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DESIGN OF HELICAL COMPRESSION SPRINGS

HENRY P. SWIESKOWSKI

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operating stress value. Direct and simplified analytical design procedures are developed for round wire and rectangular wire compression springs. Also presented are nomographs for use as design aids and detailed numerical design examples. This study combines design characteristics and stress advantages of nested spring systems versus a single spring for equivalent load conditions.

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FOREWORD

The design procedures, mathematical derivations, and spring data presented in this report are primarily a review of the material covered in the reports given in the bibliography. This report comprises three major parts, and many of the important design concepts, formulas, and charts useful to the design engineers are summarized in one source. The first part covers the development of a direct and simplified analytical procedure for the design of round wire compression springs. The second part describes the design characteristics and stress advantages of nested spring systems. The final part details the derivation of a simplified design method for rectangular wire springs.

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DESIGN OF ROUND WIRE SPRINGS

Introduction

The machine designer often faces the problem of determining which springs will work in a given space and will satisfy a specific load requirement. It is necessary therefore, to allow sufficient space at an early stage of the overall machine design in order to avoid having an overstressed spring or using a costly spring and having to make later changes in the design. Spring failures generally occur when the spring becomes overstressed because the available space is incompatible with the prescribed load requirement.

The design method described in this report is a direct procedure, and allows for the rapid determination of the minimum final stress possible within the given space/load requirements. Furthermore, it shows the amount of stress reduction that can be expected by revision of the original spring parameters.

The conventional formulas for the calculation of helical round wire compression springs are:

Load as a function of deflection (spring rate):

$$R = \frac{P}{F} = \frac{G d^4}{8 D^3 N} \quad (1)$$

Stress as a function of deflection:

$$\frac{S}{F} = \frac{G d}{\pi D^2 N} \quad (2)$$

These formulas, as well as calculation tables and nomographs derived from them, are in the proper form to calculate loads, stresses, and energies for a given spring. However, their value in designing springs to meet specific space-load relationships is limited because their independent variables of

d - wire diameter

D - mean coil diameter

N - number of coils

H_F - free height (for determinations of deflections)

are not given directly to the spring designer. Therefore, the designer must assume probable values for some of these variables which agree with one part of the required spring values. Then he must determine the unknown values by some method based on the preceding formulas and compare the results with the other part of the required values. usually, the given and calculated data do not coincide so that a second step or additional steps are necessary until the final data of the desired spring are determined. Two points to be improved with regard to the present methods of spring design are:

1. The present method is a trial and error method which should be replaced by a direct method. The system of formulas used for such a direct approach should be based on the values given directly to the spring designer, such as space figures, load requirement, etc. Therefore, formulas (1) and (2) should be converted into an equivalent system of formulas with independent variables that are readily available to the spring designer. This direct design procedure would result in a considerable saving of time.

2. The assumptions to be made at the start of the calculation should be reduced to a minimum. The favorable ranges of design parameters should be analyzed in general in order to have a basis for the initial assumptions. The first step in this direction is to group all springs into three categories by load requirement.

Basic Spring Design Considerations

When a helical compression spring* has to be designed, the following values are usually known to the designer (compare with figure 1).

G modulus of torsion for the spring material to be used, Pa. (Mean value for spring steel $G = 79,290$ megapascals)

D_H diameter of the cylindrical space in which spring must work, m

H_1 assembled height of spring, m

H_2 minimum compressed height of spring after completion of the compression stroke, m

In addition to these materials and space requirements the desired spring also must satisfy a load requirement. In practical design three basic types of load requirements can be distinguished:

*Extension springs are treated analogously.

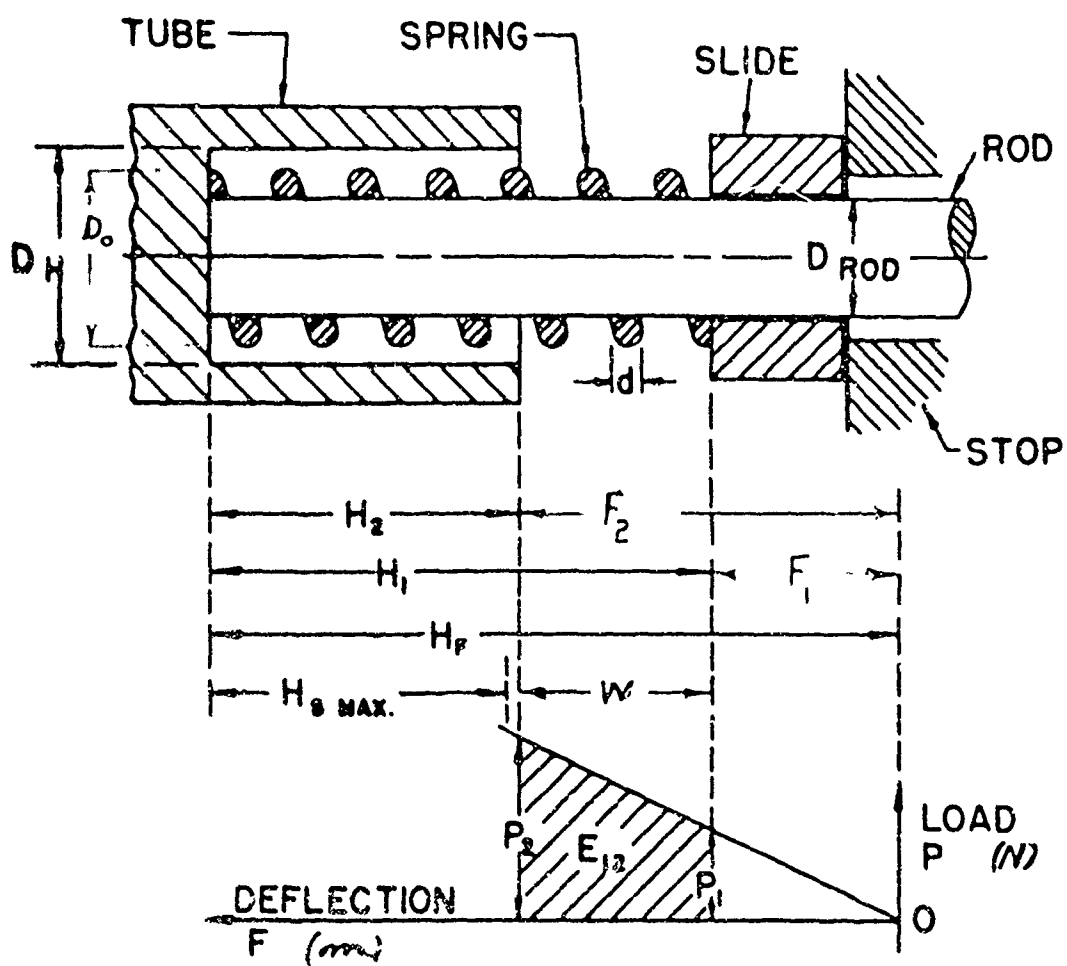


Figure 1. Spring space and spring characteristics.

1. Initial load requirement. Here the initial load at assembled height P_1 is specified; all retainer springs belong to this group.

2. Final load requirement. The final load at minimum compressed height P_2 is given.

3. Energy requirement. Here the energy capacity E of the spring over the working stroke (i.e., between assembled height and minimum compressed height) is:

$$E = \frac{P_1 + P_2}{2} \cdot W = \bar{P} \cdot W$$

With a given energy capacity over the compression stroke W , the mean load $\bar{P}$ is also determined; therefore, the energy requirement is equivalent to a mean load requirement. All springs used for stopping a moving mass by compression or for accelerating a resting mass by extension belong to this group.

According to the three types of load requirements, all compression springs can be divided into three groups. It will be shown later that there are basic differences between these three groups of springs with regard to their favorable ranges of precompression. Therefore, a separation of these groups in the analysis will be necessary. In some applications, a spring may have a double load requirement. However, for the following analysis only the three basic types mentioned above are considered.

As will be shown later, it is advantageous to use the diametral as well as longitudinal spring space in order to have a maximum amount of spring material for reduction of the final stress S_2 at minimum compressed height. Therefore, the chosen spring coil diameter should be as large as possible, considering hole diameter D_H , manufacturing tolerances, increase in coil diameter caused by compression, and final clearance necessary.

Also, the active solid height of the spring $H_s = N \cdot d$ should be selected as close as possible to the minimum compressed height H_2 considering the height of dead coils, manufacturing tolerances, and final clearance necessary. After these estimates, the desired spring must satisfy the following requirements:

G	modulus of torsion (Pa)
D	mean coil diameter (m)
$H_s = N \cdot d$	active solid height (m)
$W = H_1 - H_2$	working stroke (m)
P_1, P_2 or $\bar{P}$	load requirement (N)

These given requirements represent four independent values for the desired spring. Because a helical compression spring is completely determined by five independent values (e.g., G, d, D, N, H_F) there is still one additional spring value to be chosen; e.g., the free height H_F or the final deflection F_2 or any value which characterizes the precompression of the spring. Considering safety of spring function and spring life, this remaining spring value should be selected so that the final stress S_2 at minimum compressed height is as low as possible.

In general, the spring designer is faced with the following problems:

1. Favorable range. How shall the remaining free spring value be selected (i.e., the final deflection F_2) in order to obtain the minimum final stress S_2 ?
2. Minimum final stress. What is the value of the minimum final stress S_2 , for a given spring material, spring space, and load requirement?
3. In case of satisfactory final stress. When the calculated final stress is acceptable, what are the characteristic geometrical values of the spring?

d	wire diameter
D	mean coil diameter
N	number of active coils
H_F	free height

What is the value of the spring rate R ?

4. In event calculated final stress is too high. When the final stress is found to be too high and unacceptable, either nested springs should be used or the given requirements revised in order to reduce the excessive stress value.

a. Nested springs. Here the inner spring space is used by one or two inner springs working in a parallel with the outer one. What is the reduction of final stress that is obtained with the use of the inner space?

b. Revision of spring requirements. The spring material must be changed, the spring space increased, or the load requirement reduced. What reduction of final stress can be expected when changing a given spring value by a certain amount?

Analysis of Ranges Favorable for Spring Design

Mathematical Transformation of Conventional Spring Formulas

The conventional spring formulas 1 and 2 must be transformed into a system of formulas more suitable for the spring designer's needs. Especially the wire diameter, d , and the number of coils, N , must be eliminated. For this purpose the following typical spring values are introduced:

$$\text{Spring index } C = D/d \quad (3)$$

$$\text{Active solid height } H_s = N \cdot d \quad (4)$$

From equations 2, 3, and 4, we obtain

$$S_2 = \frac{G F_2}{\pi C^2 H_s} \quad (5)$$

From 1, 3, and 4, it follows that

$$\frac{P_2}{D^2} = \frac{G F_2}{8 C^3 H_s} \quad (6)$$

Now the two formulas 5 and 6 represent a new system of formulas for spring calculations which is equivalent to the original 1 and 2. When eliminating the spring index from formulas 5 and 6, it follows that:

$$S_2 = \frac{8^{0.4} G^{0.6}}{\pi} \left(\frac{P_2}{D^2} \right)^{0.4} \left(\frac{F_2}{H_s} \right)^{0.6}$$

or

$$S_2 = 0.731 G^{0.6} \left(\frac{P_2}{D^2} \right)^{0.4} \left(\frac{F_2}{H_s} \right)^{0.6} \quad (7)$$

Introduction of Precompression Factor

The following spring data are assumed to be given values; G , D , H_s , W , and a load requirement of either P_1 , P_2 or $\bar{P}$. In addition to these given data, one more spring value can be chosen freely; for example, the ratio of final deflection to stroke F_2/W . This ratio characterizes the precompression of the spring and, therefore, may be called the "precompression factor".

