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ON CORRELATION PARAMETERS FOR  
SOME BINARY SEQUENCES OF LENGTHS  
31 AND 63 FOR SPREAD-SPECTRUM  
MULTIPLE-ACCESS COMMUNICATIONS

DAVID WILLIAM HAGUMA GAHUTU

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OF LENGTHS 31 and 63 FOR SPREAD-SPECTRUM MULTIPLE-  
ACCESS COMMUNICATIONS

by

David William Haguma Gahutu

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OF LENGTHS 31 AND 63 FOR SPREAD-SPECTRUM MULTIPLE-ACCESS COMMUNICATIONS

by

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THESIS

Submitted in partial fulfillment of the requirements  
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Thesis Adviser: Professor M. B. Pursley

Urbana, Illinois

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Department of Electrical Engineering  
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The correlation parameters of some binary sequences of lengths 31 and 63 for spread-spectrum multiple-access (SSMA) communication are evaluated. The peak correlation parameters are the key parameters needed to determine the worst interference at any of the correlation receivers in a SSMA communications system due to the presence of other users and the multipath signals. These peak correlation parameters are readily obtained from the correlation data presented in this thesis. The performances of some new and old candidate sieves for optimal binary sequence phases are compared. When the number of binary sequences required exceeds the number in one set of a given family, one option is to use two or more sets of sequences from the family. This motivated the evaluation of the peak crosscorrelation between two different sets of Gold sequences of length 31. Finally the mean-square partial correlation parameters of some binary periodic sequences are computed.

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## CHAPTER 1

## INTRODUCTION

The key correlation parameters for binary signature sequences in an asynchronous direct-sequence spread-spectrum multiple-access (SSMA) communication system were identified in [1]. In the model there are  $K$  transmitters which are asynchronous in both time and phase. There is one correlation receiver for each transmitter. The  $K$  asynchronous signals share the same RF bandwidth.

The  $k$ -th user's data signal  $b_k(t)$  is a sequence of unit amplitude, positive and negative rectangular pulses of duration  $T_d$

$$b_k(t) = \sum_{l=-\infty}^{\infty} b_{k,l} p_{T_d}(t - lT_d)$$

where  $b_{k,l} \in \{+1, -1\}$  and  $p_{T_d}(\tau) = 1$  for  $0 \leq \tau < T_d$  and  $p_{T_d}(\tau) = 0$ , otherwise. The  $k$ -th user is assigned a code waveform  $a_k(t)$  that consists of a periodic sequence of unit amplitude, positive and negative rectangular pulses of duration  $T_c$

$$a_k(t) = \sum_{j=-\infty}^{\infty} a_j^{(k)} p_{T_c}(t - jT_c)$$

The sequence  $a_j^{(k)}$  has period  $N = T_d/T_c$  so that there is one code period  $a_0^{(k)}, a_1^{(k)}, \dots, a_{N-1}^{(k)}$  per data symbol. The sequence  $a^{(k)}$  is sometimes called the  $k$ -th user's signature sequence.

In [1] analysis of the model for an average (SNR) performance led to parameters on which signature sequence analysis, selection and evaluation can be based. It is shown in [20] that one of the parameters, the average interference parameter  $r_{k,i}$  and hence the  $SNR_i$  depend solely on the

aperiodic autocorrelation function  $C_k$ ,  $1 \leq k \leq K$ , for the  $k$ -th sequence.

$C_k$  is defined by

$$C_k(\ell) = \begin{cases} \sum_{j=0}^{N-1-\ell} a_j^{(k)} a_{j+\ell}^{(k)}, & 0 \leq \ell \leq N-1 \\ \sum_{j=0}^{N-1+\ell} a_{j-\ell}^{(k)} a_j^{(k)}, & 1-N \leq \ell < 0 \\ 0, & |\ell| \geq N. \end{cases}$$

In particular, it is shown in [20] that

$$r_{k,i} = \sum_{\ell=1-N}^{N-1} C_k(\ell) [2C_i(\ell) + C_i(\ell+1)] = 2N^2 + 4 \sum_{\ell=1}^{N-1} C_k(\ell) C_i(\ell) + \sum_{\ell=1-N}^{N-1} C_k(\ell) C_i(\ell+1).$$

Notice that this can also be written as

$$r_{k,i} = 2N^2 + 4 \sum_{\ell=1}^{N-1} C_k(\ell) C_i(\ell) + \sum_{\ell=0}^{N-2} [C_k(\ell) C_i(\ell+1) + C_k(\ell+1) C_i(\ell)].$$

The parameters for worst-case performance of the SSMA communication model depend on the peak correlation parameters which, as shown in [1], can be expressed in terms of the (discrete) aperiodic crosscorrelation function  $C_{k,i}$  for a sequence pair  $a^{(k)}$  and  $a^{(i)}$ .  $C_{k,i}$  is defined by

$$C_{k,i}(\ell) = \begin{cases} \sum_{j=0}^{N-1-\ell} a_j^{(k)} a_{j+\ell}^{(i)}, & 0 \leq \ell \leq N-1 \\ \sum_{j=0}^{N-1+\ell} a_{j-\ell}^{(k)} a_j^{(i)}, & 1-N \leq \ell < 0 \\ 0, & |\ell| \geq N. \end{cases}$$

The peak periodic correlation parameters introduced in [1] are defined by

$$\begin{aligned}\theta(k,i) &= \max \{ |\theta_{k,i}(\ell)| : 0 \leq \ell \leq N-1 \} \\ \theta_c &= \max \{ \theta(k,i) : 1 \leq k < i \leq K \}\end{aligned}$$

where  $\theta_{k,i}(\ell) = C_{k,i}(\ell) + C_{k,i}(\ell-N)$ ,  $0 \leq \ell \leq N-1$ . The periodic or even crosscorrelation function  $\theta_{k,i}(\ell)$  is also given by

$$\theta_{k,i}(\ell) = \sum_{j=0}^{N-1} a_j^{(k)} a_{j+\ell}^{(i)} .$$

The odd crosscorrelation introduced by Massey and Uhran [14] is given by  $\hat{\theta}_{k,i}(\ell) = C_{k,i}(\ell) - C_{k,i}(\ell-N)$ ,  $0 \leq \ell \leq N-1$ . The peak odd correlation parameters are given by

$$\begin{aligned}\hat{\theta}(k,i) &= \max \{ |\hat{\theta}_{k,i}(\ell)| : 0 \leq \ell \leq N-1 \} \\ \hat{\theta}_c &= \max \{ \hat{\theta}(k,i) : 1 \leq k < i \leq K \} .\end{aligned}$$

When the two sequences in the above expressions are identical and  $\ell = 0$  is excluded we get the corresponding autocorrelation parameters. Thus the periodic or even autocorrelation function  $\theta_k(\ell)$  is defined by

$$\theta_k(\ell) = \sum_{j=0}^{N-1} a_j^{(k)} a_{j+\ell}^{(k)}$$

The even autocorrelation function is also given by

$$\theta_k(\ell) = C_k(\ell) + C_k(\ell-N)$$

where  $C_k(\ell) = C_{k,k}(\ell)$ . The odd autocorrelation function

$$\hat{\theta}_k(\ell) = C_k(\ell) - C_k(\ell-N).$$

The corresponding peak parameters are as follows.

$$\theta(k) = \max [|\theta_k(\ell)|: 1 \leq \ell \leq N-1]$$

$$\theta_a = \max [\theta(k): 1 \leq k \leq K]$$

$$\hat{\theta}(k) = \max [|\hat{\theta}_k(\ell)|: 1 \leq \ell \leq N-1]$$

$$\hat{\theta}_a = \max [\hat{\theta}(k): 1 \leq k \leq K]$$

Another code parameter of interest is the sidelobe energy of the k-th

sequence, a mean-square autocorrelation parameter  $S(k) = \sum_{\ell=1}^{N-1} C_k^2(\ell)$ .

$$\text{If } \lambda_{k,i} = \max [\theta(k,i), \hat{\theta}(k,i)],$$

$$\Lambda_i = \sum_{\substack{k \neq i \\ k=1}}^K \lambda_{k,i} \text{ and } \Lambda = \max [\Lambda_i: 1 \leq i \leq K] \text{ then for worst-case}$$

performance, Pursley [1] showed that the maximum error probability for the i-th user is

$$P_{\max}(i) = 1 - \Phi([1 - (\Lambda_i/N)]\sqrt{2\mathcal{E}/N_0})$$

where  $N_0/2$  is the two-sided power spectral density of the channel noise process,  $\Phi(\cdot)$  is the standard Gaussian cumulative distribution function and  $\mathcal{E} = PT_d$  is the energy per data bit. If  $P_{\max} = \max [P_{\max}(i): 1 \leq i \leq K]$

$$\text{then } P_{\max} = 1 - \Phi([1 - (\Lambda/N)]\sqrt{2\mathcal{E}/N_0}).$$

On the other hand, the average (power) signal-to-noise ratio (SNR) at the i-th user receiver output is

$$SNR_i = [(6N^3)^{-1} \sum_{\substack{k \neq i \\ k=1}}^K r_{k,i} + \frac{N_0}{2\mathcal{E}}]^{-\frac{1}{2}}.$$

## CHAPTER 2

## BINARY SEQUENCES

In practical communication systems such as the binary direct-sequence SSMA systems, use is made of periodic sequences  $a_l^{(k)}$  of elements of  $[-1,1]$  which are derived from binary sequences  $\alpha_l^{(k)}$  of elements of  $[0,1]$  by the relation

$$a_l^{(k)} = (-1)^{\alpha_l^{(k)}}, \quad 0 \leq l \leq N-1, \quad 1 \leq k \leq K$$

where  $N$  is the length (equal to or a multiple of the period) of the periodic binary sequence and  $K$  is the number of binary sequences. Hitherto, the maximal-length linear shift-register sequences ( $m$ -sequences for short) have played a key role, indirectly as a convenient starting point for other sequences or directly for some applications. The  $m$ -sequences are a natural starting point which leads to such sequences as Gold sequences, Kasami sequences, weakly-Barker sequences and others considered in this thesis.

### 2.1 $m$ -Sequences and Primitive Polynomials over $GF(2)$

In this section introductory notation of and terminology for  $m$ -sequences is reviewed. The conventions used here closely parallel those in references [2] and [17].  $m$ -Sequences are a special class of periodic linear binary recursive sequences. In general, a linear binary recursive sequence  $\alpha_0, \alpha_1, \alpha_2, \dots$  is a sequence  $\{\alpha_i\}_{i=1}^{\infty}$  which satisfies a recursive relation of the form

$$\alpha_{n+i} = \sum_{j=1}^n c_j \alpha_{n+i-j}, \quad i = 0, 1, 2, \dots$$

where each  $\alpha_i, c_j$  belong to  $GF(2)$  and  $\alpha_0, \alpha_1, \dots, \alpha_{n-1}$  are preassigned.

$\alpha_0, \alpha_1, \dots, \alpha_{n-1}$  are often called the initial state or loading and may be specified in octal notation. An octal representation of the binary vector  $(\alpha_0, \alpha_1, \dots, \alpha_{n-1})$  is the octal vector  $(b_0, b_1, \dots, b_{L-1})$  of  $L$  elements  $b_i \in \{0, 1, \dots, 7\}$ , where  $L$  is the smallest integer greater than or equal to  $n/3$ , as shown here:

$$(\alpha_0 \alpha_1 \dots \alpha_{11}) = 111010011101 \equiv 1647 = (b_0 b_1 b_2 b_3).$$

A linear binary recursive sequence can be generated by an  $n$ -stage linear feedback shift-register (LFSR). The  $n$ -stage LFSR is represented by the polynomial (over  $GF(2)$ ) of degree  $n$ :

$$h(x) = h_0 x^n + h_1 x^{n-1} + \dots + h_{n-1} x + h_n$$

where  $h_0 = 1 = h_n$  and for  $1 \leq i \leq n-1$ ,  $h_i = 1$  if there is a feedback tap connection to the output of the  $i$ -th shift-register stage and  $h_i = 0$  otherwise. The polynomial  $h(x)$  is said to generate the sequence  $\{\alpha_i\}$ . As an example [2], the shift-register represented by  $h(x) = x^4 + x + 1$  is shown in Fig. 1. The initial loading used is  $(\alpha_0, \alpha_1, \alpha_2, \alpha_3)$ . The generating polynomial is also represented in octal form. The binary vector of its coefficients in descending degree order  $h = (h_0, h_1, \dots, h_n)$  becomes the octal vector  $H = (H_0, H_1, \dots, H_{L-1})$  where  $L$  is the smallest integer greater than or equal to  $(n+1)/3$ . As an example  $h(x) = x^4 + x + 1$  gives the binary vector 1 0 0 1 1 which is 23 in octal.

The sequences generated by linear feedback shift-registers are periodic. The period  $N$  of a sequence  $\{\alpha_j\}$  is the smallest positive integer  $N$  such that  $\alpha_{j+N} = \alpha_j$  for all  $j$ . With reference to the generating polynomial  $h(x)$ , the period  $N$  is the smallest positive integer

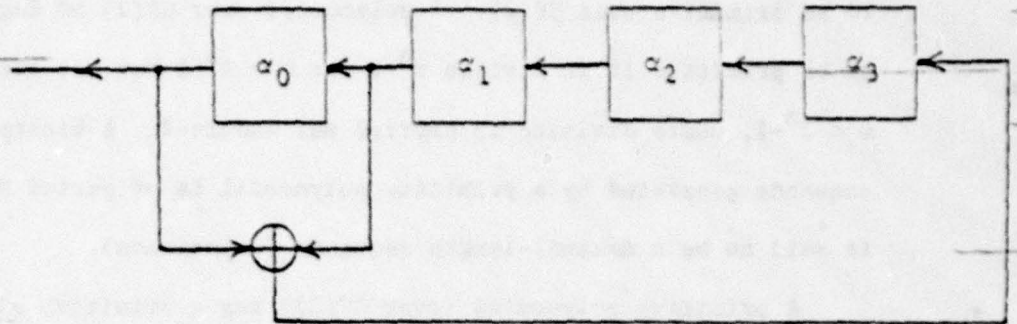


Figure 1. Shift register for  $h(x) = X^4 + X + 1$  example  $h(x) = X^4 + X + 1$  gives the binary vector 10011 which is 23 in octal.

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such that  $h(x)$  divides  $x^N - 1$ . When  $N = 2^n - 1$ , the polynomial  $h(x)$  is said to be primitive over  $GF(2)$ . A polynomial over  $GF(2)$  of degree  $n$  is said to be primitive if it divides  $x^m - 1$  for  $m = 2^n - 1$  but not for any  $m < 2^n - 1$ , where division is carried out modulo-2. A binary periodic sequence generated by a primitive polynomial is of period  $N = 2^n - 1$  and is said to be a maximal-length sequence (m-sequence).

