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ON ABSTRACT VOLTERRA EQUATIONS
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POSITIVE RESOLVENT

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ABSTRACT

We consider the nonlinear abstract Volterra equation of convolution
is considered, =
type A

$$(V) \quad u(t) + b * Au(t) \geq u_0 + b * g(t) \quad t \geq 0$$

where A is m -accretive in a Banach space X , b is a given real kernel,
 $u_0 \in D(A)$ and $g: [0, \infty) \rightarrow X$ are given.

Boundedness and asymptotic properties of the solutions are established
under the assumption that the kernel satisfies certain natural positivity
conditions. $\rightarrow$ to p. 13

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Significance and Explanation

cont

An important property of linear and nonlinear diffusion equations is that the solutions of such equations obey a "maximum principle". In particular, if the initial data and the forcing terms are nonnegative, then the solution is nonnegative.

Equation (V) stated in the Abstract is a generalization of the evolution equation

$$\begin{aligned} \text{(DE)} \quad & \frac{du}{dt} + Au = g \quad t > 0 \\ & u(0) = u_0 \end{aligned}$$

For example, if $Au(x) = -\Delta u(x)$ with Dirichlet or Neumann conditions on a bounded domain of $\mathbb{R}^n$, then equation (DE) is the standard heat equation and the solution u is nonnegative whenever u_0 and g are nonnegative.

The Volterra equation (V) is an abstraction of a mathematical model for nonlinear heat flow in a material with "memory". The kernel b in (V) can be expressed in terms of the physically meaningful heat flux and internal energy relaxation functions.

In this paper ~~we consider~~ equation (V) for a class of kernels b , ^{is considered} which insure the positivity of the solution operator. For this class of kernels we use techniques of nonlinear functional analysis to establish boundedness and asymptotic behaviour of solutions of (V) as $t \rightarrow \infty$. We show that in the special case $b \in L^1(0, \infty)$ the memory induces a damping effect on solutions of (V).

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ON ABSTRACT VOLTERRA EQUATIONS WITH
KERNELS HAVING A POSITIVE RESOLVENT

Ph. Clément

1. Introduction

Let X be a real Banach space with norm $\|\cdot\|$. Let A be a m -accretive operator in X , [3], i.e. for every $\lambda > 0$, $J_\lambda := (I + \lambda A)^{-1}$ is a non-expansive map which is everywhere defined on X . We consider the following Volterra equation of convolution type:

$$(1.1) \quad u(t) + b * Au(t) = f(t) \quad t \geq 0$$

where b is a given real kernel, f is a given function with values in X and $b * Au(t) = \int_0^t b(t-s)Au(s)ds$. Since for every $\lambda > 0$, the Yosida approximation of A , $A_\lambda := \lambda^{-1}(I - J_\lambda)$ is Lipschitz continuous, the equation

$$(1.1)_\lambda \quad u(t) + b * A_\lambda u(t) = f(t) \quad t \geq 0$$

possesses a unique solution $u_\lambda \in C([0, T]; X)$ if $b \in L^1[0, T]$ and $f \in C([0, T]; X)$, $T > 0$. In [4], Crandall and Nohel have proved that if the assumption

$$(H1) \quad \begin{cases} b \in W^{1,1}[0, T], \quad b(0) > 0, \quad \dot{b} \in BV[0, T] \\ f \in W^{1,1}[0, T; X], \quad f(0) \in \overline{D(A)} \end{cases}$$

is satisfied, then there exists $u \in C([0, T]; X)$ such that $\lim_{\lambda \rightarrow 0} u_\lambda = u$ in $C([0, T]; X)$; u is called the generalized solution of (1.1). Note that if (H1) is satisfied, then there exists a unique $u_0 \in \overline{D(A)}$ and a unique $g \in L^1(0, T; X)$ such that

$$(1.2) \quad f(t) = u_0 + b * g(t) \quad 0 \leq t \leq T.$$

Indeed $u_0 = f(0)$ and g is the unique solution of the equation

$$b(0)g(t) + \dot{b} * g(t) = \dot{f}(t) \quad 0 \leq t \leq T,$$

(where $\dot{\cdot} = d/dt$). Conversely, if $b \in W^{1,1}[0, T]$, $b(0) > 0$, $\dot{b} \in BV[0, T]$ and $u_0 \in \overline{D(A)}$, $g \in L^1(0, T; X)$, then f given by (1.2) satisfies assumption (H1).

