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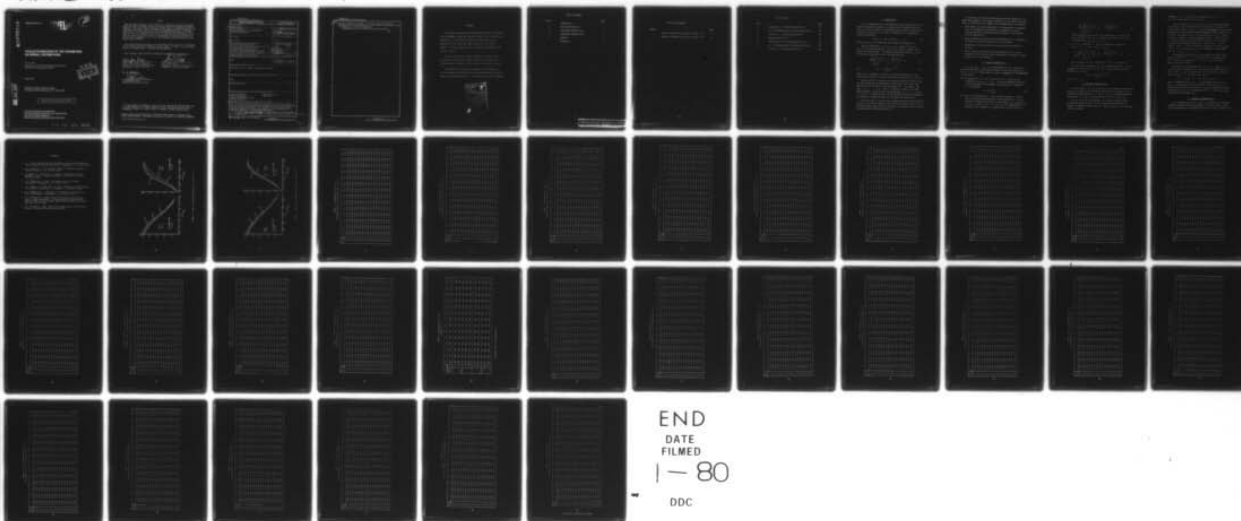
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POOLED ESTIMATIONS OF THE PARAMETERS ON WEIBULL DISTRIBUTIONS

WON J. PARK

MECHANICS AND SURFACE INTERACTIONS BRANCH
NONMETALLIC MATERIALS DIVISION

August 1979

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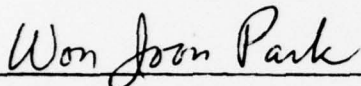
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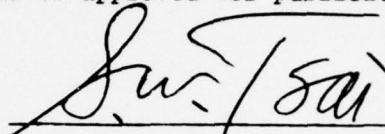
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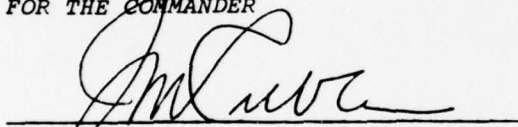


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cont.

the shape parameter, comparison of the bias factor of three pooled estimators of the shape parameter and exact confidence intervals and testing hypotheses for the scale parameters.

Examples are given for composite material applications.



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FOREWORD

This report was prepared in the Mechanics and Surface Interactions Branch (AFML/MBM), Nonmetallic Materials Division, Air Force Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio under Project No. 2303, Task No. 2303/D4. The time period covered by the effort was January 15, 1979 to June 30, 1979.

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1. INTRODUCTION

It is a common practice in metallurgical fatigue testing that m (>1) levels of stress are assumed to be employed with n specimens tested at each of m levels of stress. The underlying distribution of stress cycles to failure at each level (i^{th} level) is assumed to be a two-parameter Weibull distribution,

$$F_i(x) = 1 - \exp \{-(x/b_i)^{c_i}\}, \quad x > 0, \quad (1)$$

where c_i and b_i are the shape and scale parameters respectively.

From the fatigue data $x_{i1}, x_{i2}, \dots, x_{in}$ (under i^{th} level of stress), the unknown parameters c_i and b_i are usually estimated by the maximum likelihood method. The maximum likelihood estimators $\hat{c}_i$ and $\hat{b}_i$ are found to be solutions of the following equations;

$$\frac{\sum_{j=1}^n x_{ij}^{\hat{c}_i} \ln x_{ij}}{\sum_{j=1}^n x_{ij}^{\hat{c}_i}} - \frac{1}{\hat{c}_i} - \frac{\sum_{j=1}^n \ln x_{ij}}{n} = 0 \quad (2)$$

and
$$\hat{b}_i = \left\{ \frac{1}{n} \sum_{j=1}^n x_{ij}^{\hat{c}_i} \right\}^{1/\hat{c}_i}. \quad (3)$$

Cohen [1] has suggested a method of numerical computation of $\hat{c}_i$ and Thoman, Bain and Antle [5] have given statistical inferences on c_i and b_i .

However there is a conjecture and strong experimental evidences that the shape parameters are independent of applied level of stress (see Hahn and Kim [2] and Lipson, Sheth and Desney [3]). This implies then that $c_1 = c_2 = \dots = c_m = c$ and it is needed to estimate the common shape parameter c on the basis of the pooled data x_{ij} , $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. Statistical tests for the equality of the shape parameters c_i were given in Thoman and Bain [4] and Schafer and Sheffied [6].

Wolff and Lemon [7] have recently considered various pooled estimation methods of c to analyze composite materials data, and for the case of $m = 2$, pooling techniques were involed in the study of two-sample tests by Schafer and Sheffied [6] and Thoman and Bain [4].

The problems of estimations and testing hypotheses regarding to the common shape parameter c and scale parameters b_i in the Weibull distributions are considered in this report. The following results are obtained:

1. Three methods of pooled estimations of c (Averaging M.L. estimation, M.L.E. by normalization and Joint M.L. estimation) are given. These methods were introduced by Wolff and Lemon [7] but included here for completeness.
2. Exact confidence intervals and testing hypotheses for c , depending on the methods of pooled estimations of c , are presented.
3. Comparison of the bias factors of the three pooled estimators of c is given.
4. Exact confidence intervals and testing hypotheses for b_i (scale parameter at each level of stress), depending on the method of pooled estimators of c , are presented.

2. POOLED ESTIMATIONS OF c

Let $x_{i1}, x_{i2}, \dots, x_{in}$ be a random sample of size n taken from a Weibull distribution with the shape parameter c and scale parameter b_i , for $i = 1, 2, \dots, m$. Three methods of pooled estimation of c are presented in this section.

- (A) Averaging M.L.E. $\bar{c}$: Let $\hat{c}_i$ be the M.L.E. of c_i based on the observations $x_{i1}, x_{i2}, \dots, x_{in}$. Since the averaging procedure is a common technique in estimations, the averaging M.L. estimator $\bar{c}$ of c is defined by

$$\bar{c} = 1/m \sum_{i=1}^m \hat{c}_i \quad . \quad (4)$$

- (B) M.L.E. by normalization $\tilde{c}$: Since the shape parameter c is free from scale changes (normalization), pooled estimation of c can be obtained by normalizing the data, i.e. by letting $y_{ij} = x_{ij}/b_{i\sim}$, for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. The M.L.E. by normalization $\tilde{c}$ is a solution of the following M.L. equation for the normalized data :

$$\frac{\sum_{i=1}^m \sum_{j=1}^n y_{ij} \tilde{c}^{\lambda_n} y_{ij}}{\sum_{i=1}^m \sum_{j=1}^n y_{ij} \tilde{c}} - \frac{1}{\tilde{c}} - \frac{\sum_{i=1}^m \sum_{j=1}^n \lambda_n y_{ij}}{m n} = 0. \quad (5)$$

When normalizing the data, the scale parameters b_i are usually not known, and hence they may be replaced by the M.L.E. $\hat{b}_i$ based on the observations $x_{i1}, x_{i2}, \dots, x_{in}$.

(C) Joint M.L. estimator $\hat{c}$: The M.L. equation for pooled data x_{ij} , $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$, is

$$\sum_{i=1}^m \left[\frac{\sum_{j=1}^n x_{ij} \hat{c}^{\lambda_n} x_{ij}}{\sum_{j=1}^n x_{ij} \hat{c}} \right] - \frac{m}{\hat{c}} - \frac{\sum_{i=1}^m \sum_{j=1}^n \lambda_n x_{ij}}{n} = 0, \quad (6)$$

and a solution $\hat{c}$ of (6) is called the joint M.L. estimator of c .

For notational convenience, let c^* be representing any one of pooled estimators of c given in (A), (B) and (C). We also denote the corresponding M.L.E. of b_i by

$$b_i^* = \left[1/n \sum_{j=1}^n x_{ij} c^{*\lambda_n} \right]^{1/c^*}, \quad (7)$$

for $i = 1, 2, \dots, m$.

3. CONFIDENCE INTERVALS FOR c

In what follows, $\bar{c}_{11}$, $\tilde{c}_{11}$ and $\hat{c}_{11}$ are used to denote the pooled estimation of c , given in (A), (B) and (C), when in fact the sample is from a Weibull distribution with $b = 1$ and $c = 1$, i.e. a standard exponential distribution. Let c_{11}^* be representing any one of $\bar{c}_{11}$, $\tilde{c}_{11}$ and $\hat{c}_{11}$.

The key result for the estimation of c is the following theorem, which can be proved by using the same method given in the proof of Theorem A of Thoman, Bain and Antle [5]. Hence the proof of the following theorem is omitted.

THEOREM 1. c^*/c is distributed independently of $b_1, b_2, \dots, b_m$ and c and has the same distribution as c_{11}^* .

The distributions of $\bar{c}_{11}$, $\tilde{c}_{11}$ and $\hat{c}_{11}$ were obtained by the Monte Carlo method. These distributions were based on the simulations of 10,000 random samples of size $n \times m$ which was performed at the Air Force Aeronautical Systems Division (Wright-Patterson Air Force Base) on the CDC 6600. Percentage points of the distributions of $\bar{c}_{11}$, $\tilde{c}_{11}$ and $\hat{c}_{11}$ are given in Tables 1, 2, and 3 respectively for $m = 2, 3, 4, 5, 7$ and $n = 5, 6, 7, 8, 9, 10, 12, 14, 16, 18, 20, 25, 30, 50$. These results can be used to construct confidence intervals for c when $b_1, b_2, \dots, b_m$ are unknown. $100(1 - \gamma)$ percent confidence intervals will be of the form $(c^*/\ell_1, c^*/\ell_2)$ where ℓ_1 and ℓ_2 are from Tables 1, 2, and 3 such that $P_r\{\ell_1 < c_{11}^* < \ell_2\} = 1 - \gamma$.

The generated distributions of c_{11}^* provides the factors $B^*(n)$ such that $E[B^*(n) c^*] = c$. We note that $B^*(n)$ represents $\bar{B}(n)$, $\tilde{B}(n)$ or $\hat{B}(n)$, corresponding to $\bar{c}$, $\tilde{c}$ or $\hat{c}$. These unbiasing factors are given in Table 4.

The problem of testing hypotheses $H_0 : c = c_0$ against $H_1 : c = c_1$ with the level of significance γ , can be solved by using Tables 1, 2, and 3. When $c_1 > c_0$, the distribution of c^*/c_0 yields the critical region $(c_0 \ell_{1-\gamma}, \infty)$. The power of the test is

$$P_r\{c^* > c_0 \ell_{1-\gamma} \mid H_1\} = P_r\{c_{11}^* > \ell_{1-\gamma} c_0 / c_1\},$$

which is independent of $b_1, b_2, \dots, b_m$ and depends only on c_0/c_1 , γ and n . A similar approach can be taken for the case that $c_1 < c_0$. The powers of the test as a function of c_1/c_0 were obtained and given in Figure 1 ($m = 2, n = 5$) and Figure 2 ($m = 4, n = 14$). It is noted that $\tilde{c}$ gives most powerful test.

4. CONFIDENCE INTERVALS FOR b

Let b represent any one of $b_1, b_2, \dots, b_m$, where b_1 is the scale parameter of Weibull distribution under i^{th} level of stress. This b can be estimated by b^* , representing any one of $\bar{b}$, $\tilde{b}$ or $\hat{b}$, given in (7).

Recall that c^* ($\bar{c}$, $\tilde{c}$ or $\hat{c}$) appeared in (7) is an estimator of c from the pooled data. As before b_{11}^* will denote the M.L.E. of b , as given in (7), when in fact the sampling is from a Weibull distribution with $b = 1$ and $c = 1$.

THEOREM 2. $c^* \ln(b^*/b)$ is independent of b and c and has the same distribution as $c_{11}^* \ln(b_{11}^*)$.

The proof of Theorem 2 is omitted since it is simply done by using the same approach as in the proof of Theorem B of Thoman, Bain and Antle [5].

The distributions of $\bar{c}_{11} \ln(\bar{b}_{11})$, $\tilde{c}_{11} \ln(\tilde{b}_{11})$ and $\hat{c}_{11} \ln(\hat{b}_{11})$ were obtained by Monte Carlo methods, based on the results of 10,000 random samples of size $n \times m$. The percentage points of these distributions are given in Tables 5, 6, and 7.

100 $(1-\gamma)$ percent confidence interval for b can now be constructed and will be of the form

$$(b^* e^{-\ell_2/c^*}, b^* e^{-\ell_1/c^*}) ,$$

where ℓ_1 and ℓ_2 are obtained from Tables 5, 6 or 7 such that

$$P_r \{ \ell_1 < c_{11}^* \ln(b_{11}^*) < \ell_2 \} = 1 - \gamma .$$

5. EXAMPLES

We consider the following fatigue life data, given in Kim and Park [8], of composite material T300/5208 graphite epoxy laminate with $[0/90/+45]_s$ orientation.

