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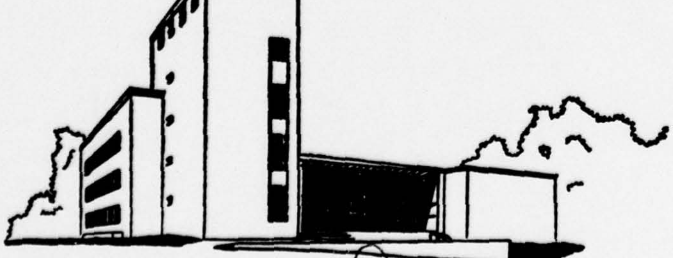

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**Ratio Goal Programming for**

**Personnel Assignments**



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## Abstract

Various optimization techniques have been applied to meet manpower planning and staffing goals. Attainment of these goals is usually accepted as a surrogate for attaining the maximum organizational effectiveness. However, the goals of most of these techniques are exogenous and the way they are determined has been generally ignored. This paper presents a novel formulation of manpower goals which is stated in terms of attaining optimal ratios of personnel desired between the various skills employed in an organization. This framework stresses the importance of the appropriate mixture of skills in the various units of an organization, not just the total number of people assigned. A technique called RATIO GOAL PROGRAMMING is presented and an example illustrates staffing an organization by minimizing a weighting of the squared deviations from the ideal ratios and the unit personnel ceilings.


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The attainment of a high level of organizational effectiveness is the motivation for most of the work done in modeling and evaluating manpower systems. However, most of the quantitative techniques used by operations researchers working in this area treat organizational effectiveness (OE) in a very cursory manner. Optimization or simulation is usually performed using measures which are assumed to be good surrogates for OE, i.e. minimize the deviation between projected on-board personnel and manning goals. Given such a formulation, programs thus devised can be no better than the goals established, and little quantitative work has been performed in evaluation of these goals. It appears to be generally assumed that knowledgeable planners have determined the best set of goals possible.

Prominent researchers from the organizational behavior area continue to debate what are good measures of OE [1],[3],[11]. The debate actually extends much deeper, to the basic question of whether OE can even be objectively evaluated or not [2]. Most researchers in this area agree that a single measure of OE is not sufficient and that multiple objectives must be used in the evaluation of OE. The spectrum as to what objectives should be used ranges from the attainment of specific goals (the "goal centered approach") to the survival of the organization (the "natural system approach")[11]. Even if a set of objectives could be selected, Goodman and Pennings [6] and others [9] maintain that various different groups of people interested in the organization, "constituencies" in their terminology, have quite different sets of weights for these objectives.

Researchers working with manpower models have long been faced with the dilemma of focusing on either distribution or utility maximization models. The distribution models are concerned primarily with filling quotas, and, in some cases, distributing manpower shortages between various units. Thus, they examine the "fill" of units. The utility maximization models are concerned with filling a set of jobs with individuals whose attributes best "fit" the attributes desired for the job. This is usually evaluated by summing individual job-man utility scores. Thus, we have "fit" and "fill" objectives, which unfortunately often pull in different directions.

Various techniques have been modified to attempt to satisfy both "fit" and "fill" requirements. Goal programming formulations [5],[13] have been attempted, using various weighting schemes to try and model the subjective value of the "fill" in different skills and different units. However, most of these goal programming models minimize the sum of the deviations from the various manning goals, where each goal is of the form "have x people in this category by year t", and no consideration is made of the actual assignments in other categories.

Others faced with the same problem have suggested the use of "successive surface optimization" [7],[8]. This is a technique of optimizing the model with respect to one objective function, such as maximize fill, then reoptimizing with another objective function, such as maximize fit. The value of the original objective function, relaxed by a small amount, is treated as a constraint in subsequent optimizations. This technique of considering conflicting objectives is a different approach, but the final solution clearly depends upon the order of the successive optimizations.

