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THE INITIAL VALUE PROBLEM FOR THE EQUATIONS OF MOTION OF COMPRE--ETC(U)

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MRC Technical Summary Report #1991 ✓

THE INITIAL VALUE PROBLEM FOR THE  
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VISCOUS AND HEAT-CONDUCTIVE FLUIDS

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September 1979

(Received July 2, 1979)

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OF COMPRESSIBLE VISCOUS AND HEAT-CONDUCTIVE FLUIDS

Akitaka Matsumura\* and Takaaki Nishida\*\*

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September 1977

ABSTRACT

↙ The initial value problem associated with the equations of motion for isotropic Newtonian fluids is investigated. The fluids are compressible, viscous and heat-conductive. It is proved that there exists a unique global solution in time, for the small initial data, and the solution has the decay rate of  $(1+t)^{-3/4}$  as  $t \rightarrow +\infty$ .

$\frac{1}{(1+t)}$  to  $3/4$  power as  $t$  approaches positive infinity.  $\rightarrow$  to page - B -

AMS(MOS) Subject Classification: 35B40, 35K55, 35M05, 76.35

Key Words: Initial value problem, Equation of motion, Compressible, viscous and heat-conductive fluids, Global solutions in time

Work Unit Number 1 - Applied Analysis

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Sponsored by the United States Army under Contract No. DAAG29-75-C-0024.

# SIGNIFICANCE AND EXPLANATION

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The motions of compressible, viscous and heat-conductive fluids are described by a system of partial differential equations which is of hyperbolic-parabolic type and highly nonlinear. One of the first mathematical problems associated with this system is the initial value problem. We obtain the existence of a unique smooth global solution in time for the initial value problem and also the decay rate of the solution as time tends to infinity.

Since the system is quasilinear with respect to the unknowns: density, velocity and temperature, we need to assume that the initial data are close to the constant equilibrium state. The proof which is necessarily quite technical, involves a combination of the estimates for the decay rate of solutions of the linearized equations as time tends to infinity, together with energy estimates in the space of square summable functions. Our method to obtain the "small" but global solution in time for this particular nonlinear problem is rather general and can be applied to many systems of nonlinear partial differential equations, if the solution of the linearized equation has an appropriate decay rate as time tends to infinity (e.g. dissipative equations, parabolic equations and so on) and if the nonlinear equations are amenable to ordinary energy estimates.

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| Accession For      | NTIS GMA&I        |  |  |   |
|                    | DOC TAB           |  |  |   |
|                    | Unannounced       |  |  |   |
|                    | JA Classification |  |  |   |
| By                 |                   |  |  |   |
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THE INITIAL VALUE PROBLEM FOR THE EQUATIONS OF MOTION  
OF COMPRESSIBLE VISCOUS AND HEAT-CONDUCTIVE FLUIDS

Akitaka Matsumura\* and Takaaki Nishida\*\*

§1. Introduction and Theorem

The motion of the general isotropic Newtonian fluids is described by the five conservation laws:

$$(1.1) \quad \begin{cases} \rho_t + (\rho u^j)_{x_j} = 0 \\ u_t^i + u^j u_{x_j}^i + \frac{1}{\rho} p_{x_i} = \frac{1}{\rho} \{ (\mu (u_{x_j}^i + u_{x_i}^j))_{x_j} + (\mu' u_{x_j}^j)_{x_i} \}, \quad i = 1, 2, 3 \\ \theta_t + u^j \theta_{x_j} + \frac{\theta p_{\theta}}{\rho c} u_{x_j}^j = \frac{1}{\rho c} \{ (\kappa \theta_{x_j})_{x_j} + \Psi \}, \end{cases}$$

