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SIZE EFFECT ON NAVIGATION USING A STRAPDOWN IMU

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Size effects in a strapdown IMU (Inertial Measurement Unit) are not always ignorable. They should be considered in the error budget for high accuracy inertial navigation. This report presents results of a study on size effects. It is found that while each individual accelerometer has only one type of size effect, an IMU has two in general, the "mounting offset size effect" and the "cluster size effect".

General formulas for determining IMU size effect errors are developed in the report. Numerical examples, laboratory test results, and flight simulation

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results are given to demonstrate the size effects and to reveal their effect on navigation error.

It is discussed that an optimum orientation for the accelerometer cluster can be chosen to minimize the navigation errors caused by size effects. It is also shown that, in principle, size effect of an IMU can be compensated using the angular rate information available from gyros. A size effect compensation concept is proposed with the associated computation algorithm given.

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1. INTRODUCTION

For an inertially navigated vehicle using a strapdown inertial measurement unit (IMU), the ideal location for the IMU's accelerometers is the vehicle's center of rotation. Deviation from this ideal situation will lead to acceleration errors caused by unwanted centripetal and tangential accelerations when the vehicle is under rotation. This phenomenon has been called "size effect."[1] To satisfy the ideal situation, two conditions must be met. First the IMU must be mounted at the vehicle's center of rotation. Second, the size of the IMU must be zero, meaning a "point IMU." The first condition can physically be met, though it is not always practical. The second condition cannot be met physically. Therefore, navigation error due to size effect always exists when an IMU is used. It should be pointed out that a gimbaled IMU also exhibits size effect, but the effect is usually not significant enough to warrant special attention.

Although the size effect of an IMU is the combined size effects of individual accelerometers, the seriousness of accelerometer size effect does not necessarily reflect the seriousness of the associated IMU size effect. Since the navigation error is usually the final concern, it is more appropriate to consider the size effect from the view point of an IMU system.

This report presents the results of a study of the size effect on an inertial navigator using a strapdown IMU. The contents of the report include an error analysis for size effect, results of testing and simulation, optimum orientation of the accelerometer cluster for minimum size effect, a proposed size effect compensation concept, and concluding remarks. Comparison to size effect in a gimbaled IMU will also be made at various places in the analysis.

Three assumptions are made. First, ideal computation is assumed, meaning no software-induced error. The result of size effect on computation error will be discussed in Section 5. Second, the IMU is assumed to possess an orthogonal accelerometer triad, with three accelerometers designated U, V and W. When any one of the three is referred to, the designation K will be used. Third, the position of a vehicle will be considered as the position of its center of rotation.

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1 M. Fernandez and G. R. Macomber, *Inertial Guidance Engineering*, Prentice-Hall, Inc., New Jersey, 1962, pp 510-511

2. SIZE EFFECT ERROR ANALYSIS

A. SIZE EFFECT ERROR OF AN ACCELEROMETER

Figure 1 shows two coordinate frames, the fixed frame (X,Y, Z) and the moving frame (x,y,z). P is a point fixed with respect to the moving frame. $\vec{R}_o$, $\vec{R}_K$ and $\vec{r}_K$ are position vectors. The angular velocity of the moving frame with respect to the fixed frame is represented by $\vec{\omega}$. Using the vector analysis notation the acceleration of the point P with respect to the fixed frame is given by

$$a = \frac{d^2\vec{R}_o}{dt^2} + \left[\left(\frac{d}{dt}\right)_m + \vec{\omega}\ X\right]\vec{v}_m + \frac{d\vec{\omega}}{dt}\ X\ \vec{r}_K$$

$$+ \vec{\omega}\ X\left[\left(\frac{d}{dt}\right)_m + \vec{\omega}\ X\right]\vec{r}_K$$

$$= \frac{d^2R_o}{dt^2} + \vec{a}_m + 2\vec{\omega}\ X\ \vec{v}_m + \dot{\vec{\omega}}\ X\ \vec{r}_K + \vec{\omega}\ X\ \vec{\omega}\ X\ \vec{r}_K \quad (1)$$

where the subscript m denotes the relativeness with respect to the moving frame and v represents velocity[2].

For a strapdown IMU, the origin Q of the XYZ-frame is a fixed point in inertial space, the origin 0 is the vehicle's center of rotation, point P is the location of the IMU's K-accelerometer, and $\vec{\omega}$ is the vehicle's angular velocity. Since the accelerometer is strapped down on the vehicle body, a_m and v_m are zero. The acceleration along the accelerometer's input axis is the dot product of Equation (1) and the unit vector U_K along the input axis of the accelerometer,

$$\vec{a}_{KI} = \vec{a}_K \cdot \vec{u}_K\ ,\ K = U,\ V,\ W \quad . \quad (2)$$

Without size effect, r_K is zero, so the desired acceleration sensed by the accelerometer is

$$a_{KI} = \frac{d^2\vec{R}_o}{dt^2} \cdot \vec{u}_K\ ,\ K = U,\ V,\ W\ . \quad (3)$$

The error in the output of the accelerometer due to size effect is

$$\boxed{\begin{gathered}\varepsilon_K = (\dot{\vec{\omega}}\ X\ r_K + \vec{\omega}\ X\ \omega\ X\ \vec{r}_K) \cdot \vec{u}_K\ . \\ K = U,\ V,\ W\end{gathered}} \quad (4)$$

2 D T. Greenwood, *Principles of Dynamics*, Prentice-Hall, Inc., New Jersey, 1965, pp 49-51

It is recognized that the first term of Equation (4) represents the effect of tangential acceleration while the second term represents the effect of centripetal acceleration. Equation (4) shows that the size effect error for each individual accelerometer is proportional to the distance between the accelerometer and the axis of rotation.