All of these models assume that when personnel manning goals are attained, organizational goals will also be attained. Often entire organizations are modeled as one macro unit, due to the time-cost or inability of higher level managers to set manning goals for the various units. This results in ignoring information on the varying demands and performances of the units and their directors or commanders. In addition, the marginal contribution of adding an additional person, if evaluated at all, is generally stated in system terms, i.e. "one unit less deviation from x manning goal", rather than as the actual impact of the incremental employee on the specific unit he is assigned to. The impact of having an additional employee with a given skill can be quite different on two different units.

Many who have approached organizational effectiveness by focusing on the individual job-man match of top executives have stressed the importance of an appropriate mix of skills and abilities in a group of managers who will work together. This paper proposes extending this line of thought to the organizational level by considering the appropriate mix of skills needed by the various units of the total organization. This skill mix is allowed to vary between units as appropriate. Rather than have higher level managers attempt to specify the desired mix, it is proposed that this mix should be obtained from unit managers, within constraints supplied by the larger unit manager.

A technique called RATIO GOAL PROGRAMMING is presented here that assigns personnel so as to come as close to these ratios as possible, considering the unit personnel ceilings. The underlying assumption is that if the desired ratio is attained, the unit will attain the maximum possible effectiveness, for a given number of individuals available to be assigned. Also it is implicitly assumed that the total organizational effectiveness can be accurately represented as the sum of the individual units' effectiveness. This technique does not consider the capital/labor ratio, nor problems of leadership and motivation, which obviously have a strong influence on unit effectiveness.

Section A presents a division of the problems of manpower planning which are then analyzed in the following sections. Section B discusses the skill ratios and how to obtain them and Section C illustrates the use of these ratios to determine an ideal inventory of personnel. Section D then discusses the use of a similar formulation for assigning an on-board inventory. The techniques of sections C and D are illustrated by following the solution of a sample problem. In Section E we describe various extensions and changes in the formulation of the model.

## Section A. Division of the Generalized Manpower Planning Problem

This section presents a simple division of the tasks necessary to effectively plan and operationally manage any manpower system. The divisions are not meant to imply any strict stepwise sequence, although initially they are probably performed in the order presented here, particularly in organizations where a production process cannot be specified explicitly (i.e. unit outputs are difficult to measure).

1. Examine the objectives of the organization and determine the total number of personnel and how many individuals are needed in each skill in each of the various units. (Remember, the amount of capital is assumed to be fixed.)

2. Allocate on-board personnel to these units based on their current skills.

3. Retrain on-board personnel having skills in excess of unit requirements to come closer to desired skill mixes in the various units.

4. Hire and train new employees for unfilled positions (present or anticipated).

5. Analyze alternative unit sizes, skill mixes and categories of new employees, considering the organization's effectiveness and the cost of the workforce.

Here it is assumed that each individual possesses one identifiable skill, and that skill mix can be modeled by the mixture of people having appropriate job classifications. The proficiency of all employees having the same skill is assumed equal.

Points 1 and 2 from above will be discussed in this paper, while points 3 and 4 will be discussed in a subsequent paper [14]. Unfortunately the first part of step one, determining the total number of personnel in the entire organization, is not approached directly, because the relation between the number of personnel and organizational effectiveness is confounded by the capital/labor ratio, leadership, motivation and other organizational attributes. Instead, the technique presented here accepts the total number of personnel in the entire organization and unit maximum sizes as given exogenous constraints, possibly set by a governing body or higher manager.

## Section B. Skill Ratios

The crux of the technique presented in this paper is the skill ratios developed within the various units. These ratios are the relative numbers of auxiliary personnel needed to support actual production workers (or front-line personnel in a military context). The ratios can be expected to vary not only between units having different missions, i.e. an infantry company versus a mechanized infantry company, but also between units with similar missions but having different environments, i.e. an infantry company operating independently versus a company participating in a joint maneuver.

