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MULTIVALENT PREFERENCE STRUCTURES.(U)

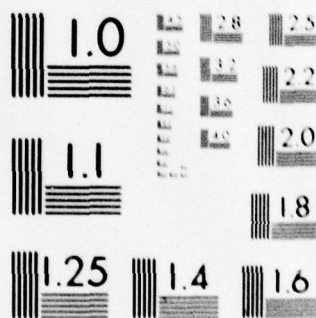
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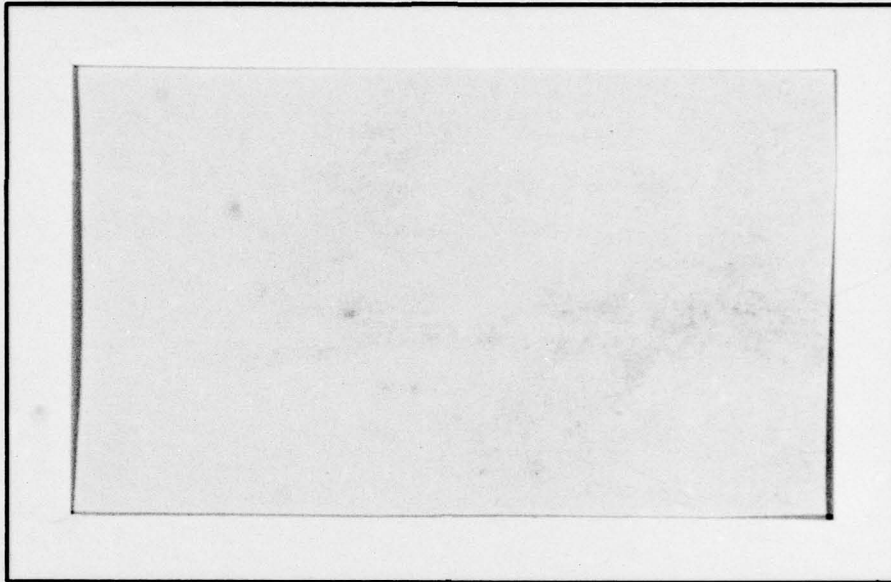
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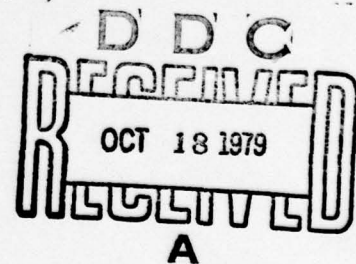


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⑥ MULTIVALENT PREFERENCE STRUCTURES

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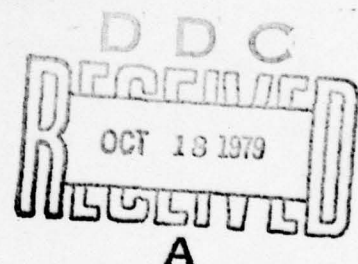
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MULTIVALENT PREFERENCE STRUCTURES

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Abstract

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This paper presents a valence approach for assessing multi-attribute utility functions. Unlike the decomposition approach which uses independence axioms on whole attributes to obtain utility representations, the valence approach partitions the elements of each attribute into classes on the basis of equivalent conditional preference orders. These partitions generate multivalent utility independence axioms that lead to additive-multiplicative and quasi-additive representation theorems for multiattribute utility functions defined over product sets of equivalence classes. Preference interdependencies are thereby reflected in these classes, so attribute interactions are readily interpreted and the functional forms of the representations are kept simple.
↙

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Key words. Decision analysis, utility theory, multiattribute, preference, interdependent, valence.

1. Introduction. The principal aim of utility analysis is to obtain a mathematical representation of preferences that will aid in the evaluation of risky decisions. A predominant theme in multiattribute utility theory for more than a decade has been the *decomposition approach*. Given various independence assumptions, this approach prescribes how to divide the assessment of a multiattribute utility function into several steps requiring the determination of scaling coefficients and the estimation of conditional utility functions involving one or more attributes. Farquhar [6] reviews independence axioms and corresponding multiattribute utility representation theorems; further details are in [3-5, 10, 13-26].

This paper presents a *valence approach* for assessing multiattribute utility functions. Unlike utility decompositions which rely on independence axioms defined on whole attributes, the valence approach partitions the elements of each attribute into classes on the basis of equivalent conditional preference orders. Multiattribute utility representations are derived over a collection of subspaces determined by these partitions.

After introducing basic terminology, multivalent preference structures, and independence axioms, we establish a representation theorem for two-attribute utility functions using multivalent utility independence. We show that a structural assumption called *uniform preferability* greatly simplifies the assessment of multivalent representations. We then extend these basic results to n-attribute representation theorems: first, by using joint multivalent utility independence assumptions, and then by using a special form of individual multivalent utility independence.

2. Multivalent preference structures. Let X denote the *outcome space* in a decision problem, and let P denote the space of all simple probability distributions (*lotteries*) over X . Let $\succ$ denote a *preference order* on P satisfying the von Neumann-Morgenstern axioms [12, 27]. Hence, there exists a real function u on X , called a *utility function* for $\succ$ on P , such that for all $p, q \in P$, $p \succ q$ iff (if and only if) $\sum_{x \in X} p(x)u(x) > \sum_{x \in X} q(x)u(x)$.

