

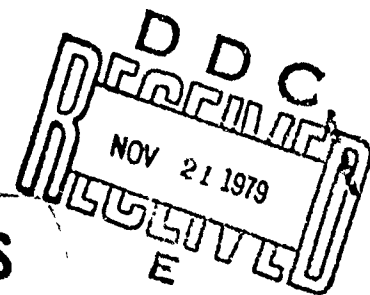
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(16) **THREE-DIMENSIONAL EDGE WAVES  
ON CURVED TOPOGRAPHIES**

by

(10) Richard Paul/Shaw/Professor

*Summary report*

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20.-  $h_M$  and a typical offshore distance  $x_M$ , solutions are given in terms of the nondimensional parameters  $h'_0$ ,  $x_M/h_M$  and  $h'_0$ .

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# ABSTRACT

Exact solutions are found to the linearized three dimensional equations for free surface gravity waves trapped against a straight coastline with a variable (perpendicular to the coastline) topography. Three families of topographies are found, one concave upwards and two convex upwards, which will support these edge waves as separable solutions to the original equations. For a given topography, specified by an initial slope,  $h'_0$ , a typical water depth,  $h_M$ , and a typical offshore distance,  $x_M$ , solutions are given in terms of the nondimensional parameters,  $h'_0 x_M / h_M$  and  $h'_0$ . The first parameter is a measure of how the initial slope  $h'_0$  compares to that for a straight topography from the origin to  $(x_M, h_M)$ , i.e.  $(h_M/x_M)$ . This parameter characterizes the families in ranges  $0 < h'_0 x_M / h_M < 1/2$ ,  $1/2 < h'_0 x_M / h_M < 1$  and  $1 < h'_0 x_M / h_M < (\ln 10)/0.9$ . (This latter constant bound is a function of the definition of offshore scale and can be modified to other values.)

The nondimensional frequency, period, vertical wave number, offshore decay rate and topography can be expressed in terms of the single parameter  $h'_0 x_M / h_M$ ; the velocities, wave height and alongshore wave number depend on both parameters.

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## INTRODUCTION

The trapping of wave energy along coastlines has been of theoretical interest since the original edge wave solution of Stokes (1846). Recently, there has been consideration of the physical reality of such trapped waves and a number of observations have led to a belief that such trapped waves may indeed play a role in beach erosion, local resonances, anisostatic response to traveling pressure disturbances, etc.; a review of these points is contained in LeBlond and Mysak (1977) and (1978). Some distinction must be made at the outset of any research as to the type of trapped waves to be discussed. Here the emphasis is on class I waves, i.e. gravity dominated, of frequencies high enough that Coriolis effects may be neglected, i.e. edge waves. These waves are basically high frequency, dispersive and can travel in either direction along a coastline; in fact, since no rotational effects are included and thus no direction is preferred, the coast line may be oriented in any direction.

The problem at hand then is to examine the existence of edge waves traveling along and trapped against a straight coastline bounding a semi-infinite ocean whose depth contours are parallel to the coastline. The ocean is considered to be a perfect homogeneous fluid, inviscid and irrotational. The purpose of the paper is to establish exact solutions for these three dimensional equations by reversing the usual question "what are the trapped solutions for a given topography" to read "what topographies will support trapped wave solutions". One solution already available for comparison is that of Stokes (1846) who considered a linear topography (constant slope) and obtained a "zeroth" mode solution which Ursell (1952) extended to a number of discrete modes plus a continuous spectrum. This was based on the previous work of Eckart (1951) who obtained higher modes using a shallow water (2D) approximation.

Other authors [e.g. Reid (1958)] added Coriolis effects to the shallow water theory. In fact, the bulk of the effort in this area has relied on the shallow water assumption [e.g. see p. 460 LeBlond and Mysak (1977)] and the lack of other exact three dimensional edge wave solutions was commented upon by Grimshaw (1974) who examined upper and lower bounds on such dispersion relationships. Some non-linear topographies have been considered. Robinson (1964) examined class II, i.e. quasi-geostrophic low frequency, waves on a linear sloping shelf of finite width terminated by a discontinuous drop to a constant depth infinite ocean. Ball (1966) used an exponential depth profile with solutions in the form of hypergeometric or Jacobi polynomials to examine both edge and continental shelf waves, i.e. both class I and class II waves. Mysak (1968) also discussed both classes, using Robinson's depth profile, where the solution is given in terms of Laguerre polynomials. Hidaka (1976-a) examined seiching due to a submarine bank described by a parabolic depth variation in terms of Mathieu functions. This is a trapped but not specifically an "edge" wave, but it is of class I nevertheless. In a related paper (Hidaka (1976-b)) he examined shelf resonances when the apex of the parabola reaches the free surface. Murty (1977) gives a general review of tsunami theory and current research, which includes theoretical and observational studies on edge waves caused by tsunamis (which are generally of high enough frequency that Coriolis effects may be neglected, i.e. class I waves) but are long enough that a two dimensional shallow water theory suffices.

Experiments performed by Ursell (1952) indicated the reality of such edge waves; further experiments by Galvin (1965) indicated that these could be excited (nonlinearly) by normally incident waves. Huntley and Bowen (1973) and Huntley (1976) have made observations which indicate the presence of edge waves

along coastlines in the U.K. while Nakamura (1962), Hatori and Takahasi (1964), Nakamura and Watanabe (1966), Aida (1967), Hatori (1967) and others have described the excitation of such edge waves by tsunamis incident onto the coast of Japan. There is other observational evidence, Wilson and Torum (1968), that some tsunami energy is trapped on the continental shelf in the generating region at least for the 1964 Alaskan Earthquake and it may be expected that some of this energy could be converted to and travel along the coastline as edge waves. Olsen and Hwang (1974) indicate that edge waves may play a significant role in near shore tsunami behavior.

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FORMULATION

The governing equations are well known, e.g. Lamb (1932) and are written here for a straight coastline,  $x = 0$ ,  $-\infty < y < +\infty$ , bounding a semi-infinite ocean,  $x > 0$  with a free surface at  $z = 0$  and a bottom topography,  $z = -h(x)$  with  $z$  positive upward:

$$(1) \quad \nabla^2 \phi(x, y, z, t) = 0; \quad x > 0, \quad -h < z < 0, \quad -\infty < y < \infty$$

$$(2) \quad \sigma^2 \phi - g \partial \phi / \partial z = 0; \quad x > 0, \quad z = 0, \quad -\infty < y < \infty$$

$$(3) \quad \partial \phi / \partial z = -(dh/dx) \partial \phi / \partial x; \quad x > 0, \quad z = -h, \quad -\infty < y < \infty$$

where a time harmonic behavior,  $\exp(-i\sigma t)$  has been assumed for the velocity potential,  $\phi$ . The Coriolis effect has been neglected and the  $y$  dependence will be taken as periodic (the edge wave assumption). Assuming separation of variables, the velocity potential is given as  $\phi = X(x) Z(z) \cos(ky - \sigma t)$ , using a real form for the  $y$  and  $z$  dependence, leading to:

$$(4) \quad X(x) = A \exp(-\ell x) + B \exp(+\ell x)$$

$$(5a) \quad Z(z) = C \sinh(nz) + D \cosh(nz)$$

$$(5b) \quad = C' \sin(mz) + D' \cos(mz)$$

where  $\ell^2$  is a separation constant and  $n^2 = k^2 - \ell^2 > 0$  and  $m^2 = \ell^2 - k^2 > 0$  provide two different solution forms for  $Z(z)$ . The coefficient  $B$  is taken to be zero to provide a "trapped" solution and  $A$  may be absorbed into  $Z(z)$ .