where  $\rho$ : density,  $u = (u^1, u^2, u^3)$ : velocity,  $\theta$ : absolute temperature,  $p = p(\rho, \theta)$ : pressure,  $\mu = \mu(\rho, \theta)$ : viscosity coefficient  $\mu' = \mu'(\rho, \theta)$ : second viscosity coefficient,  $\kappa = \kappa(\rho, \theta)$ : coefficient of heat conduction,  $c = c(\rho, \theta)$ : heat capacity at constant volume and  $\Psi = \frac{\mu}{2} (u_{x_k}^j + u_{x_j}^k)^2 + \mu' (u_{x_j}^j)^2$ : dissipation function. We consider the initial value problem for (1.1) with the initial data

$$(1.2) \quad (\rho, u, \theta)(0, x) = (\rho_0, u_0, \theta_0)(x), \quad x \in \mathbb{R}^3.$$

We seek the solutions in a neighbourhood of any constant state  $(\rho, u, \theta) = (\bar{\rho}, 0, \bar{\theta})$ , where  $\bar{\rho}, \bar{\theta}$  are any positive constants. Thus we assume the following natural conditions on the system (1.1) of hyperbolic-parabolic type throughout this paper:

(i)  $p, c, \mu, \mu'$  and  $\kappa$  are smooth functions in  $\mathcal{O} = \{(\rho, u, \theta): |\rho - \bar{\rho}|, |u|, |\theta - \bar{\theta}| < \bar{\epsilon}\}$ .

(ii)  $\partial p / \partial \rho, \partial p / \partial \theta > 0$ ,  $c, \mu, \kappa > 0$  and  $\mu' + \frac{2}{3} \mu \geq 0$  in  $\mathcal{O}$ ,

where  $\bar{\epsilon} < \min\{\bar{\rho}, \bar{\theta}\}$ .

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First rewrite the system (1.1) by the change of the unknown and known variables as follows:  $\rho \mapsto \bar{\rho} + \rho$ ,  $u \mapsto u$ ,  $\theta \mapsto \bar{\theta} + \theta$ ,  $p(\bar{\rho} + \rho, \bar{\theta} + \theta) \mapsto p(\rho, \theta)$ ,  $u(\bar{\rho} + \rho, u, \bar{\theta} + \theta) \mapsto u(\rho, u, \theta)$  and so on.

$$(1.3) \quad \begin{cases} L^0(\rho, u) \equiv \rho_t + (\bar{\rho} + \rho)u_{x_j}^j + u^j \rho_{x_j} = 0 \\ L^i(u) \equiv u_t^i - \bar{u} u_{x_j x_j}^i - (\bar{u} + u')u_{x_1 x_j}^j = G^i, \quad i = 1, 2, 3 \\ L^4(\theta) \equiv \theta_t - \bar{\kappa} \theta_{x_j x_j} = G^4 \end{cases}$$

where

$$(1.4) \quad \begin{cases} G^i \equiv -\tilde{p}_\rho \rho_{x_i} - \tilde{p}_\theta \theta_{x_i} + g^i, \quad G^4 \equiv -\tilde{p}_3 u_{x_j}^j + g^4, \\ g^i \equiv -u^j u_{x_j}^i + \{u_{x_j} (u_{x_j}^i + u_{x_1}^j) + u_{x_1} (u_{x_j}^j)\} / (\bar{\rho} + \rho) \\ g^4 \equiv -u^j \theta_{x_j} + (\kappa_{x_j} \theta_{x_j} + \psi) / (\bar{\rho} + \rho) c. \end{cases}$$

