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EXTERIOR ORIENTATION OF NEAR-VERTICAL AERIAL PHOTOGRAPHS--
COMPUTATIONAL PROCEDURE AND ILLUSTRATED EXAMPLES

VERNON A. RYBSKI
THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION

FEBRUARY 1952

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Vernon A. Rybski
The Ohio State University Research Foundation

February 1952

Photo Reconnaissance Laboratory
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**Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio**

FOREWORD

This report was prepared by the Mapping and Charting Research Laboratory of the Ohio State University Research Foundation, under USAF Contract No. AF 18(600)90. The contract was administered under the direction of the Mapping and Charting Branch, Photographic Reconnaissance Laboratory, Air Research and Development Command, Wright-Patterson Air Force Base, Dayton, Ohio with Mr. D. L. Radcliffe, Chief of the Mapping and Charting Branch, as Project Engineer.

Research and Development Order No. R683-44, "Charting, Aeronautical, Photogrammetry and Geodesy," and R683-58, "Aeronautical Charting Systems," are applicable to this report.

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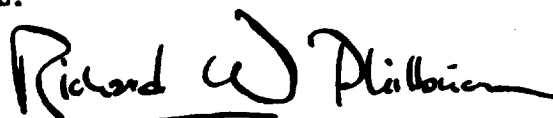
ABSTRACT

An analytical method of determining the exterior orientation of a near-vertical photograph is described. The theory, computing procedure, test results, and sample computations using fictitious photographs are presented. It is concluded that the method is rapid and meets practical accuracy needs for photographs containing tilts up to three degrees.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDING GENERAL:



f GORDON A. BLAKE
Brigadier General, USAF
Chief, Weapons Components Division

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EXTERIOR ORIENTATION OF NEAR-VERTICAL AERIAL PHOTOGRAPHS - COMPUTATIONAL PROCEDURE AND ILLUSTRATED EXAMPLES

INTRODUCTION

The analytical method of determining the elements of exterior orientation of an aerial photograph presented here is unique in that a complete determination of tilt is made without computing the space position of the photographic exposure station. Eliminating the necessity for the space position determination greatly simplifies the computing procedure.

The theory for this method of determining tilt was developed by Mr. W. O. Byrd and presented as part of Technical Paper No. 142, "Some Elementary Aspects of Computational Problems of Photogrammetry," prepared at this Laboratory. In that paper he developed several methods of determining the elements of exterior orientation and exposure station position, or combinations of both. The test results and computational procedures of these methods will be subjects for subsequent papers. In this paper the theory presented by Mr. Byrd is restated.

The computational procedure for the method presented here is shown as a computing form. Sample computations, fictitious photographs for test purposes, and a guide form indicating all computing operations are included.

SECTION I

THEORY

1.1 A restatement of the theory pertaining to this method of tilt analysis is required for a thorough study of the computational procedure and final results. The evolution of the theory and the equations, as presented here, have been taken directly from Technical Paper No. 142 by Mr. W. O. Byrd of this Laboratory.

1.2 The direction cosines relating the photographic coordinate system with the ground coordinate system are tabulated as follows:

	cos x	cos y	cos z
X	m_1	n_1	k_1
Y	m_2	n_2	k_2
Z	m_3	n_3	k_3

These nine elements comprise the basic concept governing the orientation between the two coordinate systems.

1.3 Consider now a directed line, $\overline{PL}$, that intersects the ground plane at point $P(X_P, Y_P, Z_P)$, the photo plane at $p(x_p, y_p)$, and the exposure station at $L(X_L, Y_L, Z_L)$. Then the direction cosines of this line with respect to each coordinate system are expressed as follows:

with respect to the Ground System

	cos X	cos Y	cos Z
$\overline{PL}$	P_1	P_2	P_3

and

with respect to the Photographic System

	cos x	cos y	cos z
$\overline{pL}$	P_1	P_2	P_3

where

$$\begin{aligned} P_1 &= \frac{X_L - X_P}{PL} = m_1 p_1 + n_1 p_2 + k_1 p_3 \\ P_2 &= \frac{Y_L - Y_P}{PL} = m_2 p_1 + n_2 p_2 + k_2 p_3 \\ P_3 &= \frac{Z_L - Z_P}{PL} = m_3 p_1 + n_3 p_2 + k_3 p_3 \end{aligned} \quad (I-1)$$

and

$$\begin{aligned} p_1 &= -\frac{x_p}{pL} = m_1 P_1 + m_2 P_2 + m_3 P_3 \\ p_2 &= -\frac{y_p}{pL} = n_1 P_1 + n_2 P_2 + n_3 P_3 \\ p_3 &= +\frac{f}{pL} = k_1 P_1 + k_2 P_2 + k_3 P_3 . \end{aligned} \quad (I-2)$$