Substituting the first solution, (5a), into the boundary conditions provides

$$(6) \quad C/D = \sigma^2/gn$$

$$(7) \quad dh/dx = -(n/\ell) \frac{[\tanh(nh) - \sigma^2/gn]}{[1 - (\sigma^2/gn)\tanh(nh)]}$$

$$(a) \quad = -(n/\ell) \tanh[nh - \tanh^{-1}(\sigma^2/gn)]; \quad \sigma^2/gn < 1$$

$$(b) \quad = -(n/\ell) \coth[nh - \coth^{-1}(\sigma^2/gn)]; \quad \sigma^2/gn > 1$$

Using the boundary conditions in (5b) leads to

$$(8) \quad C'/D' = \sigma^2/gm$$

$$(9) \quad dh/dx = (m/l)\tan[mh + \tan^{-1}(\sigma^2/gm)]$$

Equations (7) and (9) can be integrated to give three families of topographies that give trapped solutions as summarized below:

$$\text{Case I:} \quad k^2 > l^2; \quad \sigma^2/gm < 1$$

$$\text{Case II:} \quad k^2 > l^2; \quad \sigma^2/gm > 1$$

$$\text{Case III:} \quad k^2 < l^2; \quad \text{all } \sigma^2/gm$$

SPECIFIC SOLUTIONS

Case I leads to a concave upward topography with  $h(0)$  set equal to zero (to prevent flux across the coastline), an asymptotic constant depth,  $h_\infty = [\tanh^{-1}(\sigma^2/gn)]/n$  and an initial slope,  $h'_0 \equiv (dh/dx)_0 = \sigma^2/gl$ . Using  $\theta = \tanh^{-1}(\sigma^2/gn)$ ,

$$h(x) = [\theta - \sinh^{-1}(\sinh \theta \cdot \exp[-n^2x/l])]/n$$

with a velocity potential given as

$$\phi = D \exp(-lx) \cos(ky - \sigma t) \cosh[n(z+h_\infty)] / \cosh(nh_\infty)$$

Case II ( $k^2 > l^2$ ,  $\sigma^2/gn > 1$ , i.e. eq. 7b) leads to a convex upward topography with an infinite slope at a finite value of  $h$  and  $x$ . Using  $\theta' = \coth^{-1}(\sigma^2/gn)$ ,

$$h(x) = [\theta' - \cosh^{-1}(\cosh \theta' \cdot \exp[-n^2x/l])]/n$$

The maximum extent of the topography is  $x_M = (l/n^2) \ln[\cosh \theta']$  which occurs at  $h_M = \theta'/n$ . The velocity potential in this case is, for  $x < x_M$ ,

$$\phi = D \exp(-lx) \cos(ky - \sigma t) \sinh[n(z+h_M)] / \sinh(nh_M)$$

When  $\sigma^2/gn = 1$ , both cases I and II recover the Stokes solution, with  $h = x \tan \alpha$ ,  $l = -k \cos \alpha$ ,  $n = -k \sin \alpha$  and  $\alpha = \tan^{-1}(h'_0)$ .

Case III ( $k^2 < l^2$ , all  $\sigma^2/gn$ ; i.e. eq. 9) using  $\gamma = \tan^{-1} \frac{\sigma^2}{gm}$  leads again to a convex upward topography with an infinite slope at  $x_M = -(l/m^2) \ln[\sin \gamma]$  with depth  $h_M = (\pi/2 - \gamma)/m$

$$h(x) = [\sin^{-1}(\sin \gamma \cdot \exp[+m^2x/l]) - \gamma]/m$$

with a velocity potential for  $x < x_M$  given by

$$\phi = D' \exp(-lx) \cos(ky - \sigma t) \sin[m(h_M + z)] / \sin(mh_M)$$

These last two families of solutions both appear to be physically unrealistic in that the prescribed bottom topography does not extend past a finite distance and finite depth before "doubling back." If, however, the resulting velocity fields for these problems have decayed sufficiently with distance from the straight coastline through the term  $\exp(-lx)$  and/or if they have decayed

sufficiently with depth to have both  $u$  and  $w$  negligible at the lower boundary before these "turning points" are reached, the point is moot since the lower boundary condition, eq. (3), is essentially satisfied identically for all  $x$  and  $z$  past these turning points. Thus these solutions may provide a basis for realistic trapped waves on a topography which matches that prescribed by these solutions prior to the turning points and a flat (or any other) topography past them.

The rest of this report will examine the implications of these solutions, particularly for the various limiting cases. To do so conveniently, it is useful to nondimensionalize the parameters and variables used. The coordinates  $x$ ,  $y$  and  $z$  are nondimensionalized with respect to  $h_M$  ( $h_\infty = h_M$  for case I) as are the parameters  $k$ ,  $l$  and  $n$  ( $m$  will be counted with  $n$  generically), i.e.  $(\bar{x}, \bar{y}, \bar{z}) = (x, y, z)/h_M$  and  $(\bar{k}, \bar{l}, \bar{n}) = (k, l, n)(h_M)$  as the case requires. The frequency is scaled to  $\sigma_0 = (g/h)^{1/2}$  as is the time, i.e.  $\bar{\sigma} = \sigma/\sigma_0$  and  $\bar{t} = t \cdot \sigma_0$ . Velocities,  $\nabla = \nabla\phi$ , are scaled by  $u_0 = u(x=0, y=0, z=0, t=0)$  and the wave height,  $\xi = -(1/g)\frac{\partial\phi}{\partial t}|_{z=0}$ , by  $\xi_0 = \xi(x=0, y=\pi/2k, z=0, t=0)$  which equals  $(\sigma/gk)u_0$ .

### SOLUTIONS AND LIMITING CASES

The above solutions are defined by three dimensional parameters or two nondimensional parameters, using  $h_M$  as a typical length scale. The choice of those parameters to be prescribed is important since it establishes how simply the results may be described. One such nondimensional "fundamental" parameter will be chosen as the initial slope,  $h'_0$ . The second choice depends on which aspect of the problem is under study. For examination of various "asymptotic solutions," the parameter  $\bar{U}^2/\bar{n}$  (or  $\theta$ ,  $\theta'$  or  $\gamma$ ) arises naturally in the derivation and will therefore be used for these cases. The remaining computations for the general solutions however will be carried out using  $h'_0 \bar{x}_M$  as the second "fundamental" parameter. For case I,  $x_M$  is defined as  $h(x = .9 x_M) = .9 h_\infty$ ; actually other choices could equally well be made to define a typical offshore length, e.g.  $h(x = (1-\delta)x_M) = (1-\delta)h_\infty$ . Then  $\bar{x}_M$  is  $x_M/h_M$  and the parameter  $h'_0 \bar{x}_M$  is the ratio of the actual initial slope to the slope of a straight line from the origin to  $(x_M, h_M)$ . This parameter characterizes the three families as well as did  $\bar{U}^2/\bar{n}$ .

Case I requires  $1 < h'_0 \bar{x}_M < (\ln 10)/.9$  (or more generally,  $(\ln 1/\delta)/(1-\delta)$ ) which is a curve whose initial slope  $h'_0$  is greater than  $h_M/x_M$  giving a curve lying below the straight line from the origin to  $(.9 x_M, .9 h_M)$  for  $x < x_M$ , i.e. is concave upwards. Case II and case III require  $0.5 < h'_0 \bar{x}_M < 1$  and  $0 < h'_0 \bar{x}_M < 0.5$  respectively; these both give topographies lying above (closer to the surface) than the straight line to  $(x_M, h_M)$ , i.e. are concave downwards.

Thus the three families can be represented by a continuous range of values of  $h'_0 \bar{x}_M$ , with some question as to the cross over values of 1.0 (which corresponds to the Stokes solution) and 0.5 (which presents no actual difficulty). The main advantage to use of this parameter is that the frequency,

$\bar{\sigma}$ , the vertical "wavenumber,"  $\bar{n}$ , the (modified) offshore decay rate,  $\bar{L}/\bar{x}_M$ , and the topography,  $\bar{h}(\bar{x}/\bar{x}_M)$  can be represented as functions of  $h'_0 \bar{x}_M$  alone; i.e. they are one parameter functions. The along shore wavenumber,  $\bar{k}$ , the wave height,  $\bar{\xi}$ , and the velocities,  $\bar{u}$ ,  $\bar{v}$ ,  $\bar{w}$ , depend on both  $h'_0 \bar{x}_M$  and  $h'_0$ , i.e. are two parameter functions. Thus much of the solution can be expressed quite simply in terms of a single parameter.

CASE I:

This is in many ways the most satisfying solution with a well behaved topography. Consider  $h_0$ ,  $h'_0$  and  $x_M = (1/.9) \times (h = .9h_0)$  to be specified. In non-dimensional terms,  $\bar{n}$  is given by  $h'_0$  and  $\bar{x}_M$  since  $\bar{h}_M$  is 1 by definition:

$$\frac{\tanh \bar{n}}{\bar{n}} = \ln \left[ \frac{\sinh \bar{n}}{\sinh .9\bar{n}} \right] = .9 h'_0 \bar{x}_M$$

with  $\bar{\sigma}$ ,  $\bar{l}$  and  $\bar{k}$  given by

$$\begin{aligned}\bar{\sigma} &= [\bar{n} \tanh \bar{n}]^{1/2} \\ \bar{l} &= [\bar{n} \tanh \bar{n}] / h'_0 \\ \bar{k} &= \bar{n} [1 + (\tanh^2 \bar{n}) / h_0'^2]^{1/2}\end{aligned}$$

If  $\bar{\sigma}^2 / \bar{n}$  were specified instead of  $h'_0 \bar{x}_M$ ,  $\bar{n}$  would be simply  $\tanh^{-1}(\bar{\sigma}^2 / \bar{n})$ , eliminating the need to solve a transcendental equation for  $\bar{n}$ .