Rather than expecting the director of a larger division to be able to specify the appropriate skill mix for all of the units he controls, it is proposed that these ratios be collected from the managers of the various units, or inferred from the past actions of these managers. They are in closer contact with the individuals involved and it is felt their proximity to daily operations enhances their ability to analyze skill trade-offs. A brief discussion of some methods of collecting these ratios follows, to illustrate the feasibility of collecting the data needed by the models of sections C and D. An initial division of these techniques can be made on whether the determination of the ratios is to focus on <u>current</u> unit managers or the actions of <u>previous</u> managers.

Two methods could be used to obtain the ratios from current unit commanders. They could be directly asked questions such as: "What is the desired ratio of medics to infantrymen in your company". Or hypothetical cuts in the unit manpower ceiling could be posed to these commanders and the skill ratios then computed, based on the particular skill(s) the unit commanders decided to reduce to remain within these new ceilings.

Insights as to what the appropriate ratios should be could also be obtained by evaluating historical actions. Forced "mismatches" actually imposed by unit commanders in the past would give some indication of what the desired ratios are. By a mismatch we mean incidents where an individual of one skill category was assigned to a job requiring a different skill, either due to an excess (as determined by the unit commander) number of personnel in his original skill or because of an evaluation of the relative importance of two positions to the unit when both his original and the new skill have vacancies, and only one person is available to fill them.

Instances in the past when manpower reductions were actually made would also yield some information. Examination of the skill categories that were reduced might give some indication of the appropriate range for these ratios.

Base skill

The ratios discussed here are all related to a "base skill". This does not necessarily imply any hierarchy of importance between the base skill and other skills; the base skill is chosen simply as a convenient skill to relate ratios to. Note that n-1 two-skill ratios (i.e. medics to infantrymen, not medics to the sum of infantrymen and logistics personnel) would uniquely identify the structure of a unit with n different skills.[1] This poses two problems. If n ratios were given, it is probable that

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[1] n-1 linearly independent ratios are all that are needed, because the n-1 ratios can be expressed as:

$$\text{ratio of } i \text{ to } b = \frac{\text{fraction of total with skill } i}{\text{fraction of total with base skill}}$$

and the nth equation then is: $\Sigma$ all fractions = 1 they would be mathematically inconsistent, i.e. using n-1 ratios to compute the nth ratio would yield a different value than that furnished by the commander. Attempts to resolve this inconsistency by again querying the unit commander may result in a different inconsistency.[1] The second problem is related to the practicality of specifying an exact ratio for two skills. Although it is still proposed to ask the unit commander for a desired ratio, a permissible range for this ratio will also be requested. The use of a range is more realistic and would probably be more readily responded to by these managers. Responses of the form "the ratio of medics to infantrymen should be from 1:5 to 1:12 with 1:9 being ideal" could then be solicited and utilized in the computations of sections C and D.

Note that even though these ratios are obtained from unit commanders, their superiors may impose constraints on the range of the ratios. For example, probably few infantry commanders would object to a medic/infantry ratio of 1:2 (provided they still received a full complement of infantrymen). However, such a ratio in one unit would probably necessitate shorting medical support in another unit. Thus, higher commanders may specify constraints such as "no unit may have a medic/infantry ratio exceeding 1:4". Such constraints would work not only toward equity in support (and equity in workload of the support personnel), but also possibly toward other goals, such as

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[1] These inconsistencies may be indicators of the quality of the data gathered.

maximizing the number of infantrymen, given the larger unit has constraints on the total number of personnel assigned.

Just as many others have discussed the value of modeling even when the models are not subsequently implemented, the value of specifying the skill ratios extends beyond their use (or non-use). Probably few unit commanders have examined their personnel structure in such a context before and the exercise seems well worth the mental effort involved. Likewise, probably few major unit commanders realize the weighting their subordinate managers place on the various skill categories and such information may be of value in assessing not only the goal congruency of these lower managers, but also may give insights on workload, morale and flexibility.