A primitive polynomial (over  $GF(2)$ ) has a primitive element as a zero. A primitive element  $\beta$  of  $GF(2^n)$  (any prime number  $p$  could replace 2 here) is defined as an element of  $GF(2^n)$  which has order  $N = 2^n - 1$ , i.e.,  $\beta^N = 1$  but  $\beta^l \neq 1$  for all  $l < 2^n - 1$ . There are  $\varphi(2^n - 1)$  primitive elements in the finite extended field  $GF(2^n)$  where  $\varphi(\cdot)$  is the Euler totient ( $\varphi$ -) function. Correspondingly there are  $K = \frac{1}{n} \varphi(2^n - 1)$  distinct primitive polynomials of degree  $n$ . Note that  $K$  is even for  $n > 2$  and for  $n = 2$ ,  $K = 1$ , the polynomial of degree 2 over  $GF(2)$  is self-reciprocal. There are  $K$  m-sequences of period  $N = 2^n - 1$ .

The idea of a minimal polynomial is also useful here. The minimal polynomial of an element  $\alpha$  of  $GF(2^n)$  is the polynomial of lowest degree,  $h(x)$ , with coefficients in  $GF(2)$  such that  $h(\alpha) = 0$ .  $h(x)$  is irreducible and it divides  $x^{2^n} - x$  if the degree of  $h(x) \leq n$ . If  $h_k(x)$  denotes the minimal polynomial of  $\alpha^k$  then  $h_k(x) = \prod_{i \in C_s} (x - \alpha^i)$  where  $C_s$  is a coset of integers modulo  $2^n - 1$  with smallest element  $s$ .  $h_k(x)$  is a primitive polynomial of degree  $n$  iff it divides  $x^m - 1$  for  $m = 2^n - 1$  but for no  $m < 2^n - 1$ .

The reciprocal polynomial  $h^*(x)$  of a polynomial  $h(x)$  of degree  $n$  is defined as  $x^n h(x^{-1})$ . The degree of  $h^*(x)$  is the same as that of  $h(x)$ . If  $h(x)$  is irreducible (primitive) then so is  $h^*(x)$ .

## 2.2 Cyclotomic Cosets Modulo N

For  $N = 2^n - 1$ , the  $\varphi(N)$  integers form a multiplicative group  $G$  modulo  $N$  and the set  $H = \{1, 2, 2^2, \dots, 2^{n-1}\}$  forms a multiplicative subgroup of  $G$ . The proper cyclotomic cosets  $C_s$  of  $H$  are formed by multiplying every element of  $H$  by an element of  $G$ . There are exactly  $\frac{1}{n} \varphi(2^n - 1)$  proper cyclotomic cosets but  $\frac{1}{n} \sum_{d/n} \varphi(d) 2^{n/d} - 1$  cyclotomic cosets modulo  $2^n - 1$  in all. The cyclotomic cosets that are not proper are called improper. The latter are formed by elements  $g$  for which  $\gcd(g, 2^n - 1) \neq 1$ ,  $1 < g < 2^n - 1$ . The smallest member  $s$  of a cyclotomic coset is called the cyclotomic coset leader and the coset is denoted  $C_s$ . Table 1 shows the cyclotomic cosets modulo  $N = 15, 31$  and  $63$ .

## 2.3 Maximal Connected Sets and Preferred Pairs of Primitive Polynomials

For certain communications applications  $m$ -sequences with good crosscorrelation properties are required. Gold and Kopitzke experimentally determined sets of  $m$ -sequences called maximal connected sets. These are the largest possible subsets of  $m$ -sequences of period  $N = 2^n - 1$  such that any pair of sequences in the same set has only the three crosscorrelation values of  $-1$ ,  $-1 - 2^{\lfloor (n+2)/2 \rfloor}$  and  $-1 + 2^{\lfloor (n+2)/2 \rfloor}$  (where  $\lfloor x \rfloor$  denotes the largest integer less or equal to  $x$ ).

Table 1

Cyclotomic cosets of integers mod  $N = 15, 31$  and  $63$ .

$N = 15$	0	
	1 2 4 8	7 14 13 11
	3 6 12 9	
	5 10	
$N = 31$	0	
	1 2 4 8 16	15 30 29 27 23
	3 6 12 24 17	7 14 28 25 19
	5 10 20 9 18	11 22 13 26 21
$N = 63$	0	
	1 2 4 8 16 32	31 62 61 59 55 47
	3 6 12 24 48 33	15 30 60 57 51 39
	5 10 20 40 17 34	23 46 29 58 53 43
	7 14 28 56 49 35	
	9 18 36	27 54 45
	11 22 44 25 50 37	13 26 52 41 19 38
	21 42	

Let  $M_n$  be the number of sequences in the maximal connected set of  $m$ -sequences of period  $N = 2^n - 1$ . Gold and Kopitzke [19] determined the values of  $M_n$  for  $n \leq 13$  as follows

$$M_1 = M_2 = M_4 = M_8 = M_{12} = 0$$

$$M_3 = M_6 = M_9 = 2$$

$$M_5 = M_{10} = 3$$

$$M_7 = 6$$

$$M_{11} = M_{13} = 4$$

Clearly the known maximal connected sets of  $m$ -sequences are too small for most practical applications. As a result, effort has been directed towards obtaining alternative sequence sets. Gold [5,6] obtained particularly attractive sequence sets. In [7] the small sets and large sets of Kasami sequences are also described. The Gold sequence sets and the maximal connected sets of  $m$ -sequences are based on the notion of preferred pairs of primitive polynomials.

A pair of primitive polynomials, each of degree  $n$ , whose corresponding  $m$ -sequences have the three-level crosscorrelation function with values  $-1$ ,  $-1-2^{\lfloor (n+2)/2 \rfloor}$  and  $-1+2^{\lfloor (n+2)/2 \rfloor}$  is called a preferred pair of primitive polynomials. For values of  $n$  and integer parameter  $k$  for which

$$(a) \quad n \text{ is odd and } l = \gcd(k, n) = 1 \text{ or}$$

$$(b) \quad n \text{ is even with } n \equiv 2 \pmod{4} \text{ and } l = \gcd(k, n) = 2$$

the following theorem [18] gives preferred pairs of polynomials of degree  $n$  (see [4] for further discussion).

Theorem: Let  $\alpha$ , an element of  $GF(2^n)$ , be a root of a primitive polynomial  $h_1(x)$  of degree  $n$ . For  $0 < k < n$  let  $t = 2^k + 1$  and  $h_t(x)$  be the minimal polynomial of  $\alpha^t$ . Let  $e = \gcd(n, k)$ , then if  $\frac{n}{e}$  is odd, the crosscorrelation function of the sequences generated by the pair  $\{h_1(x), h_t(x)\}$  takes on only three values with the following frequencies:

-1 occurs for  $2^n - 2^{n-e} - 1$  different shifts

$-1 - 2^{\lfloor (n+2)/2 \rfloor}$  occurs for  $2^{n-e-1} - 2^{(n-e-2)/2}$  different shifts

$-1 + 2^{\lfloor (n+2)/2 \rfloor}$  occurs for  $2^{n-e-1} + 2^{(n-e-2)/2}$  different shifts.

In [4] Pursley showed that for  $n = 5$ , four of the maximal connected sets are  $\{h_1, h_3, h_5\}$ ,  $\{h_1, h_3, h_{11}\}$ ,  $\{h_1, h_5, h_7\}$  and  $\{h_1, h_7, h_{11}\}$ . This information can be succinctly displayed on a double polygon (here a double equilateral triangle) for all  $n > 2$  where the vertices represent the primitive polynomials. It turns out that the three vertices of the two equilateral triangles and any isosceles triangles in Fig. 2 correspond to a maximal connected set. There are eight in all. This figure is also helpful in the treatment of pairs of Gold sequence sets which is given in Section 4.3.

For  $n = 5$ ,  $N = p = 2^n - 1 = 31$ , the primitive polynomials are:

$$h_1(x) = 1 + x^2 + x^5$$

$$h_1^*(x) = h_{15}(x) = 1 + x^3 + x^5$$

$$h_3(x) = 1 + x^2 + x^3 + x^4 + x^5$$

$$h_3^*(x) = h_7(x) = 1 + x + x^2 + x^3 + x^5$$

$$h_5(x) = 1 + x + x^2 + x^4 + x^5$$

$$h_5^*(x) = h_{11}(x) = 1 + x + x^3 + x^4 + x^5$$

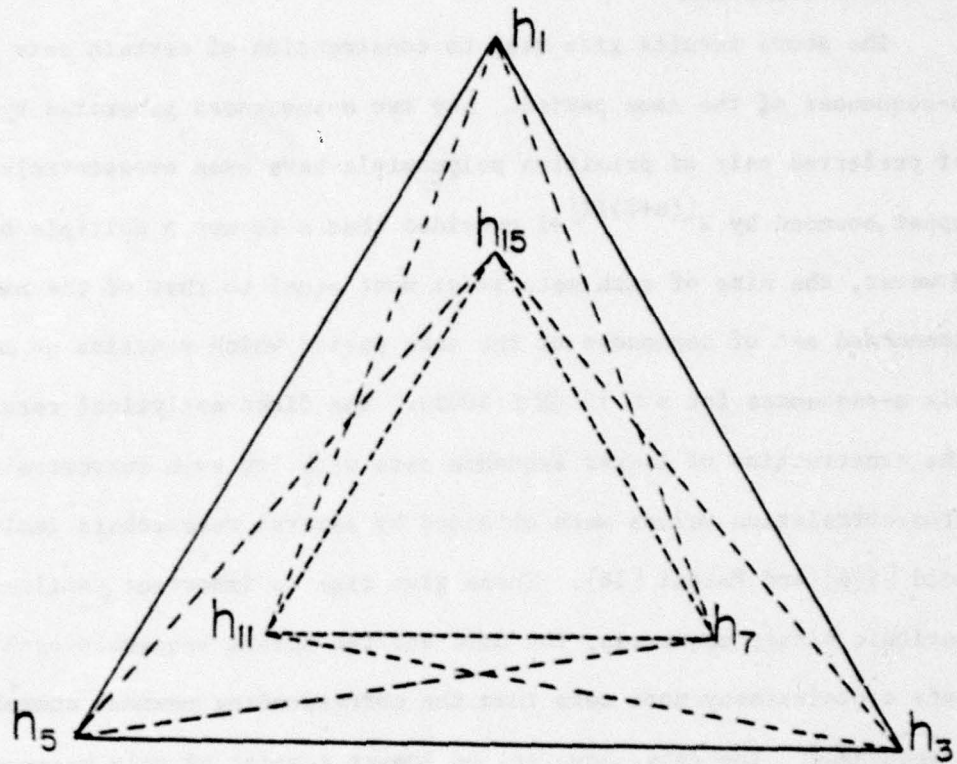


Figure 2. Primitive polynomial double triangle,  $n = 5$ .

#### 2.4 Gold Sequences

The above results give rise to construction of certain sets of  $m$ -sequences of the same period. Any two  $m$ -sequences generated by a pair of preferred pair of primitive polynomials have even crosscorrelation upper bounded by  $2^{\lfloor (n+2)/2 \rfloor} + 1$  provided that  $n$  is not a multiple of 4. However, the size of such sets is at most equal to that of the maximal connected set of sequences of the same period which contains no more than six  $m$ -sequences for  $n \leq 12$  ( $N \leq 4095$ ). The first analytical results on the construction of larger sequence sets with low even autocorrelation and crosscorrelation values were obtained by several researchers including Gold [5,6] and Kasami [16]. These give rise to important families of periodic binary sequences, the Gold and the Kasami sequences each of whose sets contains many more sets than the corresponding maximal connected sets of  $m$ -sequences. The sets, however, no longer consist of only  $m$ -sequences. This means that the excellent periodic autocorrelation properties of the  $m$ -sequences are sacrificed in exchange for larger cardinality of the sequence sets.

Gold sequences have a terse characterization. Suppose  $h_1(x)$  and  $h_2(x)$  are a preferred pair of primitive polynomials each of degree  $n$ , where  $n$  is not a multiple of 4. Then the linear shift-register with  $h_1(x)h_2(x)$  as its characteristic polynomial generates a set of  $2^n + 1$  different sequences of common period  $N = 2^n - 1$ . The even crosscorrelation function of any pair of sequences and the out-of-phase even autocorrelation function of any sequence (except for the two  $m$ -sequences) take on only the three values:  $-1$ ,  $-1 - 2^{\lfloor (n+2)/2 \rfloor}$  and  $-1 + 2^{\lfloor (n+2)/2 \rfloor}$ .

The  $2^n+1$  sequences in a Gold sequence set are obtained by initially loading the generating  $2n$ -stage linear shift-register with  $2^n+1$  different  $2n$ -tuples. As stated in [4] the  $2^n+1$  initial loadings are inferred from knowledge of the first  $2n$  bits of either  $m$ -sequence and an entire period of the other  $m$ -sequence. However, a more practical way to generate the set of Gold sequences is to modulo-2 add the  $2^n-1$  different shifts of either  $m$ -sequence to a fixed shift of the other  $m$ -sequence. Thus if  $a = (a_0, a_1, \dots, a_{N-1})$  where  $N = 2^n-1$ , define  $T^j a = (a_j, a_{j+1}, \dots, a_{N-1}, a_0, a_1, \dots, a_{j-1})$  as the  $j$ -th cyclic shift of sequence  $a$   $j$  places to the left for  $0 \leq j \leq N-1$ . Then the Gold sequence set comprises two  $m$ -sequences  $u$  and  $v$  generated by  $h_1(x)$  and  $h_2(x)$  respectively and the  $N = 2^n-1$  composite sequences  $w = u \oplus T^j v$ ,  $0 \leq j \leq 2^n-2$ . The symbol  $\oplus$  denotes modulo-2 addition.

From the theory of cyclic codes it follows that a Gold sequence set (and a Kasami sequence set) when augmented by the all zeros codeword forms a linear cyclic code. Each Gold sequence gives rise to  $N-1 = 2^n-2$  others of the form  $u^{(j)} = T^j u$ . Note that two sequences  $u$  and  $v$  are said to be cyclically equivalent if  $v = T^i u$  for some  $i$  in  $(0, N-1)$  and cyclically distinct if  $v \neq T^i u$  for any  $i$  in  $[0, N-1]$ . Thus the Gold sequence set consists of all the nonzero cyclically distinct codewords of the corresponding linear cyclic code.

### 2.5 Kasami Sequences

Another class of binary sequences with good periodic (or even) autocorrelation and crosscorrelation values is the Kasami sequences [7]. These sequences have lengths  $N = 2^n-1$  where  $n$  is even. They provide sequences of those lengths not given by Gold sequences (i.e., for

$n \equiv 0 \pmod{4}$ ). For  $n$  even, there are a small set of  $2^{n/2}$  sequences called the small set of Kasami sequences and its super set of  $2^{n/2}(2^n+1)$  sequences called the large set of Kasami sequences.

For  $n$  even,  $\alpha$  a primitive root of  $h_1(x)$  let  $\alpha^s$  be a root of the irreducible polynomial  $h_s(x)$  over  $GF(2)$  where  $s = 2^{n/2}+1$ . The linear shift-register with the product  $h_1(x)h_s(x)$  over  $GF(2)$  as its characteristic polynomial generates a set (the small set of Kasami sequences) of  $2^{n/2}$  sequences of period  $p = 2^n-1$ . The out-of-phase even autocorrelation function of any sequence and the even crosscorrelation function of any pair of sequences in the set take on the values  $-1$ ,  $-s$  and  $s-2$ .

