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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

GEOMETRY, ARTIFICIAL SATELLITES,
AND ORBIT DETERMINATION

L. G. TAFF
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ABSTRACT

The large proper motions and diurnal parallaxes of earth-bound artificial satellites coupled with the use of devices more sophisticated than the photographic plate to record their motion force one to consider methods of analysis beyond the traditional ones. In particular, it is theoretically possible to deduce from the streak made by the passage of a celestial object a complete specification of its location and velocity. Thus, for artificial satellites, one has the possibility of orbital element set construction from a complete set of initial conditions without recourse to any approximations. In this report, I show how and why this is possible. The practical situation is less hopeful however, in that the distance and radial velocity can be determined only if the streak has measureable curvature. Extremely accurate ($\pm 0.05/\text{sec}$) angular velocities should be obtainable. The positional accuracy is a function of the driving of the telescope.

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I. MOTIVATION

If one uses a solid state device camera (CID or CCD) or an electron beam tube with digitized output to observe artificial satellites, then a whole new field of optical observing and data reduction is thrown open. With such a device connected to a telescope moving at the sidereal rates the time history of the motion of a satellite can be obtained. That is, we can know the coordinates of the light deposited on the camera target and when that light was deposited. This is possible with a photographic plate, but difficult to do rapidly and accurately. Within this knowledge is a set of complete information concerning the satellite's geocentric location and velocity. Hence, an analysis of the image's location as a function of time can yield a full set of initial conditions for orbital analysis. Moreover, there should be an extremely high internal accuracy in these points as they all suffer from nearly the same systematic errors. The random and systematic errors associated with the telescope pointing enter weakly or not at all. Hence, we have the promise of more information than just the object's position and of extremely high accuracy in this additional data.

The analysis of the imaged motion is straightforward. It is given in Sections II, III, and IV. In Section V, a practical discussion of the situation reveals that the distance and radial velocity can only be determined if the motion on the camera target deviates from a straight line. The angular velocity will be very precisely ascertained and this information should be used in initial orbit construction as well as in the differential correction of orbits^{1,2}.

II. THE GEOCENTRIC GEOMETRY

We use the standard, almost inertial, spherical, geocentric coordinate system based on the earth's equator and the Vernal Equinox. A celestial object's location is given by $\underline{r} = r\underline{\ell}$, $\underline{\ell} = (\cos\delta\cos\alpha, \cos\delta\sin\alpha, \sin\delta)$. The geocentric distance is r , the geocentric declination is δ , and the geocentric right ascension is α . Over very short times, very much less than an orbital period ($\leq 0.3\%$ in practice), we regard the motion as uniform and rectilinear in this coordinate system. Since this is different from uniform "rectilinear" (e.g., along a great circle) motion on the celestial sphere, there is a coupling between the quotient of radial velocity ($\dot{r}$) and the distance (r) and the object's position (e.g., $\underline{\ell}$). To see how this comes about consider the orthonormal basis given by $\underline{\ell}$, $\underline{a}$, and $\underline{d}$; $\underline{a} = (-\sin\alpha, \cos\alpha, 0)$, $\underline{d} = (-\sin\delta\cos\alpha, -\sin\delta\sin\alpha, \cos\delta)$. We can write

$$\dot{\underline{r}} = \underline{\ell} \cdot \dot{\underline{r}}, \quad \underline{\omega} = \dot{\alpha} \cos\delta \underline{a} + \dot{\delta} \underline{d}. \quad (1)$$

In terms of the constant unit vector $\underline{n} = (0, 0, 1)$, which points towards the North Celestial Pole,

$$\dot{\underline{r}} = \underline{\ell} \cdot \dot{\underline{r}}, \quad (2a)$$

$$\dot{\alpha} \cos\delta = \underline{a} \cdot \underline{\omega} = \underline{n} \cdot (\underline{\ell} \times \dot{\underline{r}}) / (r \cos\delta), \quad (2b)$$

$$\dot{\delta} = \underline{d} \cdot \underline{\omega} = (\underline{n} \cdot \dot{\underline{r}} \sec\delta - \dot{r} \tan\delta) / r. \quad (2c)$$

By differentiating Eqs. (2) and enforcing the constraint $\ddot{\underline{r}} = \underline{0}$, we find,

$$\ddot{\underline{r}} = r\omega^2 \quad (3a)$$

$$\ddot{\alpha} = 2\dot{\alpha}(\dot{\delta}\tan\delta - \dot{r}/r), \quad (3b)$$

$$\ddot{\delta} = -\dot{\alpha}^2 \sin\delta \cos\delta - 2\dot{\delta}(\dot{r}/r). \quad (3c)$$