(1) Under stress level 345MPa, fatigue life in cycle :

293,000	443,870	661,090	923,840	1,340,070
364,200	491,800	671,540	943,300	1,367,890
367,580	539,980	704,870	996,170	1,488,150
369,890	614,960	764,680	1,013,630	1,809,060
412,200	631,230	778,380	1,104,570	3,690,560
429,960	646,370	793,340	1,333,390	

(2) Under stress level 414MPa, fatigue life in cycle :

7,180	17,950	28,440	37,330	52,350
10,190	21,270	28,760	37,560	66,410
10,300	22,080	31,110	38,400	77,130
12,740	22,400	33,690	39,480	78,720
15,760	22,550	34,970	43,680	101,300
17,230	24,570	35,470	47,640	

Denote b_1, c_1 and b_2, c_2 the scale and shape parameters of Weibull fatigue life distributions under stress levels 345MPa and 414MPa respectively. The maximum likelihood estimates of the above parameters are :

$$\begin{aligned}\hat{b}_1 &= 1,009,350 & \hat{c}_1 &= 1.58 \\ \hat{b}_2 &= 39,560 & \hat{c}_2 &= 1.71\end{aligned}$$

In testing hypotheses $H_0 : c_1 = c_2$ against $H_2 : c_1 \neq c_2$, we must accept the H_0 since the critical region is $\hat{c}_2/\hat{c}_1 > 1.48$ ($\gamma = 0.1$) according to Thoman and Bain [4].

The pooled estimates of c are : $\bar{c} = 1.645$, $\tilde{c} = 1.636$, and $\hat{c} = 1.638$. (one-sided) 95 percent confidence intervals for c are : $(1.315, \infty)$, $(1.349, \infty)$, and $(1.333, \infty)$ from Tables 1, 2, and 3 respectively.

The M.L.E. of b_i (according to (7)) are :

$$\begin{aligned}\bar{b}_1 &= 1,023,575 & \bar{b}_2 &= 39,121 \\ \tilde{b}_1 &= 1,021,587 & \tilde{b}_2 &= 39,064 \\ \hat{b}_1 &= 1,022,028 & \hat{b}_2 &= 39,077 ,\end{aligned}$$

and (one-sided) 95 percent confidence intervals for b_1 and b_2 , using Table 6 of the joint M.L.E., are : $(846,248, \infty)$ for b_1 and $(32,359, \infty)$ for b_2 .

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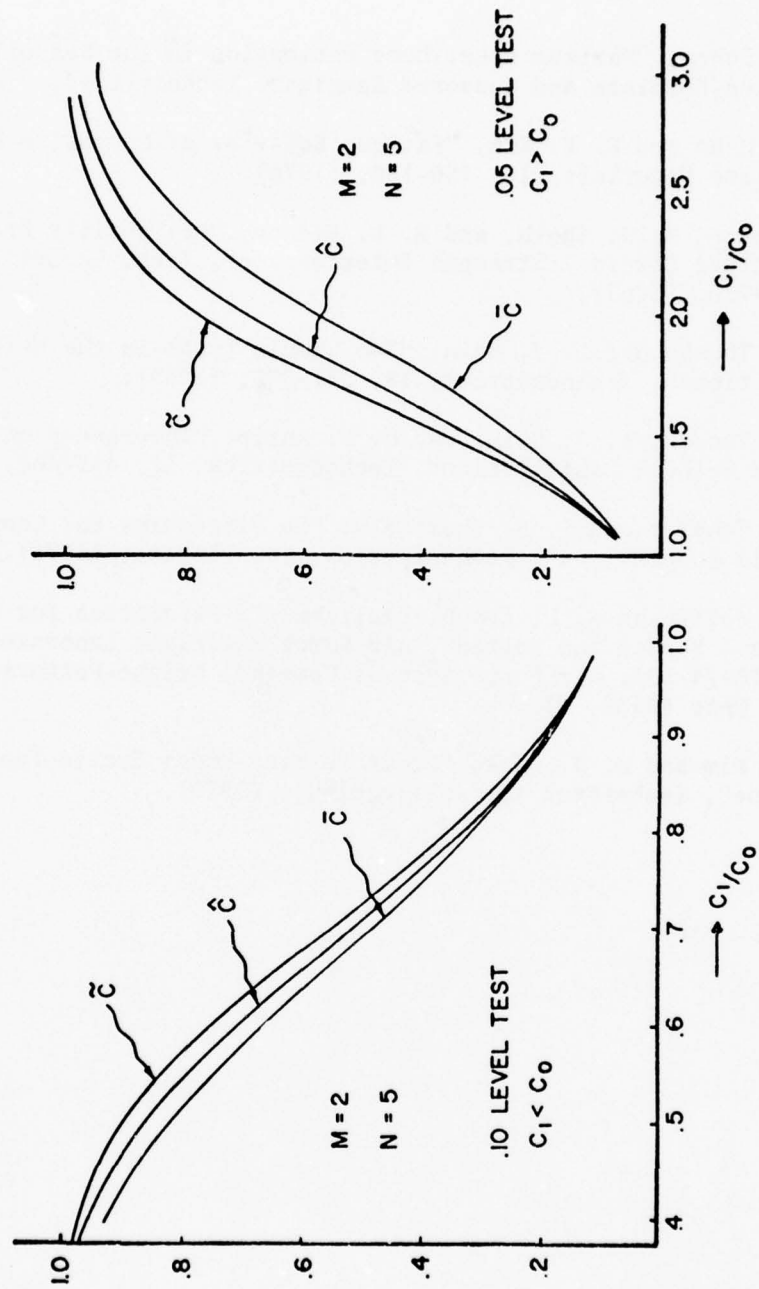


Figure 1. Power as a Function of c_1/c_0 ($m = 2$ and $n = 5$)

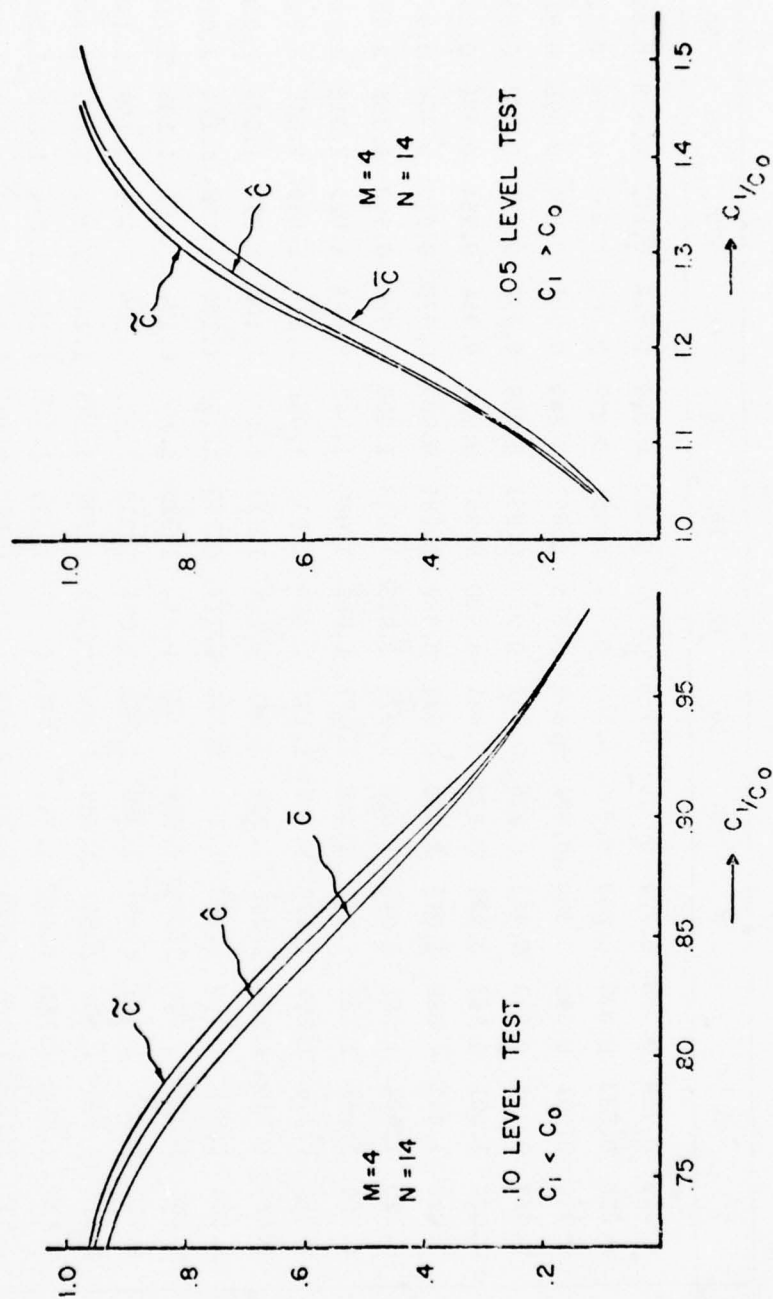


Figure 2. Power as a Function of c_1/c_0 ($m = 4$ and $n = 14$)

TABLE 1. Averaging M.L.E. ($m = 2$)
Percentage point, ℓ_{γ} , such that $P_r\{\bar{c}/c < \ell_{\gamma}\} = \gamma$

γ \ n	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.745	0.758	0.768	0.774	0.779	0.780	0.794	0.803	0.809	0.820	0.822	0.838	0.847	0.861
.05	0.828	0.833	0.829	0.840	0.835	0.839	0.847	0.851	0.857	0.860	0.867	0.874	0.882	0.890
.10	0.910	0.901	0.898	0.898	0.891	0.892	0.895	0.897	0.899	0.901	0.904	0.908	0.914	0.916
.15	0.969	0.955	0.942	0.943	0.934	0.932	0.927	0.931	0.929	0.930	0.931	0.932	0.935	0.935
.20	1.022	1.001	0.985	0.979	0.970	0.964	0.960	0.957	0.956	0.954	0.954	0.951	0.954	0.951
.25	1.070	1.043	1.026	1.015	1.002	0.994	0.987	0.985	0.980	0.976	0.972	0.969	0.970	0.964
.30	1.120	1.084	1.061	1.048	1.030	1.023	1.013	1.008	1.000	0.996	0.992	0.986	0.985	0.976
.40	1.214	1.166	1.131	1.111	1.089	1.077	1.063	1.050	1.042	1.033	1.025	1.016	1.013	0.999
.50	1.314	1.246	1.204	1.177	1.147	1.131	1.108	1.092	1.081	1.069	1.060	1.047	1.040	1.022
.60	1.423	1.338	1.285	1.246	1.211	1.191	1.159	1.139	1.121	1.108	1.097	1.079	1.068	1.046
.70	1.568	1.451	1.381	1.330	1.290	1.259	1.217	1.190	1.168	1.150	1.138	1.114	1.099	1.071
.75	1.664	1.519	1.441	1.379	1.334	1.300	1.254	1.220	1.195	1.174	1.161	1.135	1.118	1.082
.80	1.767	1.601	1.512	1.434	1.388	1.348	1.293	1.255	1.225	1.204	1.189	1.159	1.138	1.103
.85	1.912	1.705	1.601	1.507	1.453	1.404	1.345	1.299	1.265	1.240	1.221	1.187	1.161	1.122
.90	2.130	1.852	1.721	1.603	1.544	1.486	1.410	1.358	1.315	1.288	1.264	1.222	1.194	1.147
.95	2.458	2.125	1.928	1.779	1.707	1.624	1.515	1.453	1.394	1.365	1.335	1.279	1.244	1.186
.98	3.021	2.480	2.235	2.005	1.912	1.793	1.646	1.568	1.489	1.453	1.415	1.351	1.312	1.233

TABLE 1. Averaging M.L.E. (m = 3)
 Percentage point, ℓ_Y , such that $P_F\{\bar{c}/c < \ell_Y\} = Y$

$Y \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.826	0.832	0.827	0.839	0.841	0.841	0.847	0.845	0.854	0.857	0.862	0.875	0.880	0.886
.05	0.905	0.899	0.889	0.891	0.891	0.890	0.892	0.894	0.895	0.898	0.899	0.904	0.910	0.911
.10	0.979	0.966	0.953	0.948	0.942	0.940	0.936	0.932	0.932	0.931	0.934	0.933	0.936	0.934
.15	1.043	1.014	0.997	0.988	0.979	0.976	0.966	0.961	0.958	0.955	0.957	0.954	0.955	0.952
.20	1.089	1.057	1.034	1.022	1.011	1.004	0.992	0.986	0.981	0.977	0.975	0.971	0.970	0.965
.25	1.131	1.094	1.068	1.053	1.040	1.030	1.015	1.006	1.000	0.995	0.991	0.987	0.984	0.976
.30	1.174	1.128	1.099	1.080	1.066	1.053	1.037	1.026	1.018	1.011	1.007	1.001	0.997	0.987
.40	1.259	1.197	1.158	1.139	1.114	1.099	1.075	1.062	1.053	1.043	1.037	1.026	1.021	1.006
.50	1.346	1.267	1.220	1.193	1.166	1.148	1.117	1.098	1.084	1.073	1.066	1.051	1.043	1.025
.60	1.450	1.348	1.289	1.249	1.217	1.194	1.157	1.135	1.117	1.108	1.095	1.078	1.067	1.045
.70	1.569	1.445	1.368	1.316	1.280	1.249	1.206	1.178	1.157	1.142	1.128	1.107	1.093	1.067
.75	1.642	1.501	1.419	1.354	1.315	1.281	1.233	1.201	1.179	1.163	1.148	1.124	1.107	1.078
.80	1.739	1.565	1.475	1.398	1.358	1.318	1.265	1.230	1.205	1.187	1.171	1.142	1.123	1.091
.85	1.845	1.651	1.548	1.460	1.410	1.366	1.305	1.266	1.236	1.217	1.195	1.164	1.142	1.107
.90	1.998	1.771	1.642	1.538	1.480	1.428	1.357	1.315	1.275	1.254	1.228	1.192	1.168	1.128
.95	2.270	1.974	1.804	1.669	1.596	1.525	1.442	1.387	1.335	1.306	1.285	1.236	1.206	1.159
.98	2.721	2.266	2.007	1.853	1.751	1.660	1.551	1.475	1.413	1.382	1.355	1.293	1.255	1.196