If  $t = 2^{(n+2)/2}+1$ , then for  $h_t(x)$  the irreducible polynomial with root  $\alpha^t$  and for  $s$  as above the linear shift-register with  $h_1(x)h_s(x)h_t(x)$  as characteristic polynomial generates  $2^{n/2}(2^n+1)$  sequences called the large set of Kasami sequences. The even crosscorrelation and out-of-phase even autocorrelation functions of these sequences assume the values  $-1$ ,  $-s$ ,  $-t$ ,  $s-2$  and  $t-2$ . The Gold and small set of Kasami sequences are subsets of the large set of Kasami sequences.

For completeness, the case for  $n \equiv 0 \pmod{4}$  is considered. Let  $n$  be a multiple of 4 and  $h_1(x)$  and  $h_t(x)$  be as defined above. Then the linear shift-register with  $h_1(x)h_t(x)$  as characteristic polynomial generates  $2^n$  sequences whose even crosscorrelation and out-of-phase even autocorrelation functions take on values  $-1$ ,  $-s$ ,  $-t$ ,  $(s-2)$  and  $(t-2)$ .

### 2.5.1 Generation of Kasami Sequences

Consider first the small set of Kasami sequences. Fix  $n$  (even). Then the number of sequences,  $K = 2^{n/2}$ , the sequence length,  $N = 2^n - 1$ ,  $s = 2^{n/2} + 1$  and  $p_s = 2^{n/2} - 1$ . The period of the sequence,  $u$ , generated by  $h_1(x)$  is  $N$  while that of the sequence,  $v$ , generated by  $h_s(x)$  is  $p_s$ , a factor of  $N$ . The latter sequence is then repeated  $N/p_s$  times to get a derived sequence of length  $N$ . To get the  $K$  sequences the following derived sequences are used:

$$u \text{ and } u \oplus T^l v, \quad 0 \leq l \leq p_s - 1.$$

When the initial loadings are known, the product polynomial  $h_1(x)h_s(x)$  is used instead of the above method.

The large set of Kasami sequences are generated in a similar way. For fixed  $n$  (even), the number of sequences is  $K = 2^{n/2}(2^n + 1)$ .  $N = 2^n - 1$ ,  $s = 2^{n/2} + 1$  and  $t = 2^{(n+2)/2} + 1$ .

Let  $h_1(x)$  generate sequence  $u$ ,  $h_t(x)$  generate sequence  $v$  and  $h_s(x)$  generate sequence  $w$  and let all the three periodic sequences have the same length  $N = 2^n - 1$ . Then the large Kasami sequence set consists of

$u$

$w$

$$u + T^i w, \quad 0 \leq i \leq 2^n - 2$$

$$u + T^j v, \quad 0 \leq j \leq 2^{n/2} - 2$$

$$w + T^k v, \quad 0 \leq k \leq 2^{n/2} - 2$$

$$u + T^l w + T^m v, \quad 0 \leq l \leq 2^n - 2; \quad 0 \leq m \leq 2^{n/2} - 2.$$

As before this method is used when the initial loadings cannot easily be obtained. For  $n \neq 0 \pmod 4$ , the above large set of Kasami sequences is equivalent to adding the  $2^{n/2}$  codewords (after repeating each codeword  $2^{n/2}+2$  times to get its length to  $N$ )  $0, v_1, v_2, \dots, v_{p_s-1}$ ,  $p_s = 2^{n/2}$  in the cyclic code with check polynomial  $h_s(x)$  to each of the  $2^n+1$  Gold sequences generated by the characteristic polynomial  $h_1(x)h_t(x)$ .

### 2.5.2 Characteristic m-Sequences

In generating the Gold and Kasami sequences at least one of the polynomials used is a primitive polynomial. For the convenience of checking by comparison, the m-sequences are generated in their characteristic form. A sequence  $u = u_0 u_1 u_2 \dots u_{N-1} \dots$  is said to be in its characteristic form (or natural orientation) if  $u_i = u_{2i} \forall i$ . Willet [9] tabulated the characteristic form of the primitive polynomials of degree  $n$  for  $2 \leq n \leq 168$  which are listed in [15]. Willet used the following algorithm to compute the characteristic m-sequence corresponding to a given primitive polynomial. By truncating the computation the corresponding initial loading is obtained. A summary of the algorithm is:

Let  $n$  be the degree of the primitive polynomial. Find  $m = 2n-2$ .

Compare the primitive polynomial with  $h(x) = x^n + a_1 x^{n-1} + \dots + 1$

and thereby determine  $a_1, a_2, \dots, a_n \in GF(2)$ .

Let the initial loadings (unknowns, to be solved for here) be

$u_0, u_1, \dots, u_{n-1}$ .

From the linear recursion relation associated with  $h(x)$

$$u_{i+k} = a_1 u_{i+n-1} + \dots + a_n u_i \quad i = 0, 1, 2, \dots$$

find  $u_n, u_{n+1}, \dots, u_{2n-2}$  in terms of (a linear combination of the) unknowns  $a_1, a_2, \dots, a_n$ .

Finally solve the system of equations  $u_i = u_{2i}, i = 0, 1, \dots, n-1$

for the unknowns. The resulting nonzero solution is the initial loading for the characteristic form of the m-sequence of  $h(x)$ . Refer to Table 2.

### 2.6 Weakly-Barker Sequences

Barker sequences of +1 and -1 have extremely good autocorrelation properties. In particular, they have  $|C_k(l)| \leq 1$  and  $\max_l \{\max\{\hat{\theta}(l), \hat{\theta}^*(l)\}\} = 2$  for any possible length  $N$ . Also

$$C_k(l) = \begin{cases} 0 & , l \text{ odd} \\ (-1)^{(N-l)/2} & , l \text{ even} \end{cases}$$

$$\sum_{l=1-N}^{N-1} C_{k,i}(l) C_{k,i}(l+1) = 0,$$

$$r_{k,i} = 2 \sum_{l=1-N}^{N-1} C_{k,i}^2(l) = N^2 + N - 1,$$

and 
$$C_{k,i}(l) = (-1)^l C_{k,i}(-l) .$$

However, the only known Barker sequences have lengths  $N = 1, 2, 3, 4, 5, 7, 11$  and 13. Each of the Barker sequences  $u = u_0, u_1, \dots, u_{N-1}$  gives rise to three others under the transformations

Table 2

Characteristic m-sequences of length  $N = 7, 15, 31$  and  $63$ .

	Primitive Polynomial	Loading	Primitive Polynomial	Loading
$N = 7$	013	1647	015	1646
$N = 15$	023	0115	031	0753
$N = 31$	045	1131	051	1025
	075	1756	057	1114
	067	1642	073	1663
$N = 63$	103	0020	141	0772
	147	0772	163	0642
	155	0662	133	0110

$$a_i = (-1)^i u_i$$

$$b_i = (-1)^{i+1} u_i$$

$$c_i = -u_i$$

Thus not only are the lengths of the Barker sequences too short, but for any such lengths they are far too few in number for almost all applications.

Seguin [10] constructed what he called weakly-Barker (WB) sequences each of which has either good or bad even and odd autocorrelation functions. In fact he showed that  $||\theta_k(l) - \hat{\theta}_k(l)|| \leq 2$ ,  $1 \leq l \leq N-1$ . This was the motivation for examination of the correlation properties of a set of 12 "best WB-sequences of length 31". The set has extremely bad crosscorrelation parameters but extremely good autocorrelation parameters. Their autocorrelation parameters are superior to those of the Gold sequences of the same length  $N = 31$ . Incidentally Seguin [10] also showed that there exist WB-sequences of length  $n$  for every  $n$ .

## CHAPTER 3

## ALGORITHMS FOR FINDING SUBOPTIMAL SEQUENCE PHASES

The maximal connected sets for  $m$ -sequences, the Gold sequences and the Kasami sequences guarantee only good even crosscorrelation and even autocorrelation values. Their odd correlation parameters, which are at least as important as the even correlation parameters for some applications, may be too large. Moreover, whereas most of the results on the even correlation functions have been derived using the theory of linear cyclic codes, odd correlation functions are not known to admit such analysis. Consequently, there are only scanty analytical results on the odd correlation parameters. We have evaluated complete correlation parameter tables for some phases (shifts) of Gold, Kasami, and  $m$ -sequence sets among others.

The only analytical method known for constructing a set of sequences with a known (designed) upper bound on the odd crosscorrelation function is due to Massey and Uffner [14]. They showed that for  $f(x)$  not divisible by  $1 + x$  if the sequences generated by  $h(x) = (1 + x) f(x)$  have maximum even crosscorrelation  $b$ , then the set of sequences generated by  $f(x)$  have correlation parameters  $\theta_a \leq b$  and  $\hat{\theta}_a \leq (L + b + 4)/2$  where  $L$  is the LCM of the periods of the sequences generated by  $f(x)$ .

### 3.1 Old and New Algorithms for Finding Optimal Sequence Phases

To obtain optimal sequence phases of a given sequence set, an exhaustive search would always suffice. However, the amount and time of computation is so prohibitive even for the shortest sequence lengths of practical interest that one must resort to suboptimal techniques. Some of the methods that have been used are presented in what follows.

### 3.1.1 Auto-Optimal Sieve

The following algorithm is credited to Massey and Uhran. Choose a family of  $K$  cyclically distinct shifts of sequences for which  $\theta(k) = \max\{|\theta_k(l)| : 1 \leq l \leq N-1, 1 \leq k \leq K\}$  is relatively small, for example, the Gold or Kasami sequences. This first step applies to all the sieves considered herein. Then choose a best phase of each of the  $K$  sequences in the sense of minimizing the maximum out-of-phase magnitude of its odd autocorrelation function and (when several shifts of the same sequence have the same such maximum) the number of occurrences of this maximum. Finally order the  $K$  auto-optimal sequences (those with smaller  $\hat{\theta}_1$  preceding those with larger values).

### 3.1.2 AO/LSE Sieve

This and the next sieves were introduced by Pursley and Roefs [2]. The AO/LSE sieve is similar to the previous sieve except that ties are resolved by choosing the (first) shift which has least sidelobe energy, LSE,

$$S(k) = \sum_{l=1}^{N-1} C_k^2(l) .$$

### 3.1.3 LSE/AO Sieve

Choose a shift of each of the  $K$  sequences which corresponds to the least sidelobe energy (LSE) and, in case of ties, select the (first) LSE-optimal shift which, further, is auto-optimal.

### 3.1.4 Mean-Square Odd Autocorrelation Sieve

This is the first of the sieves introduced here. For each of the  $K$  sequences that phase with the least value of

$$\sum_{l=1}^{\lfloor N/2 \rfloor} [\hat{\theta}(l)]^2$$

is chosen. Ties are resolved by selecting the (first) optimal shift which also has greatest number of occurrences of  $C_k(l) = 0$ ,  $0 \leq l \leq N-1$ .

### 3.1.5 A Terminal Autocorrelation Sieve

For a binary sequence  $u = u_0, u_1, \dots, u_{N-1}$ , the terminal sequence is defined as

$$t_k = \sum_{j=0}^k u_j u_{N-j-1}.$$

For each of the  $K$  sequences choose the (first) phase for which

$$T = \sum_{k=0}^{\lfloor N/2 \rfloor - 1} |t_k|^2$$

is smallest.

### 3.1.6 Partial Correlation Sieve

From the definition of the mean-squared partial correlation function [12] for a fixed window length  $n$  ( $\approx O(\log_2 N)$ )

$$N^{-2} \nu_{x,y}(n) = n + 2 \sum_{i=1}^{n-1} (n-i) \theta_x(i) \theta_y(i)$$

for binary sequences a sieve function

$$S(n) = \sum_{l=1}^{n-1} (n-l) (\theta_k(l))^2$$

is deduced. The first phase of each of the  $K$  sequences with least  $S(n)$  is selected.

### 3.1.7 Concave Exponential Sieves

In contrast to the LSE/AO sieve which is convex, it was felt that a concave sieve which explicitly emphasizes the zero values of  $C_k(l)$  would be better. The conjecture was arrived by consideration of the average interference parameter

$$r_{k,i} = 2N^2 + 4 \sum_{l=1}^{N-1} C_k(l)C_i(l) + \sum_{l=0}^{N-2} [C_k(l)C_i(l+1) + C_k(l+1)C_i(l)]$$

The first concave sieve used was  $S(l) = A \exp(-C_k^2(l)/2\sigma^2)$  where A is a constant and  $\sigma = \lceil N/3 \rceil$  or  $\sqrt{50}$ . This time it is that first shift of a sequence which maximizes  $\sum_{l=1}^{N-1} S(l)$  which is selected. Ties are resolved by choosing the best phase which, in addition has the greatest occurrences of  $C_k(l) = 0$ .

### 3.1.8 Concave Quadratic and Higher Order Sieves

Two concave quadratic sieves have been used. One is  $S(l) = -(C_k(l))^2$ .

For each sequence shift the sum

$$\sum_{l=0}^{N-1} S(l)$$

is evaluated and that shift with largest value of this sum is chosen.

When more than one such shift occur, the first shift with the greatest number of occurrences of  $C_k(l) = 0$  is chosen.

The second sieve uses a general quadratic form and, by including the linear term, incorporates the negativeness of the  $C_k(l)$ . Here  $S(l) = N(N+1) - C_k(l) - (C_k(l))^2$ . The rest of the algorithm is as in the preceding sieve. A cubic sieve  $S(l) = -|C_k(l)|^3$  and a quartic sieve  $S(l) = -(C_k(l))^4$  were also tried. Ties are resolved as in the concave quadratic sieve.

The quality of the sequence shifts obtained by the various sieves was measured by the self interference parameter  $r_{k,k}$ . Results obtained from application of the sieves on Gold sequence sets of LSE/AO sequences of length  $N = 31$  generated by the characteristic polynomials  $h_1 h_t = 3013$  and  $h_1 h_t = 3551$  are given in Tables 4 and 5 where the following notation is used:

- A: LSE/AO
- B:  $\sum_{l=1}^{15} (\hat{\theta}_k(l))^2$
- C:  $\sum_{k=0}^{14} \left| \sum_{j=0}^{k-1} u_j u_{N-j-1} \right|^2$
- D:  $\sum_{l=1}^4 (5-l) (\theta_k(l))^2$
- E:  $\exp\{- (C_k(l))^2 / 242\}$
- F:  $242 \exp\{- (C_k(l))^2 / 242\}$
- G:  $100 \exp\{- (C_k(l))^2 / 100\}$
- H:  $-(C_k(l))^2$
- I:  $992 - C_k(l) - (C_k(l))^2$
- J:  $-|C_k(l)|^3$
- K:  $-(C_k(l))^4$

The initial loadings of the LSE/AO Gold sequences generated by  $h_1 h_t = 3013$  and  $h_1 h_t = 3551$  are given in Table 3.

