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NOTES ON ALTITUDE DELAY SETTINGS

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WRIGHT AIR DEVELOPMENT CENTER

NOTICES

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The Ohio State University Research Foundation

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FOREWORD

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ABSTRACT

In many of the current radar sets, ground-range presentation is not available. However, partial compensation for the slant-range distortion can be obtained by changing all of the ranges by a constant amount, known as the "altitude delay." A basis for selecting the magnitude of this delay is described in this paper and a graph is provided for its determination.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDING GENERAL:

Richard W Philbin

for GORDON A. BLAKE
Brigadier General, USAF
Chief, Weapons Components Division

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I. Statement of the Problem

1.1 Various methods for compensating for the effect of the slant range presentation of the customary radar set have been proposed. The most satisfactory method, at least in theory, is the inclusion in the radar set of circuits which automatically compute and present the correct ground range.

1.2 In many of the current radar sets, however, partial compensation for this effect is obtained by changing all the ranges by a constant amount, known as the "altitude delay." Considerable arbitrariness exists in the choice of the value to be used for this delay.

1.3 If the slant ranges are changed by an amount equal to the altitude of the aircraft, the "altitude hole" is eliminated. The resulting presentation is neither "ground range" nor "slant range." When the altitude delay is selected arbitrarily, this last statement usually holds.

1.4 Now, we know that for many combinations of scope setting and altitude the difference between slant and ground ranges is so small, over certain portions of the scope face, that it is incapable of detection. Thus, over these portions of the scope, we may assume that we have a ground range presentation. These considerations suggest the following criterion for selecting the altitude delay.

1.5 For a particular combination of altitude and range setting on the scope, the altitude delay is to be chosen so as to maximize the area on the scope in which the assumption of ground range presentation is valid.

1.6 That this criterion completely determines the altitude delay as a function of aircraft altitude and range setting may be seen in the following section.

II. Determination of the Delay

2.1 Due to the difficulties inherent in the processes of identification and measurement on a radar scope photograph, it seems safe to assume that we cannot determine the position of a "point" on a photograph with a precision exceeding 0.01 inch. Thus, since the radius of the scope face is about 2.5 inches, we cannot determine the range at a "point" on a photograph with a precision better than $(\frac{0.01}{2.5})R_m$, where R_m is the range setting of the scope.

2.2 Let h be the aircraft altitude, d the altitude delay, and let R and r denote the slant and ground ranges, respectively, to a point P on the scope photograph. Then the condition that the assumption of ground range presentation be valid at P is

$$|R-d-r| \leq \frac{R_m}{250},$$

and the condition that this assumption be valid on an interval $R_1 \leq R \leq R_2$ is $|R-d-r| \leq \frac{R_m}{250}$ for $R_1 \leq R \leq R_2$.

2.3 Since our conditions do not involve the azimuth, it follows that the problem of maximizing the area on the scope in which the assumption of ground range presentation is valid is equivalent to maximizing the length of the interval $R_1 \leq R \leq R_2$; i.e., it is equivalent to maximizing the difference $R_2 - R_1$.

2.4 Now,

$$R-r = R-\sqrt{R^2-h^2}$$

is a decreasing function of R. Hence, R_1, R_2, and d satisfy the inequalities

$$\left.\begin{aligned} &R_1-\sqrt{R_1^2-h^2}-d \leq kR_m \\ &d-R_2+\sqrt{R_2^2-h^2} \leq kR_m \\ &h \leq R_1,\ R_2 \leq R_m, \\ &\text{where } k = \frac{1}{250}. \end{aligned}\right\} \quad (1)$$

2.5 Eliminating d from the first two inequalities,

$$R_1 - \sqrt{R_1^2 - h^2} \leq R_2 - \sqrt{R_2^2 - h^2} + 2kR_m . \tag{2}$$

2.6 Hence, if

$$h \leq R_m(1 + 2k) - \sqrt{R_m^2 - h^2} , \tag{3}$$

then

$$R_1 - \sqrt{R_1^2 - h^2} \leq h \leq R_m(1+2k) - \sqrt{R_m^2 - h^2} \leq R_2 - \sqrt{R_2^2 - h^2} + 2kR_m , \tag{4}$$

and the inequality (2) is satisfied by all values of R_1 and R_2 lying between h and R_m . In this case, the difference $R_2 - R_1$ is maximized if we take $R_1 = h$ and $R_2 = R_m$. The value of d is not uniquely determined. Any value of d satisfying

$$h - kR_m \leq d \leq R_m(1 + k) - \sqrt{R_m^2 - h^2} \tag{5}$$

will satisfy the inequalities (1). In order to have a single value for d, we choose the smaller of the two extreme values; i.e.,

$$d = \begin{cases} h - kR_m , & kR_m < h < R_m(1+2k) - \sqrt{R_m^2 - h^2} \\ 0 , & h \leq kR_m \end{cases} \tag{6}$$

