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ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPE--ETC(U)
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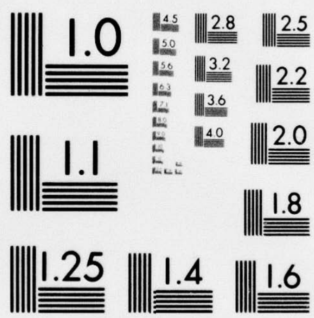
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AMMRC TR 79-46

ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPECIMEN WITH CONSTRAINED ENDS

OSCAR L. BOWIE and FRANCIS I. BARATTA
MECHANICS OF MATERIALS DIVISION

July 1979

ARMY MATERIALS AND MECHANICS RESEARCH CENTER
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ABSTRACT

Relationships are presented between the net force, the end displacement, and the stress intensity factor for cracked rectangular panels subjected to uniform displacement. Two configurations are considered: an edge-cracked panel and a center-cracked panel. If the stress intensity factor is already known in terms of the normal displacement, then the resulting formulation allows the determination of that stress intensity in terms of the applied load and vice versa.

1. INTRODUCTION

This paper is concerned with the derivation of a relationship between the net force, the end displacement, and the stress intensity for cracked rectangular specimens with constrained ends. Two cracked-panel configurations are considered: an edge crack and a center crack. The relationship is generally derived using the edge crack case as an example and via a change in appropriate geometry parameters the center crack case is also solved.

2. EDGE CRACK

As an example, in Reference 1 the case of an edge crack in a rectangular panel, Figure 1, was considered with displacement end conditions

$$V = 0,\ U = U_0 \text{ (constant) on CD}$$
$$V = 0,\ U = -U_0 \text{ on AB} \tag{1}$$

The results were normalized with respect to an averaged applied stress σ_A, where

$$\sigma_A = (1/h)\int_D^C \sigma_x dy. \tag{2}$$

In Reference 2, similar results were presented for the center-cracked rectangular panel. Yet if σ_A is regarded as fixed (with respect to L), then $U_0 = U(L)$ and vice versa. If the alternate normalization with respect to U_0 is preferable, then a relationship between U_0, σ_A, and the stress intensity factor K is necessary for the interpretation of the results presented in References 1 and 2.

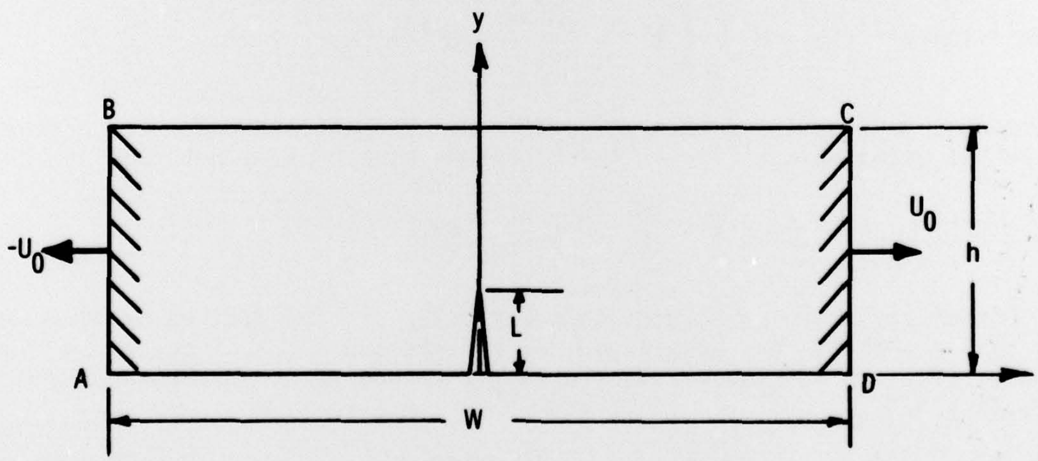

Figure 1. Edge crack in a rectangular panel.

1. BOWIE, O. L., et al. *Solution of Plane Problems of Elasticity Utilizing Partitioning Concepts.* J. Applied Mechanics, v. 95, 1973, p. 767-772.
2. BOWIE, O. L. *Methods of Analysis and Solutions of Crack Problems (Chapter I).* G. C. Sih, ed., Noordhoff International Publishing, Leyden, 1973.