This discussion of methods of obtaining skill ratios has been presented for descriptive reasons, to show how one could gather such data. The major purpose of the rest of this paper is to present the methodology of using these ratios to staff an organization, once the ratios have been collected.

## Section C. Distant Planning

This section discusses a distant planning model, distant planning in the sense that an on-board inventory of personnel is not considered. The goal of the model is to determine an ideal inventory and the allocation of the total number of personnel among skill specialties and units. The distribution among specialties is determined by using the desired skill ratios for the units and then computing the total system-wide number of each skill category needed, given the sizes of the various units. These sizes are input variables, based on the system managers' evaluations of the expected scenarios and the relevant strategies. Output from this model is the number of personnel of the various skill categories needed to attain the desired skill ratios, when all the units are staffed at their maximum strengths.

Consider a large organization with M subordinate units and I skill categories. The base skill category, the category that appears in the denominator of all the skill ratios, will be designated by subscript b, where $b \in I$.

Let:

$x_{im}$ = number of personnel in skill category i in unit m

$x_{bm}$ = number of personnel in base skill in unit m

$y_m$ = maximum personnel strength for unit m

$d_{im}$ = desired ratio of personnel in skill category i to personnel in skill category b in unit m

$f_{im}$ = fraction of unit m personnel ceiling ($y_m$) filled by skill i personnel at the desired ratio ($d_{im}$)

Thus, given we are constrained by the total number of personnel to be allocated to unit m, $y_m$, we can form the system:

$$\sum_i x_{im} = y_m \qquad \text{(the unit will be fully manned)} \qquad (1)$$

$$x_{im}/x_{bm} = d_{im} \qquad \text{(the desired ratios will be attained)} \qquad (2)$$

$$x_{im} = f_{im}\, y_m \qquad \forall\, i \text{ and } \forall\, m \qquad (3)$$

$$\sum_i f_{im} = 1 \qquad \forall\, m \qquad (4)$$

In order to perform the summation across units to obtain overall skill requirements as a function of unit size, we need to identify the fraction of each unit total strength that is assigned to each skill. From (2) and (3) we can write:

$$d_{im} = \frac{f_{im}}{f_{bm}} \qquad \forall\, i \quad \text{and} \quad \forall\, m \qquad (5)$$

If we denote the total number assigned to skill i as S , we can then write:

$$S_i = \sum_m f_{im}\, y_m \qquad (6)$$

The system planners have provided $y_m$ (the unit manning levels) and they are interested in the subsequent distribution of $S_i$ (the total number needed in each skill category for the entire organization).

To obtain $S_i$ , the values of $f_{im}$ must be determined. Note the desired ratios, $d_{im}$, yield i - 1 equations of the form of

(5), and (4) furnishes the $i^{th}$ equation needed to uniquely determine the fractions $f_{im}$ . Equation (6) can then be used to solve for the $S_i$ .

A brief example with 3 skills and 2 units follows:

External information:

desired ratios:

| unit: skill | 1 | 2 |
|---|---|---|
| 1 | .250 | .500 |
| 2 | .333 | .250 |
| 3* | 1.000 | 1.000 |

* skill 3 is the base skill and its ratios are 1.0 by definition.

unit ceilings:

| unit: | 1 | 2 |
|---|---|---|
| ceiling: | 200 | 200 |

Solve for $f_{im}$:

for unit 1:

$$\left.\begin{aligned} f_{11}/f_{31} &= d_{11} = .250 \\ f_{21}/f_{31} &= d_{21} = .333 \\ f_{11} + f_{21} + f_{31} &= 1 \end{aligned}\right\} \quad \begin{aligned} f_{11} &= .158 \\ f_{21} &= .210 \\ f_{31} &= .632 \end{aligned}$$

for unit 2:

$$\left.\begin{aligned} f_{12}/f_{32} &= d_{12} = .500 \\ f_{22}/f_{32} &= d_{22} = .250 \\ f_{12} + f_{22} + f_{32} &= 1 \end{aligned}\right\} \quad \begin{aligned} f_{12} &= .286 \\ f_{22} &= .143 \\ f_{32} &= .571 \end{aligned}$$