The topography is given by

$$\bar{h}(\bar{x}) = 1 - (1/\bar{n}) \sinh^{-1} [\sinh \bar{n} \cdot \exp(-\bar{n}^2 \bar{x} / \bar{l})]$$

and the velocities and wave height by

$$\begin{aligned}\bar{u} &= \exp(-\bar{l}\bar{x}) \cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t}) \cosh[\bar{n}(1+\bar{z})] / \cosh \bar{n} \\ \bar{v} &= (\bar{k}/\bar{l}) \exp(-\bar{l}\bar{x}) \sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t}) \cosh[\bar{n}(1+\bar{z})] / \cosh \bar{n} \\ \bar{w} &= -(\bar{n}/\bar{l}) \exp(-\bar{l}\bar{x}) \cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t}) \sinh[\bar{n}(1+\bar{z})] / \cosh \bar{n} \\ \bar{\xi} &= \exp(-\bar{l}\bar{x}) \sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\end{aligned}$$

Note that, as  $\bar{n}$  approaches zero,  $h'_0 \bar{x}_M$  approaches 1 while as  $\bar{n}$  approaches infinity,  $h'_0 \bar{x}_M$  approaches  $(\ln 10) / .9$ . Furthermore,  $\bar{n}$  and  $\bar{\sigma}$  depend only on the combination  $h'_0 \bar{x}_M$ , while  $\bar{l}$ ,  $\bar{k}$ ,  $\bar{h}$  and the velocities require separate values for  $h'_0$  and  $\bar{x}_M$ . However,  $\bar{l}/\bar{x}_M$  and  $\bar{h}(\bar{x}/\bar{x}_M)$  depend only on  $h'_0 \bar{x}_M$ .

A: Consider the limiting case  $\bar{\sigma}^2/\bar{n} = \epsilon \ll 1$ . Approximations for the governing parameters are found to be

$$\bar{n} \approx \epsilon$$

$$\bar{l} \approx \epsilon^2/h'_0$$

$$\bar{\sigma} \approx \epsilon$$

$$\bar{k} \approx \epsilon[1 + (\epsilon/h'_0)^2]^{1/2}$$

$$h'_0 \bar{x}_M \approx (1/.9)[\ln 10 - \epsilon^2((\ln 10)/3 - 1/)]$$

The restriction on  $\bar{\sigma}^2/\bar{n}$  implies  $\sigma \ll \sigma_0$ , i.e. that a shallow water wave theory be used leading to the same topography for all applicable frequencies. The topography is given by

$$\bar{h}(\bar{x}) = 1 - \exp[-h'_0 \bar{x}]$$

which in turn is the same as that used by Ball (1967). Within the limitations of this case, the velocity potential is

$$\phi = D \exp[-\kappa^2 \bar{x}/h'_0] \cos[\kappa(1 + (\kappa/h'_0)^2)^{1/2} \bar{y} - \bar{\sigma} \bar{t}]$$

with  $\kappa \equiv \bar{\sigma}$ . This corresponds to Ball's solution for the case of zero rotation and mode number  $(n) = 0$ , i.e. the first term in his hypergeometric polynomial which then reduces to a simple exponential decay in  $\bar{x}$ . The corresponding velocity field has no vertical variation, an exponential decay with distance perpendicular to the shore line and a periodic wave behavior along the shore line, both of which depend on the nondimensional frequency and the initial beach slope. Since this form has already been obtained by Ball, in far greater generality, it will not be discussed further here.

B: Consider next the case when  $\bar{\sigma}^2/\bar{n}$  is near 1, i.e.  $\bar{\sigma}^2/\bar{n} = 1-\epsilon$ ;  $\epsilon \ll 1$ .

Then  $\bar{n} \approx \frac{1}{2} \ln(2/\epsilon)$  is a large number. The remaining parameters follow as

$$\bar{\sigma} \approx [\frac{1}{2} \ln(2/\epsilon)]^{1/2}$$

$$\bar{l} \approx (1/2 h'_0) \ln(2/\epsilon)$$

$$\bar{k} \approx (1/2) \ln(2/\epsilon) [1 + (1/h'_0)^2]^{1/2}$$

and

$$h'_0 \bar{x}_M \approx 1 - \ln(1 - (\epsilon/2)^{1/2}) / (.45 \ln(2/\epsilon))$$

i.e.  $h'_0 \bar{x}_M$  is slightly larger than 1.

The topography is given by

$$\bar{h}(\bar{x}) = 1 - (1/2 \ln(2/\epsilon))^{-1} \sinh^{-1} \left[ (c/2)^{(h'_0 \bar{x} - 1)/2} \right]$$

For  $h'_0 \bar{x} \ll 1$ , this reduces to a linear topography,  $\bar{h} = h'_0 \bar{x}$ .

Since  $h'_0 \bar{x}_M$  is close to 1, this implies that  $\bar{x} \ll \bar{x}_M$  for this linear topography. For these small values of  $\bar{x}$ , the velocities and wave height are

$$\bar{u} \approx (2/\epsilon)^{1/2} (\bar{z} - \bar{x}/h'_0) \cos [\bar{y} \cdot (1/2 \ln(2/\epsilon) (1 + (1/h'_0)^2)^{1/2} - (1/2 \ln(2/\epsilon))^{1/2} \bar{z}]$$

$$\bar{v} \approx (1 + h_0'^2) (2/\epsilon)^{1/2} (\bar{z} - \bar{x}/h'_0) \sin [ \quad \quad \quad " \quad \quad \quad ]$$

$$\bar{w} \approx -h'_0 (2/\epsilon)^{1/2} (\bar{z} - \bar{x}/h'_0) \cos [ \quad \quad \quad " \quad \quad \quad ]$$

$$\bar{\xi} \approx (2/\epsilon)^{-\bar{x}/2h'_0} \sin [ \quad \quad \quad " \quad \quad \quad ]$$

This solution recovers the classical Stokes' solution, but only for large  $\bar{n}$  and  $\bar{\sigma}$ . However, if the general solution is examined, the complete Stokes' solution (for all  $\bar{\sigma}$ ) is obtained for  $\bar{\sigma}^2/\bar{n}$  exactly equal to 1.0. This limiting process for case I can only provide the high frequency Stokes' solution since the regular Stokes' solution is actually neither in class I nor in class II but on their boundary. Since  $\bar{z} \leq 0$ , these velocities die out rapidly with increasing depth and distance offshore.

## CASE II:

This, together with case III, compose the solutions for  $h'_0 \bar{x}_M < 1$ , i.e. the concave downward topographies;  $\bar{n}$  is related to  $h'_0 \bar{x}_M$  through

$$h'_0 \bar{x}_M = \frac{\coth \bar{n}}{\bar{n}} \cdot \ln(\cosh \bar{n})$$

with

$$\bar{\sigma} = (\bar{n} \coth \bar{n})^{1/2}$$

$$\bar{l} = (\bar{n} \coth \bar{n})/h'_0$$

$$\bar{k} = \bar{n}[1 + (\coth^2 \bar{n})/h_0'^2]^{1/2}$$

and

$$\bar{h}(\bar{x}) = 1 - (1/\bar{n})\cosh^{-1}[\cosh \bar{n} \cdot \exp(-\bar{n}^2 \bar{x}/\bar{l})]$$

The velocities are then given by

$$\bar{u} = \exp(-\bar{l}\bar{x})\cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\sinh[\bar{n}(1 + \bar{z})]/\sinh \bar{n}$$

$$\bar{v} = (\bar{k}/\bar{l})\exp(-\bar{l}\bar{x})\sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\sinh[\bar{n}(1 + \bar{z})]/\sinh \bar{n}$$

$$\bar{w} = -(\bar{n}/\bar{l})\exp(-\bar{l}\bar{x})\cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t})\cosh[\bar{n}(1 + \bar{z})]/\sinh \bar{n}$$

and the wave height by

$$\bar{\xi} = \exp(-\bar{l}\bar{x})\sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t})$$

As  $\bar{n}$  approaches zero,  $h'_0 \bar{x}_M$  approaches 1/2 while as  $\bar{n}$  approaches infinity,  $h'_0 \bar{x}_M$  approaches 1 from below. Again,  $\bar{\sigma}$ ,  $\bar{l}/\bar{x}_M$ ,  $\bar{h}(\bar{x}/\bar{x}_M)$  and  $\bar{n}$  depend only on the single parameter  $h'_0 \bar{x}_M$ .