Although the basic results of this paper can be arrived at by compliance arguments, we choose a more interesting approach from an analytical point of view by utilizing an extension of the "weight function" arguments of Bueckner[3] and Rice.[4]

First, we summarize the original weight function argument with Rice's notation as follows: Neglecting body forces and considering only those configurations consistent with Mode I behavior, we assume that a solution has been found for a given geometrical configuration "1", with boundary τ, crack length L, for an applied load with stress vector $\bar{t}^{(1)}(x,y)$, a function of position only. The corresponding stress intensity factor will be denoted by $K^{(1)}(L)$ and displacement vector by $\bar{U}^{(1)}(x,y,L)$. It is assumed that $\bar{t}^{(1)}(x,y)$ is chosen such that $K^{(1)}(L) \neq 0$. Then, for the same geometrical configuration and any load system "2" with an applied stress vector $\bar{t}^{(2)}(x,y)$ consistent, of course, with Mode I behavior,

$$2K^{(1)}(L)\ K^{(2)}(L) = H \int_\tau \bar{t}^{(2)} \cdot \frac{d\bar{U}^{(1)}}{dL}\, d\tau, \tag{3}$$

where $H = E/(1 - \nu^2)$ for plane strain and $H = E$ for plane stress. In Equation 3, it is assumed that $\sqrt{\pi}$ is included in the definitions of $K^{(1)}(L)$ and $K^{(2)}(L)$, i.e., in the vicinity of the crack tip

$$\sigma \approx K(L)\ (2\pi r)^{-1/2}. \tag{4}$$

For the problems of end constraints, the stress vector on the ends is a function of the parameter L as well as position, thus Equation 3 is not applicable. The Reciprocal Theorem of Betti and Rayleigh used in deriving Equation 3 can be easily extended to the case when $\bar{t}^{(1)}(x,y,L)$ is a function of both position and the parameter L. In fact,

$$2K^{(1)}(L)\ K^{(2)}(L) = H \int_\tau \left\{ \bar{t}^{(2)} \cdot \frac{d\bar{U}^{(1)}}{dL} - \bar{U}^{(2)} \cdot \frac{d\bar{t}^{(1)}}{dL} \right\} d\tau, \tag{5}$$

which reduces to Equation 3 when $d\bar{t}^{(1)}/dL = 0$. We shall be concerned primarily with the situation when $\bar{t}^{(1)} = \bar{t}^{(2)} = \bar{t}$. Then, with obvious notation

$$2\left[K(L)\right]^2 = H \int_\tau \left\{ \bar{t} \cdot \frac{d\bar{U}}{dL} - \bar{U} \cdot \frac{d\bar{t}}{dL} \right\} d\tau. \tag{6}$$

Consider now the displacement boundary value problem defined by Equation 1. Assume that a solution has been found for a fixed end force, i.e., σ_A is independent of L. Then U_0 must be considered as a function of L. Utilizing symmetry and Equation 6,

$$H \int_0^h \left\{ \sigma_x \frac{dU_0}{dL} - U_0 \frac{d\sigma_x}{dL} \right\} dy = \left[K^{(1)}(L) \right]^2 \tag{7}$$

3. BUECKNER, H. F. *A Novel Principle for the Computation of Stress Intensity Factors.* Z. Angew. Math. Mech., v. 50, 1970, p. 529-546.
4. RICE, J. R. *Some Remarks on Elastic Crack-Tip Stress Fields.* International Journal Solids Structures, v. 8, no. 6, June 1972, p. 751-758.

or

$$H \frac{dU_0}{dL} \int_0^h \sigma_x dy - HU_0 \frac{d}{dL} \left\{ \int_0^h \sigma_x dy \right\} = \left[K^{(1)}(L) \right]^2 ,$$

$$Hh\sigma_A \frac{dU_0}{dL} = \left[K^{(1)}(L) \right]^2 \tag{8}$$

$$Hh\sigma_A U_0(L) = \int_0^L \left[K^{(1)}(\xi) \right]^2 d\xi + C_1,$$

where the constant of integration C_1 must be chosen such that Equation 8 is consistent at $L = 0$.