When introducing the precompression factor F_2/W into formula 7, it follows that

$$S_2 = 0.731 G^{0.6} \left(\frac{P_2}{D^2} \right)^{0.4} \left(\frac{W}{H_s} \right)^{0.6} \left(\frac{F_2}{W} \right)^{0.6} \quad (8)$$

Here the final stress is a function of the given values G , D , W , H_s , a load requirement, and the selected precompression factor F_2/W . Now the following questions are raised: How will the final stress S_2 change when varying the last free value; i.e., the precompression factor F_2/W ? What is the "optimum precompression factor" which gives the minimum final stress possible, $S_{2 \min}$? The answer to these questions depends on the particular load requirement to be satisfied; therefore, the following analysis is suitably divided into three cases.

Springs with Initial Load Requirement, P_1

Given values: G, D, H_s, W, P_1

The final load is related to the initial load in terms of the precompression factor as follows:

$$P_2 = \frac{\frac{F_2}{W}}{\frac{F_2}{W} - 1} \cdot P_1 \quad (9)$$

Combining equations 8 and 9 gives the following expression for the final stress.

$$S_2 = 0.731 G^{0.6} \left(\frac{P_1}{D^2} \right)^{0.4} \left(\frac{W}{H_s} \right)^{0.6} \frac{F_2/W}{(F_2/W - 1)^{0.4}} \quad (10)$$

Differentiating the above equation with respect to the precompression factor shows that the final stress reaches its minimum value when

$$F_2/W = 5/3 = 1.67$$

$$\text{with } \frac{F_2/W}{(F_2/W - 1)^{0.4}} = \frac{5/3}{(5/3 - 1)^{0.4}} = 1.960 \quad (11)$$

From equations 10 and 11 it is established that

$$S_{2 \min} = 1.433 G^{0.6} \left(\frac{P_1}{D^2} \right)^{0.4} \left(\frac{W}{H_s} \right)^{0.6} \quad (12)$$

Therefore, it is concluded that the final stress reaches its minimum value when $F_2/W = 1.67$. However, the whole range from $1.35 \leq F_2/W \leq 2.25$ should be considered favorable for design because within this range the final stress is less than 5 percent above the minimum possible, $S_{2 \min}$.

Springs with Final Load Requirement, P_2

Given values G, D, H_s, W, P_2

From equation 8

$$S_2 = 0.731 G^{0.6} \left(\frac{P_2}{D^2} \right)^{0.4} \left(\frac{W}{H_s} \right)^{0.6} \left(\frac{F_2}{W} \right)^{0.6} \quad (8)$$

The final stress reaches its minimum, $S_{2 \min}$, for a precompression factor of $F_2/W = 1$

$$S_{2 \min} = 0.731 G^{0.6} \left(\frac{P_2}{D^2} \right)^{0.4} \left(\frac{W}{H_s} \right)^{0.6} \quad (13)$$

the range of $1 \leq F_2/W \leq 1.1$ can be considered favorable for spring design because there the final stress is less than 6 percent above the minimum final stress $S_{2 \min}$.

Springs with Energy (Mean Load) Requirement, $\bar{P}$

Given values $G, D, H_s, W, \bar{P}$

The final load is related to the mean load as follows:

$$P_2 = \frac{\frac{F_2}{W}}{\frac{F_2}{W} - 0.5} \cdot \bar{P} \quad (14)$$

Equations 8 and 14 give the following expression:

$$S_2 = 0.731 G^{0.6} \left(\frac{\bar{P}}{D^2} \right)^{0.4} \left(\frac{W}{H_s} \right)^{0.6} \frac{F_2/W}{(F_2/W - 0.5)^{0.4}} \quad (15)$$

The optimum precompression factor again equals 1; therefore,

$$S_2 \text{ min} = 0.965 G \cdot \left(\frac{\bar{P}}{D^2} \right)^{0.4} \left(\frac{W}{H_s} \right)^{0.6}$$

However, the range of $1 \leq F_2/W \leq 1.2$ can be considered favorable for design because within this range the final stress is less than 5 percent above $S_2 \text{ min}$.

Results

For all three classes of springs, it has been established that there is an optimum precompression factor for which the final stress is at a minimum. This optimum value is 1.67 for springs with initial load requirement and 1 for springs with final load or energy requirement.

The range of F_2/W , favorable for spring design, is different for each class of springs:

P_1 required. The range F_2/W from 1.35 to 2.25 is recommended. Then the final stress S_2 is less than 5 percent above $S_2 \text{ min}$.

P_2 required. When the range of F_2/W is from 1 to 1.1, S_2 is less than 6 percent above $S_2 \text{ min}$.

Energy required. When the range of F_2/W is from 1 to 1.2, S_2 is less than 5 percent above $S_2 \text{ min}$.

In the Event the Final Stress is Too High

When the final stress is too excessive to insure satisfactory function, the following methods of stress reduction may be possible.

Nested Springs

Here the inner spring space is used by adding inner springs working in parallel with the outer springs. The percentage of stress reduction obtained with nested springs is directly proportional to the spring index of the single spring. Not only is a reduction in the final stress obtained, but the use of nested spring effects a similar reduction

in the stress range. Figure 2 shows the variation of the percentage of reduction in the final stress and in the stress range with respect to the spring index of the single spring. The graph is based on the conditions that the nested springs and the single spring have the same values for

active solid height, H_s

load-deflection rate, R

final load, P_2

modulus of torsion, G

outside coil diameter, D_o

Revisions of Spring Requirements

When nested springs are impossible because of limited inner space or when they give insufficient stress reduction, then the given spring requirements must be revised in order to have the spring work at a reasonable stress level. When keeping G and F_2/W constant, one or more of the following parameters should be changed:

D - coil diameter

H_s - active solid height

W - stroke

P_1 , P_2 or $\bar{P}$ - load required

From equations 8, 10, and 15, the minimum final stress depends on the design parameter as follows:

S_2 increases in proportion to $D^{-0.8}$; i.e., 10 percent increase in the coil diameter results in a stress reduction of 7.3 percent.

S_2 increases in proportion to $H_s^{-0.6}$; i.e., a 10 percent increase in the active solid height reduces the final stress by 5.6 percent.

S_2 increases in proportion to $W^{0.6}$; i.e., a 10 percent increase in the stroke raises the final stress by 5.9 percent.

S_2 increases in proportion to $P_1^{0.4}$, $P_2^{0.4}$ or $\bar{P}^{0.4}$; i.e., a 10 percent increase in the load increases the stress by 3.9 percent.

General Remarks

It was assumed in the previous analysis that the mean coil diameter D was known, rather than the outside diameter D_o , for simplification of the calculation. However, in practical spring design, the outer diameter is usually known or closely approximated by the formula $D_o = 0.96 D_m$. It has been established that, in considering the outer diameter in the analysis, the favorable F_2/W design ranges remain essentially the same as those that were obtained based on a mean coil diameter. The major difference is that the precompression factor for the case of the initial load requirement is increased slightly from 1.67 to 1.75. The optimum F_2/W ratio for the case where the final load P_2 or mean load $\bar{P}$ is required still remains at the value 1.

Generally in designing springs to satisfy a final load or a mean load requirement, the precompression factor F_2/W should be somewhat larger than 1. The reason for this is that some precompression of the spring at assembled height is always practical. It prevents the spring from getting loose and compensates for spring set that may occur when working at high stress levels. Another factor, which will cause a deviation from the optimum F_2/W value, is that for manufacturing reasons a wire diameter corresponding to a standard wire gage should be selected.

When designing springs to a specific initial load, the range $1.35 \leq F_2/W \leq 2.25$ is recommended. There the final stress is less than 5 percent above the minimum stress possible. However, springs with factors in the upper part of this range have the advantage of thinner wires (that can withstand higher stress levels) and a narrower stress range ($S_2 - S_1$). However, they will be more susceptible to buckling and will require closer guidance.

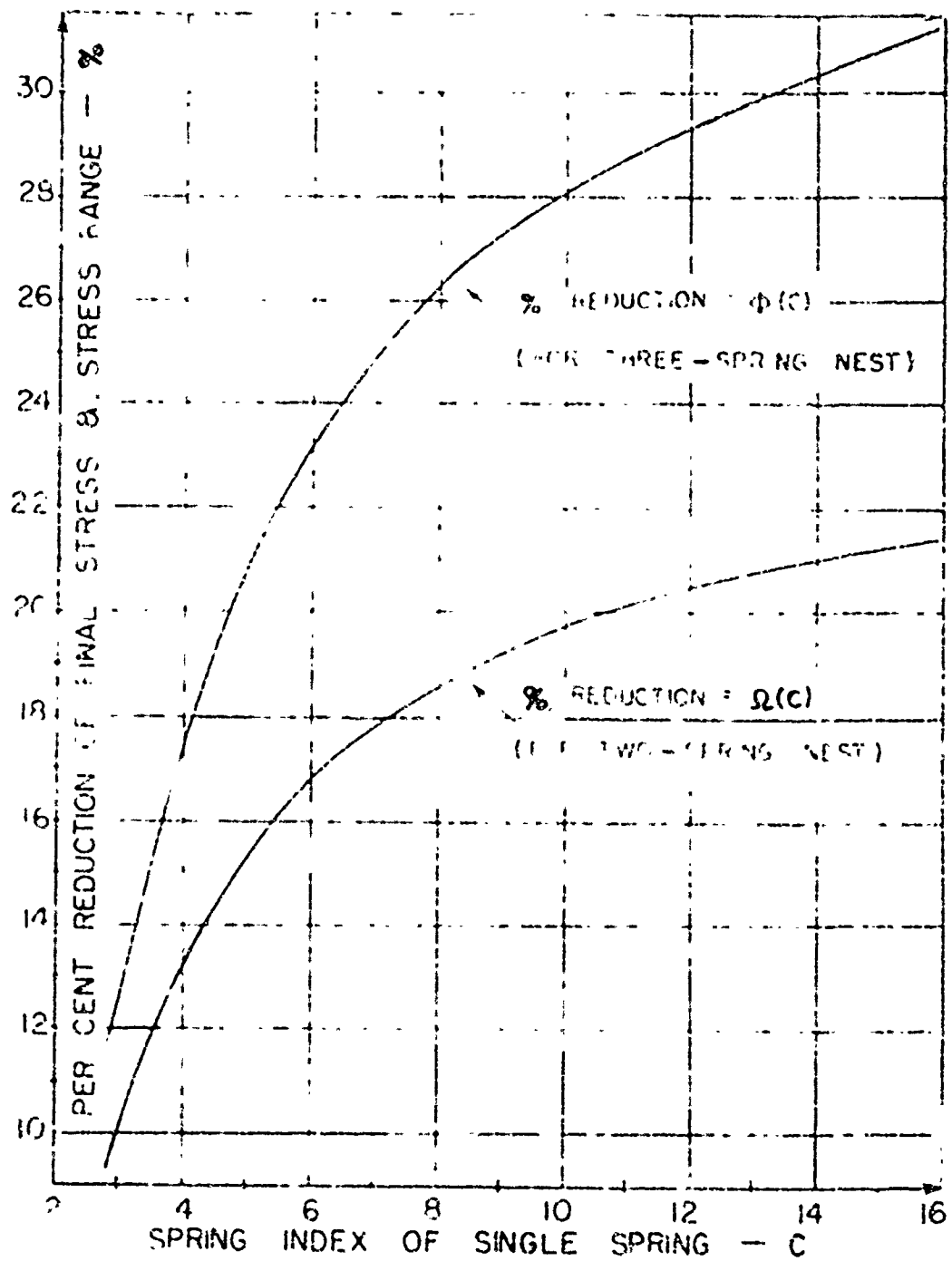


Figure 2. Percentage reduction of final stress and stress range versus index of single spring

Numerical Example - Initial Load Requirement

Given Values

Modulus of torsion, $G = 79290 \text{ MP}_a$

Assembled height of spring, $H_1 = 0.2870 \text{ m}$

Minimum compressed height of spring, $H_2 = 0.1346 \text{ m}$

Diameter of hole in which spring must work, $D_H = 0.0137 \text{ m}$

Initial load requirement, $P = 160 \text{ N}$

Step 1. Select the active solid height H_s as close as possible to the minimum compressed height H , considering the height of dead coils, manufacturing tolerances, and, final longitudinal clearance desired.