The proof in [4] of the existence of a generalized solution of (1.1), shows that (1.1) is closely related to the equation

$$(1.3) \quad \begin{cases} \dot{u}(t) + Au(t) = g(t) & 0 < t \leq T \\ u(0) = u_0 \end{cases}$$

which is (1.1) with $b \equiv 1$. It is known [1], that if u_1 and u_2 are the generalized solutions of (1.3) corresponding to the data $u_{0,1}, u_{0,2}$ and g_1, g_2 , then the following estimate, which implies continuous dependence of solutions of (1.3) holds:

$$(1.4) \quad \|u_1(t) - u_2(t)\| \leq \|u_{0,1} - u_{0,2}\| + b * \|g_1 - g_2\|(t)$$

on $[0, T]$, with $b \equiv 1$. In this paper we consider a class of kernels satisfying (H1), containing the kernel $b \equiv 1$, for which the estimate (1.4) still holds. Such class of kernels was introduced in [2, assumptions H4, H5]. Moreover, we prove that if the kernel b belongs to this class and is in $L^1(0, \infty)$, then the generalized solution of (1.1) converges strongly to a limit u_∞ provided that g itself is bounded and converges to a limit g_∞ . If $b \notin L^1(0, \infty)$, it is well-known that u may not converge to a limit. (Take $X = \mathbb{R}^2$ with the Euclidean norm, $A := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $b \equiv 1$, $g = 0$, $u_0 \neq 0$). Work is in progress on an analogous result in the case $b \notin L^1(0, \infty)$ and $A = \omega I + B$ ($\omega > 0$, B m -accretive).

In order to state our main assumption on the kernel b we need the following definitions. For $b \in L^1(0, T)$, let us denote by $r(b)$ the resolvent of b , i.e. the unique solution in $L^1(0, T)$ of the equation

$$(1.5) \quad r + b * r = b \quad 0 \leq t \leq T,$$

and by $s(b)$, the unique solution in $AC[0, T]$ of the equation

$$(1.6) \quad s + b * s = 1 \quad 0 \leq t \leq T.$$

Our basic assumption on the kernel b is

$$(H2) \quad \begin{cases} \text{For every } \lambda > 0, \quad r(\lambda b) \geq 0 \text{ a.e. on } [0, T] \\ \text{and } s(\lambda b) \geq 0 \text{ on } [0, T]. \end{cases}$$

It is known [7], [5], [2] that if $b \in L^1(0, T)$, is positive, nonincreasing and if $\log b$ is convex on $(0, T)$, then b satisfies (H2). Observe that if b is completely monotonic on $(0, \infty)$, then $\log b$ is convex [7]. Observe also that (H2) implies $b \geq 0$. In order to avoid trivialities we shall assume that b is not identically equal to 0. In connection with this class of kernels we mention the following "positivity" result:

Theorem [2; Theorem 5] Let b, f satisfy (H1) and (H2) on $[0, T]$ with $f = u_0 + b * g$. Let P be a closed convex cone in X . If $J_\lambda(P) \subseteq P$ for every $\lambda > 0$, $u_0 \in P$ and $g(t) \in P$ a.e. on $[0, T]$, then u the generalized solution of (1.1) satisfies $u(t) \in P$, $t \in [0, T]$.

2. Statement of results

We first give the generalization of (1.4) to (1.1) with kernels b satisfying (H2).

Theorem 1. Let b, f_1, f_2 satisfy (H1) and (H2) on $[0, T]$, with $f_i = u_{0,i} + b * g_i$, $i = 1, 2$. Let u_1, u_2 be the corresponding generalized solutions of (1.1) on $[0, T]$. Then

$$(2.1) \quad \|u_1(t) - u_2(t)\| \leq \|u_{0,1} - u_{0,2}\| + b * \|g_1 - g_2\|(t)$$

$0 \leq t \leq T$ holds.

Our main result concerns the asymptotic behaviour of solutions of (1.1) as $t \rightarrow \infty$. For results in this direction, in the scalar case, but for more general kernels b , we refer the reader to [6].