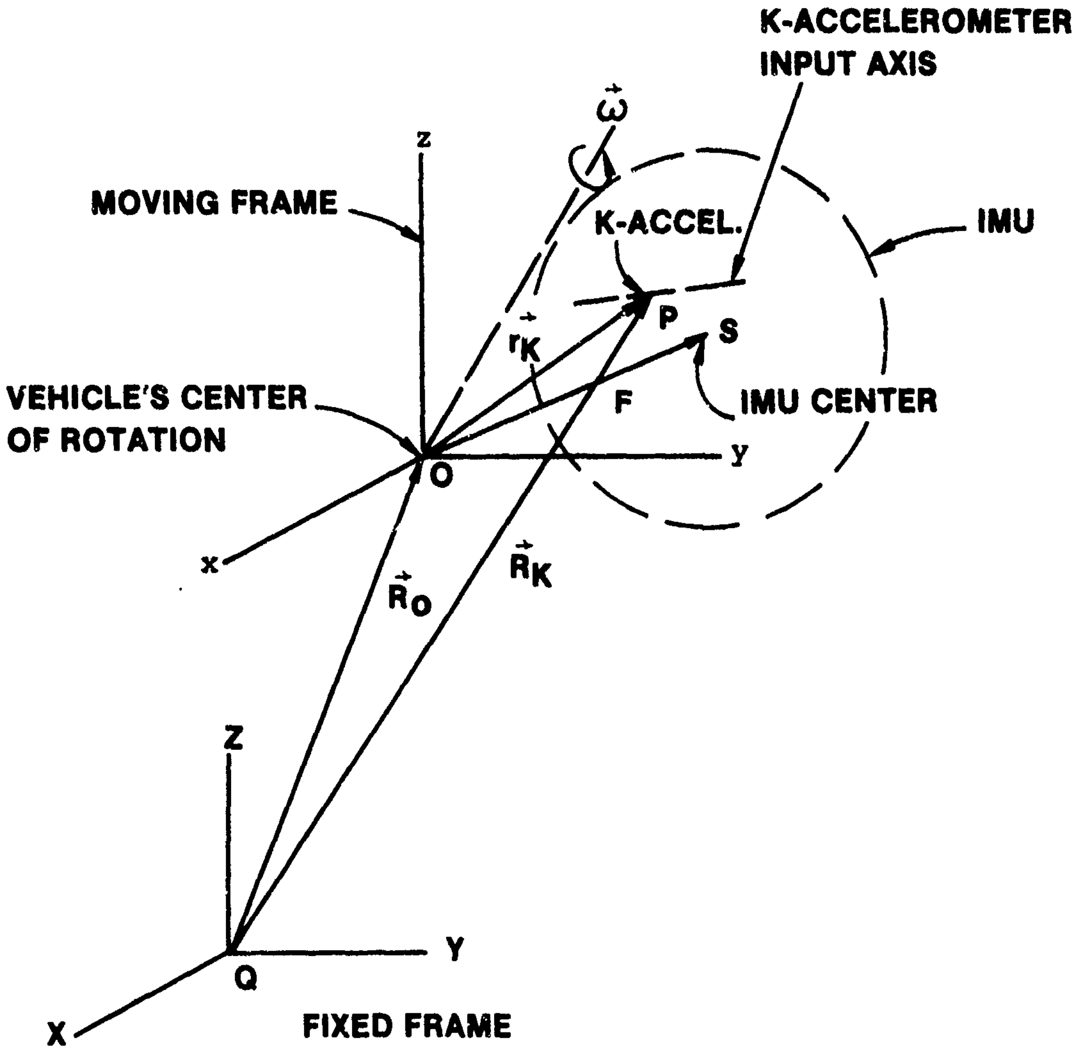


Figure 1. Geometry for size effect analysis.

A rotational oscillation test was performed on an accelerometer to show the size effect of one individual accelerometer. The main test equipment is a oscillation table as sketched in *Figure 2.* An accelerometer is fastened to a mounting block which, in turn, is mounted on the table. The accelerometer's input axis is at an angle of 45 deg from the table's axis of rotation, and its center of percussion is at a distance of 1 in. from the table axis.

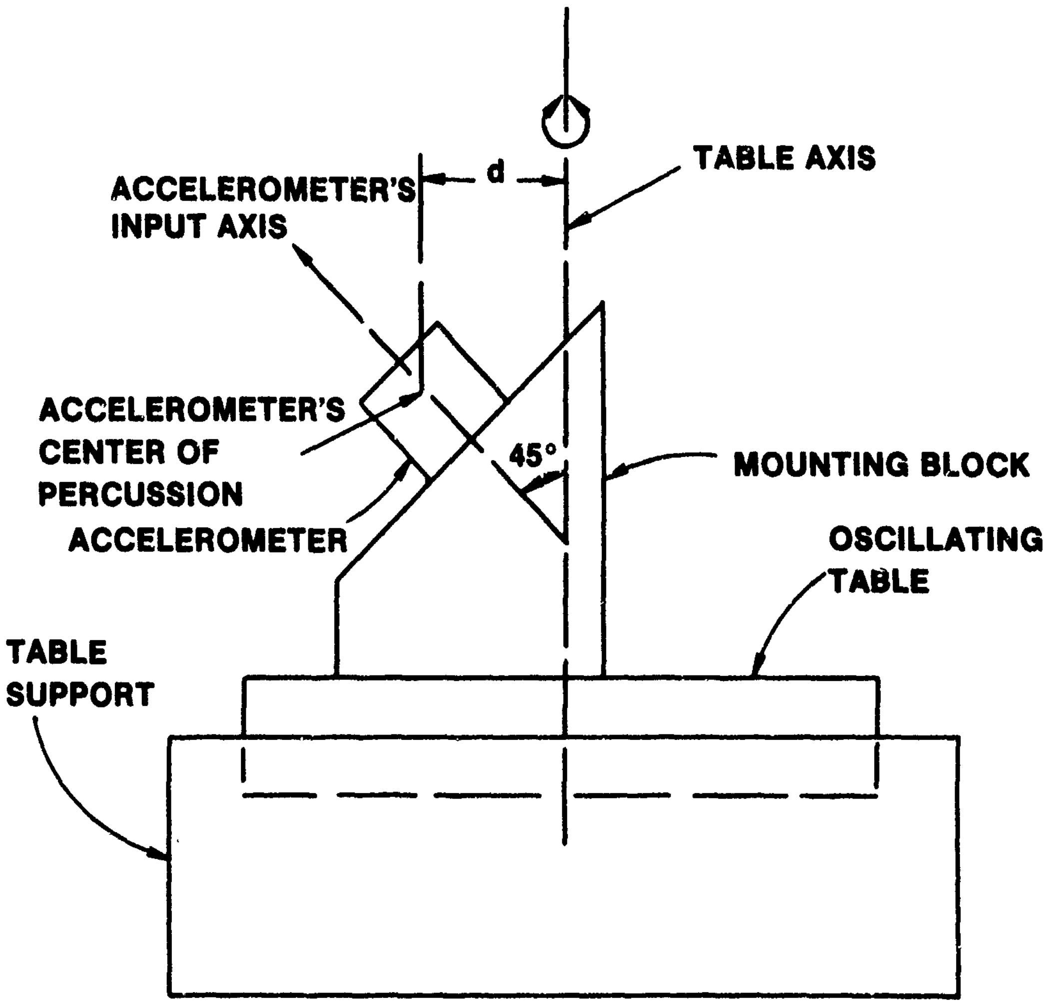


Figure 2. Accelerometer size effect test.

The table is driven in an oscillatory motion with a rate amplitude of 100 deg/sec and a frequency of 8 Hz. Angular amplitude under this condition is about 2 deg. The result of the test is plotted in *Figure 3*. The output of the accelerometer is shown in solid line in terms of millivolts. It agrees with the computed value. Also shown in *Figure 3* are the computed centripetal and tangential components, designated a_c and a_T, respectively, of the acceleration in mV. The scale factor is 500 mV/g.

