0
6	4	QUADRATIC	X14	14	X4	4	[illegible]	10	-.1[illegible]	0.0

ARRAY OF CONSTANTS CC1(.)
****

LOCATION	VALUE
1	0.240000 02
2	0.120000 02
3	0.500000 00
4	0.400000 00
5	0.100000 00
6	0.150000 00
7	-0.150000 00
8	0.110000 02
9	-0.240000 02
10	0.0
11	0.100000 01
12	0.122500 02
13	0.700000 00
14	-0.200000 00
15	0.300000 01
16	0.500000 00
17	-0.100000 03
18	-0.200000 01
19	0.500000 00
20	-0.250000 02
21	0.500000 00
22	-0.500000 02
23	-0.100000 02
24	0.100000 03
25	0.500000 00
26	-0.300000 02
27	0.500000 00
28	0.100000 03
29	-0.100000 02
30	-0.100000 01
31	-0.200000 02
32	-0.100000 01
33	0.250000 05
34	0.340000 05
35	0.500000 00
36	0.900000 00

BOUNDSING36(...) : LOWER BOUND HAS, MEANS SIGN

CVF	CVF#	VALUE OF BOUND	LOCATION IN CC1(.)
1 XII	12	0.250000 05	33

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5	X1	1	0.0	-10
6	X2	2	0.0	-10
7	X3	3	0.0	-10
8	X4	4	0.0	-10
9	X5	5	0.0	-10
10	X6	6	0.0	-10
11	X7	7	0.0	-10
12	X8	8	0.0	-10

RELATIVE LOCATION:ITAB(...)

CVF#	SEPARABLE	QUADRATIC
9	1	1
10	1	2
11	1	5
12	7	6
13	10	7
14	11	7
15	18	10

SEPARABLE DEFINITIONS : IEPTAB(...)

	ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN CC1	C2	LOC.IN CC1
1	X1	1	3	LIN	0.2400D 02	1	0.0	10
2	X2	2	3	LIN	0.1200D 02	2	0.0	10
3	X3	3	3	LIN	0.8000D 00	3	0.0	10
4	X4	4	3	LIN	0.4000D 00	4	0.0	10
5	X10	10	3	LIN	0.1000D 00	5	0.0	10
6	X9	9	3	LIN	0.1500D 00	6	0.0	10
7	X1	1	3	LIN	0.1225D 02	12	0.0	10
8	X10	10	2	IDN	0.0	10	0.0	10
9	X9	9	3	LIN	0.7000D 00	13	0.0	10
10	X6	6	33	QDR	0.1000D 01	11	0.3000D 01	15
11	X1	1	11	POW	0.5000D 00	16	-.1000D 03	17
12	X2	2	3	LIN	-.2000D 01	18	0.0	10
13	X3	3	11	POW	0.5000D 00	19	-.2500D 02	20
14	X4	4	11	POW	0.5000D 00	21	-.5000D 02	22
15	X5	5	3	LIN	-.1000D 02	23	0.0	10
16	X13	13	3	LIN	0.1000D 03	24	0.0	10
17	X8	8	3	LIN	0.1000D 03	28	0.0	10

QUADRATIC DEFINITIONS : IQADTB(...) : EACH LINE REFERS TO A PAIR OF ARGUMENTS

	FIRST ARGUMENT									SECOND ARGUMENT							
	ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN CC1	C2	LOC.IN CC1		ARG	ARG#	TRAN#	TRANSF	C1	LOC.IN	C2	LOC.IN
1	X4	4	2	IDN	0.0	10	0.0	10	*	X6	6	2	IDN	0.0	10	0.0	10
2	X3	3	2	IDN	0.0	10	0.0	10	*	X6	6	2	IDN	0.0	10	0.0	10
3	X9	9	3	LIN	-.1500D 00	7	0.0	10	*	X5	5	2	IDN	0.0	10	0.0	10
4	X1	1	3	LIN	0.1100D 02	8	0.0	10	*	X7	7	2	IDN	0.0	10	0.0	10
5	X1	1	3	LIN	-.2400D 02	9	0.0	10	*	X5	5	2	IDN	0.0	10	0.0	10
6	X9	9	3	LIN	-.2000D 03	14	0.0	10	*	X5	5	2	IDN	0.0	10	0.0	10
7	X1	1	11	POW	0.5000D 00	25	-.3000D 02	26	*	X7	7	15	POW	0.5000D 00	27	0.0	10
8	X1	1	3	LIN	-.1000D 02	29	0.0	10	*	X3	3	10	POW	-.1000D 01	30	0.0	10
9	X1	1	3	LIN	-.2000D 02	31	0.0	10	*	X4	4	10	POW	-.1000D 01	32	0.0	10

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NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4 7/13/71

     N=  8       M= 12       MZ=  3

       MAX. TIME= 0.3333000J 03     R= 0.1000000J 03     RATIO= 0.1000000J 02     EPSILON= 0.1000000J-03     THETA= 0.1000000J-04

 OPTIONS SELECTED
     3       2       2       1       1       1       1       2       1       1

 TOLERANCES
      0.1000000J-02        0.1000000J-02

 SECOND SET OF OPTIONS
     1       1
       F= -0.1041543J 05      P=  0.0                 G=  0.0                 RSIGMA=  0.0                 H=  0.0
     THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------
    X1=  0.1800000J 03      X2=  0.7400000J 02      X3=  0.2680000J 04      X4=  0.1465000J 05      X5=  0.4000000J 01
    X6=  0.2300000J 01      X7=  0.6000000J 03      X8=  0.2000000J 03
      THE CONSTRAINT VALUES
       0.3590500J 03        -0.4740100J 04         0.1000000J 03          0.1000000J 00          0.1400000J 05          0.7400000J 03
       0.2680000J 04         0.1465000J 05         0.4000000J 03          0.2300000J 01          0.6000000J 03          0.2000000J 03

       ORTHOGONAL MOVE

       DEL P VECTOR
 0.5619926BJ 01  -0.3207J1780 J1   0.2106147BJ 01   0.1263768?J 01  -0.3081327?J 04   0.1136461BJ 05  -0.3347J6?6J 03
 0.703175?5J 03

       SECOND ORDER MOVE VECTOR
-0.1235912?J 04  -0.1751?57?J 05  -0.8480436?J 05   0.4987?59?J 06  -0.2?67?7?2J 01  -0.9?6?37?4J 00  -0.1?0000?J 01
 0.0

*****************************************
         POINT=   1        OUTT=  0.5006309J 06       RHO=  0.1000000J 01       MAGNITUDE=  0.1271432J 05       PHASE= 1
       F=  0.0                 P= -0.3508601J 04       G= -0.1200000J 04       RSIGMA= -0.3508601J 04       H=  0.0
     THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------
    X1=  0.1536920?J 03      X2=  0.3670955?J 03      X3=  0.?748?34?J 03      X4=  0.2?2?7?4?J 05      X5=  0.?42?96?J 00
    X6=  0.2278991?J 01      X7=  0.5781?77?J 00      X8=  0.2000?00?J 00
      THE CONSTRAINT VALUES
       0.1561637J 03        0.6239?26J 04         0.1574?37J 00          0.1000000J 00          0.1536?21J 03          0.3670955J 03
       0.8748346J 03        0.2526?04J 05         0.3425963J 00          0.2278992J 01          0.5737136J 00          0.2000000J 00

*****THE FEASIBLE STARTING POINT TO BE USED IS ...
       F= -0.1037741J 05      P= -0.3508601J 04      G=  0.0                 RSIGMA= -0.3508601J 04      H=  0.0
     THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------
    X1=  0.1536920?J 03      X2=  0.3670955?J 03      X3=  0.?748?34?J 03      X4=  0.2?2?7?4?J 05      X5=  0.?42?96?J 00
    X6=  0.2278991?J 01      X7=  0.5781?77?J 00      X8=  0.2000?00?J 00
      THE CONSTRAINT VALUES
       0.1561637J 03        0.6239?26J 04         0.1574?37J 00          0.1000000J 00          0.1536?21J 03          0.3670955J 03
       0.8748346J 03        0.2526?04J 05         0.3425963J 00          0.2278992J 01          0.5737136J 00          0.2000000J 00

       ORTHOGONAL MOVE

       DEL P VECTOR
-0.1055?36?J 02  -0.?+1162?0? J1   0.+125?37?J-01  -0.?157?6?J 00   0.?????????? J1  -0.1??????? J4  -0.???????? J3
 0.1762?1?6J 04