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Theorem 2. Let b, f satisfy (H1) and (H2) on $[0, T]$ for every $T > 0$, with $f = u_0 + b * g$ and $b \neq 0$. If $b \in L^1(0, \infty)$, $g \in L^\infty(\mathbb{R}^+, X)$ and $\lim_{t \rightarrow \infty} g(t) = g_\infty$ exists in X , then

$$(2.2) \quad \|u(t) - u_\infty\| \leq \frac{\int_0^\infty b(s) ds}{\int_0^\infty b(s) ds} \|u_0 - u_\infty\| + b * \|g - g_\infty\|(t)$$

holds for $t > 0$, where u is the generalized solution of (1.1) and $u_\infty = (I + \bar{b}A)^{-1}(u_0 + \bar{b}g_\infty)$ with $\bar{b} = \int_0^\infty b(s) ds$.

3. Proofs.

In the proofs we shall use the fact that if $v \in L^1(0, T; X)$ satisfies

$$(3.1) \quad v(t) + b * v(t) = u_0 + b * g(t) \quad 0 \leq t \leq T$$

with $b \in L^1(0, T)$, $u_0 \in X$ and $g \in L^1(0, T; X)$ then

$$(3.2) \quad v(t) = s(b)(t)u_0 + r(b) * g(t) \quad 0 \leq t \leq T$$

holds.

Proof of Theorem 1.

We first establish (2.1) with A replaced by A_λ , $\lambda > 0$ and then we pass to the limit as $\lambda \downarrow 0$. For $\lambda > 0$, let $u_{i,\lambda}$ satisfy

$$(3.3) \quad u_{i,\lambda} + b * A_\lambda u_{i,\lambda} = u_{0,i} + b * g_i \quad t \in [0, T], \quad i = 1, 2.$$

From the definition of A_λ , we have

$$(3.4) \quad u_{i,\lambda} + \lambda^{-1} b * u_{i,\lambda} = \lambda^{-1} b * J_\lambda u_{i,\lambda} + u_{0,i} + b * g_i, \quad i = 1, 2.$$

Using (3.2) we get

$$(3.5) \quad u_{i,\lambda} = r(\lambda^{-1}b) * J_\lambda u_{i,\lambda} + s(\lambda^{-1}b)u_{0,i} + \lambda r(\lambda^{-1}b) * g_i, \quad i = 1, 2.$$

Next we estimate $\|u_{1,\lambda} - u_{2,\lambda}\|$. Since J is nonexpansive, $s(\lambda^{-1}b)$ and $r(\lambda^{-1}b)$ are nonnegative, we obtain:

$$(3.6) \quad \|u_{1,\lambda} - u_{2,\lambda}\| \leq r(\lambda^{-1}b) * \|u_{1,\lambda} - u_{2,\lambda}\| + s(\lambda^{-1}b) \|u_{0,1} - u_{0,2}\| + \lambda r(\lambda^{-1}b) * \|g_1 - g_2\|.$$

We take the convolution of (3.6) with $\lambda^{-1}b$ (which is nonnegative) and we add (3.6). We have

$$(3.7) \quad \begin{aligned} & \|u_{1,\lambda} - u_{2,\lambda}\| + \lambda^{-1}b * \|u_{1,\lambda} - u_{2,\lambda}\| \leq \\ & (r(\lambda^{-1}b) + \lambda^{-1}b * r(\lambda^{-1}b)) * \|u_{1,\lambda} - u_{2,\lambda}\| \\ & + (s(\lambda^{-1}b) + \lambda^{-1}b * s(\lambda^{-1}b)) \|u_{0,1} - u_{0,2}\| \\ & + \lambda(r(\lambda^{-1}b) + \lambda^{-1}b * r(\lambda^{-1}b)) * \|g_1 - g_2\|. \end{aligned}$$

From the definition of $r(\lambda^{-1}b)$ and $s(\lambda^{-1}b)$, we obtain $\|u_{1,\lambda} - u_{2,\lambda}\| \leq \|u_{0,1} - u_{0,2}\| + b * \|g_1 - g_2\|$. The conclusion of Theorem 1 follows by letting λ go to 0.

Proof of Theorem 2.

As in the proof of Theorem 1, we first prove the result with A replaced by A_λ , $\lambda > 0$ and then we pass to the limit as $\lambda \downarrow 0$.