$H = 0.90 H_2$ is recommended

$$= 0.90 (0.1346) = 0.121 \text{ m}$$

Step 2. Calculate the approximate value of the mean coil diameter; usually $D = (0.75 - 0.80) D_H$ is a practical choice.

$$\text{Let } D = 0.80 D_H = 0.80 (0.0137) = 0.0254 \text{ m}$$

Step 3. Let the precompression factor $F_2/W = 1.67$

Having selected a proper value for F_2/W , the spring is now completely defined. The next stage is to determine the values for final stress S_2 , load deflection rate R , wire diameter d , number of active coils N , and the free height H_F . The following is one of several procedures of calculation that can be used:

From equation 10

$$\begin{aligned} S_2 &= 0.731 G^{0.6} \left(\frac{P_1}{D^2} \right)^{0.4} \left(\frac{W}{H_s} \right)^{0.6} \frac{F_2/W}{(F_2/W - 1)^{0.4}} \\ &= 0.731 (79,290 \times 10^6)^{0.6} \left(\frac{160}{0.0254^2} \right)^{0.4} \\ &\quad \left(\frac{0.1524}{0.121} \right)^{0.6} \frac{1.67}{(1.67-1)^{0.4}} = 820 \text{ MP}_a \end{aligned}$$

$$P_2 = \left(\frac{F_2/W}{F_2/W - 1} \right) P_1 = \left(\frac{1.67}{0.67} \right) 160 = 400 \text{ N}$$

$$\text{Load deflection rate, } R = \frac{P_2 - P_1}{W} = \frac{400 - 160}{0.1524} = 1,575 \text{ N/m}$$

$$d = \sqrt[5]{\frac{8RD^3 H_s}{G}} = \sqrt[5]{\frac{8(1575)(0.0254)^3(0.121)}{79,290 \times 10^6}} = 0.0032 \text{ m}$$

$$N = H_s/d = 0.121/0.0032 = 38$$

$$F_1 = P_1/R = 160/1575 = 0.1016 \text{ m}$$

$$H_F = H_1 + F_1 = 0.2870 + 0.1016 = 0.3886 \text{ m}$$

$$D_o = D + d = 0.0254 + 0.0032 = 0.0286 \text{ m}$$

Complete data for this spring design are shown in table 1.

Numerical Example - Energy (Mean Load) Requirement

Given Values

Modulus of torsion, $G = 68,950 \text{ MP}_a$

Assembled height of spring, $H_1 = 0.1422 \text{ m}$

Minimum compressed height of spring, $H_2 = 0.1016 \text{ m}$

Diameter of hole in which spring must work, $D_H = 0.0190 \text{ m}$

Energy required, $E = 27.1 \text{ m}\cdot\text{N}$

Let $H_s = 0.90 H_2 = 0.90(0.1016) = 0.0914 \text{ m}$

Select $D = 0.75 D_H = 0.75 (0.019) = 0.0143 \text{ m}$

Allow some precompression of the spring at assembled height; therefore, let $F_2/W = 1.1$.

Table 1. Specification table for spring--initial load requirement

Wire size (m)	0.0032
Outside diameter (m)	0.0286
Total coils	40
Type of ends	Closed and ground
Free height, approx (m)	0.3886
Mean assembled height (m)	0.2870
Load at mean assembled height (N)	160
Minimum operating height (m)	0.1346
Load at minimum operating height (N)	400
Load-deflection rate (N/m)	1575
Maximum solid height (m)	0.1308
Spring helix	Optional
Material	Music wire, QQ-W-470

Calculate the mean load value

$$\bar{P} = \frac{E}{W} = \frac{27.1}{0.0406} = 667 \text{ N}$$

Determine the final stress from equation 15

$$\begin{aligned} S_2 &= 0.731 \cdot G^{0.6} \left(\frac{\bar{P}}{D^2} \right)^{0.4} \left(\frac{W}{H_s} \right)^{0.6} \frac{F_2/W}{(F_2/W - .5)^{0.4}} \\ &= 0.731 (68,950 \times 10^6)^{0.6} \left(\frac{667}{0.0143^2} \right)^{0.4} \left(\frac{0.0406}{0.0914} \right)^{0.6} \frac{1.1}{(1.1 - .5)^{0.4}} \\ &= 779 \text{ MP}_a \end{aligned}$$

From equation 14 calculate P_2

$$\begin{aligned} P_2 &= \left(\frac{F_2/W}{F_2/W - 0.5} \right) \bar{P} = \left(\frac{1.1}{0.6} \right) 667 = 1223 \text{ N} \\ R &= \frac{P_2 - \bar{P}}{W/2} = \frac{1,223 - 667}{0.0203} = 27,390 \text{ N/m} \end{aligned}$$

Determine the value of the wire diameter

$$d = \sqrt[5]{\frac{8RD^3 H_s}{G}} = \sqrt[5]{\frac{8(27,390)(0.0143)^3 (0.0914)}{68,950 \times 10^6}} = 0.0038 \text{ m}$$

$$N = H_s/d = 0.0914/0.0038 = 24$$

$$F_2 = P_2/R = 1223/27,390 = 0.0447 \text{ m}$$

$$H_F = H_2 + F_2 = 0.102 + 0.045 = 0.147 \text{ m}$$

$$D_o = D + d = 0.152 + 0.152 + 0.014 + 0.0038 = 0.0178 \text{ m}$$

Complete data for this spring design are shown in table 2.

Table 2. Specification table for spring--energy requirement

Wire size (m)	0.0038
Outside diameter (m)	0.0178
Total coils	26
Type of ends	Closed and ground
Free height, approx (m)	0.147
Mean assembled height (m)	0.1422
Load at mean assembled height (N)	111
Minimum operating height (m)	0.1016
Load at minimum operating height (N)	1,223
Load-deflection rate N/m	27,390
Maximum solid height (m)	0.100
Spring helix	Optional
Material	Stainless steel, QQ-W-423, FS302

NESTED SPRING SYSTEMS

Introduction

A frequent problem in spring design is how to decrease the working stress in a helical compression spring and still maintain the given load-height requirement within the overall dimensions of height and diameter. In other words, assume that the available spring space in both the longitudinal and radial directions has been fully used, so that the outer diameter and solid height of the single spring are as large as possible. In this case, where the final stress of the single spring is considered too high and it is not possible to revise the given requirements, it would be practical to use nested springs in order to obtain a stress reduction.

The nested spring system is not a new principle, but it has lacked a clean-cut design approach. A set of equations is developed here that simplifies the analysis, and design curves are offered which quickly give the amount of stress reduction possible for a given spring index of the single spring. This indicates immediately whether sufficient stress savings are possible. The curves show that a two-spring nest offers a reduction of approximately 18% and a three-spring nest, a reduction of about 25%.

The analysis also shows that the stress reduction varies in direct proportion to the spring index of the single spring and that the use of nested springs not only produces a reduction in the final stress but also effects a similar reduction in the stress range--an important consideration in fatigue life.

Discussion of Analysis

General Formulas and Basic Assumptions

The analysis on nested springs is based primarily on equations 5 and 6.

$$S_2 = \frac{GF_2}{\pi C^2 H_s} \quad (5)$$

$$\frac{P_2}{D^3} = \frac{GF_2}{8C^3 H_s} \quad (6)$$

In order to make a valid comparison between the single spring and the nested springs, both should have the same values for

active solid height, H_S

load deflection rate, R

free height, H_F

modulus of torsion, G

outside diameter, D_o (i.e., the outside diameter of the single spring should equal the outside diameter of the outer spring in the nested design.)

To simplify the analysis, it is assumed that there is no diametral clearance between the nested springs. Furthermore, for practical design, the stresses of the individual springs in a nest are equal:

$$S^O = S^i, \quad S^A = S^B = S^C$$

Based on the above equality conditions placed on the values of S , G , H_F , and H_S , an examination of equation 5 shows that the indices of the nested springs are equal.

To convert equation 6 into terms of the outside spring diameter in place of the mean spring diameter, the following expression is used:

$$D_o = \frac{C+1}{C} D \quad (16)$$

Hence, equation 6 becomes

$$\frac{F_o}{D_o^3} = \frac{GF_o}{8H_S^3(C+1)^3} \quad (17)$$

Replacement of a Single Spring by a Nest of Two Springs

Percentage of Stress Reduction

The physical characteristics of the single spring are given by equations 5 and 17. Similarly, the outer spring of the nest is described by

$$\frac{P_2^0}{(D_0^0)^2} = \frac{GF_2}{8H_S C'^3 (C'+1)^2} \quad (18)$$

$$S_2^1 = \frac{GF_2}{\pi C'^2 H_S} \quad (19)$$

and the inner spring of the nest, by equation 19 and

$$\frac{P_2^1}{(D_0^1)^2} = \frac{GF_2}{8H_S C'^3 (C'+1)^2} \quad (20)$$

The relationship between the outer diameters of the nest is

$$D_0^1 = D_0^0 \left(\frac{C'-1}{C'+1} \right) \quad (21)$$

Hence equation 20 can be written in the following form:

$$\frac{P_2^1}{(D_0^0)^2} = \frac{GF_2}{8H_S C'^3 [C'+1]^2} \left[\frac{C'-1}{C'+1} \right]^2 \quad (22)$$

which expresses the relationship between the final load of the inner spring and the outside diameter of the outer spring.

It follows from equations 17, 18, and 22 that the relationship of the indexes between the single spring and the two-spring nests is

$$\frac{1}{C^3(C+1)^2} = \frac{1}{C'^3(C'+1)^2} \left[1 + \frac{C'-1}{C'+1} \right]^2 \quad (23)$$

Although this equation appears complex, its curve is practically a straight line. Equation 23 is plotted in figure 3.

The ratio S_2/S_2^1 of the final stress of the single spring to that of the nested spring is obtained by combining equations 5 and 19

$$\frac{S_2}{S_2^1} = \frac{C'^2}{C^2} \quad (24)$$

It can be seen from equation 24 that the reduction in stress obtainable with the use of nested springs increases with increase in the spring indexes. The percentage reduction is plotted against the index of the single spring (fig 2). Note that, even when there are low index values, consideration should be given to nested springs, because it is possible in this range to obtain approximately a 10% stress reduction. Furthermore, the stress advantage increases with the spring index so that in the commonly used range ($C = 5$ to $C = 9$) a stress reduction of about 18% can be realized.