Thus, even though the space motion is along a geodesic, the position of the object as a function of time changes because of its purely radial motion. This term is known as the foreshortening term in astrometry. It can be measured for some natural celestial objects. To convince the reader that $\dot{r}/r$ is not numerically much smaller than $\dot{\alpha}$ or $\dot{\delta}$ for artificial satellites consider a satellite with argument of perigee = 270° , inclination = 60° , eccentricity = $1/\sqrt{2}$. These numbers are typical of the Molniya class satellite of which, with their associated rocket bodies, there are hundreds. For such a satellite observed at the equator $\dot{\alpha} = n\sqrt{2}$, $\dot{\delta} = n\sqrt{6}$, $\dot{r}/r = 2n$ where n is the mean motion (for which I'll use 2 rev/day when it's needed). Clearly, the three quantities are comparable and, since $\delta = 0$, the foreshortening terms drive the acceleration of the position.

To derive the rigorous results we use the equations of motion

$$\underline{r}(t) = \underline{r}(t_0) + \dot{\underline{r}}(t_0)(t - t_0), \quad (4a)$$

$$\underline{t}(t) = \underline{t}(t_0). \quad (4b)$$

By manipulating the component forms of Eqs. (4), we can obtain

$$(d = r(t)/r(t_0), \mu_\alpha = \dot{\alpha}(t_0), \mu_\delta = \dot{\delta}(t_0), v = \dot{r}(t_0)/r(t_0), T = t - t_0, \delta_0 = \delta(t_0), \alpha = \alpha(t), \text{ etc.}).$$

$$d^2 = 1 + 2vT + (v^2 + \omega^2)T^2, \quad (5a)$$

$$\tan(\alpha - \alpha_0) = \mu_\alpha T / [1 + (v - \mu_\delta \tan \delta_0)T], \quad (5b)$$

$$\tan[(\delta - \delta_0)/2] = \frac{\mu_\delta T + (1 - d + vT)\tan \delta_0}{d + \{[1 + (v - \mu_\delta \tan \delta_0)T]^2 + \mu_\alpha^2 T^2\}^{1/2}}, \quad (5c)$$

$$\dot{r}/r = \dot{d}/d = [v + (v^2 + \omega^2)T]/d^2, \quad (6a)$$

$$\dot{\alpha} = \mu_\alpha / \{[1 + (v - \mu_\delta \tan \delta_0)T]^2 + \mu_\alpha^2 T^2\}, \quad (6b)$$

$$\dot{\delta} = \frac{\mu_\delta (1 + vT) - \omega^2 T \tan \delta_0}{d^2 \{[1 + (v - \mu_\delta \tan \delta_0)T]^2 + \mu_\alpha^2 T^2\}^{1/2}}. \quad (6c)$$

These equations were first correctly derived by Eichhorn and Rust³.

Equations (5 and 6) give a complete description of the motion in the form we'll need it. The position at any time depends on the five initial conditions α_0 , δ_0 , v , μ_α , and μ_δ . To recover the sixth quantity we need to refer the motion to the observer's location instead of the center of the earth. Before we do this, we could put the physics back into the problem. My main purpose in considering uniform rectilinear motion was to bring to the reader's attention the geometrical effect

that the foreshortening terms do influence the position. Equations (5 and 6) could now be replaced by the formulas of two-body Keplerian motion. The development of the next two sections is independent of the physics, or lack thereof, in Eqs. (5 and 6). It should also be clear that, since $\dot{r}/r$ enters first into $\ddot{\alpha}$ and $\ddot{\delta}$, that it can only be determined if the imaged path has measureable curvature. We'll calculate the radius of curvature in §V and find it to be essentially infinite. Hence, as a practical matter, the physics doesn't matter.

III. DIURNAL PARALLAX

I shall, except for time variables, uniformly use the corresponding upper case letter to denote a topocentric quantity. Thus, $A = A(t)$ = the topocentric right ascension at time t , $M_\delta = \dot{\Delta}_o = \dot{\Delta}(t_o)$ = the proper motion in the topocentric declination at time t_o , $D = R/R_o$ etc. Let $\underline{\rho} = \rho(\cos\phi'\cos\tau, \cos\phi'\sin\tau, \sin\phi')$ be the observer's geocentric location. Here ρ is the observer's geocentric distance, ϕ' his geocentric latitude, and τ is the local mean sidereal time. The translation to the observer is accomplished by

$$\underline{r} = \underline{R} + \underline{\rho}. \quad (7)$$

If we write the right-hand side of this in component form, and use Eqs. (4) similarly expressed, one can derive the results analogous to those in Eqs. (5 and 6). To persuade the reader that we don't want to follow this course one finds, for the location,

$$\begin{aligned} D^2 + 2DScosZ &= 1 + 2VT + (V^2 + \Omega^2)T^2 + 2ScosZ_o \\ &+ 2ST(VcosZ_o + dcosZ_o/dt) + (S\dot{T}cos\phi')^2 \\ &- 2S\dot{T}^2cos\phi'cos\Delta_o\sin H_o(V - M_\delta\tan\Delta_o - M_\alpha\cot H_o), \end{aligned} \quad (8a)$$

$$\tan(A - A_o) = \frac{M_\alpha T + S_1}{1 + (V - M_\delta\tan\Delta_o)T + S_2}, \quad (8b)$$

$$\tan[(\Delta - \Delta_0)/2] = [M_0 T + (1 - D + VT)\tan\Delta_0]/(D + C). \quad (8c)$$