Limiting values of  $\bar{\sigma}^2/\bar{n}$  may again be examined.

A: Consider the limiting case  $\bar{\sigma}^2/\bar{n} = 1 + \epsilon$  where  $\epsilon \ll 1$ . Here,  $\bar{n} \approx 1/2 \ln(2/\epsilon)$  again and  $h'_0 \bar{x}_M \approx 1 - \frac{2 \ln 2}{\ln(2/\epsilon)}$  with

$$\bar{\sigma} \approx [1/2 \ln(2/\epsilon)]^{1/2}$$

$$\bar{\tau} \approx (\ln(2/\epsilon))/2h'_0$$

$$\bar{k} \approx (1/2h'_0) \ln(2/\epsilon) (1 + h_0'^2)^{1/2}$$

and, for  $\bar{x}h'_0 < 1$  (as it must be for  $\bar{x} < \bar{x}_M$ )

$$\bar{h}(\bar{x}) \approx h'_0 \bar{x}$$

with

$$\bar{u} \approx (2/\epsilon)^{(\bar{z} - \bar{x}/h'_0)/2} \cos[\bar{y} \cdot (1/2 \ln(2/\epsilon) \cdot (1 + (1/h'_0)^2)^{1/2} - (1/2 \ln(2/\epsilon))^{1/2} \bar{z}]$$

$$\bar{v} \sim (1 + h_0'^2)^{1/2} (2/\epsilon)^{(\bar{z} - \bar{x}/h'_0)/2} \sin[ \quad \quad \quad ]$$

$$\bar{w} \sim -h'_0 \cdot (2/\epsilon)^{(\bar{z} - \bar{x}/h'_0)/2} \cos [ \quad \quad \quad ]$$

$$\bar{\xi} \sim (2/\epsilon)^{-\bar{x}/2h'_0} \sin [ \quad \quad \quad ]$$

For small  $\bar{x}$ ,  $\bar{z}$ , the Stokes solution for large  $\bar{\sigma}$  is found again as would be expected. In fact, this solution is identical to I-B except for  $h'_0 \bar{x}_M$  which is greater than 1 for I-B and less than 1 here for non-zero  $\epsilon$ .

B: Next consider the limiting case  $\bar{\sigma}^2/\bar{n} = 1/c$  where  $\epsilon \ll 1$ . Here,  $\bar{n}$  is approximately  $\epsilon + (1/3)\epsilon^3$ ,  $h'_0 \bar{x}_M = 1/2 + \epsilon^2/12$  and

$$\bar{\sigma} \approx 1 + \epsilon^2/6$$

$$\bar{z} = (1/h'_0)(1 + \epsilon^2/3)$$

$$\bar{k} = [\epsilon^2 + (1/h'_0)^2]^{1/2}(1 + \epsilon^2/3)$$

and

$$\bar{h}(\bar{x}) = 1 - (1 - 2h'_0 \bar{x})^{1/2}$$

which for small  $h'_0 \bar{x}$  is linear again with slope  $h'_0$ . The velocity field and wave height are given by

$$\bar{u} \approx (1 + \bar{z})\exp(-\bar{x}/h'_0)\cos\{\bar{y} \cdot (\epsilon^2 + (1/h'_0)^2)^{1/2} - \bar{t}\}$$

$$\bar{v} \approx (1 + (\epsilon h'_0)^2)^{1/2}(1 + \bar{z})\exp(-\bar{x}/h'_0)\sin[ \quad ]$$

$$\bar{w} = h'_0 \cdot \exp(-\bar{x}/h'_0)\cos[ \quad ]$$

$$\bar{\xi} = \exp(-\bar{x}/h'_0)\sin[ \quad ]$$

Note that, to the lowest order in  $\epsilon$ ,  $h'_0 \bar{x}_M$ ,  $\bar{\sigma}$  and  $\bar{z}$  are essentially independent of  $\epsilon$ , as are  $\bar{k}$ ,  $\bar{u}$ ,  $\bar{v}$  and  $\bar{w}$  if  $h'_0 \ll 1$  as well.

The topography is parabolic, with a maximum horizontal extent of  $1/2 h'_0$  as expected. The horizontal velocities decrease linearly with depth, but do not vanish at the bottom until  $\bar{h}$  equals  $\bar{h}_M$ , i.e.  $\bar{z}$  can reach -1.

The vertical velocity is constant with depth and is  $O(h'_0)$  compared to the horizontal velocities; this is then a small value unless  $\bar{x}_M$  is itself of order  $O(1)$ , i.e. the initial slope is  $45^\circ$  or greater which is physically unlikely.

CASE III: ( $\bar{k}^2 < \bar{\ell}^2$ )

Here the range of  $h'_0 \bar{x}_M$  is from 0, as  $\bar{m}$  approaches  $\pi/2$  to  $1/2$ , as  $\bar{m}$  approaches zero with the general relation

$$-\frac{\cot \bar{m} \cdot \ln(\cos \bar{m})}{\bar{m}} = h'_0 \bar{x}_M$$

with

$$\bar{\sigma} = (\bar{m} \cot \bar{m})^{1/2}$$

$$\bar{\ell} = (\bar{m} \cot \bar{m})/h'_0$$

$$\bar{k} = \bar{m} \left[ \left( \frac{\cot \bar{m}}{h'_0} \right)^2 - 1 \right]^{1/2}$$

and

$$\bar{h}(\bar{x}) = \frac{\sin^{-1}[\sin \gamma \cdot \exp(+\bar{m}^2 \bar{x}/\bar{\ell})] - \gamma}{\bar{m}}$$

with

$$\gamma = -\bar{m} + \pi/2 \quad \text{and}$$

$$\bar{u} = \exp(-\bar{\ell}\bar{x}) \cdot \cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t}) \cdot \sin[\bar{m}(1 + \bar{z})]/\sin \bar{m}$$

$$\bar{v} = (\bar{k}/\bar{\ell}) \exp(-\bar{\ell}\bar{x}) \sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t}) \sin[\bar{m}(1 + \bar{z})]/\sin \bar{m}$$

$$\bar{w} = -(\bar{m}/\bar{\ell}) \exp(-\bar{\ell}\bar{x}) \cos(\bar{k}\bar{y} - \bar{\sigma}\bar{t}) \cos[\bar{m}(1 + \bar{z})]/\sin \bar{m}$$

$$\bar{\xi} = \exp(-\bar{\ell}\bar{x}) \sin(\bar{k}\bar{y} - \bar{\sigma}\bar{t})$$

A: The first limiting case here is  $\bar{\sigma}^2/\bar{m} = \epsilon \ll 1$ . Then  $\gamma \approx \epsilon$  and

$$\bar{m} \approx \pi/2 - \epsilon \quad \text{with} \quad h'_0 \bar{x}_M \approx -\frac{2\epsilon \ln \epsilon}{\pi}$$

$$\bar{\sigma} \approx (\pi \epsilon/2)^{1/2}$$

$$\bar{\ell} \approx \epsilon \pi/2 h'_0$$

$$\bar{k} \approx (\pi/2) [(\epsilon/h'_0)^2 - 1]^{1/2}$$

and

$$\bar{h}(\bar{x}) \approx 2/\pi [\sin^{-1}(\exp[-\pi(\bar{x}_M - \bar{x})h'_0/2\epsilon]) - \epsilon]$$

Clearly  $\epsilon > h'_0$  in order to have real  $\bar{k}$ , i.e. a progressive wave solution.