B. TWO TYPES OF IMU SIZE EFFECT

Adding the U, V and W components of Equation (4) yields the size effect error for the IMU.

$$\vec{\varepsilon} = \sum_{K = U, V, W} \left[(\dot{\vec{\omega}} \times \vec{r}_K + \vec{\omega} \times \vec{\omega} \times \vec{r}_K) \cdot \vec{u}_K \right] \vec{u}_K \quad . \tag{5}$$

The position vector $\vec{r}_K$ for each accelerometer can be resolved into two components $\vec{r}$ and $\vec{d}_K$ as shown in *Figure 4*. The $\vec{r}$ component is the position vector of points S, the center of the accelerometer cluster, in the body frame; and the $\vec{d}_k$ component is the position vector of the K-accelerometer relative to the cluster center. Thus,

$$\vec{r}_K = \vec{r} + \vec{d}_K \; , \quad K = U, V, W \; . \tag{6}$$

Substituting Equation (6) into Equation (4) and separating the result into two components, one involves $\vec{r}$, and the other involves $\vec{d}_k$.

$$\begin{aligned} \vec{\varepsilon} &= \sum_{K = U, V, W} \left[(\dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r}) \cdot \vec{u}_K \right] \vec{u}_K \\ &+ \sum_{K = U, V, W} \left[(\dot{\vec{\omega}} \times \vec{d}_K + \vec{\omega} \times \vec{\omega} \times \vec{d}_K) \cdot \vec{u}_K \right] \vec{u}_K \\ &= \vec{\varepsilon}_m + \vec{\varepsilon}_c \quad . \end{aligned} \tag{7}$$

The first term of Equation (7) is due to the non-zero value of $\vec{r}$. Its effect will be called the "mounting offset size effect of the IMU" and will be designated $\vec{\varepsilon}_M$. This term can be simplified to give

$$\boxed{\vec{\varepsilon}_m = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r}} \tag{8}$$

The second term of Equation (7) is due to non-zero values of $\vec{d}_k$. The effect will be called the "cluster size effect of the IMU" and will be designated $\vec{\varepsilon}_c$. Thus

$$\boxed{\vec{\varepsilon}_c = \sum_{k=u,v,w} \left[(\dot{\vec{\omega}} \times \vec{d}_K + \vec{\omega} \times \vec{\omega} \times \vec{d}_K) \cdot \vec{u}_K \right] \vec{u}_K \quad .} \tag{9}$$

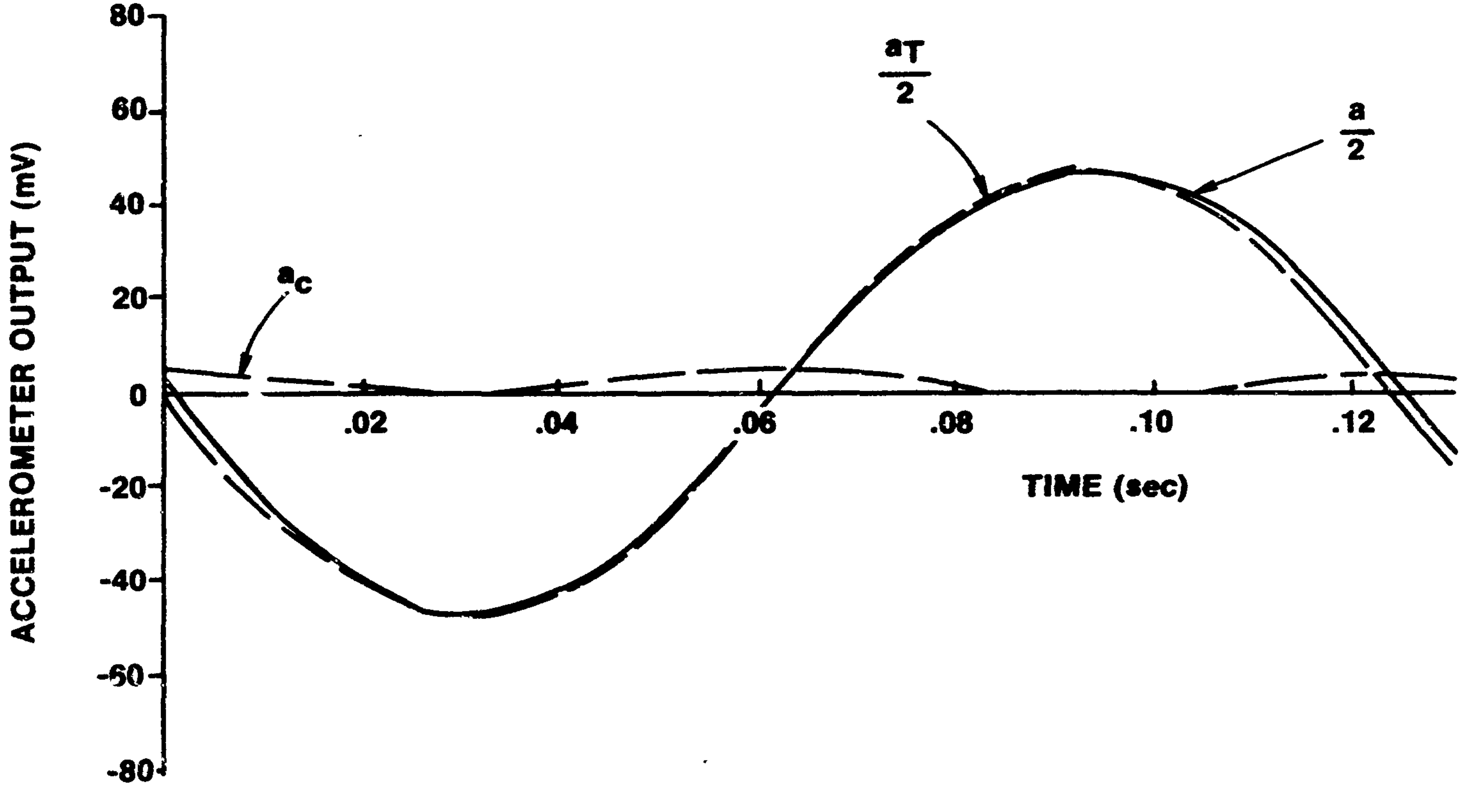


Figure 3. Accelerometer size effect response.