Here we also use the abbreviations

$$\bar{u} = u / (\bar{\rho} + \rho), \quad \bar{u}' = u' / (\bar{\rho} + \rho), \quad \tilde{p}_\rho = p_\rho / (\bar{\rho} + \rho)$$

$$\tilde{p}_\theta = p_\theta / (\bar{\rho} + \rho), \quad \tilde{p}_3 = (\bar{\theta} + \theta) p_\theta / (\bar{\rho} + \rho) c \quad \text{and}$$

$$\bar{\kappa} = \kappa / (\bar{\rho} + \rho) c.$$

Let  $H^\ell$  ( $\ell = 1, 2, \dots, 5$ ) be the Sobolev space with the norm  $\| \cdot \|_\ell$  of the  $L_2$ -functions having all the  $\ell$ -th derivatives of  $L_2$ -functions. Define  $\varepsilon$  by the Sobolev's lemma so that for  $\|f\|_2 < \varepsilon$  we have  $\max |f| \leq C \|f\|_2 < \bar{\varepsilon}$ . Denote  $D^\ell f = \{\partial^\ell f / \partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \partial x_3^{\alpha_3} \text{ for all } \alpha, \alpha_1 + \alpha_2 + \alpha_3 = \ell\}$ ,  $\ell = 1, \dots, 4$ . The initial data for (1.3) are given by

$$(1.5) \quad (\rho, u, \theta)(0) = (\rho_0, u_0, \theta_0) \in H^\ell \cap L_1 \quad \text{for } \ell = 3 \text{ or } 4.$$

The solution is sought in the space of functions  $X^k(0, \infty; E)$  for some  $E < \epsilon$ ,  $k = 3$  or  $4$ , where for  $0 \leq t_1 < t_2 \leq \infty$

$$\begin{aligned} X^k(t_1, t_2; E) = \{(\rho, u, \theta)(t) : \rho(t, x) \in C^0(t_1, t_2; H^k) \cap C^1(t_1, t_2; H^{k-1}) \\ u^i(t, x), \theta(t, x) \in C^0(t_1, t_2; H^k) \cap C^1(t_1, t_2; H^{k-2}) \cap \\ (1.6) \quad \cap L_2(t_1, t_2; H^{k+1}), \quad i = 1, 2, 3, \text{ and} \\ \sup_{t_1 \leq t \leq t_2} \|(\rho, u, \theta)(t)\|_k^2 + \int_{t_1}^{t_2} \|\rho(s)\|_k^2 + \|(u, \theta)(s)\|_{k+1}^2 ds \leq E^2\} . \end{aligned}$$

#### Theorem

Consider the initial value problem (1.3) (1.5) and let the initial data have the norm for  $k = 4$

$$(1.7) \quad E_k = \|(\rho, u, \theta)(0)\|_k + \|(\rho, u, \theta)(0)\|_{L_1} < \infty .$$

Then there exist positive constants  $\delta_0$  and  $C_0 < \infty$  ( $C_0 \delta_0 \leq \epsilon$ ) such that if  $E_k < \delta_0$ , then the problem (1.3) (1.5) has the unique solution  $(\rho, u, \theta)(t)$  in large time such that

$$(\rho, u, \theta)(t) \in X^k(0, \infty; C_0 E_k)$$

and it has the decay rate

$$(1.8) \quad \|(\rho, u, \theta)(t)\|_2 \leq C_0 E_k / (1+t)^{3/4} .$$

In particular if  $\mu, \mu'$  and  $\kappa$  do not depend on  $\rho$ , then the above assertion holds for  $k = 3$  also.

In [1] we obtain the same type of result in the more restricted case of a polytropic gas. We also refer the reader to [1] for the bibliography of other known results.

## §2. Proof of Theorem

Theorem is proved by a combination of a local existence theorem and a priori estimates for the solution in  $X^\ell$ .

### Theorem 2.1 (local existence)

Consider the initial value problem (1.3),  $t \geq t_1$ . Let the initial data

$$(\rho, u, \theta)(t_1) \in H^\ell \text{ for } \ell = 3 \text{ or } 4.$$

Then there exist three constants  $\delta_1 > 0$ ,  $C_1 < \infty$  ( $C_1 \delta_1 < \epsilon$ ) and  $\tau > 0$  which are independent of  $t_1$  such that if  $\|(\rho, u, \theta)(t_1)\|_\ell < \delta_1$ , then the problem (1.3)

(1.5) has the unique solution

$$(\rho, u, \theta)(t) \in X^\ell(t_1, t_1 + \tau; C_1 \|(\rho, u, \theta)(t_1)\|_\ell).$$

The proof for  $\ell = 4$  is the same as that for polytropic gas in [1]. We need an approximation of the initial data in  $H^4$  and the  $L_2$  energy estimate for the case  $\ell = 3$ .