       SECOND ORDER MOVE VECTOR
-0.3?3?85?3J 03   0.1?????+?? J?  -0.???151??? J?   0.1??+?????? J?   0.?????????? J?   0.1?????00 J1   0.1?????00?J-01
 0.0
```

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```
**********
     POINT=    1        DOTT=  0.1722[illegible]D 04        RHO=  0.1000000D 03        MAGNITUDE=  0.5[illegible]D 04        PHASE= 2
   F= -0.1103533D 05      P= -0.1455299D 05      G= -0.1223548D 05      PSI,MA= -0.[illegible]D 04      H=  0.0
  THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
----------
  X1=  0.1490949D 03     X2=  0.[illegible]D 01     X3=  0.[illegible]D 05     X4=  0.2727175D 05     X5=  0.4593757D 00
  X6=  0.2414319D 01     X7=  0.6977074D 00     X8=  0.2100000D 00
   THE CONSTRAINT VALUES
    0.3268649D 03        0.9938983D 04        0.4092422D-01        0.1000000D 01        0.1143995D 03        0.3364442D 03
    0.8109178D 03        0.2727176D 05        0.4593757D 00        0.2414320D 01        0.6977074D 00        0.2100000D 00

    DEL P VECTOR
-0.1345300D 01  -0.1414597D 01   0.3284813D 00  -0.1163467[illegible]-01   0.5697[illegible]D 03  -0.4581[illegible]D 03  -0.4864[illegible]D 02
 0.2371126D 03

    SECOND ORDER MOVE VECTOR
 0.1539124D 02  -0.1088910D 04   0.1103639D 04   0.3223[illegible]D 05  -0.[illegible]D-02  -0.3286[illegible]D 01  -0.[illegible]D-01
 0.1528867D-01

**********
        POINT=    2        DOTT=  0.2684272D 04        RHO=  0.1000000D 03        MAGNITUDE=  0.7937607D 03        PHASE= 2
     F= -0.1160368D 05      P= -0.1501796D 05      G= -0.1280368D 05      PSI,MA= -0.3414281D 04      H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
----------
  X1=  0.1178896D 03     X2=  0.1151277D 03     X3=  0.1089149D 04     X4=  0.3539871D 05     X5=  0.4568693D 00
  X6=  0.1585839D 01     X7=  0.6869886D 00     X8=  0.2038543D 00
   THE CONSTRAINT VALUES
    0.9159097D 03        0.3337487D 04        0.4313066D-01        0.9614563D-01        0.1178836D 03        0.1151277D 03
    0.1089150D 04        0.3539871D 05        0.4568693D 00        0.1585834D 01        0.6869886D 00        0.2038543D 00

    DEL P VECTOR
 0.4063882D 01   0.1558432D 01   0.4342756D 00   0.1068403D 01  -0.1506669D 04   0.6972975D 03   0.2034746D 03
-0.3406302D 03

    SECOND ORDER MOVE VECTOR
-0.1202432D 02  -0.4914147D 02   0.1122305D 04   0.3616477D 04   0.3[illegible]D-04  -0.1591224D 00  -0.1[illegible]D-01
 0.6255631D-02

**********
        POINT=    3        DOTT=  0.6333819D 03        RHO=  0.1000000D 03        MAGNITUDE=  0.1736635D 04        PHASE= 2
     F= -0.1194359D 05      P= -0.1523868D 05      G= -0.1314359D 05      PSI,MA= -0.3293294D 04      H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
----------
  X1=  0.1114518D 03     X2=  0.8881662D 02     X3=  0.1690038D 04     X4=  0.3733605D 05     X5=  0.4568835D 00
  X6=  0.1500639D 01     X7=  0.6815810D 00     X8=  0.2072035D 00
   THE CONSTRAINT VALUES
    0.2115046D 03        0.4001311D 04        0.4311444D-01        0.9279641D-01        0.1114518D 03        0.8881663D 02
    0.1690036D 04        0.3733605D 05        0.4568856D 00        0.1500643D 01        0.6815810D 00        0.2072036D 00

    ORTHOGONAL MOVE

    DEL P VECTOR
-0.5421827D 01  -0.2547719D 01   0.4571366D-01  -0.9203767D-01   0.1002794D 04  -0.5415612D 03  -0.2411122D 03
 0.5696628D 03

    SECOND ORDER MOVE VECTOR
-0.1205758D 03   0.5260615D 01   0.8306454D 02   0.4274344D 04   0.1[illegible]D-02  -0.1179923D 00   0.1000000D 01
 0.0

**********
        POINT=    4        DOTT=  0.7553[illegible]D 02        RHO=  0.1000000D 03        MAGNITUDE=  0.1[illegible]D 04        PHASE= 2
     F= -0.1192424D 05      P= -0.1525[illegible]D 05      G= -0.1313[illegible]D 05      PSI,MA= -0.3331[illegible]D 04      H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
----------
  X1=  0.[illegible]D 03     X2=  0.[illegible]D 02     X3=  0.1711[illegible]D 04     X4=  0.[illegible]D 05     X5=  0.[illegible]D 00
```


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**********************************************
        POINT=  24        DELTA=  0.4557075D-24        RMS=  0.1000000D-01        PENALTY=  0.2778330D 04        PHASE= 2
     F= -0.1257374D 05      P= -0.1257374D 05      G= -0.1257374D 05      RSIGMA=  0.1177760D 00      H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------
  X1=  0.1192159D 03    X2=  0.8891666D-02    X3=  0.4759226D 04    X4=  0.3999971D 05    X5=  0.4999994D 00
  X6=  0.1133622D 01    X7=  0.1870306D 00    X8=  0.2999995D 03
    THE CONSTRAINT VALUES
     0.3432454D-01        0.4999618D 00        0.5729999D-05        0.1401866D-04        0.1921E3D 03        0.8334666D-02
     0.4759222D 04        0.3999372D 05        0.4999943D 00        0.1133622D 01        0.1870306D 00        0.2999995D 00
APPARENTLY ROUNDOFF ERRORS PREVENT A MORE ACCURATE DETERMINATION OF THE MINIMUM OF THIS SUBPROBLEM.


   2ND  ORDER ESTIMATES
     F= -0.1257384D 05      P= -0.1257372D 05      G= -0.1257384D 05      RSIGMA=  0.0                H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------
  X1=  0.1192295D 03    X2=  0.8429172D-04    X3=  0.4759193D 04    X4=  0.3999389D 05    X5=  0.4999995D 00
  X6=  0.1133597D 01    X7=  0.1870180D 00    X8=  0.2999997D 03
    THE CONSTRAINT VALUES
     0.4333422D-03        0.5324138D-02        0.9368838D-07        0.2184000D-06        0.1922255D 03        0.8429173D-04
     0.4759190D 04        0.3998898D 05        0.4999999D 00        0.1133597D 01        0.1870181D 00        0.2999997D 00


   1ST  ORDER ESTIMATES
     F= -0.1257384D 05      P= -0.1257371D 05      G= -0.1257384D 05      RSIGMA=  0.0                H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------
  X1=  0.1192288D 03    X2=  0.8684010D-04    X3=  0.4759188D 04    X4=  0.3999901D 05    X5=  0.4999999D 00
  X6=  0.1133597D 01    X7=  0.1870181D 00    X8=  0.2999997D 03
    THE CONSTRAINT VALUES
     0.4153131D-03        0.4797492D-02        0.9368183D-07        0.2199293D-06        0.1922283D 03        0.8634010D-04
     0.4759188D 04        0.3999902D 05        0.4999999D 00        0.1133597D 01        0.1870182D 00        0.2999999D 00


  LAGRANGE MULTIPLIERS
     F= -0.1257384D 05      P= -0.1257371D 05      G= -0.1257384D 05      RSIGMA=  0.0                H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-----------------------------------------------------
  X1=  0.1192288D 03    X2=  0.8684010D-04    X3=  0.4759188D 04    X4=  0.3999901D 05    X5=  0.4999999D 00
  X6=  0.1133597D 01    X7=  0.1870181D 00    X8=  0.2999997D 03
    THE CONSTRAINT VALUES
     0.2913367D 00        0.2176033D-01        0.1745216D 04        0.7133337D 03        0.3381190D-04        0.1452903D 01
     0.2101184D-05        0.2505719D-06        0.2000023D-01        0.8821274D-02        0.3345286D-01        0.3333489D-01


SENSITIVITY OF SOLUTION VECTOR WRT PROBLEM PARAMETERS
**********************************************

PROBLEM PARAMETERS ARE
======================

  A1            A2            A3            A4            A5            A6            A7            A8            A9

     DEL P VECTOR
 0.1192369D 03   0.6947316D-02   0.4759034D 04   0.3999854D 05   0.4999994D 00   0.1133604D 01   0.1869743D 00
 0.2999850D 03

     SECOND ORDER MOVE VECTOR
 0.4630155D 02  -0.2995019D-03  -0.1753776D 03  -0.1917446D 04   0.4722747D-03   0.2813552D-01  -0.7347374D-01
 0.4803439D-05


THE ESTIMATES OF ZERO-TH ORDER SENSITIVITY OF  A1  WRT PROBLEM PARAMETERS A1, A2 RESPECTIVELY :
-----------------------------------------------------------------------------------------------
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[illegible] 0.155153D 06 0.132370 05 0.152943 06 [illegible] 0.792353 02 0.113573 03

SENSITIVITY OF X1 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

[illegible] 0.196853 06 0.202520 05 0.169943 06 [illegible] 0.333253 02 0.133343 03

ARE :

0.903563 02 0.503493 03 0.169383 02 0.213733 01 -0.136853 01 0.773673 01 -0.261773 02

[illegible] 0.475904733 04 0.399083493 05 0.499994163 03 0.113362453 01 0.186929133 00

VECTOR
[illegible] -0.113236613-01 -0.102472373 03 0.579662033-11 0.239012523-05 -0.696377173-07

OF ZEROTH ORDER SENSITIVITY OF X2 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

[illegible] -0.537723-01 -0.448723 00 -0.651033-01 -0.543443 00 -0.271723 00 -0.253353-03 -0.405323-03

OF FIRST ORDER SENSITIVITY OF X2 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

[illegible] -0.391423-01 -0.328343 00 -0.439283-01 -0.368533 00 -0.184263 00 -0.181633-03 -0.293453-03

ELASTICITIES ARE :

[illegible] 0.229223 00 -0.341123 01 -0.190773 02 -0.633653 03 -0.802373 01 0.401453 01 -0.293203 00 0.102333 01

VECTOR
[illegible] 0.694731653D-02 0.475904733 04 0.399083493 05 0.499994163 00 0.113362453 01 0.186929133 00

SECOND ORDER MOVE VECTOR
[illegible] -0.113236613-01 0.436395283 05 -0.575903433 05 -0.435543023-05 -0.362753223 00 -0.381124883-01

OF ZEROTH ORDER SENSITIVITY OF X3 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

[illegible] -0.703953-01 -0.643853 05 -0.413503 06 -0.727123 05 -0.463133 03 -0.232153 00 -0.229453 03 -0.373043 03

OF FIRST ORDER SENSITIVITY OF X3 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

[illegible] -0.774593-01 -0.693993 05 -0.457973 06 -0.787363 05 -0.514933 03 -0.257473 00 -0.254363 03 -0.413133 03

ELASTICITIES ARE :

[illegible] -0.195313-03 -0.881223 01 -0.334713 02 -0.165443 01 -0.162333 02 0.411503 01 -0.547413 03 0.208813 01

VECTOR
[illegible] 0.694731653D-02 0.475904733 04 0.399083493 05 0.499994163 00 0.113362453 01 0.186929133 00

ORDER MOVE VECTOR

-0.662254530-04

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X4 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.103030 05 -0.590350 00 -0.412370 06 -0.304220 07 -0.466320 06 -0.412170 07 -0.206350 07 -0.191240 04 -0.330220 04

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X4 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.114040 05 -0.636140 00 -0.456100 06 -0.400870 07 -0.516190 06 -0.458720 07 -0.228360 07 -0.211740 04 -0.342100 04

RESPECTIVE ELASTICITIES ARE :

-0.685790 01 -0.191280-03 -0.685710 01 -0.401790 02 -0.129340 01 -0.170530 02 0.352650 01 -0.583620 00 0.205730 01

DEL P VECTOR
0.119236980 03 0.694731650-02 0.475904730 04 0.399085490 05 0.499994160 00 0.113362450 01 0.186929130 00
0.299985610 00

SECOND ORDER MOVE VECTOR
0.692087460-08 0.579862000-11 -0.435544020-05 0.446722370-04 0.277212100-08 -0.155394300-10 0.134013910-09
-0.419223380-14