Dimensions of Two-Spring Nest

The next step is to determine the physical characteristics of the two-spring nest. A comparison of the single spring with the two-spring nest is shown in figure 4. A single spring is completely defined when the following dimensions are known:

- d - wire diameter
- N - number of coils
- D_o - outside diameter
- H_F - free height
- G - modulus of torsion

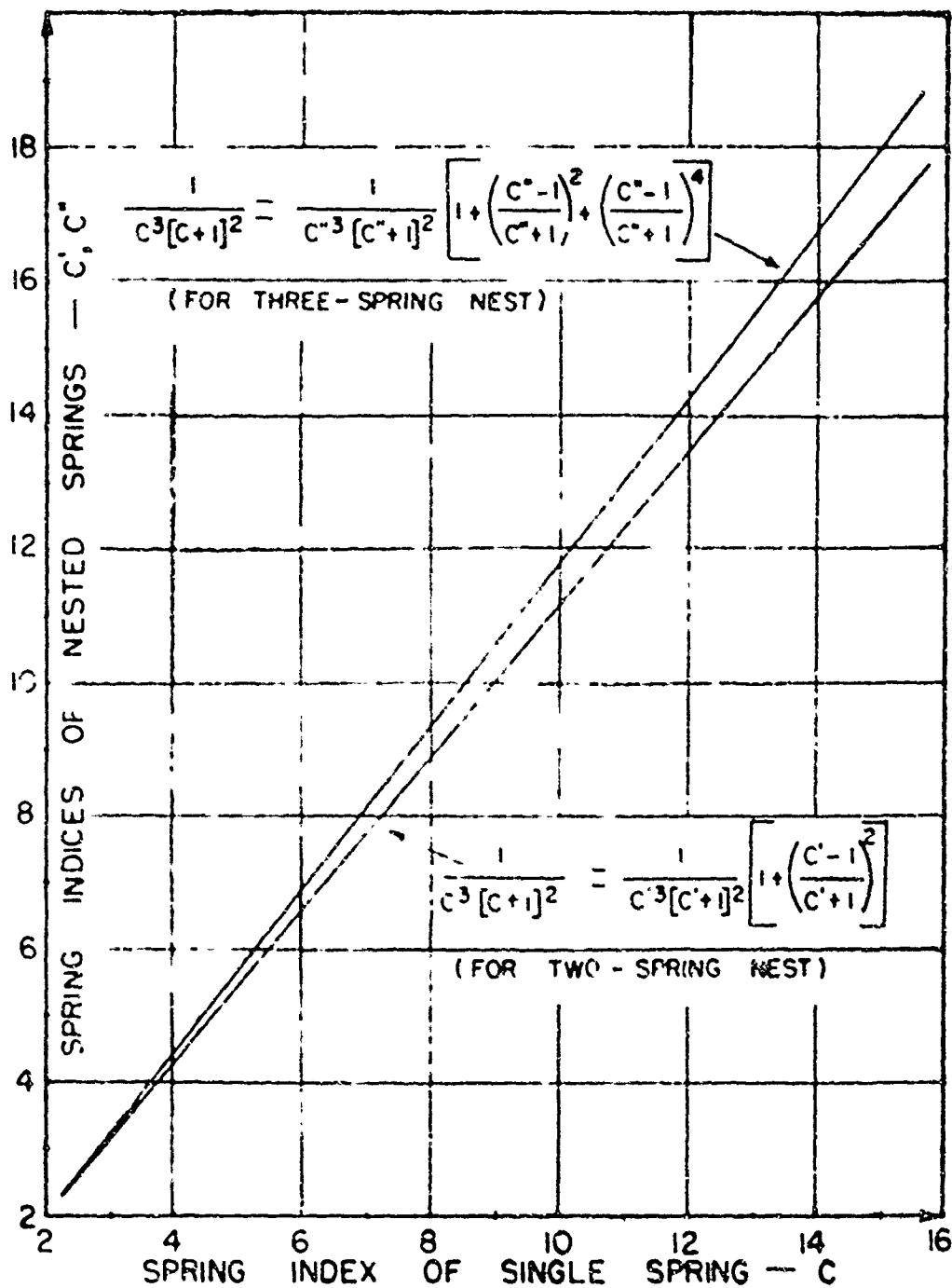


Figure 3. Spring indices of nested springs versus spring index of single spring.

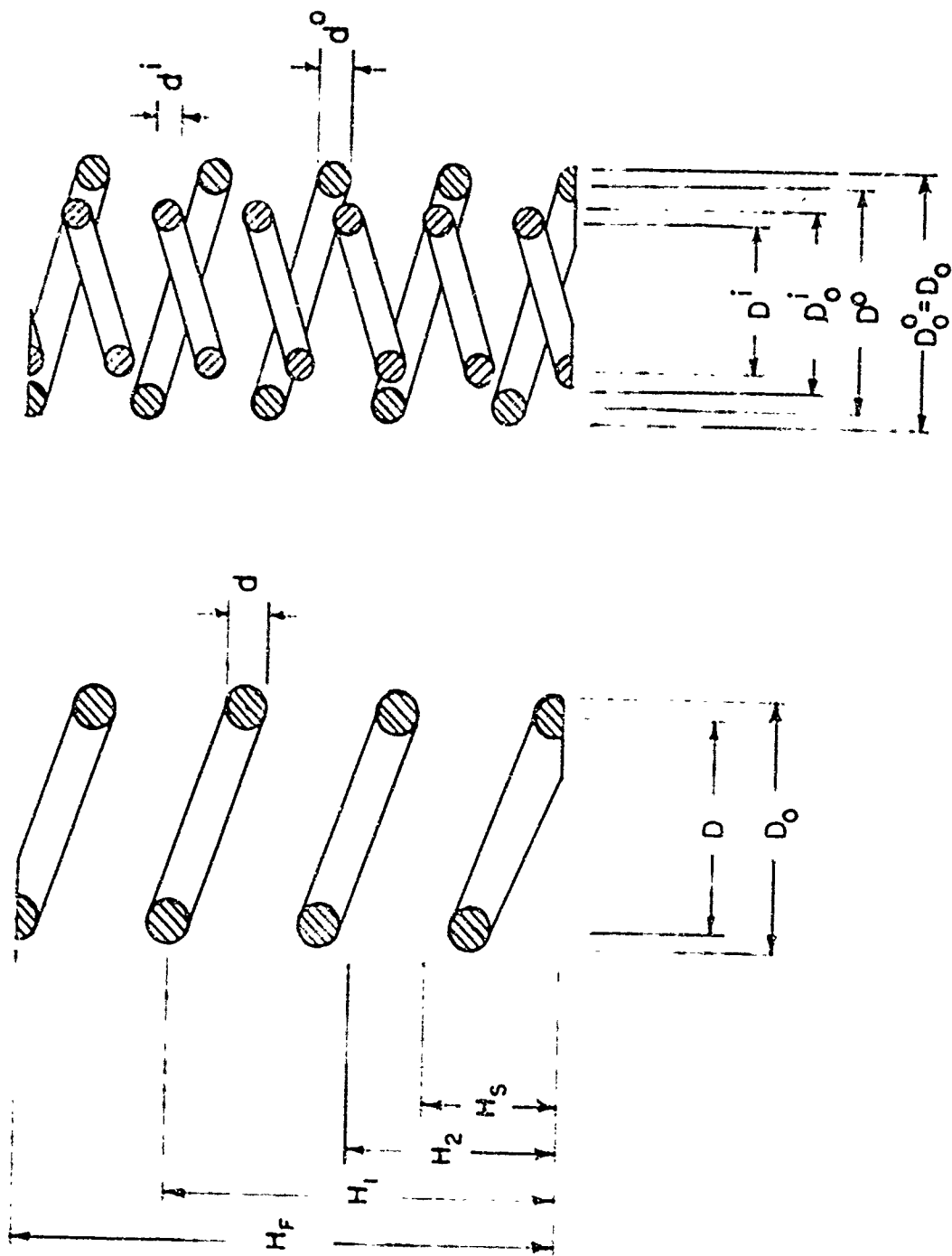


Figure 4. Comparison of single spring and two-spring nest.

The wire diameter of the outer spring is related to the wire diameter of the single spring by:

$$d^0 = \left[\frac{C+1}{C'+1} \right] d \quad (25)$$

Therefore, the wire size of the outer spring can be determined by multiplying the wire diameter of the single spring by the ratio

$$\frac{C+1}{C'+1}$$

From equations 23 and 25 the wire diameter of the outer spring is also a function of only the spring index and wire diameter of the single spring.

$$d^0 = F(d, C) \quad (25A)$$

This function cannot be expressed specifically because of the implicit form of equation 23. However, a design chart of equation 25A showing the relationship of the three variables is given in figure 5. The wire diameter of the outer spring is mainly a function of the wire diameter of the single spring and the spring index has comparatively little influence on the variation of d^0 .

For determination of the wire size of the inner spring, the following relationship is used:

$$d^i = \frac{n_o^i}{C'+1} = \frac{D_o^0 - 2d^0}{C'+1} \quad (26)$$

The number of active coils in the nested springs can be quickly calculated by

$$N^0 = H_S / d^0 \quad (27)$$

$$N^i = H_S / d^i \quad (28)$$

The outside diameter of the outer spring is the same as the single spring. The outside diameter of the inner spring is given by the numerator in equation 26 as

$$D_o^i = D_o^0 - 2d^0 \quad (29)$$

The modulus of torsion and the free height are the same for the single spring and the nested springs, as stated in the basic assumptions.

Problem 1 - Double-Nest Design

Given the following single spring:

Dimensions

- d - wire diameter = 0.0025 m
- D_o - outside diameter = 0.0209 m
- C - spring index = $D/d = 7.24$
- N - number of active coils = 10
- H_s - active solid height = $d \cdot N = 0.0254$ m
- H_f - free height = 0.0826 m
- H_1 - mean assembled height = 0.0610 m
- H_2 - minimum operating height = 0.0318 m
- G - modulus of torsion = 79,290 MP_a

Loads and Stress

- P_1 - load at $H_1 = 142.3$ N
- P_2 - load at $H_2 = 338.1$ N
- R - load-deflection rate = 6,655 N/m
- S/F - stress-deflection rate = $18,640 \times 10^6$ N/m³

The problems are: (1) to determine the stress reduction obtained by the replacement of a single helical spring with a two-spring nest and (2) to determine the dimensions of the nested springs.

From figure 2, for a single spring with an index value of 7.24, an 18% stress reduction is obtainable with a nested design. This means that for the given single spring with a final stress of 965 MP_a the final stress of a nested design will be 793 MP_a.