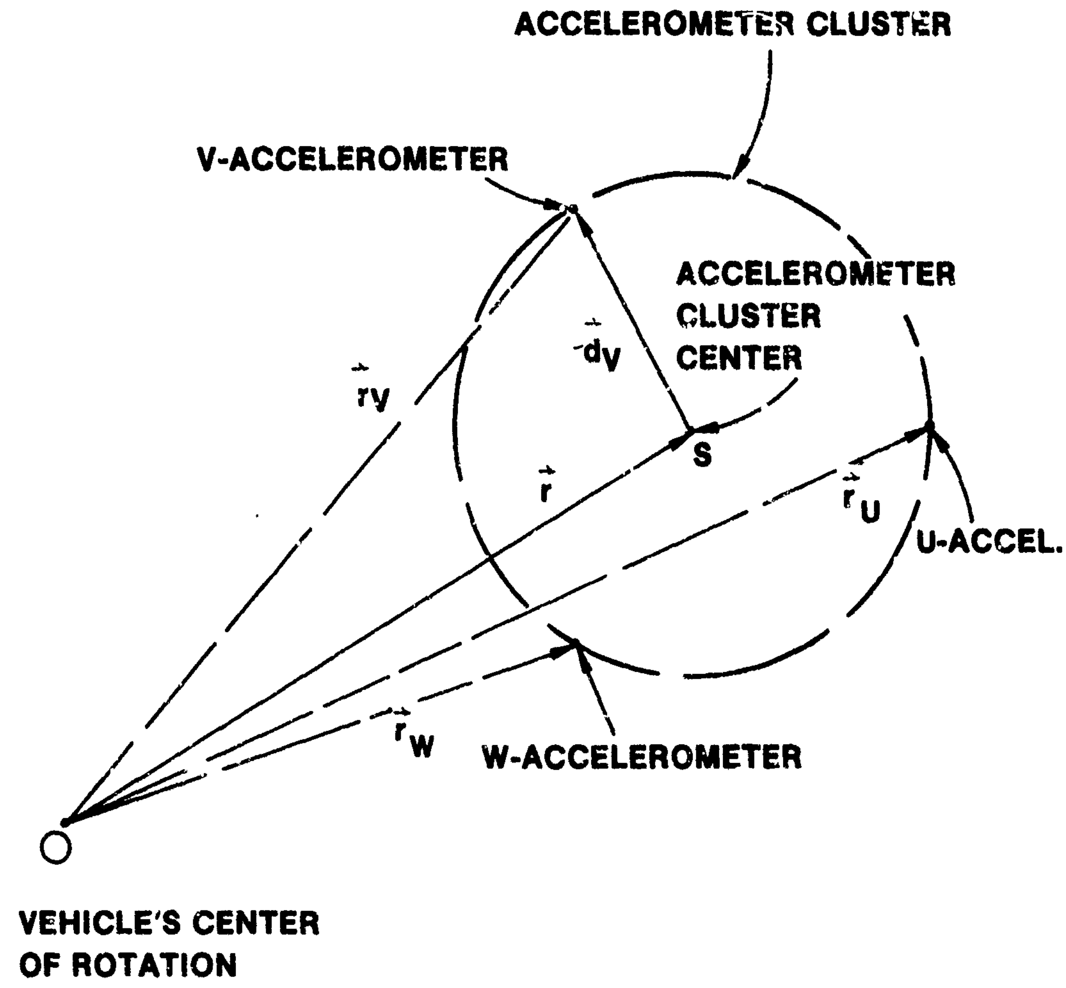


Figure 4. Mounting offset and accelerometer cluster.

While an IMU has two types of size effect, the individual accelerometer has only one, the mounting offset size effect. The nature of each type of the IMU's size effect error will be further analyzed in the following.

C. MOUNTING OFFSET SIZE EFFECT OF IMU

The mounting offset size effect is the same for both strapdown and gimbaled IMU's as long as a mounting offset exists between the vehicle's center of rotation and the vehicle's center of IMU. It results in an acceleration of point S (*Figure 1*) with respect to point 0 when the vehicle is under angular motion. One can also consider point S to be the position of a "point IMU" which has zero physical size. The acceleration error due to mounting offset size effect is given by Equation (8). The navigation error caused by this size effect is reversible, meaning that the net navigation error is zero when the initial and final orientations of point S with respect to point 0 are the same. It is physically clear that the maximum navigation error is 2r, twice the distance between points S and 0. The following example will demonstrate this fact.

Consider a case shown in *Figure 5* where the fixed frame has axes N, E and D. The angular motion of point S is parallel to the NE-plane. The centripetal and tangential accelerations are

$$a_C = -r\dot{\theta}^2 \tag{10}$$

$$a_T = r\ddot{\theta} \tag{11}$$

and their components along the fixed frame are

$$a_N = a_C \cos\theta - a_T \sin\theta \tag{12}$$

$$a_E = a_C \sin\theta + a_T \cos\theta \ . \tag{13}$$

Let $\dot{\theta} = \Omega$, a constant, then $\ddot{\theta} = 0$. By Equations (10) and (11), $a_c = -r\Omega^2$ and $a_T = 0$, using Equations (12) and (13), gives

$$a_N = -r\ \Omega^2 \cos \Omega t \tag{14}$$

$$a_E = -r\ \Omega^2 \sin \Omega t \ . \tag{15}$$

Integrating Equations (14) and (15) twice yields the distances

$$D_N = r \cos \Omega t \tag{16}$$

$$D_E = r \sin \Omega t \ . \tag{17}$$

The net change of distances between time t and the initial time t_o is

$$\Delta D_N = r\ (\cos \Omega t - \cos \Omega t_o) \tag{18}$$

$$\Delta D_E = r\ (\sin \Omega t - \sin \Omega t_o) \ . \tag{19}$$

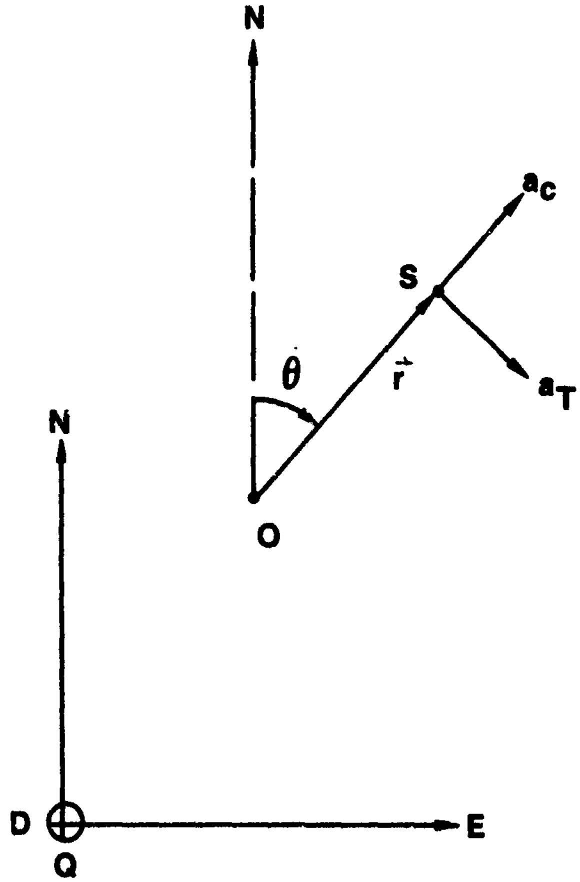


Figure 5. Components of acceleration.

The resultant distance change is

$$\Delta D = \sqrt{\Delta D_N^2 + \Delta D_E^2}$$

$$= r\sqrt{2 - 2\cos(\Omega t - \Omega t_o)} \quad . \tag{20}$$

As expected,

$$\text{Max } \Delta D = 2\, r \quad . \tag{21}$$

In general, the magnitude of r is much smaller when compared to the desired accuracy of inertial navigation. Therefore, mounting offset size effect can often be ignored in the error budget of a guidance scheme.