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X5 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.301160-06 -0.189650-10 -0.106730-04 -0.113160-03 -0.120780-04 -0.128100-03 -0.130590-05 -0.560970-07 -0.903450-07

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X5 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.828070-06 -0.280530-09 -0.333490-04 -0.274490-03 -0.378130-04 -0.311230-03 -0.145240-03 -0.154250-06 -0.248350-06

RESPECTIVE ELASTICITIES ARE :

-0.397480-04 -0.673280-08 -0.400190-04 -0.219600-03 -0.756230-05 -0.953710-04 0.447740-04 -0.339360-05 0.117210-04

DEL P VECTOR
0.119236980 03 0.694731650-02 0.475904730 04 0.399085490 05 0.499994160 00 0.113362450 01 0.186929130 00
0.299985610 00

SECOND ORDER MOVE VECTOR
0.231385520-01 0.299012520-05 -0.302756220 00 -0.617016760 00 -0.155394300-10 0.171004230-04 0.421401280-04
-0.193694250-09

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X6 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

0.126990 00 0.685170-05 0.544430 01 0.449470 02 0.614540 01 0.507310 02 0.253650 02 0.222940-01 0.330950-01

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X6 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

0.139210 00 0.715970-05 0.593160 01 0.490340 02 0.669770 01 0.551640 02 0.276820 02 0.245730-01 0.417510-01

RESPECTIVE ELASTICITIES ARE :

0.244720-01 0.757440-04 0.815740 01 0.173020 02 0.599420 01 0.122570 01 -0.360210 01 0.233460 00 -0.364120 00

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DEL P VECTOR
0.119236980D 03 0.69473165D-02 0.47590473D 04 0.39903549D 05 0.49959416D 00 0.11336249D 01 0.18692913D 00
0.29998561D 00

SECOND ORDER MOVE VECTOR
-0.73470074D-01 -0.69557717D-07 -0.38112486D-01 -0.44577320D 00 0.13401591D-09 0.42140128D-04 0.13386954D-02
-0.70282823D-08

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X7 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.155+00 01 -0.93407D-04 -0.62345D 02 -0.52665D 03 -0.71501D 02 -0.39185D 02 -0.29892D 01 -0.32855D 00 -0.46017D 00

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X7 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.17501D 01 -0.11304D-03 -0.70669D 02 -0.59256D 03 -0.80171D 02 -0.67223D 03 -0.33611D 03 -0.36516D 00 -0.52501D 00

RESPECTIVE ELASTICITIES ARE :
==========

-0.22452D 03 -0.72539D-02 -0.22665D 03 -0.12670D 04 -0.42353D 02 -0.53399D 03 0.26744D 03 -0.21471D 02 0.67352D 02

DEL P VECTOR
0.11923698D 03 0.69473165D-02 0.47590473D 04 0.39903549D 05 0.49959416D 00 0.11336249D 01 0.18692913D 00
0.29998561D 00

SECOND ORDER MOVE VECTOR
0.48834897D-05 -0.12074599D-10 -0.81317911D-05 -0.66225493D-04 -0.41922338D-14 -0.17369425D-09 -0.70282823D-08
0.15503281D-07

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X8 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

0.73779D-04 0.43464D-08 0.30045D-02 0.25194D-01 0.34062D-02 0.28566D-01 0.14281D-01 0.14047D-04 0.21594D-04

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X8 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

0.92423D-04 0.96113D-08 0.36731D-02 0.30801D-01 0.41641D-02 0.34918D-01 0.17459D-01 0.17204D-04 0.27347D-04

RESPECTIVE ELASTICITIES ARE :
==========

0.73942D-02 0.38449D-06 0.73466D-02 0.41078D-01 0.13581D-02 0.17463D-01 -0.17297D-02 0.65099D-03 -0.22219D-02

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EXAMPLE-3 : SOLUTION AND SESITIVITY ANALYSIS OF CORRUGATED BULKHEAD

SYMPUT PARAMETER CARD
M= 6 NN= 45 M= 19 MZ= 0 ITEST= 1 IPR= 0 ITES= 0 NSWIT= 0 IID= 15

TYPE NAME	TYPE #	TYPE DESCRIP.	VAR. NAME	NUMBR. ASSND.	ARG. VARIAB. NAME	NUMBR.	FUNCT. TYPE	NUMBR. ASSND.	CONSTANT C1	CONSTANT C2
V		ORIG. VARIABLE	X1	1	0.458000D 02					
V		ORIG. VARIABLE	X2	2	0.432000D 02					
V		ORIG. VARIABLE	X3	3	0.305000D 02					
V		ORIG. VARIABLE	X4	4	0.120000D 01					
V		ORIG. VARIABLE	X5	5	0.120000D 01					
V		ORIG. VARIABLE	X6	6	0.130000D 01					
P		SENSITIVITY PAR	LT					1	0.495000D 03	
P		SENSITIVITY PAR	LM					2	0.385000D 03	
P		SENSITIVITY PAR	LB					3	0.315000D 03	
P		SENSITIVITY PAR	TTM					4	0.105000D 01	
P		SENSITIVITY PAR	TMM					5	0.105000D 01	
P		SENSITIVITY PAR	TBM					6	0.105000D 01	
P		SENSITIVITY PAR	K2					7	0.150000D 00	
D		DATA DEFINITION	E						0.800000000 00	
D		DATA DEFINITION	GB						0.373660000 04	
D		DATA DEFINITION	KHT						0.345600000-04	
D		DATA DEFINITION	KHM						0.651000000-04	
D		DATA DEFINITION	KHB						0.893800000-04	
D		DATA DEFINITION	CT						0.111770000-01	
D		DATA DEFINITION	CM						0.137650000-01	
D		DATA DEFINITION	CB						0.155660000-01	
S	3	SEPARABL	Z1	7	X2	2	POW	10	0.200000000 01	0.0
S	3	SEPARABL	Z1	7	X3	3	POW	11	0.200000000 01	-.100000000 01
S	3	SEPARABL	Z2	8	X1	1	IDN	2	0.0	0.0
S	3	SEPARABL	Z2	8	Z1	7	POW	10	0.500000000 00	0.0
Q	4	QUADRATC	Z3	9	X3	3	LIN	3	0.500000000 00	0.0
Q	4	QUADRATC	Z3	9	X4	4	IDN	2	0.0	0.0
Q	4	QUADRATC	Z4	10	X3	3	LIN	3	0.500000000 00	0.0
Q	4	QUADRATC	Z4	10	X5	5	IDN	2	0.0	0.0
Q	4	QUADRATC	Z5	11	X3	3	LIN	3	0.500000000 00	0.0
Q	4	QUADRATC	Z5	11	X6	6	IDN	2	0.0	0.0
S	3	SEPARABL	Z6	12	X2	2	LIN	3	0.333333330 00	0.0
S	3	SEPARABL	Z6	12	X1	1	LIN	3	0.800000000 00	0.0
Q	4	QUADRATC	Z7	13	Z3	9	IDN	2	0.0	0.0
Q	4	QUADRATC	Z7	13	Z6	12	IDN	2	0.0	0.0
Q	4	QUADRATC	Z8	14	Z4	10	IDN	2	0.0	0.0
Q	4	QUADRATC	Z8	14	Z6	12	IDN	2	0.0	0.0
Q	4	QUADRATC	Z9	15	Z5	11	IDN	2	0.0	0.0
Q	4	QUADRATC	Z9	15	Z6	12	IDN	2	0.0	0.0
S	3	SEPARABL	Z10	16	Z2	8	LIN	3	0.345600000-04	0.0
S	3	SEPARABL	Z11	17	Z2	8	LIN	3	0.651000000-04	0.0
S	3	SEPARABL	Z12	18	Z2	8	LIN	3	0.893800000-04	0.0
S	3	SEPARABL	Z13	19	Z10	16	LIN	3	0.495000000 03 LT	0.0
S	3	SEPARABL	Z14	20	Z11	17	LIN	3	0.385000000 03 LM	0.0
S	3	SEPARABL	Z15	21	Z12	18	LIN	3	0.315000000 03	0.0

S	3	SEPARABL	Z16	22	Z13	19	LIN	3	0.495000000D 03 0.0
									LT
S	3	SEPARABL	Z17	23	Z14	20	LIN	3	0.385000000D 03 0.0
									LM
S	3	SEPARABL	Z18	24	Z15	21	LIN	3	0.315000000D 03 0.0
									LB
S	3	SEPARABL	G1	25	X2	2	IDN	2	0.0 0.0
S	3	SEPARABL	G1	25	X3	3	LIN	3	-.100000000D 01 0.0
S	3	SEPARABL	G2	26	Z7	13	IDN	2	0.0 0.0
S	3	SEPARABL	G2	26	Z16	22	LIN	3	-.100000000D 01 0.0
S	3	SEPARABL	G3	27	Z8	14	IDN	2	0.0 0.0
S	3	SEPARABL	G3	27	Z17	23	LIN	3	-.100000000D 01 0.0
S	3	SEPARABL	G4	28	Z9	15	IDN	2	0.0 0.0
S	3	SEPARABL	G4	28	Z18	24	LIN	3	-.100000000D 01 0.0
Q	4	QUADRATC	G5	29	Z7	13	IDN	2	0.0 0.0
Q	4	QUADRATC	G5	29	X3	3	LIN	3	0.500000000D 00 0.0
S	3	SEPARABL	G5	29	Z16	22	PDK	11	0.133333330 01 -.220000000D 01
Q	4	QUADRATC	G6	30	Z8	14	IDN	2	0.0 0.0
Q	4	QUADRATC	G6	30	X3	3	LIN	3	0.500000000D 00 0.0
S	3	SEPARABL	G6	30	Z17	23	PDK	11	0.133333330 01 -.220000000D 01
Q	4	QUADRATC	G7	31	Z9	15	IDN	2	0.0 0.0
Q	4	QUADRATC	G7	31	X3	3	LIN	3	0.500000000D 00 0.0
S	3	SEPARABL	G7	31	Z18	24	PDK	11	0.133333330 01 -.220000000D 01
S	3	SEPARABL	G8	32	X4	4	LIN	4	-.100000000D 01 0.105000000D 01
									TTM
S	3	SEPARABL	G9	33	X4	4	LIN	3	-.100000000D 01 0.0
S	3	SEPARABL	G9	33	X1	1	LIN	4	0.111770000-01 0.150000000 00
									K2
S	3	SEPARABL	G10	34	X4	4	LIN	3	-.100000000D 01 0.0
S	3	SEPARABL	G10	34	X2	2	LIN	4	0.111770000-01 0.150000000 00
									K2
S	3	SEPARABL	G11	35	X5	5	LIN	4	-.100000000D 01 0.105000000D 01
									TMM
S	3	SEPARABL	G12	36	X5	5	LIN	3	-.100000000D 01 0.0
S	3	SEPARABL	G12	36	X1	1	LIN	4	0.137650000-01 0.150000000 00
									K2
S	3	SEPARABL	G13	37	X5	5	LIN	3	-.100000000D 01 0.0
S	3	SEPARABL	G13	37	X2	2	LIN	4	0.137650000-01 0.150000000 00
									K2
S	3	SEPARABL	G14	38	X6	6	LIN	4	-.100000000D 01 0.105000000D 01
									TBM
S	3	SEPARABL	G15	39	X6	6	LIN	3	-.100000000D 01 0.0
S	3	SEPARABL	G15	39	X1	1	LIN	4	0.155660000-01 0.150000000 00
									K2
S	3	SEPARABL	G16	40	X6	6	LIN	3	-.100000000D 01 0.0
S	3	SEPARABL	G16	40	X2	2	LIN	4	0.155660000-01 0.150000000 00
									K2
S	3	SEPARABL	Z19	41	X1	1	IDN	2	0.0 0.0
S	3	SEPARABL	Z19	41	X2	2	IDN	2	0.0 0.0
S	3	SEPARABL	Z20	42	Z19	41	LIN	3	0.375660000 04 0.0
S	3	SEPARABL	Z21	43	X4	4	LIN	3	0.495000000 03 0.0
									LT
S	3	SEPARABL	Z21	43	X5	5	LIN	3	0.385000000D 03 0.0
									LM
S	3	SEPARABL	Z21	43	X6	6	LIN	3	0.315000000D 03 0.0
									LB
Q	4	QUADRATC	Z22	44	Z20	42	IDN	2	0.0 0.0
Q	4	QUADRATC	Z22	44	Z21	43	IDN	2	0.0 0.0
Q	4	QUADRATC	OBJ	45	Z22	44	IDN	2	0.0 0.0
Q	4	QUADRATC	OBJ	45	Z2	8	PDK	10	-.100000000D 01 0.0