The missing initial condition is $S = \rho/R_0$. The other quantities are defined by (Z = topocentric zenith distance, H = topocentric hour angle)

$$\cos Z = \sin\phi' \sin\Delta + \cos\phi' \cos\Delta \cos H, \quad (9a)$$

$$H = \tau - A, \quad (9b)$$

$$\begin{aligned} C^2 = & [1 + (V - M_0 \tan\Delta_0)T]^2 + M_\alpha^2 T^2 + 2\{S_1 M_\alpha T \\ & + S_2[1 + (V - M_0 \tan\Delta_0)T]\} + (S \cos\phi' \sec\Delta_0)^2 \{(\dot{\tau}T)^2 \\ & + 2[1 - \cos(\tau - \tau_0) - \dot{\tau}T \sin(\tau - \tau_0)]\}, \end{aligned} \quad (9c)$$

$$S_1 = S \cos\phi' \sec\Delta_0 [\sin H_0 + \dot{\tau}T \cos H_0 - \sin(\tau - A_0)], \quad (9d)$$

$$S_2 = S \cos\phi' \sec\Delta_0 [\cos H_0 - \dot{\tau}T \sin H_0 - \cos(\tau - A_0)]. \quad (9e)$$

Necessarily as $S \rightarrow 0$ $D \rightarrow d$ etc. and Eqs. (8) reduce to Eqs. (5). The analog of Eqs. (6) can be obtained from Eqs. (8) by differentiation with respect to the time. Equations (8 and 9) are rigorous.

It seems simpler to me to interpret Eqs. (7) geometrically. Then one finds⁴

$$\tan(\alpha - A) = \tan(H - h) = \frac{(\sec\phi' \sec\delta/d) \sinh}{[1 - (\sec\phi' \sec\delta/d) \cosh]}, \quad (10a)$$

$$\tan(\delta - \Delta) = \frac{(\sin\phi' \csc\gamma/d) \sin(\gamma - \delta)}{[1 - (\sin\phi' \csc\gamma/d) \cos(\gamma - \delta)]}, \quad (10b)$$

$$D = d \sin(\delta - \gamma) \csc(\Delta - \gamma), \quad (10c)$$

where

$$\tan\gamma = \tan\phi' \cos[(\alpha - A)/2] \sec[h + (\alpha - A)/2]. \quad (10d)$$

Equations (5, 10) completely specify the topocentric location of the object in terms of the initial conditions $s = \rho/r_o$, α_o , δ_o , $v = \dot{r}_o/r_o$, $\mu_\alpha = \dot{\alpha}_o$, and $\mu_\delta = \dot{\delta}_o$. When the telescope/camera combination images the motion, with the telescope in sidereal drive, on a photographic plate or camera target, the resulting streak contains, albeit implicitly, a complete description of the motion. The last step then is to project a portion of the celestial sphere onto a plane.

IV. STANDARD COORDINATES

If the camera is equivalent to a pinhole camera, then the projection of the imaged portion of the celestial sphere onto a plane will be a gnomonic projection. The resulting coordinate system on the plane is a rectangular one known as the standard coordinate system. To relate standard coordinates, symbolized by ξ , η , to right ascension and declination we choose the unit of length for the standard coordinates to be the telescope's focal length and we need to know the coordinates where a plane, parallel to the plane of the plate, touches the celestial sphere. This is known as the tangential point. One end of the optical axis of the telescope pierces the celestial sphere at this point, the other end intersects the center of the plate. The positive η axis points toward the North Celestial Pole. The positive ξ axis points toward the east point⁴.