Suppose for simplicity that $X = Y \times Z$, where Y and Z are *attribute sets* each containing at least two elements. Let P_Y denote the set of all simple probability distributions on Y . The (single-element) *conditional preference order* $\succ_z$ induced on P_Y by the preference order $\succ$ on P and a fixed element $z \in Z$ is defined by

$$p_Y \succ_z q_Y \quad \text{iff} \quad (p_Y, z) \succ (q_Y, z), \quad (1)$$

where $p_Y, q_Y \in P_Y$.

One way of describing how preferences for lotteries on Y depend on elements in Z is to partition Z into classes corresponding to the distinct conditional preference orders induced on P_Y .

Definition 1:¹ The *multivalent preference structure* of Y given Z is defined by $(Y, \Omega_Z, [Z])$ where for some nonempty index set $\mathbb{Z}$,

(1) $\Omega_Z \equiv \{ \succ_j : j \in \mathbb{Z} \}$ denotes a collection of distinct preference orders, called *base orders*, on P_Y ; and

¹Our terminology is motivated by certain theories of molecular structure in physical chemistry.

(11) $[Z] \equiv \{\hat{Z}^j: j \in \mathbb{Z}\}$ denotes a partition of Z into nonempty classes, called *orbitals*, such that $\succ_z = \succ_j$ for all $z \in \hat{Z}^j$ and $j \in \mathbb{Z}$.

Thus two elements z' and z'' in Z belong to the same orbital iff $\succ_{z'} = \succ_{z''}$ on P_Y . Instead of using conditional preference orders to determine $[Z]$, we can obtain the same partition by using strategically equivalent conditional utility functions on Y [8].

Valence refers to the cardinality of $[Z]$. At one extreme, the preference structure is *univalent* if $[Z] = \{Z\}$; Y is utility independent of Z in this case (see Definition 2). At the other extreme, the preference structure exhibits complete utility dependence of Y on Z if $[Z]$ consists of all single-element subsets of Z . Multivalent preference structures, therefore, cover an entire spectrum of interdependencies between attributes.

Practical illustrations of multivalent preference structures are easy to find: for example, the evaluation of changes in a portfolio when balance or complementarity among the items is important [3, 7, 9]; the evaluation of multi-period income streams given past income levels when inter-temporal dependencies exist [1, 11, 13, 22, 23]; and many others [e.g., 3, 7, 8, 9, 11, 16, 22].

3. Independence axioms. Pollak [25], Keeney [17-21], Raiffa [26], and others have used the following independence axiom to develop multiattribute utility decompositions.

Definition 2: Y is *utility independent* of Z , denoted $Y(UI)Z$, iff there exists a preference order $\succsim_1$ on P_Y such that $\succsim_z = \succsim_1$ for all $z \in Z$.

We note that if Y is not utility independent of Z , then there are at least two distinct conditional preference orders on P_Y .

Definition 3: Y is *multivalent utility independent* of $[Z]$, denoted by $Y(UI)[Z]$, iff there exists a collection of base orders Ω_Z such that Y given Z has the multivalent preference structure $(Y, \Omega_Z, [Z])$.

An analogous definition holds for $Z(UI)[Y]$.

Since von Neumann-Morgenstern utility functions are unique up to positive linear transformations and since $Y(UI)[Z]$ implies Y is utility independent of the restriction of Z to $\hat{Z}$ for all $\hat{Z} \in [Z]$, it is trivial to establish

Lemma 1: $Y(UI)[Z]$ iff $[Z]$ is a partition of Z such that for all $\hat{Z} \in [Z]$,

$$z \in \hat{Z} \text{ iff } u(y, z) = \alpha_2(z) + \beta_2(z)u(y, \hat{z}) \quad \text{for all } y \in Y, \quad (2)$$

where $\hat{z}$ is fixed arbitrarily in $\hat{Z}$, and α_2 and β_2 are real functions on Z with $\beta_2 > 0$.

Analogously, $Z(UI)[Y]$ iff $[Y]$ is a partition of Y such that for all $\hat{Y} \in [Y]$,

$$y \in \hat{Y} \text{ iff } u(y, z) = \alpha_1(y) + \beta_1(y)u(\hat{y}, z) \quad \text{for all } z \in Z, \quad (3)$$

where $\hat{y}$ is fixed arbitrarily in $\hat{Y}$, and α_1 and β_1 are real functions on Y with $\beta_1 > 0$.

4. Multivalent utility representations with two attributes. This section establishes a representation theorem for multivalent preference structures involving two attributes.²

THEOREM 1: Let u be a von Neumann-Morgenstern utility function on the outcome space $Y \times Z$. Suppose $Y(UI)[Z]$ and $Z(UI)[Y]$. Then there exist real functions α_1 and β_1 on Y with $\beta_1 > 0$, real functions α_2 and β_2 on Z with $\beta_2 > 0$, and constants $\hat{k}$ depending on only the sets $\hat{Y} \times \hat{Z}$, where $\hat{Y} \in [Y]$ and $\hat{Z} \in [Z]$, such that u has one of the following *additive-multiplicative representations* for all $y \in \hat{Y}$ and $z \in \hat{Z}$:

$$u(y, z) = \alpha_1(y) + \alpha_2(z) + u(\hat{y}, \hat{z}), \quad (4a)$$

$$u(y, z) = \alpha_1(y) + \beta_1(y)u(\hat{y}, \hat{z}), \quad (4b)$$

$$u(y, z) = \alpha_2(z) + \beta_2(z)u(\hat{y}, \hat{z}), \quad (4c)$$

$$u(y, z) = \hat{k} + \beta_1(y)\beta_2(z)[u(\hat{y}, \hat{z}) - \hat{k}]. \quad (4d)$$