ARRAY OF CONSTANTS CCII.)
****

LOCATION VALUE

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```
NONLINEAR PROGRAMMING ROUTINE-334. VERSION 4 08/13/71

     N=  6       M= 19       MZ=  0

      MAX. TIME= 0.60000000 03     R= 0.10000000 03     RATIO= 0.40000000 01      EPSILON= 0.10000000-04     THETA= 0.10000000-05

 OPTIONS SELECTED
     3       2       2       1       1       1       1       2       1       1

 TOLERANCES
      0.10000000-02        0.10000000-02

 SECOND SET OF OPTIONS
     1       1
       F=  0.63796300 07      P=  0.0                 G=  0.0                 RSIGMA=  0.0                 H=  0.0
     THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------------
    X1=  0.45800000 02     X2=  0.43200000 02     X3=  0.30500000 02     X4=  0.12000000 01     X5=  0.12000000 01
    X6=  0.13000000 01
      THE CONSTRAINT VALUES
       0.45800000 02        0.43200000 02        0.30500000 02        0.12700000 02        0.23712310 03        0.19687260 03
       0.33435060 03        0.19352510 04       -0.40602190 03        0.23396720 04        0.15000000 00        0.53809340 00
       0.56715360 00        0.15000000 00        0.41956300 00        0.45535200 00        0.25000000 00        0.43737720 00
       0.47754880 00

       ORTHOGONAL MOVE

       DEL P VECTOR
-0.31385337D 02  -0.29407003D 03   0.13432732D 04   0.19132730D 04   0.13589973D 05   0.15783284D 04

       SECOND ORDER MOVE VECTOR
 0.79081510D 01   0.19889044D 01   0.10000000D 01   0.0              0.0              0.0

**************************************************
          POINT=   1        DOTI=  0.51019610 03       RHO= 0.10000000 07       MAGNITUDE= 0.13687040 05       PHASE= 1
       F=  0.0                 P= -0.41387020 04      G= -0.19000000 04      RSIGMA= -0.41387020 04      H=  0.0
     THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------------
    X1=  0.53708151D 02     X2=  0.45188904D 02     X3=  0.31500000 02     X4=  0.12000000 01     X5=  0.12000000 01
    X6=  0.13000000 01
      THE CONSTRAINT VALUES
       0.53708150 02        0.45188900 02        0.31500000 02        0.13688900 02        0.36758450 03        0.26385720 03
       0.42448010 03        0.28851640 04        0.88734440 02        0.33566560 04        0.15000000 00        0.44970400 00
       0.54492360 00        0.15000000 00        0.31370730 00        0.42797470 00        0.25000000 00        0.31397890 00
       0.44658950 00

*****THE FEASIBLE STARTING POINT TO BE USED IS ...
       F=  0.62892580 07      P= -0.41387020 04      G=  0.0                 RSIGMA= -0.41387020 04      H=  0.0
     THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------------
    X1=  0.53708151D 02     X2=  0.45188904D 02     X3=  0.31500000 02     X4=  0.12000000D 01     X5=  0.12000000 01
    X6=  0.13000000 01
      THE CONSTRAINT VALUES
       0.53708150 02        0.45188900 02        0.31500000 02        0.13688900 02        0.36758450 03        0.26385720 03
       0.42448010 03        0.28851640 04        0.88734440 02        0.33566560 04        0.15000000 00        0.44970400 00
       0.54492360 00        0.15000000 00        0.31370730 00        0.42797470 00        0.25000000 00        0.31397890 00
       0.44658950 00

       ORTHOGONAL MOVE

       DEL P VECTOR
 0.94097440 04   0.37347440 03  -0.60387020 05  -0.41224000 04  -0.64444010 07  -0.13532440 07
```

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-0.905380710 01 -0.231636930 01 -0.352927020 00 -0.100000000 01 0.0 0.0

****

POINT= 1 DOTT= 0.173860D 07 RHO= 0.100000D 03 MAGNITUDE= 0.300054D 07 PHASE= 2
F= 0.59924760 07 P= 0.59993320 07 G= 0.59935750 07 SIGMA= -0.31432330 04 H= 0.0
THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.52350550 02 X2= 0.44841570 02 X3= 0.31447080 02 X4= 0.10500920 01 X5= 0.12000000 01
X6= 0.13000000 01
THE CONSTRAINT VALUES

0.52350550 02	0.44841570 02	0.31447080 02	0.13394490 02	0.22425390 03	0.25802630 03
0.41380710 03	0.71309830 03	0.14923360 03	0.33321100 04	0.52280670-04	0.31493010 00
0.39885810 00	0.15000000 00	0.32939460 00	0.43275560 00	0.25000000 00	0.33511130 00
0.45199610 00					

ORTHOGONAL MOVE

DEL P VECTOR
0.939105310 04 0.379133390 05 -0.687793270 05 -0.216364250 06 -0.166730620 07 -0.135520450 07

SECOND ORDER MOVE VECTOR
0.65330939D 01 0.18812360D 02 0.16147683D 02 -0.44514847D-05 -0.10000000D 01 0.0

****

POINT= 2 DOTT= 0.130939D 07 RHO= 0.100000D 03 MAGNITUDE= 0.214552D 07 PHASE= 2
F= 0.57933220 07 P= 0.57907850 07 G= 0.57914220 07 SIGMA= -0.25374070 04 H= 0.0
THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.53330011 02 X2= 0.47661965 02 X3= 0.33867980 02 X4= 0.10500516 01 X5= 0.10500775 01
X6= 0.13000000 01
THE CONSTRAINT VALUES

0.53330010 02	0.47661970 02	0.33867980 02	0.13793990 02	0.30555240 03	0.20295670 03
0.51857400 03	0.30223350 04	0.24410730 03	0.62902370 04	0.51613290-04	0.30398210 00
0.36733380 00	0.77531280-04	0.16598590 00	0.24401060 00	0.25000000 00	0.31986510 00
0.40809380 00					

ORTHOGONAL MOVE

DEL P VECTOR
0.931062490 04 0.372880300 05 -0.667137360 05 -0.211445920 06 -0.379362190 06 -0.136702860 07

SECOND ORDER MOVE VECTOR
-0.44541843D 01 0.17454048D 01 0.24181495D 01 -0.13894792D-05 -0.71351486D-05 -0.10000000D 01

****

POINT= 3 DOTT= 0.122931D 07 RHO= 0.100000D 03 MAGNITUDE= 0.143511D 07 PHASE= 2
F= 0.54864190 07 P= 0.54830840 07 G= 0.54827190 07 SIGMA= -0.15349420 04 H= 0.0
THE CURRENT VALUE OF THE PROB-M(I.E.X(I)) VARS.ARE;

X1= 0.52216326D 02 X2= 0.48098175D 02 X3= 0.34472321D 02 X4= 0.10500513D 01 X5= 0.10500757D 01
X6= 0.10500812D 01
THE CONSTRAINT VALUES

0.52216830 02	0.48098170 02	0.34472320 02	0.13826950 02	0.32001390 03	0.21572070 03
0.28567940 03	0.36721970 04	0.94103330 03	0.27596700 04	0.51266030-04	0.31642380 00
0.36245830 00	0.75745070-04	0.18131110 00	0.23800440 00	0.81164640-04	0.67274050-01
0.15138500 00					

ORTHOGONAL MOVE

DEL P VECTOR
0.925815640 04 0.369710250 05 -0.664650860 05 -0.211564890 06 -0.378342940 06 -0.141465910 07

SECOND ORDER MOVE VECTOR
-0.77821637D 00 -0.12036125D 01 -0.10730000D 01 0.0 0.0 0.0

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**********************************************
        POINT=  33        DUIT=  0.1183357D-02        RHO=  0.7050250-01        MAGNITUDE=  0.1183334D 00        PHASE= 2
     F=  0.5248023D 07     P=  0.5248027D 07     G=  0.5248021D 07     RSIGMA=  0.4257422D 01     H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------
  X1=  0.5781833D 02    X2=  0.5781834D 02    X3=  0.3568432D 02    X4=  0.1050000D 01    X5=  0.1050000D 01
  X6=  0.1050000D 01
    THE CONSTRAINT VALUES
     0.5781833D 02        0.5781833D 02        0.3568433D 02        0.2213003D 02        0.3529272D 03        0.2308912D 03
     0.3115359D 03        0.3501210D 04        0.3591984D-02        0.2330933D 04        0.4770149D-07        0.2537646D 00
     0.2537646D 00        0.9526797D-07        0.1041308D 00        0.1041308D 00        0.4884491D-06        0.3077094D-06
     0.1243014D-06
APPARENTLY ROUNDOFF ERRORS PREVENT A MORE ACCURATE DETERMINATION OF THE MINIMUM OF THIS SUBPROBLEM.


   2ND  ORDER ESTIMATES
     F=  0.5248022D 07     P=  0.5248023D 07     G=  0.5248022D 07     RSIGMA=  0.0               H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------
  X1=  0.5781832D 02    X2=  0.5781832D 02    X3=  0.3568833D 02    X4=  0.1050000D 01    X5=  0.1050000D 01
  X6=  0.1050000D 01
    THE CONSTRAINT VALUES
     0.5781832D 02        0.5781832D 02        0.3568833D 02        0.2213003D 02        0.3529272D 03        0.2308840D 03
     0.3115352D 03        0.3501210D 04        0.8764775D-04        0.2330920D 04        0.1194681D-08        0.2537646D 00
     0.2537646D 00        0.2325524D-08        0.1041308D 00        0.1041308D 00        0.1195437D-07        0.7511207D-08
     0.3038892D-08


   1ST  ORDER ESTIMATES
     F=  0.5248022D 07     P=  0.5248023D 07     G=  0.5248022D 07     RSIGMA=  0.0               H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------
  X1=  0.5781832D 02    X2=  0.5781832D 02    X3=  0.3568829D 02    X4=  0.1050000D 01    X5=  0.1050000D 01
  X6=  0.1050000D 01
    THE CONSTRAINT VALUES
     0.5781832D 02        0.5781832D 02        0.3568830D 02        0.2213003D 02        0.3529272D 03        0.2308890D 03
     0.3115352D 03        0.3501210D 04        0.7764443D-04        0.2330920D 04        0.1032981D-08        0.2537646D 00
     0.2537646D 00        0.2064054D-08        0.1041308D 00        0.1041308D 00        0.1052740D-07        0.6658967D-08
     0.2689538D-08


  LAGRANGE MULTIPLIERS
     F=  0.5248022D 07     P=  0.5248023D 07     G=  0.5248022D 07     RSIGMA=  0.0               H=  0.0
   THE CURRENT VALUE OF THE PROB-M(I.E.X(I) ) VARS.ARE;
-------------------------------------------------------
  X1=  0.5781832D 02    X2=  0.5781832D 02    X3=  0.3568829D 02    X4=  0.1050000D 01    X5=  0.1050000D 01
  X6=  0.1050000D 01
    THE CONSTRAINT VALUES
     0.1669019D-02        0.1689019D-02        0.2736366D-02        0.4412435D-02        0.2737035D-03        0.4229719D-03
     0.3134671D-03        0.2789211D-04        0.2718728D 02        0.4189276D-04        0.2067208D 07        0.3848302D 00
     0.3848304D 00        0.1025969D 07        0.9378245D 00        0.9378250D 00        0.1758845D 06        0.3173652D 06
     0.7856437D 06


SENSITIVITY OF SOLUTION VECTOR WRT PROBLEM PARAMETERS
**********************************************

PROBLEM PARAMETERS ARE
======================

  LF          LM          LB          ITA         ITB         E A         KZ

  - DELTA VECTOR
 0.5781833D 02   0.5781834D 02   0.3568433D 02   0.1050000D 01   0.1050000D 01   0.1050000D 01
```