Table 3

Initial loadings of the LSE/AO Gold sequences generated by characteristic polynomial  $h_1 h_t$

	$h_1 h_t = 3551, N = 31$	$h_1 h_t = 3013$
1	1174	1476
2	1734	1416
3	1632	0364
4	0711	0713
5	1644	1366
6	1077	1266
7	1603	0705
8	1253	0271
9	0601	1105
10	1622	1610
11	0625	0204
12	0340	0051
13	1060	1020
14	0577	1273
15	0710	1150
16	1552	1110
17	1611	1732
18	1712	1227
19	1246	0420
20	1310	1003
21	1232	0216
22	0221	0067
23	1014	1243
24	0746	0640

Table 3  
(Continued)

25	1570	1664
26	0610	1547
27	0254	0650
28	0153	1475
29	1256	0061
30	0056	0647
31	1616	0647
32	1742	1103
33	1600	1312

Table 4

Selfinterference parameter of optimal sequences generated by various sieves,  
 $h_1 h_t = 3551$ .

	A	B	C	D	E	F	G	H	I	J	K
1	2182	2382	2182	2182	2182	2182	2182	2182	2182	2382	2382
2	2206	2206	2206	2206	2206	2206	2206	2206	2206	2206	2206
3	2590	2814	2590	3814	2590	2662	2590	2590	2590	2814	2814
4	3598	3726	3598	3942	3598	3942	3702	3598	3598	3598	3598
5	2982	2982	3150	2982	2982	2982	2982	2982	2982	3262	3262
6	3494	3494	3862	3646	3494	3646	3494	3494	3494	3614	3614
7	3662	3774	3662	3958	3790	3958	3790	3662	3662	3766	3766
8	2462	2462	2462	2462	2462	2462	2462	2462	2462	2606	2606
9	3646	3646	4134	4102	3646	4102	3646	3646	3646	3646	3646
10	2974	3382	2974	3366	2974	3366	2974	2974	2974	2902	2902
11	3006	3006	3006	3006	3006	3006	3006	3006	3006	3006	3006
12	3670	3670	4070	3110	3670	3110	3670	3670	3670	3670	3670
13	3102	3102	3654	3342	3102	3342	3102	3102	3102	3222	3222
14	2782	2782	2782	2902	2782	2902	2838	2782	2782	2782	2782
15	2966	2966	2918	3374	2966	3374	2966	2966	2966	2886	2886
16	2494	2494	2494	2646	2494	2646	2494	2494	2494	2494	2494
17	3206	3198	3102	4030	3206	4030	3230	3206	3206	3206	3206
18	4102	4054	4022	4302	3950	4302	3950	4102	4102	3950	3950
19	2606	2606	2870	2838	2606	2838	2510	2606	2606	2606	2606
20	3118	3134	3494	4862	3118	3486	3118	3118	3118	3134	3134
21	3118	3278	3398	3566	3150	3566	3150	3118	3118	3118	3118
22	3222	3286	3134	3678	3222	3678	3254	3222	3222	3110	3110
23	4238	4246	4358	4862	4238	4862	4238	4238	4238	4246	4246
24	3190	3190	3478	3646	3190	3646	3190	3190	3190	3190	3390

Table 4  
(Continued)

25	3406	3406	4358	4358	3406	4358	3406	3406	3406	3326	3326
26	3126	3078	3126	3910	3126	3910	3126	3126	3126	3126	3126
27	2910	2950	3102	3886	2910	3150	2910	2910	2910	2950	2950
28	3054	3030	3182	3966	3030	3183	3183	3054	3054	3054	3054
29	2638	2814	3798	2838	2638	2638	2638	2638	2638	2638	2814
30	3662	3822	3662	4574	3662	3790	3662	3662	3662	3662	3814
31	4462	4486	4462	5022	4462	4486	4486	4462	4462	4462	4462
32	4486	4486	4486	5270	4486	4702	4486	4486	4486	4486	4558
33	4622	4590	4622	5310	4622	5166	4590	4622	4622	4622	4622



Table 5  
(Continued)

	A	B	C	D	E	F	G	H	I	J	K
25	2734	2942	2734	3070	2670	2670	2670	2670	2670	2734	2734
26	4110	4110	4110	4542	4110	4110	4110	4110	4110	4110	4110
27	2758	2678	2758	2758	2678	2678	2678	2678	2678	2678	2678
28	4182	4182	4182	5118	4054	4054	4054	4054	4054	4182	4182
29	4010	4014	4014	4222	4046	4046	4046	4014	4014	4038	4014
30	2678	2678	2678	2702	2678	2678	2646	2678	2678	2678	2678
31	3398	3398	3398	4406	3398	3398	3398	3398	3398	3398	3398
32	2838	2814	2838	3134	2814	2838	2838	2814	2814	2886	2886
33	3310	3462	3310	3598	3358	3358	3358	3310	3310	3310	3310

### 3.2 A Binary Even-Odd Correlation Parameter Transformation

Define "randomizing" sequences of odd length  $N$  as  $r = 0101\dots010$  and  $\bar{r} = 1010\dots101$ . For a given sequence set  $X$  of  $K$  sequences,  $a_i$ , each of length  $N$  form the set

$$Y = \{b_i = a_i \oplus s, \quad a_i \text{ in } X \text{ and } s = r \text{ or } \bar{r}\}$$

where here  $\oplus$  denotes the term-by-term modulo-2 addition. If set  $X$  has good even autocorrelation and/or crosscorrelation parameters, then  $Y$  will have good odd correlation parameters and conversely. This is due to a property of the aperiodic correlation function as shown in [17].

As an example, it is pointed out that this transformation can be applied to exchange the odd crosscorrelation functions for the even cross-correlation functions of the LSE-optimal  $m$ -sequences of length  $N = 255$  [13] where  $39 \leq \hat{\theta}(k) \leq 79$  and  $31 \leq \theta(k) \leq 95$ .

## CHAPTER 4

## SOME NUMERICAL RESULTS

Several numerical results have been obtained via digital computation. Some of these results and, where possible, their analyses are presented. The peak even and odd auto- and cross-correlation parameters are the main focus. Even more important perhaps is the aforementioned transformation whose significance is with respect to these correlation properties. In the main, sets of 33 Gold sequences of length 31 have been examined but the following sequence families are some of the other cases also considered: the 8 Kasami sequences of length 63, the 12 weakly Barker sequences of length 31, the 6 m-sequences of length 31, the 6 m-sequences of length 63, the 6 cross-optimal Gold sequences (of Sywyk [11]) and some "transformed" Gold sequences of length 31. Because of space limitations, in general, only peak correlation parameter tables will be included. In these tables, the peak even and odd crosscorrelation parameters are respectively below and above the main diagonal and the peak odd autocorrelation parameters are on the main diagonal.

Where the sequence set is generated by modulo-2 addition of a fixed sequence,  $u$ , and successive shifts of another sequence,  $v$ , the following convention regarding sequence ordering is adopted throughout:  $u, v, w_l = u \oplus T^l v, 0 \leq l \leq N-1$  where  $N$  is the common sequence length.

4.1 Correlation Parameters of m-Sequences (in Characteristic Form)

The correlation parameters of the 6 m-sequences shown in Tables 6, 7, and 8 for lengths 31 and 63 respectively are analyzed in this section.

For  $N = 31$  the correlation functions are:

$$\theta_1(l) = \begin{cases} N, & l \equiv 0 \pmod{N} \\ -1, & l \not\equiv 0 \pmod{N} \end{cases}$$

for all the 6 m-sequences. The in-phase odd autocorrelation function has a constant value of N while the out-of-phase odd autocorrelation function takes on the values 1, 3, 5, 7, 9, 11 and their negatives. The even crosscorrelation function

$$\theta_{k,i}(\ell) = \begin{cases} -9, 11 & \ell \in C_0 \\ -1, 3, 7, -9 & \ell \in C_1 \cup C_3 \cup C_5 \\ -1, 3, -5, 7, -9 & \ell \in C_{15} \cup C_7 \cup C_{11} \end{cases}$$

for all  $1 \leq k < i \leq 6$ . Moreover, for any fixed pair  $(k,i)$ ,  $\theta_{k,i}(\ell)$  is constant for all values  $\ell$  in any cyclotomic coset mod N. The in-phase odd crosscorrelation function also takes on only values -9 and 11. The out-of-phase values are 1, 3, 5, 7, 9, 11, 13, 15 and their negatives as well as -17.

From Table 7 of the peak correlation parameters we have  $\theta_c = 11$  and  $\hat{\theta}_c = 17$ .

The correlation parameters of the 6 m-sequences of length  $N = 63$  in their characteristic form are analyzed next.

$$\theta_i(\ell) = \begin{cases} N, & \ell \equiv 0 \pmod{N} \\ -1, & \ell \not\equiv 0 \pmod{N} \end{cases}$$

for all 6 m-sequences. The in-phase odd autocorrelation takes on the value of N while the out-of-phase values are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19 and their negatives. The even crosscorrelation function

Correlation parameters of 6 m-sequences in characteristic form. N = 31.

# 1 1001011001111100011011101010000
# 2 1000010101110110001111100110100
# 3 111110111000101010100001100100
# 4 100100110000101101010001110111
# 5 1110100010010101100001110011011
# 6 111011001110000110101001001011

ODD AUTOCORRELATION FUNCTION

Table with 31 columns (Lp # 1-31) and 6 rows of data representing odd autocorrelation function values.

EVEN (PERIODIC) AUTOCORRELATION FUNCTION

Table with 31 columns (Lp # 1-31) and 6 rows of data representing even (periodic) autocorrelation function values.

EVEN CROSSCORRELATION FN BETWEEN SEQUENCES K AND I

Table with 31 columns (Lp # 1-31) and 6 rows of data representing even crosscorrelation function values between sequences K and I.

ODD CROSSCORRELATION FN BETWEEN SEQUENCES K AND I

Table with 31 columns (Lp # 1-31) and 6 rows of data representing odd crosscorrelation function values between sequences K and I.

Table 7

Peak correlation parameters of 6 m-sequences in their characteristic form,  
 $N = 31$ .

9	17	11	11	15	11
11	9	15	15	15	13
9	9	7	13	9	11
9	9	11	7	11	13
9	9	9	9	11	13
9	9	9	9	11	11

Table 8

Peak correlation parameters of 6 m-sequences in characteristic form,  
 $N = 63$ .

11	19	15	17	19	19
15	11	19	19	19	19
17	23	19	23	15	17
23	17	15	15	17	19
23	17	17	23	11	25
17	23	23	17	15	11

$$\theta_{k,i}(\ell) = \begin{cases} -9, 15 & , \ell \in C_0 \\ -1, -5, -9, -13, -17, 7 & , \ell \in C_1 \\ -1, -9, 3 & , \ell \in C_3 \cup C_{15} \\ -1, -3, -9, -17, 7, 15 & , \ell \in C_5 \\ -1 & , \ell \in C_7 \\ -1, -9, 7, 15 & , \ell \in C_9 \cup C_{27} \\ -1, -9, -17, 7, 11, 15 & , \ell \in C_{11} \\ -1, -5, -9, -13, -17, 7, 15 & , \ell \in C_{13} \\ -1, 11, 23 & , \ell \in C_{23} \\ -1, -5, -9, -17, 7, 11, 15 & , \ell \in C_{23} \cup C_{31} \end{cases}$$

for all  $1 \leq k < i \leq 6$ . The greatest number of distinct values assumed by the even crosscorrelation function  $\theta_{k,i}(\ell)$  of any two m-sequences of common period  $N = 63$  is 7 here. For any fixed pair  $(k,i)$ ,  $\theta_{k,i}(\ell)$  is constant for all values of  $\ell$  in any cyclotomic coset.

$\hat{\theta}_{k,i}(0)$  is -9 or 15 for  $1 \leq k, i \leq 6$ . The out-of-phase values for the odd crosscorrelation function are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, their negatives, -23 and 25.  $\theta_c = 23$  and  $\hat{\theta}_c = 25$ . Table 8 shows the corresponding peak correlation parameters.

#### 4.2 Correlation Parameters of the Gold Sequence Family of Length $N = 31$ Generated by $h_1 h_t = 3551$

The correlation functions of the family of 33 Gold sequences in a special form and in some of their suboptimal phases/shifts are tabulated and, where possible, analyzed. A convenient notation for the sequences is as follows. The two generating m-sequences will be denoted  $u$  and  $v$  and the other sequences by  $w_j$  where  $w_j = u \oplus T^j v$ ,  $0 \leq j \leq 30$ .

#### 4.2.1 The Two Generating m-Sequences in Their Characteristic Form

From Table 28 of the correlation parameters the following analysis is made.

$$\theta_i(0) = \hat{\theta}_i(0) = N \quad \text{for any of the sequences,}$$

$$\theta(1) = \begin{cases} 1, & i = u, v \\ 7, & i = w_0 \\ 9, & i = w_j, \quad 1 \leq j \leq 30 \end{cases}$$

$$\max\{\theta_i(l) : 1 \leq l \leq 30\} = \begin{cases} -1, & i = u, v, w_0 \\ 7, & i = w_j, \quad 1 \leq j \leq 30 \end{cases}$$

For any fixed  $l$ ,  $1 \leq l \leq 30$ ,

$$\theta_i(l) = \begin{cases} -1 & \text{for 17 sequences} \\ 7 & \text{for 10 sequences} \\ -9 & \text{for 6 sequences} \end{cases}$$

$\hat{\theta}(1)$  takes values in the set  $\{7, 9, 11, 13, 15\}$

$$\theta_{u,v}(l) = \begin{cases} -9, & l \in C_0 \cup C_5 \\ -1, & l \in C_1 \cup C_3 \cup C_{15} \\ 7, & l \in C_7 \cup C_{11} \end{cases}$$

$$\theta_{u,w_0}(l) = \begin{cases} -9, & l \in C_3 \\ -1, & l \in C_0 \cup C_5 \cup C_7 \cup C_{11} \\ 7, & l \in C_1 \cup C_{15} \end{cases}$$

$$\theta_{v,w_0}(l) = \begin{cases} -9, & l \in C_{15} \\ -1, & l \in C_0 \cup C_3 \cup C_5 \cup C_{11} \\ 7, & l \in C_1 \cup C_7 \end{cases}$$

$\theta_{\alpha,\beta}(l) \in \{-9, -1, 7\}$  where  $\alpha$  and  $\beta$  are any sequences as shown in Table 28.

$$\theta_{u,w_i}(0) = -1 \quad , \quad 0 \leq i \leq 30$$

$$\theta_{v,w_i}(0) = \begin{cases} -9 & , \quad i \in C_1 \\ -1 & , \quad i \in C_5 \cup C_7 \cup C_{11} \\ 7 & , \quad i \in C_3 \cup C_{15} \end{cases}$$

$$\theta_{w_i,w_j}(0) = -1 \quad , \quad 0 \leq i < j \leq 30$$

$$\theta_{u,\alpha}(0) = \begin{cases} -9 & , \quad \alpha = v \\ -1 & , \quad \alpha = w_i \end{cases} \quad 0 \leq i \leq 30$$

$$\hat{\theta}_{v,w_i}(0) = \begin{cases} -9 & , \quad i \in C_1 \\ -1 & , \quad i \in C_5 \cup C_7 \cup C_{11} \\ 7 & , \quad i \in C_3 \cup C_{15} \end{cases}$$

$$\theta_{w_i,w_j}(0) = -1 \quad , \quad 0 \leq i < j \leq 30$$

It is shown in Table 9 of the peak correlation parameters that the maximum magnitude of the odd crosscorrelation function is six-valued, it takes on values 7,9,11,13,15 and 17. The maximum magnitude even crosscorrelation function is two-valued whereas the maximum magnitude odd autocorrelation function takes on the five values: 7,9,11,13 and 15.