If, on the other hand, a solution has been found normalized with respect to a constant value of U_0, Equation 6 becomes

$$-HU_0 \int_0^h \frac{d\sigma_x}{dL} dy = \left[K^{(2)}(L) \right]^2 , \tag{9}$$

or

$$HU_0 h \; \sigma_A(L) = - \int_0^L \left[K^{(2)}(\xi) \right]^2 d\xi + C_1', \tag{10}$$

where for a fixed value of U_0, $\sigma_A(L)$ is a decreasing function of L. In fact, $\sigma_A(h) = 0$ and

$$C_1' = \int_0^h \left[K^{(2)}(\xi) \right]^2 d\xi. \tag{11}$$

The constants of integration C_1 and C_1' cannot be derived in a direct rigorous manner owing to an unknown distribution of shear stresses on the ends. For practical purposes, however, its value can be established by the following argument: Consider the two load systems in Figure 2. We choose

$$\sigma_x^{(1)} = \sigma_A = h^{-1} \int_{-h/2}^{h/2} \sigma_x^{(2)} dy, \tag{12}$$

and

$$\sigma_y^{(1)} = [(H/2\mu) - 1] \; \sigma_A, \tag{13}$$

where μ is the shear modulus. For this load system, the displacement V in system "1" vanishes and

$$U^{(1)} = [\mu^{-1} - H/4\mu^2] \; \sigma_A x. \tag{14}$$

Applying the Reciprocal Theorem,

$$U_0 h \sigma_A = \sigma_A \left[\mu^{-1} - H/4\mu^2 \right] \frac{hW}{2} - [(H/2\mu) - 1] \; \sigma_A \int_{-W/2}^{W/2} V^{(2)} dx. \tag{15}$$

Although we do not know $V^{(2)}$ explicitly it is evidently a measure of the vertical contraction of System "2". It is evident physically that if we assume $V^{(2)} = 0$, a lower bound on U_0 is obtained from Equation 15,

$$U_0 h\sigma_A \geq \sigma_A \left[\mu^{-1} - H/4\mu^2\right] \frac{hW}{2}, \tag{16}$$

with equality for $W/h \ll 1$.

On the other hand, a reasonably accurate upper bound can be found by assuming a uniform lateral contraction $V^{(2)} = [H^{-1} - 1/2\mu]\sigma_A h/2$, which leads to

$$U_0 h\sigma_A \leq \sigma_A^2 \, Wh/2H. \tag{17}$$

On the basis of numerical results for $W/h = 1$, the equality sign in Equation 17 can be taken with satisfactory accuracy. On physical grounds, the accuracy of this approximation increases with increasing W/h. Thus,

$$C_1 \simeq \sigma_A^2 \, Wh/2, \; W/h > 1. \tag{18}$$

By introducing the function $\left[M_{\sigma_A}(L/h)\right]$ where

$$K(L/h) = \left[M_{\sigma_A}(L/h)\right] \sigma_A(\pi L)^{\frac{1}{2}}. \tag{19}$$