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0.125706020-07 0.900249740-08 0.167239550-08 0.142723750-17 -0.929278770-17 0.140132980-09

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X1 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

0.146720-06 0.124830-05 -0.449360-06 0.612560-04 -0.959480-04 0.572890 02 -0.572840 02

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X1 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.255490-07 -0.376380-06 0.772000-07 -0.253500-04 0.486530-04 0.540540 02 -0.540610 02

RESPECTIVE ELASTICITIES ARE :

-0.218740-06 -0.250620-05 0.420590-06 -0.478520-06 0.883550-06 0.981640 00 -0.140250 00

DEL P VECTOR
0.578183340 02 0.578183460 02 0.356883030 02 0.105000000 01 0.105000010 01 0.105000050 01

SECOND ORDER MOVE VECTOR
0.900249740-08 0.958474730-08 0.173435800-08 0.134015490-17 -0.881938310-17 0.140132980-09

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X2 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

0.136830-06 0.122840-05 -0.449270-06 0.575140-04 -0.948950-04 0.572890 02 -0.572830 02

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X2 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.239940-07 -0.355040-06 0.787570-07 -0.247440-04 0.464250-04 0.540540 02 -0.540610 02

RESPECTIVE ELASTICITIES ARE :

-0.205420-06 -0.236410-05 0.429070-06 -0.449360-06 0.843130-06 0.981640 00 -0.140250 00

DEL P VECTOR
0.578183340 02 0.578183460 02 0.356883030 02 0.105000000 01 0.105000010 01 0.105000050 01

SECOND ORDER MOVE VECTOR
0.167239550-08 0.173435800-08 0.337523720-09 0.220355200-13 -0.119285080-11 0.254468070-10

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X3 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

0.224980-07 0.933420-01 -0.839320-07 0.945750-05 -0.128330 02 0.104030 02 -0.104020 02

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X3 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.394690-08 0.880920-01 0.147110-07 -0.470930-05 0.121120 02 0.981970 01 -0.981730 01

RESPECTIVE ELASTICITIES ARE :

-0.547430-07 0.950330 00 0.[illegible] -0.[illegible] 0.[illegible] 0.[illegible] 0.[illegible] 01

DEL P VECTOR
0.578183340 02 0.578183460 02 0.356883030 02 0.[illegible] 01 0.[illegible] 01 0.[illegible] 01

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0.1421237D-17 0.1341134D-17 0.2033333D-15 0.2077[illegible]D-15 0.3476633[illegible]D-21 0.1777218D-17

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X4 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.866950-10 -0.304900-10 -0.027702-10 0.871695 00 0.415260-08 0.415277-08 -0.815197-08

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X4 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

0.152020-10 0.130950-10 0.340830-15 0.841490 00 -0.180210-08 -0.350940-08 0.350120-08

RESPECTIVE ELASTICITIES ARE :

0.716680-03 0.480150-08 0.103450-12 0.841490 00 -0.180210-03 -0.350940-08 0.350120-09

DEL P VECTOR
0.578183340 02 0.578183460 02 0.356883030 02 0.105000000 01 0.105000010 01 0.105000050 01

SECOND ORDER MOVE VECTOR
-0.929278370-17 -0.881938310-17 -0.119235080-11 0.349650070-21 0.823727800-13 -0.129659330-18

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X5 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

0.397810-10 -0.436090-10 0.397730-10 0.167240-07 0.891700 00 -0.550070-07 0.536460-07

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X5 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.695430-11 -0.120300-10 -0.693400-11 -0.718790-08 0.841490 00 0.259580-07 -0.364870-07

RESPECTIVE ELASTICITIES ARE :

-0.327850-08 -0.441080-08 -0.208170-08 -0.718790-08 0.841480 00 0.259540-07 -0.435530-08

DEL P VECTOR
0.578183340 02 0.578183460 02 0.356883030 02 0.105000000 01 0.105000010 01 0.105000050 01

SECOND ORDER MOVE VECTOR
0.140132930-09 0.140132980-09 0.254468070-10 0.199421840-19 -0.129659330-18 0.218131080-11

THE ESTIMATES OF ZEROTH ORDER SENSITIVITY OF X6 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

0.203610-08 0.181510-07 -0.708720-08 0.866910-05 -0.139510-04 0.891700 00 0.641780-05

THE ESTIMATES OF FIRST ORDER SENSITIVITY OF X6 WRT PROBLEM PARAMETERS ARE RESPECTIVELY :

-0.357100-09 -0.523430-05 0.124230-08 -0.366230-06 0.832900-06 0.841410 01 -0.242110-05

RESPECTIVE ELASTICITIES ARE :

-0.164350-06 -0.191410-05 0.572100-06 -0.[illegible]230-06 0.832900-06 0.[illegible] 01 -0.417[illegible]-05

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APPENDIX 2

"Sensitivity Analysis in Nonlinear
Programming Using Factorable
Symbolic Input"
(borrowed directly from [4])

THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Institute for Management Science and Engineering

SENSITIVITY ANALYSIS IN NONLINEAR PROGRAMMING USING FACTORABLE SYMBOLIC INPUT

by

Anura de Silva
Garth P. McCormick

1. Introduction

This paper develops a methodology to utilize the input to nonlinear programs in symbolic factorable form to perform first order sensitivity analysis on the solution vector $x(y) \in E^n$ of a general nonlinear program, where $y \in E^k$ is the parametric vector. In Section 2 we discuss the key results of sensitivity analysis theory in nonlinear programming. In Section 3 we develop the formulae underlying the new methodology to utilize the symbolic factorable input to perform sensitivity analysis on the solution vector $x(y)$.

The above are implemented via the SUMT algorithm [4] for nonlinear programming which uses a penalty function technique. Section 4 briefly discusses the penalty function-based implementation of the sensitivity analysis methodology. An extrapolation result based on the penalty function parameter r is stated and incorporated in the computer code.

2. Relevant Results from NLP Sensitivity Theory

Consider the problem

$$\left.\begin{array}{ll} \underset{x \in E^n}{\text{MIN}} & F(x,y) \\ \text{s.t.} & G(x,y) \geq 0 \\ & H(x,y) = 0 \\ \text{where} & G : E^n \times E^k \to E^m \\ & H : E^n \times E^k \to E^p \end{array}\right\} \text{Problem M(y)}$$

By sensitivity information we mean the rates of change of the (locally) optimal objective function value and the corresponding optimizing vector with respect to changes in the parametric vector. If $x(y)$ is an n-vector and y a k-vector, the first order sensitivity information for Problem M(y) is:

(a) The rate of change of the solution value with respect to the parametric vector, $D_y F[x(y),y] \in E^k$.