For $\lambda > 0$, let u_λ satisfy

$$(3.8) \quad u_\lambda + b * A_\lambda u_\lambda = u_0 + b * g.$$

From the definition of A_λ and (3.2) we have:

$$(3.9) \quad u_\lambda = r(\lambda^{-1}b) * J_\lambda u_\lambda + s(\lambda^{-1}b)u_0 + \lambda r(\lambda^{-1}b) * g.$$

Since A is m -accretive, A_λ is also m -accretive and there is a unique $u_{\lambda\infty}$ satisfying

$$(3.10) \quad u_{\lambda\infty} + \bar{b} A_\lambda u_{\lambda\infty} = u_0 + \bar{b} g_\infty$$

where $\bar{b} = \int_0^\infty b(s)ds$.

Using again the fact that $b \in L^1(0,T)$, we can rewrite (3.10) as

$$(3.11) \quad u_{\lambda\infty} + b * A_\lambda u_{\lambda\infty} = u_0 + b * g + b * (g_\infty - g) - \xi w_\lambda$$

where

$$(3.12) \quad \xi(t) := \int_t^\infty b(s)ds$$

and

$$(3.13) \quad w_\lambda := A_\lambda u_{\lambda\infty} - g_\infty.$$

Let η satisfy

$$(3.14) \quad \eta + \lambda^{-1}b * \eta = \xi.$$

Then obviously ηw_λ satisfies

$$(3.15) \quad \eta w_\lambda + \lambda^{-1}b * \eta w_\lambda = \xi w_\lambda.$$

Using (3.11), (3.15), (3.2) and the definition of A_λ we obtain

$$(3.16) \quad \begin{aligned} u_{\lambda\infty} &= r(\lambda^{-1}b) * J_\lambda u_{\lambda\infty} + s(\lambda^{-1}b)u_0 \\ &\quad + \lambda r(\lambda^{-1}b) * g + \lambda r(\lambda^{-1}b) * (g_\infty - g) - \eta w_\lambda. \end{aligned}$$

Subtracting (3.16) from (3.9) and using the fact that J_λ is nonexpansive, $s(\lambda^{-1}a)$, $r(\lambda^{-1}a)$ are nonnegative, we get:

$$(3.17) \quad \|u_\lambda - u_{\lambda\infty}\| \leq r(\lambda^{-1}b) * \|u_\lambda - u_{\lambda\infty}\| + \lambda r(\lambda^{-1}b) * \|g_\infty - g\| + |\eta| \|w_\lambda\|.$$

Next we take the convolution of (3.17) with $\lambda^{-1}b$ (which is nonnegative) and we add (3.17); we obtain

$$(3.18) \quad \|u_\lambda - u_{\lambda\infty}\| \leq b * \|g - g_\infty\| + (|\eta| + \lambda^{-1}b * |\eta|) \|w_\lambda\|.$$

We claim that η is nonnegative. Indeed η satisfies (3.14) with

$\xi(t) = \bar{b} - \int_0^t b(s)ds$. Thus η satisfies (3.1) for every $T > 0$, with $X = \mathbb{R}$, b replaced by $\lambda^{-1}b$, u_0 replaced by $\bar{b}$ and g replaced by $-\lambda \mathbb{1}$, where $\mathbb{1}(t) \equiv 1$. From (3.2) we get

$$(3.19) \quad \eta(t) = s(\lambda^{-1}b)(t)\bar{b} - \lambda \int_0^t r(\lambda^{-1}b)(\tau)d\tau \quad t > 0.$$

By using the identity

$$(3.20) \quad s(\lambda^{-1}b)(t) + \int_0^t r(\lambda^{-1}b)(\tau)d\tau = 1 \quad t \geq 0$$

we have

$$(3.21) \quad \dot{\eta}(t) = -\bar{b} r(\lambda^{-1}b)(t) - \lambda r(\lambda^{-1}b)(t) \quad t \geq 0.$$

The fact that $\bar{b}, \lambda$ are positive and assumption (H2) imply that η is nonincreasing.

It remains to prove that $\lim_{t \rightarrow \infty} \eta(t) \geq 0$.