If  $\bar{x}$  is small enough such that  $\exp(\pi\bar{x}h'_0/2\epsilon) \approx 1 + \pi\bar{x}h'_0/2\epsilon$  the linear topography solution is found again, i.e.

$$\bar{h} \approx h'_0 \bar{x}$$

The velocities and wave height are

$$\begin{aligned}\bar{u} &\approx \exp(-\epsilon\pi\bar{x}/2h'_0) \cdot \cos[\bar{y} \cdot \pi/2 \cdot ((\epsilon/h'_0)^2 - 1)^{1/2} - (\pi\epsilon/2)^{1/2} \bar{z}] \sin[\pi/2(1 + \bar{z})] \\ \bar{v} &\approx [1 - (h'_0/\epsilon)^2]^{1/2} \cdot \exp(-\epsilon\pi\bar{x}/2h'_0) \sin[\quad] \sin[\pi/2(1 + \bar{z})] \\ \bar{w} &\approx -(h'_0/\epsilon) \exp(-\epsilon\pi\bar{x}/2h'_0) \cos[\quad] \cos[\pi/2(1 + \bar{z})] \\ \bar{\xi} &\approx \exp(-\epsilon\pi\bar{x}/2h'_0) \sin[\quad]\end{aligned}$$

Such a solution requires extremely small values of the initial slope,  $h'_0$ .

This in turn leads to a rapid decay of wave heights and velocities in the offshore direction due to the  $\exp[-\epsilon\pi\bar{x}/2h'_0]$ , e.g. for  $\epsilon$  equal to  $h'_0$  (such that  $k$  is zero corresponding to no alongshore variation), the velocities have decayed to 4% of their shoreline values when  $x$  has reached  $2h'_M$ . The wave periods are very long, which could cause Coriolis effects to become significant and thus invalidate this solution.

B: The final limiting case,  $\bar{\sigma}^2/\bar{M} = 1/\epsilon$ ,  $\epsilon \ll 1$  implies  $\gamma \approx \pi/2 - \epsilon$ ,  $\bar{m} \approx \epsilon$  and  $h'_0 \bar{x}_M \approx 1/2$  with  $\bar{\sigma} \approx 1$ ,  $\bar{\ell} \approx (1/h'_0)$ ,  $\bar{k} \approx (1 - (\epsilon h'_0)^2)^{1/2}/h'_0$  and  $\bar{h}(\bar{x}) \approx 1 - (1 - \pi\bar{x}h'_0)^{1/2}$  which reduces to the linear form when  $\bar{x}h'_0$  is small. The velocities and wave height in this case are

$$\bar{u} \approx (1 + \bar{z}) \exp(-\bar{x}/h'_0) \cos[\bar{y}(1 - (ch'_0)^2)^{1/2}/h'_0 - \bar{t}]$$

$$\bar{v} \approx (1 - (ch'_0)^2)^{1/2} (1 + \bar{z}) \exp(-\bar{x}/h'_0) \sin[ \quad " \quad ]$$

$$\bar{w} \approx -h'_0 \exp(-\bar{x}/h'_0) \cos[ \quad " \quad ]$$

$$\bar{z} \approx \exp(-\bar{x}/h'_0) \sin[ \quad " \quad ]$$

which match the limiting case II-B as they must since both of these correspond to the case  $h'_0 \bar{x}_M = 1/2$  which is common to both case II and case III.

Apart from the alongshore velocity,  $\bar{v}$ , and the corresponding alongshore dependence on  $\bar{ky}$ , this solution is independent of  $c$ . Furthermore, as  $c$  passes through zero, the match between case II-E and case III-B is complete, i.e. the transition from case II to case III is smooth.

## DISCUSSION OF GENERAL SOLUTION

Although the problem is completely specified by three dimensional (e.g.  $h'_0$ ,  $x_M$  and  $h_M$ ) or two nondimensional (e.g.  $h'_0$  and  $\bar{x}_M$ ), it is of practical interest to express results in terms of as few parameters as possible. This may be done for a portion of the present problem. The quantities  $\bar{n}$ ,  $\bar{\sigma}$ ,  $\bar{l}/\bar{x}_M$  are functions only of the single parameter ( $h'_0 \bar{x}_M$ ) and the topography  $\bar{h}$  is a function of the parameter  $h'_0 \bar{x}_M$  and the spatial variable  $\bar{x}/\bar{x}_M$ . Thus although the computer program, given as appendix A, uses three dimensional inputs,  $h'_0$ ,  $h_M$  and  $x_M$ , the results may be stated in a fairly compact form.

The vertical "wavenumber,"  $\bar{n}$ , is related to the single parameter  $h'_0 \bar{x}_M$  by  $h'_0 \bar{x}_M = F(\bar{n})$  where  $F(\bar{n})$  has different forms for the three cases considered and is plotted in Figure (1). Thus  $\bar{n}$  may be read from this figure for any value of  $h'_0 \bar{x}_M$  between 0 and  $(\ln 10)/0.9$  (or calculated through the program). Figure (2) provides the (nondimensional) period,  $\bar{T}$ , as a function of the single parameter,  $h'_0 \bar{x}_M$ . Since the nondimensionalization involves  $h_M$ , dimensional periods require knowledge of both  $h_M$  and  $h'_0 \bar{x}_M$ . At  $h'_0 \bar{x}_M$  equal to 1, there is an anomaly in the original solution. At this value, equation (7) reduces to  $dh/dx$  equal to a constant  $(-n/l)$  and  $h'_0 \bar{x}_M$  is 1 for all values of  $h'_0$ . In a sense, this is a "fourth" solution family, the well known Stokes solution, in that it can not be obtained from the other solutions, except as a high frequency limit. Thus Figure (2) has zero period at  $h'_0 \bar{x}_M$  equal to 1 with the Stokes solution indicated as a dashed line. The Stokes solution,  $h = + \tan \alpha \cdot x$ , requires  $\sigma^2 = gk \sin \alpha$ . Thus for any  $h'_0 = \tan \alpha$ , there is a single specific frequency,  $\sigma$ , at least for the lowest separable mode. Ursell's extension (1952) to higher modes is not comparable here since

those higher modes were not separable solutions and could not be obtained by this present approach.

A question then arises as to the appropriate solution for an "almost" linear topography. The examples here, which represent the only separable solutions--but clearly not necessarily the only solutions--will only support very high frequency trapped waves when  $h'_0 \bar{x}_M$  is near to 1 but not equal to 1.

The offshore decay parameter,  $\bar{\ell}$ , depends on both  $h'_0 \bar{x}_M$  and  $\bar{x}_M$  but in a manner such that  $\bar{\ell}/\bar{x}_M$  is a function of  $h'_0 \bar{x}_M$  only. This is shown in Figure (3) where  $\bar{\ell}/\bar{x}_M$  is given for  $h_M = 200$  meters and  $x_M = 200$  kilometers for various values of  $h'_0$  but valid of course for any choices of  $h_M$  and  $x_M$ . This figure indicates the limited range of practical solutions since all terms decay like  $\exp(-\bar{\ell}\bar{x})$  or equivalently  $\exp(-(\bar{\ell}/\bar{x}_M)(\bar{x}/\bar{x}_M)\bar{x}_M^2)$ . To have this exponential greater than 0.04 compared to a value of 1 at  $\bar{x} = 0$ ,  $(\bar{\ell}/\bar{x}_M) < \pi/(\bar{x}/\bar{x}_M)(\bar{x}_M)^2$ . Even for  $\bar{x}/\bar{x}_M$  up to 0.1 representing the region of measurable wave heights,  $(\bar{\ell}/\bar{x}_M) < 31.4/\bar{x}_M^2$ . Realistic choices for  $\bar{x}_M$  are of the order of 40 to 1000 for the continental slope and shelf respectively; these limit  $\bar{\ell}/\bar{x}_M < 2 \times 10^{-2}$  and  $3 \times 10^{-5}$  which in turn imply that  $h'_0 \bar{x}_M$  must be close to either zero or  $(\ln 10)/0.9$  in either case. Thus for realistic ocean topographic scales, the energy must be trapped very near shore.

Figure (4) shows the topography,  $h$ , as a function of  $\bar{x}/\bar{x}_M$  for several values of the parameter  $h'_0 \bar{x}_M$ , based on a choice of  $h_M$  equal to 200 meters and  $x_M$  equal to 200 kilometers but again valid in general. Those topographies for  $h'_0 \bar{x}_M > 1$  are forced to pass through (0.9, 0.9) by the choice of definition for  $x_M$ ; other choices would simply rescale these curves to pass through (1- $\delta$ , 1- $\delta$ ), i.e. appear steeper for  $\delta < 0.1$  or shallower for  $\delta > 0.1$ . As  $h'_0 \bar{x}_M$  approaches 1.0, the topography appears

almost linear except near  $\bar{x}/\bar{x}_M$  equal to 1. For values of  $h'_0 \bar{x}_M$  less than 1, convex upward topographies are found with an infinite vertical slope at  $\bar{x}/\bar{x}_M$  equal to 1.0. The transition across  $h'_0 \bar{x}_M$  equal to 0.5 is smooth with no ascertainable peculiarities.