1.4 From the above relationships, equations may be written for the determination of either the photographic coordinates or the ground coordinates in terms of the other coordinate system and the nine direction cosine elements. By ratioing, substituting, and reducing the above relationships, the photographic coordinates are expressed thus,

$$\begin{aligned} -\frac{x_p}{f} &= \frac{m_1 X_P + m_2 Y_P + m_3 Z_P + m_4}{k_1 X_P + k_2 Y_P + k_3 Z_P + k_4} \\ -\frac{y_p}{f} &= \frac{n_1 X_P + n_2 Y_P + n_3 Z_P + n_4}{k_1 X_P + k_2 Y_P + k_3 Z_P + k_4} \end{aligned} \quad (I-3)$$

where

$$m_4 = - (m_1 X_L + m_2 Y_L + m_3 Z_L)$$

$$n_4 = - (n_1 X_L + n_2 Y_L + n_3 Z_L)$$

$$k_4 = - (k_1 X_L + k_2 Y_L + k_3 Z_L) .$$

Rewriting the above equations, we have

$$\begin{aligned} m_1 X_P + m_2 Y_P + m_3 Z_P + m_4 + \frac{k_1}{f} (x_p X_P) \\ + \frac{k_2}{f} (x_p Y_P) + \frac{k_3}{f} (x_p Z_P) + \frac{k_4}{f} x_p = 0 \end{aligned} \quad (I-4)$$

and

$$\begin{aligned} n_1 X_P + n_2 Y_P + n_3 Z_P + n_4 + \frac{k_1}{f} (y_p X_P) \\ + \frac{k_2}{f} (y_p Y_P) + \frac{k_3}{f} (y_p Z_P) + \frac{k_4}{f} y_p = 0 . \end{aligned} \quad (I-5)$$

1.5 In order to eliminate m_4 and n_4 , substitute into I-4 and I-5 the coordinates of two points. Let the ground coordinates of the points be $A(X_A, Y_A, Z_A)$ and $B(X_B, Y_B, Z_B)$, and the photographic coordinates are then $a(x_a, y_a)$ and $b(x_b, y_b)$. Subtracting the two equations will form the following:

$$\begin{aligned} m_1(X_B - X_A) + m_2(Y_B - Y_A) + m_3(Z_B - Z_A) \\ + \frac{k_1}{f} (x_b X_B - x_a X_A) + \frac{k_2}{f} (x_b Y_B - x_a Y_A) \\ + \frac{k_3}{f} (x_b Z_B - x_a Z_A) \\ + \frac{k_4}{f} (x_b - x_a) = 0 \end{aligned} \quad (I-6)$$

and

$$\begin{aligned} & n_1(X_B - X_A) + n_2(Y_B - Y_A) + n_3(Z_B - Z_A) \\ & + \frac{k_1}{f} (y_b X_B - y_a X_A) + \frac{k_2}{f} (y_b Y_B - y_a Y_A) \\ & + \frac{k_3}{f} (y_b Z_B - y_a Z_A) + \frac{k_4}{f} (y_b - y_a) = 0. \end{aligned} \quad (I-7)$$

Eliminating $\frac{k_4}{f}$ by multiplying (I-6) by $(y_b - y_a)$ and (I-7) by $(x_b - x_a)$, then subtracting (I-7) from (I-6), we have

$$\begin{aligned} & m_1(y_b - y_a)(X_B - X_A) + m_2(y_b - y_a)(Y_B - Y_A) \\ & + m_3(y_b - y_a)(Z_B - Z_A) - n_1(x_b - x_a)(X_B - X_A) \\ & - n_2(x_b - x_a)(Y_B - Y_A) - n_3(x_b - x_a)(Z_B - Z_A) \\ & + \frac{k_2}{f} (x_a y_b - x_b y_a) (X_B - X_A) \\ & + \frac{k_2}{f} (x_a y_b - x_b y_a) (Y_B - Y_A) \\ & + \frac{k_3}{f} (x_a y_b - x_b y_a) (Z_B - Z_A) = 0. \end{aligned} \quad (I-8)$$

1.6 For near-vertical photography, the elements of exterior orientation are defined as

$$(\alpha - s) = 180 + \beta$$

$$t_x$$

$$t_y$$

where

$$s = \tan^{-1} \frac{t_y}{t_x}$$

$$t = \sqrt{t_x^2 + t_y^2},$$

then, the nine direction cosines relating the coordinate systems in terms of the small angles β , t_x , and t_y are determined and tabulated thus,

Near Vertical

$$(\alpha - \delta) = 180 + \beta$$

	cos x	cos y	cos z
X	$+1 - \frac{\beta^2}{2} - \frac{t_y^2}{2}$	$+\beta - \frac{t_x t_y}{2}$	$+t_y + \beta t_x$
Y	$-\beta - \frac{t_x t_y}{2}$	$+1 - \frac{\beta^2}{2} - \frac{t_x^2}{2}$	$+t_x - \beta t_y$
Z	$-t_y$	$-t_x$	$+1 - \frac{t_x^2}{2} - \frac{t_y^2}{2}$

The complete derivation for each of these elements to terms of the second order is shown in Technical Paper 142, pages 111-112.