If (α^*, δ^*) are the coordinates of the tangential point then the relationship between ξ , η and a corresponding α , δ is given by

$$\xi = \sec\delta \sin(\alpha - \alpha^*) / [\tan\delta \tan\delta^* + \cos(\alpha - \alpha^*)], \quad (11a)$$

$$\eta = [\tan\delta - \tan\delta^* \cos(\alpha - \alpha^*)] / [\tan\delta \tan\delta^* + \cos(\alpha - \alpha^*)]. \quad (11b)$$

The inverse of Eqs. (11) is

$$\cot\delta \sin(\alpha - \alpha^*) = \xi / (\sin\delta^* + \eta \cos\delta^*), \quad (12a)$$

$$\cot\delta\cos(\alpha - \alpha^*) = (\cos\delta^* - \eta\sin\delta^*)/(\sin\delta^* + \eta\cos\delta^*). \quad (12b)$$

The formula,

$$\sin\delta = (\sin\delta^* + \eta\cos\delta^*)/(1 + \xi^2 + \eta^2)^{1/2}. \quad (12c)$$

is useful near the equator.

The problem is now completely solved. Since we obviously observe topocentrically, we replace α, δ in Eqs. (11) by A, Δ computed from Eqs. (10). The geocentric coordinates appearing in Eqs. (10) are given as a function of time by Eqs. (5). Since we measure the properties of the streak imaged on the plate, we have an implicit problem for the determination of the location and velocity of the moving object. Once these have been solved for we can compute the orbital elements. It's also clear from Eqs. (11, 12) that it is simplest if the position of the telescope is not changed during the exposure of the plate.

V. APPROXIMATE FORMULA

By combining all of the formulas of the preceding three sections, we find that, through all terms of the second order

$$\begin{aligned} \xi = \{E_{\alpha} + M_{\alpha}T + [2M_{\alpha}(M_{\delta}\tan\Delta_0 - V) + St^2\cos\phi' \sec\Delta_0\sin H_0]T^2/2\}\cos\Delta_0 \\ - M_{\delta}T(E_{\alpha} + M_{\delta}T)\sin\Delta_0, \end{aligned} \quad (13a)$$

$$\begin{aligned} \eta = E_{\delta} + M_{\delta}T - [M_{\alpha}^2\sin\Delta_0\cos\Delta_0 + 2VM_{\delta} + St^2\cos\phi'\sin\Delta_0\cos H_0]T^2/2 \\ + [(E_{\alpha} + M_{\alpha}T)^2/2]\sin\Delta_0\cos\Delta_0, \end{aligned} \quad (13b)$$

where

$$E_{\alpha} = A_0 - \alpha^*, \quad E_{\delta} = \Delta_0 - \delta^*. \quad (13c)$$

The reader is reminded again that these expressions are not series in S but rather series in E_{α} , E_{δ} , $M_{\alpha}T$, $M_{\delta}T$, and VT . It's clear that all of these quantities are limited by the field-of-view of the telescope/camera combination. It should also be clear that neither $\alpha_0 - \alpha^*$ nor $\delta_0 - \delta^*$ is limited by the field of view and that they aren't necessarily small. (Compare with references 5, 6, and 7.)

The first thing we shall compute is the curvature of the streak. If the radius of curvature is effectively infinite, then the quadratic terms

in the time can be dropped from Eqs. (13) and, as a practical matter, the distance and radial velocity are unobtainable. The leading term in the curvature is

$$K = (St^2 \cos \phi' / \Omega^3) |M_\delta \sin H_0 + M_\alpha \sin \Delta_0 \cos \Delta_0 \cos H_0|. \quad (14a)$$

As $S \rightarrow 0$ the curvature becomes

$$K \rightarrow 2 |(VE_\alpha / \Omega) \sin \Delta_0|. \quad (14b)$$

For the satellite considered earlier, if its equator crossing occurs on the meridian, $K \sim 1/400$. For any reasonable focal length the streak will, therefore, be indistinguishable from a straight line.

The high internal accuracy of the measured values of ξ , η imply that $E_\alpha \cos \Delta_0$, E_δ , $M_\alpha \cos \Delta_0$, and M_δ will be very well determined. The external systematic errors in the angular velocity will also be negligible. However, the external systematic and random errors in the position will reflect those of the telescope pointing. In fact, in the simplest situation, the standard deviation of $E_\alpha \cos \Delta_0$ and E_δ will be $\sqrt{N}$ down from those of ξ and η (N = number of observations) while the standard deviations of $M_\alpha \cos \Delta_0$ and M_δ will $\sim$ (total exposure duration) down from these. This may well be a few hundredth's of a second of arc/second. The length of the streak, when $K \approx 0$, is just Ω times the duration of the exposure. This may be useful as a constraint but because of its relatively large

standard deviation is not useful for the overall analysis. Finally, had only the developed streak been analyzed, we can see from Eqs. (13), that the model would have been

$$\xi = (E_{\alpha} M_{\delta} - E_{\delta} M_{\alpha} \cos \Delta_o) / M_{\delta} + (M_{\alpha} \cos \Delta_o / M_{\delta}) \eta. \quad (15)$$

Hence, unless the streak passes through the origin and the length of the streak is employed, all of the information would have been lost.

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