Calculations of velocities and waveheights produced extremely small values for any offshore distance that itself was not a very small fraction of  $x_M$ . For example, with  $h'_0 = 0.002556$ ,  $h_M = 200$  meters,  $x_M = 200$  kilometers the velocity and waveheight values had reached 4% of their maximum values when  $x$  reached 0.2% of  $x_M$ , e.g. about 400 meters--at which distance the water depth was approximately one meter. This implies that the edge wave was completely trapped in the region where the model is the poorest representation of the overall topography and is therefore inadequate. For this reason, no results are shown for this case.

For much smaller values of  $\bar{x}_M$  however, it is possible to illustrate the depth dependence for velocities. Figures (5) and (6) show results for  $\bar{x}_M = 1.0$ . The first plots the offshore velocity,  $\bar{u}$ , as a function of  $(z/\bar{x}_M)$  and  $\bar{z}$  for  $h'_0 = 1.01$ . The second illustrates the different coastline values for  $\bar{v}$  and  $\bar{w}$  as compared to the reference value of 1 for  $\bar{u}$ , as a function of  $h'_0 \bar{x}_M$ . There is a cutoff at  $h'_0 = 0.2854$  for  $\bar{x}_M = 1.0$  below which  $k$  becomes imaginary. This corresponds to the requirement that  $\bar{m} > \ln(\sec \bar{m})$ . Similarly near  $h'_0 \bar{x}_M = (\ln 10)/0.9$ ,  $\bar{v}$  behaves like  $(1 + (h'_0/\epsilon)^2)^{1/2}$ , as defined in case I-A, while  $\bar{w}$  behaves like  $-h'_0$ .

## CONCLUSION

The bulk of observations have been for shallow water-long wavelength situations such that the question of vertical variation does not enter, as befits the general emphasis on two dimensional theory for edge waves. All tsunami observations are of this form as were the observations of Huntley and Bowen (1973) who found evidence for the lowest mode edge wave of Ball's (1967) solution corresponding to case I-A here. Ursell (1952) described experiments on a linear topography but did not give any details of vertical structure. Thus it appears that separate observations and/or experiments may be necessary to detect the three dimensionality of those edge waves described here. Whether such observations are warranted is another question. Wavelengths that are significant enough to carry appreciable energy alongshore may also need to be long enough to allow a two dimensional theory to apply. Such a theory allows for far more general topographies than does the three dimensional theory, as discussed in appendix B. This is clearly a question for further consideration on physical grounds.

Assuming this research to be physically justified, there are still several mathematical questions left unanswered. First, are there non-separable solutions, analogous to Ursell's (1952) solutions, for these or related topographies and if so how may they be found? Second, what is the appropriate three dimensional edge wave solution for an "almost" linear topography? Is it close to the original Stokes solution or must it be close to the high frequency limit given here?

One extension of this work would be towards alongshore variations in topography. If the topography varied periodically along the coastline (i.e. in  $y$ ) the edge waves would be filtered as Floquet waves, e.g. Brillouin (1953), and only certain frequencies and wavelengths could

propagate. Alternatively, the effect of a small irregularity in the otherwise straight coastline could be studied, e.g. as by Fuller and Mysak (1977).

The extension to lower frequencies such that Coriolis effects become significant does not appear to be appropriate in context of a three dimensional theory since these lower frequencies would imply shallow water-long wave theory to be a good approximation in the real ocean. This then brings up the question of the applicability of two dimensional theories to the present examples. Case I-A is essentially shallow water theory but the remaining asymptotic solutions do have some depth variation as does the general solution. Thus two dimensional theories give quite different results than those given here as seen in appendix B. Even case III-A, which is a low frequency solution, requires very small  $h'_0$  such that a two dimensional solution may apply for  $\bar{x} \ll \bar{x}_M$  eventually (for  $\bar{x}$  near  $\bar{x}_M$ ) vertical variation is required.

However over the regions of interest, i.e. where the waves have not decayed to negligible values, even the asymptotic solutions do not exhibit appreciable vertical variations for realistic values of  $h'_0$  and  $\bar{x}_M$  since  $\bar{z}$  never got appreciably different from 1 (the free surface). There are clearly values for the input parameters where these vertical variations may be significant, but they do not appear to be physically realistic, e.g.  $\bar{x}_M < 1$ .

## REFERENCES

- Aida, I., 1967. Water level oscillations on the continental shelf in the vicinity of Miyagi-Enoshima. *Bull. Earthquake Res. Inst.*, 45: 61-78.
- Ball, F.K., 1967. Edge waves in an ocean of finite depth. *Deep Sea Res.*, 14: 79-88.
- Bateman, H. (Bateman Manuscript Project), Erdelyi, A. (Editor), 1954. *Higher Transcendental Functions*, 3 Vol.. McGraw Hill Book Co., Inc., New York.
- Brillouin, L., 1953. *Wave Propagation in Periodic Structures*. Dover, New York, 255 pp.
- Eckart, C., 1951. Surface waves in water of variable depth. *Marine Physical Laboratory of the Scripps Institute of Oceanography, LaJolla, Cal.*, Wave Rep. No. 100, S.I.O. Ref. 51-12, 99 pp.
- Fuller, J.D. and Mysak, L.A., 1977. Edge waves in the presence of an irregular coastline, *J. Phys. Oceanogr.*, 7: 846-855.
- Galvin, C.J., 1965. Resonant edge waves on laboratory beaches. *Trans. Am. Geophys. Union*, 46: 112 (abstract).
- Grimshaw, R., 1974. Edge waves: a long-wave theory for oceans of finite depth. *J. Fluid Mech.*, 62: 775-791.
- Hatori, T., 1967. The wave form of the tsunami on the continental shelf. *Bull. Earthquake Res. Inst.*, 45: 79-90.
- Hatori, T. and Takahasi, R., 1964. On the Iturup tsunami of October 13, 1963 as observed along the coast of Japan. *Bull. Earthquake Res. Inst.*, 42: 543-554.
- Hidaka, K. (Bretschneider, C., Editor), 1976. Seiches Due to a Submarine Bank, *Topics in Ocean Eng. III*. Gulf Pub. Co., Houston, Tx., 137-140.
- Hidaka, K. (Bretschneider, C., Editor), 1976. A Theory of Shelf Seiches, *Topics in Ocean Eng. III*. Gulf Pub. Co., Houston, Tx., 141-143.
- Huntley, D.A., 1976. Long-period waves on a natural beach. *J. Geophys. Res.*, 81: 6441-6449.
- Huntley, D.A. and Bowen, A.J., 1973. Field observations of edge waves. *Nature*, 243 (5403): 160-162.
- Lamb, H., 1945. *Hydrodynamics* (6th Edition). Dover, New York.
- LeBlond, P.H. and Mysak, L.A., 1978. *Waves in the Ocean*, Elsevier Oceanography Series 20. Elsevier Scientific Pub. Co., New York.
- LeBlond, P.H. and Mysak, L.A. (Goldberg, McCave, O'Brien and Steele, Editors), 1977. Trapped Coastal Waves and their Role in Shelf Dynamics. *Wiley-Interscience*, 6: 459-495.
- Murty, T.S., 1977. *Seismic Sea Waves--Tsunamis*. Fisheries and Environment, Bulletin 198, Ottawa, Canada.
- Mysak, L.A., 1968. Edge waves on a gently sloping continental shelf of finite width. *J. Mar. Res.*, 26 (1): 24-33.
- Nakamura, K., 1962. The generation of edge waves by cylindrical waves impinging from the outer sea. *Sci. Rep., Tohoku Univ., Ser. 5*, 14: 27-40.
- Nakamura, K. and Watanabe, H., 1966. Edge waves and tsunamis. *Proc. 11th Pac. Sci. Cong.*, Tokyo, Japan, 1-4, 163.
- Olsen, K.H. and Hwang, L.S. (Heath, R.A. and Cresswell, M.M., Editors), 1974. Edge wave generation and associated bay responses. *Tsunami Research Symposium*, Roy. Soc. of New Zealand, Wellington, N.Z., *Bull.* 15, 253 (abstract).