### Theorem 2.2 (a priori estimates)

Suppose that for the initial data having the norm  $E_\ell < \infty$  for  $\ell = 4$ , there is a solution

$$(\rho, u, \theta)(t) \in X^\ell(0, T; E)$$

for some  $T > 0$  and some  $E < \infty$ . Then there exist positive constants  $\epsilon_2 (< \epsilon)$ ,  $\delta_2$  and  $C_2$  ( $C_2 \delta_2 < \epsilon$ ) such that if  $E < \epsilon_2$  and  $E_\ell < \delta_2$ , then the solution has the a priori estimates

$$(\rho, u, \theta)(t) \in X^\ell(0, T; C_2 E_\ell),$$

where  $\epsilon_2, \delta_2, C_2$  do not depend on  $T$ . In particular in the case of (1.9) the above estimates are true for  $\ell = 3$  also.

Proof of Theorem

Take  $\delta_0 = \min \{\delta_1, \delta_2, \varepsilon_2/C_1, \delta_1/C_2, \varepsilon_2/C_1 C_2\}$  and  $C_0 = C_2$ . We use the standard continuation argument of local solution on  $[0, n\tau]$ ,  $n = 1, 2, \dots$  to get the global solution. In fact by the local existence theorem, the definition of  $\delta_0$  and the assumption  $(E_k < \delta_0)$  we have a positive constant  $\tau$  and a local solution

$$(\rho, u, \theta)(t) \in X^k(0, \tau; C_1 E_k) .$$

By  $C_1 E_k < C_1 \delta_0 \leq \varepsilon_2$  and  $E_k < \delta_0 \leq \delta_2$ , a priori estimates gives

$$(\rho, u, \theta)(t) \in X^k(0, \tau; C_2 E_k) .$$

But by  $C_2 E_k < C_2 \delta_0 \leq \delta_1$  and the local existence theorem, we have again

$$(\rho, u, \theta)(t) \in X^k(\tau, 2\tau; C_1 C_2 E_k) .$$

Now by  $C_1 C_2 E_k < C_1 C_2 \delta_0 \leq \varepsilon_2$  and  $E_k < \delta_0 \leq \delta_2$ , a priori estimate shows

$$(\rho, u, \theta)(t) \in X^k(0, 2\tau; C_2 E_k) .$$

Thus we can continue the same arguments on  $[n\tau, (n+1)\tau]$  and  $[0, (n+1)\tau]$  successively  $n = 2, 3, \dots$ .

### §3. A Priori Estimates

We present here a general method to obtain a priori estimates for small solutions of equations with dissipation, which is a combination of the linear spectral theory and the  $L_2$ -energy method. First we rewrite the system (1.3) so that all the non-linear terms appear at the right hand side of equations:

$$(3.1) \quad \begin{cases} \rho_t + \bar{p} u_{x_j}^j = f^0, \\ u_t^i + \bar{p}_1 \rho_{x_i} + \bar{p}_2 \theta_{x_i} - \bar{\mu} u_{x_j x_j}^i - (\bar{\mu} + \bar{\mu}') u_{x_i x_j}^j = f^i, \quad i = 1, 2, 3, \\ \theta_t + \bar{p}_3 u_{x_j}^j - \bar{\kappa} \theta_{x_j x_j} = f^4, \end{cases}$$

where  $f = \{f^i, i = 0, \dots, 4\}$  is at least quadratic functions of  $(\rho, u, \theta)$  and their first and second derivatives, and  $\bar{p}_1 = \bar{p}_\rho(0, 0)$ ,  $\bar{p}_2 = \bar{p}_\theta(0, 0)$ ,  $\bar{p}_3 = \bar{p}_3(0, 0)$ ,  $\bar{\mu} = \bar{\mu}(0, 0, 0)$ ,  $\bar{\mu}' = \bar{\mu}'(0, 0, 0)$ ,  $\bar{\kappa} = \bar{\kappa}(0, 0, 0)$  are positive constants.