Proof: For simplicity, let $\alpha_1, \beta_1, \alpha_2, \beta_2, \hat{u}$, and u represent $\alpha_1(y), \beta_1(y), \alpha_2(z), \beta_2(z), u(\hat{y}, \hat{z})$, and $u(y, z)$, respectively. Since $Y(UI)[Z]$ and $Z(UI)[Y]$, Lemma 1 yields (2) and (3). If we let $z = \hat{z}$ in (3) and substitute the result into (2), and likewise let $y = \hat{y}$ in (2) and substitute the result into (3), we obtain the following equations on $\hat{Y} \times \hat{Z}$:

$$u = \alpha_2 + \beta_2\alpha_1 + \beta_1\beta_2\hat{u} = \alpha_1 + \beta_1\alpha_2 + \beta_1\beta_2\hat{u}. \quad (5)$$

²Results essentially equivalent to Theorem 1 were obtained independently by Farquhar [3] and, in a different context, by Meyer [23]. The proof of Theorem 1 given here, however, is substantially shorter and simpler than earlier proofs; more importantly, it generalizes from two to n attributes to yield further representation theorems.

The proof involves four cases which depend on whether or not β_1 or β_2 equals one for equivalent elements:

Case 1 ($\beta_1 \equiv 1$ on $\hat{Y}$ and $\beta_2 \equiv 1$ on $\hat{Z}$): The equations in (5) give $u = \alpha_1 + \alpha_2 + \hat{u}$ when $\beta_1 \equiv 1$ and $\beta_2 \equiv 1$, so representation (4a) is immediate.

Case 2 ($\beta_1 \neq 1$ on $\hat{Y}$ and $\beta_2 \equiv 1$ on $\hat{Z}$): Since $\beta_2 \equiv 1$, (5) yields $\beta_1 \alpha_2 = \alpha_2$. But $\beta_1 \neq 1$ on $\hat{Y}$ implies that $\alpha_2 \equiv 0$ on $\hat{Z}$. Thus (5) gives $u = \alpha_1 + \beta_1 \hat{u}$, which is (4b).

Case 3 ($\beta_1 \equiv 1$ on $\hat{Y}$ and $\beta_2 \neq 1$ on $\hat{Z}$): By analogy with Case 2, we obtain $\alpha_1 \equiv 0$ on $\hat{Y}$. Thus (5) gives $u = \alpha_2 + \beta_2 \hat{u}$, which is (4c).

Case 4 ($\beta_1 \neq 1$ on $\hat{Y}$ and $\beta_2 \neq 1$ on $\hat{Z}$): In this case, (5) can be rewritten as

$$\frac{\alpha_1}{1 - \beta_1} = \frac{\alpha_2}{1 - \beta_2} . \quad (6)$$

Since the left side depends on only y , the right side on only z , and equality holds throughout $\hat{Y}$ and $\hat{Z}$, both sides of (6) must equal a constant, say $\hat{k}$.

Thus (6) implies $\alpha_1 = \hat{k}(1 - \beta_1)$ on $\hat{Y}$ and $\alpha_2 = \hat{k}(1 - \beta_2)$ on $\hat{Z}$. These results combine with (5) to give $u = \hat{k} + \beta_1 \beta_2 [\hat{u} - \hat{k}]$, which is (4d). ■

5. Interpretation of the β 's. The multivalent utility representations in Theorem 1 require the assessment of the functions $\alpha_1(y)$, $\beta_1(y)$, $\alpha_2(z)$, and $\beta_2(z)$. We present the assessment for just α_2 and β_2 , since the assessment for α_1 and β_1 is completely analogous.

Except in the trivial case where $\succ_j = \phi$ on P_Y (and a representation for u is therefore immediate), we can choose y_0^j and y_1^j in Y such that $(y_1^j, \hat{z}^j) \succ (y_0^j, \hat{z}^j)$ where $\hat{z}^j$ is arbitrarily fixed in $\hat{Z}^j$ for all $j \in \mathbb{Z}$. Successive substitution of y_0^j and y_1^j into (2) yields two equations that can be solved for α_2 and β_2 to give

$$\alpha_2(z) = a_2^j u(y_1^j, z) + (1 - a_2^j) u(y_0^j, z) \quad \text{for } z \in \hat{Z}^j, \quad (7a)$$

$$\beta_2(z) = b_2^j (u(y_1^j, z) - u(y_0^j, z)) \quad \text{for } z \in \hat{Z}^j, \quad (7b)$$

where $a_2^j \equiv -u(y_0^j, \hat{z}^j) / (u(y_1^j, \hat{z}^j) - u(y_0^j, \hat{z}^j))$ and $b_2^j \equiv 1 / (u(y_1^j, \hat{z}^j) - u(y_0^j, \hat{z}^j))$ for $j \in \mathbb{Z}$.