Substituting the near-vertical orientation elements in equation I-8, grouping the first and second order terms, and letting

$$X_{AB} = X_B - X_A$$

$$Y_{AB} = Y_B - Y_A$$

$$Z_{AB} = Z_B - Z_A$$

$$x_{ab} = x_b - x_a$$

$$y_{ab} = y_b - y_a$$

$$w_{ab} = x_a y_b - x_b y_a$$

gives the final equation as

$$\begin{aligned}
 & - (x_{ab} X_{AB} + y_{ab} Y_{AB}) \beta + (x_{ab} Z_{AB} + \frac{w_{ab} Y_{AB}}{f}) t_x \\
 & + (\frac{w_{ab} X_{AB}}{f} - y_{ab} Z_{AB}) t_y \\
 & + (y_{ab} X_{AB} - x_{ab} Y_{AB} + \frac{w_{ab} Z_{AB}}{f}) + G_{AB} = 0
 \end{aligned} \tag{I-9}$$

where

$$\begin{aligned}
 G_{AB} = & + (\frac{w_{ab} X_{AB}}{f}) \beta t_x - (\frac{w_{ab} Y_{AB}}{f}) \beta t_y \\
 & + (x_{ab} Y_{AB} - y_{ab} X_{AB}) \frac{\beta^2}{2} \\
 & + (x_{ab} Y_{AB} - \frac{w_{ab} Z_{AB}}{f}) \frac{t_x^2}{2} \\
 & - (y_{ab} X_{AB} + \frac{w_{ab} Z_{AB}}{f}) \frac{t_y^2}{2} \\
 & + (x_{ab} X_{AB} - y_{ab} Y_{AB}) \frac{t_x t_y}{2} .
 \end{aligned}$$

The minimum data required to solve for β , t_x , and t_y is three control points. Given the coordinates of these points, three linear non-homogeneous equations can be formed from I-9 by letting G_{ab} equal zero in the first solution. If a second solution is required, the constant terms in the equations can be corrected by using the first solution values for β , t_x , and t_y and computing and applying G_{ab} as illustrated in the appendix (page 17).

1.7 If desired, the space position of the exposure station can easily be determined by the following sequence of equations:

$$Z_L = Z_P - X_P \frac{m_3(-x_p) + n_3(-y_p) + k_3 f}{m_1(-x_p) + n_1(-y_p) + k_1 f} \quad (I-10)$$

or

$$Z_L = Z_P - Y_P \frac{m_3(-x_p) + n_3(-y_p) + k_3 f}{m_2(-x_p) + n_2(-y_p) + k_2 f} .$$

Using the computed Z_L the other coordinates of the exposure station are then determined by

$$X_L = X_P + (Z_L - Z_P) \frac{m_1(-x_p) + n_1(-y_p) + k_1 f}{m_3(-x_p) + n_3(-y_p) + k_3 f} \quad (I-11)$$

$$Y_L = Y_P + (Z_L - Z_P) \frac{m_2(-x_p) + n_2(-y_p) + k_2 f}{m_3(-x_p) + n_3(-y_p) + k_3 f} .$$

For any further investigation requiring the use of the photographs, it will be necessary to compute the nine direction cosine elements from the determined values of exterior orientation. Knowing these values, the coordinates of the space position can readily be determined from I-10 and I-11. The equations for evaluating the nine cosines are given on page 25 of Technical Paper No. 142.

SECTION II

FICTITIOUS PHOTOGRAPHS

2.1 In order to test this method for determining tilt, a set of three overlapping fictitious photographs were prepared. The overlapping condition is not needed in this case, but was incorporated for use in future tests. In Figure 1 the exterior orientation elements and space position of each photograph are listed with a diagram showing the approximate position of the computed photo points. In Table I the ground coordinates and corresponding computed photographic coordinates of each point are listed.