- Reid, R.O., 1958. Effect of coriolis force on edge waves: Part I: Investigation of normal modes. J. Marine Res., 16: 109-144.
- Robinson, A.R., 1964. Continental shelf waves and the response of sea level to weather systems. J. Geop. Res., 69: 367-368.
- Stokes, G., 1846. Report on recent researches in hydrodynamics. Brit. Ass. Rep. (also "Papers" 1, 167).
- Ursell, F., 1952. Edge waves on a sloping beach. Proc. Roy. Soc. London, A 214, 79-97.
- Wilson, B.W. and Torum, A., 1968. The tsunami of the Alaskan earthquake, 1964: engineering evaluation. Tech Memo No. 25, C.E.R.C., U.S. Army Corps of Eng..

#### ADDITIONAL REFERENCES FOR APPENDIX

- Shaw, R.P., 1977. Long Waves Obliquely Incident on a Continental Slope and Shelf with a Partially Reflecting Coastline. Proc. of IUGG Tsunami Symposium, Ensenada, Mexico, 1977 (Manuscript Report #48, Dept. of Fisheries and Environment, Ottawa, Canada) - also Marine Geodesy, Vol 2(1), pp1-14, 1979

## APPENDIX A

The following computer program accepts a single card IJOB as the number of jobs (input data sets) and then several "jobs" each with  $h'_0$ ,  $h'_M$  and  $x_M$  (in cm) as input. The card IVEL = 0 behind IVEL = 1 will surpass velocity output while the reverse will provide velocities at  $\bar{x}/\bar{x}_M = 0.0$  (0.1) 1.0 and  $\bar{z}/\bar{h} = 0.0$  (0.2) 1.0. The remaining output is labeled; the symbol D in front of a variable means the dimensional form (in cgs units). The symbol HPX is  $h'_0 \bar{x}_M$ , HOP is  $h'_0$ , SIGMA0 is  $\sigma_0$ , etc.. Typical running times for twenty different input cards is about 2.5 seconds on a CDC CYBER 173.

```

1  INPUT TAPE5,ELL (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
2  CALL H2(100),H2(100),H2(100),H3(101)
3  TIME SIG=U(41,41),V(41,41),W(41,41)
4  SIGMSIG=F1(21),F2(21),F3(21)
5  REAL K1,K2,K3,M,L1,L2,L3
6  ATANH(X)=0.5*ALOG((1+X)/(1-X))
7  ACOTH(X)=0.5*ALOG((1+X)/(X-1))
8  COSH(X)=ALOG(X+SQRT(X**2+1))
9  SINH(X)=ALOG(X+SQRT(X**2+1))
10 TANH(X)=0.5*(EXP(X)-EXP(-X))
11 COSH(X)=0.5*(EXP(X)+EXP(-X))
12 TANH(X)=SINH(X)/COSH(X)
13 COT(X)=1./TANH(X)
14 FF1(X)=TANH(X)*ALOG(SINH(X)/SINH(X/10.))/(X*0.9)
15 FF2(X)=ALOG(COSH(X))/(X*TANH(X))
16 FF3(X)=-ALOG(COS(X))/(X*TAN(X))
17 C=437.
18 PI=3.1415926
19 IV=L=
20 IVCL=
21 IRR1=
22 IRR2=
23 IRR3=
24 IRR4=
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      XH = (XR + XL)/2.
      FXM1 = FF1(XH)
60      IF( XL .EQ. 0.0 ) FXL1 = F1(1)
      IF( XL .GT. 0.0 ) FXL1 = FF1(XL)
      C PRINT*, 5H YH = ,XM, 10H FF1(YH) = ,FXM1 10H FF1(XL) = ,FXL1
      TL = ( FXM1 - HPX )*( FXL1 - HPX )
      IF( TL .LT. 0.0 ) XP = XM
65      IF( TL .GE. 0.0 ) YL = XM
      IF ( ABS(XR-XL) .LE. ERRX ) GO TO 74
73      CONTINUE
      PRINT*, 40H F1 HALVING ROUTINE DID NOT PRODUCE ROOT
      PRINT*, 5H XR = ,XR, 5H XL = ,XL
70      GO TO 55
74      N1 = (YR + XL)/2.
      GO TO 64
67      N1 = 10.
      GO TO 64
75      60 CONTINUE
      PRINT*, 55H THIS VALUE OF HPX CORRESPONDS TO N1.GT. 1
      N1 = 10.
      DO 76 J = 1,21
      OLDN1 = N1
80      N1 = -0.3+4LOG( 1.0-EXP(-0.9+.1*(HPX - 1.0)))
      IF( ABS( (N1-OLDN1)/N1 ) .LT. 0.0001 ) GO TO 64
76      CONTINUE
      PRINT*, 45H "1 ITERATION .GT. 1" DOES NOT CONVERGE
      PRINT*, 5H N1 = ,N1, 5H DN1 = ,OLDN1
85      GO TO 55
64      CONTINUE
      PRINT*, 6H N1 = ,N1
      IF(N1 .GT. 49.) S1 = SQRT (N1)
      IF(N1 .LE. 49.)
90      S1 = SQRT( N1+TANH(N1))
      L1 = S1**2/HPX
      K1 = SQRT(N1**2 + L1**2)
      PRINT*, 5H N1 = ,N1, 5H L1 = ,L1, 5H K1 = ,K1, 5H S1 = ,S1, 5H XH1 =
      DN1 = N1/HPX
95      DL1 = L1/HPX
      DK1 = K1/HPX
      DS1 = SIGMA0*S1
      PRINT*, 6H DN1 = ,DN1, 6H DL1 = ,DL1, 6H DK1 = ,DK1, 6H DS1 = ,DS1
      T1 = 2.*PI/S1
      DT1 = 2.*PI/DS1
100      PRINT*, 6H DT1 = ,DT1 5H T1 = ,T1
      DO 15 I = 1,11
      X = (I-1)*XH1/10.
      ARG1 = SINH(N1) * EXP(-N1**2**7/L1)
105      H1(I) = 1. - ASINH(ARG1)/N1
      Y = X/XH1
      PRINT*, 4H X = ,Y, 5H H1 = ,H1(I)
      IF( IVEL.EQ. 0 ) GO TO 2H
      DO 155 IZ = 1,6
110      DZ = H1(I)/ 5.
      Z = -(IZ-1)*DZ
      XND = X/XH1
      YL1 = X*L1
      IF(XL1.GT. 100.) FAC = 1.

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117      IF(XL1.LT. 1.0) FAC = EXP(-XL1)/COSH(N1)
      U(I,IZ) = FAC*COSH(V1*(1.+Z))
      V(I,IZ) = (V1/L1)*FAC*COSH(N1*(1.+Z))
      W(I,IZ) = -(V1/L1)*FAC*SINH(N1*(1.+Z))
      WRITE(6,95) Y,XND,Z,U(I,IZ),V(I,IZ),W(I,IZ)
120      95 FORMAT(15.5)
121      CONTINUE
122      CONTINUE
123      CONTINUE
      GO TO 95
124      IF(HPX.LT. 0.5) GO TO 29
125      PRINT*, 'ON CASE 2 HPX .LT. 1.0 AND .GT. 0.5'
      XND = 0.0
      CO(1) = 0.5
      DO 11 J = 2,11
126      Y = J-1.
      CO(J) = FF2(Y)
      PRINT*,4,X=,Y,5H F2(J)=,F2(J)
127      11 CONTINUE
      IF(F2(11) - HPX) 86,87,88
128      DO 11 J = 1,11
      TEST = (F2(J) - HPX)*(F2(J+1) - HPX)
      IF(TEST.LT. 0.) GO TO 92
      IF(F2(J) .EQ. HPX) N2 = J - 1
      IF(F2(J) .GT. HPX) GO TO 84
129      11 CONTINUE
      PRINT*,4,H CASE FIND ROOT TO F2 IN RANGE
      GO TO 95
130      YL = J-1.
      XR = J
      DO 12 I = 1,21
      XM = (XR + YL)/2.
      FXM = FF2(XM)
      IF(XL .GT. 0.) FYL2 = F2(1)
      IF(YL .LT. 0.) FYL2 = FF2(YL)
      FXM2 = (FXM2 - HPX)*(FXL2 - HPX)
131      IF(FXM2 .LT. 0.) XR = XM
      IF(FXM2 .GT. 0.) XL = XM
      IF(ABS(XR - YL) .LE. EPRX) GO TO 94
132      12 CONTINUE
      PRINT*,4,H F2 HALVING ROUTINE DID NOT PRODUCE ROOT
      PRINT*,4,X=,XR,5H YL=,YL
      GO TO 95
133      12 = (XR + YL)/2.
      GO TO 12
134      12 = 1.0
      GO TO 84
135      CONTINUE
      PRINT*,4,H THIS VALUE OF HPX CORRESPONDS TO N2.GT. 10
      N2 = ALN(2.)/(1. - HPX)
      DO 121 I = 1,21
      OLPN2 = 0.2
      IF(N2 .LT. 49.) EFAC = EXP(-2.*N2)
      IF(N2 .GT. 49.) EFAC = 0.
      12 = (-ALN(2.) + EFAC) / (HPX/(1. + 2.*EFAC) - 1.)
      IF(ABS(12 - OLPN2) .LT. FRKN) GO TO 121

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120 CONTINUE
PRINT*,4TH M2 ROOT .GT. 10 DUEFS NOT CONVERGE
PRINT*,5H M2= ,N2,8H OLON2= ,OLON2
GO TO 94
175
121 PRINT*,5H M2= ,N2
84 CONTINUE
PRINT*,5H M2= ,N2
IF(N2 .EQ. 0.0) GO TO 55
IF( M2 .GT. 49.) S2 = SQRT(M2)
IF(N2 .LT. 49.)
180
1S2 = SQRT(M2/TANH(N2) )
L2 = S2*.2/400
K2 = SQRT( M2*.2 + L2*.2)
195
PRINT*,5H M2= ,N2,5H L2= ,L2,5H K2= ,K2,5H S2= ,S2,5H X2= ,X
DS2 = S2*SIGMA0
DN2 = N2/HM
DL2 = L2/HM
DK2 = K2/HM
190
PRINT*,4H DN2= ,DN2,4H DL2= ,DL2,4H DK2= ,DK2,5H DS2= ,DS2
T2 = 2.*PI/S2
DT2 = 2.*PI/DS2
PRINT*,4H DT2 = ,DT2 ,5H T2= ,T2
DO 25 I = 1,11
195
X = (I-1)*X2/10.
IF(1.EQ.11) Y = .9999*X2
ARG2 = COSH(N2)*EXP(-N2*.2*X/L2)
H2(I) = 1. - ACOSH(ARG2)/2
Y = X/X2
200
PRINT*,4H Y= ,Y,5H H2= ,H2(I)
IF(1VEL.EQ.1) GO TO 15
DO 255 IZ = 1,6
DZ = H2(I) / 5.
Z = -(I7-1)*DZ
205
XND = X/X2
XL2 = X*L2
IF(XL2.GT. 100.) FAC = 0.
IF(XL2.LE. 100.) FAC = EXP(-XL2)/SINH(N2)
U(I,I7) = FAC*SINH(N2*(1.+Z))
210
V(I,I7) = (K2/L2)*FAC*SINH(N2*(1.+Z))
W(I,I7) = -(N2/L2)*FAC*COSH(N2*(1.+Z))
WRITE(6,45) X,XND,Z,U(I,I7),V(I,I7),W(I,I7)
255 CONTINUE
25 CONTINUE
29 CONTINUE
215
IF( HPY .GT. 0.5) GO TO 55
PRINT*, 50H CASE 3 HPY .LT. 0.5
XM3 = XM/HM
F3(1) = 0.5
220
DO 101 J = 2,11
X = (J-1)*PI/20.
F3(J) = FF3(Y)
C PRINT*,4H Y= ,X,4H F3(J)= ,F3(J)
101 CONTINUE
225
DO 111 J = 1,10
TEST = (F3(J) -HPX)*( F3(J+1) - HPX)
IF(TEST .LT. 0.) GO TO 112
IF( F3(J) .EQ. HPX ) X = J-1

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237      IF ( F3(1) .EQ. HPX ) GO TO 56
238      111 CONTINUE
      PRINT*,44 = CANT FIND ROOT TO F3 IN THIS RANGE
      GO TO 55
239      112 XL = (J-1)*PI/20.
      XR = J*PI/20.
      DO 113 I = 1,21
      XM = (XR + XL)/2.
      F3MT = FF3(XM)
      IF (XL .EQ. 1.0) FXL3 = F3(1)
      IF (XL .EQ. 1.0) FXL3 = FF3(XL)
240      C      PRINT*,44 YN= ,YM, 14H FF3(YM)= ,FXM3, 10H FF3(XL)= ,FXL3
      TL = (F3MT - HPX)*(FXL3 - HPX)
      IF (TL .LT. 0.0) XR = XM
      IF (TL .GT. 0.0) XL = YM
      IF (ABS(XR - XL) .LE. ERRK) GO TO 114
241      113 CONTINUE
      PRINT*,44 F3 HALVING ROUTINE DID NOT PRODUCE ROOT
      PRINT*,44 XR= ,XR,5H XL= ,XL
      GO TO 51
242      114 I = (XR + XL)/2.
243      56 CONTINUE
      PRINT*,44 M= ,M
      IF ( M .EQ. 1.0 ) GO TO 55
      S3 = SIN(TAN(M))
      L3 = ST*0.5/400
      IF (L3 .LT. 0) GO TO 45
      K3 = SIN(L3*2 - M*2)
      PRINT*,44 M= ,M,5H L3= ,L3,5H K3= ,K3,5H S3= ,S3,6H XM3= ,
      GAMMA = -1/L3 - M
      CM = 1/M
      DL3 = L3/CM
      DK3 = K3/CM
      DS3 = ST*SICHAQ
      PRINT*,44 DM= ,DM,6H DL3= ,DL3,6H DK3= ,DK3,6H DS3= ,DS3
      T3 = 0.001/S3
      DT3 = 0.001/DS3
      PRINT*,44 DT3= ,DT3 ,5H T3= ,T3
      DO 25 I = 1,11
      Y = (I-1)*XM3/10.
      IF (1.EC(11)) Y = 0.999*XM3
      F7(I) = (4*SIN( SIN(GAMMA)*EXP(M*2*Y/L3)) - GAMMA) /M
      Y = Y/X
      PRINT*,44 Y= ,Y,5H HZ= ,HZ(1)
      IF (VEL .EQ. 0) GO TO 35
      DO 255 I7 = 1,6
      DZ = HZ(I)/5.
      Z = -(I7-1)*DZ
      YMD = Y/YM3
      YL7 = Y*L3
      IF (YL7 .GT. 10.0) FAC = 0.
      IF (XL3 .LT. 1.0) FAC = EXP(-XL3)/SIN(M)
      U(I,I7) = FAC*SIN(M*(1.+Z))
      V(I,I7) = (K3/L3)*FAC*SIN(M*(1.+Z))
      W(I,I7) = -(L/L3)*FAC*COS(M*(1.+Z))
      PRINT*,44 X,XMD,Z,U(I,I7),V(I,I7),W(I,I7)
244      255 CONTINUE

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35 CONTINUE
GO TO 55
45 PRINT*,L3,M,40H L3.LT.M
55 CONTINUE
290 ICOUNT = ICOUNT + 1
IF(ICOUNT .LT. 1006) GO TO 1
END