2.7 Now, suppose

$$h \geq R_m(1+2k) - \sqrt{R_m^2 - h^2} \tag{7}$$

Then there are values of R_1 and R_2 for which the sign of equality holds in (2). The inequality (2) may be transformed into the inequality

$$R_2 - R_1 \leq \frac{\sqrt{R_2^2 - h^2} - kR_m}{1 + \dfrac{R_2 - \sqrt{R_2^2 - h^2}}{2\,kR_m}} \tag{8}$$

The numerator of this fraction increases with R_2 , while the denominator decreases as R_2 increases. Hence, the fraction increases as R_2 increases and the maximum value of $R_2 - R_1$ is obtained for $R_2 = R_m$;

$$\max.(R_2 - R_1) = \frac{\sqrt{R_m^2 - h^2} - kR_m}{1 + \dfrac{R_m - \sqrt{R_m^2 - h^2}}{2\,kR_m}} = R_m - R_1 . \tag{9}$$

2.8 Let $h_0 = R_m(1+2k) - \sqrt{R_m^2 - h^2}$. (10)

Then

$$R_1 = \frac{R_m^2(1+2k+2k^2) - R_m(1+2k)\sqrt{R_m^2-h^2}}{R_m(1+2k) - \sqrt{R_m^2-h^2}}$$

$$= \frac{h^2 + h_0^2}{2\,h_0}$$

$$\geq h \,, \tag{11}$$

with the equality sign holding if, and only if, $h = h_0$. Thus, the solution given by (9) is valid for $h \geq h_0$, but does not hold when $h < h_0$ (since then $R_1 = h$).

2.9 When $h \geq h_0$,

$$R_1 - \sqrt{R_1^2 - h^2} = \frac{h^2+h_0^2}{2\,h_0} - \sqrt{\left(\frac{h^2+h_0^2}{2\,h_0}\right)^2 - h^2} = h_0 \,; \tag{12}$$

so that, from (1), d must satisfy

$$h_0 - kR_m \leq d \leq h_0 - kR_m ,$$

whence

$$d = h_0 - kR_m = R_m(1+k) - \sqrt{R_m^2 - h^2} \tag{13}$$

2.10 Recapitulating, we have

$$R_2 = R_m \,, \tag{14}$$

$$R_1 = \begin{cases} h \,, & \text{when } h \leq h_0 = R_m(1+2k) - \sqrt{R_m^2 - h^2} \,; \\ \dfrac{h^2+h_0^2}{2\,h_0}, & \text{when } h \geq h_0 \end{cases} \tag{15}$$

$$d = \begin{cases} 0 \,, & \text{when } h \leq kR_m \\ h - kR_m \,, & \text{when } kR_m < h \leq h_0 \\ h_0 - kR_m \,, & \text{when } h_0 \leq h \end{cases} \tag{16}$$

2.11 The curves contained in Figures 1 and 2, pages 5 and 6, illustrate these functions. To facilitate the computations of these curves, the following parameters were introduced:

$$\sin\theta = \frac{h}{R_m} \tag{17}$$

$$p = 1 + \frac{1}{k}\sin^2\frac{\theta}{2} \tag{18}$$

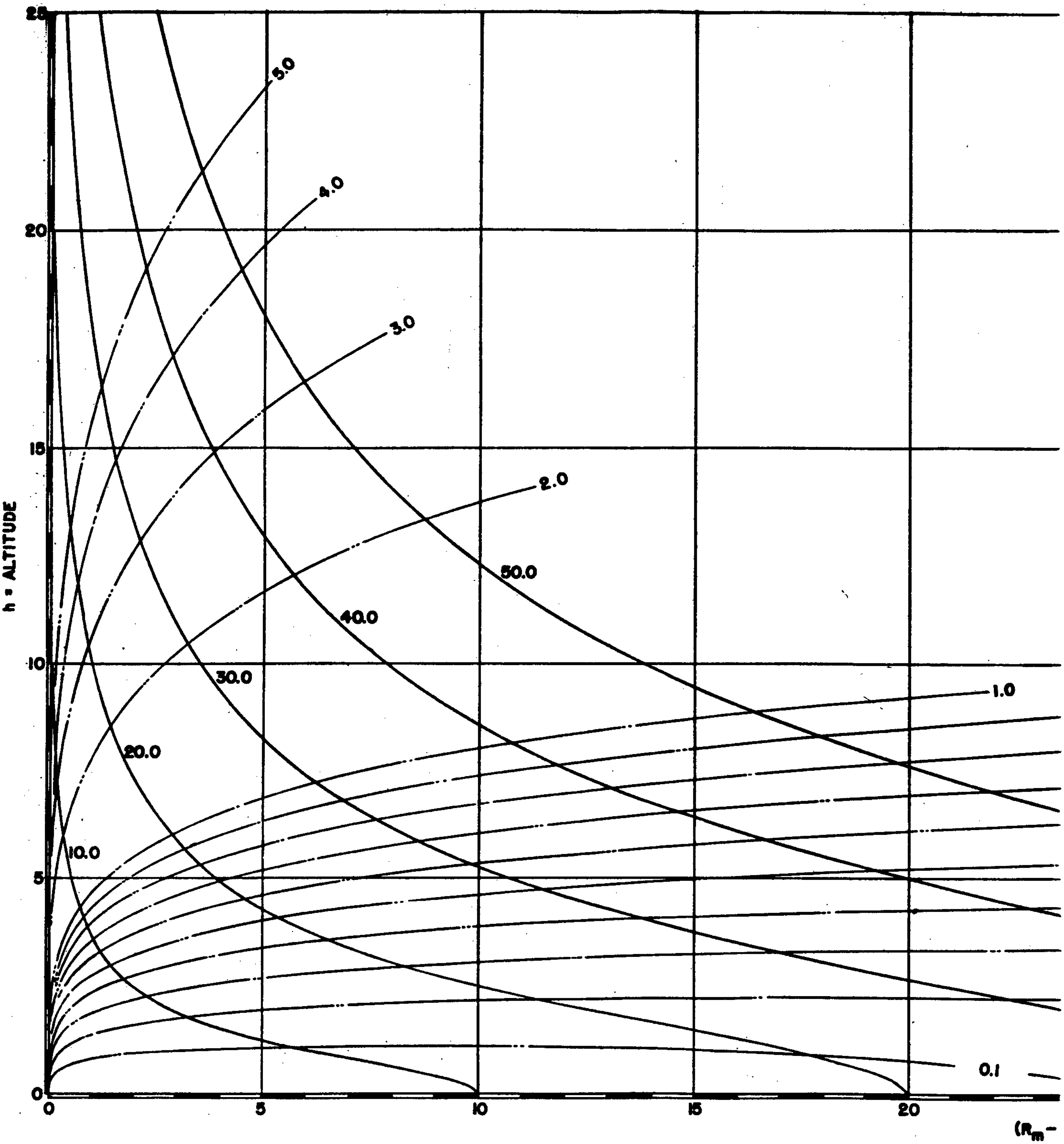
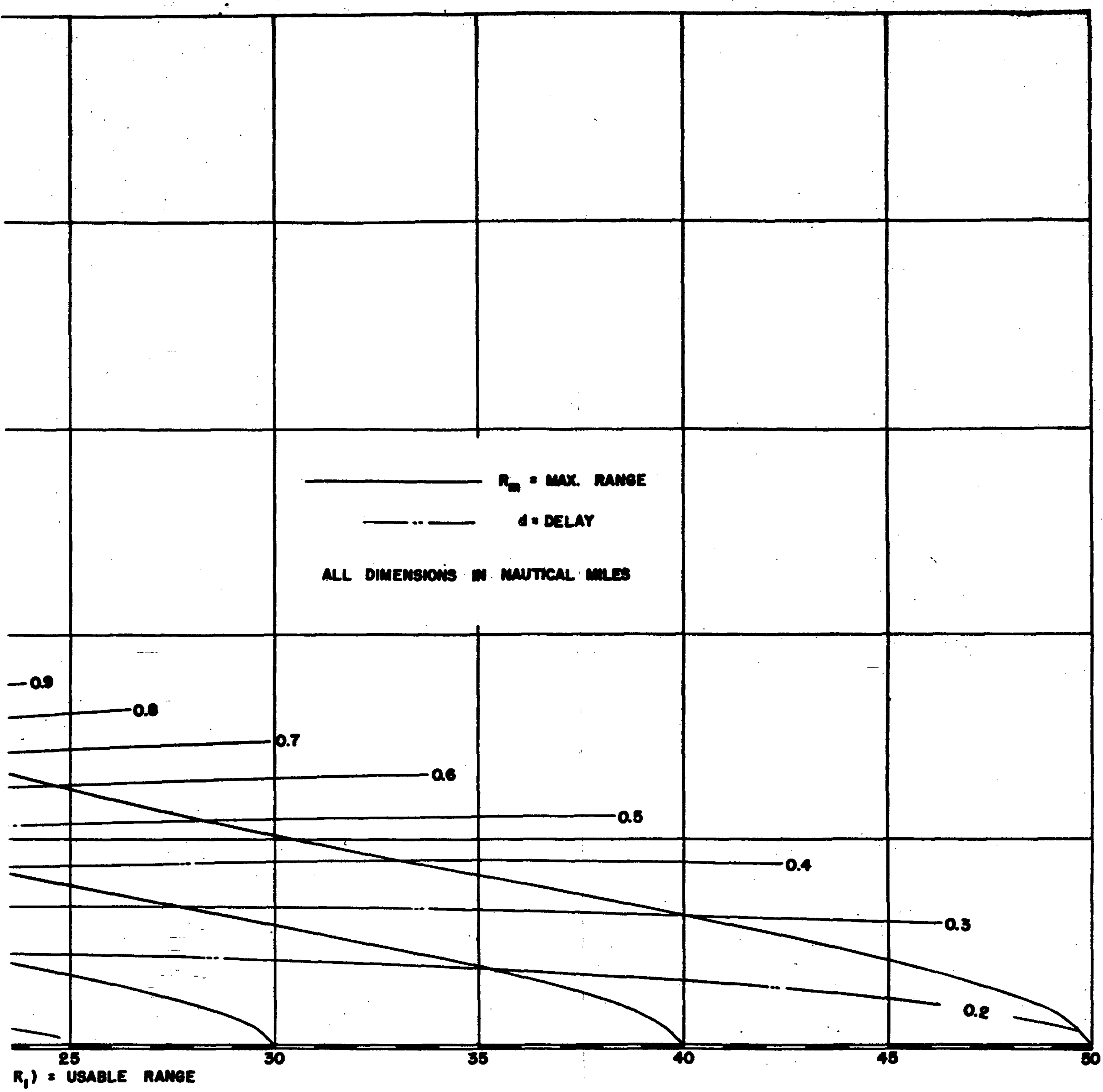
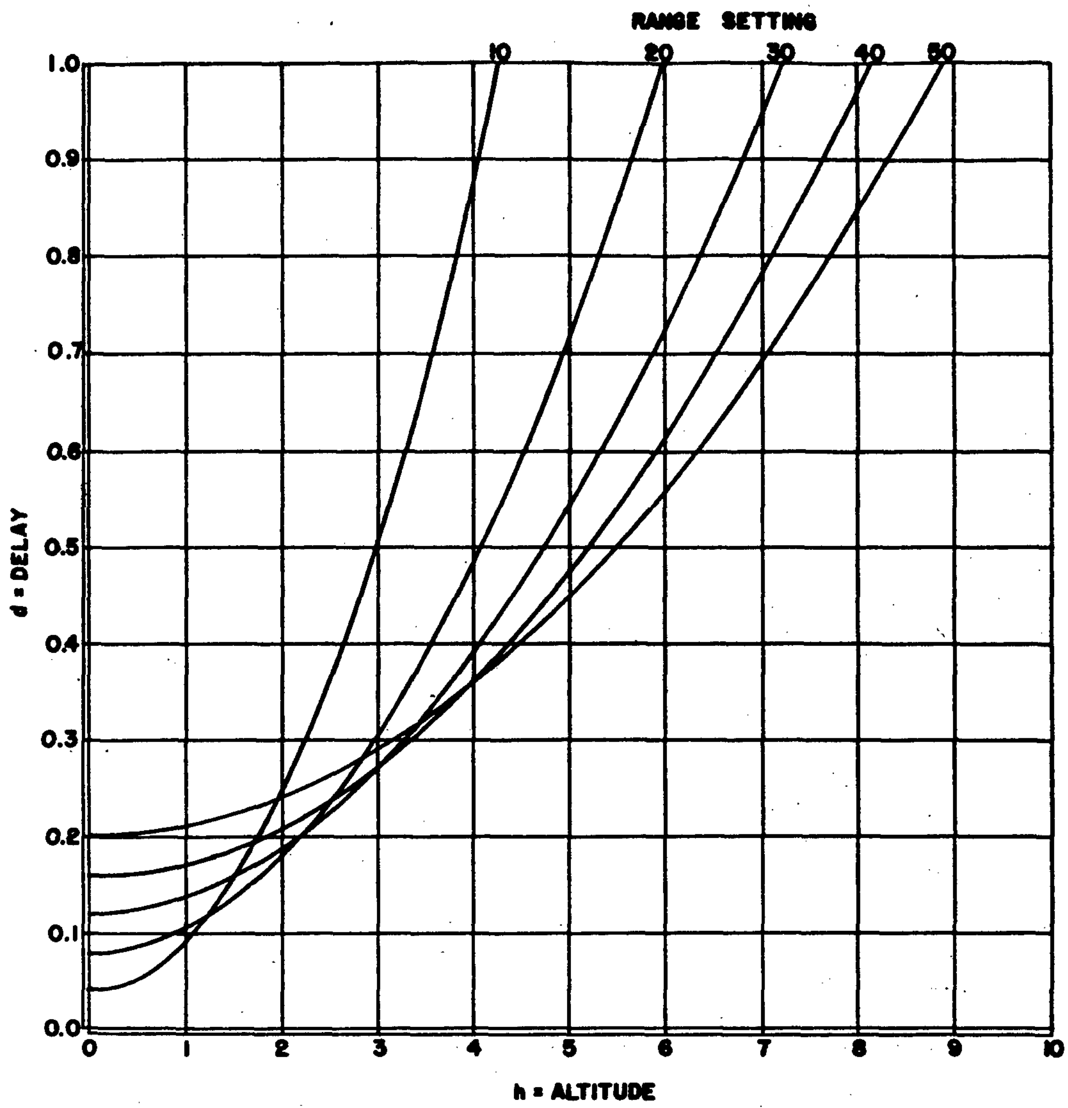