TABLE 1. Averaging M.L.E. ($m = 4$)
Percentage point, ℓ_Y , such that $P\{\bar{c}/c < \ell_Y\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.886	0.878	0.868	0.872	0.875	0.873	0.877	0.881	0.882	0.886	0.890	0.898	0.901	0.899
.05	0.955	0.942	0.933	0.927	0.928	0.924	0.918	0.918	0.918	0.917	0.921	0.926	0.928	0.924
.10	1.028	1.002	0.989	0.979	0.973	0.968	0.959	0.953	0.953	0.948	0.949	0.950	0.950	0.945
.15	1.083	1.051	1.031	1.016	1.007	1.000	0.988	0.980	0.976	0.971	0.970	0.969	0.967	0.959
.20	1.129	1.090	1.066	1.047	1.034	1.025	1.011	1.002	0.995	0.990	0.988	0.984	0.980	0.972
.25	1.170	1.126	1.096	1.077	1.059	1.047	1.032	1.020	1.012	1.005	1.003	0.997	0.993	0.982
.30	1.206	1.161	1.124	1.102	1.082	1.068	1.050	1.037	1.029	1.021	1.016	1.010	1.004	0.991
.40	1.285	1.224	1.178	1.150	1.125	1.108	1.087	1.070	1.060	1.049	1.044	1.031	1.024	1.009
.50	1.367	1.288	1.235	1.200	1.168	1.148	1.122	1.102	1.089	1.077	1.068	1.054	1.044	1.026
.60	1.456	1.356	1.294	1.251	1.213	1.191	1.157	1.137	1.118	1.105	1.093	1.077	1.065	1.043
.70	1.558	1.435	1.363	1.308	1.270	1.240	1.198	1.175	1.151	1.138	1.121	1.101	1.088	1.062
.75	1.619	1.485	1.405	1.342	1.297	1.269	1.221	1.195	1.171	1.156	1.139	1.115	1.100	1.072
.80	1.695	1.544	1.453	1.382	1.337	1.302	1.247	1.218	1.194	1.176	1.158	1.131	1.114	1.084
.85	1.795	1.610	1.517	1.436	1.381	1.341	1.281	1.246	1.218	1.199	1.181	1.151	1.131	1.097
.90	1.932	1.714	1.601	1.498	1.444	1.394	1.327	1.284	1.252	1.228	1.212	1.175	1.154	1.115
.95	2.185	1.887	1.729	1.615	1.544	1.472	1.395	1.344	1.308	1.278	1.257	1.216	1.186	1.143
.98	2.505	2.126	1.913	1.753	1.655	1.592	1.486	1.417	1.375	1.338	1.309	1.260	1.226	1.177

TABLE 1. Averaging M.L.E. ($m = 5$)
Percentage point, ℓ_Y , such that $P_r\{\bar{c}/c < \ell_Y\} = Y$

$\begin{array}{c} n \\ Y \end{array}$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.931	0.915	0.904	0.903	0.902	0.902	0.898	0.897	0.903	0.903	0.903	0.910	0.915	0.910
.05	0.996	0.980	0.964	0.956	0.948	0.944	0.938	0.935	0.936	0.936	0.938	0.937	0.939	0.933
.10	1.064	1.039	1.013	1.007	0.993	0.983	0.976	0.970	0.965	0.964	0.963	0.963	0.961	0.952
.15	1.113	1.081	1.052	1.038	1.024	1.014	1.003	0.995	0.987	0.985	0.982	0.979	0.977	0.966
.20	1.157	1.116	1.083	1.066	1.051	1.038	1.023	1.014	1.006	1.001	0.998	0.992	0.988	0.978
.25	1.197	1.146	1.112	1.090	1.073	1.057	1.043	1.031	1.022	1.016	1.011	1.003	0.999	0.987
.30	1.234	1.177	1.140	1.114	1.095	1.078	1.060	1.047	1.037	1.029	1.024	1.015	1.010	0.995
.40	1.309	1.237	1.192	1.158	1.135	1.115	1.092	1.076	1.064	1.053	1.047	1.034	1.028	1.011
.50	1.382	1.294	1.240	1.201	1.174	1.153	1.124	1.104	1.089	1.080	1.069	1.055	1.045	1.026
.60	1.464	1.359	1.293	1.247	1.215	1.191	1.156	1.133	1.116	1.106	1.092	1.075	1.064	1.041
.70	1.554	1.433	1.352	1.301	1.263	1.235	1.195	1.164	1.147	1.133	1.118	1.098	1.085	1.058
.75	1.613	1.475	1.390	1.334	1.289	1.260	1.216	1.184	1.165	1.148	1.134	1.100	1.096	1.067
.80	1.682	1.529	1.432	1.370	1.322	1.291	1.240	1.207	1.184	1.165	1.151	1.123	1.109	1.079
.85	1.766	1.593	1.487	1.412	1.363	1.326	1.268	1.233	1.208	1.188	1.172	1.140	1.124	1.091
.90	1.878	1.678	1.559	1.477	1.414	1.371	1.308	1.268	1.236	1.214	1.197	1.163	1.144	1.108
.95	2.090	1.829	1.676	1.569	1.502	1.450	1.373	1.321	1.290	1.258	1.237	1.196	1.174	1.131
.98	2.366	2.048	1.812	1.687	1.608	1.544	1.459	1.385	1.344	1.310	1.287	1.239	1.207	1.163

TABLE 1. Averaging M.L.E. (m = 7)
Percentage Point, ℓ_Y , such that $P_Y\{\bar{c}/c < \ell_Y\} = Y$

$Y \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.988	0.964	0.951	0.946	0.939	0.935	0.933	0.931	0.928	0.930	0.929	0.931	0.935	0.927
.05	1.056	1.019	1.003	0.990	0.979	0.974	0.965	0.964	0.960	0.956	0.954	0.955	0.955	0.945
.10	1.116	1.071	1.050	1.033	1.018	1.012	0.997	0.992	0.986	0.981	0.979	0.976	0.974	0.963
.15	1.160	1.112	1.082	1.064	1.049	1.038	1.020	1.012	1.005	1.000	0.995	0.990	0.987	0.975
.20	1.196	1.148	1.111	1.089	1.071	1.058	1.040	1.029	1.021	1.014	1.011	1.003	0.997	0.984
.25	1.233	1.174	1.136	1.111	1.092	1.076	1.057	1.044	1.034	1.026	1.022	1.013	1.007	0.993
.30	1.267	1.201	1.158	1.131	1.110	1.094	1.072	1.057	1.046	1.037	1.033	1.022	1.016	1.001
.40	1.331	1.252	1.202	1.170	1.144	1.126	1.100	1.082	1.070	1.060	1.052	1.040	1.032	1.014
.50	1.392	1.303	1.246	1.209	1.179	1.157	1.128	1.106	1.091	1.081	1.072	1.057	1.047	1.027
.60	1.459	1.354	1.293	1.248	1.215	1.192	1.156	1.133	1.115	1.103	1.093	1.074	1.062	1.039
.70	1.541	1.417	1.345	1.293	1.257	1.226	1.186	1.160	1.141	1.127	1.115	1.092	1.079	1.054
.75	1.592	1.458	1.376	1.321	1.280	1.248	1.205	1.175	1.155	1.139	1.128	1.103	1.088	1.062
.80	1.645	1.502	1.413	1.351	1.307	1.273	1.225	1.193	1.172	1.155	1.141	1.114	1.098	1.071
.85	1.716	1.551	1.457	1.385	1.339	1.302	1.250	1.216	1.192	1.172	1.157	1.130	1.111	1.081
.90	1.816	1.625	1.515	1.435	1.382	1.340	1.283	1.244	1.217	1.196	1.177	1.147	1.127	1.095
.95	1.989	1.748	1.612	1.515	1.455	1.404	1.335	1.290	1.256	1.232	1.209	1.175	1.153	1.115
.98	2.210	1.910	1.732	1.627	1.549	1.475	1.396	1.347	1.302	1.275	1.249	1.209	1.181	1.135

TABLE 2. M.L.E. by normalization ($m = 2$)
Percentage point, ℓ_Y , such that $P\{\tilde{c}/c < \ell_Y\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.679	0.697	0.716	0.726	0.735	0.746	0.764	0.775	0.783	0.798	0.801	0.819	0.831	0.851
.05	0.740	0.757	0.765	0.776	0.784	0.793	0.807	0.820	0.826	0.833	0.842	0.855	0.864	0.881
.10	0.802	0.812	0.822	0.832	0.832	0.837	0.851	0.862	0.866	0.872	0.877	0.888	0.897	0.907
.15	0.851	0.856	0.862	0.870	0.868	0.872	0.882	0.890	0.896	0.898	0.903	0.910	0.918	0.925
.20	0.892	0.892	0.894	0.901	0.901	0.901	0.908	0.913	0.919	0.921	0.924	0.929	0.934	0.940
.25	0.928	0.926	0.922	0.929	0.929	0.927	0.933	0.936	0.940	0.940	0.942	0.945	0.949	0.954
.30	0.964	0.959	0.954	0.957	0.954	0.951	0.955	0.958	0.959	0.959	0.958	0.960	0.963	0.966
.40	1.029	1.019	1.012	1.008	1.002	1.001	0.997	0.998	0.995	0.993	0.991	0.989	0.990	0.988
.50	1.102	1.083	1.070	1.061	1.052	1.047	1.040	1.036	1.031	1.026	1.023	1.017	1.015	1.011
.60	1.184	1.153	1.136	1.118	1.106	1.096	1.083	1.077	1.070	1.062	1.056	1.048	1.042	1.034
.70	1.280	1.236	1.209	1.185	1.166	1.155	1.135	1.123	1.112	1.102	1.094	1.082	1.071	1.058
.75	1.333	1.287	1.253	1.223	1.204	1.191	1.166	1.150	1.136	1.122	1.116	1.100	1.089	1.073
.80	1.397	1.346	1.305	1.271	1.250	1.229	1.200	1.180	1.163	1.149	1.141	1.123	1.107	1.089
.85	1.489	1.418	1.376	1.331	1.305	1.277	1.242	1.219	1.196	1.181	1.172	1.148	1.131	1.108
.90	1.609	1.523	1.461	1.406	1.377	1.343	1.298	1.269	1.241	1.222	1.209	1.182	1.162	1.132
.95	1.817	1.692	1.604	1.528	1.493	1.450	1.390	1.353	1.312	1.291	1.273	1.236	1.208	1.171
.98	2.098	1.901	1.798	1.706	1.647	1.593	1.494	1.456	1.399	1.369	1.349	1.308	1.271	1.216

TABLE 2. M.L.E. by normalization ($m = 3$)
Percentage point, ℓ_{γ} , such that $P\{\tilde{c}/c < \ell_{\gamma}\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.717	0.735	0.749	0.764	0.774	0.783	0.798	0.805	0.816	0.825	0.831	0.847	0.858	0.872
.05	0.770	0.784	0.798	0.807	0.816	0.824	0.833	0.845	0.852	0.860	0.864	0.876	0.886	0.898
.10	0.823	0.832	0.844	0.851	0.859	0.865	0.872	0.880	0.885	0.890	0.897	0.903	0.911	0.921
.15	0.864	0.868	0.880	0.882	0.888	0.894	0.898	0.904	0.909	0.911	0.918	0.922	0.928	0.937
.20	0.898	0.901	0.908	0.908	0.912	0.919	0.921	0.925	0.929	0.929	0.934	0.938	0.942	0.950
.25	0.928	0.929	0.932	0.933	0.935	0.939	0.940	0.942	0.946	0.945	0.949	0.952	0.955	0.961
.30	0.957	0.954	0.955	0.955	0.957	0.958	0.959	0.959	0.961	0.962	0.963	0.964	0.967	0.972
.40	1.012	1.002	1.001	0.997	0.998	0.996	0.993	0.992	0.990	0.990	0.990	0.989	0.990	0.990
.50	1.066	1.052	1.045	1.040	1.038	1.032	1.026	1.022	1.020	1.017	1.015	1.012	1.010	1.008
.60	1.124	1.106	1.093	1.083	1.079	1.071	1.062	1.055	1.049	1.046	1.042	1.035	1.032	1.027
.70	1.195	1.166	1.150	1.135	1.126	1.117	1.102	1.091	1.082	1.079	1.071	1.063	1.056	1.048
.75	1.235	1.204	1.183	1.166	1.152	1.141	1.122	1.111	1.101	1.096	1.089	1.078	1.070	1.059
.80	1.281	1.250	1.222	1.200	1.183	1.171	1.149	1.135	1.124	1.116	1.107	1.093	1.084	1.072
.85	1.345	1.305	1.268	1.242	1.220	1.207	1.181	1.165	1.150	1.140	1.131	1.114	1.103	1.088
.90	1.432	1.377	1.328	1.298	1.274	1.254	1.222	1.204	1.185	1.173	1.162	1.139	1.126	1.108
.95	1.576	1.493	1.429	1.390	1.357	1.331	1.291	1.263	1.241	1.223	1.208	1.179	1.163	1.139
.98	1.737	1.647	1.558	1.494	1.465	1.422	1.369	1.343	1.303	1.285	1.271	1.233	1.204	1.174