The expression for β_2 in (7b) reveals the presence (or absence) of a restricted form of additivity in the multivalent representations of Theorem 1. For example, if $\beta_2(z) \equiv 1$ on $\hat{Z}$, then (7b) implies

$$u(y_1, z) - u(y_0, z) = u(y_1, \hat{z}) - u(y_0, \hat{z}), \quad (8)$$

for all $z, \hat{z} \in \hat{Z}$. Since Y is utility independent of $\hat{Z}$, it is easy to verify that (8) holds for all $y_0, y_1 \in Y$. On the other hand if $\beta_2 \neq 1$ on $\hat{Z}$, then there is at least one pair $y_0, y_1 \in Y$ for which equality fails in (8). Therefore given the assumptions of Theorem 1, Y and $\hat{Z}$ are *additive independent* [6, 7, 10, 12, 14, 20, 22] iff $\beta_2(z) \equiv 1$ on $\hat{Z}$. A similar statement holds for β_1 as well.

One readily notes the effect of this restricted form of additivity in the four representations of Theorem 1. The additive result in (4a) is obtained when $\beta_1 \equiv 1$ and $\beta_2 \equiv 1$, while the multiplicative result in (4d) is

obtained when $\beta_1 \neq 1$ and $\beta_2 \neq 1$. The results in (4b) and (4c) are additive-multiplicative representations obtained when exactly one of the two β 's is unity.

6. Simplifying the assessments. The representations in Theorem 1 have simple forms, but complete assessment of a two-attribute utility function is complicated by the number of conditional utility functions required to determine the α and β functions. In an extreme case, one pair of elements in Y serves all orbitals in $[Z]$, so only two conditional utility functions are needed on Z .

Definition 4: For $y_0, y_1 \in Y$, y_1 is *uniformly preferable* to y_0 , denoted by $y_1 \gg y_0$, iff $(y_1, z) \succ (y_0, z)$ for all $z \in Z$.

An analogous definition obviously holds for $z_1 \gg z_0$.

We observe that uniform preferability is much weaker than the axiom of *preference independence* [6, 15, 22]. For example, Y preference independent of Z implies that the preference (or indifference) relation between any pair of elements in Y holds uniformly for all elements in Z ; uniform preferability, however, considers only one pair of elements in Y . Furthermore when $Y(UI)[Z]$, we note that $y_1 \gg y_0$ iff $(y_1, \hat{z}^j) \succ (y_0, \hat{z}^j)$ where $\hat{z}^j$ is arbitrarily fixed in $\hat{Z}^j$ for all $j \in Z$. Thus instead of checking all elements in Z , uniform preferability can be tested by considering just one element from each orbital in $[Z]$ when $Y(UI)[Z]$.

We now state a fundamental result.

COROLLARY 1: Let $Y(UI)[Z]$ and $Z(UI)[Y]$. If there exist $y_0, y_1 \in Y$ and $z_0, z_1 \in Z$ such that $y_1 \gg y_0$ and $z_1 \gg z_0$, then a von Neumann-Morgenstern utility function u on $Y \times Z$ has one of the additive-multiplicative representations in (4) on each $\hat{Y} \times \hat{Z}$, where $\hat{Y} \in [Y]$ and $\hat{Z} \in [Z]$, and u is completely specified by four conditional utility functions $u(y_0, z)$, $u(y_1, z)$, $u(y, z_0)$, $u(y, z_1)$, and the utilities assigned to the representative outcomes $(\hat{y}, \hat{z}) \in \hat{Y} \times \hat{Z}$ for each $\hat{Y} \in [Y]$ and $\hat{Z} \in [Z]$.

Proof: Since Theorem 1 holds, $y_1 \gg y_0$ implies that equations (7a) and (7b) can be simplified as follows,

$$\alpha_2(z) = \hat{a}_2 u(y_1, z) + (1 - \hat{a}_2) u(y_0, z) \quad \text{for } z \in \hat{Z}, \quad (9a)$$

$$\beta_2(z) = \hat{b}_2 (u(y_1, z) - u(y_0, z)) \quad \text{for } z \in \hat{Z}, \quad (9b)$$

where $\hat{a}_2 \equiv -u(y_0, \hat{z}) / (u(y_1, \hat{z}) - u(y_0, \hat{z}))$ and $\hat{b}_2 \equiv 1 / (u(y_1, \hat{z}) - u(y_0, \hat{z}))$ for a fixed representative $\hat{z} \in \hat{Z}$, for all $\hat{Z} \in [Z]$. Thus α_2 and β_2 are completely specified by $u(y_0, z)$ and $u(y_1, z)$ on Z . Similarly, α_1 and β_1 are completely specified by $u(y, z_0)$ and $u(y, z_1)$ on Y when $z_1 \gg z_0$. Therefore, the representations in (4) are determined by these four conditional utility functions and the utilities of $(\hat{y}, \hat{z}) \in \hat{Y} \times \hat{Z}$ for each $\hat{Y} \in [Y]$ and $\hat{Z} \in [Z]$. ■

If $Y(UI)Z$ in Corollary 1, then only $u(y, z_0)$ is needed on Y , because $u(y, z_1)$ can be derived from $u(y, z_0)$ by an appropriate positive, linear transformation. Similarly, if $Z(UI)Y$ in Corollary 1, then only $u(y_0, z)$ is needed on Z .