It should be pointed out that mounting offset size effect does not exist if the position of a vehicle is defined to be the position of its IMU center.

D. CLUSTER SIZE EFFECT ERROR

Cluster size effect depends on the manner in which accelerometers are mounted in the vehicle body and the direction of body rotation. Consider a typical accelerometer arrangement as depicted in *Figure 6*. An orthogonal accelerometer triad is mounted in a skewed configuration with respect to the body frame in such a way that the origin of the accelerometer triad is on the roll axis of the body frame, and the roll axis of the body frame makes equal angle with all these axes of the accelerometer triad. Let the body rotation be a roll motion. Under this condition $\vec{\omega} x \vec{d}_k$ is perpendicular to $\vec{\mu}_k$ so

$$(\vec{\omega} \times \vec{d}_K) \cdot \vec{u}_K = 0 \quad K = U, V, W \quad . \tag{22}$$

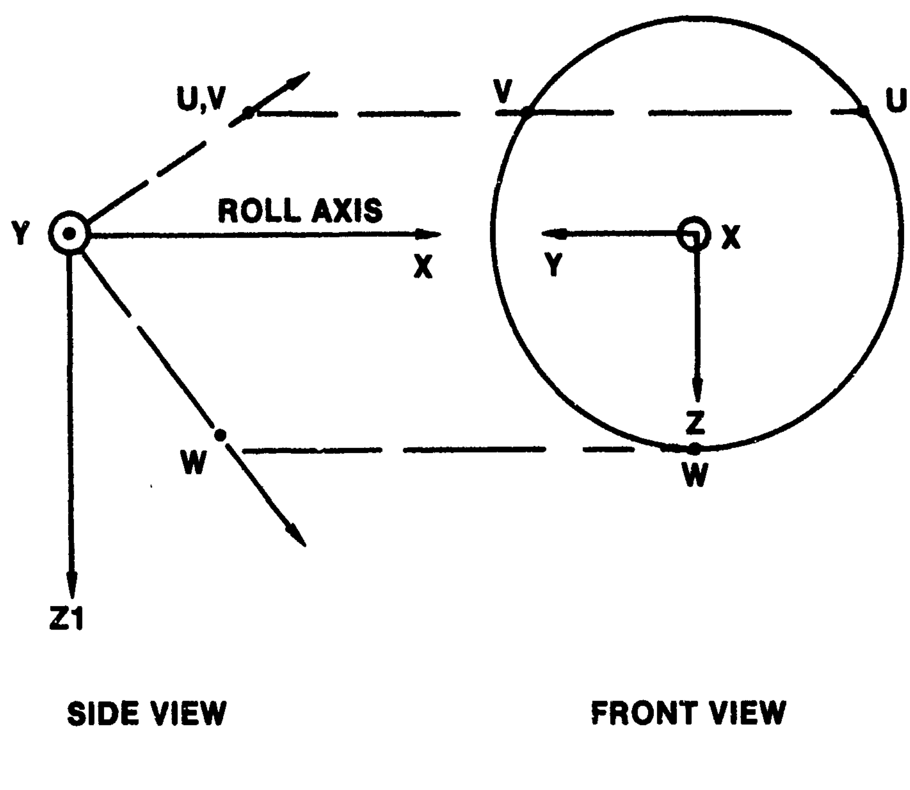


Figure 6. A typical accelerometer cluster orientation.

Therefore Equation (9) gives the IMU's cluster size effect error as

$$\vec{\varepsilon}_C = \sum_{K = U,\ V,\ W} \varepsilon_{KC}\ \vec{u}_K \tag{23}$$

where

$$\varepsilon_{CK} = (\vec{\omega} \times \vec{\omega} \times \vec{d}_K) \cdot \vec{u}_K \qquad K = U,\ V,\ W \tag{24}$$

is the component of ε_c contributed by the K-accelerometer.

Equation (24) can also be expressed in matrix form as

$$\varepsilon_{CK} = \underline{u}_K^T\ \Omega^2\ \underline{d}_K \quad . \tag{25}$$

The correspondences between vector analysis notation and matrix notation are as follows.

$$\vec{\omega} \longleftrightarrow \Omega = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & 0 & 0 \end{bmatrix} \tag{26}$$

$$\vec{d}_K \longleftrightarrow \underline{d}_K = \begin{bmatrix} d_{Kx} \\ d_{Ky} \\ d_{Kz} \end{bmatrix} \tag{27}$$

$$\vec{u}_K \longleftrightarrow \underline{u}_K = \begin{bmatrix} U_{KX} \\ U_{KY} \\ U_{KZ} \end{bmatrix} \tag{28}$$

The superscript T denotes the transpose of a matrix.

For the sample case of *Figure 4*,

$$\underline{d}_u = \begin{bmatrix} 0.79 \\ 0.96 \\ -0.56 \end{bmatrix}, \quad \underline{d}_v = \begin{bmatrix} 0.79 \\ -0.96 \\ -0.56 \end{bmatrix}, \quad \underline{d}_\omega = \begin{bmatrix} 0.79 \\ 0 \\ 1.11 \end{bmatrix} \tag{29}$$

$$\underline{u}_u = \begin{bmatrix} -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} \end{bmatrix} \underline{u}_v = \begin{bmatrix} -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} \end{bmatrix} \quad \underline{u}_\omega = \begin{bmatrix} -\frac{1}{\sqrt{3}} \\ 0 \\ -\frac{2}{\sqrt{6}} \end{bmatrix} . \tag{30}$$

The unit for d_k is inches.

For roll motion only, $\omega_y = \omega_z = 0$. Because of mounting symmetry, Equation (25) gives

$$\varepsilon_{CK} = -(d_{Ky}\ u_{Ky} + d_{Kz}\ u_{Kz})\ \omega_x^2 = 0.9072\ \omega_x^2 \tag{31}$$

for all K. The unit for Equation (31) is in/sec^2 when ω_x is expressed in rad/sec. The projection of ε_{CK} on roll axis is

$$\varepsilon_{CKX} = 0.577\ \varepsilon_{KC} = 0.5238\ \omega_x^2\ \text{m/sec}^2 . \tag{32}$$

The IMU's cluster size effect error is along its roll axis, due to the cluster symmetry, and is given by

$$\varepsilon_C = 3\ \varepsilon_{CKX} = 1.5714\ \omega_x^2\ \ \text{m/sec}^2 . \tag{33}$$

In terms of ft/sec^2,

$$\varepsilon_C = 0.131\ \omega_x^2\ \text{ft/sec}^2 . \tag{34}$$

Equation (34) shows that, in general, the cluster size effect error is not reversible since ω_x^2 does not change sign. This fact indicates a cumulative navigation error which increases with time.