(b) The rate of change of the optimizing vector with respect to the parametric vector, $\frac{dx(y)}{dy}$, where

$$\frac{dx(y)}{dy} = \begin{bmatrix} \frac{\partial x_1(y)}{\partial y_1} & \cdots & \frac{\partial x_1(y)}{\partial y_k} \\ \vdots & & \vdots \\ \frac{\partial x_n(y)}{\partial y_1} & \cdots & \frac{\partial x_n(y)}{\partial y_k} \end{bmatrix} \qquad (1)$$

The first order sensitivity results for the general nonlinear programming problem were first stated by Fiacco and McCormick in [3], Theorem 6, where they give conditions under which a smooth (once continuously differentiable, abbreviated OCD) trajectory of the optimizing Kuhn-Tucker triple $[x(y),u(y),w(y)]$ exist for y close to an initial value y_0 . The u and w are the Lagrange multiplier vectors associated with the inequality

constraints G and equality constraints H of Problem M(y), in the Lagrangian,

$$L(x,y,u,w) = F(x,y) - \sum_{i=1}^{m} u_i G_i(x,y) + \sum_{j=1}^{p} w_j H_j(x,y) \qquad (2)$$

where $u_i, G_i; i = 1,\ldots,m$ and $w_j, H_j : j = 1,\ldots,p$ are the components of the mappings G and H and the Lagrange multipliers u and w. Here, $u(y)$ and $w(y)$ are the values of these multipliers associated with the optimal value $x(y)$. They assume the following to hold; (a) twice continuous differentiability (abbreviated TCD) of $L(x,y,u,w)$ in (x,y) in a neighborhood of $(x(y_0),y_0)$, (b) second order sufficiency conditions (abbrev. SOSC) at $(x(y_0),y_0)$, (c) linear independence of binding constraints (abbrev. LIB) at $(x(y_0),y_0)$, (d) strict complementary slackness (abbrev. SCS). Here, (c) and (d) are regularity conditions they invoke to obtain the key result in a useful form. In a more recent paper Fiacco [2] obtains a similar result for somewhat more general parametric problem, such as M(y).

The crux of the theory is the application of the implicit function theorem (see [6] for a general treatment of the implicit function theorem) to the first order Kuhn-Tucker system of M(y), which is

$$\begin{aligned} \nabla_x L(x,y,u,w) &= 0 \\ u_i G_i(x,y) &= 0 \,, \quad i = 1,\ldots,m \\ H_j(x,y) &= 0 \,, \quad j = 1,\ldots,p \,. \end{aligned} \qquad (3)$$

Consider a system of equations

$$D(x,y) = 0 \,, \quad x \in E^n \text{ and } y \in E^k \qquad \text{Problem P(y)} \,,$$

where $D : E^n \times E^k \to E^\ell$, ℓ being the number of scalar valued functions in D, which are assumed OCD. This ensures the existence of the Jacobians of $D(x,y)$ with respect to x and y, denoted $J_x D$ and $J_y D$, respectively. With the sensitivity analysis context in view, the implicit function theorem could be stated as follows:

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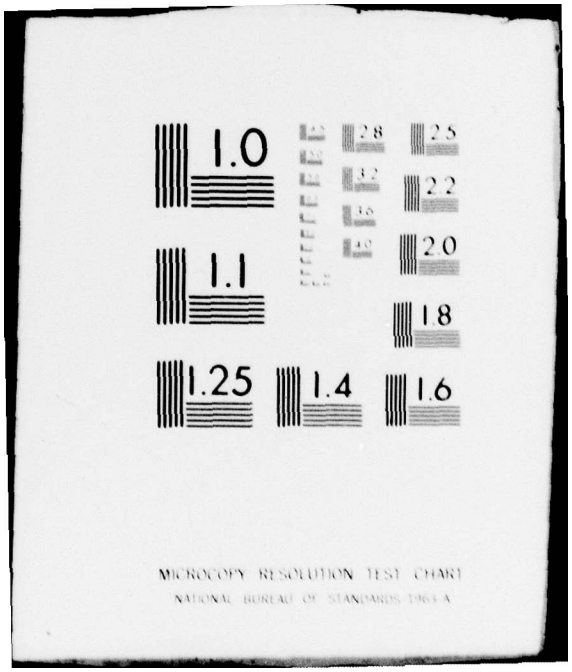
1.0
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MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

Theorem 1. Suppose $D : E^n \times E^k \to E^\ell$; $(x(y_0),y_0)$ is a zero of $D(x,y)$; $D(x,y)$ is OCD near $(x(y_0),y_0)$; $J_x D$ is nonsingular at $(x(y_0),y_0)$.

Then, there exist open sets $S_x(x(y_0)) \subset E^n$ and $S_y(y_0) \subset E^k$ such that for any $y \in \bar{S}_y(y_0)$ there exists a unique solution for x in terms of y , i.e., $x = x(y) \in S_x(x(y_0))$, which is continuous in y . Moreover $x(y)$ is OCD at y_0 and

$$\nabla_y x(y_0) = - \left[J_x D_0\right]^{-1} \left[J_y D_0\right] ,$$

where the subscript 0 on D denotes evaluation at $(x(y_0),y_0)$.

If $P(y)$ is compared with (3) the role of Theorem 1 in the context of sensitivity analysis of the solution vector becomes evident. The assumed TCD of the problem functions gives the necessary OCD for the system in (3). The additional conditions of linear independence of the binding constraint gradients and SCS were assumed by Fiacco and McCormick [3] and Fiacco [2] to produce desirable results such as the uniqueness of the Lagrange multipliers $[u(y),w(y)]$ associated with the optimal $x(y)$ and the invariance of the set of inequality constraints binding at $x(y_0)$ in a neighborhood of y_0 . The following representation for the Kuhn-Tucker triple,

$$X \triangleq (x,u,w)$$

$$\text{and} \quad X(y) \triangleq [x(y),u(y),w(y)]$$

will be adopted for notational convenience. For the same reason, the first order Kuhn-Tucker system of (3) will be denoted as follows.

$$R(x,u,w,y) = 0$$

or (3b)

$$\bar{R}(X,y) = 0 .$$

Differentiating (3b) partially with respect to X yields the Jacobian of (3b) or (3) with respect to X,

$$J_X\bar{R}(X,y) = \left[\begin{array}{c|ccc|ccc} \nabla_{xx}^2 L & -\nabla_x G_1 & \cdots & -\nabla_x G_m & \nabla_x H_1 & \cdots & \nabla_x H_p \\ \hline u_1 \nabla_x^T G_1 & G_1 & & 0 & & & \\ \vdots & & \ddots & & & 0 & \\ u_m \nabla_x^T G_m & 0 & & G_m & & & \\ \hline \nabla_x^T H_1 & & & & & & \\ \vdots & & 0 & & & 0 & \\ \nabla_x^T H_p & & & & & & \end{array}\right] \tag{4}$$

Differentiating (3b) partially with respect to y yields the Jacobian with respect to this parametric vector,

$$J_y\bar{R}(X,y) = \left[\nabla_{yx}^2 L^T,\ u_1 \nabla_y G_1, \ldots, u_m \nabla_y G_m,\ \nabla_y H_1, \ldots, \nabla_y H_p\right]^T . \tag{5}$$

At a Kuhn-Tucker point, (3b) could be written as

$$\bar{R}[X(y),y] = 0 . \tag{3c}$$

Totally differentiating (3c) with respect to y, the following relationship is obtained.

$$J_X\bar{R}(X(y),y)\ \frac{dX(y)}{dy} + J_y\bar{R}(X(y),y) = 0 . \tag{6}$$

The principal result immediately follows.

$$\frac{dX(y)}{dy} = -\ J_X\bar{R}(X(y),y)^{-1} \cdot J_y\bar{R}(X(y),y) . \tag{7}$$

Equation (7) is almost the same as the result stated under Theorem 1 but is stated with respect to the entire Kuhn-Tucker triple, X . Note that (7) is stated for y in a neighborhood of y_0 , whereas the result of Theorem 1 was only at y_0 . This is possible due to the slightly stronger assumptions made by Fiacco and McCormick [3] and Fiacco [2].

The above result is the basis for the development of a methodology for sensitivity analysis in nonlinear programming. The computational technique used here is the penalty function method. The same result and technique were used by Armacost and Mylander [1] to compute (1), the principal difference in their methodology being that they approximate $J_y\bar{R}$ by central differencing techniques, and compute all other derivatives manually. The relevant formulae for the penalty function implementation using the logarithmic barrier-quadratic loss penalty function of the SUMT algorithm [4] are discussed in Section 3.

3. Sensitivity Analysis With Symbolic Input

The factorable input technique and computer code to compute gradient vectors and Hessian matrices of problem functions was developed by McCormick [5] to facilitate the use of nonlinear programming computer codes. The code itself has since been extended by de Silva and McCormick to accept input in a symbolic format. This symbolic code has been used to solve several general nonlinear programming problems and the ease with which a problem input can be supplied or modifications done is very encouraging. A sensitivity analysis option turned out to be a natural extension to the symbolic computer code.

A reduced version of the matrix in (1) which provides substantial computational economy, results from the formulae which are developed below; a brief discussion of the factorable input philosophy precedes this development.

In the factorable input method, the problem functions are built up recursively as sums, or sums of pairwise products, of simple transformations of previously defined functions. Initially, these simple transformations

are applied to the original problem variables x_i , $i = 1,\ldots,n$, and the recursive application of the above procedure (shown mathematically in Equation (8)) generates new functions X^i , $i = n+1,\ldots,N$.

McCormick [5] gives a detailed exposition illustrated by an example. He sets $X^i = x_i$, $i = 1,\ldots,n$ and refers to the entire set of X^i , $i = 1,\ldots,N$ as concomitant variable functions (abbrev. CVFs). The general equation for the creation of a new CVF is:

$$X^i(x) = \sum_{p=1}^{i-1} T_p^i[X^p(x)] + \sum_{p=1}^{i-1}\sum_{q=1}^{p} V_{q,p}^i[X^p(x)] \cdot U_{p,q}^i[X^q(x)] \ . \qquad (8)$$

In the presence of a parametric vector y , the equation becomes

$$X^i(x,y) = \sum_{p=1}^{i-1} T_p^i[X^p(x,y),y] + \sum_{p=1}^{i=1}\sum_{q=1}^{p} V_{q,p}^i[X^p(x,y),y] \cdot U_{p,q}^i[X^q(x,y),y] \ . \qquad (9)$$

The terms in the first summation of (8) or (9) are referred to as <u>separable</u> terms comprising the new CVF $X^i(\cdot)$ and the terms of the second summation of pairwise products are called <u>quadratic</u> terms. A point of note in (9) is that the arguments of the simple transformation T_p^i , $V_{q,p}^i$ or $U_{p,q}^i$ contain y directly as well as indirectly via the argument CVF. The direct presence of the parametric vector y is restricted to, at most, two components because all the simple transformations used in the methodology and code contain, at most, two constant terms. The parametric components are a subset of these constants.