7. One-way utility independence. This section illustrates the cross-over effect that a univalent preference structure on Y has on the preference

structure on Z . Let $Y(UI)Z$, but not $Z(UI)Y$. If Y is *essential* (i.e.,

$\succ_z \neq \phi$ on P_Y for some $z \in Z$), then there exist $y_0, y_1 \in Y$ such that $y_1 \gg y_0$.

Without loss of generality, let z_0 be a representative of Z and scale u so that $u(y_0, z_0) = 0$ and $u(y_1, z_0) = 1$. Thus (9a,b) gives $\alpha_2(z) = u(y_0, z)$ and $\beta_2(z) = u(y_1, z) - u(y_0, z)$ for $z \in Z$, so by (2) and Corollary 1,

$$u(y, z) = u(y_0, z) + (u(y_1, z) - u(y_0, z))u(y, z_0), \quad (10)$$

for all $y \in Y$ and $z \in Z$. One conditional utility function on Y and two conditional utility functions on Z therefore determine u when $Y(UI)Z$ and Y is essential. (See also Keeney [18, 19, 22] and Nahas [24].)

Before stating the next theorem, we make the following definitions.

Let the *dual order* $\succ^*$ of a preference order $\succ$ on P be defined by $p \succ^* q$ iff $q \succ p$ where $p, q \in P$. Z is *generalized utility independent* of Y , denoted $Z(GUI)Y$, iff there exists a nonempty preference order $\succ_0$ on P_Z such that $\succ_y \in \{\succ_0, \succ_0^*, \phi\}$ on P_Z , for all $y \in Y$. Generalized independence thus allows conditional preference orders which are either identical to a given base order, complete reversals, or complete indifference.

THEOREM 2: Let $Y(UI)Z$. Suppose there exist $y_0, y_1 \in Y$ satisfying (i) *uniform preferability*: $(y_1, z_0) \succ (y_0, z_0)$ for some $z_0 \in Z$, and (ii) *strategic duality*: $\succ_{y_1} \in \{\succ_{y_0}, \succ_{y_0}^*, \phi\}$ on P_Z . Then the preference structure $(Z, \Omega_Y, [Y])$ is at most trivalent and $\Omega_Y \subseteq \{\succ_0, \succ_0^*, \phi\}$, where $\succ_0$ is a nonempty preference order on P_Z .

Proof: $Y(UI)Z$ and $(y_1, z_0) \succ (y_0, z_0)$ for some $z_0 \in Z$ imply $(y_1, z) \succ (y_0, z)$ for all $z \in Z$, hence (10) holds. If $\succ_{y_1} \in \{\succ_{y_0}, \succ_{y_0}^*, \phi\}$, then

there exist constants a and b such that $u(y_1, z) = a + b u(y_0, z)$ for all $z \in Z$ [6]. By Corollary 1 in Keeney [19], these two results imply

$$u(y, z) = u(y, z_0) + u(y_0, z) + k u(y, z_0)u(y_0, z), \quad (11)$$

for all $y \in Y$ and $z \in Z$, where k is a constant. By Theorem 2.1 in Fishburn [14], the representation in (11) implies $Z(\text{GUI})Y$; therefore, $\Omega_Y \subseteq \{>_0, >_0^*, \phi\}$ for some nonempty preference order $>_0$ on P_Z by earlier definitions. ■

8. Multiattribute representations using joint multivalent utility independence.

This section extends the additive-multiplicative representations in Theorem 1 from two attributes to n attributes. Let $X \equiv X_1 \times \dots \times X_n$ be the outcome space, and let $N \equiv \{1, \dots, n\}$. Let $X_{\bar{1}} \equiv X_1 \times \dots \times X_{i-1} \times X_{i+1} \times \dots \times X_n$ for $i \in N$. With $Y = X_{\bar{1}}$ and $Z = X_i$ for $i \in N$, Lemma 1 gives the following:

$X_{\bar{1}}(\text{UI})[X_i]$ iff $[X_i]$ is a partition of X_i such that for all $\hat{x}_i \in [X_i]$,

$$x_i \in \hat{x}_i \text{ iff } u(x_i, x_{\bar{1}}) = \alpha_i(x_i) + \beta_i(x_i)u(\hat{x}_i, x_{\bar{1}}) \quad (12)$$

for all $x_{\bar{1}} \in X_{\bar{1}}$, where $\hat{x}_i$ is fixed arbitrarily in $\hat{x}_i$, and α_i and β_i are real functions on X_i with $\beta_i > 0$. By definition, *joint multivalent utility independence* holds on X iff $X_{\bar{1}}(\text{UI})[X_i]$ for all $i \in N$.

Let $x^0 = (x_1^0, \dots, x_n^0)$ and $x^1 = (x_1^1, \dots, x_n^1)$ denote distinct outcomes in X . Let $x_{(\bar{1})}^\sigma \equiv (x_1^\sigma, \dots, x_{i-1}^\sigma, x_{i+1}^\sigma, \dots, x_n^\sigma)$ where $\sigma \in \{0, 1\}$, for $i \in N$. Then $x_{(\bar{1})}^1$ is *uniformly preferable* to $x_{(\bar{1})}^0$, written $x_{(\bar{1})}^1 \gg x_{(\bar{1})}^0$, iff $(x_i, x_{(\bar{1})}^1) \succ (x_i, x_{(\bar{1})}^0)$ for all $x_i \in X_i$.