Consider a roll oscillation at an angular velocity of

$$\omega_x = 0.35 \cos 2\pi(6)t . \tag{35}$$

Then, for the typical case considered, Equation (34) gives

$$\begin{aligned} \varepsilon_C &= 0.131 \times 0.35^2 \cos^2 12\pi\ t \\ &= 8 \times 10^{-3}\ (1 + \cos 24\pi\ t) . \end{aligned} \tag{36}$$

Integrating Equation (36) twice to give the distance error as

$$\Delta D = 8 \times 10^{-3}\ \left(\frac{t^2}{2} + \frac{1 - \cos 24\pi\ t}{24\pi}\right) . \tag{37}$$

In practice, the second term of Equation (37) can be neglected, yielding

$$\Delta D = 4 \times 10^{-3}\ t^2\ \text{ft} \quad . \tag{38}$$

For a navigation time of 120 sec, the navigation error is

$$\Delta D = 57.6\ \text{ft} \tag{39}$$

which often needs to be included in the navigation error budget.

The flight simulation of a typical missile was performed to reveal the size effect error. The same accelerometer cluster as for the above sample calculation was used. A typical profile of the roll angular velocity is shown in *Figure 7*. The maximum amplitude of the roll oscillator is about 0.35 rad/sec, and the maximum oscillator frequency is about 6 Hz. A size effect error of 6 ft in the down-range direction was given by the simulation. Comparing to Equation (39), this error is much smaller. This is expected, since in the sample case the roll oscillator at the rate amplitude of 0.35 rad/sec was maintained over the entire flight, while in the simulation the roll oscillation was much less. In fact, the roll motion was nearly nil over two-thirds of the simulated flight.

A gimbaled IMU also experiences the cluster size effect but in a different nature from that of a strapdown IMU. First, the angular velocity involved is the turning rate of the trajectory, which is considerably smaller than the body turning rate. Second, the cluster size effect is reversible for a gimbaled IMU, meaning that the net effect is zero when the vehicle returns to its initial position for a closed trajectory. The maximum navigation error induced by this effect is

$$\text{Max}\ \Delta D = 2d \tag{40}$$

where d is the diameter of the accelerometer cluster. For d = 3 in, ΔD = 6 in. which is insignificant when compared to Equation (39).

However, cluster size effect can be serious for a gimbaled IMU if the bandwidth of its platform stabilization loop is not sufficiently wide. Under this condition, the platform will respond to high frequency body vibrations in a strapdown fashion. As a result, the strapdown kind of cluster size effect error is generated. Equations (23) and (24) can be used to evaluate the size effect acceleration in this high frequency range. These two accelerations, in conjunction with the body rate information from strapdown gyros, are resolved into erroneous accelerations in the coordinate frame of the analytic platform. However, these

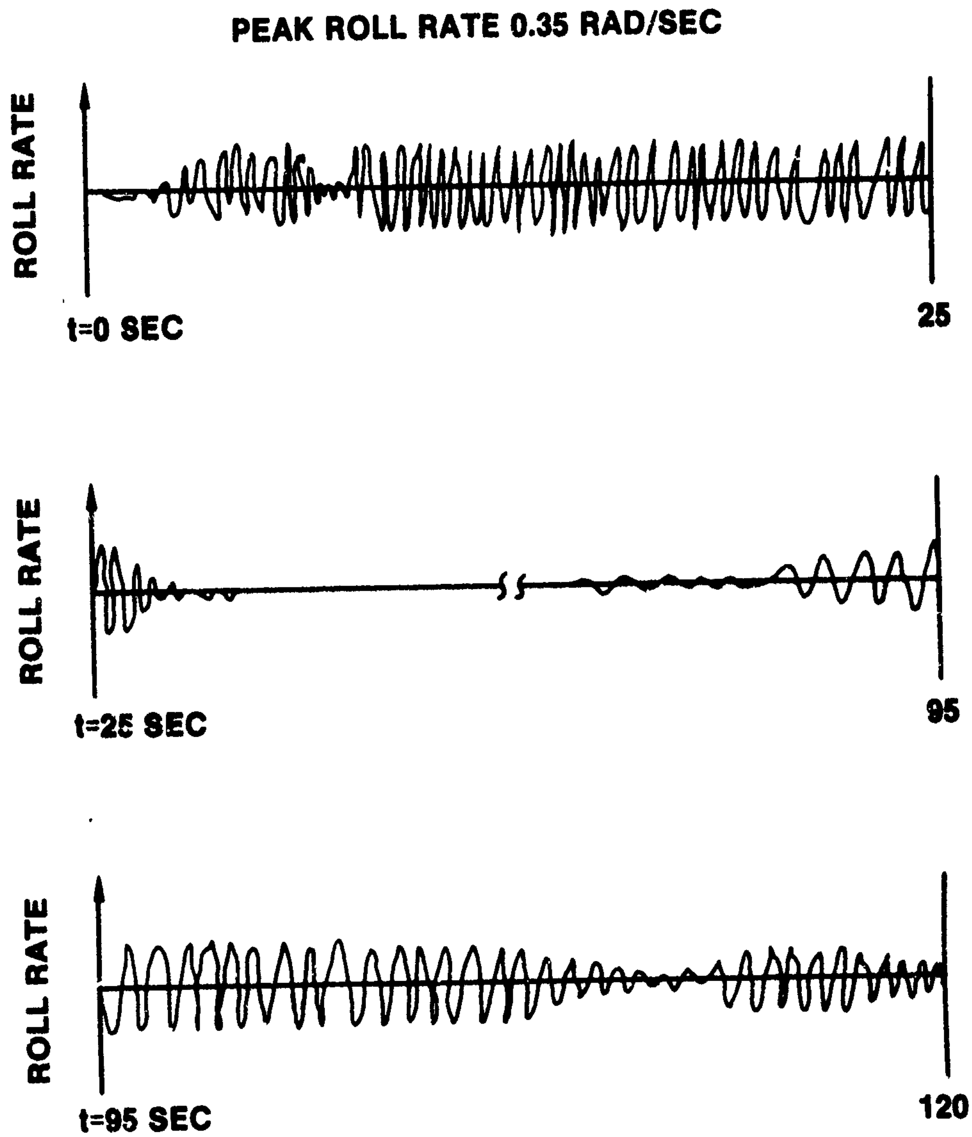


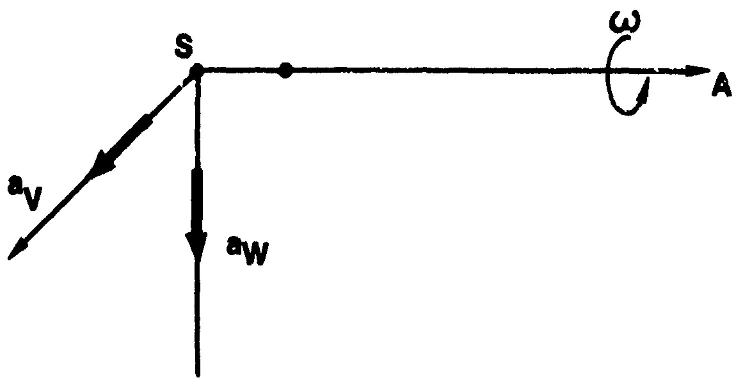
Figure 7. Roll rate profile for a typical missile.