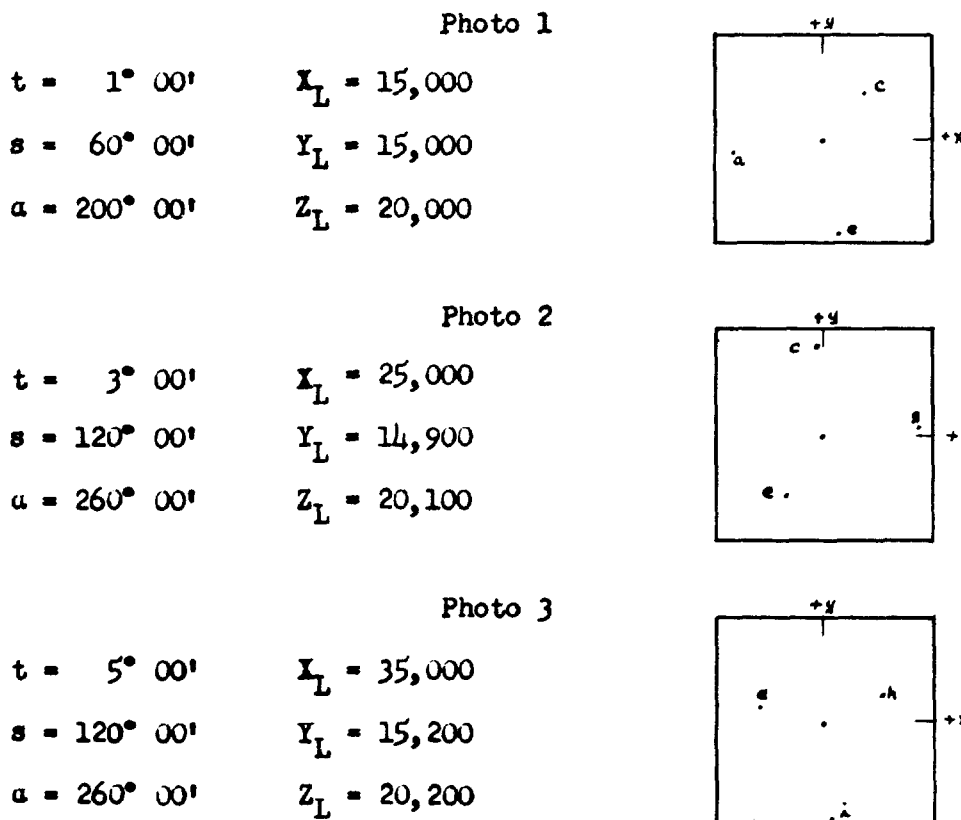


Figure 1 - Orientation Elements and Exposure
Station Coordinates in Feet

Point	Ground Coordinates (feet)			Photographic Coordinates (millimeters)			
	X	Y	Z	Photo 1		Photo 2	
				x	y	x	y
a - 1	5,000	5,000	100	- 102.7739	- 7.8870		
c - 5	15,000	25,000	300	+ 51.6387	+ 60.1116	- 1.9404	+ 101.3111
e - 1	25,000	5,000	500	+ 11.6937	- 106.5058	- 41.7523	- 61.7749
g - 5	35,000	25,000	200			+ 117.1065	+ 6.2006
h - 5	40,000	25,000	100			+ 59.1888	+ 50.3569
e - 2	25,000	10,000	500			+ 89.6699	+ 26.4577
i - 1	45,000	5,000	600			- 69.1607	+ 11.4203
						+ 20.3766	- 120.1842

Table 1 - Photo and Ground Coordinates

2.2 A focal length of 150 mm was used for all fictitious photo computations, and the photo coordinates were computed by the method presented in Technical Paper No. 34, "Fictitious Photographs for Research," by W. O. Byrd of this Laboratory.

SECTION III

COMPUTING FORM

3.1 The final equation (I-9) on which this method is based can be conveniently adapted to a computing form which clearly illustrates the computing procedures. A form for the complete tilt determination is shown in the appendix (page 17). As noted in the form, an attempt has been made to place and group all values, terms, and operations in the most advantageous position for the computer's use.

3.2 All but one of the computing operations are shown on the form. This one exception, the solution of the three non-homogeneous linear equations, can be accomplished by several different methods. In this particular case, the solution was performed by determinants according to Cramer's rule. By this method the second solution values, as indicated on the form, can be conveniently obtained and with a minimum number of computing operations. In order to illustrate the method used in solving the three operations, a guide form (on page 18) was prepared. Since the computing form indicates all other operations, the guide form, in most cases, will be used by the computer only to become familiar with the equation solution. However, all operations are shown on the guide form.

3.3 According to the assumption $(u - s) = 180 + \beta$, angle β must be small; therefore, it may be necessary to rotate the photo coordinates through an angle θ . The angle θ can be determined computationally or graphically, whichever is preferred, and the rotation of coordinates is then performed as indicated on the form. If θ is determined graphically,

the plot can be used to check approximately the magnitude and sign of the rotated photo coordinates. To determine θ graphically, a plot of the ground points, at some convenient scale, is prepared on vellum or tracing paper. On the photo, radial lines from the principal point are drawn through the image points. The vellum plot of the ground points is superimposed on the radial lines of the photo images; θ is the angle between the photo and ground x-axes.

3.4 For convenience of computation, the factor of 10^{-3} was used. Since each term of the basic equation (I-9) contains a ground coordinate value, the introduction of this factor has no effect. Also, in the formation of the three equations, the factor merely displaces the decimal point in all terms.

SECTION IV

TEST RESULTS

4.1 Computations were made for each of the three fictitious photographs, with angle β the same magnitude as the tilt; also, computations were made for Photo 3a, with angle β changed to one degree. A complete set of computations are included in the appendix. The results of the computations were tabulated as follows:

Photo 1

<u>Actual Values</u>	<u>Computed Values</u>			
	First Solution	Error	Second Solution*	Error
$\beta = 1^\circ 00'$	$0^\circ 59.9'$	- 0.1'		
$t = 1^\circ 00'$	$0^\circ 59.5'$	- 0.5'		
$s = 60^\circ 00'$	$59^\circ 39.1'$	- 20.9'		
$\alpha = 200^\circ 00'$	$199^\circ 39.2'$	- 20.8'		
$\theta = 39^\circ 00'$				