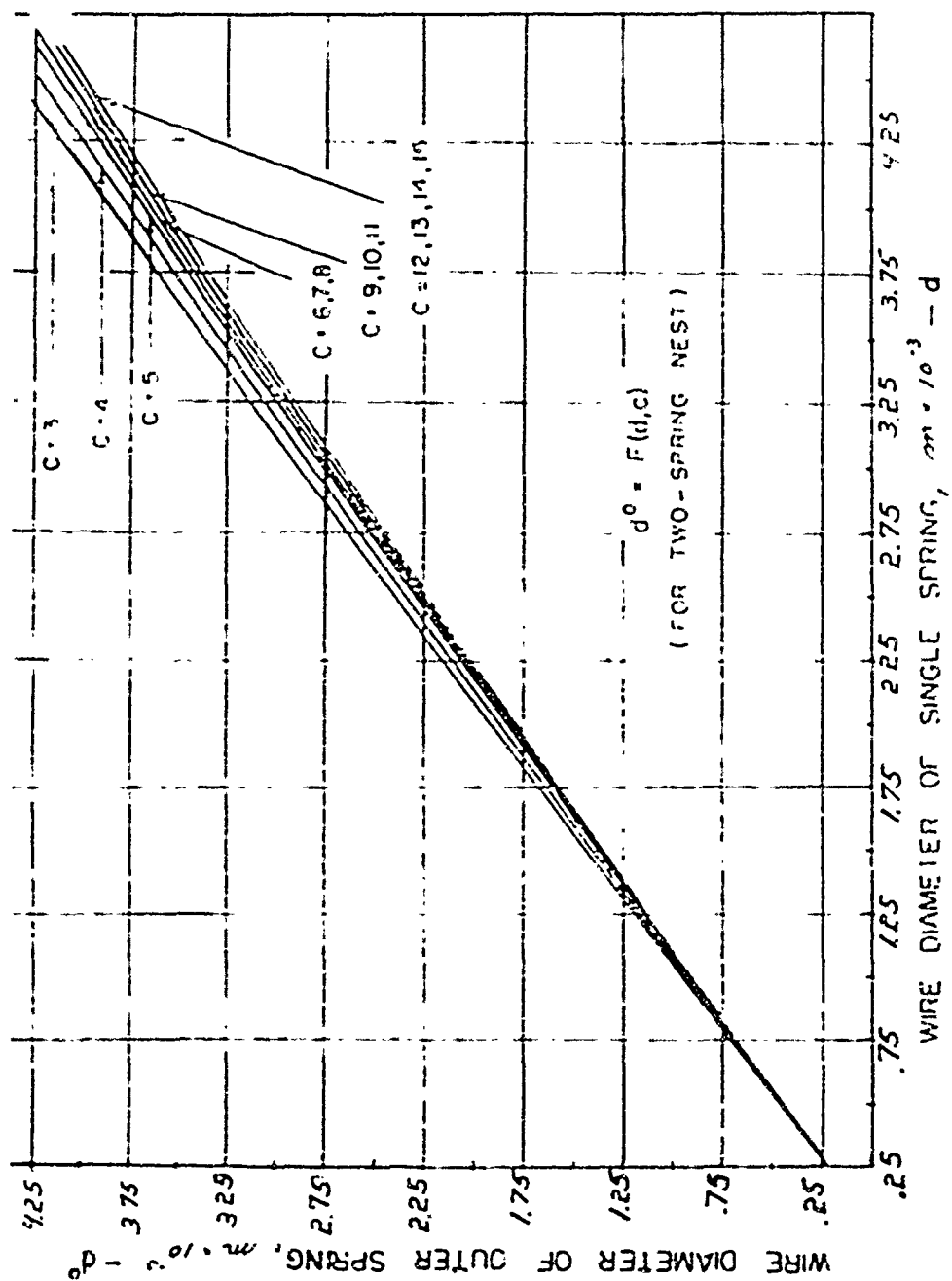


Figure 5. Wire diameters of outer spring versus single spring.

The spring index of the nested design is obtained from figure 3. For this particular example, $C' = 8.0$. The wire size of the outside spring, d^o , is calculated from equation 25 to be 0.0023 m; this value can also be obtained directly from figure 5. Also, for the outer spring

$$N^o = H_s/d^o = 0.025/0.0023 = 11 \text{ coils}$$

$$D_o^o = 0.0209 \text{ m}$$

$$H_F = 0.0826 \text{ m}$$

From equations 26, 28, and 29 the dimensions of the inner spring are

$$d^i = \frac{0.0209 - 2(0.0023)}{8+1} = 0.0018 \text{ m}$$

$$N^i = 0.0254/0.0018 = 14$$

$$D_o^i = 0.0209 - 2(0.0023) = 0.0162 \text{ m}$$

$$H_F = 0.0826 \text{ m}$$

The load-deflection rates for the double-nest springs are 4,203 and 2,452 N/m, the total of which equals the load-deflection rate of the single spring, 6,655 N/m.

Complete data on the single and nested springs are given in table 3. The combined functional loads of the nested springs are equal to those of the single spring (i.e., $89 + 53 = 142 \text{ N}$ load at assembled height and $213 + 125 = 338 \text{ N}$ load at minimum operating height), but that the final stress of both springs in the nested design has been reduced to 793 MP_a.

Replacement of a Single Spring by a Nest of Three Springs

Percentage of Stress Reduction

For the outer spring

$$\frac{P_2^A}{(D_o^A)^2} = \frac{GF_2}{8H_s C'^3 [C'+1]^2} \quad (30)$$

Table 3. Comparative data of single spring and nested springs

	Single spring	Two-spring nest Outer	Two-spring nest Inner	Three-spring nest Outer	Three-spring nest Center	Three-spring nest Inner
Wire size (m)	0.0025	0.0023	0.0018	0.0022	0.0017	0.0014
Outside diameter (m)	0.0209	0.0209	0.0162	0.0209	0.0164	0.0130
Active coils (n)	10.0	11	14.0	11.4	14.9	18.1
Free height (m)	0.0926	0.0826	0.0826	0.0826	0.0826	0.0826
Modulus of torsion (MPa)	79,290	79,290	79,290	79,290	79,290	79,290
Active solid height (m)	0.0254	0.0254	0.0254	0.0254	0.0254	0.0254
Spring index	7.24	8.0	8.0	8.4	8.4	8.4
Load-deflection rate (N/m)	6655	4203	2452	3327	2102	1226
Mean Assembled height (m)	0.061	0.061	0.061	0.061	0.061	0.061
Load at mean assembled height (N)	142	89	53	71	44	27
Stress at mean assembled height (MPa)	407	338	338	303	303	303
Minimum operating height (m)	0.0318	0.0318	0.0318	0.0318	0.0318	0.0318
Load at minimum operating height (N)	338	213	125	169	107	62
Stress at minimum operating height (MPa)	965	793	793	717	717	717

For the center spring

$$\frac{P_2^B}{(D_o^B)^2} = \frac{GF_2}{8H_S C''^3 [C''+1]^2} \quad (31)$$

The relationship between the diameters of the outer and center springs is

$$D_o^B = D_o^A \left[\frac{C''-1}{C''+1} \right]$$

Hence, equation 31 becomes

$$\frac{P_2^B}{(D_o^A)^2} = \frac{GF_2}{8H_S C''^3 [C''+1]^2} \left[\frac{C''-1}{C''+1} \right]^2 \quad (32)$$

For the inner spring

$$\frac{P_2^C}{(D_o^C)^2} = \frac{GF_2}{8H_S C''^3 [C''+1]^2} \quad (33)$$

With the successive transference of outer diameters, equation 33 becomes

$$\frac{P_2^C}{(D_o^A)^2} = \frac{GF_2}{8H_S C''^3 [C''+1]^2} \left[\frac{C''-1}{C''+1} \right]^4 \quad (34)$$

The final stress in each of the three springs is given by

$$S_2'' = \frac{GF_2}{\pi C''^2 H_S} \quad (35)$$

When the load-diameter equations 30, 32, and 34 are compared with equation 17, the relationship of the index of the single spring with the nexted springs is obtained:

$$\frac{1}{C^3 [C+1]^2} = \frac{1}{C''^3 [C''+1]^2} \left\{ 1 + \left[\frac{C''-1}{C''+1} \right]^2 + \left[\frac{C''-1}{C''+1} \right]^4 \right\} \quad (36)$$

Equation 36 is plotted in figure 3. As in the case of the index equation associated with the two-spring nest, equation 36 also approximates a straight line.

Analogous to the double nest, the following similar relationship exists between the spring indexes and final stresses of the single spring and the triple nest (from equations 5 and 35):

$$\frac{S_2}{S_2''} = \frac{C''^2}{C^2} \quad (37)$$

The percentage reduction in final stress, obtained by the substitution of a triple nest for a single spring, is shown in figure 2. A stress reduction of about 25% is achieved with common values for the spring index (from $C = 5$ to $C = 9$). The percentage gain of the third spring represents approximately an additional 7% stress reduction beyond an equivalent double nest ($5 < C < 9$). For values of C less than 5, the stress advantage beyond a double nest steadily decreases. Therefore, $C = 4$ should be considered the lower limit for the practical use of a three-spring nest.

Dimensions of Three-Spring Nest

From equations 5, 30, 32, 34, and 36 the following expressions are obtained which equate the wire diameters of the triple nest to the wire diameter of the single:

$$d^A = \frac{C+1}{C''+1} d \quad (38)$$

$$d^B = \frac{[C+1] [C''-1]}{[C''+1]^2} d \quad (39)$$

$$d^C = \frac{[C+1] [C''-1]^2}{[C''+1]^3} d \quad (40)$$

The number of active coils in each spring is

$$N_A = \frac{H_s}{d^A} \quad (41)$$

$$N^B = \frac{H_s}{d^B} \quad (42)$$

$$N^C = \frac{H_s}{d^C} \quad (43)$$

and the outer diameters are determined by

$$D_o^A = D_o \quad (44)$$

$$D_o^B = D_o^A - 2d^A \quad (45)$$

$$D_o^C = D_o^B - 2d^B \quad (46)$$

Problem 2 - Triple-Nest Design

When the same single spring is considered (as defined in the first problem with a spring index = 7.24), figure 2 shows that slightly more than 25% stress reduction is possible with the substitution of a three-spring nest.

From figure 3, the spring index of the triple nest is $i'' = 8.4$. The wire sizes of the three springs are (from equations 38, 39, and 40)

$$d^A = \frac{(7.24+1)(0.0025)}{(8.4+1)} = 0.0022 \text{ m}$$

$$d^B = \frac{(7.24+1)(8.4-1)(0.0025)}{(8.4+1)^2} = 0.0017 \text{ m}$$

$$d^C = \frac{(7.24+1)(8.4-1)^2(0.0025)}{(8.4+1)^3} = 0.0014 \text{ m}$$

The numbers of active coils are (from equations 41 through 43)

$$D_o^A = D_o = 0.0209 \text{ m}$$

$$D_o^B = 0.0164 \text{ m}$$

$$D_o^C = 0.0130 \text{ m}$$

Dimensions of the individual springs for the triple-nest design are listed in table 3 for quick comparison with the double nest and the single spring. The functional loads of the single spring are maintained and are equal to the combined load of the triple nest. Thus, the combined loads at the assembled height = $71 + 44 + 27 = 142 \text{ N}$, which is equal to that of the single spring. Also, the combined loads at the minimum operating height = $169 + 107 + 62 = 338 \text{ N}$, equal to that of the single spring. However, the final stress of 965 MP_a for a single spring has been reduced to 717 MP_a for the triple nest.

Reduction in Fatigue Stress

Numerous tests have shown that decreasing the stress range (S_2-S_1) directly increases the fatigue life of a spring. A substantial gain in life is obtained by the reduced stress range of nested springs. This reduction is proportional to the reduction that has been obtained in the final stress.