An examination of (7) suggests that second partial derivatives of the problems functions with respect to x and the cross second partial derivatives with respect to x and y are required. This is because $\bar{R}$ already contains first partial derivatives with respect to x . What is developed below will yield the matrix in (1) reduced by postmultiplying it

with a known column vector $\beta \in E^k$ or premultiplying it with a known row vector $\sigma \in E^n$. The postmultiplication produces an n-vector, each component being a weighted sum of the k partial derivatives of a particular $x_i(y)$, with respect to the components of the parametric vector. The weights are the components of $\beta \in E^k$. The ith component of this resulting n-vector can be looked upon as the "total sensitivity" of the variable $x_i(y)$ $(i=1,\ldots,n)$ with respect to all the y_j, $j = 1,\ldots k$, weighted according to β_j, $j = 1,\ldots,k$. The rth component of the n-vector is $\sum_{j=1}^{k} \frac{x_r(y)}{\partial y_j} \beta_j$. The premultiplication produces a k-component vector, the jth component being the "total sensitivity" of a composite of all n variables $x_i(y)$, $i = 1,\ldots,n$, weighted according to σ_i, $i = 1,\ldots,n$, with respect to $y_j(j=1,\ldots,k)$. The ℓth component of this k-vector would be $\sum_{i=1}^{n} \frac{\partial x_i}{\partial y_\ell} \sigma_i$.

For clarity of exposition, (9) will be simplified below to a separable portion of one term or a quadratic term of one product.

Consider the separable case first. Suppose

$$X^i(x,y) = T[X^p(x,y),y] \ . \tag{10}$$

Letting

$$\dot{T} \triangleq \dot{T}\,[X^p(x,y),y] \triangleq \frac{\partial T[X^p(x,y),y]}{\partial X^p} \ , \tag{11}$$

we have

$$\nabla_x X^i(x,y) = \nabla_x X^p(x,y) \cdot \frac{\partial T[X^p(x,y),y]}{\partial X^p} \ . \tag{12a}$$

$$= \dot{T} \cdot \nabla_x X^p(x,y) \ . \tag{12b}$$

In (12b) $\dot{T}$ is readily available since all transformations are simple ones, and $\nabla_x X^p(x,y)$ is built up inductively from its argument CVF.

The expressions for $\nabla^2_{xx} X^i(x,y)$ are already developed and coded in [5], and will not be repeated here. Note that

$$\nabla^2_{yx} X^i(x,y) = \nabla_y \nabla_x X^i(x,y) \tag{13a}$$

$$= D_y D_x \, T[X^p(x,y),y] \; . \tag{13b}$$

In (13b) D denotes total differentiation, although in the previous step the partial differentiation ∇ was used. This is due to the fact that the partial differentiation in (13a) would capture the total variation due to x and y of the expression in (12a), but not so in (13b). Continuing the differentiation with respect to y of (12b), as in (13b), we get

$$\nabla^2_{yx} X^i(x,y) = \nabla^2_{yx} X^p(x,y) \cdot \dot{T} + \nabla_x X^p(x,y) \left\{ \ddot{T}\,[X^p(x,y),y] \; \nabla^T_y X^p(x,y) + \nabla^T_y \dot{T} \right\} . \tag{14}$$

In the above, $\ddot{T} \triangleq \ddot{T}\,[X^p(x,y),y] \triangleq \dfrac{\partial^2 T[X^p(x,y),y]}{\partial (X^p)^2}$. (15)

In (14), $\dot{T}$ and $\ddot{T}$ are readily available scalar quantities. The quantity $\nabla^T_y \dot{T}\,[X^p(x,y),y]$ is the true partial derivative unlike in (13b). This is easy to compute because y enters the $\dot{T}$ argument directly, at most, in two nonzero components. The expression for $\nabla_x X^p(x,y)$ has been derived already in (12a). The only remaining terms are $\nabla^2_{yx} X^p(x,y)$ which is built up inductively, and $\nabla^T_y X^p(x,y)$ which is discussed below.

Now

$$\nabla_y X^i(x,y) = D_y\, T[X^p(x,y),y]$$
$$= \nabla_y\, T[X^p(x,y),y] + \dot{T} \cdot \nabla_y X^p(x,y) \ . \tag{16}$$

Note $\nabla_y T(X^p(x,y),y)$ is the true partial derivative and will have at most two nonzero components in E^k which are easily computable. Thus (16) permits the computation of the $\nabla_y^T X^p(x,y)$ term of (14) by induction. Since the formulae of (12b), (14), (16) are initially applied to the original problem variables $X_i = x^i$, $i = 1,\ldots,n$, we have, for $p \leq n$:

$$\nabla_x X^i(x,y) = e_p \cdot \dot{T} \tag{17}$$

where e_j is a unit vector $\in E^n$ with unity in the jth position.

Also,
$$\nabla^2_{yx} X^i(x,y) = \nabla^2_{yx} X^p + e_p[\dot{T} \cdot \nabla^T X^p + \nabla_y^T \dot{T}]$$
$$= e_p\, \nabla_y^T \dot{T} \ . \tag{18}$$

Finally,
$$\nabla_y X^i(x,y) = \nabla_y T + \nabla_y X^p \dot{T}$$
$$= \nabla_y T \ . \tag{19}$$

On page 6 , we spoke of dimensionally reducing the matrix of (1) in two separate ways. The case of premultiplying by $\alpha \in E^n$ is considered below.

Thus, from (17) and (18) letting α_p denote the pth component of α , we get

$$\alpha^T \nabla_x X^i(x,y) = \alpha_p \cdot \dot{T} \ . \tag{20}$$

$$\alpha^T \nabla^2_{yx} X^i(x,y) = \alpha_p \nabla_y^T \dot{T} \ . \tag{21}$$

The summation of (21) will have, at most, two nonzero components due to the nature of the simple transformations used. Next, consider the case of $p > n$. Here,

$$a^T \nabla_x X^i(x,y) = a^T \nabla_x X^p(x,y) \cdot \dot{T} \cdot \tag{22}$$

Next,

$$a^T \nabla^2_{yx} X^i(x,y) = a^T \nabla^2_{yx} X^p(x,y) \cdot \dot{T}$$

$$+ a^T \nabla_x X^p(x,y) \left\{ \dot{T} \nabla^T_y X^p(x,y) + \nabla^T_y \dot{T} \right\} . \tag{23}$$

Equation (22) works purely inductively and does not require the deliberate introduction of a, as this has been done in (20). It is a scalar quantity for each i. The first term on the right-hand side of (23) is also an inductive term. As far as the second term is concerned, (the scalar multiplier $a^T \nabla_x X^p(x,y)$, the only presence of a there,) it is built up inductively. The expression within the curly bracket is a k-vector, the first term being an inductive one as demonstrated in (16) and the second possessing, at most, two nonzero components.

In summary, we have a scalar quantity for the reduced version of the gradient vector of each CVF i, and (20) (or (17), if $p \leq n$) has to be repeated for every p corresponding to the separable terms of X^i, and added together. The reduced second cross partial of each CVF i (as shown in (21) or (18) depending on whether $p \gtrless n$) is a k-vector for each component p, which must be added together as before.

The quadratic components of (9) will be addressed next. Considering the creation of an X^i, $i = n+1,\ldots,N$, using a single product, we have

$$X^i(x,y) = V[X^p(x,y),y] * U[X^q(x,y),y] , \tag{24}$$

then,

$$\nabla_x X^i(x,y) = D_x V[X^p(x,y),y] * U[X^q(x,y),y]$$

$$+ V[X^p(x,y),y] * D_x U[X^q(x,y),y] \tag{25}$$

$$= \dot{V}U \cdot \nabla_x X^p(x,y) + V\dot{U} \cdot \nabla_x X^q(x,y) . \tag{26}$$

Now $\dot{V}$, $\dot{U}$ are defined analogously to $\dot{T}$ (see (11)). Since V , $\dot{V}$, U , $\dot{U}$ are scalar quantities, the premultiplication by $\alpha \in E^n$ could be done as in (20) and (22) depending on whether p and q are $\leq$ or $>$ n . The separate terms in (25) must be added together. The result is

$$\alpha^T \nabla_x X^i(x,y) = \dot{V}U\ \alpha^T \nabla_x X^p(x,y) + V\dot{U}\ \alpha^T \nabla_x X^q(x,y) \ . \tag{26}$$

When p (or q) is $> n$, $\alpha^T \nabla_x X^p$ (or $\alpha^T \nabla_x X^q$) enters purely inductively. Otherwise, the term simplifies to $\dot{V}U\ \alpha_p$ (or $V\dot{U}\ \alpha_q$). The second cross partial derivative is obtained by differentiating (26) with respect to y . The arguments of X^p and X^q are omitted for convenience of representation. Finally,

$$\begin{aligned} \nabla^2_{yx} X^i(x,y) = U &\left\{ \dot{V} \cdot \nabla^2_{yx} X^p + \nabla_x X^p [\dot{\dot{V}} \cdot \nabla_y^T X^p + \nabla_y^T \dot{V}] \right\} \\ &+ \dot{V} \cdot \nabla_x X^p \cdot D_y U \\ &+ V \left\{ \dot{U} \cdot \nabla^2_{yx} X^q + \nabla_x X^q [\dot{\dot{U}} \cdot \nabla_y^T X^q + \nabla_y^T \dot{U}] \right\} \\ &+ \dot{U} \cdot \nabla_x X^q \cdot D_y V \ . \end{aligned} \tag{27}$$

Premultiplying (27) by $\alpha \in E^n$, and rearranging,

$$\begin{aligned} \alpha^T \nabla^2_{yx} X^i(x,y) = U &\left\{ \alpha^T \nabla^2_{yx} X^p \cdot \dot{V} + \alpha^T \nabla_x X^p [\dot{\dot{V}} \cdot \nabla_y^T X^p + \nabla_y^T \dot{V}] \right\} \\ &+ V \left\{ \alpha^T \nabla^2_{yx} X^q \cdot \dot{U} + \alpha^T \nabla_x X^q [\dot{\dot{U}} \cdot \nabla_y^T \dot{U}] \right\} \\ &+ \alpha^T \nabla_x X^p \cdot D_y U \cdot \dot{V} + \alpha^T \nabla_x X^q \cdot \nabla_y D \cdot \dot{U} \ . \end{aligned} \tag{28}$$

The evaluation of the portions within the curly brackets in (28) is identical to the evaluation of the cross second partials in the separable analysis, i.e., the right-hand side of (23). The last two terms of (26) contain two quantities $\alpha^T \nabla_x X^p$ and $\alpha^T \nabla_x X^q$ which are developed by induction as demonstrated by (12) and (20). The quantities $D_y U$ and $D_y V$

are analogous to $D_y T$ whose computation is shown in (16). The only remaining expression to be developed is that for $\nabla_y X^i$,

$$\begin{aligned} \nabla_y X^i(x,y) &= D_y\{V[X^p(x,y),y]\ U[X^q(x,y),y]\} \\ &= U\{\dot{V}\cdot\nabla_y X^p+\nabla_y V\} + V\{\dot{U}\ \nabla_y X^q+\nabla_y U\} \ . \end{aligned} \tag{29}$$

Examination of (23) and (28) shows that it is not necessary to premultiply the expressions of (16) and (29) by α^T , since $\nabla_y X$ never gets directly premultiplied by it.