Photo 2

<u>Actual Values</u>	<u>Computed Values</u>			
	First Solution	Error	Second Solution*	Error
$\beta = 3^\circ 00'$	$3^\circ 00.2'$	+ 0.2'	$2^\circ 59.8'$	- 0.2'
$t = 3^\circ 00'$	$2^\circ 57.6'$	- 2.4'	$2^\circ 59.4'$	- 0.6'
$s = 120^\circ 00'$	$119^\circ 43.8'$	- 16.2'	$119^\circ 59.2'$	- 0.8'
$\alpha = 260^\circ 00'$	$259^\circ 43.6'$	- 16.4'	$259^\circ 59.4'$	- 0.6'
$\theta = 37^\circ 00'$				

*Second order term solution

Photo 3

<u>Actual Values</u>	<u>Computed Values</u>			
	First Solution	Error	Second Solution*	Error
$\beta = 5^{\circ} 00'$	$4^{\circ} 58.1'$	- 1.9'	$4^{\circ} 59.4'$	- 0.6'
$t = 5^{\circ} 00'$	$5^{\circ} 29.1'$	+ 29.1'	$4^{\circ} 53.4'$	- 6.6'
$s = 120^{\circ} 00'$	$113^{\circ} 35.1'$	+ $6^{\circ} 24.9'$	$117^{\circ} 40.0'$	- $2^{\circ} 20.0'$
$\alpha = 260^{\circ} 00'$	$253^{\circ} 37.0'$	- $6^{\circ} 23.0'$	$257^{\circ} 40.6'$	- $2^{\circ} 19.4'$
$\theta = 37^{\circ} 00'$				

Photo 3a

<u>Actual Values</u>	<u>Computed Values</u>			
	First Solution	Error	Second Solution*	Error
$\beta = 1^{\circ} 00'$	$0^{\circ} 57.0'$	- 3.0'	$1^{\circ} 00.2'$	+ 0.2'
$t = 5^{\circ} 00'$	$5^{\circ} 29.1'$	+ 29.1'	$4^{\circ} 53.4'$	- 6.6'
$s = 120^{\circ} 00'$	$117^{\circ} 41.6'$	- $2^{\circ} 18.2'$	$120^{\circ} 27.4'$	+ $27.4'$
$\alpha = 260^{\circ} 00'$	$257^{\circ} 44.3'$	- $2^{\circ} 15.2'$	$260^{\circ} 27.2'$	+ $27.2'$
$\theta = 39^{\circ} 00'$				

*Second order term solution

In Table II only the errors of the final results are tabulated for comparison.

Photo	<u>Actual Values</u>			<u>Errors - Computed Values</u>		
	β	t	s	β	t	s
1	1° 00'	1° 00'	60° 00'	- 0.1'	- 0.5'	- 20.9'
2	3° 00'	3° 00'	120° 00'	- 0.2'	- 0.6'	- 0.8'
3	5° 00'	5° 00'	120° 00'	- 0.6'	- 6.6'	- 2° 20.0'
3a	1° 00'	5° 00'	120° 00'	+ 0.2'	- 6.6'	+ 27.4'

Table II - Errors in Final Results

Since α ($\alpha = 180 + \beta + s$) is defined in terms of β and s , the magnitude of error in α will be the sum of errors in β and s . Hence, in Table II α was omitted.

SECTION V

CONCLUSIONS

5.1 The development of the general basic theory, which is presented in Technical Paper No. 142, should be emphasized because of its significant importance to research problems in which photographs are used. In this particular case, the theory has given rise to a practical and rapid analytical method of determining the photographic elements of exterior orientation.

5.2 For photographs with tilts up to three degrees, as indicated by the results shown in Table II, the method meets practical accuracy needs for a determination of exterior orientation elements. Since the majority of aerial photography flown today contains less than $3^{\circ} 00'$ and usually closer to $1^{\circ} 00'$ tilts, this method is applicable and will prove to be a useful tool in photogrammetric research and mapping problems.

5.3 The time required for performing a complete solution, including second order terms, is about two hours. However, due to the small tilts in a majority of the photographs, the second solution will be unwarranted and the computing time will be reduced considerably.

5.4 The results of Photo 3a show that an improvement of $4^{\circ} 00'$ in β angle has little effect on the resulting tilt angle, but does increase the accuracy of the swing angle.

5.5 The method as presented does not provide a check on the computations. However, the completed computations can be checked by substituting β , t_x , and t_y in the original equations.

APPENDIX I

EXTERIOR ORIENTATION COMPUTATION GUIDE FORM

PHOTO NO. _____ CAMERA NO. _____ FOCAL LENGTH _____ DATE _____ COMPUTER _____

Point	PHOTO COORDINATES		ROTATED PHOTO COORDINATES		GROUND COORDINATES	
	x	y	$x' = x \cos \theta - y \sin \theta$	$y' = y \cos \theta + x \sin \theta$	$X \cdot 10^{-3}$	$Z \cdot 10^{-3}$
A	x_a	y_a	$x_a \cos \theta - y_a \sin \theta$	$x_a \sin \theta + y_a \cos \theta$	$X_A \cdot 10^{-3}$	$Z_A \cdot 10^{-3}$
B	x_b	y_b	$x_b \cos \theta - y_b \sin \theta$	$x_b \sin \theta + y_b \cos \theta$	$X_B \cdot 10^{-3}$	$Z_B \cdot 10^{-3}$
C	x_c	y_c	$x_c \cos \theta - y_c \sin \theta$	$x_c \sin \theta + y_c \cos \theta$	$X_C \cdot 10^{-3}$	$Z_C \cdot 10^{-3}$