Thus

$$\frac{S_2 - S_1}{S_2' - S_1'} = \frac{C'^2}{C^2} \quad \text{and} \quad \frac{S_2 - S_1}{S_2'' - S_1''} = \frac{C''^2}{C^2}$$

Furthermore, the percentage reduction in the stress range is equal to the percentage reduction in the final stress, as shown graphically in figure 2. For example, in comparing the single spring with the two-spring nest, the percentage reduction in final stress is

$$\Delta S = \frac{S_2 - S_2'}{S_2} 100 = \frac{965 \times 10^6 - 793 \times 10^6}{965 \times 10^6} 100 = 18\%$$

and the percentage reduction in stress range is

$$\Delta(S_2 - S_1) = \frac{[S_2 - S_1] - [S_2' - S_1']}{[S_2 - S_1]} 100 = \frac{[558 \times 10^6 - 407 \times 10^6]}{[558 \times 10^6]} 100 = 18\%$$

Other benefits of nested springs are: (1) the use of thinner wire sizes that have higher maximum design stresses and (2) smaller Wahl correction factors, which are important when corrected stresses are considered. The stress formula, equation 5, does not include the Wahl factor. If corrected stresses were to be considered, the percentage reduction in the final stress and in the stress range would be a few points higher.

DESIGN OF RECTANGULAR WIRE SPRINGS

Introduction

Use of rectangular cross sectional material is advantageous in applications in which the radial space is restricted because the spring must work within a specified hole diameter and also over a fixed rod size. In these applications, where round wire cannot meet the loading requirements, the designer must resort to the use of a rectangular section. A typical example of this situation is the buffer spring for the M85 caliber .50 machine gun. Here, the spring operates within a guide tube with an inside diameter of 0.0269 m and over the bolt drive spring with outside diameter of 0.0165 m. Further more, the assembled height of the buffer spring is 0.1295 m and must have an energy capacity of 27 m·N over a stroke of 0.0095 m. After allowances are made for adequate diametral clearance and for manufacturing tolerances, there results a maximum round wire size of 0.0047 m. This small diameter prohibits a satisfactory round wire design; therefore, a practical recourse is to consider rectangular wire. The actual cross section of the M85 buffer spring material is 0.0047 and 0.0142 m.

The object of this investigation is to develop a direct and simplified analytical design method for rectangular wire springs. It also includes the special case of springs fabricated from square wire. The analytical technique that is applied in this section is analogous to that used in the previous study on round wire springs. The procedure that is established enables the designer to rapidly determine the spring that has the minimum final stress and satisfies the space-load requirements. Design recommendations are determined for two important load requirements: Case 1 - initial load P_1 is specified; Case 2 - energy capacity E is specified. A further distinction is made in that the two basic configurations of rectangular wire springs, edge-wound and flat-wound springs (fig 6), are treated separately.

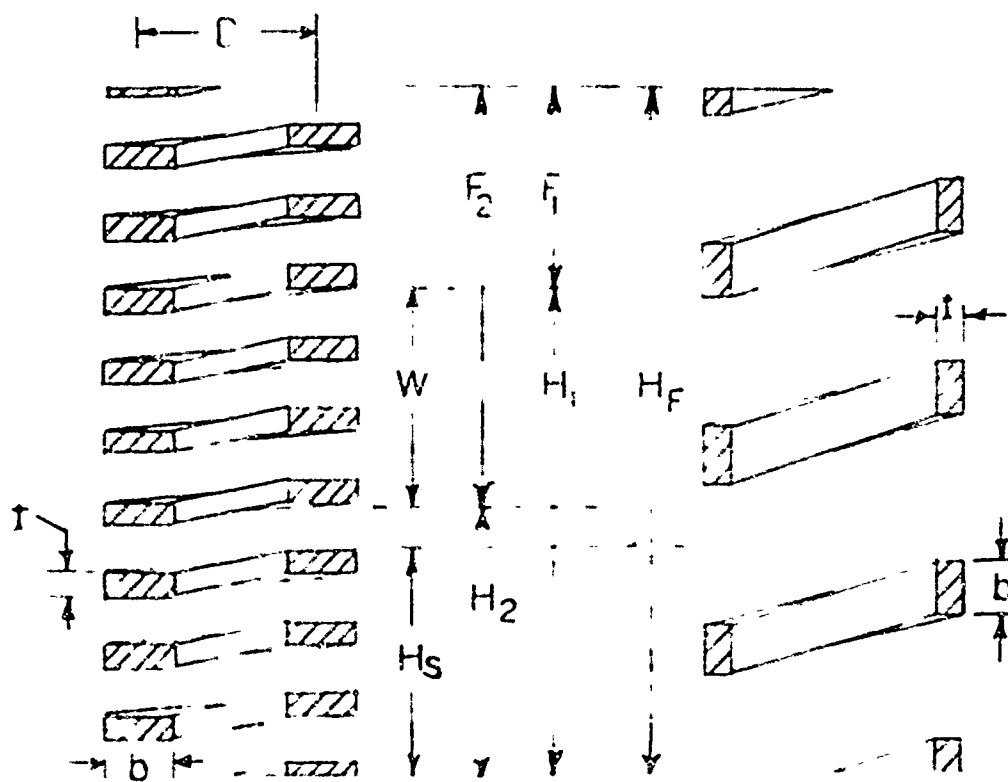
Discussion of Analysis

The usual design method based on the following conventional equations

$$R = \frac{P}{F} = \frac{K_1 G b t^3}{D^3 N} \quad (47)$$

$$S = \frac{K_1 G t F}{K_2 D^2 N} \quad (48)$$

(values for K_1 and K_2 are taken from figure 7)



EDGE - WOUND

FLAT - WOUND

Figure 6. Rectangular wire springs.

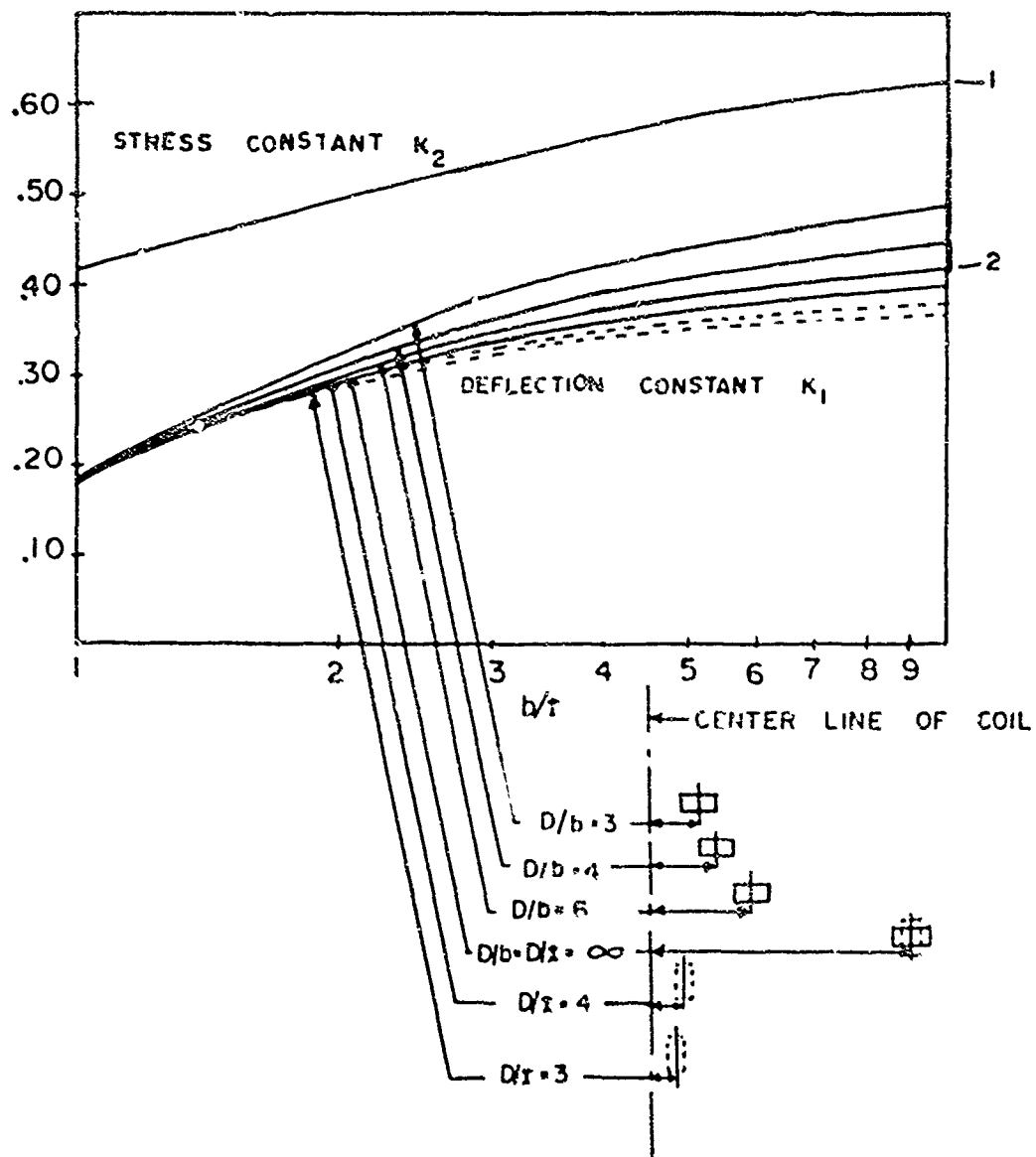


Figure 7. Constants for rectangular wire springs.

is a trial and error procedure and cumbersome. The system of formulas for the direct approach should be based on independent variables of spring space values and load requirement. These values are either given directly to the designer or defined within narrow limits.

Before a design analysis is possible, algebraic expressions that closely approximate the values of K_1 and K_2 must be determined. By application of the method of least squares to curves 1 and 2 in figure 7, the following expressions are obtained:

$$K_1 = 0.202 (b/t)^{0.451} \quad (49)$$

$$K_2 = 0.416 (b/t)^{0.228} \quad (50)$$

Since the majority of rectangular wire springs have b/t values between 1 to 5, the curve fitting process was confined to this interval to maintain a high degree of accuracy.

The following equations are used in the development of the design method:

$$\text{For edge-wound springs, active solid height } H_S = Nt \quad (51)$$

$$\text{For flat-wound springs, active solid height } H_S = Nb \quad (52)$$

$$\text{Precompression ratio} = \frac{F_2}{W} = \frac{\text{total deflection}}{\text{working stroke}} \quad (53)$$

Final load is related to the initial load as

$$P_2 = \frac{P_1 (F_2/W)}{F_2/W - 1} \quad (54)$$

Springs Coiled on Edge

Springs with Initial Load Requirement, P_1

Equations 47, 49, and 51 give

$$\frac{P}{F} = \frac{.202(b/t)^{1.451} Gt^5}{D^3 H_S} \quad (55)$$

The combination of equations 48, 49, 50, and 51 yields

$$S = \frac{0.485 (b/t)^{0.223} G F t^2}{D^2 H_S} \quad (56)$$

From equations 53, 54, 55, and 56, it follows that

$$S_2 = \frac{0.92 G^{0.6} W^{0.6} P_1^{0.4} (F_2/W)}{(b/t)^{0.357} D^{0.8} H^{0.6} (F_2/W-1)^{0.4}} \quad (57)$$

Equation 57 shows the variation of the final stress with the known values of torsion modulus, initial load, and working stroke, and with the approximated values of mean coil diameter and active solid height.

It is important to determine the effect that the precompression ratio has on the final stress and, in particular, what is the optimum precompression ratio that gives the minimum final stress. By differentiation of equation 57 with respect to F_2/W and by setting the resulting expression equal to zero, the minimum final stress is obtained by $F_2/W = 5/3$. To show in detail the variation of the final stress with the precompression ratio, a modified form of equation 57 is plotted in figure 8. A family of curves is obtained by letting the ratios b/t act as a parameter with the values 2, 3, 4, and 6. Although the final stress is at a minimum when $F_2/W = 5/3$, figure 8 shows that values of $1.5 \leq F_2/W \leq 2.0$ are a favorable design interval because within this range the final stress is less than 3% above the minimum value. Figure 8 also shows how the final stress varies with different values of b/t . In applications where diametral space is available, it is recommended that the ratio b/t should be large as possible.