#### 4.2.2 The Gold Sequences Shifted Into Their LSE/AO Phases

The Gold sequences of length 31 and generated by  $h_1 h_t = 3551$  are each shifted into its LSE/AO phase and the correlation parameters are evaluated. The even autocorrelation function of the sequence set has the values: -1,-9,7,31 and



$$\theta(i) = \begin{cases} 1 & , \quad i = u, v \\ 7 & , \quad i = w_0 \\ 9 & , \quad i = w_1, \quad 1 \leq i \leq 30 \end{cases}$$

and

$$\hat{\theta}(i) = \begin{cases} 9 & , \quad i = u \\ 7 & , \quad i = v \\ 5 & , \quad i = w_0 \\ 7, 9 & , \quad i = w_j, \quad j \in C_1 \cup C_7 \\ 5, 7, 9 & , \quad i = w_j, \quad j \in C_3 \cup C_5 \cup C_{11} \cup C_{15} \end{cases}$$

The associated peak correlation parameters are shown in Table 10.

#### 4.2.3 The Gold Sequences Shifted Into Their AO/LSE Phases

The Gold sequences of length 31 and generated by  $h_1 h_t = 3551$  are each shifted into its AO/LSE phase (Table 11) and the correlation parameters of the resulting sequences are found. For any fixed  $l$ ,  $1 \leq l \leq 30$ ,

$$\theta_l(0) = \begin{cases} -1 & \text{for 17 sequences} \\ 7 & \text{for 10 sequences} \\ -9 & \text{for 6 sequences} \end{cases}$$

$$\theta(i) = \begin{cases} 1 & , \quad i = u, v \\ 7 & , \quad i = w_0 \\ 9 & , \quad i = w_k, \quad 1 \leq k \leq 30 \end{cases}$$

Table 12 shows the peak correlation parameters for the AO/LSE phases (shown in Table 11). The maximum magnitude odd crosscorrelation value is still 21.



Table 11

AO/LSE Gold sequences,  $h_1 h_c = 3551, N = 31$ .

# 1	11001111020110111210100013010
# 2	111101110001010110100011001001
# 3	1110011010001101101111110110101
# 4	0111001001100111011110011111110
# 5	0011111101001001011111010111101
# 6	100011111111101100110101110001
# 7	1110000011110111011111110101001
# 8	0110111011001011110111101010101
# 9	011000000111101000100001000110
# 10	1110010010100100000000011010101
# 11	0110010101010000010000111001100
# 12	0011100000100111011000110100000
# 13	000111010000101000110000010101
# 14	010111111100001100010010101001
# 15	0111001000001101100010111010111
# 16	1101101010010011010100010011110
# 17	1110001001100001110101101101100
# 18	100101000011111001000111000111
# 19	1001011010101001100010100000010
# 20	1001000100010010011100001001101
# 21	1010011010010111000000100100001
# 22	0100100100010111100010000010110
# 23	1000001100001000000110011110101
# 24	0111100110010110000101101001110
# 25	1101111000010101110010110110000
# 26	0110001000111110010111011010001
# 27	0010101100101001110101011000111
# 28	0001101011010110011001101101001
# 29	10101011101000011101110100101000
# 30	0011101001100100001011100110111
# 31	1110001110100000111011011100001
# 32	111110001011011110110000000101
# 33	111000000100010100110111111011



#### 4.2.4 The Gold Sequences Shifted Into Certain Other Optimal Phases

The correlation parameters of the optimal sequences obtained from the same Gold sequence family of length 31 and generated by  $h_1 h_t = 3551$  using the terminal aperiodic correlation, the concave exponential and the partial correlation sieves do not admit the same statistical regularity as the preceding optimal sequences. Here only tables of the corresponding peak correlation parameters are presented.

Tables 13, 14 and 15 show the peak correlation parameters of the optimal sequences generated by the concave exponential, the partial correlation and the terminal aperiodic correlation sieve respectively. Also see Table 16.

#### 4.3 Peak Crosscorrelation Parameters Between Some Pairs of Gold Sequence Sets, $N = 31$

In some applications it may be necessary to maintain a sequence length  $N$  but require more than  $K = N + 2$  Gold sequences. Selection is then made among groups of Gold sequences. Of course, the requirement that the correlation parameters have to be minimized is still in force. Following are some results on the peak crosscorrelation parameters obtained between pairs of Gold sequences.

Let  $(\alpha, \beta)$  represent the Gold sequence set generated by the characteristic polynomial  $h_\alpha h_\beta$  and  $XY$  represent the pair of Gold sequence sets generated by characteristic polynomials  $X$  and  $Y$  (the two generating  $m$ -sequences are in their characteristic form). Then if  $A = (1,3)$ ,  $B = (1,5)$ ,  $C = (3,5)$ ,  $D = (1,11)$ ,  $E = (3,11)$ ,  $F = (1,7)$ ,  $G = (5,7)$  and  $H = (7,11)$  Table 17 shows the values of the peak crosscorrelation parameters of the indicated sequence set pairs.







Table 16

Comparison of the peak correlation values for various optimal sequences derived from sequences generated by  $h_1 h_c = 3551$ ,  $N = 31$ .

"Sieve"	$\theta(1)$	$\hat{\theta}(1)$	$\theta(k,1)$	$\hat{\theta}(k,1)$
Generating m-sequences in characteristic form	1,7,9	7,9,11,13,15	7,9	7,9,11,13,15,17
AO/LSE	1,7,9	5,7,9	7,9	9,11,13,15,17,19,21
LSE/AO	1,7,9	5,7,9	7,9	9,11,13,15,19,21
Terminal aperiodic correlation sieve	1,7,9	5,7,9	7,9	9,11,13,15,17,19,21
Concave experimental sieve	1,7,9	5,7,9,11	7,9	9,11,13,15,17,19,21
Partial correlation sieve	1,7,9	5,7,9,11,13,15,17	7,9	7,9,11,13,15,17,19

Table 17

Values of the peak crosscorrelation parameters of pairs of Gold sequence sets.

XY	Spectra of Peak Crosscorrelation Parameters	
	Even	Odd
AB	9,15,17	9,11,13,15,17,19,21
AC	9,15,17	9,11,13,15,17,19,21
AD	9,15,17	9,11,13,15,17,19,21,23
AE	9,15,17	9,11,13,15,17,19,21
AF	7,9,11,13,15,21	7,9,11,13,15,17,19,21
BC	9,15,17	9,11,13,15,17,19
BF	9,15,17	9,11,13,15,17,19,21
BG	9,15,17	9,11,13,15,17,19,21
DE	9,15,17	9,11,13,15,17,19,21
DF	9,15,17	9,11,13,15,17,19,21
DH	9,15,17	9,11,13,15,17
GH	7,9,11,13,15,21	7,9,13,15,17,19,21,23
GF	9,15,17	9,11,13,15,17,19

From the data it is concluded that the peak even crosscorrelation function between two Gold sequence families whose generating polynomials are products of polynomials belonging to the same maximal connected set takes on only the three values: 9, 15 and 17 for  $N = 31$  while the peak odd crosscorrelation function takes values in the set  $\{9, 11, 13, 15, 17, 19, 21, 23\}$  provided the sequence pair is expurgated of repeated sequences. When the two Gold sequence sets do not belong to the same maximal connected set, the peak even and odd crosscorrelation function take values in the sets  $\{7, 9, 11, 13, 15, 21, 31\}$  and  $\{7, 9, 13, 15, 17, 19, 21, 23, 31\}$  respectively. Thus it is apparent that, provided that peak even crosscorrelation parameters as high as 17 are tolerated, the number of Gold sequences can be substantially increased beyond  $K = N + 2$ . The full peak even and odd crosscorrelation parameters for the pair DH are presented in Tables 18-20.

#### 4.4 Mean-Square Partial Correlation Parameters of Some Sequence Families of Lengths $N = 31$ and $N = 63$

The acquisition of the epoch of a long sequence involves correlation over a relatively short window of the sequence since acquisition time is relatively short. In [12] Pursley showed that one of the key correlation partial correlation parameters is a mean-squared partial correlation  $v_{x,y}^{(w)}$  for window length  $w = O(\log_2 N)$ .

$$v_{x,y}^{(w)} = \sum_{i=1-w}^{w-1} (w - |i|) \theta_x(i) \theta_y^*(i)$$

which for binary sequences of length  $N$  reduces to

$$v_{x,y}^{(w)} = wN^2 + 2 \sum_{i=0}^{w-1} (w-i) \theta_x(i) \theta_y(i)$$

Table 18

Initial loadings for sequences in the pair DH.

	D	H
1	1131	1114
2	1663	1663
3	0752	0777
4	0476	0453
5	0227	0202
6	1705	1720
7	0541	0564
8	0051	0074
9	1270	1255
10	1632	1617
11	0737	0712
12	0524	0501
13	0103	0126
14	1154	1171
15	1063	1046
16	1215	1230
17	1760	1745
18	0413	0436
19	0375	0350
20	1421	1404
21	0310	0335
22	1573	1556
23	0034	0011
24	1322	1307

Table 18  
(Continued)

25	1516	1533
26	0166	0143
27	1006	1023
28	1347	1362
29	1444	1461
30	0242	0267
31	1657	1672
32	0665	0640
33	0600	0625



Table 20  
Peak odd crosscorrelation parameters for the pair DH.

11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95	97	99	101	103	105	107	109	111	113	115	117	119	121	123	125	127	129	131	133	135	137	139	141	143	145	147	149	151	153	155	157	159	161	163	165	167	169	171	173	175	177	179	181	183	185	187	189	191	193	195	197	199	201	203	205	207	209	211	213	215	217	219	221	223	225	227	229	231	233	235	237	239	241	243	245	247	249	251	253	255	257	259	261	263	265	267	269	271	273	275	277	279	281	283	285	287	289	291	293	295	297	299	301	303	305	307	309	311	313	315	317	319	321	323	325	327	329	331	333	335	337	339	341	343	345	347	349	351	353	355	357	359	361	363	365	367	369	371	373	375	377	379	381	383	385	387	389	391	393	395	397	399	401	403	405	407	409	411	413	415	417	419	421	423	425	427	429	431	433	435	437	439	441	443	445	447	449	451	453	455	457	459	461	463	465	467	469	471	473	475	477	479	481	483	485	487	489	491	493	495	497	499	501	503	505	507	509	511	513	515	517	519	521	523	525	527	529	531	533	535	537	539	541	543	545	547	549	551	553	555	557	559	561	563	565	567	569	571	573	575	577	579	581	583	585	587	589	591	593	595	597	599	601	603	605	607	609	611	613	615	617	619	621	623	625	627	629	631	633	635	637	639	641	643	645	647	649	651	653	655	657	659	661	663	665	667	669	671	673	675	677	679	681	683	685	687	689	691	693	695	697	699	701	703	705	707	709	711	713	715	717	719	721	723	725	727	729	731	733	735	737	739	741	743	745	747	749	751	753	755	757	759	761	763	765	767	769	771	773	775	777	779	781	783	785	787	789	791	793	795	797	799	801	803	805	807	809	811	813	815	817	819	821	823	825	827	829	831	833	835	837	839	841	843	845	847	849	851	853	855	857	859	861	863	865	867	869	871	873	875	877	879	881	883	885	887	889	891	893	895	897	899	901	903	905	907	909	911	913	915	917	919	921	923	925	927	929	931	933	935	937	939	941	943	945	947	949	951	953	955	957	959	961	963	965	967	969	971	973	975	977	979	981	983	985	987	989	991	993	995	997	999
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The window lengths used for  $N = 31$  are 4, 5, 6 and 7. The parameters were normalized to

$$N^{-2}v_{x,y}(w) = w + 2N^{-2} \sum_{i=0}^{w-1} (w-i)\theta_x(i) \theta_y(i)$$

called the mean-squared partial correlation for window length  $w$ . The values are given in Tables 34 to 37 from which it is clear that  $N^{-2}v_{x,y}(w) \approx w$  for Gold sequences. Partial correlation values were computed for some sequence families of lengths 31 and 63 and are shown in Tables 38 to 40 and the corresponding extremal values in Tables 21-24, 26 and 27.

For Gold sequences, the partial correlation for the LSE/AO phases are identical to those for AO/LSE phases. However, those for the Gold sequence set for which the two generating  $m$ -sequences are in their characteristic form are different as shown in Table 22.

The partial correlation parameters of the set of Gold sequences obtained by modulo-2 adding the sequence  $r = 0101\dots 01$  of the same length 31 to each sequence of the AO/LSE Gold sequence set are shown in Table 23.

Thus, since it is desirable that the partial correlation function  $v_{x,y}(w)$  be as small as possible, it is advantageous to "randomize" the sequences - the peak partial correlation values are decreased but also the least values are increased slightly.

For the 12 best weakly-Barker sequences of length 31 obtained in [10] and shown in Table 25, the partial correlation parameters are given in Table 24.

For length,  $N = 63$  partial correlation parameters were computed for Sywyk's 6 cross-optimal Gold sequences and for the 8 Kasami sequences.

Table 21

Extended partial correlation parameters: AO/LSE Gold sequences:  $h_1 h_t = 3551$ ,  $N = 31$ .

Window length, $w$	Maximum, $\alpha$	Minimum, $\beta$	$\frac{\alpha-\beta}{w} \times 100\%$
4	4.9	3.4	37
5	6.4	4.2	44
6	8.1	4.8	55
7	9.9	5.5	62.9

Table 22

Extended partial correlation parameters: Gold sequences generated by  $h_1 h_t = 3551$ , the two generating m-sequences in characteristic form,  $N = 31$ .

Window length, $w$	Maximum, $\alpha$	Minimum, $\beta$	$\frac{\alpha-\beta}{w} \times 100\%$
4	4.9	3.3	40
5	6.4	4.1	46
6	8.1	4.8	55
7	9.9	5.6	61.4

Table 23

Extremal partial correlation parameters: "randomized" AO/LSE Gold sequences:  $h_1 h_t = 3551$ ,  $N = 31$ .

Window length, $w$	Maximum, $\alpha$	Minimum, $\beta$	$\frac{\alpha-\beta}{w} \times 100\%$
4	4.7	3.6	27.5
5	6.1	4.4	34.0
6	7.5	5.5	33.3
7	8.9	6.2	38.6

Table 24

Extremal partial correlation parameters of the 12 best WB-sequences,  $N = 31$ .

Window length, $w$	Maximum, $\alpha$	Minimum, $\beta$	$\frac{\alpha-\beta}{w} \times 100\%$
4	4.2	3.9	7.5
5	5.4	4.9	10
6	6.5	5.9	10
7	7.7	6.8	12.9

Table 25

Best WB-sequences of length  $N = 31$ .