THEOREM 3: Suppose there exist $x^0, x^1 \in X$ such that $x_{(\bar{1})}^1 \gg x_{(\bar{1})}^0$ for all $i \in N$. If $X_{\bar{1}}(\text{UI})[X_i]$ for all $i \in N$, then a von Neumann-Morgenstern utility

function u on X has one of the following representations on each $\hat{X}_1 \times \dots \times \hat{X}_n$,
for $\hat{X}_i \in [X_i]$, $i \in N$:

$$u(x_1, \dots, x_n) = \alpha_1(x_1) + \dots + \alpha_n(x_n) + u(\hat{x}_1, \dots, \hat{x}_n), \quad (13a)$$

$$u(x_1, \dots, x_n) = \alpha_t(x_t) + \beta_t(x_t)u(\hat{x}_t, x_t^-) \text{ for some } t \in N, \quad (13b)$$

$$u(x_1, \dots, x_n) = \hat{k} + \beta_1(x_1) \dots \beta_n(x_n)[u(\hat{x}_1, \dots, \hat{x}_n) - \hat{k}], \quad (13c)$$

where α_i and β_i satisfy

$$\alpha_i(x_i) = \frac{u(x_i, x_{(\bar{i})}^0)u(\hat{x}_i, x_{(\bar{i})}^1) - u(x_i, x_{(\bar{i})}^1)u(\hat{x}_i, x_{(\bar{i})}^0)}{u(\hat{x}_i, x_{(\bar{i})}^1) - u(\hat{x}_i, x_{(\bar{i})}^0)}, \text{ and} \quad (14a)$$

$$\beta_i(x_i) = \frac{u(x_i, x_{(\bar{i})}^1) - u(x_i, x_{(\bar{i})}^0)}{u(\hat{x}_i, x_{(\bar{i})}^1) - u(\hat{x}_i, x_{(\bar{i})}^0)}, \quad (14b)$$

for $x_i \in \hat{X}_i$, where $\hat{k}$ is a constant depending on only the set $\hat{X}_1 \times \dots \times \hat{X}_n$.

Proof: Let $\alpha_i(x_i)$, $\beta_i(x_i)$, $u(x)$, $u(\hat{x}_i, x_{\bar{i}}^-)$, $u(\hat{x}_i, \hat{x}_j, x_{\bar{i}\bar{j}}^-)$ and $u(\hat{x}_1, \dots, \hat{x}_n)$ be represented by α_i , β_i , u , $\hat{u}_i$, $\hat{u}_{ij}$, and $\hat{u}$, respectively. Since $X_{\bar{i}}(UI)[X_i]$ for all $i \in N$, induction on (12) for $i \in N$ gives

$$u = \alpha_1 + \beta_1\alpha_2 + \beta_1\beta_2\alpha_3 + \dots + \beta_1 \dots \beta_{n-1}\alpha_n + \beta_1 \dots \beta_n \hat{u}. \quad (15)$$

Following the proof of Theorem 1 one can show that for any distinct $i, j \in N$,

(12) implies

$$\alpha_i + \beta_i\alpha_j = \alpha_j + \beta_j\alpha_i. \quad (16)$$

The proof divides into three cases:

Case 1 ($\beta_i \equiv 1$ on $\hat{X}_i$ for all $i \in N$): Equation (15) gives (13a) immediately.

Case 2 ($\beta_i \equiv 1$ on $\hat{X}_i$ for all $i \in N$ except $i = t$): If $\beta_i \equiv 1$ and $j = t$ in (16), then $\alpha_i = \beta_t \alpha_i$ for all $i \neq t$. Since $\beta_t \neq 1$ on $\hat{X}_t$, it follows that $\alpha_i \equiv 0$ on $\hat{X}_i$ for all $i \neq t$. Thus (15) yields $u = \alpha_t + \beta_t \hat{u}_t$, which is (13b).

Case 3 ($\beta_i \equiv 1$ on $\hat{X}_i$ for all $i \in N$ except $i \in T$, where $|T| \geq 2$): With no loss in generality, relabel the attributes so $\beta_i \neq 1$ for $i \in \{1, \dots, r\}$ and $\beta_i \equiv 1$ for $i \in \{r+1, \dots, n\}$, where $|T| = r$ and $2 \leq r \leq n$. When $i = 1$ and $j \in \{r+1, \dots, n\}$, (16) yields $\beta_1 \alpha_j = \alpha_j$. Since $\beta_1 \neq 1$, $\alpha_j \equiv 0$ on $\hat{X}_j$ by previous arguments. On the other hand, for distinct $i, j \in \{1, \dots, r\}$, (16) gives $\alpha_i / (1 - \beta_i) = \alpha_j / (1 - \beta_j)$ which equals a constant, say $\hat{k}$, for the reasons following (6). Thus, $\alpha_i \equiv \hat{k}(1 - \beta_i)$ on $\hat{X}_i$ for $i \in \{1, \dots, r\}$ and $\alpha_i \equiv 0$ on $\hat{X}_i$ for $i \in \{r+1, \dots, n\}$. Substituting these results into (15) yields $u = \hat{k}(1 - \beta_1) + \beta_1 \hat{k}(1 - \beta_2) + \dots + \beta_1 \dots \beta_{r-1} \hat{k}(1 - \beta_r) + \beta_1 \dots \beta_r \hat{u}$, which reduces to $u = \hat{k} + \beta_1 \dots \beta_r (\hat{u} - \hat{k})$. Since $\beta_{r+1} \equiv 1, \dots, \beta_n \equiv 1$, an equivalent expression is $u = \hat{k} + \beta_1 \dots \beta_n (\hat{u} - \hat{k})$, which is (13c).