Set  $U = \begin{pmatrix} t \\ \sqrt{\bar{p}/\bar{p}_1} \rho, u, \sqrt{\bar{\theta}/c(0, 0, 0)} \theta \end{pmatrix}$  and write (3.1) in the form

$$(3.2) \quad U_t + AU = F(U).$$

The Fourier transform  $\hat{A}(\xi)$  of the linear partial differential operator  $A$  is the  $5 \times 5$  matrix

$$(3.3) \quad \hat{A}(\xi) = \begin{pmatrix} 0 & -ia\xi_k & 0 \\ -ia\xi_j & -\bar{\mu}\delta^{jk}|\xi|^2 - (\bar{\mu} + \bar{\mu}')\xi_j\xi_k & -ib\xi_j \\ 0 & -ib\xi_k & -\bar{\kappa}|\xi|^2 \end{pmatrix},$$

where  $a = \sqrt{\bar{p}_\rho(0, 0)}$ ,  $b = \bar{p}_2 \sqrt{\bar{\theta}/c(0, 0, 0)}$  and  $j, k$  run from 1 to 3. The eigenvalues  $\lambda_j, j = 1, \dots, 4$  of  $\hat{A}$  and their projections  $P_j, j = 1, \dots, 4$ , on the eigenspaces are analyzed by

Lemma 3.1

- (i)  $\lambda_j$  depends on  $i|\xi|$  only and  $\lambda_j = 0$  iff  $|\xi| = 0, j = 1, \dots, 4$ .
- (ii)  $\lambda_j \neq \lambda_k, j \neq k$ , for all  $|\xi|$  except at most four points of  $|\xi| > 0$ .
- (iii) There exist positive constants  $r_1 < r_2$  such that  $\lambda_j$  has a Taylor (Laurent) series expansion for  $|\xi| < r_1$   $(|\xi| > r_2, \text{ respectively})$ . Specifically, the Taylor series has the form

$$(3.4) \quad \begin{cases} \lambda_1 = \sqrt{a^2 + b^2} i |\xi| + \frac{(a^2 + b^2)(2\bar{\mu} + \bar{\mu}') + b^2 \bar{\kappa}}{2(a^2 + b^2)} (i |\xi|)^2 + \dots \\ \lambda_2 = \lambda_1^* \text{ (complex conjugate)} \\ \lambda_3 = \frac{a^2 \bar{\kappa}}{a^2 + b^2} (i |\xi|)^2 + \frac{a^2 b^2 \bar{\kappa}^2 ((a^2 + b^2)(2\bar{\mu} + \bar{\mu}') - a^2 \bar{\kappa})}{(a^2 + b^2)^4} (i |\xi|)^4 + \dots \\ \lambda_4 = \bar{\mu} (i |\xi|)^2 \end{cases}$$

- (iv)  $\text{rank } (\lambda_4 - \hat{A}) = 3$  for all  $|\xi| > 0$ .
- (v) The matrix exponential has the spectral resolution

$$(3.5) \quad e^{t\hat{A}(\xi)} = \sum_{j=1}^4 e^{t\lambda_j(\xi)} P_j(\xi)$$

for all  $|\xi|$  except at most four points of  $|\xi| > 0$ .

$$(3.6) \quad \|P_j(\xi)\| \leq C \text{ for } |\xi| \leq r_1.$$

It has the estimate by the modification of the right hand side of (3.5) near the points of a multiple eigenvalue

$$(3.7) \quad \|e^{t\hat{A}(\xi)}\| \leq C(1+t)^3 e^{-\beta t}$$

for  $|\xi| > r_1$  and a positive constant  $\beta$ .