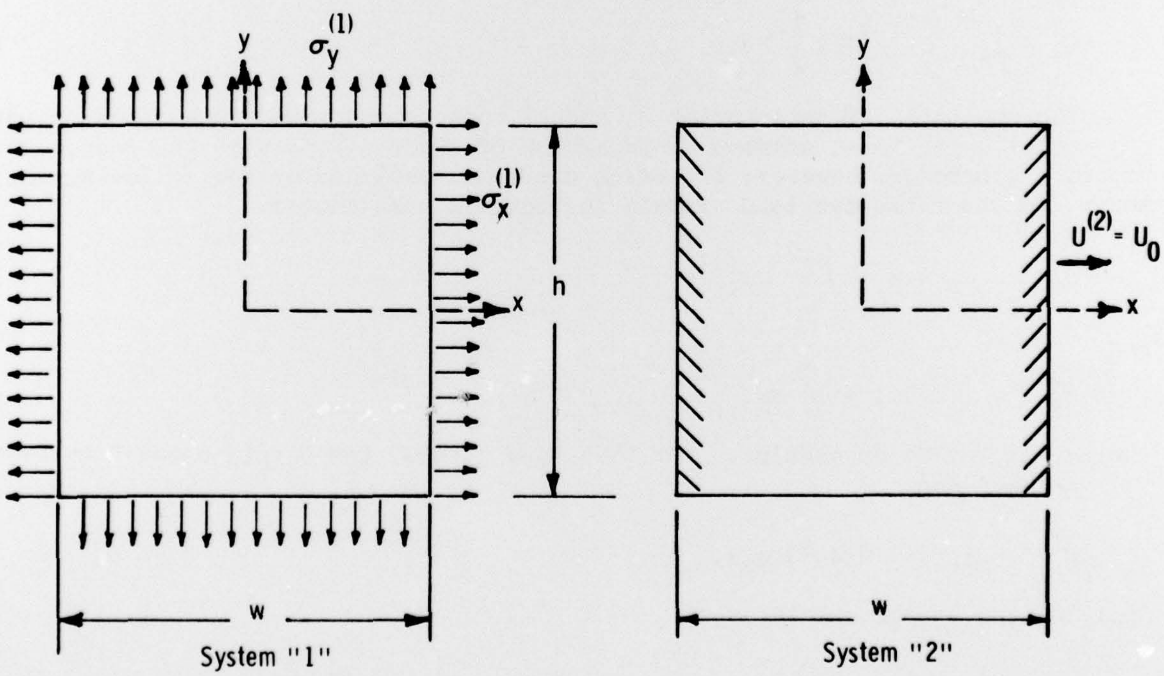

Figure 2. Load systems "1" and "2".

Equations 8 and 18 can be written in this notation as

$$\frac{K}{H\left(\frac{2U_0}{W}\right)(\pi L)^{\frac{1}{2}}} = \frac{\left[M_{\sigma_A}(L/h)\right]}{1 + 2\pi(h/W)\int_0^{L/h}\left[M_{\sigma_A}(L/h)\right]^2 (L/h)\ d(L/h)} . \tag{20}$$

Regarding the constant C_1', when U_0 is kept fixed, we have from the geometry of Figure 1 that when $L = 0$

$$C' = HhU_0\sigma_A(0) = HhU_0\left(\frac{2HU_0}{W}\right) \tag{21}$$

or

$$C_1' = 2H^2U_0^2\ (h/W).$$

Similarly, if the function $\left[M_{U_0}(L/h)\right]$ is defined as

$$K(L/h) = \left[M_{U_0}(L/h)\right] H\left(\frac{2U_0}{W}\right)(\pi L)^{\frac{1}{2}} \tag{22}$$

Equations 10 and 21 can be written as

$$\frac{K}{\sigma_A(\pi L)^{\frac{1}{2}}} = \frac{M_{U_0}(L/h)}{1 - 2\pi(h/W)\int_0^{L/h}\left[M_{U_0}(L/h)\right]^2 (L/h)\ d(L/h)} . \tag{23}$$

Note when $h/W \to 0$ or $L/h \to 0$ from Equation 20 or 23 we have

$$\left[M_{U_0}(L/h)\right] \equiv \left[M_{\sigma_A}(L/h)\right], \tag{24}$$

as expected.

The validity of the approximations to C_1 and C_1' in Equations 18 and 21 is implemented through the use of Equations 20 and 23 and illustrated in Table 1 using results of Freese.* His data corresponds to the plane stress constrained end problem with $h/W = 0.983$ and $\nu = 0.35$.

Equation 20 was used to obtain $\left[M_{U_0}(L/h)\right]$ from the data given in Reference 1, where $\left[M_{\sigma_A}(L/h)\right]$ was given for the loading case described by Figure 1, for $\nu = 1/4$. These results are shown in Table 2.

Shown in Table 3, for comparison, are some other known results for this problem from A. S. Kobayashi and S. Mall reported in Reference 5, for $W/h = 3$. It is seen that the greatest difference between the results reported here and those given in Reference 5 is approximately 6% occurring at $L/h = 0.70$.