Since $x_{(\bar{i})}^1 \gg x_{(\bar{i})}^0$, the expressions for α_i and β_i in (14a,b) are obtained by solving the equations generated by alternately putting $x_{\bar{i}} = x_{(\bar{i})}^0$ and $x_{\bar{i}} = x_{(\bar{i})}^1$ in (12). ■

The assumption of uniform preferability in Theorem 3 can be deleted in a straightforward manner, but this generalization is not pursued here. We note that the additive-multiplicative representations derived from joint multivalent utility independence and uniform preferability assumptions require

at most two conditional utility functions, $u(x_i, x_{(\bar{i})}^0)$ and $u(x_i, x_{(\bar{i})}^1)$, for each of n attributes. The utilities $u(\hat{x}_1, \dots, \hat{x}_n)$ are needed for consistent scaling, too.

9. Multiattribute representations using correlative multivalent utility

independence. *Individual multivalent utility independence* holds on X iff $X_i(UI)[X_{\bar{i}}]$ for all $i \in N$. Individual independence, however, allows partitions of $X_{\bar{i}}$ that may not correspond to any set of partitions for the single attributes. *Correlative multivalent utility independence* holds on X iff $X_i(UI)[X_{(\bar{i})}]$ for all $i \in N$, where $[X_{(\bar{i})}] \equiv [X_1] \times \dots \times [X_{i-1}] \times [X_{i+1}] \times \dots \times [X_n]$ for $i \in N$, where $[X_1], \dots, [X_n]$ are partitions of the individual attributes. Correlative independence simplifies the derivation and interpretation of multivalent utility representations because a meaningful orbital structure is assumed on each attribute.

In the next theorem, let $(x_i, \hat{x}_{(i)})$ denote $(\hat{x}_1, \dots, \hat{x}_{i-1}, x_i, \hat{x}_{i+1}, \dots, \hat{x}_n)$, where $x_i \in X_i$ and, for $j \neq i$, $\hat{x}_j \in \hat{X}_j$ and $\hat{X}_j \in [X_j]$.

THEOREM 4: Suppose there exist $x_i^0, x_i^1 \in X_i$ such that $x_i^1 \gg x_i^0$ for all $i \in N$. If $X_i(UI)[X_{(\bar{i})}]$ for all $i \in N$, then a von Neumann-Morgenstern utility function u on X has a *multivalent quasi-additive representation* on each $\hat{X}_1 \times \dots \times \hat{X}_n$, where $\hat{X}_i \in [X_i]$ for $i \in N$,

$$u(x_1, \dots, x_n) = \sum \{c_{i_1 \dots i_r} u_{i_1}(x_{i_1}, \hat{x}_{(\bar{i}_1)}) \cdot \dots \cdot u_{i_r}(x_{i_r}, \hat{x}_{(\bar{i}_r)}) :$$

$$1 \leq i_1 < \dots < i_r \leq n, 1 \leq r \leq n\}, \quad (17)$$

where the standard scaling constants $c_{i_1 \dots i_r}$ are defined by

$$c_{i_1 \dots i_r} \equiv \sum \{ (-1)^{r+\sum \sigma_j} u(x_1^{\sigma_1}, \dots, x_n^{\sigma_n}) : \sigma_j \in \{0, 1\} \text{ if } j \in \{i_1, \dots, i_r\}, \sigma_j = 0 \text{ otherwise} \}, \quad (18)$$

and the normalized conditional utility functions $u_i(x_i, \hat{x}_{(-i)})$ on X_i are given by

$$u_i(x_i, \hat{x}_{(-i)}) = \frac{u(x_i, \hat{x}_{(-i)}) - u(x_i^0, \hat{x}_{(-i)})}{u(x_i^1, \hat{x}_{(-i)}) - u(x_i^0, \hat{x}_{(-i)})}. \quad (19)$$

Proof: By Lemma 1, $X_i(\text{UI})[X_{(-i)}]$ for each $i \in N$ implies

$$u(x_1, \dots, x_n) = \alpha_i(x_{-i}) + \beta_i(x_{-i})u(x_i, \hat{x}_{(-i)}), \quad (20)$$

on $X_i \times \hat{X}_{(-i)}$, where $\hat{x}_j$ is a representative of $\hat{X}_j$ for $j \neq i$.