h = ALTITUDE
25
20
15
10
5
0
0
5
10
15
20
(Rm-
5.0
4.0
3.0
2.0
1.0
0.1
10.0
20.0
30.0
40.0
50.0

FIGURE 1



R_m = MAX. RANGE
d = DELAY
ALL DIMENSIONS IN NAUTICAL MILES
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
25
30
35
40
45
50
R_1) = USABLE RANGE

2



ALL DIMENSIONS IN NAUTICAL MILES

FIGURE 2

In terms of these parameters, equations (14), (15), and (16) become

$$\frac{R_m - R_1}{R_m} = \begin{cases} 1 - \sin\theta, \text{ when } \sin\theta \le 2kp \\ \dfrac{1+k}{p} - 2k, \text{ when } \sin\theta \ge 2kp, \end{cases} \tag{19}$$

$$\frac{d}{R_m} = \begin{cases} 0 \quad , \text{ when } \sin\theta \le k \\ \sin\theta - k, \text{ when } k < \sin\theta \le 2kp \\ k(2p-1) \quad , \text{ when } \sin\theta \ge 2kp , \end{cases} \tag{20}$$

where $k = \dfrac{1}{250}$,

2.12 The critical value of θ, θ_o, for which

$$h = R_m(1+2k) - \sqrt{R_m^2 - h^2} \tag{21}$$

may be obtained as follows: Dividing through by R_m,

$$\sin\theta_o = 1 + 2k - \cos\theta_o \ , \tag{22}$$

or

$$\sin(\theta_o + 45^\circ) = \frac{1+2k}{\sqrt{2}} \ , \tag{23}$$

whence,

$$\theta_o = \left(\sin^{-1}\frac{1+2k}{\sqrt{2}}\right) - 45^\circ \ , \tag{24}$$

and

$$\sin\theta_o = \frac{1+2k-\sqrt{1-4k-4k^2}}{2} \ , \tag{25}$$

$$\cos\theta_o = \frac{1+2k+\sqrt{1-4k-4k^2}}{2} \ . \tag{26}$$

2.13 The following values illustrate the reading of the respective figures.

In Figure 1, when $h = 15.0$ and $R_m = 50.0$, $d = 2.5$ and $R_m - R_1 = 7.0$.