Determine the distribution of skills needed from $S_i = \sum_m f_{im} y_m$ :

$$S_1 = .158(200) + .286(200) = 88.7$$

$$S_2 = .210(200) + .143(200) = 70.7$$

$$S_3 = .632(200) + .143(200) = 240.6$$

Thus, the ideal assignment would be: $(x_{im} = f_{im}y_m)$

| unit: skill | 1 | 2 | total |
|---|---|---|---|
| 1 | 31.6 | 57.1 | 88.7 |
| 2 | 42.1 | 28.6 | 70.7 |
| 3 | 126.3 | 114.3 | 240.6 |
| total | 200.0 | 200.0 | |

Note this ideal assignment yields exactly the desired ratios. Because this model is to be used for planning purposes, fractional values for assignments are acceptable. Any differences in ratios that would result from rounding to integer increments are far overshadowed by the inherent vagueness in the specification of the desired ratios.

To summarize the distant planning model, this section has presented a technique where, given that desired skill ratios have been determined, system planners can focus on what level to man various units at, instead of focusing on the number of people needed in each skill category. The latter focus is often misleading, because the incremental value of one person of a particular skill varies with the unit he is assigned to. The focus of this model is more realistic for the military manpower system, as the overall manning constraint (the sum of all unit strengths) is generally set by Congress. The system planners' attention can then be directed toward weighing the contribution of various units toward defense (i.e. setting unit manning levels), and letting the model determine the ideal skill requirements.

## Section D. An Operational Algorithm

This section describes an algorithm which applies the ratio-effectiveness concept in an operational sense, considering an on-board inventory of skilled personnel. Retraining and new entries to the system are not considered in this formulation.

Redefining $S_i$ as the total number of on-board personnel in skill category i (which may or may not be equal to the ideal numbers derived in the preceding section), and introducing $\ell_{im}$ and $h_{im}$, the least acceptable and highest acceptable ratios of skill i to the base skill b in unit m, the formal model can be stated as:

$$\min \beta \sum_i \sum_m \left(\frac{x_{im}}{x_{bm}} - d_{im}\right)^2 + (1 - \beta) \sum_m \left(\sum_i x_{im} - y_m\right)^2 \qquad (7)$$

subject to:

$$\sum_i x_{im} \leq y_m \qquad \forall m \qquad (8)$$

$$\sum_m x_{im} \leq S_i \qquad \forall i \qquad (9)$$

$$\ell_{im} \leq x_{im}/x_{bm} \leq h_{im} \qquad \forall i \text{ and } \forall m \qquad (10)$$

$$x_{im} \geq 0 \qquad (11)$$

$$x_{bm} > 0 \qquad (12)$$

The objective function (7) is to minimize a weighted sum of the squared deviation from the ideal skill ratios and the squared deviation from the unit ceilings. This is both a "fit" and "fill" maximizing function, where fit is defined as attaining the desired ratios and fill is defined as attaining the unit personnel ceilings. The coefficient $\beta$, satisfying $0 \leq \beta \leq 1$, is specified by the user and represents the relative importance of the fit and fill objectives. A quadratic form was chosen so as to penalize large deviations greater than a series of small deviations.

Constraint set (8) assures that the exogenously determined unit strengths are not violated. Set (9) constrains the model to assign only individuals currently in the inventory, and constraint set (10) assures that the final solution has actual skill ratios that fall between the least acceptable and highest acceptable values for all units and all support skills. Constraints (11) and (12) impose the nonnegativity requirements.