*FREESE, C. E., Army Materials and Mechanics Research Center, private communication, 1975.

5. QUACKENBUSH, C. L., and FRECHETTE, V. D. *Crack-Front Curvature and Glass Slow Fracture.* J. Am. Ceram. Soc., v. 61, nos. 9-10, September-October 1978, p. 402-406.

Table 1. NORMALIZED STRESS INTENSITY FACTORS

	$\left[M_{U_0}(L/h)\right] = \frac{K}{H\left(\frac{2U_0}{W}\right)(\pi L)^{1/2}}$		$\left[M_{\sigma_A}(L/h)\right] = \frac{K}{\sigma(\pi L)^{1/2}}$	
L/h	Freese Data	Eq. 20	Freese Data	Eq. 23
0				
0.10	1.03	1.00	1.03	1.03
.20	0.86	0.85	0.95	0.95
.30	.74	.73	.92	.92
.40	.67	.65	.93	.93
.50	.59	.58	.98	.98
.60	.54	.53	1.09	1.09
.70	.50	.49	1.28	1.28
.80	.46	.46	1.63	1.63

Table 2. NORMALIZED STRESS INTENSITY FACTOR FOR AN EDGE CRACK IN A RECTANGULAR PANEL

$$\left[M_{U_0}(L/h)\right] = \frac{K}{H\left(\frac{2U_0}{W}\right)(\pi L)^{1/2}}$$

$\nu = 1/4$

L/h \ W/h	1	2	3	4	8	12	16
0							
0.05	1.01	1.11	1.13	1.13	1.13	1.13	1.13
.10	0.97	1.10	1.14	1.15	1.16	1.18	1.18
.20	.85	1.06	1.14	1.20	1.27	1.30	1.32
.30	.73	0.99	1.14	1.23	1.41	1.49	1.53
.40	.64	.91	1.10	1.24	1.55	1.70	1.79
.50	.58	.83	1.04	1.21	1.65	1.90	2.07
.60	.53	.75	0.95	1.12	1.67	2.04	2.32
.70	.49	.69	.86	1.02	1.58	2.03	2.40

Table 3. COMPARISON OF $\left[M_{U_0}(L/h)\right]$ WHEN W/h = 3

L/h	From Eq. 20 and Ref. 1 $\nu = 1/4$	From Ref. 5 $\nu = 0.23$	Difference (%)
0			
0.05	1.13	1.11	-2
.10	1.14	1.11	-3
.20	1.14	1.11	-3
.30	1.14	1.09	-4
.40	1.10	1.05	-5
.50	1.04	0.99	-5
.60	0.95	.90	-5
.70	.86	.81	-6

3. CENTRAL CRACK

When we apply the Rice type of argument to the central crack shown in Figure 3, we fix end A and consider a perturbation of the end B. Equation 7 holds with the obvious change in notation

$$H \int_{-h/2}^{h/2} \left\{ \sigma_y \frac{dU_0}{da} - \frac{U_0 d\sigma_y}{da} \right\} dx = [K(a)]^2, \tag{25}$$

where K(a) is the stress intensity at B. Now suppose we introduce the conventional notation a = 2L, then K(a) is equivalent to the conventional K(L). Note, however, that now

$$H \int_{-h/2}^{h/2} \left\{ \sigma_y \frac{dU_0}{d(2L)} - U_0 \frac{d\sigma_y}{d(2L)} \right\} dx = [K(L)]^2, \tag{26}$$

leading to

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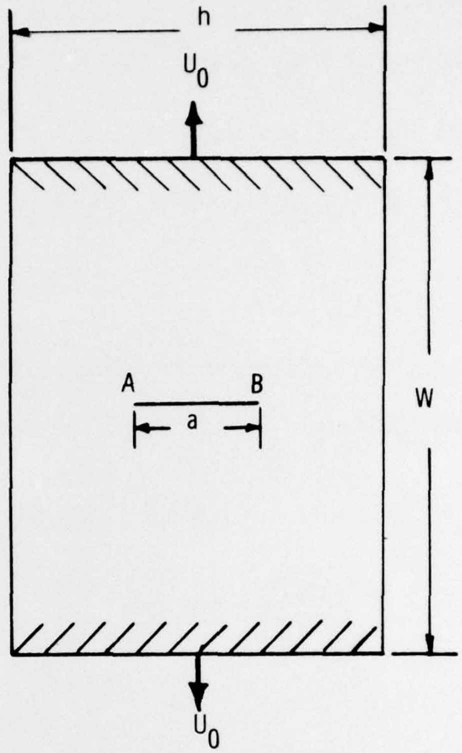