Lemma 3.2

There is a constant  $C = C(\epsilon)$  such that

$$(3.8) \quad \begin{aligned} & \|F(U)\|_{L_1}, \quad \|F(U)\| \leq C \|U\|_2^2 \\ & \|D^k F(U)\| \leq C \|U\|_2 \|U\|_{k+2} \quad \text{for } k = 1, 2. \end{aligned}$$

In particular in the case of (1.9)

$$(3.9) \quad \|D^2 F(U)\| \leq C \|U\|_2 (\|U\|_3 + \|u, \theta\|_4)$$

Proposition 3.3

There exist  $\delta_3, \epsilon_3$  and  $C_3$  such that if  $E_\ell < \delta_3$  and  $E < \epsilon_3$ , then  $U(t)$  satisfying (3.2) has the estimates

$$(3.10) \quad \begin{cases} \|U(t)\|_2 \leq C_3 E_\ell (1+t)^{-3/4} \\ \int_0^t \|U(s)\|_2^2 ds \leq C_3 E_\ell, \end{cases}$$

where  $\ell = 4$  in general and  $\ell = 3$  for the case (1.9).

The Proposition is a consequence of Lemmas 3.1 and 3.2. In fact we have

$$\begin{cases} \|U(t)\| \leq C_0 E_\ell (1+t)^{-3/4} + C \int_0^t (1+t-s)^{-3/4} \|U(s)\|_2^2 ds \\ \|D^k U(t)\| \leq C_0 E_\ell (1+t)^{-5/4} + C \int_0^t (1+t-s)^{-5/4} \|U(s)\|_2 \cdot \\ \cdot (\|U(s)\|_2 + \|U(s)\|_4) ds, \quad k = 1, 2. \end{cases}$$

Therefore for  $M(t) = \sup_{0 \leq s \leq t} (1+s)^{3/4} \|U(s)\|_2$  we have  $M(t) \leq C_0 E_\ell + C M(t)^2 +$

$C E M(t)$ , where  $E$  is the norm (1.6) assumed on the solution. Thus we get the conclusion of Proposition for  $\ell = 4$ .

Next we have to obtain the estimates for the higher derivatives, which is given by

Proposition 3.4

There exist  $\delta_4, \epsilon_4$  and  $C_4$  such that if  $E_\ell < \delta_4, E < \epsilon_4$  and the solution  $U(t)$  satisfies the estimates (3.10), then the energy estimates hold

$$(3.11) \quad \|D^k(u, \theta)(t)\|^2 + \int_0^t \|D^{k+1}(u, \theta)(s)\|^2 ds \leq C_4 E_\ell^2 \quad \text{for } 2 \leq k \leq \ell,$$

$$(3.12) \quad \|D^m \rho(t)\|^2 + \int_0^t \|D^{m+1} \rho(s)\|^2 ds \leq C_4 E_\ell^2 \quad \text{for } 3 \leq m \leq \ell.$$

Since we have already obtained the necessary estimates (3.10) for the lower order derivatives of the solution, the proof of Proposition 3.4 is given successively with respect to  $k$  and  $m$  in the same way as that for polytropic gases in [1]. In fact let us remind the operators  $L^i, i = 0, \dots, 4$  in (1.3) and note the estimates for the nonlinear terms  $g$  in the right hand side of (1.3).