accelerations are reversible and contribute to zero net acceleration after each complete rotation. As a result, the size effect navigation error is limited to a distance of

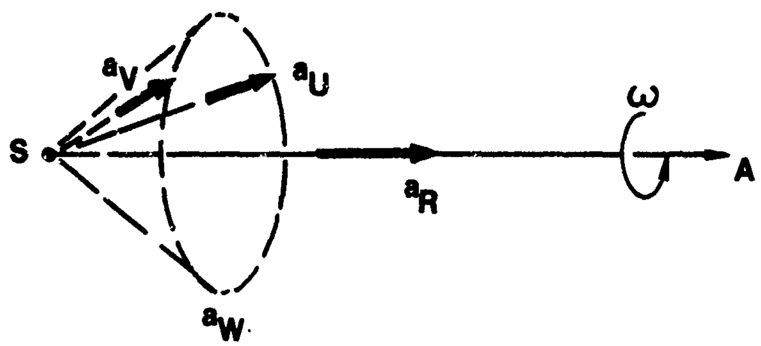
$$\Delta D = 2.828 \ d \tag{41}$$

which is not significant in practice.

In *Figure 8(b)*, the cluster is so oriented that all three input axes make the same angle of 54.74 deg with the axis of rotation. Under this condition, each accelerometer senses an acceleration of



(a)



(b)

Figure 8. Two accelerometer cluster orientations.

$$a_K = d\,\omega^2 \sin^2 54.74 = 0.667\, d\, \omega^2 \quad . \tag{42}$$

Because of symmetry, the resultant acceleration a_r is along the axis of rotation, with a magnitude of

$$a_R = 3a_K \cos 54.74 = 1.152\, d\, \omega^2 \quad . \tag{43}$$

This acceleration is not reversible. It is falsely utilized by the analytic platform to produce a size effect navigation error of

$$\Delta D = 1/2\, a_R\, t^2 = 0.576\, d\, \omega^2\, t^2 \tag{44}$$

which grows rapidly with time.

From the above considerations, it is seen that the optimum orientation for an accelerometer cluster is to place one accelerometer input axis along the direction where body rotation is maximum. Such orientation for minimum size effect may not be optimum for other considerations.

3. SIZE EFFECT COMPENSATION

Size effect of an IMU can be compensated by using the angular rate information available from the gyros. For any IMU $\vec{d}_k$, the position vector of each accelerometer is known. If the vehicle is an aircraft, its center of rotation is fixed, so $\vec{r}$, the position vector of the IMU is also known. Therefore, the position vector $\vec{r}_k = \vec{r} + \vec{d}_k$ is known. With known body angular rate, Equation (4) can be used to generate needed size effect compensation for accelerometer outputs. The compensation algorithm is simply

$$a_{KC} = a_K - \varepsilon_K \qquad K = U,\ V,\ W \tag{45}$$

where a_{KC} is the compensated acceleration, a_K is sensed acceleration, and ε_k is the compensation given by Equation (4).

For a missile, the center of rotation changes during the flight. The change is usually approximately known. One can either ignore the mounting offset size effect and compensate only for the cluster size effect, since the effect of the former is often negligible. The compensation algorithm is still given by Equation (45), except that $\vec{r}_K$ is replaced by $\vec{d}_k$ in Equation (4). Or, one can use a varying $\vec{r}_k$ in Equation (4) for Equation (45), which is programmed to change in a statistically determined way. *Figure 9* is a proposed size effect compensation concept, where other types of sensor compensations are excluded.

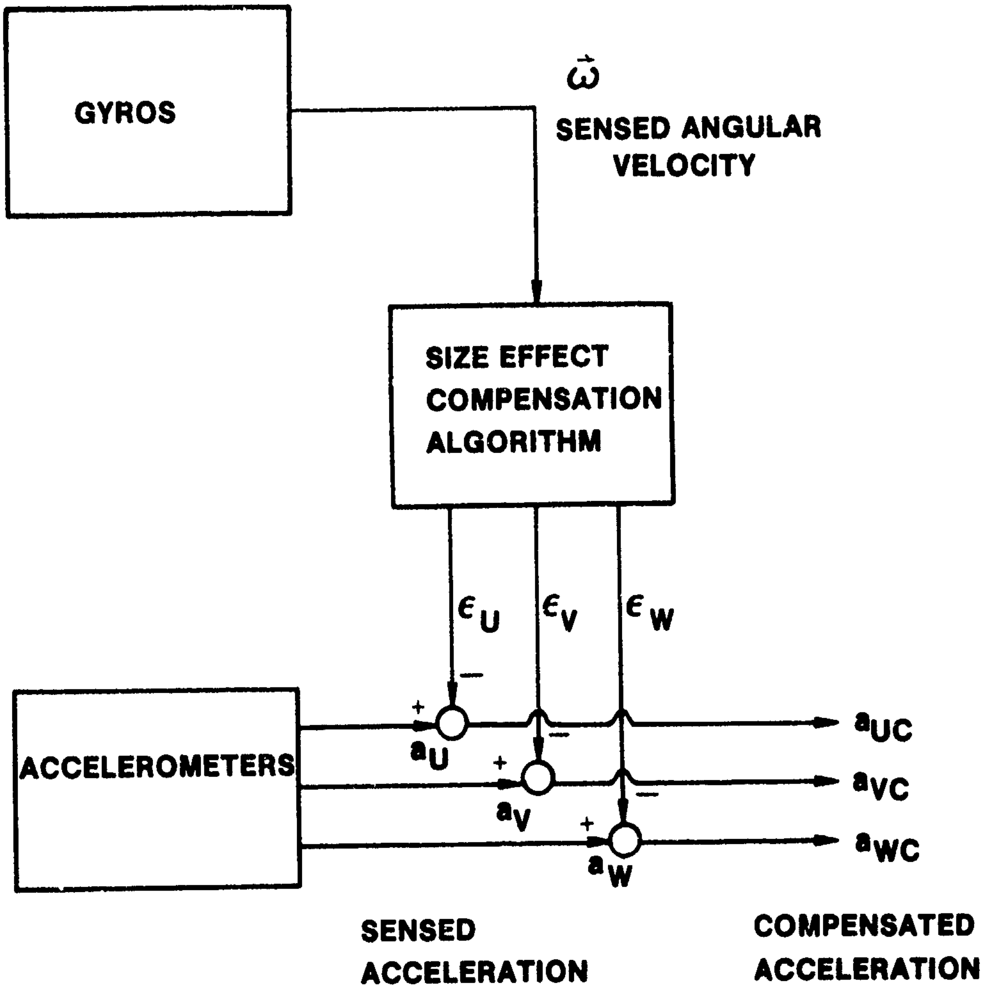


Figure 9. A size effect compensation scheme.