Figure 3. Center crack in a rectangular panel.

$$Hh\sigma_A \frac{dU}{2dL} = [K(L)]^2. \tag{27}$$

Thus Equation 8 now becomes

$$Hh\sigma_A U_0(L) = 2 \int_0^L [K(\xi)]^2 \, d\xi + C_1. \tag{28}$$

The constant $C_1 \approx \sigma_A^2 \, Wh/2$, $W/h > 1$ so that

$$Hh\sigma_A U_0(L) = \sigma_A^2 \, Wh/2 + 2 \int_0^L [K(\xi)]^2 \, d\xi + C_1. \tag{29}$$

Substituting C_1 into Equation 29 and proceeding as before, but with

$$K = \left[M_{\sigma_A}(2L/h)\right] \sigma_A (\pi L)^{\frac{1}{2}}, \tag{30}$$

for the center-cracked panel, we can rewrite Equation 29 as

$$\frac{K}{H\left(\frac{2U_0}{W}\right)(\pi L)^{\frac{1}{2}}} = \frac{\left[M_{\sigma A}(2L/h)\right]}{1 + \pi(h/W)\int_0^{2L/h}\left[M_{\sigma_A}(2L/h)\right]^2 (2L/h)\, d(2L/h)}. \tag{31}$$

Regarding the constant C_1' for the center crack case, but allowing for a factor of 2, i.e.,

$$HU_0 h\sigma_A(L) = C_1' - 2\int_0^L [K(\xi)]^2\, d\xi, \tag{32}$$

and when U_0 is fixed but letting L = 0 from Figure 3, we have

$$C_1' = Hh\ U_0\sigma_A(0) = Hh\ U_0\left(\frac{2HU_0}{W}\right)$$

or

$$C_1' = 2H^2U_0^2\ (h/W),$$

as determined before for the edge crack case.

Substituting C_1' into Equation 32 and defining K for the center-cracked panel as

$$K = \left[M_{U_0}(2L/h)\right] H \left(\frac{2U_0}{W}\right)(\pi L)^{\frac{1}{2}}, \tag{33}$$

we finally obtain

$$\frac{K}{\sigma(L)(\pi L)^{\frac{1}{2}}} = \frac{\left[M_{U_0}(2L/h)\right]}{1 - \pi(h/W)\int_0^{2L/h}\left[M_{U_0}(2L/h)\right]^2 (2L/h)\, d(2L/h)}\ . \tag{34}$$

Again as expected, it is noted that when $h/W \to 0$ or $2L/h \to 0$, Equations 31 and 34 reduce to

$$\left[M_{U_0}(2L/h)\right] \equiv \left[M_{\sigma_A}(2L/h)\right]. \tag{35}$$

4. CLOSING COMMENTS

Equation 24 or 35 can serve as a check on the accuracy of known results when $h/W \to 0$ or $L/h \to 0$ for the edge crack, and $2L/h \to 0$ for the center-cracked panel subjected to uniform normal displacement. Further, Equation 11 can be used to provide a check on the limiting end point, i.e., L/h = 1.0 for the edge crack or 2L/h = 1.0 for the center crack, if $\left[M_{U_0}(L/h)\right]$ or $\left[M_{U_0}(2L/h)\right]$ are known.

It is expected that the method outlined in sections 2 and 3, although specifically applicable to an edge- or center-cracked panel, can be generally applied to other cracked-body configurations subjected to displacement type loadings. The results of the method described here are particularly useful if the stress intensity factor is known only as a function of the applied load and it is desired as a function of the normal displacement, or vice versa.

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