In Figure 2, when $h = 3.0$ and $R_m = 30.0$; $d = 0.27$.

III. The Increase in Usable Range

3.1 Figure 3, page 10, illustrates the increase in usable range given by the delays determined from Figure 2 when compared to zero delay (slant range presentation). When no altitude delay is used all ranges satisfying the inequalities

$$\left.\begin{array}{l} R - \sqrt{R^2 - h^2} \leq kR_m, \\ h \leq R \leq R_m \end{array}\right\} \tag{27}$$

may be assumed to be ground ranges. If $h \leq kR_m$, then

$$R - \sqrt{R^2 - h^2} \leq h \leq kR_m, \tag{28}$$

and all ranges from h to R_m are usable as ground ranges. When $h \geq kR_m$, all ranges satisfying

$$R_0 \leq R \leq R_m , \tag{29}$$

where
$$R_0 - \sqrt{R_0^2 - h^2} = kR_m , \tag{30}$$

may be assumed to be ground ranges. We find

$$R_0 = \frac{h^2}{2kR_m} + \frac{kR_m}{2} . \tag{31}$$

3.2 Thus,

$$R_0 = \begin{cases} h & , \quad h \leq kR_m \\ \dfrac{h^2}{2kR_m} + \dfrac{kR_m}{2} , & h \geq kR_m . \end{cases} \tag{32}$$

3.3 The requirement $R_0 \leq R_m$ has yet to be satisfied. We find

$$h^2 \leq R_m^2 (2k - k^2) ,$$

$$\text{or} \quad h \leq R_m \sqrt{2k - k^2} . \tag{33}$$

3.4 Thus, for the usable portion of the scope range, we have

$$R_m - R_0 = \begin{cases} R_m - h & , \text{ for } h \leq kR_m \\ R_m(1 - \frac{k}{2}) - \dfrac{h^2}{2kR_m} ; & \text{for } kR_m \leq h \leq R_m\sqrt{2k - k^2} \\ 0 & , \text{ for } h \geq R_m \sqrt{2k - k^2} ; \end{cases} \tag{34}$$

or in terms of θ ,

$$\frac{R_m - R_o}{R_m} = \begin{cases} 1 - \sin\theta & , \text{ for } \sin\theta \leq k \\ 1 - \dfrac{k}{2} - \dfrac{\sin^2\theta}{2k} & , \text{ for } k \leq \sin\theta \leq \sqrt{2k - k^2} \\ 0 & , \text{ for } \sin\theta \geq \sqrt{2k - k^2} \end{cases} \qquad (35)$$

3.5 The improvement in usable range, measured by $R_o - R_1$, is then given by

$$\frac{R_o - R_1}{R_m} = \begin{cases} 0 & , \text{ for } 0 \leq \sin\theta \leq k \\ \dfrac{k}{2} + \dfrac{\sin^2\theta}{2k} - \sin\theta & , \text{ for } k \leq \sin\theta \leq \sin\theta_o \\ \dfrac{1+k}{p} + \dfrac{\sin^2\theta}{2k} - 1 - \dfrac{3}{2}k & , \text{ for } \sin\theta_o \leq \sin\theta \leq \sqrt{2k - k^2} \\ \dfrac{1+k}{p} - 2k & , \text{ for } \sqrt{2k - k^2} \leq \sin\theta \leq 1 \end{cases} \qquad (36)$$