TABLE 2. M.L.E. by normalization ($m = 4$)
Percentage point, ℓ_Y , such that $P_F\{\tilde{c}/c < \ell_Y\} = Y$

$\begin{array}{c} n \\ Y \end{array}$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.746	0.764	0.775	0.783	0.798	0.801	0.816	0.829	0.842	0.852	0.865	0.875	0.885	
.05	0.793	0.807	0.820	0.826	0.833	0.842	0.852	0.861	0.867	0.872	0.879	0.891	0.899	0.908
.10	0.837	0.851	0.862	0.866	0.872	0.877	0.885	0.892	0.898	0.900	0.905	0.914	0.920	0.928
.15	0.872	0.882	0.890	0.896	0.898	0.903	0.909	0.915	0.918	0.920	0.924	0.931	0.935	0.942
.20	0.901	0.908	0.913	0.919	0.921	0.924	0.929	0.933	0.935	0.937	0.939	0.945	0.949	0.954
.25	0.927	0.933	0.936	0.940	0.940	0.942	0.946	0.948	0.950	0.951	0.953	0.957	0.960	0.964
.30	0.951	0.955	0.958	0.959	0.959	0.958	0.961	0.962	0.963	0.963	0.966	0.969	0.970	0.973
.40	1.001	0.997	0.998	0.995	0.993	0.991	0.990	0.989	0.989	0.988	0.989	0.989	0.989	0.991
.50	1.047	1.040	1.036	1.031	1.026	1.023	1.020	1.017	1.016	1.012	1.011	1.009	1.008	1.006
.60	1.096	1.083	1.077	1.070	1.062	1.056	1.049	1.045	1.041	1.037	1.034	1.030	1.026	1.023
.70	1.155	1.135	1.123	1.112	1.102	1.094	1.082	1.075	1.070	1.065	1.058	1.052	1.047	1.041
.75	1.191	1.166	1.150	1.136	1.122	1.116	1.101	1.095	1.086	1.081	1.073	1.065	1.058	1.050
.80	1.229	1.200	1.180	1.163	1.149	1.141	1.124	1.114	1.103	1.098	1.091	1.079	1.072	1.062
.85	1.277	1.242	1.219	1.196	1.181	1.172	1.150	1.138	1.125	1.118	1.111	1.097	1.087	1.074
.90	1.343	1.298	1.269	1.241	1.222	1.209	1.185	1.169	1.155	1.145	1.137	1.120	1.108	1.092
.95	1.450	1.390	1.353	1.312	1.291	1.273	1.240	1.216	1.200	1.187	1.177	1.155	1.139	1.120
.98	1.593	1.494	1.456	1.399	1.369	1.349	1.303	1.274	1.252	1.239	1.226	1.194	1.174	1.150

TABLE 2. M.L.E. by normalization ($m = 5$)
Percentage point, ℓ_Y , such that $P_{\tilde{c}/c} \{ \tilde{c} < \ell_Y \} = \gamma$

γ	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.766	0.783	0.791	0.801	0.813	0.819	0.831	0.839	0.852	0.858	0.865	0.877	0.887	0.894
.05	0.810	0.824	0.831	0.842	0.848	0.855	0.864	0.871	0.879	0.886	0.891	0.902	0.910	0.916
.10	0.854	0.865	0.869	0.877	0.882	0.888	0.897	0.900	0.905	0.911	0.914	0.924	0.928	0.934
.15	0.885	0.894	0.899	0.903	0.906	0.910	0.918	0.920	0.924	0.928	0.931	0.938	0.943	0.947
.20	0.911	0.919	0.922	0.924	0.926	0.929	0.934	0.937	0.939	0.942	0.945	0.950	0.954	0.959
.25	0.935	0.940	0.942	0.942	0.945	0.945	0.949	0.951	0.953	0.955	0.957	0.960	0.964	0.967
.30	0.955	0.958	0.960	0.958	0.961	0.960	0.963	0.964	0.966	0.967	0.969	0.970	0.971	0.976
.40	0.996	0.996	0.994	0.991	0.992	0.989	0.990	0.989	0.989	0.990	0.989	0.990	0.989	0.990
.50	1.038	1.032	1.027	1.023	1.022	1.017	1.015	1.013	1.011	1.010	1.009	1.007	1.007	1.004
.60	1.082	1.071	1.065	1.056	1.054	1.048	1.042	1.038	1.034	1.032	1.030	1.025	1.023	1.019
.70	1.131	1.117	1.104	1.094	1.087	1.082	1.071	1.065	1.058	1.056	1.052	1.045	1.042	1.035
.75	1.161	1.141	1.128	1.116	1.106	1.100	1.089	1.081	1.073	1.070	1.065	1.056	1.052	1.044
.80	1.194	1.171	1.154	1.141	1.128	1.123	1.107	1.099	1.091	1.084	1.079	1.070	1.063	1.055
.85	1.234	1.207	1.188	1.172	1.158	1.148	1.131	1.120	1.111	1.103	1.097	1.084	1.078	1.067
.90	1.286	1.254	1.228	1.209	1.193	1.182	1.162	1.147	1.137	1.126	1.120	1.104	1.095	1.083
.95	1.379	1.331	1.295	1.273	1.250	1.236	1.208	1.189	1.177	1.163	1.155	1.134	1.122	1.105
.98	1.488	1.422	1.381	1.349	1.319	1.308	1.271	1.241	1.226	1.204	1.194	1.172	1.154	1.134

TABLE 2. M.L.E. by normalization ($m = 7$)
Percentage point, ℓ_Y , such that $P\{\tilde{c}/c < \ell_Y\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.791	0.805	0.818	0.829	0.833	0.839	0.855	0.864	0.871	0.878	0.883	0.893	0.902	0.907
.05	0.831	0.845	0.853	0.861	0.866	0.871	0.882	0.889	0.895	0.898	0.905	0.914	0.921	0.925
.10	0.869	0.880	0.887	0.892	0.896	0.900	0.908	0.915	0.919	0.922	0.926	0.933	0.938	0.943
.15	0.899	0.904	0.910	0.915	0.918	0.920	0.926	0.932	0.935	0.937	0.941	0.947	0.950	0.954
.20	0.922	0.925	0.928	0.933	0.934	0.937	0.941	0.946	0.948	0.950	0.952	0.957	0.960	0.963
.25	0.942	0.942	0.944	0.948	0.950	0.951	0.954	0.958	0.959	0.961	0.963	0.966	0.969	0.971
.30	0.960	0.959	0.960	0.962	0.963	0.964	0.966	0.969	0.970	0.971	0.973	0.975	0.976	0.978
.40	0.994	0.992	0.990	0.989	0.989	0.989	0.988	0.990	0.990	0.990	0.990	0.990	0.991	0.992
.50	1.027	1.022	1.018	1.017	1.015	1.013	1.010	1.010	1.009	1.008	1.007	1.006	1.005	1.003
.60	1.065	1.055	1.048	1.045	1.043	1.038	1.032	1.030	1.027	1.026	1.024	1.021	1.019	1.016
.70	1.104	1.091	1.083	1.075	1.071	1.065	1.059	1.053	1.049	1.046	1.044	1.038	1.033	1.029
.75	1.128	1.111	1.100	1.095	1.088	1.081	1.072	1.065	1.061	1.057	1.055	1.047	1.042	1.036
.80	1.154	1.135	1.122	1.114	1.107	1.099	1.088	1.080	1.075	1.069	1.066	1.058	1.052	1.046
.85	1.188	1.165	1.151	1.138	1.129	1.120	1.106	1.098	1.090	1.085	1.081	1.070	1.063	1.056
.90	1.228	1.204	1.186	1.169	1.158	1.147	1.130	1.121	1.109	1.105	1.099	1.087	1.079	1.069
.95	1.295	1.263	1.239	1.216	1.205	1.189	1.168	1.157	1.142	1.137	1.126	1.111	1.101	1.087
.98	1.381	1.343	1.310	1.274	1.259	1.241	1.214	1.201	1.181	1.174	1.158	1.138	1.127	1.109

TABLE 3. Joint M.L. Estimation ($m = 2$)Percentage point, ℓ_Y , such that $P\{\hat{c}/c < \ell_Y\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.704	0.722	0.739	0.746	0.753	0.762	0.777	0.789	0.792	0.807	0.811	0.825	0.838	0.857
.05	0.778	0.792	0.795	0.804	0.807	0.814	0.826	0.835	0.840	0.847	0.852	0.865	0.873	0.885
.10	0.849	0.854	0.854	0.863	0.858	0.862	0.871	0.878	0.883	0.886	0.890	0.897	0.906	0.910
.15	0.901	0.900	0.899	0.901	0.899	0.899	0.904	0.908	0.912	0.911	0.916	0.920	0.926	0.929
.20	0.948	0.939	0.937	0.938	0.932	0.930	0.932	0.933	0.935	0.936	0.937	0.939	0.943	0.945
.25	0.992	0.978	0.970	0.966	0.963	0.959	0.957	0.957	0.958	0.956	0.954	0.955	0.958	0.959
.30	1.032	1.015	1.004	0.996	0.991	0.984	0.980	0.980	0.978	0.976	0.973	0.971	0.973	0.971
.40	1.115	1.085	1.066	1.054	1.042	1.037	1.025	1.021	1.015	1.011	1.007	1.002	1.000	0.993
.50	1.198	1.155	1.132	1.110	1.096	1.086	1.070	1.062	1.054	1.044	1.039	1.030	1.026	1.016
.60	1.291	1.235	1.202	1.174	1.155	1.140	1.117	1.103	1.092	1.081	1.074	1.063	1.054	1.039
.70	1.401	1.327	1.288	1.248	1.221	1.202	1.173	1.158	1.136	1.123	1.113	1.097	1.084	1.063
.75	1.472	1.382	1.336	1.291	1.262	1.239	1.203	1.180	1.162	1.146	1.135	1.116	1.101	1.078
.80	1.557	1.448	1.393	1.342	1.310	1.282	1.241	1.212	1.189	1.173	1.161	1.138	1.120	1.095
.85	1.661	1.536	1.466	1.404	1.368	1.335	1.288	1.252	1.224	1.206	1.191	1.164	1.145	1.113
.90	1.802	1.656	1.569	1.487	1.442	1.408	1.347	1.305	1.272	1.249	1.232	1.201	1.175	1.139
.95	2.060	1.844	1.741	1.635	1.581	1.520	1.441	1.396	1.341	1.322	1.298	1.252	1.224	1.177
.98	2.404	2.133	1.973	1.820	1.746	1.678	1.559	1.502	1.430	1.401	1.382	1.324	1.288	1.222

TABLE 3. Joint M.L. Estimation ($m = 3$)
 Percentage point, ℓ_Y , such that $P\{\hat{c}/c < \ell_Y\} = Y$

$\frac{n}{Y}$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.763	0.779	0.783	0.795	0.803	0.805	0.820	0.823	0.834	0.840	0.846	0.859	0.868	0.879
.05	0.828	0.837	0.842	0.847	0.849	0.851	0.861	0.868	0.873	0.876	0.880	0.889	0.897	0.904
.10	0.893	0.890	0.890	0.894	0.896	0.898	0.900	0.904	0.905	0.909	0.913	0.916	0.922	0.927
.15	0.941	0.930	0.931	0.928	0.928	0.931	0.930	0.929	0.930	0.930	0.934	0.936	0.940	0.944
.20	0.980	0.968	0.961	0.957	0.956	0.958	0.952	0.950	0.952	0.950	0.952	0.952	0.955	0.957
.25	1.019	1.000	0.994	0.983	0.983	0.980	0.973	0.971	0.970	0.967	0.967	0.967	0.969	0.968
.30	1.053	1.030	1.019	1.010	1.006	1.001	0.993	0.989	0.986	0.985	0.982	0.981	0.980	0.978
.40	1.117	1.089	1.069	1.057	1.049	1.040	1.031	1.024	1.018	1.014	1.010	1.005	1.004	0.997
.50	1.182	1.142	1.121	1.106	1.093	1.082	1.066	1.055	1.049	1.043	1.038	1.030	1.026	1.015
.60	1.255	1.207	1.177	1.157	1.140	1.125	1.104	1.089	1.078	1.073	1.065	1.054	1.048	1.035
.70	1.343	1.283	1.240	1.212	1.192	1.173	1.146	1.129	1.113	1.107	1.097	1.082	1.073	1.056
.75	1.399	1.327	1.278	1.245	1.221	1.201	1.170	1.151	1.135	1.126	1.114	1.097	1.086	1.067
.80	1.458	1.381	1.327	1.286	1.257	1.233	1.200	1.175	1.159	1.146	1.133	1.115	1.101	1.080
.85	1.530	1.442	1.380	1.329	1.299	1.273	1.232	1.208	1.187	1.172	1.158	1.135	1.120	1.096
.90	1.642	1.534	1.457	1.391	1.360	1.329	1.279	1.252	1.223	1.207	1.189	1.161	1.144	1.116
.95	1.811	1.677	1.569	1.497	1.452	1.406	1.351	1.318	1.281	1.260	1.241	1.204	1.181	1.147
.98	2.049	1.852	1.723	1.620	1.574	1.515	1.438	1.397	1.347	1.324	1.309	1.259	1.224	1.183