# 1	00001100001101110111010010101
# 2	1111001111001000100010101101010
# 3	0101101011100010001000001111111
# 4	1010011010011101110111111000000
# 5	00000011111011101110010110101
# 6	1111110000001000100011010011010
# 7	0101011010100010001001111001111
# 8	101010010101110111011000110000
# 9	0000001111001001110010110101010
# 10	111111000011011000101010101010
# 11	01010010110001101100001111111
# 12	101010110100111001001111000000

Table 26

Extremal partial correlation parameters for Sywyk's 6 crossoptimal sequences.

Window length, w	Maximum, $\alpha$	Minimum, $\beta$	$\frac{\alpha-\beta}{w} \times 100\%$
5	5.2	5.0	4
6	6.7	5.9	13.3
7	7.9	6.9	14.3
8	9.1	7.9	15

Table 27

Extremal partial correlation parameters for 8 LSE/AO small Kasami sequences  
 $h_1 = 103, h_t = 015, h_1 h_t = 1527, N = 63.$ 

Window length, w	Maximum, $\alpha$	Minimum, $\beta$	$\frac{\alpha-\beta}{w} \times 100\%$
5	5.3	4.7	12
6	6.6	5.6	16.7
7	7.8	6.4	20
8	9.0	7.3	21.3

## CHAPTER 5

## CONCLUSIONS

Full periodic correlation parameters of some special binary sequence families including the Gold sequences of length 31 have been computed. The associated peak correlation parameters were also readily found. Sieves for cyclically shifting a given set of binary sequences into desired phases were examined. Performances of old and new candidate sieves for optimal sequence phases were compared. No sieve examined was found to be optimal in the sense of being uniformly better than any other.

The peak crosscorrelation parameters of some pairs of Gold sequence sets of length 31 were evaluated. They exhibited a set membership pattern. A binary transformation which exchanges the peak even and odd periodic correlation parameters of any binary sequence set was exhibited. Finally the mean-square partial correlation parameters of some binary sequences were computed.

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## APPENDIX A

CORRELATION PARAMETERS OF GOLD SEQUENCES,  $h_1 h_t = 3551$

Table 28

Correlation parameters of the Gold sequence set generated by  $h_{1h_c} = 3551$  with the two generating m-sequences (#1 and #2) in their characteristic form,  $N = 31$ .

```

# 1 100101100111110001101111011000
# 2 11110111100110111100111101100
# 3 11110111110110111110111101100
# 4 11110111101110111110111101100
# 5 11110111101110111110111101100
# 6 11110111101110111110111101100
# 7 11110111101110111110111101100
# 8 11110111101110111110111101100
# 9 11110111101110111110111101100
#10 11110111101110111110111101100
#11 11110111101110111110111101100
#12 11110111101110111110111101100
#13 11110111101110111110111101100
#14 11110111101110111110111101100
#15 11110111101110111110111101100
#16 11110111101110111110111101100
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#26 11110111101110111110111101100
#27 11110111101110111110111101100
#28 11110111101110111110111101100
#29 11110111101110111110111101100
#30 11110111101110111110111101100
#31 11110111101110111110111101100
#32 11110111101110111110111101100
#33 11110111101110111110111101100
    
```

DDD AUTOCORRELATION FUNCTION

L	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
# 1	31	1	-1	-3	-1	-7	3	1	-1	1	-4	5	7	5	3	9	-9	-3	-5	-7	-5	9	-1	1	-1	-3	7	1	3	1	-1
# 2	31	1	3	1	3	5	-1	-3	7	1	-1	5	7	1	-5	-7	7	5	-1	-7	-5	1	-1	-7	3	1	-5	-3	-1	-3	-1
# 3	31	-3	-1	13	-5	1	15	-3	-1	13	-5	1	15	-7	-5	9	-9	5	7	-15	-1	5	-13	1	3	-15	-1	5	-13	1	2
# 4	31	1	-13	-3	-1	9	-1	-3	3	-3	3	1	-9	1	11	5	-5	-11	-1	9	-1	-3	3	-3	3	1	-9	1	3	13	-1
# 5	31	1	-1	-7	-13	-3	3	5	-1	5	3	-1	-7	-5	-3	3	5	7	1	3	-3	-5	1	-5	-3	3	13	7	1	1	-1
# 6	31	-7	7	-7	7	-3	7	-11	11	-7	7	-3	3	11	-7	7	-11	-1	-3	3	-7	7	-11	11	-7	3	7	7	7	7	-1
# 7	31	1	3	1	3	-7	-5	-3	-13	1	-5	1	3	5	-1	-3	3	1	-5	-3	-1	5	-1	13	3	5	7	-3	-1	-3	-1
# 8	31	5	-13	-7	3	9	3	1	-1	-7	-5	5	7	1	-9	7	9	-1	-7	-5	5	7	1	-1	-3	-9	-3	7	13	-5	-1
# 9	31	-3	-5	1	7	-11	-5	5	3	-3	3	-3	-1	-7	-5	-7	7	5	7	1	3	-3	3	-3	-5	5	11	-7	-1	5	2
#10	31	-7	3	5	-5	9	3	-3	7	1	-1	1	-5	1	3	1	-1	-3	-1	5	-1	1	-1	-7	3	-3	-9	5	-5	-3	7
#11	31	-7	3	-7	-1	1	-1	-3	7	5	-1	1	-9	5	-9	9	-9	9	-5	9	-1	1	-5	-7	3	1	-1	1	7	-9	7
#12	31	5	7	1	7	5	-1	1	-1	1	-1	1	7	5	-5	-3	3	5	-5	-7	-1	1	-1	1	-1	1	-5	-7	-1	-7	-5
#13	31	1	3	9	-9	-3	-5	-7	-1	-3	3	1	11	5	3	5	-5	-3	-5	-11	-1	-3	3	1	7	5	3	9	-9	-3	-1
#14	31	9	3	-3	-1	-3	-5	-7	-5	5	7	1	-5	1	-1	1	-1	1	-1	5	-1	-7	-5	5	7	5	3	1	7	-3	-9
#15	31	5	-5	-11	-5	-3	-1	13	11	-1	-13	-11	3	5	7	1	-1	-7	-5	-3	11	13	-1	-11	-13	1	3	5	11	5	-5
#16	31	-3	-1	-3	11	5	3	-7	7	5	-1	-7	3	1	-1	5	-5	1	-1	-3	7	1	-5	-7	7	-3	-5	-11	3	1	3
#17	31	-3	-9	9	-1	-7	3	5	-9	1	11	-7	3	13	-9	-7	7	9	-13	-3	7	-11	-1	9	-5	-3	7	1	-9	9	3
#18	31	-3	7	-3	-1	5	-9	9	-5	1	3	1	-1	-7	-1	1	-1	1	7	1	-1	-3	-1	5	-9	9	-5	1	3	-7	3
#19	31	-11	-5	1	3	5	-9	5	-1	1	5	-5	-7	11	-3	3	-11	7	5	-9	1	-1	1	-5	9	-5	-3	-1	5	11	5
#20	31	9	7	5	7	-3	-1	5	-1	-7	-5	-3	-5	-11	-1	3	3	1	11	5	3	5	7	1	-5	1	3	-7	-5	-7	-9
#21	31	-11	3	1	7	-7	-1	1	3	-3	-1	13	-5	1	-1	13	-13	1	-1	5	-13	1	3	-3	-1	1	7	-7	-1	-9	11
#22	31	5	-1	1	3	1	-1	5	-5	-11	-9	1	-1	-11	-1	-3	3	1	11	1	-1	9	11	5	-5	1	-1	-3	-1	1	-5
#23	31	5	3	-3	3	9	11	-7	-5	-7	-1	5	-1	-3	5	-3	3	5	3	1	-5	1	7	5	7	-11	-9	-3	3	-3	-5
#24	31	1	3	-7	-5	5	-1	9	-5	1	-1	1	7	5	3	1	-1	-3	-5	-7	-1	1	-1	5	-9	1	-5	5	7	-5	-1
#25	31	1	7	5	3	1	-1	1	-5	-3	3	-11	-5	-3	-5	-3	3	5	3	5	11	-3	3	5	-1	1	-1	-3	-5	-7	-1
#26	31	9	3	-3	-5	-3	-9	-7	-1	1	3	1	-1	1	-1	5	-5	1	-1	1	-1	-3	-1	1	7	9	3	5	3	-3	-9
#27	31	5	7	5	-1	1	-5	-3	3	1	-1	9	-5	1	3	-3	3	-3	-1	5	-9	1	-1	3	3	5	-1	1	-5	-7	-5
#28	31	1	-1	5	-5	-11	3	5	-5	13	7	-3	-1	-3	-1	-7	7	1	3	1	3	-7	-13	5	-5	-3	11	5	-5	1	1
#29	31	5	-1	1	-5	-3	3	1	3	-3	11	9	3	-3	-5	-7	7	5	3	-3	-9	-11	3	-3	-1	-3	3	5	-1	1	-5
#30	31	1	-9	-7	3	1	-1	1	-1	5	-1	-11	-5	9	1	1	-1	1	-9	5	11	1	-5	1	-1	1	-1	-3	7	9	1
#31	31	-3	-5	9	-9	5	11	-3	7	5	-9	-3	-1	-5	7	5	-5	-7	-5	1	3	9	-5	7	3	-11	-5	9	-5	5	3
#32	31	-11	3	5	-1	1	-1	-7	7	-11	3	-3	-5	-3	3	-7	-3	3	5	3	-3	11	-7	7	1	-1	-1	1	-5	-3	11
#33	31	1	-1	1	7	1	-5	1	-5	1	3	5	-1	1	7	5	-3	-7	-1	1	-5	-3	-1	5	-1	5	-1	-7	-1	1	-1





Table 28

(Continued)

Table with multiple columns of numerical data (likely +1, -1, or 0) and row labels (e.g., 3 18, 3 19, 3 20, ..., 5 21).

Table 28

(Continued)

Table with 29 rows and 29 columns. Each cell contains a number, likely a digit '7' or '-1', representing a data point in a grid. The values are arranged in a pattern that is difficult to discern due to the high density and repetition of characters.

Table 28  
(Continued)

Table with 18 columns and 40 rows of numerical data. The data consists of integers ranging from -1 to 9. The rows are labeled on the left with numbers such as 7 30, 7 31, 7 32, 7 33, 8 4, 8 10, 8 11, 8 12, 8 13, 8 14, 8 15, 8 16, 8 17, 8 18, 8 19, 8 20, 8 21, 8 22, 8 23, 8 24, 8 25, 8 26, 8 27, 8 28, 8 29, 8 30, 8 31, 8 32, 8 33, 9 10, 9 11, 9 12, 9 13, 9 14, 9 15, 9 16, 9 17, 9 18, 9 19, 9 20, 9 21, 9 22, 9 23, 9 24, 9 25, 9 26, 9 27, 9 28, 9 29, 9 30, 9 31, 9 32, 9 33, 10 11, 10 12, 10 13, 10 14, 10 15, 10 16, 10 17, 10 18.

80











Table 28

(Continued)

Table with 25 columns and 40 rows of numerical data. The data is organized in a grid format with values ranging from -15 to 15.

Table 28

(Continued)

Table with 30 columns and 30 rows of numerical data. The table contains a dense grid of integers, mostly ranging from -5 to 5, with occasional larger values like 15. The data appears to be organized in a regular grid pattern across the page.





Table 28

(Continued)

A dense grid of numbers, likely a statistical or tabular data set, consisting of rows and columns of numerical values. The grid is organized into a regular pattern of rows and columns, with each cell containing a numerical entry. The numbers are integers, mostly ranging from -5 to 5, with some larger values like -13 and 15 appearing. The table is a continuation of Table 28 from the previous page.

90





Table 28

(Continued)

Table with 25 columns and 40 rows of numerical data, including integers and negative values.

APPENDIX B  
CORRELATION PARAMETERS OF SOME SEQUENCE FAMILIES

Table 29

12 Best WB-sequences of length N = 31.

ODD (PERIODIC) AUTOCORRELATION FUNCTION																															
L <sub>n</sub>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
# 1	31	-3	3	-3	3	1	-1	1	-1	1	-5	-3	-5	5	-5	1	-1	5	-5	5	3	5	-1	1	-1	1	-1	-3	3	-3	3
# 2	31	-3	3	-3	3	1	-1	1	-1	1	-5	-3	-5	5	-5	1	-1	5	-5	5	3	5	-1	1	-1	1	-1	-3	3	-3	3
# 3	31	3	3	3	3	-3	-1	-3	-1	-3	-5	5	-5	-3	-5	1	-1	5	3	5	-5	5	3	1	3	1	3	-3	-5	-3	-5
# 4	31	5	5	5	5	-3	-1	-3	-1	-3	-5	5	-5	-3	-5	1	-1	5	3	5	-5	5	3	1	3	1	3	-3	-5	-3	-5
# 5	31	5	5	5	5	-3	-1	-3	-1	-3	-5	5	-5	-3	-5	1	-1	5	3	5	-5	5	3	1	3	1	3	-3	-5	-3	-5
# 6	31	5	5	5	5	-3	-1	-3	-1	-3	-5	5	-5	-3	-5	1	-1	5	3	5	-5	5	3	1	3	1	3	-3	-5	-3	-5
# 7	31	-3	3	-3	3	1	-1	1	-1	1	-5	-3	-5	5	-5	1	-1	5	-5	5	3	5	-1	1	-1	1	-1	-3	3	-3	3
# 8	31	-3	3	-3	3	1	-1	1	-1	1	-5	-3	-5	5	-5	1	-1	5	-5	5	3	5	-1	1	-1	1	-1	-3	3	-3	3
# 9	31	1	3	1	-5	1	-5	1	3	1	3	1	-5	1	3	1	-1	-3	-1	5	-1	-3	-1	-3	-1	5	-1	5	-1	-3	-1
# 10	31	1	3	1	-5	1	-5	1	3	1	3	1	-5	1	3	1	-1	-3	-1	5	-1	-3	-1	-3	-1	5	-1	5	-1	-3	-1
# 11	31	1	3	1	-5	1	-5	1	3	1	3	1	-5	1	3	1	-1	-3	-1	5	-1	-3	-1	-3	-1	5	-1	5	-1	-3	-1
# 12	31	1	3	1	-5	1	-5	1	3	1	3	1	-5	1	3	1	-1	-3	-1	5	-1	-3	-1	-3	-1	5	-1	5	-1	-3	-1
EVEN (PERIODIC) AUTOCORRELATION FUNCTION																															
L <sub>n</sub>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
# 1	31	-5	3	-5	3	3	-1	3	-1	3	-5	-5	-5	3	-5	-1	-1	-5	3	-5	-5	-5	3	-1	3	-1	3	3	-5	3	-5
# 2	31	-5	3	-5	3	3	-1	3	-1	3	-5	-5	-5	3	-5	-1	-1	-5	3	-5	-5	-5	3	-1	3	-1	3	3	-5	3	-5
# 3	31	3	3	3	3	-1	-1	-1	-1	-1	-5	3	-5	-5	-5	-1	-1	-5	-5	-5	3	-5	-1	-1	-1	-1	-1	3	3	3	3
# 4	31	3	3	3	3	-1	-1	-1	-1	-1	-5	3	-5	-5	-5	-1	-1	-5	-5	-5	3	-5	-1	-1	-1	-1	-1	3	3	3	3
# 5	31	3	3	3	3	-1	-1	-1	-1	-1	-5	3	-5	-5	-5	-1	-1	-5	-5	-5	3	-5	-1	-1	-1	-1	-1	3	3	3	3
# 6	31	3	3	3	3	-1	-1	-1	-1	-1	-5	3	-5	-5	-5	-1	-1	-5	-5	-5	3	-5	-1	-1	-1	-1	-1	3	3	3	3
# 7	31	-5	3	-5	3	3	-1	3	-1	3	-5	-5	-5	3	-5	-1	-1	-5	3	-5	-5	-5	3	-1	3	-1	3	3	-5	3	-5
# 8	31	-5	3	-5	3	3	-1	3	-1	3	-5	-5	-5	3	-5	-1	-1	-5	3	-5	-5	-5	3	-1	3	-1	3	3	-5	3	-5
# 9	31	-1	3	-1	-5	-1	-5	-1	3	-1	3	-1	-5	-1	3	-1	-1	3	-1	-5	-1	3	-1	3	-1	-5	-1	-5	-1	3	-1
# 10	31	-1	3	-1	-5	-1	-5	-1	3	-1	3	-1	-5	-1	3	-1	-1	3	-1	-5	-1	3	-1	3	-1	-5	-1	-5	-1	3	-1
# 11	31	-1	3	-1	-5	-1	-5	-1	3	-1	3	-1	-5	-1	3	-1	-1	3	-1	-5	-1	3	-1	3	-1	-5	-1	-5	-1	3	-1
# 12	31	-1	3	-1	-5	-1	-5	-1	3	-1	3	-1	-5	-1	3	-1	-1	3	-1	-5	-1	3	-1	3	-1	-5	-1	-5	-1	3	-1