From (3.20) and assumption (H2), it follows that $s(\lambda^{-1}b)(\infty) := \lim_{t \rightarrow \infty} s(\lambda^{-1}b)(t)$ exists and

$$(3.22) \quad s(\lambda^{-1}b)(\infty) = 1 - \int_0^{\infty} r(\lambda^{-1}b)(\tau) d\tau$$

holds.

Letting t go to infinity in (3.19) we get

$$(3.23) \quad \lim_{t \rightarrow \infty} \lambda^{-1} \eta(t) = s(\lambda^{-1}b)(\infty) \lambda^{-1} \bar{b} - \int_0^{\infty} r(\lambda^{-1}b)(\tau) d\tau$$

hence from (3.22), we obtain

$$(3.24) \quad \lim_{t \rightarrow \infty} \lambda^{-1} \eta(t) = s(\lambda^{-1}b)(\infty) (1 + \lambda^{-1} \bar{b}) - 1.$$

Next we observe that

$$(3.25) \quad s(\lambda^{-1}b)(\infty) (1 + \lambda^{-1} \bar{b}) = 1.$$

Indeed from the definition of $s(\lambda^{-1}b)$ we have

$$s(\lambda^{-1}b)(t) + \lambda^{-1} \bar{b} * s(\lambda^{-1}b)(t) = 1.$$

But $\lim_{t \rightarrow \infty} s(\lambda^{-1}b)(t)$ exists, $0 \leq s(\lambda^{-1}b)(t) \leq 1$ for every $t \geq 0$, and $b \in L^1(0, \infty)$, thus (3.25) follows. Consequently $\lim_{t \rightarrow \infty} \eta(t) = 0$ and $\eta(t) \geq 0$ for every $t \geq 0$. Replacing $|\eta|$ by η in (3.18) and using (3.14), we obtain:

$$(3.26) \quad \|u_{\lambda} - u_{\lambda\infty}\| \leq \xi \|w_{\lambda}\| + b * \|g - g_{\infty}\|.$$

Since $\bar{b} \geq 0$, using (3.13) we have

$$(3.27) \quad \xi(t) \|w_{\lambda}\| = \frac{\xi(t)}{\bar{b}} \|\bar{b} A_{\lambda} u_{\lambda\infty} - \bar{b} g_{\infty}\|.$$

Finally using (3.10), (3.12), (3.26) and (3.27) we get

$$(3.28) \quad \|u_{\lambda}(t) - u_{\lambda\infty}\| \leq \frac{\int_0^t b(s) ds}{\int_0^{\infty} b(s) ds} \|u_0 - u_{\lambda\infty}\| + b * \|g - g_{\infty}\|(t) \quad t \geq 0.$$

Observe that (3.28) is the conclusion of the theorem with A replaced by A_{λ} .

Since A is m -accretive we have

$$\lim_{\lambda \rightarrow 0} u_{\lambda\infty} := \lim_{\lambda \rightarrow 0} (I + \bar{b} A_{\lambda})^{-1} (u_0 + \bar{b} g_{\infty})$$

$$= \lim_{\lambda \rightarrow 0} (I + \bar{b} A)^{-1} (u_0 + \bar{b} g_{\infty}) =: u_{\infty}.$$

Using assumption (H1), $\lim_{\lambda \downarrow 0} u_\lambda = u$ in $C([0,T];X)$, thus (2.2) follows from (3.28) by letting λ go to 0.

Remark. It is clear from the proofs of the theorems that the assumption (H1) has been used only to insure that $\lim_{\lambda \downarrow 0} u_\lambda$ exists in $C([0,T];X)$, for every $T > 0$. Indeed Theorems 1 and 2 are valid for A replaced by A_λ , $\lambda > 0$, if the assumption (H1) is replaced by the assumption

$$(H1') \quad \begin{cases} a \in L^1(0,T) \\ u_0 \in X, \quad g \in L^1(0,T;X) \end{cases}$$

It has been proved in [2, Th. 1(ii), Th. 2(i), Remark 2.3] that under the assumptions (H1') and (H2), if A is linear m -accretive with $D(A)$ dense in X that $\lim_{\lambda \downarrow 0} u_\lambda$ exists in $L^1(0,T;X)$. Therefore in the linear case, Theorems 1 and 2 are true with (H1) replaced by (H1'). Then pointwise inequalities (2.1) and (2.2) have to be replaced by a.e. inequalities.

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