TABLE 3. Joint M.L. Estimation ($m = 4$)Percentage point, ℓ_Y , such that $P\{\hat{c}/c < \ell_Y\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.797	0.814	0.817	0.818	0.831	0.833	0.844	0.851	0.857	0.863	0.869	0.880	0.887	0.892
.05	0.860	0.866	0.868	0.870	0.874	0.876	0.882	0.885	0.891	0.893	0.897	0.908	0.912	0.916
.10	0.919	0.918	0.917	0.917	0.915	0.917	0.918	0.919	0.922	0.921	0.925	0.930	0.934	0.936
.15	0.960	0.955	0.950	0.947	0.945	0.945	0.944	0.944	0.944	0.941	0.944	0.948	0.949	0.949
.20	0.996	0.985	0.978	0.976	0.969	0.967	0.966	0.961	0.961	0.960	0.961	0.962	0.962	0.962
.25	1.029	1.014	1.004	0.998	0.992	0.987	0.983	0.978	0.977	0.974	0.975	0.975	0.974	0.972
.30	1.062	1.041	1.029	1.020	1.013	1.006	1.000	0.995	0.993	0.989	0.989	0.986	0.985	0.981
.40	1.118	1.091	1.074	1.061	1.050	1.042	1.032	1.026	1.020	1.015	1.013	1.008	1.004	0.999
.50	1.178	1.142	1.120	1.104	1.087	1.077	1.065	1.053	1.047	1.040	1.035	1.029	1.024	1.015
.60	1.242	1.198	1.169	1.146	1.125	1.114	1.095	1.084	1.075	1.068	1.060	1.050	1.043	1.032
.70	1.314	1.259	1.223	1.195	1.171	1.156	1.132	1.119	1.105	1.097	1.086	1.073	1.065	1.050
.75	1.360	1.293	1.255	1.224	1.197	1.182	1.152	1.138	1.122	1.113	1.101	1.086	1.077	1.060
.80	1.410	1.333	1.293	1.252	1.227	1.209	1.176	1.160	1.141	1.130	1.120	1.101	1.090	1.071
.85	1.469	1.387	1.336	1.291	1.265	1.241	1.205	1.185	1.166	1.153	1.140	1.121	1.106	1.084
.90	1.554	1.457	1.393	1.343	1.314	1.288	1.242	1.214	1.196	1.181	1.166	1.144	1.127	1.102
.95	1.704	1.563	1.497	1.430	1.393	1.357	1.304	1.266	1.245	1.226	1.207	1.182	1.160	1.130
.98	1.872	1.704	1.620	1.534	1.480	1.442	1.373	1.331	1.304	1.279	1.264	1.223	1.178	1.160

TABLE 3. Joint M.L. Estimateion (m = 5)
Percentage point, ℓ_Y , such that $P_r\{\hat{c}/c < \ell_Y\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.829	0.841	0.841	0.845	0.852	0.852	0.861	0.865	0.873	0.878	0.883	0.892	0.900	0.902
.05	0.886	0.889	0.886	0.887	0.891	0.895	0.897	0.899	0.903	0.909	0.912	0.917	0.923	0.923
.10	0.943	0.939	0.933	0.931	0.930	0.928	0.932	0.931	0.930	0.934	0.935	0.941	0.943	0.942
.15	0.981	0.973	0.964	0.960	0.957	0.955	0.954	0.953	0.952	0.953	0.953	0.957	0.957	0.955
.20	1.012	1.001	0.991	0.983	0.981	0.976	0.972	0.971	0.970	0.968	0.968	0.969	0.970	0.967
.25	1.042	1.024	1.015	1.005	1.000	0.995	0.989	0.986	0.984	0.982	0.982	0.980	0.979	0.976
.30	1.071	1.049	1.037	1.025	1.018	1.011	1.005	1.000	0.998	0.994	0.993	0.989	0.988	0.984
.40	1.124	1.097	1.076	1.062	1.054	1.042	1.034	1.027	1.022	1.018	1.014	1.009	1.006	1.000
.50	1.176	1.141	1.116	1.098	1.087	1.075	1.062	1.053	1.045	1.040	1.036	1.028	1.024	1.014
.60	1.232	1.188	1.160	1.137	1.122	1.109	1.091	1.078	1.069	1.063	1.057	1.047	1.041	1.028
.70	1.293	1.242	1.206	1.180	1.160	1.146	1.122	1.107	1.096	1.090	1.081	1.068	1.061	1.045
.75	1.331	1.272	1.233	1.205	1.182	1.166	1.141	1.125	1.113	1.103	1.095	1.079	1.071	1.055
.80	1.374	1.309	1.264	1.234	1.209	1.191	1.163	1.144	1.130	1.119	1.110	1.094	1.084	1.065
.85	1.431	1.354	1.304	1.269	1.242	1.221	1.188	1.168	1.152	1.138	1.129	1.109	1.097	1.078
.90	1.505	1.412	1.353	1.313	1.280	1.258	1.221	1.199	1.177	1.162	1.152	1.128	1.116	1.094
.95	1.620	1.505	1.438	1.385	1.348	1.321	1.275	1.244	1.222	1.201	1.190	1.160	1.144	1.116
.98	1.763	1.625	1.536	1.485	1.430	1.399	1.344	1.299	1.273	1.247	1.230	1.199	1.178	1.145

TABLE 3. Joint M.L. Estimation ($m = 7$)
Percentage point, ℓ_Y , such that $P\{\hat{c}/c < \ell_Y\} = Y$

$Y \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	0.872	0.874	0.874	0.877	0.877	0.878	0.888	0.892	0.895	0.900	0.902	0.909	0.915	0.916
.05	0.922	0.916	0.917	0.915	0.915	0.914	0.919	0.920	0.923	0.924	0.926	0.932	0.936	0.934
.10	0.972	0.961	0.956	0.951	0.948	0.949	0.946	0.949	0.949	0.948	0.949	0.953	0.954	0.951
.15	1.005	0.991	0.984	0.976	0.972	0.971	0.967	0.967	0.966	0.964	0.964	0.966	0.967	0.963
.20	1.034	1.015	1.004	0.998	0.992	0.989	0.985	0.982	0.979	0.977	0.977	0.978	0.977	0.972
.25	1.061	1.038	1.023	1.015	1.009	1.005	0.998	0.996	0.992	0.989	0.989	0.987	0.985	0.980
.30	1.084	1.060	1.042	1.033	1.025	1.020	1.012	1.008	1.003	1.000	0.999	0.996	0.994	0.988
.40	1.126	1.099	1.077	1.064	1.054	1.047	1.037	1.030	1.024	1.020	1.018	1.012	1.009	1.001
.50	1.169	1.136	1.112	1.096	1.083	1.074	1.060	1.051	1.045	1.039	1.036	1.028	1.023	1.014
.60	1.216	1.176	1.149	1.129	1.114	1.101	1.085	1.073	1.066	1.059	1.054	1.043	1.038	1.026
.70	1.272	1.220	1.188	1.164	1.148	1.132	1.112	1.099	1.088	1.081	1.074	1.061	1.053	1.040
.75	1.302	1.245	1.210	1.185	1.166	1.151	1.127	1.112	1.102	1.092	1.085	1.071	1.062	1.047
.80	1.335	1.275	1.237	1.209	1.189	1.171	1.144	1.128	1.116	1.106	1.098	1.082	1.072	1.056
.85	1.377	1.312	1.271	1.237	1.215	1.194	1.165	1.146	1.132	1.123	1.113	1.096	1.083	1.067
.90	1.431	1.361	1.316	1.276	1.249	1.224	1.192	1.171	1.154	1.143	1.131	1.113	1.099	1.080
.95	1.518	1.437	1.374	1.332	1.302	1.273	1.233	1.211	1.188	1.175	1.159	1.137	1.123	1.101
.98	1.636	1.535	1.461	1.402	1.365	1.330	1.286	1.260	1.229	1.212	1.196	1.170	1.149	1.120

TABLE 4. Unbiasing Factors for c
 $E[B^*(n)c] = c$

n	5	6	7	8	9	10	12	14	16	18	20	25	30	50
$\bar{B}(n)$	.694	.749	.787	.816	.838	.856	.881	.898	.911	.922	.930	.944	.953	.973
$\tilde{B}(n)$	.856	.881	.897	.912	.922	.931	.943	.949	.957	.962	.965	.973	.977	.984
	.906	.922	.934	.943	.948	.953	.962	.967	.972	.974	.977	.982	.985	.988
	.931	.943	.949	.957	.962	.965	.972	.975	.979	.981	.983	.986	.989	.991
	.945	.953	.960	.965	.969	.973	.977	.981	.983	.985	.986	.989	.990	.993
	.960	.967	.972	.975	.978	.981	.984	.986	.988	.989	.990	.992	.993	.995
$\hat{B}(n)$	.781	.821	.846	.869	.883	.896	.915	.926	.937	.944	.950	.960	.967	.979
	.810	.843	.868	.886	.897	.909	.925	.936	.945	.951	.956	.965	.971	.981
	.823	.855	.875	.893	.906	.916	.931	.941	.948	.955	.960	.967	.973	.983
	.831	.861	.882	.898	.910	.920	.934	.944	.951	.957	.961	.969	.974	.984
	.840	.869	.889	.904	.915	.925	.938	.946	.953	.959	.963	.970	.975	.985

Note that $\bar{B}(n)$ is independent of m.

TABLE 5. Averaging M.L.E. ($m = 2$)
Percentage point, ℓ_Y , such that $P\{\bar{c}\ell_n(\bar{b}/b) < \ell_Y\} = \gamma$

γ	n	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02		-1.611	-1.355	-1.122	-1.017	-0.913	-0.858	-0.719	-0.653	-0.615	-0.564	-0.527	-0.456	-0.410	-0.309
.05		-1.165	-0.997	-0.866	-0.785	-0.705	-0.646	-0.562	-0.510	-0.480	-0.431	-0.420	-0.354	-0.325	-0.248
.10		-0.857	-0.735	-0.652	-0.579	-0.531	-0.498	-0.434	-0.390	-0.367	-0.333	-0.320	-0.276	-0.245	-0.190
.15		-0.671	-0.570	-0.516	-0.457	-0.420	-0.398	-0.342	-0.313	-0.289	-0.268	-0.256	-0.219	-0.197	-0.153
.20		-0.531	-0.451	-0.419	-0.365	-0.341	-0.319	-0.277	-0.251	-0.235	-0.216	-0.205	-0.180	-0.159	-0.123
.25		-0.416	-0.353	-0.332	-0.290	-0.268	-0.253	-0.219	-0.200	-0.185	-0.171	-0.165	-0.145	-0.127	-0.099
.30		-0.318	-0.269	-0.258	-0.222	-0.208	-0.195	-0.171	-0.156	-0.142	-0.133	-0.129	-0.112	-0.097	-0.078
.40		-0.159	-0.127	-0.125	-0.103	-0.099	-0.091	-0.078	-0.070	-0.069	-0.066	-0.062	-0.057	-0.043	-0.035
.50		-0.004	0.006	-0.003	0.003	-0.004	0.009	0.001	0.004	0.001	-0.002	0.001	-0.001	0.005	0.003
.60		0.146	0.138	0.119	0.109	0.095	0.098	0.079	0.080	0.073	0.066	0.063	0.056	0.054	0.041
.70		0.318	0.288	0.241	0.224	0.202	0.201	0.162	0.157	0.145	0.134	0.131	0.113	0.108	0.078
.75		0.407	0.364	0.309	0.291	0.258	0.255	0.211	0.200	0.187	0.174	0.167	0.149	0.136	0.100
.80		0.509	0.460	0.392	0.364	0.328	0.319	0.271	0.255	0.236	0.218	0.205	0.185	0.170	0.126
.85		0.639	0.573	0.488	0.451	0.410	0.392	0.339	0.317	0.290	0.267	0.256	0.226	0.208	0.153
.90		0.826	0.713	0.624	0.562	0.511	0.491	0.422	0.396	0.357	0.326	0.313	0.280	0.257	0.189
.95		1.137	0.953	0.843	0.741	0.668	0.633	0.557	0.524	0.466	0.416	0.405	0.360	0.327	0.250
.98		1.563	1.283	1.125	0.972	0.883	0.821	0.719	0.667	0.571	0.534	0.513	0.455	0.408	0.313

TABLE 5. Averaging M.L.E. (m = 3)