EQUATIONS

Line	$\frac{d}{dx}$	$\frac{d}{dy}$	$\frac{d}{dz}$	$\frac{d}{d\theta}$	$\frac{d}{d\phi}$	$\frac{d}{d\psi}$	$\frac{d}{d\tau}$	$\frac{d}{d\delta}$	$\frac{d}{d\epsilon}$
1	AB	$-(1.4+2.3) \cdot 10^{-3}$	$(3.6+4.5) \cdot 10^{-3}$	$(1.6-2.5) \cdot 10^{-3}$	$(1.2-3.4+5.6) \cdot 10^{-3}$	AB	g	$g + G_{AB}$	$g + G_{AB}$
2	BC		Same	for points BC					
3	AC		Same	for points AC					

2ND ORDER TERMS

Coefficients - 1st Solution Values	AB	BC	AC
$\frac{d^2}{dx^2}$	$(1.6) \cdot 2B \cdot 1_x$		
$\frac{d^2}{dy^2}$	$(3.6) \cdot 2B \cdot 1_y$		
$\frac{d^2}{dz^2}$	$(3.4-1.2) \cdot \beta^2$	same	same
$\frac{d^2}{d\theta^2}$	$(3.4-5.6) \cdot \lambda_{AB}^2$	for	for
$\frac{d^2}{d\phi^2}$	$-(1.2+5.6) \cdot \lambda_{AB}^2$	points	points
$\frac{d^2}{d\psi^2}$	$(1.4-2.3) \cdot \lambda_{AB}^2$	BC	AC
SUM = 0'			
$\theta = \frac{1}{2} \cdot 10^{-3}$			

*For convenience of computation select as point A the smallest and point C the largest Z value.

*Check: $M_d - M_d + M_d = 0$, $M_e - M_e + M_e = 0$, $M_f - M_f + M_f = 0$

$M_g - M_g + M_g = 0$, $M_h - M_h + M_h = 0$, $M_i - M_i + M_i = 0$

* β , λ , λ , and τ are in radians — conversion factor: 1 minute = $2.90888 \cdot 10^{-4}$ radians.

SOLUTION

	MINORS		0
	M	N	
1	$e_2 f_3 - f_2 e_3$	$d_2 f_3 - f_2 d_3$	$d_2 e_3 - e_2 d_3$
2	$e_1 f_3 - f_1 e_3$	$d_1 f_3 - f_1 d_3$	$d_1 e_3 - e_1 d_3$
3	$e_1 f_2 - f_1 e_2$	$d_1 f_2 - f_1 d_2$	$d_1 e_2 - e_1 d_2$
D	$M_d - M_d + M_d$	$M_e - M_e + M_e$	$M_f - M_f + M_f$
β	$(M_d - M_d + M_d) \cdot \frac{1}{\beta}$	$(M_e - M_e + M_e) \cdot \frac{1}{\beta}$	$(M_f - M_f + M_f) \cdot \frac{1}{\beta}$
λ	$(M_g - M_g + M_g) \cdot \frac{1}{\lambda}$	$(M_h - M_h + M_h) \cdot \frac{1}{\lambda}$	$(M_i - M_i + M_i) \cdot \frac{1}{\lambda}$
τ	$(M_j - M_j + M_j) \cdot \frac{1}{\tau}$	$(M_k - M_k + M_k) \cdot \frac{1}{\tau}$	$(M_l - M_l + M_l) \cdot \frac{1}{\tau}$
ϕ	$(M_m - M_m + M_m) \cdot \frac{1}{\phi}$	$(M_n - M_n + M_n) \cdot \frac{1}{\phi}$	$(M_o - M_o + M_o) \cdot \frac{1}{\phi}$
ψ	$(M_p - M_p + M_p) \cdot \frac{1}{\psi}$	$(M_q - M_q + M_q) \cdot \frac{1}{\psi}$	$(M_r - M_r + M_r) \cdot \frac{1}{\psi}$
δ	$(M_s - M_s + M_s) \cdot \frac{1}{\delta}$	$(M_t - M_t + M_t) \cdot \frac{1}{\delta}$	$(M_u - M_u + M_u) \cdot \frac{1}{\delta}$
ϵ	$(M_v - M_v + M_v) \cdot \frac{1}{\epsilon}$	$(M_w - M_w + M_w) \cdot \frac{1}{\epsilon}$	$(M_x - M_x + M_x) \cdot \frac{1}{\epsilon}$

RESULTS

$S_A = 100 \cdot \frac{1}{\lambda}$			
$\tau = (1 + \phi) \cdot \frac{1}{\tau}$			
$S = S_A + \phi$			
$\phi = 180 + S_A + \beta$			