The above Equations (20) through (29) have developed the necessary components to perform the computations necessary to produce a row k-vector $\alpha^T[dx(y)/dy]$. The next section describes how these are implemented via a penalty function method.

4. Computation of Sensitivity Results

The SUMT computer code [4] is used in conjunction with the factorable symbolic sensitivity analysis code to yield $[\alpha^T dx(y)/dy]$. The code uses an interior-exterior penalty function with a single parameter r , and generates a sequence of subproblems which will be denoted as $M(y,r)$, which tend to $M(y)$ as $r \to 0$. In straight nonlinear optimization SUMT uses an extrapolation based on r to estimate the relevant values at $r = 0$. A similar extrapolation technique is used here to provide first order estimates of $[\alpha^T dx(y)/dy]$.

A logarithmic barrier quadratic loss penalty function is considered below. For $M(y)$, this penalty function is

$$P[x,y,r] = F(x,y) - r \sum_{i=1}^{m} \ell n\ G_i(x,y) + \sum_{j=1}^{p} \frac{H_j^2(x,y)}{r} \ . \tag{30}$$

The problem solved at the ℓth stage of the sequential procedure, for a value $r = r^\ell$ is

$$\underset{x\in R^0}{\text{MIN}} \quad P[x,y,r^\ell] \qquad\qquad \text{Problem } M(r,y)$$

where

$$R^0 \equiv \{x \mid G_i(x,y) > 0 \ , \ i=1,\ldots,m\} \ .$$

The optimality condition is that the gradient with respect to x vanishes, i.e.,

$$\nabla_x P[x,y,r^\ell] = 0 \ . \tag{31}$$

At local optima, for a general value of r we can write

$$\nabla_x P[x(y,r),y,r] \equiv 0 \ . \tag{32}$$

Applying this to (30),

$$\nabla_x F[x(y,r),y] - \sum_{i=1}^{m} \frac{r}{G_i[x(y,r),y]} \cdot \nabla_x G_i[x(y,r),y] + \sum_{i=1}^{p} \frac{2\, H_j[x(y,r),y]}{r} \nabla_x H_j[x(y,r),y] \equiv 0 \ . \tag{33}$$

Differentiating (33) partially with respect to y, we get

$$\begin{aligned} \nabla^2_{yx} P[x(y,r),y,r] &= \nabla^2_{yx} F[x(y,r),y] - \sum_{i=1}^{m} \nabla^2_{yx} G_i[x(y,r),y] \cdot \frac{r}{G_i[x(y,r),y]} \\ &+ \sum_{i=1}^{m} \frac{r}{G_i[x(y,r),y]^2} \cdot \nabla_x G_i[x(y,r),y] \cdot \nabla^T_y G_i[x(y,r),y] \\ &+ \sum_{j=1}^{p} \nabla^2_{yx} H_j[x(y,r),y] \cdot \frac{2\, H_j[x(y,r),y]}{r} \\ &+ \sum_{j=1}^{p} \left(\frac{2}{r}\right) \nabla_x H_j[x(y,r),y] \cdot \nabla^T_y H_j[x(y,r),y] \ . \end{aligned} \tag{34}$$

Differentiating (33) totally with respect to y,

$$\nabla^2_{xx} P[x(y,r),y,r] \frac{dx(y,r)}{dy} + \nabla^2_{yx} P[x(y,r),y,r] \equiv 0 \ . \tag{35}$$

The argument variables within the square brackets will be omitted when referring to the terms in the above.

$\nabla^2_{xx}P$ has been developed elsewhere by McCormick [5]. Those for $\nabla^2_{xy}P$ are developed using the Equation (34). Thus,

$$\frac{d(x(y,r))}{dy} = - \nabla^2_{xx}P[x(y,r),y,r]^{-1} \cdot \nabla^2_{yx}P[x(y,r),y,r] \ . \tag{36}$$

The similarity between (7) and (36) is to be noted. Suppose (36) is premultiplied by some $\sigma \in E^n$. Then, we get

$$\sigma^T \frac{dx(y,r)}{dy} = - \alpha^T \cdot \nabla^2_{yx}P[x(y,r),y,r] \ , \tag{37}$$

where

$$\alpha^T = \sigma^T \cdot \nabla^2_{xx}P[x(y,r),y,r]^{-1} \ . \tag{38}$$

If σ is a known vector, α is determined because the Hessian of the penalty function with respect to x is known by [5]. Then (37) is determined by premultiplying (34) by this α , and using (20), (21), (22), (23), (27) and (29) to determine the individual components of the resulting equation. This can be done because every problem function F , G_i ; $i = 1,\ldots,m$, H_j ; $j = 1,\ldots p$, can be identified with a CVF X^i , $i = 1,\ldots,n$, $n+1,\ldots,N$.

The final aspect remaining to be discussed in the context of the sensitivity methodology is the improvement of the estimate of $\sigma^T \frac{dx(y,r)}{dy}$ by extrapolating on r , at the ℓth sequential unconstrained problem, to $r = 0$. A Taylor series approximation applied at the ℓth and $(\ell-1)$th subproblems yields

$$\frac{dx(y,r^{\ell})}{dy} \doteq \frac{dx(y,0)}{dy} + r^{\ell}\,\frac{d}{dr}\left[\frac{dx(y,0)}{dy}\right]. \tag{39}$$

$$\frac{dx(y,r^{\ell-1})}{dy} \doteq \frac{dx(y,0)}{dy} + r^{\ell-1}\,\frac{d}{dr}\left[\frac{dx(y,0)}{dy}\right]. \tag{40}$$

Eliminating the last term in (39) and (40)

$$\frac{d(x(y,0))}{dy}\left(r^{\ell-1} - r^{\ell}\right) \doteq r^{\ell-1} \cdot \frac{dx(y,r^{\ell})}{dy} - r^{\ell} \cdot \frac{dx(y,r^{\ell-1})}{dy} \tag{41}$$

from which, (letting $r^{\ell-1} = c \cdot r^{\ell}$) we get

$$\frac{dx(y,0)}{dy} \doteq \frac{1}{c-1} \cdot \left[c\,\frac{dx(y_1 r^{\ell})}{dy} - \frac{dx(y_1 r^{\ell-1})}{dy}\right]. \tag{42}$$

Hence by performing the computation for successive subproblems when r becomes sufficiently small, (42) can be used to estimate the value for the original problem, i.e., $r = 0$.

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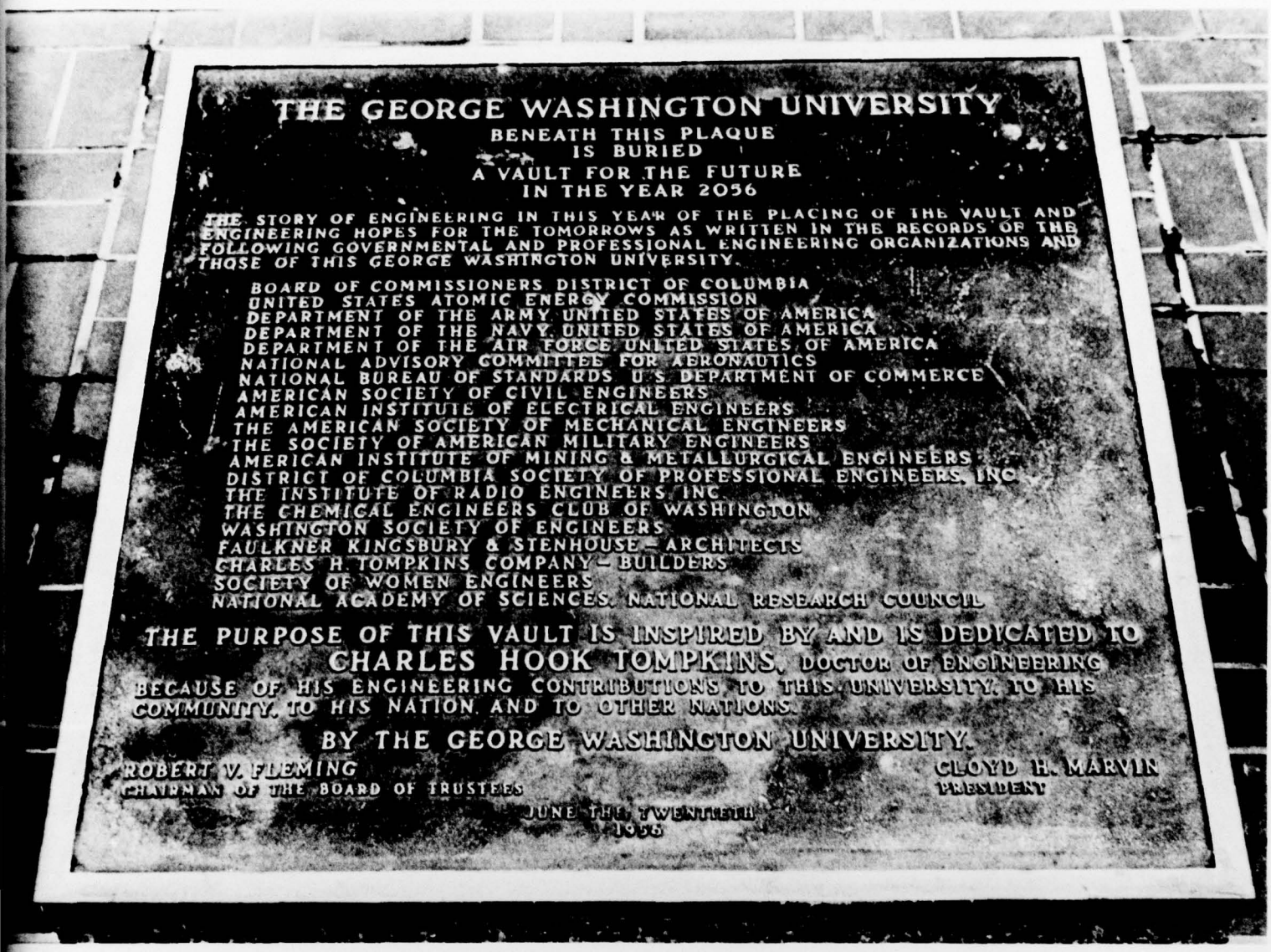

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