Percentage point, ℓ_Y , such that $P\{\bar{c}\ell n(\bar{b}/b) < \ell_Y\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.613	-1.335	-1.104	-1.021	-0.906	-0.869	-0.726	-0.655	-0.599	-0.581	-0.547	-0.463	-0.418	-0.314
.05	-1.183	-0.993	-0.852	-0.780	-0.690	-0.659	-0.559	-0.517	-0.469	-0.435	-0.426	-0.362	-0.326	-0.250
.10	-0.859	-0.737	-0.633	-0.585	-0.527	-0.506	-0.430	-0.398	-0.360	-0.336	-0.328	-0.280	-0.254	-0.191
.15	-0.660	-0.581	-0.498	-0.462	-0.424	-0.398	-0.344	-0.315	-0.287	-0.271	-0.259	-0.223	-0.202	-0.154
.20	-0.514	-0.461	-0.396	-0.368	-0.337	-0.319	-0.275	-0.256	-0.232	-0.218	-0.209	-0.182	-0.161	-0.126
.25	-0.402	-0.356	-0.310	-0.286	-0.266	-0.249	-0.215	-0.201	-0.184	-0.174	-0.166	-0.144	-0.128	-0.101
.30	-0.306	-0.269	-0.234	-0.218	-0.204	-0.194	-0.160	-0.153	-0.139	-0.130	-0.127	-0.113	-0.102	-0.079
.40	-0.131	-0.124	-0.104	-0.098	-0.095	-0.090	-0.072	-0.068	-0.064	-0.056	-0.060	-0.052	-0.049	-0.038
.50	0.027	0.019	0.020	0.011	0.004	0.004	0.013	0.008	0.009	0.010	-0.004	0.003	0.003	-0.002
.60	0.175	0.154	0.138	0.123	0.104	0.099	0.096	0.088	0.082	0.076	0.067	0.057	0.052	0.036
.70	0.342	0.294	0.264	0.236	0.213	0.197	0.182	0.168	0.156	0.142	0.135	0.114	0.104	0.078
.75	0.436	0.374	0.334	0.301	0.275	0.251	0.234	0.212	0.192	0.178	0.171	0.146	0.131	0.102
.80	0.540	0.463	0.414	0.377	0.344	0.313	0.291	0.260	0.238	0.222	0.210	0.181	0.163	0.124
.85	0.661	0.567	0.511	0.463	0.427	0.384	0.355	0.320	0.292	0.270	0.252	0.224	0.202	0.151
.90	0.848	0.708	0.639	0.575	0.533	0.476	0.435	0.394	0.359	0.334	0.314	0.274	0.248	0.185
.95	1.116	0.945	0.834	0.742	0.690	0.622	0.557	0.509	0.461	0.435	0.394	0.354	0.316	0.240
.98	1.519	1.258	1.074	0.965	0.879	0.788	0.702	0.635	0.590	0.549	0.495	0.446	0.403	0.304

TABLE 5. Averaging M.L.E. ($m = 4$)
 Percentage point, ℓ_Y , such that $P\{\bar{c}\ell_n(\bar{b}/b) < \ell_Y\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.605	-1.292	-1.098	-1.005	-0.925	-0.847	-0.745	-0.664	-0.615	-0.568	-0.528	-0.453	-0.421	-0.301
.05	-1.184	-0.986	-0.837	-0.768	-0.712	-0.660	-0.584	-0.511	-0.480	-0.441	-0.416	-0.357	-0.332	-0.242
.10	-0.866	-0.723	-0.626	-0.564	-0.526	-0.503	-0.440	-0.388	-0.366	-0.335	-0.317	-0.275	-0.255	-0.188
.15	-0.682	-0.566	-0.488	-0.441	-0.414	-0.400	-0.346	-0.306	-0.296	-0.270	-0.254	-0.224	-0.204	-0.152
.20	-0.533	-0.450	-0.383	-0.345	-0.326	-0.323	-0.276	-0.240	-0.239	-0.214	-0.204	-0.181	-0.164	-0.122
.25	-0.418	-0.346	-0.296	-0.270	-0.254	-0.261	-0.220	-0.188	-0.185	-0.172	-0.161	-0.142	-0.130	-0.096
.30	-0.313	-0.258	-0.220	-0.201	-0.193	-0.198	-0.164	-0.144	-0.142	-0.131	-0.124	-0.107	-0.099	-0.074
.40	-0.136	-0.109	-0.089	-0.086	-0.083	-0.093	-0.068	-0.062	-0.064	-0.062	-0.055	-0.047	-0.046	-0.034
.50	0.025	0.029	0.033	0.025	0.026	0.008	0.011	0.012	0.006	-0.000	0.006	0.008	0.003	0.004
.60	0.193	0.160	0.154	0.134	0.123	0.102	0.094	0.089	0.073	0.064	0.069	0.061	0.050	0.041
.70	0.374	0.310	0.278	0.251	0.231	0.205	0.181	0.165	0.146	0.138	0.132	0.116	0.105	0.081
.75	0.467	0.394	0.354	0.316	0.290	0.261	0.228	0.212	0.186	0.177	0.168	0.148	0.134	0.102
.80	0.576	0.483	0.432	0.384	0.351	0.323	0.280	0.262	0.230	0.220	0.210	0.181	0.164	0.126
.85	0.700	0.583	0.519	0.471	0.424	0.399	0.343	0.317	0.284	0.272	0.256	0.220	0.200	0.156
.90	0.866	0.717	0.637	0.585	0.518	0.496	0.427	0.394	0.351	0.332	0.313	0.273	0.246	0.191
.95	1.139	0.937	0.816	0.738	0.674	0.636	0.544	0.499	0.460	0.428	0.397	0.353	0.318	0.239
.98	1.492	1.200	1.040	0.929	0.859	0.793	0.688	0.632	0.566	0.525	0.495	0.445	0.397	0.301

TABLE 5. Averaging M.L.E. ($m = 5$)
Percentage point, ℓ_Y , such that $P\{\bar{c}\ell n(\bar{b}/b) < \ell_Y\} = \gamma$

γ	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.578	-1.282	-1.114	-1.011	-0.912	-0.856	-0.727	-0.662	-0.590	-0.577	-0.526	-0.461	-0.423	-0.317
.05	-1.186	-0.979	-0.852	-0.777	-0.719	-0.664	-0.573	-0.515	-0.469	-0.447	-0.419	-0.362	-0.330	-0.250
.10	-0.873	-0.722	-0.640	-0.580	-0.536	-0.497	-0.426	-0.384	-0.360	-0.341	-0.322	-0.275	-0.258	-0.191
.15	-0.680	-0.572	-0.500	-0.454	-0.423	-0.390	-0.339	-0.311	-0.288	-0.274	-0.256	-0.220	-0.206	-0.150
.20	-0.539	-0.453	-0.397	-0.363	-0.337	-0.310	-0.264	-0.250	-0.233	-0.218	-0.202	-0.177	-0.164	-0.121
.25	-0.425	-0.347	-0.311	-0.285	-0.268	-0.242	-0.207	-0.199	-0.185	-0.171	-0.157	-0.141	-0.131	-0.097
.30	-0.312	-0.261	-0.229	-0.211	-0.205	-0.176	-0.160	-0.151	-0.144	-0.133	-0.118	-0.109	-0.102	-0.074
.40	-0.128	-0.104	-0.096	-0.088	-0.093	-0.074	-0.064	-0.068	-0.067	-0.057	-0.051	-0.049	-0.050	-0.034
.50	0.039	0.032	0.031	0.024	0.011	0.023	0.017	0.008	0.008	0.010	0.009	0.008	-0.000	0.004
.60	0.204	0.161	0.150	0.133	0.112	0.116	0.100	0.086	0.078	0.076	0.072	0.061	0.050	0.038
.70	0.376	0.315	0.277	0.246	0.221	0.212	0.190	0.168	0.151	0.144	0.136	0.119	0.106	0.078
.75	0.477	0.386	0.347	0.306	0.280	0.268	0.238	0.214	0.195	0.181	0.171	0.152	0.134	0.099
.80	0.588	0.478	0.423	0.376	0.342	0.333	0.294	0.266	0.242	0.222	0.211	0.187	0.166	0.123
.85	0.718	0.584	0.513	0.457	0.416	0.396	0.357	0.323	0.293	0.270	0.258	0.226	0.200	0.151
.90	0.878	0.718	0.636	0.564	0.515	0.488	0.433	0.394	0.362	0.334	0.312	0.275	0.251	0.185
.95	1.147	0.923	0.834	0.715	0.659	0.615	0.550	0.506	0.454	0.420	0.395	0.349	0.322	0.241
.98	1.482	1.200	1.044	0.910	0.844	0.765	0.681	0.621	0.559	0.522	0.492	0.428	0.391	0.299

TABLE 5. Averaging M.L.E. ($m = 7$)Percentage point, ℓ_Y , such that $P\{\bar{c}\ell n(\bar{b}/b) < \ell_Y\} = Y$

$\frac{n}{Y}$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.558	-1.315	-1.123	-0.973	-0.921	-0.829	-0.726	-0.663	-0.612	-0.554	-0.523	-0.453	-0.407	-0.305
.05	-1.175	-0.994	-0.849	-0.764	-0.704	-0.637	-0.573	-0.504	-0.478	-0.432	-0.408	-0.359	-0.324	-0.239
.10	-0.867	-0.730	-0.635	-0.580	-0.544	-0.480	-0.439	-0.387	-0.355	-0.324	-0.308	-0.275	-0.251	-0.186
.15	-0.677	-0.575	-0.498	-0.453	-0.429	-0.374	-0.351	-0.307	-0.284	-0.258	-0.247	-0.222	-0.204	-0.147
.20	-0.523	-0.445	-0.391	-0.357	-0.341	-0.296	-0.278	-0.245	-0.225	-0.204	-0.198	-0.180	-0.162	-0.119
.25	-0.394	-0.342	-0.305	-0.278	-0.265	-0.227	-0.218	-0.189	-0.176	-0.162	-0.155	-0.142	-0.127	-0.092
.30	-0.286	-0.259	-0.222	-0.209	-0.201	-0.173	-0.168	-0.143	-0.132	-0.120	-0.118	-0.111	-0.098	-0.069
.40	-0.109	-0.109	-0.084	-0.090	-0.085	-0.071	-0.069	-0.059	-0.054	-0.046	-0.052	-0.051	-0.043	-0.031
.50	0.050	0.029	0.039	0.027	0.023	0.027	0.011	0.016	0.018	0.021	0.011	0.003	0.008	0.006
.60	0.211	0.167	0.157	0.133	0.120	0.119	0.097	0.088	0.092	0.082	0.075	0.059	0.530	0.041
.70	0.382	0.312	0.290	0.246	0.225	0.221	0.180	0.169	0.163	0.152	0.139	0.112	0.102	0.080
.75	0.480	0.397	0.360	0.307	0.288	0.277	0.230	0.213	0.203	0.190	0.175	0.144	0.132	0.101
.80	0.585	0.480	0.438	0.380	0.350	0.334	0.286	0.257	0.246	0.230	0.214	0.179	0.165	0.125
.85	0.707	0.577	0.535	0.466	0.432	0.405	0.347	0.319	0.298	0.275	0.256	0.220	0.200	0.151
.90	0.859	0.705	0.641	0.565	0.533	0.499	0.425	0.392	0.360	0.338	0.313	0.271	0.246	0.186
.95	1.100	0.918	0.806	0.721	0.677	0.632	0.538	0.497	0.455	0.429	0.397	0.340	0.311	0.233
.98	1.375	1.157	1.027	0.922	0.833	0.776	0.653	0.613	0.557	0.527	0.495	0.422	0.389	0.293

TABLE 6. M.L.E. by Normalization ($m = 2$)
Percentage point, λ_Y , such that $P\{\tilde{c}l_n(\tilde{E}/b) < \lambda_Y\} = Y$

$Y \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.224	-1.108	-0.976	-0.909	-0.827	-0.784	-0.679	-0.630	-0.598	-0.552	-0.520	-0.452	-0.410	-0.311
.05	-0.975	-0.875	-0.788	-0.730	-0.664	-0.620	-0.545	-0.504	-0.474	-0.430	-0.420	-0.357	-0.328	-0.252
.10	-0.761	-0.688	-0.613	-0.554	-0.514	-0.489	-0.431	-0.392	-0.369	-0.335	-0.322	-0.280	-0.251	-0.194
.15	-0.617	-0.547	-0.499	-0.450	-0.416	-0.396	-0.347	-0.318	-0.294	-0.274	-0.262	-0.226	-0.202	-0.157
.20	-0.508	-0.443	-0.416	-0.366	-0.346	-0.325	-0.284	-0.260	-0.242	-0.223	-0.212	-0.186	-0.165	-0.128
.25	-0.412	-0.357	-0.339	-0.300	-0.277	-0.263	-0.230	-0.210	-0.193	-0.179	-0.172	-0.150	-0.133	-0.103
.30	-0.332	-0.283	-0.272	-0.235	-0.222	-0.208	-0.184	-0.167	-0.152	-0.144	-0.136	-0.119	-0.104	-0.081
.40	-0.189	-0.158	-0.150	-0.127	-0.117	-0.109	-0.094	-0.084	-0.081	-0.077	-0.072	-0.065	-0.051	-0.039
.50	-0.056	-0.042	-0.041	-0.027	-0.029	-0.018	-0.020	-0.013	-0.016	-0.014	-0.011	-0.010	-0.003	-0.001
.60	0.073	0.074	0.067	0.067	0.059	0.067	0.052	0.059	0.053	0.049	0.050	0.045	0.045	0.036
.70	0.211	0.200	0.176	0.169	0.154	0.159	0.130	0.132	0.122	0.112	0.114	0.100	0.097	0.073
.75	0.281	0.264	0.233	0.225	0.205	0.208	0.176	0.171	0.161	0.151	0.146	0.133	0.125	0.094
.80	0.357	0.334	0.298	0.289	0.261	0.264	0.225	0.218	0.207	0.193	0.182	0.167	0.158	0.119
.85	0.450	0.421	0.377	0.360	0.335	0.327	0.288	0.274	0.256	0.236	0.229	0.207	0.191	0.144
.90	0.570	0.531	0.474	0.452	0.418	0.403	0.360	0.344	0.313	0.388	0.281	0.258	0.236	0.180
.95	0.775	0.682	0.629	0.582	0.540	0.517	0.465	0.448	0.407	0.373	0.366	0.334	0.301	0.238
.98	0.971	0.864	0.822	0.722	0.683	0.676	0.594	0.567	0.498	0.471	0.454	0.413	0.376	0.296