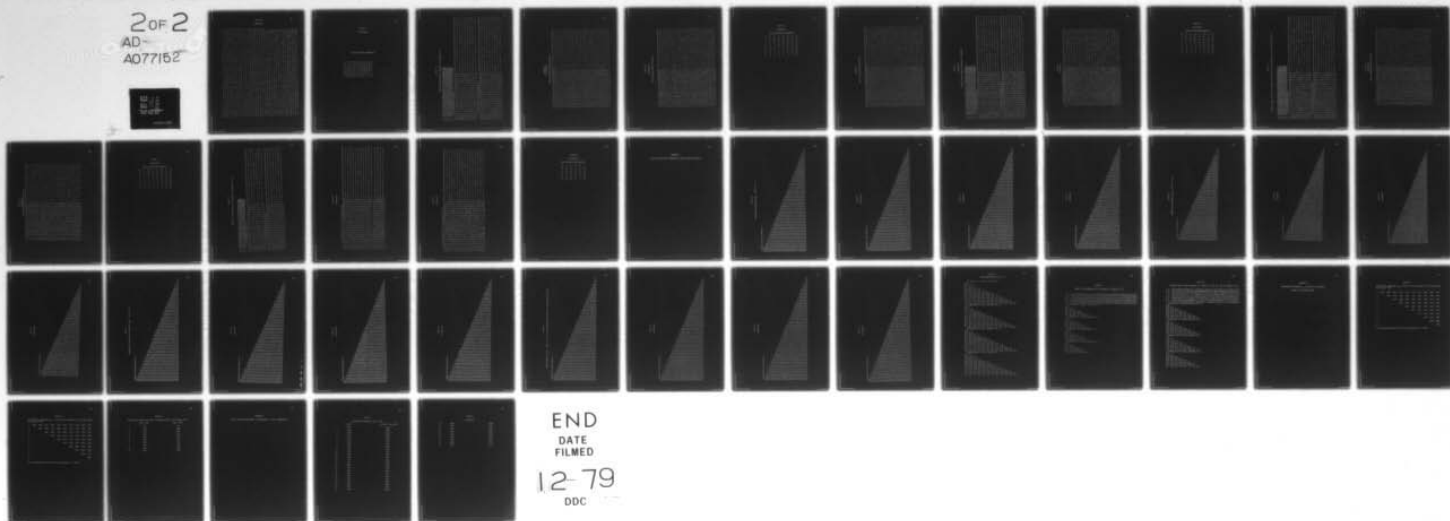


AD-A077 152

ILLINOIS UNIV AT URBANA-CHAMPAIGN COORDINATED SCIENCE LAB F/G 17/2.1  
ON CORRELATION PARAMETERS FOR SOME BINARY SEQUENCES OF LENGTHS --ETC(U)  
MAY 79 D W GAHUTU DAAB07-72-C-0259  
R-845 NL

UNCLASSIFIED

2 OF 2  
AD-A077152



END  
DATE  
FILMED  
12-79  
DDC



Table 29  
(Continued)

Peak correlation parameters

5	31	13	13	15	15	13	13	13	13	15	15
31	5	13	13	15	15	13	13	13	13	15	15
13	13	5	31	13	13	15	15	15	15	15	15
13	13	31	5	13	13	15	15	15	15	15	15
15	15	13	13	5	31	13	13	15	15	15	15
15	15	13	11	31	5	13	13	15	15	15	15
13	13	15	15	13	13	5	31	15	15	13	13
13	13	15	15	13	13	31	5	15	15	13	13
15	15	15	15	13	13	15	15	5	31	11	11
15	15	15	15	13	13	15	15	31	5	11	11
15	15	13	13	15	15	15	15	11	11	5	31
15	15	13	13	15	15	15	15	11	11	31	5





Table 30

(Continued)

Odd crosscorrelation function

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																																																			
42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92																																									
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92

Table 30  
(Continued)

Peak correlation parameters.

11	21	21	15	17	17	21	21
9	13	19	19	21	17	19	15
9	9	11	23	23	15	19	23
9	9	9	11	17	19	21	17
9	9	9	9	15	17	15	17
9	9	9	9	9	13	21	21
9	9	9	9	9	9	13	23
9	9	9	9	9	9	9	11







Table 31  
(Continued)

Peak correlation parameters.

11	21	21	15	21	19	17	21
9	13	19	19	21	23	17	15
9	9	11	23	17	17	15	23
9	9	9	11	19	19	17	17
9	9	9	9	11	17	23	21
9	9	9	9	9	13	21	21
9	9	9	9	9	9	11	21
9	9	9	9	9	9	9	11





Table 32  
(Continued)  
Odd crosscorrelation function.

4	11-05	1	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																
5	-0-13	-13	-11	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
6	2-11	2-10	2-9	2-8	2-7	2-6	2-5	2-4	2-3	2-2	2-1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41			
7	3-9	3-8	3-7	3-6	3-5	3-4	3-3	3-2	3-1	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41						
8	4-7	4-6	4-5	4-4	4-3	4-2	4-1	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41									
9	5-5	5-4	5-3	5-2	5-1	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41												
10	6-3	6-2	6-1	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41															
11	7-1	7-0	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																	
12	8-0	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																			
13	9-0	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																				
14	10-0	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																					
15	11-0	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																						
16	12-0	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																							
17	13-0	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																								
18	14-0	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																									
19	15-0	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																										
20	16-0	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																											
21	17-0	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																												
22	18-0	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																													
23	19-0	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																														
24	20-0	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																															
25	21-0	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																																
26	22-0	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																																	
27	23-0	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																																		
28	24-0	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																																			
29	25-0	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																																				
30	26-0	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																																					
31	27-0	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41																																						
32	28-0	28	29	30	31	32	33	34	35	36	37	38	39	40	41																																							
33	29-0	29	30	31	32	33	34	35	36	37	38	39	40	41																																								
34	30-0	30	31	32	33	34	35	36	37	38	39	40	41																																									
35	31-0	31	32	33	34	35	36	37	38	39	40	41																																										
36	32-0	32	33	34	35	36	37	38	39	40	41																																											
37	33-0	33	34	35	36	37	38	39	40	41																																												
38	34-0	34	35	36	37	38	39	40	41																																													
39	35-0	35	36	37	38	39	40	41																																														
40	36-0	36	37	38	39	40	41																																															
41	37-0	37	38	39	40	41																																																

Table 32

(Continued)

Peak correlation parameters							
11	21	19	21	23	19	21	25
9	21	19	15	17	21	23	23
9	9	19	15	19	19	21	25
9	9	9	19	17	21	15	15
9	9	9	9	15	17	21	23
9	9	9	9	9	15	23	15
9	9	9	9	9	9	15	19
9	9	9	9	9	9	9	21

Table 33  
Sywyk's crossoptimal Gold sequences of length N = 63.

Code	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42																																												
0 1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42																																												
0 2	31	32	33	34	35	36	37	38	39	40	41	42	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42																																
0 3	41	42	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
0 4	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42		
0 5	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42		
0 6	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42		



Table 33

(Continued)

GOOD CROSS-CORRELATIONS FROM THEIR SEQUENCES R AND I

K	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41			
42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62																								
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41				
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41				
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41				
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41				

Table 33

(Continued)

## Peak correlation parameters

13	17	17	19	17	17
17	19	17	17	17	19
17	15	13	17	17	21
17	17	17	15	15	17
17	17	17	17	9	19
17	15	15	17	17	15

APPENDIX C

PARTIAL CORRELATION PARAMETERS OF SOME SEQUENCE FAMILIES

















Table 36

"Randomized" AO/LSE Gold sequences  $h_{1t}$ ,  $N = 31$ .

CORRELATION WINDOW LENGTH, No 4

0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10.0	10.1	10.2	10.3	10.4	10.5	10.6	10.7	10.8	10.9	11.0	11.1	11.2	11.3	11.4	11.5	11.6	11.7	11.8	11.9	12.0	12.1	12.2	12.3	12.4	12.5	12.6	12.7	12.8	12.9	13.0	13.1	13.2	13.3	13.4	13.5	13.6	13.7	13.8	13.9	14.0	14.1	14.2	14.3	14.4	14.5	14.6	14.7	14.8	14.9	15.0	15.1	15.2	15.3	15.4	15.5	15.6	15.7	15.8	15.9	16.0	16.1	16.2	16.3	16.4	16.5	16.6	16.7	16.8	16.9	17.0	17.1	17.2	17.3	17.4	17.5	17.6	17.7	17.8	17.9	18.0	18.1	18.2	18.3	18.4	18.5	18.6	18.7	18.8	18.9	19.0	19.1	19.2	19.3	19.4	19.5	19.6	19.7	19.8	19.9	20.0	20.1	20.2	20.3	20.4	20.5	20.6	20.7	20.8	20.9	21.0	21.1	21.2	21.3	21.4	21.5	21.6	21.7	21.8	21.9	22.0	22.1	22.2	22.3	22.4	22.5	22.6	22.7	22.8	22.9	23.0	23.1	23.2	23.3	23.4	23.5	23.6	23.7	23.8	23.9	24.0	24.1	24.2	24.3	24.4	24.5	24.6	24.7	24.8	24.9	25.0	25.1	25.2	25.3	25.4	25.5	25.6	25.7	25.8	25.9	26.0	26.1	26.2	26.3	26.4	26.5	26.6	26.7	26.8	26.9	27.0	27.1	27.2	27.3	27.4	27.5	27.6	27.7	27.8	27.9	28.0	28.1	28.2	28.3	28.4	28.5	28.6	28.7	28.8	28.9	29.0	29.1	29.2	29.3	29.4	29.5	29.6	29.7	29.8	29.9	30.0	30.1	30.2	30.3	30.4	30.5	30.6	30.7	30.8	30.9	31.0	31.1	31.2	31.3	31.4	31.5	31.6	31.7	31.8	31.9	32.0	32.1	32.2	32.3	32.4	32.5	32.6	32.7	32.8	32.9	33.0	33.1	33.2	33.3	33.4	33.5	33.6	33.7	33.8	33.9	34.0	34.1	34.2	34.3	34.4	34.5	34.6	34.7	34.8	34.9	35.0	35.1	35.2	35.3	35.4	35.5	35.6	35.7	35.8	35.9	36.0	36.1	36.2	36.3	36.4	36.5	36.6	36.7	36.8	36.9	37.0	37.1	37.2	37.3	37.4	37.5	37.6	37.7	37.8	37.9	38.0	38.1	38.2	38.3	38.4	38.5	38.6	38.7	38.8	38.9	39.0	39.1	39.2	39.3	39.4	39.5	39.6	39.7	39.8	39.9	40.0	40.1	40.2	40.3	40.4	40.5	40.6	40.7	40.8	40.9	41.0	41.1	41.2	41.3	41.4	41.5	41.6	41.7	41.8	41.9	42.0	42.1	42.2	42.3	42.4	42.5	42.6	42.7	42.8	42.9	43.0	43.1	43.2	43.3	43.4	43.5	43.6	43.7	43.8	43.9	44.0	44.1	44.2	44.3	44.4	44.5	44.6	44.7	44.8	44.9	45.0	45.1	45.2	45.3	45.4	45.5	45.6	45.7	45.8	45.9	46.0	46.1	46.2	46.3	46.4	46.5	46.6	46.7	46.8	46.9	47.0	47.1	47.2	47.3	47.4	47.5	47.6	47.7	47.8	47.9	48.0	48.1	48.2	48.3	48.4	48.5	48.6	48.7	48.8	48.9	49.0	49.1	49.2	49.3	49.4	49.5	49.6	49.7	49.8	49.9	50.0	50.1	50.2	50.3	50.4	50.5	50.6	50.7	50.8	50.9	51.0	51.1	51.2	51.3	51.4	51.5	51.6	51.7	51.8	51.9	52.0	52.1	52.2	52.3	52.4	52.5	52.6	52.7	52.8	52.9	53.0	53.1	53.2	53.3	53.4	53.5	53.6	53.7	53.8	53.9	54.0	54.1	54.2	54.3	54.4	54.5	54.6	54.7	54.8	54.9	55.0	55.1	55.2	55.3	55.4	55.5	55.6	55.7	55.8	55.9	56.0	56.1	56.2	56.3	56.4	56.5	56.6	56.7	56.8	56.9	57.0	57.1	57.2	57.3	57.4	57.5	57.6	57.7	57.8	57.9	58.0	58.1	58.2	58.3	58.4	58.5	58.6	58.7	58.8	58.9	59.0	59.1	59.2	59.3	59.4	59.5	59.6	59.7	59.8	59.9	60.0	60.1	60.2	60.3	60.4	60.5	60.6	60.7	60.8	60.9	61.0	61.1	61.2	61.3	61.4	61.5	61.6	61.7	61.8	61.9	62.0	62.1	62.2	62.3	62.4	62.5	62.6	62.7	62.8	62.9	63.0	63.1	63.2	63.3	63.4	63.5	63.6	63.7	63.8	63.9	64.0	64.1	64.2	64.3	64.4	64.5	64.6	64.7	64.8	64.9	65.0	65.1	65.2	65.3	65.4	65.5	65.6	65.7	65.8	65.9	66.0	66.1	66.2	66.3	66.4	66.5	66.6	66.7	66.8	66.9	67.0	67.1	67.2	67.3	67.4	67.5	67.6	67.7	67.8	67.9	68.0	68.1	68.2	68.3	68.4	68.5	68.6	68.7	68.8	68.9	69.0	69.1	69.2	69.3	69.4	69.5	69.6	69.7	69.8	69.9	70.0	70.1	70.2	70.3	70.4	70.5	70.6	70.7	70.8	70.9	71.0	71.1	71.2	71.3	71.4	71.5	71.6	71.7	71.8	71.9	72.0	72.1	72.2	72.3	72.4	72.5	72.6	72.7	72.8	72.9	73.0	73.1	73.2	73.3	73.4	73.5	73.6	73.7	73.8	73.9	74.0	74.1	74.2	74.3	74.4	74.5	74.6	74.7	74.8	74.9	75.0	75.1	75.2	75.3	75.4	75.5	75.6	75.7	75.8	75.9	76.0	76.1	76.2	76.3	76.4	76.5	76.6	76.7	76.8	76.9	77.0	77.1	77.2	77.3	77.4	77.5	77.6	77.7	77.8	77.9	78.0	78.1	78.2	78.3	78.4	78.5	78.6	78.7	78.8	78.9	79.0	79.1	79.2	79.3	79.4	79.5	79.6	79.7	79.8	79.9	80.0	80.1	80.2	80.3	80.4	80.5	80.6	80.7	80.8	80.9	81.0	81.1	81.2	81.3	81.4	81.5	81.6	81.7	81.8	81.9	82.0	82.1	82.2	82.3	82.4	82.5	82.6	82.7	82.8	82.9	83.0	83.1	83.2	83.3	83.4	83.5	83.6	83.7	83.8	83.9	84.0	84.1	84.2	84.3	84.4	84.5	84.6	84.7	84.8	84.9	85.0	85.1	85.2	85.3	85.4	85.5	85.6	85.7	85.8	85.9	86.0	86.1	86.2	86.3	86.4	86.5	86.6	86.7	86.8	86.9	87.0	87.1	87.2	87.3	87.4	87.5	87.6	87.7	87.8	87.9	88.0	88.1	88.2	88.3	88.4	88.5	88.6	88.7	88.8	88.9	89.0	89.1	89.2	89.3	89.4	89.5	89.6	89.7	89.8	89.9	90.0	90.1	90.2	90.3	90.4	90.5	90.6	90.7	90.8	90.9	91.0	91.1	91.2	91.3	91.4	91.5	91.6	91.7	91.8	91.9	92.0	92.1	92.2	92.3	92.4	92.5	92.6	92.7	92.8	92.9	93.0	93.1	93.2	93.3	93.4	93.5	93.6	93.7	93.8	93.9	94.0	94.1	94.2	94.3	94.4	94.5	94.6	94.7	94.8	94.9	95.0	95.1	95.2	95.3	95.4	95.5	95.6	95.7	95.8	95.9	96.0	96.1	96.2	96.3	96.4	96.5	96.6	96.7	96.8	96.9	97.0	97.1	97.2	97.3	97.4	97.5	97.6	97.7	97.8	97.9	98.0	98.1	98.2	98.3	98.4	98.5	98.6	98.7	98.8	98.9	99.0	99.1	99.2	99.3	99.4	99.5	99.6	99.7	99.8	99.9	100.0
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Table 36  
(Continued)