Define constants $\hat{a}_i \equiv -u(x_i^0, \hat{x}_{(-i)}) / (u(x_i^1, \hat{x}_{(-i)}) - u(x_i^0, \hat{x}_{(-i)}))$ and $\hat{b}_i \equiv 1 / (u(x_i^1, \hat{x}_{(-i)}) - u(x_i^0, \hat{x}_{(-i)}))$, and abbreviate the functions $u(x_i^0, x_{-i})$, $u(x_i^1, x_{-i})$ by u_i^0 and u_i^1 , respectively. By alternately putting $x_i = x_i^0$ and $x_i = x_i^1$ in (20) and then solving the resulting pair of equations, we obtain $\alpha_i(x_{-i}) = \hat{a}_i u_i^1 + (1 - \hat{a}_i) u_i^0$ and $\beta_i(x_{-i}) = \hat{b}_i (u_i^1 - u_i^0)$ for $x_{-i} \in \hat{X}_{(-i)}$ and $i \in N$. If we denote $u_i(x_i, \hat{x}_{(-i)})$ by $\hat{u}_i$ in (19), then (20) gives

$$\begin{aligned} u(x_1, \dots, x_n) &= [\hat{a}_i u_i^1 + (1 - \hat{a}_i) u_i^0] + [\hat{b}_i (u_i^1 - u_i^0)] u(x_i, \hat{x}_{(-i)}) \\ &= [\hat{a}_i + \hat{b}_i u(x_i, \hat{x}_{(-i)})] u_i^1 + [1 - (\hat{a}_i + \hat{b}_i u(x_i, \hat{x}_{(-i)}))] u_i^0 \\ &= \hat{u}_i u(x_i^1, x_{-i}) + (1 - \hat{u}_i) u(x_i^0, x_{-i}), \end{aligned} \quad (21)$$

for $x_i \in X_i$, $x_{-i} \in \hat{X}_{(-i)}$, $i \in N$.

We finish the proof by successively substituting (21) into itself for $i = 1, 2, \dots, n$. First, put $i = 1$ in (21) to obtain

$$u(x_1, \dots, x_n) = \hat{u}_1 u(x_1^1, x_2, \dots, x_n) + (1 - \hat{u}_1) u(x_1^0, x_2, \dots, x_n) \quad (22)$$

Then put $i = 2$ in (21) and substitute the result into (22) to get

$$\begin{aligned} u(x_1, \dots, x_n) = & \hat{u}_1 [\hat{u}_2 u(x_1^1, x_2^1, x_3, \dots, x_n) + (1 - \hat{u}_2) u(x_1^1, x_2^0, x_3, \dots, x_n)] \\ & + (1 - \hat{u}_1) [\hat{u}_2 u(x_1^0, x_2^1, x_3, \dots, x_n) + (1 - \hat{u}_2) u(x_1^0, x_2^0, x_3, \dots, x_n)] \\ = & \hat{u}_1 \hat{u}_2 u(x_1^1, x_2^1, x_3, \dots, x_n) + \hat{u}_1 (1 - \hat{u}_2) u(x_1^1, x_2^0, x_3, \dots, x_n) \\ & + (1 - \hat{u}_1) \hat{u}_2 u(x_1^0, x_2^1, x_3, \dots, x_n) \\ & + (1 - \hat{u}_1) (1 - \hat{u}_2) u(x_1^0, x_2^0, x_3, \dots, x_n). \end{aligned} \quad (23)$$

By induction on $i \in N$, we obtain

$$u(x_1, \dots, x_n) = \sum \{ \hat{u}_1^{\sigma_1} \dots \hat{u}_n^{\sigma_n} u(x_1^{\sigma_1}, \dots, x_n^{\sigma_n}) : \sigma_i \in \{0, 1\}, i \in N \}, \quad (24)$$

where $x_i \in \hat{X}_i$, and $\hat{u}_i^1 \equiv \hat{u}_i$, $\hat{u}_i^0 \equiv 1 - \hat{u}_i$, for $i \in N$. The form in (24) is equivalent to the quasi-additive representation in (17) [6, 20]. ■

If the valence of $[X_j]$ is ω_j , then at most $\prod_{j \neq i} \omega_j$ conditional utility functions on attribute X_i for $i \in N$ are needed to assess the quasi-additive representation in (17). On the other hand, the additive-multiplicative

representations in (13) require a total of no more than $2n$ conditional utility functions regardless of the valences.

10. Conclusions. The valence approach differs from previous methods of obtaining decompositions of multiattribute utility functions. By partitioning the elements of each attribute into orbitals on the basis of equivalent conditional preference orders, we obtain utility representations over a collection of subspaces determined by the orbitals. Preference interdependencies are reflected in the orbitals, so attribute interactions are readily interpreted and the functional forms of the representations are kept simple. We illustrate this approach with multivalent utility independence axioms that generate several utility representation theorems. In one case, we use a minor structural assumption and joint multivalent independence to derive a set of additive-multiplicative representations that require at most two conditional utility functions on each attribute. Several other results are also established. Further research on multivalent preference structures with other independence axioms is in [8].

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report presents a valence approach for assessing multiattribute utility functions. The valence approach partitions the elements of each attribute into classes on the basis of equivalent conditional preference orders. These partitions generate the axioms that lead to additive-multiplicative and quasi-additive representation theorems for multi-attribute utility functions defined over product sets of equivalence classes. This approach should help in decisions with interdependent attributes.		

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