TABLE 6. M.L.E. by Normalization ($m = 3$)Percentage point, ℓ_Y , such that $P\{\tilde{c}\ell n(\tilde{b}/b) < \ell_Y\} = Y$

$\frac{n}{Y}$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.196	-1.088	-0.942	-0.904	-0.822	-0.795	-0.688	-0.623	-0.580	-0.564	-0.534	-0.462	-0.416	-0.316
.05	-0.952	-0.852	-0.752	-0.711	-0.647	-0.625	-0.544	-0.508	-0.462	-0.436	-0.423	-0.364	-0.330	-0.253
.10	-0.740	-0.670	-0.592	-0.553	-0.511	-0.493	-0.428	-0.398	-0.362	-0.340	-0.331	-0.285	-0.258	-0.195
.15	-0.593	-0.545	-0.482	-0.453	-0.420	-0.396	-0.349	-0.321	-0.295	-0.279	-0.267	-0.229	-0.210	-0.159
.20	-0.488	-0.444	-0.398	-0.371	-0.346	-0.326	-0.284	-0.265	-0.243	-0.228	-0.218	-0.190	-0.169	-0.131
.25	-0.398	-0.362	-0.321	-0.300	-0.281	-0.261	-0.230	-0.213	-0.197	-0.186	-0.178	-0.154	-0.136	-0.106
.30	-0.323	-0.290	-0.257	-0.239	-0.225	-0.212	-0.177	-0.168	-0.154	-0.144	-0.140	-0.122	-0.104	-0.085
.40	-0.181	-0.165	-0.144	-0.132	-0.128	-0.116	-0.095	-0.090	-0.080	-0.072	-0.074	-0.065	-0.059	-0.045
.50	-0.054	-0.047	-0.035	-0.035	-0.035	-0.032	-0.017	-0.016	-0.013	-0.009	-0.013	-0.011	-0.009	-0.008
.60	0.070	0.065	0.064	0.060	0.054	0.054	0.059	0.057	0.055	0.054	0.048	0.041	0.038	0.029
.70	0.197	0.175	0.168	0.163	0.147	0.140	0.138	0.130	0.123	0.116	0.113	0.095	0.089	0.070
.75	0.262	0.240	0.230	0.219	0.200	0.190	0.181	0.169	0.157	0.150	0.144	0.126	0.115	0.093
.80	0.340	0.308	0.288	0.282	0.260	0.241	0.231	0.212	0.198	0.188	0.181	0.160	0.146	0.115
.85	0.426	0.390	0.368	0.347	0.325	0.301	0.288	0.266	0.248	0.234	0.221	0.198	0.180	0.140
.90	0.533	0.491	0.454	0.430	0.407	0.373	0.354	0.327	0.307	0.288	0.275	0.243	0.226	0.173
.95	0.686	0.636	0.595	0.549	0.523	0.485	0.455	0.426	0.400	0.375	0.348	0.317	0.290	0.226
.98	0.846	0.795	0.737	0.672	0.660	0.603	0.569	0.523	0.503	0.475	0.436	0.402	0.364	0.282

TABLE 6. M.L.E. by Normalization ($m = 4$)Percentage point, ℓ_Y , such that $P_{\mathbf{r}}\{\tilde{\ell}n(\tilde{E}/b) < \ell_Y\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.188	-1.050	-0.946	-0.882	-0.828	-0.778	-0.704	-0.636	-0.593	-0.554	-0.518	-0.450	-0.419	-0.304
.05	-0.951	-0.849	-0.748	-0.704	-0.658	-0.624	-0.564	-0.497	-0.473	-0.439	-0.417	-0.359	-0.334	-0.246
.10	-0.735	-0.651	-0.584	-0.539	-0.510	-0.491	-0.434	-0.389	-0.369	-0.340	-0.323	-0.282	-0.261	-0.192
.15	-0.607	-0.529	-0.474	-0.436	-0.415	-0.399	-0.353	-0.313	-0.305	-0.278	-0.262	-0.232	-0.212	-0.157
.20	-0.497	-0.433	-0.386	-0.354	-0.336	-0.330	-0.289	-0.253	-0.250	-0.227	-0.214	-0.190	-0.172	-0.127
.25	-0.412	-0.352	-0.313	-0.285	-0.272	-0.274	-0.238	-0.205	-0.199	-0.186	-0.174	-0.153	-0.140	-0.103
.30	-0.337	-0.287	-0.251	-0.228	-0.216	-0.221	-0.185	-0.162	-0.160	-0.146	-0.139	-0.119	-0.110	-0.081
.40	-0.196	-0.158	-0.135	-0.129	-0.120	-0.125	-0.096	-0.087	-0.084	-0.080	-0.073	-0.060	-0.059	-0.040
.50	-0.069	-0.047	-0.032	-0.033	-0.027	-0.034	-0.023	-0.017	-0.021	-0.023	-0.014	-0.007	-0.010	-0.004
.60	0.059	0.061	0.071	0.063	0.064	0.050	0.052	0.051	0.044	0.037	0.045	0.043	0.036	0.032
.70	0.192	0.176	0.170	0.160	0.157	0.141	0.131	0.125	0.110	0.108	0.105	0.096	0.087	0.070
.75	0.266	0.237	0.226	0.216	0.204	0.186	0.173	0.164	0.146	0.144	0.138	0.124	0.115	0.091
.80	0.337	0.305	0.291	0.273	0.257	0.241	0.217	0.209	0.189	0.185	0.180	0.156	0.143	0.115
.85	0.426	0.381	0.360	0.340	0.317	0.305	0.271	0.259	0.235	0.230	0.220	0.192	0.178	0.143
.90	0.524	0.472	0.448	0.426	0.392	0.379	0.342	0.324	0.295	0.283	0.270	0.241	0.219	0.177
.95	0.667	0.603	0.575	0.537	0.504	0.484	0.441	0.413	0.388	0.367	0.342	0.314	0.290	0.223
.98	0.829	0.769	0.716	0.669	0.628	0.606	0.552	0.512	0.479	0.450	0.428	0.392	0.358	0.278

TABLE 6. M.L.E. by Normalization ($m = 5$)
Percentage point, ℓ_Y , such that $P_r \{ \tilde{c} \ell n(\tilde{b}/b) < \ell_Y \} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.190	-1.055	-0.960	-0.898	-0.825	-0.783	-0.689	-0.637	-0.575	-0.559	-0.519	-0.457	-0.423	-0.320
.05	-0.943	-0.835	-0.765	-0.709	-0.657	-0.624	-0.554	-0.506	-0.467	-0.446	-0.419	-0.365	-0.336	-0.254
.10	-0.737	-0.647	-0.595	-0.550	-0.517	-0.487	-0.425	-0.385	-0.363	-0.347	-0.328	-0.281	-0.264	-0.197
.15	-0.597	-0.538	-0.480	-0.444	-0.420	-0.394	-0.344	-0.320	-0.296	-0.284	-0.264	-0.229	-0.215	-0.157
.20	-0.497	-0.441	-0.395	-0.365	-0.345	-0.324	-0.277	-0.263	-0.245	-0.231	-0.214	-0.187	-0.173	-0.127
.25	-0.415	-0.358	-0.328	-0.304	-0.285	-0.260	-0.225	-0.217	-0.201	-0.187	-0.171	-0.154	-0.141	-0.104
.30	-0.333	-0.285	-0.257	-0.239	-0.230	-0.203	-0.182	-0.172	-0.161	-0.149	-0.135	-0.122	-0.112	-0.081
.40	-0.194	-0.162	-0.143	-0.131	-0.131	-0.113	-0.096	-0.093	-0.091	-0.079	-0.070	-0.064	-0.063	-0.042
.50	-0.065	-0.051	-0.040	-0.035	-0.041	-0.026	-0.022	-0.024	-0.020	-0.017	-0.013	-0.010	-0.016	-0.005
.60	0.061	0.056	0.059	0.058	0.050	0.058	0.054	0.048	0.045	0.046	0.046	0.041	0.034	0.029
.70	0.185	0.168	0.166	0.153	0.141	0.143	0.133	0.120	0.110	0.109	0.107	0.095	0.088	0.067
.75	0.254	0.230	0.221	0.203	0.190	0.192	0.175	0.161	0.151	0.144	0.140	0.127	0.115	0.088
.80	0.330	0.296	0.276	0.254	0.244	0.245	0.222	0.209	0.195	0.180	0.175	0.161	0.143	0.111
.85	0.418	0.372	0.345	0.319	0.306	0.301	0.280	0.261	0.239	0.227	0.216	0.196	0.178	0.138
.90	0.522	0.469	0.434	0.403	0.382	0.370	0.343	0.322	0.301	0.282	0.268	0.242	0.222	0.171
.95	0.659	0.599	0.562	0.519	0.496	0.469	0.435	0.415	0.379	0.357	0.340	0.311	0.291	0.224
.98	0.822	0.745	0.696	0.649	0.615	0.583	0.538	0.510	0.471	0.446	0.424	0.380	0.354	0.275

TABLE 6. M.L.E. by Normalization ($m = 7$)
Percentage point, ℓ_Y , such that $P_{\Gamma} \{ \tilde{\text{cln}}(\tilde{b}/b) < \ell_Y \} = \gamma$

γ	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.182	-1.070	-0.959	-0.866	-0.833	-0.761	-0.679	-0.634	-0.590	-0.543	-0.514	-0.450	-0.406	-0.307
.05	-0.935	-0.843	-0.756	-0.696	-0.657	-0.603	-0.552	-0.497	-0.474	-0.432	-0.411	-0.362	-0.328	-0.244
.10	-0.727	-0.650	-0.589	-0.549	-0.524	-0.470	-0.436	-0.387	-0.360	-0.329	-0.315	-0.285	-0.258	-0.191
.15	-0.597	-0.535	-0.481	-0.444	-0.428	-0.380	-0.357	-0.315	-0.295	-0.268	-0.256	-0.232	-0.212	-0.154
.20	-0.489	-0.438	-0.390	-0.362	-0.347	-0.312	-0.290	-0.259	-0.240	-0.220	-0.211	-0.192	-0.171	-0.126
.25	-0.396	-0.361	-0.319	-0.296	-0.285	-0.249	-0.237	-0.209	-0.192	-0.177	-0.171	-0.154	-0.138	-0.100
.30	-0.317	-0.288	-0.252	-0.240	-0.230	-0.200	-0.193	-0.165	-0.153	-0.139	-0.136	-0.124	-0.111	-0.077
.40	-0.181	-0.170	-0.139	-0.135	-0.128	-0.112	-0.105	-0.089	-0.079	-0.071	-0.073	-0.068	-0.058	-0.039
.50	-0.063	-0.064	-0.040	-0.038	-0.034	-0.024	-0.030	-0.020	-0.014	-0.007	-0.014	-0.016	-0.008	-0.004
.60	0.053	0.048	0.060	0.052	0.051	0.056	0.045	0.045	0.052	0.051	0.046	0.037	0.035	0.031
.70	0.182	0.159	0.164	0.147	0.139	0.144	0.125	0.118	0.121	0.115	0.105	0.086	0.082	0.069
.75	0.246	0.222	0.220	0.195	0.187	0.188	0.165	0.155	0.157	0.149	0.139	0.117	0.110	0.089
.80	0.321	0.290	0.277	0.253	0.245	0.238	0.212	0.199	0.196	0.185	0.174	0.151	0.141	0.112
.85	0.399	0.359	0.349	0.323	0.307	0.295	0.265	0.251	0.243	0.230	0.215	0.187	0.176	0.136
.90	0.497	0.444	0.433	0.405	0.390	0.371	0.332	0.314	0.298	0.282	0.267	0.237	0.219	0.170
.95	0.633	0.572	0.544	0.514	0.500	0.482	0.424	0.405	0.379	0.361	0.343	0.299	0.279	0.217
.98	0.786	0.718	0.675	0.635	0.620	0.581	0.524	0.501	0.468	0.443	0.422	0.372	0.351	0.272