PHOTO NO. _____ / CAMERA NO. _____ FOCAL LENGTH 150 mm DATE _____ COMPUTER 1/e

Point	PHOTO COORDINATES		ROTATED PHOTO COORDINATES		GROUND COORDINATES	
	X	Y	X' = X cos θ - Y sin θ	Y' = X sin θ + Y cos θ	X · 10 ⁻³	Y · 10 ⁻³
A/a-1	-102.774	-7887	●	39° 00'	-74.907	-70.807
B/b-5	+51.639	+60.142	SIN ●	.62932039	+2.282	+79.237
C/c-1	+11.694	-106.506	COS ●	.77714596	+76.114	-75.411

SOLUTION			MINORS	
	M	N		O
1	+1.263659	-6.801425	+2.311209	
2	-1.086623	-6.812821	-2.506516	
3	+6.37948	-0.146210	-6.066417	
1st Solution				
D=10.857323	✓			
$\frac{A}{B} = -1.189336$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{C}{D} = -0.017438$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{E}{F} = 59.9$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
2nd Solution				
$\frac{A}{B} = -1$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{C}{D} =$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{E}{F} =$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\theta_A = 100 \cdot \frac{1}{2}$	37690	20° 39'
$\theta = (100 + \frac{1}{2})\%$	01731	59.5'
$\theta = \theta_A + \theta_1$	59	39.1'
$\theta = 100 + \theta_A + \theta$	199	39.2'

EQUATIONS					
Lens	d	g	f	g	g'
1 AB	-3.772770	-754.402	-4.14929	-057038	
2 BC	-3.831280	$+841846$	-382610	-078111	
3 AC	-3.020420	$+00408$	$+1.473602$	-062645	

2nd ORDER TERMS			
Case Moments - 1st Section Values	AB	BC	AC
$(0-0)$	$\frac{0^2}{2}$		
$(-1)(3-0)$	$\frac{0^2}{2}$		
$(3-4-1-2)$	$\frac{3^2}{2}$		
$(3-4-5-0)$	$\frac{1^2}{2}$		
$(-1)(1-2+3-0)$	$\frac{1^2}{2}$		
$(1-4-2-3)$	$\frac{1}{2} \cdot \frac{4}{2}$		
SUM = 0			
$0 = \frac{1}{2} \cdot 10^3$			

Power	(1)	(2)
1 1 2	$X_2 - X_1$	$\frac{(2)}{2-1}$
A B	+10.000	+150.044
B C	+10.000	-154.648
A C	+20.000	-4.604
	(3)	(4)
	$X_3 - X_1$	$\frac{(4)}{3-1}$
A B	+20.000	+77.189
B C	-20.000	+73.832
A C	0	+151.021
	(5)	(6)
	$X_5 - X_1$	$\frac{(6)}{5-1}$
A B	+200	-38.492
B C	+200	-41.354
A C	+400	+73.588

For coverage of construction start on point A the resistors and point C the largest Z value.

PHOTO NO 2 CAMERA NO. _____
FOCAL LENGTH 150 mm DATE _____
COMPUTER ✓

POINT	PHOTO COORDINATES		ROTATED PHOTO COORDINATES		GROUND COORDINATES	
	x	y	$x'' = x \cos \theta - y \sin \theta$	$y'' = x \sin \theta + y \cos \theta$	x · 10 ⁻³	y · 10 ⁻³
A/5	+117.106	+6.201	●	+87.793	+75.428	
B/5						.200
C/5	+1.940	+101.314	SN ●	-62.532	+79.745	
						.300
C-1	-41.752	-61.775	CS ●	+3.832	-74.463	
						.500

Line	d	g	f	g	g'
1 AB	$(-1.41 \pm 2.8) \times 10^{-3}$	$(3.9 \pm 4.9) \times 10^{-3}$	$(1.9 - 2.4) \times 10^{-3}$	$(1.2 - 3.4) \times 10^{-3}$	$g + g$
2 BC	-3.046300	-0.05231	-1.583952	-0.78422	-0.77148
3 AC	-3.747700	-5.66729	$+3.20842$	-2.09200	-2.08979
4	-3.857430	$+9.04252$	$+5.09987$	-2.342261	-2.33889

2ND ORDER TERMS

Coefficients	Δ	Solution Values	AB	BC	AC
$(4-0) = .000685$	$\frac{1}{2}$		+1.08471	-0.19865	-0.31854
$(-1(3-0)) = .005373$	$\frac{1}{2}$		0	-3.11634	+4.99710
$(3(4-12)) = .002748$	$\frac{1}{2}$		+0.23726	+0.57082	+0.60541
$(3(4-0)) = .000043$	$\frac{1}{2}$		-0.00034	-0.05731	+0.07453
$(-1(1.2+5.0)) = .002626$	$\frac{1}{2}$		+0.20594	+4.03427	-3.89950
$(1.4-2.3) = .000335$	$\frac{1}{2}$		+1.02051	-0.81091	-0.71630
SUM = 6'			+2.54808	+0.44188	+0.74270
$e = \frac{1}{2} \cdot 10^{-3}$			+4.001274	+0.00221	+4.000372

* For convenience of computation select as point A the smallest and point C the largest Z value.