3.6 The maximum value of $\left(\frac{R_o - R_1}{R_m}\right)$ occurs for $\sin\theta = \sqrt{2k - k^2}$. We find

$$\max\left(\frac{R_o - R_1}{R_m}\right) = \frac{2}{3}(1 - 2k) . \qquad (37)$$

3.7 Equations (36) and (37) were used to compute the curves appearing in Fig. 3.

3.8 The following values illustrate the reading of Figure 3: when $h = 2.37$ and $R_m = 30.0$, $(R_o - R_1) = 15.0$.

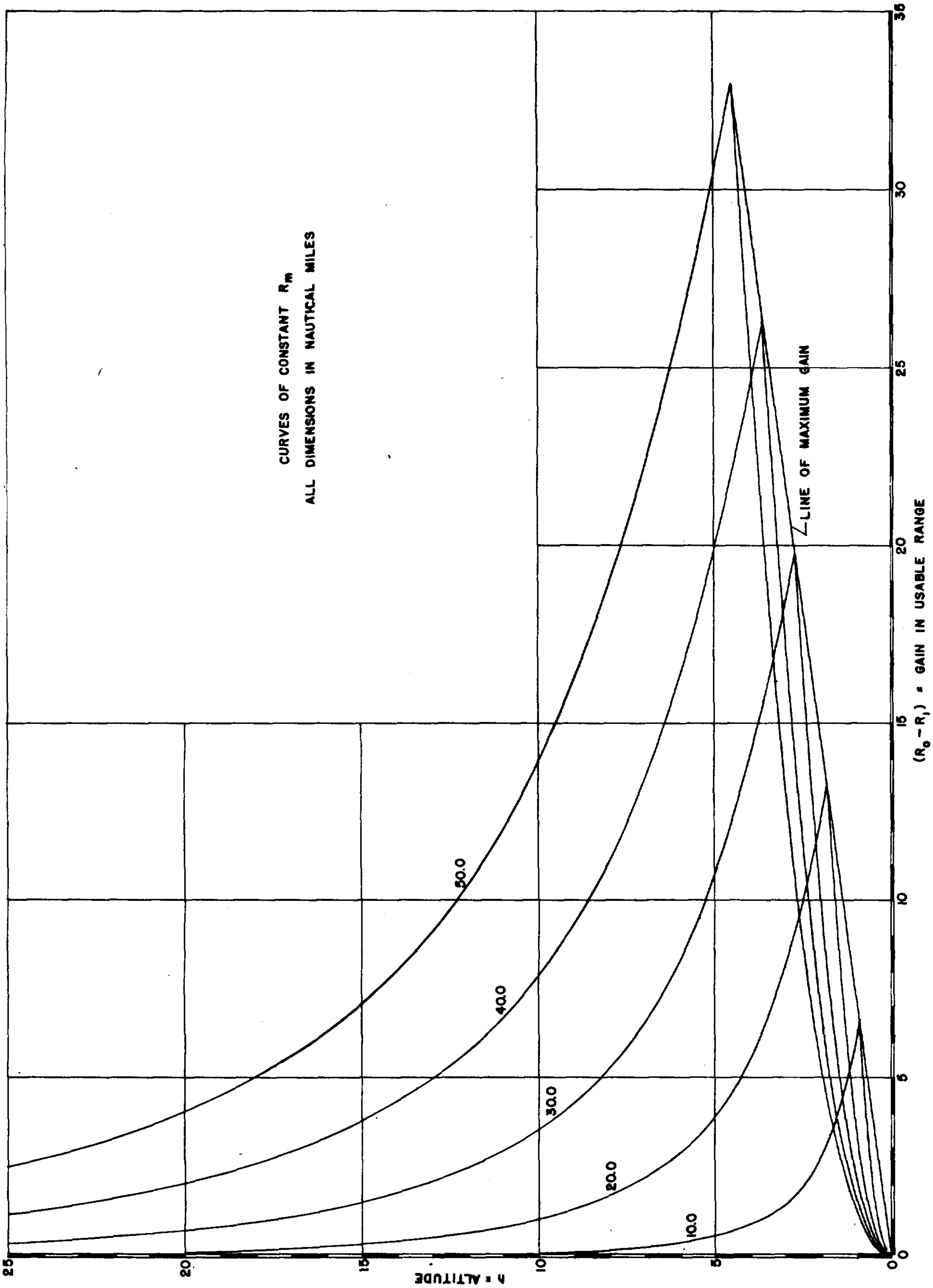

CURVES OF CONSTANT R_m
ALL DIMENSIONS IN NAUTICAL MILES
LINE OF MAXIMUM GAIN
50.0
40.0
30.0
20.0
10.0
$(R_o - R_i)$ = GAIN IN USABLE RANGE
0
5
10
15
20
25
30
35
h = ALTITUDE
0
5
10
15
20
25

FIGURE 3

IV. Conclusion

When the more desirable ground-range presentation is not available and the radar set does permit the use of a constant delay, the magnitude of the delay to be used has an optimum value predicated upon the problem stated in Section I. This optimum value may be obtained directly from Figure 2 with sufficient accuracy.

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