TABLE 7. Joint M.L.E. ($m = 2$)
Percentage point, ℓ_Y , such that $P_r\{\hat{C}n(\hat{b}/b) < \ell_Y = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.460	-1.272	-1.081	-0.993	-0.889	-0.827	-0.707	-0.648	-0.608	-0.561	-0.526	-0.455	-0.410	-0.310
.05	-1.081	-0.944	-0.840	-0.768	-0.690	-0.636	-0.557	-0.507	-0.479	-0.431	-0.421	-0.356	-0.327	-0.250
.10	-0.814	-0.715	-0.638	-0.573	-0.525	-0.498	-0.434	-0.392	-0.368	-0.334	-0.322	-0.278	-0.249	-0.192
.15	-0.645	-0.563	-0.511	-0.453	-0.421	-0.400	-0.346	-0.317	-0.291	-0.272	-0.260	-0.223	-0.200	-0.155
.20	-0.520	-0.449	-0.418	-0.368	-0.346	-0.324	-0.282	-0.257	-0.239	-0.221	-0.210	-0.184	-0.163	-0.127
.25	-0.416	-0.359	-0.338	-0.298	-0.275	-0.262	-0.227	-0.207	-0.191	-0.177	-0.170	-0.149	-0.131	-0.102
.30	-0.331	-0.281	-0.270	-0.232	-0.218	-0.204	-0.181	-0.163	-0.149	-0.141	-0.134	-0.117	-0.102	-0.080
.40	-0.182	-0.151	-0.142	-0.122	-0.111	-0.104	-0.091	-0.080	-0.078	-0.074	-0.070	-0.063	-0.048	-0.038
.50	-0.044	-0.030	-0.029	-0.017	-0.022	-0.012	-0.013	-0.008	-0.011	-0.010	-0.007	-0.008	-0.001	0.000
.60	0.093	0.096	0.083	0.080	0.071	0.076	0.060	0.065	0.059	0.055	0.054	0.049	0.048	0.037
.70	0.241	0.230	0.199	0.187	0.171	0.174	0.142	0.140	0.129	0.120	0.119	0.105	0.101	0.075
.75	0.326	0.301	0.263	0.246	0.223	0.227	0.189	0.182	0.172	0.160	0.155	0.139	0.129	0.096
.80	0.413	0.378	0.331	0.321	0.287	0.287	0.245	0.232	0.220	0.203	0.193	0.174	0.163	0.121
.85	0.527	0.479	0.419	0.396	0.364	0.354	0.313	0.293	0.271	0.249	0.241	0.216	0.198	0.148
.90	0.679	0.615	0.536	0.499	0.462	0.439	0.388	0.367	0.335	0.307	0.296	0.271	0.246	0.184
.95	0.925	0.799	0.727	0.660	0.602	0.578	0.517	0.489	0.439	0.397	0.386	0.347	0.317	0.245
.98	1.225	1.042	0.968	0.847	0.788	0.758	0.665	0.624	0.536	0.506	0.489	0.438	0.396	0.309

TABLE 7. Joint M.L.E. ($m = 3$)
Percentage point, ℓ_Y , such that $P_r \{ \hat{\ell}_n(\hat{\theta}/b) < \ell_Y \} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.416	-1.245	-1.044	-0.981	-0.880	-0.842	-0.717	-0.642	-0.590	-0.577	-0.544	-0.462	-0.418	-0.315
.05	-1.075	-0.923	-0.816	-0.752	-0.677	-0.645	-0.553	-0.515	-0.466	-0.438	-0.426	-0.362	-0.328	-0.251
.10	-0.804	-0.706	-0.617	-0.571	-0.523	-0.501	-0.430	-0.400	-0.361	-0.339	-0.330	-0.282	-0.256	-0.193
.15	-0.623	-0.565	-0.492	-0.459	-0.423	-0.398	-0.349	-0.319	-0.292	-0.275	-0.264	-0.227	-0.206	-0.157
.20	-0.503	-0.454	-0.401	-0.372	-0.345	-0.324	-0.281	-0.263	-0.240	-0.224	-0.215	-0.187	-0.166	-0.129
.25	-0.406	-0.364	-0.319	-0.298	-0.276	-0.258	-0.226	-0.209	-0.193	-0.182	-0.173	-0.150	-0.133	-0.104
.30	-0.321	-0.286	-0.252	-0.234	-0.217	-0.207	-0.171	-0.163	-0.149	-0.139	-0.136	-0.119	-0.107	-0.083
.40	-0.169	-0.154	-0.132	-0.120	-0.119	-0.108	-0.087	-0.083	-0.075	-0.066	-0.069	-0.060	-0.056	-0.043
.50	-0.032	-0.028	-0.019	-0.021	-0.024	-0.021	-0.006	-0.007	-0.005	-0.003	-0.008	-0.007	-0.005	-0.006
.60	0.103	0.091	0.087	0.079	0.070	0.069	0.073	0.066	0.063	0.062	0.054	0.047	0.043	0.031
.70	0.245	0.214	0.203	0.187	0.173	0.161	0.154	0.145	0.134	0.126	0.121	0.101	0.095	0.073
.75	0.322	0.284	0.264	0.249	0.228	0.212	0.201	0.186	0.171	0.159	0.154	0.134	0.120	0.096
.80	0.409	0.367	0.331	0.317	0.291	0.267	0.254	0.228	0.214	0.200	0.194	0.168	0.153	0.119
.85	0.512	0.457	0.421	0.394	0.368	0.332	0.316	0.288	0.268	0.249	0.236	0.209	0.189	0.144
.90	0.655	0.578	0.533	0.490	0.461	0.417	0.391	0.358	0.331	0.309	0.293	0.257	0.237	0.178
.95	0.875	0.768	0.701	0.639	0.595	0.548	0.505	0.469	0.432	0.405	0.373	0.338	0.303	0.234
.98	1.116	0.997	0.905	0.816	0.751	0.710	0.643	0.581	0.545	0.515	0.471	0.428	0.386	0.295

TABLE 7. Joint M.L.E. ($m = 4$)
Percentage point, ℓ_Y , such that $P\{\hat{\ell}_n(b/b) < \ell_Y\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.409	-1.185	-1.028	-0.953	-0.887	-0.819	-0.730	-0.659	-0.609	-0.565	-0.526	-0.453	-0.421	-0.303
.05	-1.075	-0.922	-0.792	-0.738	-0.691	-0.646	-0.575	-0.506	-0.477	-0.441	-0.416	-0.359	-0.333	-0.244
.10	-0.799	-0.687	-0.610	-0.555	-0.518	-0.498	-0.439	-0.389	-0.368	-0.339	-0.320	-0.279	-0.258	-0.190
.15	-0.644	-0.550	-0.485	-0.442	-0.416	-0.401	-0.351	-0.310	-0.303	-0.276	-0.259	-0.228	-0.209	-0.155
.20	-0.516	-0.444	-0.387	-0.353	-0.335	-0.330	-0.285	-0.248	-0.246	-0.223	-0.210	-0.187	-0.169	-0.125
.25	-0.422	-0.353	-0.312	-0.281	-0.267	-0.270	-0.232	-0.199	-0.194	-0.181	-0.169	-0.150	-0.135	-0.100
.30	-0.333	-0.280	-0.242	-0.219	-0.209	-0.213	-0.178	-0.155	-0.154	-0.141	-0.132	-0.114	-0.106	-0.079
.40	-0.182	-0.144	-0.122	-0.114	-0.107	-0.114	-0.086	-0.078	-0.077	-0.074	-0.067	-0.055	-0.054	-0.038
.50	-0.042	-0.021	-0.013	-0.014	-0.009	-0.021	-0.010	-0.008	-0.012	-0.015	-0.007	-0.001	-0.005	-0.001
.60	0.100	0.094	0.100	0.088	0.084	0.069	0.068	0.064	0.055	0.047	0.054	0.049	0.041	0.035
.70	0.253	0.221	0.207	0.191	0.182	0.162	0.149	0.140	0.125	0.120	0.115	0.105	0.094	0.074
.75	0.334	0.292	0.271	0.252	0.237	0.214	0.193	0.183	0.160	0.157	0.149	0.133	0.122	0.096
.80	0.420	0.370	0.342	0.315	0.292	0.272	0.242	0.231	0.206	0.200	0.192	0.167	0.153	0.120
.85	0.524	0.457	0.422	0.390	0.361	0.343	0.302	0.282	0.257	0.248	0.235	0.205	0.187	0.148
.90	0.656	0.568	0.521	0.488	0.445	0.429	0.379	0.351	0.322	0.304	0.290	0.256	0.230	0.183
.95	0.849	0.736	0.679	0.628	0.581	0.550	0.492	0.452	0.424	0.398	0.368	0.330	0.305	0.231
.98	1.094	0.955	0.862	0.781	0.736	0.700	0.612	0.568	0.527	0.491	0.464	0.422	0.381	0.291

TABLE 7. Joint M.L.E. ($m = 5$)
Percentage point, ℓ_Y , such that $P\{\mathcal{C}n(\delta/p) < \ell_Y\} = Y$

$\frac{n}{Y}$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.378	-1.171	-1.049	-0.968	-0.882	-0.825	-0.710	-0.652	-0.583	-0.572	-0.525	-0.459	-0.423	-0.318
.05	-1.058	-0.910	-0.808	-0.750	-0.691	-0.648	-0.565	-0.510	-0.469	-0.447	-0.420	-0.363	-0.333	-0.252
.10	-0.802	-0.686	-0.620	-0.567	-0.529	-0.493	-0.427	-0.385	-0.362	-0.344	-0.326	-0.278	-0.262	-0.195
.15	-0.637	-0.556	-0.493	-0.450	-0.422	-0.394	-0.343	-0.316	-0.293	-0.280	-0.261	-0.225	-0.211	-0.155
.20	-0.518	-0.450	-0.399	-0.367	-0.344	-0.321	-0.274	-0.259	-0.240	-0.226	-0.209	-0.183	-0.169	-0.125
.25	-0.424	-0.357	-0.322	-0.299	-0.281	-0.254	-0.217	-0.210	-0.195	-0.181	-0.166	-0.149	-0.137	-0.101
.30	-0.331	-0.280	-0.249	-0.229	-0.222	-0.194	-0.174	-0.164	-0.155	-0.143	-0.128	-0.117	-0.108	-0.079
.40	-0.177	-0.145	-0.128	-0.116	-0.117	-0.101	-0.085	-0.085	-0.083	-0.071	-0.064	-0.058	-0.058	-0.039
.50	-0.033	-0.024	-0.016	-0.016	-0.023	-0.010	-0.008	-0.013	-0.010	-0.007	-0.005	-0.004	-0.010	-0.002
.60	0.105	0.092	0.090	0.083	0.070	0.079	0.070	0.062	0.057	0.057	0.056	0.049	0.040	0.033
.70	0.248	0.216	0.206	0.188	0.170	0.166	0.155	0.137	0.125	0.123	0.117	0.104	0.095	0.071
.75	0.327	0.284	0.264	0.239	0.222	0.222	0.199	0.181	0.169	0.160	0.152	0.137	0.122	0.093
.80	0.419	0.359	0.331	0.299	0.282	0.280	0.251	0.232	0.211	0.196	0.190	0.171	0.153	0.116
.85	0.529	0.456	0.407	0.372	0.351	0.342	0.312	0.287	0.263	0.245	0.233	0.210	0.187	0.143
.90	0.648	0.568	0.513	0.466	0.436	0.416	0.381	0.353	0.327	0.305	0.288	0.256	0.235	0.178
.95	0.836	0.731	0.674	0.606	0.569	0.537	0.487	0.457	0.417	0.385	0.367	0.330	0.306	0.233
.98	1.063	0.942	0.852	0.762	0.710	0.671	0.607	0.566	0.513	0.487	0.455	0.405	0.374	0.288

TABLE 7. Joint M.L.E. ($m = 7$)
Percentage point, k_Y , such that $P_r\{\hat{\alpha}n(\hat{b}/b) < k_Y\} = \gamma$

$\gamma \backslash n$	5	6	7	8	9	10	12	14	16	18	20	25	30	50
.02	-1.364	-1.186	-1.037	-0.930	-0.881	-0.799	-0.705	-0.648	-0.606	-0.550	-0.520	-0.450	-0.407	-0.306
.05	-1.046	-0.917	-0.801	-0.731	-0.680	-0.621	-0.561	-0.502	-0.477	-0.432	-0.411	-0.361	-0.326	-0.242
.10	-0.792	-0.682	-0.612	-0.565	-0.534	-0.477	-0.439	-0.389	-0.358	-0.327	-0.313	-0.281	-0.255	-0.189
.15	-0.635	-0.554	-0.493	-0.451	-0.430	-0.379	-0.355	-0.313	-0.292	-0.265	-0.253	-0.228	-0.209	-0.151
.20	-0.509	-0.448	-0.395	-0.362	-0.348	-0.307	-0.287	-0.254	-0.235	-0.213	-0.207	-0.187	-0.167	-0.123
.25	-0.401	-0.359	-0.315	-0.290	-0.278	-0.242	-0.230	-0.202	-0.187	-0.171	-0.165	-0.150	-0.134	-0.097
.30	-0.311	-0.280	-0.243	-0.231	-0.221	-0.190	-0.184	-0.156	-0.146	-0.131	-0.129	-0.119	-0.106	-0.074
.40	-0.160	-0.151	-0.121	-0.118	-0.114	-0.098	-0.092	-0.078	-0.070	-0.062	-0.065	-0.060	-0.052	-0.036
.50	-0.028	-0.036	-0.012	-0.015	-0.013	-0.007	-0.016	-0.008	-0.002	0.003	-0.004	-0.009	-0.002	0.000
.60	0.103	0.088	0.095	0.079	0.076	0.079	0.064	0.060	0.067	0.063	-0.058	0.046	0.043	0.035
.70	0.248	0.212	0.208	0.184	0.170	0.174	0.146	0.138	0.138	0.131	-0.119	0.097	0.090	0.074
.75	0.317	0.283	0.271	0.237	0.224	0.221	0.190	0.178	0.175	0.165	-0.153	0.129	0.118	0.094
.80	0.408	0.357	0.336	0.300	0.287	0.277	0.242	0.222	0.216	0.203	-0.190	0.162	0.151	0.118
.85	0.513	0.440	0.420	0.380	0.354	0.342	0.298	0.281	0.267	0.249	-0.232	0.202	0.187	0.143
.90	0.629	0.538	0.507	0.466	0.448	0.424	0.372	0.347	0.323	0.307	-0.286	0.251	0.232	0.177
.95	0.803	0.702	0.643	0.600	0.574	0.543	0.471	0.448	0.415	0.396	-0.368	0.317	0.294	0.225
.98	1.003	0.890	0.827	0.755	0.710	0.675	0.586	0.548	0.513	0.482	-0.457	0.398	0.370	0.283