*Checks: $M_d - M_d + M_d = (M_g - M_g + M_g) = 0.1 - 0.1 + 0.1 = D$

Checks: $m_1\phi_1 - m_2\phi_2 + m_3\phi_3 = m_2\phi_2 - m_3\phi_3 + m_4\phi_4 = 0$
 $m_1\phi_1 - m_2\phi_2 + m_3\phi_3 = m_2\phi_2 - m_3\phi_3 + m_4\phi_4 = 0$
 $m_1\phi_1 - m_2\phi_2 + m_3\phi_3 = m_2\phi_2 - m_3\phi_3 + m_4\phi_4 = 0$

$$M_1, M_2, M_3 = M_1, M_2, M_3 = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99$$

SOLUTION			
MINORS			
	M	N	D
1	-579146	-673653	-5574983
2	+1424524	-7663557	-2813375
3	-902558	-6913558	+1669345

1st Solution	
$D = +10.584495$	$\frac{1}{D}$
$B = -4.554862$	$\frac{1}{B} = +.069190$
$B = -.062422$	$\frac{1}{B} = +.006537$
$B = 3 \quad 00.2$	$(\frac{1}{B} + \frac{1}{B})^2 = 2.0576$
2nd Solution	
$B = -4.553474$	$\frac{1}{B} = +.067454$
$B = -.052291$	$\frac{1}{B} = +.006373$
$B = 2 \quad 59.8$	$\frac{1}{B} = +.051800$

$S_N = \tan^{-1} \frac{1}{N}$	+ 8.12804	82°	59.2'
$t = (S_N^2 + \phi)^{\frac{1}{2}}$	.05219	2°	59.4'
$S = S_N + \phi$	119	59.2'	
$D = 180^\circ + S + \phi$	259	59.4'	

EXTERIOR ORIENTATION COMPUTATION

PHOTO NO. 3 CAMERA NO. FOCAL LENGTH 150mm DATE 1/20/55 COMPUTER 1/2

Point	PHOTO COORDINATES		ROTATED PHOTO COORDINATES		GROUND COORDINATES	
	x	y	$x' = x \cos \theta - y \sin \theta$	$y' = x \sin \theta + y \cos \theta$	X · 10 ⁻³	Z · 10 ⁻³
A	+89.670	+26.458	35.000	+73.106	40.000	100
B	-69.161	+11.420	.57357644	-30.314	25.000	500
C	+20.277	-120.184	.81915204	-86.761	45.000	600

EQUATIONS

Line	d	e	f	g	g'
1 AB	(5.1-4.23)10 ⁻³	(3.4+4.5)10 ⁻³	(1.8-2.8)10 ⁻³	(1.8-3.4+5.8)10 ⁻³	0 + 0
2 BC	-3.373530	-3.333983	-2.44022	-2.263320	-2.269525
3 AC	-3.258855	-2.544427	+1.082885	-3.77399	-3.72725
	-3.334085	+1.522495	-2.297272	-2.290076	-2.281805

2ND ORDER TERMS

Coefficients	1st Solution Values	AB	BC	AC
(1.8-0)	-0.01643	+0.46890	-1.76991	+0.61975
(-1.8-0)	-0.08138	-2.32250	-2.19164	+12.27878
(3.4-1.2)	+0.07520	+2.03739	+2.89358	+1.89771
(3.4-0.8)	+0.00359	+0.65145	-0.26908	-0.18202
(-1.2+5.8)	+0.08806	-13.72776	+9.59402	+7.37111
(1.4-2.3)	+0.01778	+0.48121	+4.79062	-5.44174
SUM = 0				
$\sigma = \frac{1}{2} \cdot 10^{-3}$				

*For convenience of computation select as point A the smallest and point C the largest Z value.

*Checks: $M_1d_1 + M_2d_2 + M_3d_3 = (N_1e_1 + N_2e_2 + N_3e_3) = 0$; $N_1d_1 + N_2d_2 + N_3d_3 = 0$; $N_1e_1 + N_2e_2 + N_3e_3 = 0$

* σ , σ_x , σ_y , and σ_z are in radians — conversion factor: 1 minute = 2.90888 · 10⁻⁴ radians.

SOLUTION

MINORS	N	
	M	Q
1	-1.573053	+4.579197
2	+4.70806	+1.89266
3	-4.23751	-4.448397
1st Solution		
D	+8.253852	✓
E	+7.5760	$\frac{1}{2} \sqrt{4 + 152.386}$
F	-0.86718	$\frac{1}{2} \sqrt{4 + 152.386}$
G	4	$\frac{1}{2} \sqrt{4 + 152.386}$
2nd Solution		
D	+7.18073	$\frac{1}{2} \sqrt{4 + 152.386}$
E	-0.87095	$\frac{1}{2} \sqrt{4 + 152.386}$
F	4	$\frac{1}{2} \sqrt{4 + 152.386}$

RESULTS

$S_R = \tan^{-1} \frac{y}{x}$	7.77105	82°	40.0'
$\theta = (\frac{1}{2} + \frac{1}{2}) \frac{1}{2}$	.08535	4°	53.4'
$S = S_R + \theta$	117	40.0	
$\sigma = 180^\circ + S_R + \theta$	257	40.6	

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