Numerical example: Usually, the designer is given the following information:

assembled height, H_1	0.1500 m
load at assembled height, P_1	413.7N
minimum compressed height, H_2	0.0889 m
modulus of torsion, G	79,290 MP _a

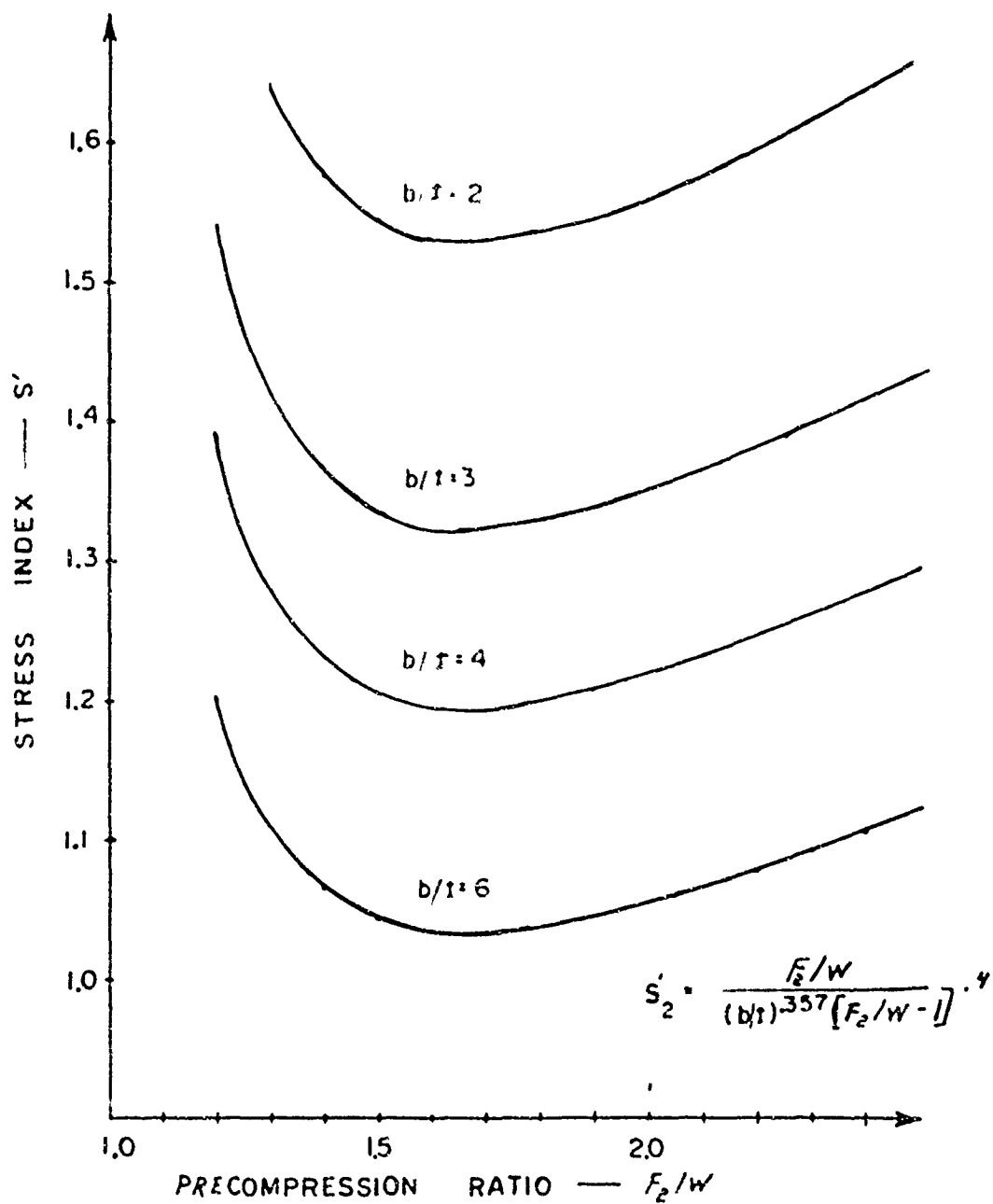


Figure 8. Variation of final stress for springs coiled on edge and initial load P_1 required versus precompression ratio.

hole diameter in which spring must fit, D_H 0.0538 m

maximum final stress, S_2 690 MPa

Step 1. Estimate realistic values for the mean coil diameter and the active solid height.

The mean coil diameter for rectangular wire springs is usually between $0.75 D_H$ and $0.90 D_H$ (which roughly corresponds with spring indexes of 3 to 9). For edge-wound springs, let $D = 0.85 D_H = 0.85 (0.0538) = 0.0457$ m.

For ends closed and ground $H_S = 0.8 H_2$ is a good choice.
 $H_S = 0.8 (0.0889) = 0.0711$ m.

Step 2. Solve for b/t using equation 57.

For minimum final stress let $F/W = 5/3$.

$$(b/t)^{.357} = \frac{0.92(79,290 \times 10^6)^{0.6} (0.0611)^{0.6} (413.7)^{0.4} (1.667)}{690 \times 10^6 (0.0457)^{0.8} (0.0711)^{0.6} (0.667)^{0.4}}$$

and $b/t = 1.263$

Step 3. Determine the load deflection rate; based on $F_2/W = 5/3$ and $P_1 = 413.7$ N, then $P_2 = 1,032$ N from equation 54.

Load deflection rate

$$R = \frac{P_2 - P_1}{W} = 10,157 \text{ N/m}$$

Step 4. Compute the thickness t from equation 55.

$$t = \left[\frac{10,157 (0.0457)^3 (0.0711)}{0.202 (1.263)^{1.451} (79,290 \times 10^6)} \right]^{1/5} = 0.0050 \text{ m}$$

$$\text{and } b = 1.263 (0.202(0.0050)) = 0.0063 \text{ m}$$

Step 5. Calculate the number of active coils, outside coils diameter, and free height.

$$N = H_s/t = 0.0711/0.0050 = 14.2$$

$$D_o = D + b = 0.0457 + 0.0050 = 0.0507 \text{ m}$$

$$H_F = H_2 + F_2 = 0.0889 + 0.1016 = 0.1905 \text{ m}$$

Complete specifications for this spring are listed in table 4 in the column titled "Spring 1".

Table 4. Specification table for rectangular-wire springs

	Spring 1	Spring 2
Free height, H_F (m)	0.1904	0.2259
Thickness, t (m)	0.0050	0.0058
Width, b (m)	0.0063	0.0173
Outside coil diameter, D_o (m)	0.0507	0.0350
Assembled height, H_1 (m)	0.1500	0.2243
Load at assembled height, P_1 (N)	414	614
Minimum compressed height, H_2 (m)	0.0889	0.2083
Load at minimum compressed height, P_2 (N)	1032	6784
Total coils, N_T	16.2	11.6
Type of ends	Closed and ground	Closed and ground
Maximum solid height H_S (m)	0.0825	0.2019
Load-deflection rate (N/m)	10,157	385,300
Material	FS-9260	FS-9260
Maximum final stress, S_2 (MP _a)	690	621

Springs with Energy (Mean Load) Requirement, $\bar{P}$

The energy content of a spring is related to the initial load as

$$P_1 = \frac{(F_2/W-1)E}{(F_2/W-0.5)W} \quad (58)$$

Using the above relationship, equation 57 is rewritten

$$S_2 = \frac{0.920 G^{0.6} (F_2/W) W^{0.2} E^{0.4}}{(b/t)^{0.357} n^{0.8} H_S^{0.6} (F_2/W-0.5)^{0.4}} \quad (59)$$

A nomograph based on equation 59 is shown in figure 9. The stress reaches a minimum for a precompression factor of $F_2/W = 1$. However, the range from $F_2/W = 1$ to $F_2/W = 1/2$ can be considered favorable for spring design because the final stress within this range is not more than 5% above the minimum. Again, it is recommended that the ratio b/t be made as large as the available space permits since the stress is inversely proportional to b/t .

Springs Coiled on Flat

Springs with Initial Load Requirement, P_1

Equations 47, 49, and 52 combined give the following expression:

$$\frac{P}{F} = \frac{0.202 (b/t)^{2.451} G t^5}{n^3 H_S} \quad (60)$$

From equations 48, 49, 50, and 52 the following relationship is obtained:

$$S = \frac{0.485 (b/t)^{1.223} G F t^2}{n^3 H_S} \quad (61)$$

The relationship between the final stress and the initial load derived from equations 53, 54, 60, and 61 is

$$S_2 = \frac{0.920 (b/t)^{0.243} G^{0.6} W^{0.6} (F_2/W) P_1^{0.4}}{n^{0.8} H_S^{0.6} (F_2/W-1)^{0.4}} \quad (62)$$

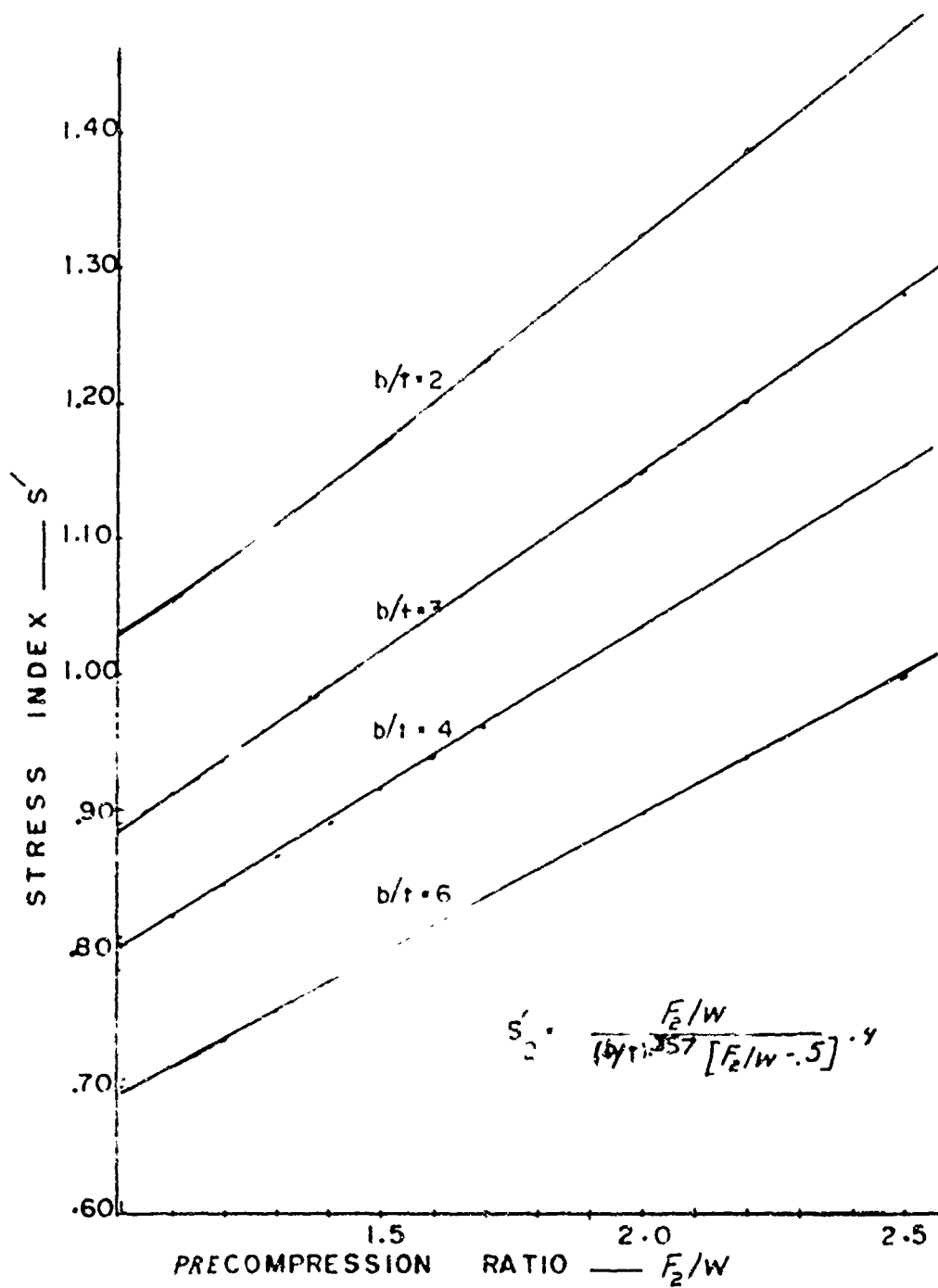


Figure 9. Variation of final stress for springs coiled on edge and mean load $\bar{P}$ required versus precompression ratio.