4. SIZE EFFECT AND COMPUTATION ERROR

It should be emphasized that ideal computation does not eliminate size effects. Ideal compensation means infinite data sampling rate, zero truncation error in the strapdown algorithm, zero round-off error from the computer, infinite computation frequency, and zero time-delay due to computation. The relationship between size effects and computation is analyzed below.

Size effects often expose IMU sensors, gyros as well as accelerometers, to the high frequency components of the vehicle dynamics. If the data sampling frequency and the computation frequency are not sufficiently high, serious aliasing effect will result, causing grave computation errors.[3]

The combined effect of size effects and computation frequency has been studied via computer simulation[4]. A qualitative picture of the effect is shown in *Figure 10.* The IMU was subject to a constant amplitude oscillatory motion, and the computation frequency was fixed. *Figure 10* shows that at low oscillation frequencies, size effect induced computation error is independent of the frequency (to the left of point A). At a certain range of oscillation frequency (between points A and B), computation error increases with the frequency linearly. In this frequency range the computation frequency is greater than twice the oscillation frequency. Beyond this frequency range (above point B), the computation frequency is less than the frequency of oscillation, and the computation errors become independent of the algorithm used.

5. CONCLUDING REMARKS

Several remarks can be made from the experience gained in this study.

Size effects in a strapdown IMU are not always ignorable. They should be considered in the error budget for a high accuracy inertial navigation system.

There are two types of size effects: the mounting offset size effect and the cluster size effect. Mounting offset size effect does not cause serious navigation error, not more than twice the distance between the vehicle's center of rotation and center of IMU. This size effect can be removed completely simply by defining the position of a vehicle to be its IMU center. The cluster size effect always exists even with ideal software, and cannot be physically eliminated. It is proportional to the separation among the three accelerometers and to the square of the angular rate of the vehicle body. The squaring of the angular rate generates a rectification type of size effect which, in turn, generates a navigation error that increases with the square of time.

The optimum orientation of the accelerometer cluster for minimum cluster size effect is to place one of the accelerometer input axes along the direction where rotation motion is maximum.

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3 R. W. Hamming, *Digital Filters*, Prentice-Hall, Inc., New Jersey, 1977, pp. 16-18.

4. J. J. Sullivan, "Evaluation of the Computation Errors of Strapdown Navigation Systems," *AIAA Journal* Vol. 6 No. 2, 1968, pp. 312-319. (Same as NASA Contract Report CR-968, April, 1968.)

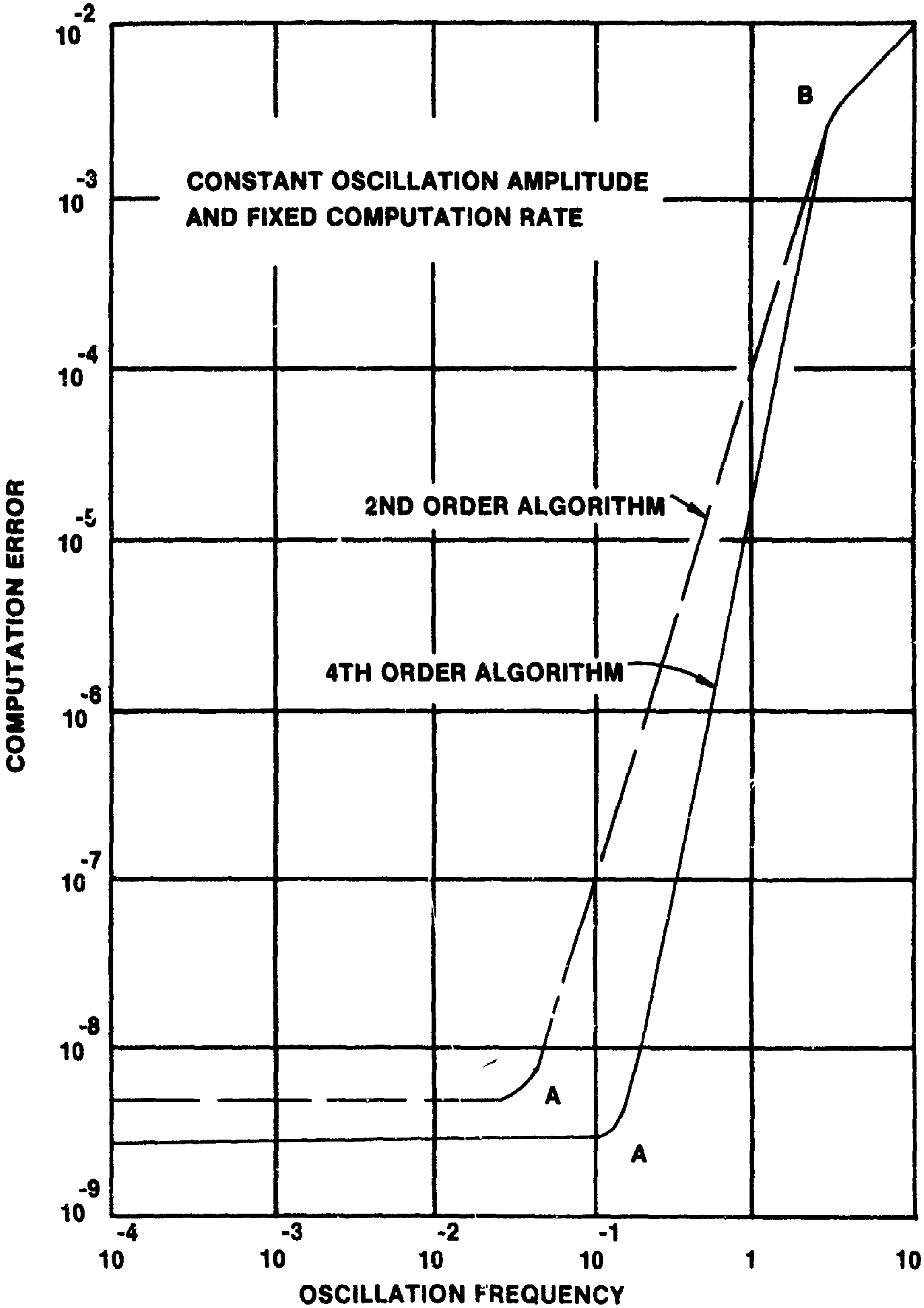

Figure 10. A qualitative picture of size effect induced computation errors.

In principle, size effect of an IMU can be compensated by software using the angular rate information available from gyros. A size effect compensation concept is proposed with the associated computation algorithm given.

A gimbaled IMU also experiences the mounting offset size effect exactly the same way as that experienced by a strapdown IMU. However, it experiences much less serious cluster size effect because its accelerometers are not subject to the vehicle's body angular rate.

Size effect often exposes the IMU to the high frequency components of the vehicle dynamics. If the data sampling frequency and the computation frequency are not sufficiently high, serious aliasing effects will result, causing grave computation errors.

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