Lemma 3.5

We have the estimates for  $k = 0, 1, \dots, 4$

$$(3.13) \quad \|D^k g\| \leq C \|\rho, u, \theta\|_3 \|D(\rho, u, \theta)\|_k.$$

The estimate (3.11) for  $k = 2$  is given by the integration on  $x \in \mathbb{R}^3, 0 \leq t \leq T$  of the equality

$$(3.14) \quad D^k(L^i(u) - G^i) \cdot D^k u^i + D^k(L^4(\theta) - G^4) \cdot D^k \theta = 0.$$

Integrate by parts, use (3.10) and Lemma 3.5. The estimate (3.12) for  $m = 3$  is obtained by the integration on  $x \in \mathbb{R}^3, 0 \leq t \leq T$  of the equality

$$\begin{aligned} & D^{m-1}\{L^0(\rho, u)\}_{x_i} \cdot D^{m-1} \rho_{x_i} + \\ & + \frac{\bar{p} + \rho}{2\bar{\mu} + \bar{\mu}'} \{D^{m-1}\{L^i(u) + \tilde{p}_\rho \rho_{x_i} - (g^i + \tilde{p}_\theta \theta_{x_i})\} \cdot D^{m-1} \rho_{x_i} = 0 \end{aligned}$$

Integrate by parts, use the equation (1.3) and (3.10), (3.11) for  $k = 2$  and Lemma 3.5.

We can proceed to get (3.11) for  $k = 3$  by (3.14) and so on. The detailed arguments

using the Friedrichs mollifier and the estimates for composite functions are the same as that in [1], and so we omit them here.

#### REFERENCE

- [1] A. Matsumura and T. Nishida, "The initial value problem for the equation of motion of viscous and heat-conductive gases" to appear in Journal of Mathematics, Kyoto University, 1979.

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| 1. REPORT NUMBER<br>1991                                                                                                                                                                                                                                                                                                                                                                                                                         | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER                                                                        |
| 4. TITLE (and Subtitle)<br>The Initial Value Problem for the Equations of Motion of Compressible Viscous and Heat-conductive Fluids                                                                                                                                                                                                                                                                                                              |                       | 5. TYPE OF REPORT & PERIOD COVERED<br>Summary Report - no specific reporting period                  |
| 7. AUTHOR(s)<br>Akitaka Matsumura and Takaaki Nishida                                                                                                                                                                                                                                                                                                                                                                                            |                       | 6. PERFORMING ORG. REPORT NUMBER                                                                     |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>Mathematics Research Center, University of Wisconsin<br>610 Walnut Street<br>Madison, Wisconsin 53706                                                                                                                                                                                                                                                                                             |                       | 8. CONTRACT OR GRANT NUMBER(s)<br>DAAG29-75-C-0024                                                   |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>U. S. Army Research Office<br>P. O. Box 12211<br>Research Triangle Park, North Carolina 27709                                                                                                                                                                                                                                                                                                         |                       | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS<br>Work Unit Number 1 - Applied Analysis |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)<br>Technical summary rept.                                                                                                                                                                                                                                                                                                                                           |                       | 12. REPORT DATE<br>September 1979                                                                    |
| 16. DISTRIBUTION STATEMENT (of this Report)<br>Approved for public release; distribution unlimited.<br>MRC-TSR-1991                                                                                                                                                                                                                                                                                                                              |                       | 13. NUMBER OF PAGES<br>10                                                                            |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)                                                                                                                                                                                                                                                                                                                                                       |                       | 15. SECURITY CLASS. (of this report)<br>UNCLASSIFIED                                                 |
| 18. SUPPLEMENTARY NOTES                                                                                                                                                                                                                                                                                                                                                                                                                          |                       | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE                                                           |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number)<br>Initial value problem, Equation of motion, Compressible, viscous and heat-conductive fluids, Global solutions in time                                                                                                                                                                                                                                      |                       |                                                                                                      |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>The initial value problem associated with the equations of motion for isotropic Newtonian fluids is investigated. The fluids are compressible, viscous and heat-conductive. It is proved that there exists a unique global solution in time, for the small initial data, and the solution has the decay rate of $(1+t)^{-3/4}$ as $t \rightarrow +\infty$ . |                       |                                                                                                      |