Noting that $x_{im}$ and $x_{bm}$ are the decision variables, the model is not linear, nor are the first order conditions linear. Thus a nonlinear pattern search technique originally proposed by Hooke and Jeeves [10] was used to solve the problem. As this technique is designed for problems which are unconstrained except for upper and lower bounds on the variables, the following penalty value model was formed from (7) - (12):

$$\min \beta \sum_i \sum_m \left(\frac{x_{im}}{x_{bm}} - d_{im}\right)^2 + (1 - \beta) \sum_m \left(\sum_i x_{im} - y_m\right)^2 \quad (13)$$

$$+ \sum_m P_1 \max \{0, (\sum_i x_{im} - y_m)\} \quad (14)$$

$$+ \sum_m P_2 \max \{0, (\sum_m x_{im} - S_i)\} \quad (15)$$

$$+ \sum_i \sum_m P_3 \max \{0, (\ell_{im} - x_{im}/x_{bm})\} \quad (16)$$

$$+ \sum_i \sum_m P_4 \max \{0, (x_{im}/x_{bm} - h_{im})\} \quad (17)$$

subject to: $x_{im} \geq 0$
$x_{bm} > 0$

In this reformulation, the constraints (8)-(10) of the ratio goal program (RGP) model are enforced by imposing penalties, $P_j$ , whenever a candidate solution point would violate one of these constraints. Dividing the objective function into parts, it can be seen that (14) corresponds to (8), the constraints on the overall number assigned to unit m ; (15) corresponds to (9), the inventory constraints; and (16) and (17) correspond to (10), and enforce the upper and lower bounds on the ratios. If sufficiently large values are chosen for the penalties, then the optimum value of (13)-(17) will correspond to the optimum of the original problem (7)-(12).

In [10], Hooke and Jeeves clearly state that they have not found generalizable sufficient conditions to guarantee optimality of their pattern search algorithm, although it has proven to be a reliable and robust technique. Thus, this technique has been compared with other formulations and approximations of the RGP model in [15] and the pattern search technique has achieved sufficient accuracy in all comparisons to date.

It is well known that the execution times of most, if not all, solution techniques are highly dependent upon having a good initial point to start from. Thus, the following heuristic procedure was developed to determine an initial feasible solution for the original RGP problem (7)-(12). Essentially it can be viewed as finding a "scaled down" solution of the ideal assignments determined in Section C. This scaled down solution has been scaled so that none of the inventory constraints are violated. The following steps can be used to determine the overall scale factor, $\gamma$:

a) Find the quantities $\alpha_i$ (scale factors for each skill i) such that:

$$\sum_m d_{im} \alpha_i ( f_{bm} y_m ) = S_i \qquad (18)$$

b) Choose $\gamma$, the overall scale factor, by:

$$\gamma = \min_i \{ \alpha_i , 1.0 \} \qquad (19)$$

Note that the quantity $f_{bm} y_m$ in (18) is actually the number of base skill personnel, $x_{bm}$, that would be assigned if no inventory constraints were binding. Thus, $\alpha_i$ is the factor that the inventory in skill i constrains the base skill assignment by. $\gamma$ is then the minimum of all the scale factors, or 1.0 if $\alpha_i > 1.0 \; \forall i$. (When $\alpha_i > 1.0$ , then $\sum_m f_{im} y_m < S_i$ and the inventory in skill i is not binding for an ideal assignment.)

After determining $\gamma$, an initial feasible solution can be easily obtained by:

c) Let $x_{bm} = \gamma f_{bm} y_m \quad \forall m$

d) Let $x_{im} = d_{im} x_{bm} \quad \forall m$ and $\forall i | i \in I \backslash \{b\}$

When using this heuristic to obtain a starting point, the pattern search technique was found to converge quickly.

Four conditions may exist at termination, categorized by either meeting or failing to meet maximum unit strengths:

1. Unit strengths are attained and all $d_{im}$ are met (the ideal solution). Any inventory then left unassigned is truly excess.