In the case of flat-wound springs, the final stress is directly proportional to the ratio b/t . This is shown graphically in figure 10 where smaller b/t ratios result in decreased stress values. The optimum design range for the precompression ratio remains the same as for edge-wound springs, $1.5 \leq F_2/W \leq 2.0$.

Springs with Energy (Mean Load) Requirement, $\bar{P}$

The substitution of equation 58 into equation 62 results in

$$S_2 = \frac{0.920(b/t)^{0.243} G^{0.6} W^{0.2} (F_2/W) E^{0.4}}{D^{0.8} H_s^{0.6} (F_2/W - 0.5)^{0.4}} \quad (63)$$

Again, the minimum final stress is obtained when $F_2/W = 1$; however, the range $1 \leq F_2/W \leq 1.2$ is considered favorable for spring design.

Numerical example: Typical known design parameters are

assembled height, H_1 ,	0.2243 m
minimum compressed height, H_2	0.2083 m
energy capacity, E	59.2 m·N
modulus of torsion, G	79,290 MP _a
hole diameter, D_H	.0365 m
maximum final stress, S_2	621 MP _a
working stroke, $W = H_1 - H_2$	.0160 m

Step 1. Estimate values for the mean coil diameter and the active solid height.

For flat-wound springs a reasonable choice for the mean coil diameter is $D = 0.8D_H = 0.8 (0.0365) = 0.0292$ m

For ends closed and ground let $H_s = 0.8 H_2 = 0.8 (0.2083) = 0.1666$ m.

To have some precompression at assembled height, let $F_2/W = 1.1$.

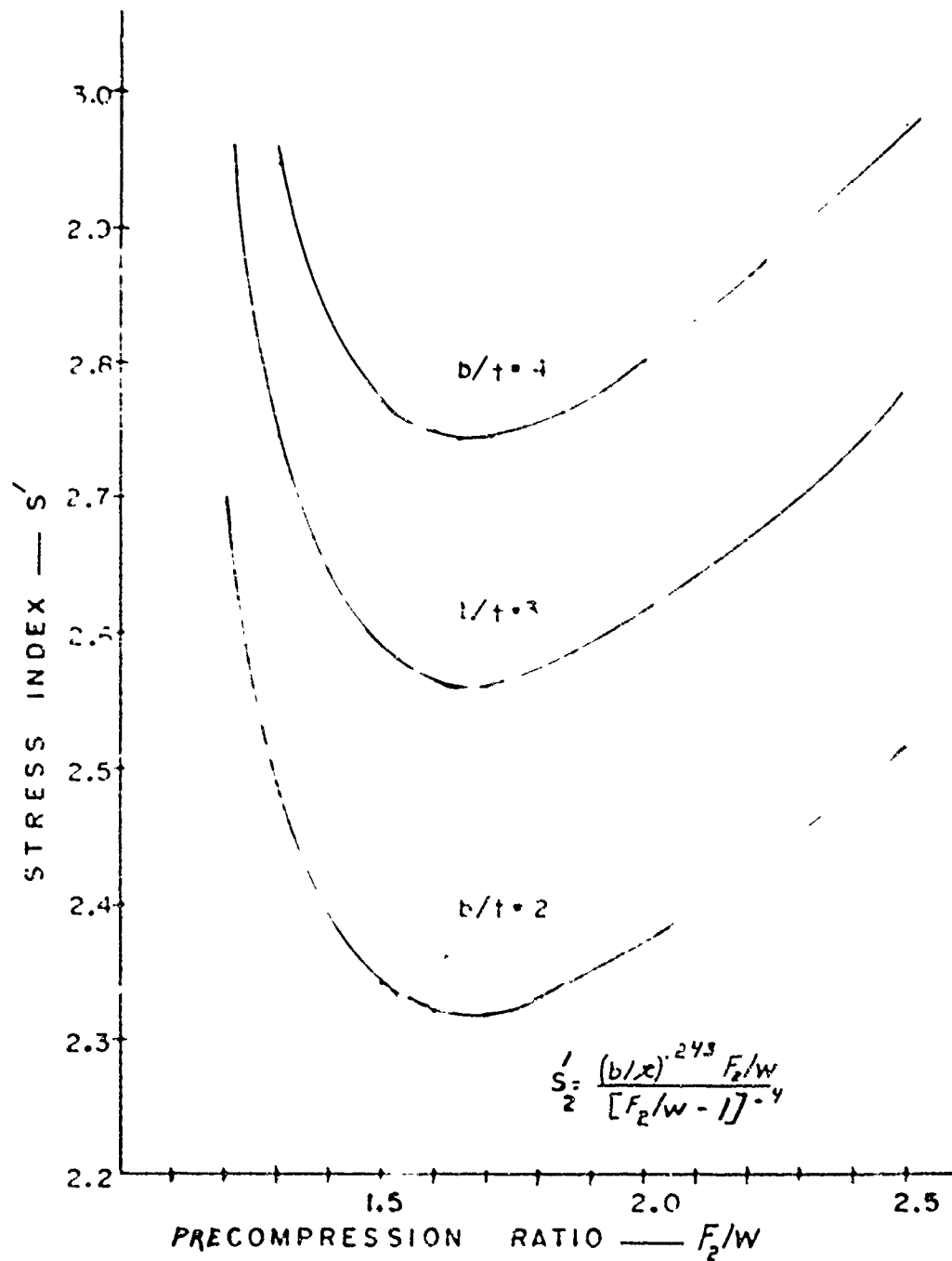


Figure 10. Variation of final stress for springs coiled on flat and initial load P_1 required versus precompression ratio.

Step 2. Solve for b/t using equation 63

$$(b/t)^{0.243} = \frac{621 \times 10^6 (0.0292)^{0.8} (0.1666)^{0.6} (0.6)^{0.4}}{0.92 (79,290 \times 10^6) (0.0160)^{0.2} 1.1 (59.2)^{0.4}}$$

and $b/t = 2.953$

Step 3. Determine the load-deflection rate. For $F_2/W = 1.1$,

$F_1 = F_2 - W = 0.0176 - 0.0160 = 0.0016$; for $F = 59.2 \text{ m} \cdot W$, the mean load $\bar{F}$ equals $F/W = 59.2/0.016 = 3700 \text{ N}$

$$\text{Load-deflection rate } R = \frac{\bar{F}}{F_1 + \frac{W}{2}} = \frac{3700}{0.0016 + \frac{0.016}{2}} = 385,300 \text{ N/m}$$

Step 4. Calculate the thickness t with equation 60

$$t = \left[\frac{385,300 (0.0292)^3 (0.1666)}{0.202 (2.953)^{2.451} 79,290 \times 10^6} \right]^{1/5} = 0.0058 \text{ m}$$

and $b = 2.953 (0.0058) = .0173 \text{ m}$

Step 5. Compute the number of active coils, the outside coil diameter, and the free height

$$N = H_s/b = 0.1666/0.0173 = 9.6$$

$$D_o = D + t = 0.0292 + 0.0058 = 0.0350 \text{ m}$$

$$H_f = H + F_s = 0.2083 + 0.0176 = 0.2259 \text{ m}$$

Complete design data for this spring are shown in table 1 in the column titled "Spring 2".

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LIST OF SYMBOLS

Symbols	Definitions
G	Modulus of torsion for spring material, P_a
d	Wire diameter, m
D_H	Diameter of spring hole, m
$D_o = D + d$	Outer diameter of spring, m
D	Mean coil diameter of spring, m
$D_I = D - d$	Inner diameter of spring, m
$C = D/d$	Spring index
N	Number of active coils
N_T	Total number of coils
H_F	Free height of spring, m
H_1	Assembled height of spring, m
H_2	Minimum compressed height of spring, m
$H_s = Nd$	Active solid height, m
F	Any deflection of spring from free height, m
$F_1 = H_F - H_1$	Initial deflection, m
$F_2 = H_F - H_2$	Final deflection, m
$W = H_1 - H_2$ $= F_2 - F_1$	Working stroke of spring from assembled height to minimum compressed height, m
P	Spring load at deflection F , N
P_1	Initial load at assembled height, N
P_2	Final load at minimum compressed height, N

Symbols

Definitions

$\bar{P} = \frac{P_1 + P_2}{2}$	Mean load over working stroke, N
F	Energy absorbed or delivered by the spring over the working stroke, m·N
$R = F/F$	Load deflection rate, N/m
S	Spring stress at deflection F, P_a
S_1	Initial stress at assembled height, P_a
S_2	Final stress at minimum compressed height, P_a
F_2/w	Precompression factor of spring, i.e., ratio of final deflection to working stroke

NOTE: For illustration of some of the above symbols, see figure 1.

For Two-Spring Nest

Superscripts o and i denote dimensions of outer spring and inner spring, respectively.

P_2^o, P_2^i	Load at minimum operating height, N
S_2^1	Stress at minimum operating height, P_a
d^o, d^i	Wire diameter, m
D_o^o, D_o^i	Outer diameter, m
D^o, D^i	Mean coil diameter, m
N^o, N^i	Number of active coils
$C^1 = \frac{D^o}{d^o} \frac{D^i}{d^i}$	Spring index

For Three-Spring Nest

Superscripts A, B, and C denote dimensions of outer, center, and inner springs, respectively.

P_2^A, P_2^B, P_2^C

Load at minimum operating height, N

S_2^{11}

Stress at minimum operating height, P_a

d^A, d^B, d^C

Wire diameter, m

D_o^A, D_o^B, D_o^C

Outside diameter, m

D^A, D^B, D^C

Mean coil diameter, m

N^A, N^B, N^C

Number of active coils

$$C^{11} = \frac{D^A}{d^A} = \frac{D^B}{d^B} = \frac{D^C}{d^C}$$

Spring index

For Rectangular-Wire Springs

b

Width of rectangular wire (long side), m

K_1

Deflection constant

K_2

Stress constant

t

Thickness of rectangular wire (short side) m

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