CORRELATION WINDOW LENGTH, No 6

6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9	4.8	4.7	4.6	4.5	4.4	4.3	4.2	4.1	4.0
4.2	4.1	4.0	3.9	3.8	3.7	3.6	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8	2.7	2.6	2.5	2.4	2.3
6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9	4.8	4.7	4.6	4.5	4.4	4.3	4.2	4.1
6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9	4.8	4.7	4.6	4.5	4.4	4.3	4.2
6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9	4.8	4.7	4.6	4.5	4.4	4.3
6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9	4.8	4.7	4.6	4.5	4.4
6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9	4.8	4.7	4.6	4.5
6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9	4.8	4.7	4.6
6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9	4.8	4.7
6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9	4.8
6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0	4.9
6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1	5.0
7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2	5.1
7.1	7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3	5.2
7.2	7.1	7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3
7.3	7.2	7.1	7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4
7.4	7.3	7.2	7.1	7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5
7.5	7.4	7.3	7.2	7.1	7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6
7.6	7.5	7.4	7.3	7.2	7.1	7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8	5.7
7.7	7.6	7.5	7.4	7.3	7.2	7.1	7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9	5.8
7.8	7.7	7.6	7.5	7.4	7.3	7.2	7.1	7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	5.9
7.9	7.8	7.7	7.6	7.5	7.4	7.3	7.2	7.1	7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0
8.0	7.9	7.8	7.7	7.6	7.5	7.4	7.3	7.2	7.1	7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1











Table 38  
12 Best WB/sequences, N = 31.

CORRELATION WINDOW LENGTH, N = 4												
4.2												
4.2	4.2											
3.9	3.9	4.1										
3.9	3.9	4.1	4.1									
3.9	3.9	4.1	4.1	4.1								
3.9	3.9	4.1	4.1	4.1	4.1							
4.2	4.2	3.9	3.9	3.9	3.9	4.2						
4.2	4.2	3.9	3.9	3.9	3.9	4.2	4.2					
4.1	4.1	4.0	4.0	4.0	4.0	4.1	4.1	4.0				
4.1	4.1	4.0	4.0	4.0	4.0	4.1	4.1	4.0	4.0			
4.1	4.1	4.0	4.0	4.0	4.0	4.1	4.1	4.0	4.0	4.0		
4.1	4.1	4.0	4.0	4.0	4.0	4.1	4.1	4.0	4.0	4.0	4.0	
4.1	4.1	4.0	4.0	4.0	4.0	4.1	4.1	4.0	4.0	4.0	4.0	4.0
CORRELATION WINDOW LENGTH, N = 5												
5.4												
5.4	5.4											
4.9	4.9	5.2										
4.9	4.9	5.2	5.2									
4.9	4.9	5.2	5.2	5.2								
4.9	4.9	5.2	5.2	5.2	5.2							
5.4	5.4	4.9	4.9	4.9	4.9	5.4						
5.4	5.4	4.9	4.9	4.9	4.9	5.4	5.4					
5.1	5.1	5.0	5.0	5.0	5.0	5.1	5.1	5.1				
5.1	5.1	5.0	5.0	5.0	5.0	5.1	5.1	5.1	5.1			
5.1	5.1	5.0	5.0	5.0	5.0	5.1	5.1	5.1	5.1	5.1		
5.1	5.1	5.0	5.0	5.0	5.0	5.1	5.1	5.1	5.1	5.1	5.1	
CORRELATION WINDOW LENGTH, N = 6												
6.5												
6.5	6.5											
5.9	5.9	6.3										
5.9	5.9	6.3	6.3									
5.9	5.9	6.3	6.3	6.3								
5.9	5.9	6.3	6.3	6.3	6.3							
6.5	6.5	5.9	5.9	5.9	5.9	6.5						
6.5	6.5	5.9	5.9	5.9	5.9	6.5	6.5					
6.1	6.1	6.0	6.0	6.0	6.0	6.1	6.1	6.2				
6.1	6.1	6.0	6.0	6.0	6.0	6.1	6.1	6.2	6.2			
6.1	6.1	6.0	6.0	6.0	6.0	6.1	6.1	6.2	6.2	6.2		
6.1	6.1	6.0	6.0	6.0	6.0	6.1	6.1	6.2	6.2	6.2	6.2	
CORRELATION WINDOW LENGTH, N = 7												
7.7												
7.7	7.7											
6.8	6.8	7.3										
6.8	6.8	7.3	7.3									
6.8	6.8	7.3	7.3	7.3								
6.8	6.8	7.3	7.3	7.3	7.3							
7.7	7.7	6.8	6.8	6.8	6.8	7.7						
7.7	7.7	6.8	6.8	6.8	6.8	7.7	7.7					
7.1	7.1	7.0	7.0	7.0	7.0	7.1	7.1	7.3				
7.1	7.1	7.0	7.0	7.0	7.0	7.1	7.1	7.3	7.3			
7.1	7.1	7.0	7.0	7.0	7.0	7.1	7.1	7.3	7.3	7.3		
7.1	7.1	7.0	7.0	7.0	7.0	7.1	7.1	7.3	7.3	7.3	7.3	

Table 39

Sywyk's 6 crossoptimal Gold sequences of length  $N = 63$ .

```

# 1 1000100111110000110110101100101111011101001000000101000111001
# 2 01011110111110011001111111001010101000101011111010101100010
# 3 1110000101011011001111100111100100111110111011010110101110010
# 4 100101010000001111101111001110101100001011100011011010010001001
# 5 011011101001110101111111011011000001101101101011101000110011
# 6 11111101110010110110011010010111011011100110101101000010011
CORRELATION WINDOW LENGTH, N = 5

```

5.0

5.0 5.5

5.0 5.0 5.0

5.0 5.0 5.0 5.0

5.0 5.0 5.0 5.0 5.2

5.0 5.0 5.0 5.0 5.2 5.2

CORRELATION WINDOW LENGTH, N = 6

6.0

6.0 6.7

6.0 6.0 6.0

6.0 6.0 6.0 6.0

6.0 5.9 6.0 6.0 6.3

6.0 5.9 6.0 6.0 6.3 6.5

CORRELATION WINDOW LENGTH, N = 7

7.0

6.9 7.9

7.0 6.9 7.0

7.0 6.9 7.0 7.0

7.0 6.9 7.0 7.0 7.5

7.0 6.9 7.0 7.0 7.4 7.7

CORRELATION WINDOW LENGTH, N = 8

8.0

7.9 9.1

8.0 7.9 8.1

8.0 7.9 8.0 8.0

8.0 7.9 8.1 8.0 8.7

7.9 7.9 7.9 7.9 8.5 8.4

Table 40

8 LSE/AO small Kasami sequences of length  $h_1 = 103$ ,  $h_2 = 015$  of length  $N = 63$ .

```
# 1 00001000011000101001111010001110010010110111011001101010111110
# 2 011111011011110001011101000000010011011100010000001010000011001
# 3 00111111001111110111001010110110101111001011000000001110110100
# 4 10010100011011111111010010011111010101000101010010111001101101
# 5 011101110101110111101100001110001111011111000110011001000010110
# 6 100001001010101011000111010100110011110011000110100010010010001
# 7 010011010010100100000101111010110101000010001000111110010001011
# 8 110101100100110000011110000101011100001101100010011100111000000
```

CORRELATION WINDOW LENGTH,  $N = 5$

```
5.0
5.0 5.2
5.0 5.2 5.3
5.0 5.0 5.0 5.3
5.0 5.0 4.9 4.7 5.3
5.0 4.7 4.7 5.0 5.1 5.4
5.0 5.0 5.0 5.3 4.7 5.0 5.3
5.0 4.9 5.0 4.7 5.3 5.0 4.8 5.3
```

CORRELATION WINDOW LENGTH,  $N = 6$

```
6.0
6.0 6.4
6.0 6.2 6.4
6.0 6.1 5.9 6.5
6.0 5.9 5.9 5.6 6.5
6.0 5.6 5.6 6.0 6.2 6.6
6.0 5.9 6.1 6.3 5.6 5.9 6.5
6.0 5.9 6.0 5.6 6.3 6.0 5.7 6.5
```

CORRELATION WINDOW LENGTH,  $N = 7$

```
7.0
6.9 7.6
7.0 7.2 7.6
7.0 7.2 6.8 7.6
7.0 6.9 6.8 6.6 7.7
7.0 6.6 6.4 7.6 7.3 7.8
7.0 6.8 7.2 7.3 6.4 6.9 7.7
7.0 6.5 7.0 6.5 7.3 7.0 6.6 7.8
```

CORRELATION WINDOW LENGTH,  $N = 8$

```
8.0
7.9 8.7
8.0 8.2 8.6
8.0 8.3 7.8 8.6
8.0 7.9 7.7 7.5 8.8
8.1 7.5 7.3 8.0 8.4 9.0
8.0 7.8 8.4 8.3 7.3 7.8 8.9
8.1 7.6 8.0 7.4 8.3 7.9 7.6 9.0
```

## APPENDIX D

INTERFERENCE PARAMETER  $r_{k,i}$  FOR BEST 10 SEQUENCES  
AMONG ALL THE SIEVES USED

Table 41

Interference parameters  $r_{k,i}$  of best 10 Gold sequences for all the sieves used,  $h_1 h_t = 3013$ .

1	2182	1826	1546	1742	1794	1702	1774	1686	1626	1802
2		2318	1622	1594	1766	1714	1906	1762	1966	1982
3			2478	1658	1662	1370	1906	1658	1870	1934
4				2494	1554	1622	1622	1446	1778	1842
5					2646	1810	1482	1602	1678	1646
6						2670	1534	1558	1258	1714
7							2710	1670	2050	1786
8								2678	1330	1586
9									2838	1846
10										2918

The corresponding worst-case total interference,  $r = 16138$

Table 42

Interference parameters  $r_{k,i}$  of best 10 Gold sequences for all the sieves used,  $h_1 h_t = 3551$ .

1	2182	1890	1538	1850	1598	1634	1762	1794	1866	1778
2		2206	1590	1742	1586	1702	1750	1878	1782	1814
3			2462	1590	1714	1822	1542	1710	1622	1590
4				2494	1314	1518	1718	1510	1798	1750
5					2510	1850	1530	2018	1546	1498
6						2590	1438	1734	1646	1542
7							2638	1486	1566	1598
8								2782	1886	1494
9									2902	1702
10										2910

The corresponding worst-case total interference,  $r = 15734$

Table 43

The initial loadings for best 10 sequences for all the sieves used.

	$h_1 h_t = 3551$	$h_1 h_t = 3013$
1	1174	1174
2	1734	0163
3	1253	0204
4	1552	1243
5	0455	1516
6	1632	1321
7	1259	1335
8	0577	1213
9	0065	0364
10	0254	1366

APPENDIX E

EFFECT ON SELF-INTERFERENCE OF RANDOMIZING A (GOLD) SEQUENCE SET

Table 44

AO/LSE Gold sequences,  $h_1 h_c = 3551$ 

	$r_{k,k}(u)$	$r_{k,k}(u+z) = r_{k,k}(u+\bar{z})$
1	2382	2318
2	2206	2366
3	2590	2878
4	3598	2830
5	3262	3294
6	3494	2518
7	3662	2510
8	2606	4078
9	3446	2526
10	2974	3294
11	3006	3390
12	3670	2662
13	3310	3566
14	2782	3198
15	2966	3110
16	2494	3614
17	3206	3190
18	3950	2734
19	2614	4422
20	3134	3710
21	3118	3758
22	3094	3622
23	4238	2798
24	3190	3558

Table 44

(Continued)

25	3406	3086
26	3126	3302
27	2910	4286
28	3054	4622
29	2638	5006
30	3822	3790
31	4462	2894
32	4486	2742
33	4622	2734