2. Unit strengths are attained and some $d_{im}$ are not met. This indicates an imbalance in the skills available in the inventory and retraining or hiring is required if this imbalance is to be corrected.

3. Unit strengths are not attained and some inventory is unassigned. This occurs when the final solution is constrained by the lower or upper bounds on the ratios ($\ell_{im}$ or $h_{im}$). Again an imbalance in the skills available in the inventory is indicated and retraining or new hires are called for.

4. Unit strengths are not attained and no inventory is left. This case indicates insufficient inventory in one or more skill categories. The hiring of additional personnel is then called for.

An Example

The following example is based on the same data as the example in section C.

External information:

On-board inventory:

| skill | 1 | 2 | 3 |
|---|---|---|---|
| inventory (S ) | 130 | 50 | 235 |
| ideal assignment (from section C.) | 88.7 | 70.6 | 240.6 |

unit ceilings:

| unit: | 1 | 2 |
|---|---|---|
| ceiling: | 200 | 200 |

Since the ideal assignment is infeasible (insufficient inventory exists in skills 2 and 3), compute weighting factors ($\alpha_i$) for each skill from (18), resulting in:

$$\alpha_1 = 1.465 \qquad \alpha_2 = .708 \qquad \alpha_3 = .977$$

Then determine $\gamma = \min\{\alpha_i, 1.0\} = .708$ and perform steps c and d to yield an initial feasible point with the following inventory-constrained assignments:

| unit: | 1 | 2 | total |
|---|---|---|---|
| skill | | | |
| 1 | 22.4 | 40.4 | 62.8 |
| 2 | 29.8 | 20.2 | 50.0 |
| 3 | 89.4 | 80.9 | 170.3 |
| total | 141.6 | 141.5 | |

These assignments are ideal in the sense of max "fit" (they yield 0 deviation from the desired ratios), but are not in the sense of "fill", as the units are considerably below their maximum manning levels.

The pattern search technique is then applied, using these assignments as a starting point. Upon solution the final assignments are then (the inventory is again shown for comparison):

| unit: | 1 | 2 | total | inventory |
|---|---|---|---|---|
| skill | | | | |
| 1 | 41.6 | 66.1 | 107.7 | 130 |
| 2 | 30.1 | 19.9 | 50.0 | 50 |
| 3 | 124.9 | 110.1 | 235.0 | 235 |
| total | 196.6 | 196.1 | | |

and the lower, actual, upper and ideal ratios are:

| skill | unit | $\ell_{im}$ | actual | $h_{im}$ | $d_{im}$ |
|---|---|---|---|---|---|
| 1 | 1 | .167 | .333 | .333 | 250 |
| 1 | 2 | .400 | .600 | .600 | .500 |
| 2 | 1 | .200 | .241 | .500 | .333 |
| 2 | 2 | .167 | .181 | .333 | .250 |

The final solution is an assignment that reflects the relative weighting of fit and fill as specified by the $\beta$ and $(1-\beta)$ coefficients. (An equal weighting was used in the above example, i.e. $\beta = .5$) The solution can be visually checked for feasibility by noting that all the actual ratios lie between the lower and upper bounds ( $\ell_{im}$ and $h_{im}$). Note that the actual ratios for skill 1 in both units are at their upper bounds. These ratios could be reduced by either assigning fewer individuals in skill 1 to both units or assigning more individuals in the base skill, skill 3. However, assigning fewer individuals is counter to the "fill" portion of the objective function (as neither unit is completely staffed) and the assignment of more base skill personnel is blocked by the inventory constraint on the base skill. (All the base skill personnel are already assigned.) The ratios for skill 2 in both units are below their desired values, but this is due to the fact that the inventory constraint is also binding on this skill.

Finally, note that neither unit is completely staffed, even though unassigned inventory exists in skill 1. This is because assigning any more skill 1 individuals to either unit would cause the skill 1 ratios to exceed their respective upper bounds.

## Section E. Extensions and Reformulations of the Model

Several extensions and reformulations of the ratio goal programming (RGP) model are possible. Kornbluth [12] considered the problem

$$\min \sum_{i=1}^{m} \left| \frac{c_i^T x + \alpha_i}{\alpha_i x + \beta_i} - \delta_i \right|$$

Noting that $\alpha$, $\beta$ and $\delta$ are constants (for each i), the analogy to a ratio and its goals is obvious. Although this exact formulation also results in a nonlinear problem, Charnes and Cooper [4] suggest substituting the Chebychev metric and show how it can be solved by access to linear programming developments. This LP alternative may be quite valuable, as we have not yet applied the nonlinear formulation presented in (13)-(17) to large scale problems.

Several points should also be mentioned about the objective function presented in (7), repeated below:

$$\min \beta \sum_i \sum_m \left( \frac{x_{im}}{x_{bm}} - d_{im} \right)^2 + (1 - \beta) \sum_m \left(\sum_i x_{im} - y_m\right)^2 \qquad (7)$$

As the desired ratios are specified with respect to an arbitrarily selected base skill, some of the ratios may be greater than one and some may be less than one. The quadratic nature of the objective function may then produce an undesirable weighting among the different skills. This could possibly be resolved by pre-processing the desired ratios to yield ratios and goals that are all less than one.

In addition, a similar problem will arise due to the different $y_m$ values. The "fill" of units with large personnel ceilings, and consequently large initial deviations in the second term of (7), may contribute more to the objective function value than was intended by the specified $\beta$ and $(1 - \beta)$ values. Thus, consideration should be given to either specifying fit versus fill trade-offs for each unit, i.e. determining a $\beta_m$ for each unit, or reformulating the second term of (7) into

$$(1 - \beta) \sum_m \left( \frac{\sum_i x_{im}}{y_m} - 1 \right)^2$$

where the fill ratio would now always be $\leq 1$, more in line with the fit ratios.

One other reformulation of the models may be of value. Constraint (8) of the initial formulation constrained the solution to not exceed the unit ceilings, $y_m$ . In many realistic cases, all the personnel available must be assigned. If this is so, then constraint (8) could be dropped from the problem, permitting "over manning", and constraint (9) should be changed to a strict equality, forcing the model to assign all the individuals in the inventory. The corresponding changes in the penalty value formulation are to let $P_1$, the penalty for overmanning a unit, equal zero and to add

$$\sum_i P_5 \ (S_i - \sum_m x_{im})$$

to the objective function. It may occur that it is then impossible to guarantee that the upper and lower bounds are not violated, but an appropriate choice of the penalties $P_3$ and $P_4$ in (16) and (17) will still yield solutions that have violated these bounds in the "least possible" manner.

Whether or not these reformulations are appropriate depends, of course, on the specific application at hand. The model as presented in (7)-(12) is easily extended to "hardware-driven" situations, where major crew-serviced machines determines the skill ratios.

The logical extensions are to next include retraining of the current inventory, hiring of additional employees and optimal "drawdowns" if strength reductions are required. These extensions are covered in a following paper [14].

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Various optimization techniques have been applied to meet manpower planning and staffing goals. Attainment of these goals is usually accepted as a surrogate for attaining the maximum organizational effectiveness. However, the goals of most of these techniques are exogenous and the way they are determined has been generally ignored. This paper presents a novel formulation of manpower goals which is stated in terms of attaining optimal ratios of personnel desired between the various skills employed in an organization. (cont'd)

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This framework stresses the importance of the appropriate mixture of skills in the various units of an organization, not just the total number of people assigned. A technique called RATIO GOAL PROGRAMMING is presented and an example illustrates staffing an organizations by minimizing a weighting of the squared deviations from the ideal ratios and the unit personnel ceilings.