```

## SYMBOLIC REFERENCE MAP (R=1)

Y POINTS  
1 MAXWELL

| TABLES | SN     | TYPE    | RELOCATION |       |        |         |
|--------|--------|---------|------------|-------|--------|---------|
| 3      | ARG1   | REAL    |            | 3757  | ARG1   | REAL    |
| 27     | DK1    | REAL    |            | 3754  | DK1    | REAL    |
| 70     | DK3    | REAL    |            | 3726  | DL1    | REAL    |
| 33     | DL2    | REAL    |            | 3767  | DL3    | REAL    |
| 56     | QM     | REAL    |            | 3725  | DL1    | REAL    |
| 52     | DN2    | REAL    |            | 3731  | DS1    | REAL    |
| 51     | DS2    | REAL    |            | 3771  | DS3    | REAL    |
| 32     | DT1    | REAL    |            | 3755  | DT2    | REAL    |
| 73     | DT3    | REAL    |            | 3736  | DZ     | REAL    |
| 47     | EFAC   | REAL    |            | 3721  | ENPV   | REAL    |
| 00     | ERRN1  | REAL    |            | 3677  | ENRV   | REAL    |
| 42     | FAC    | REAL    |            | 3721  | FAL1   | REAL    |
| 45     | FAL2   | REAL    |            | 3767  | FAL3   | REAL    |
| 20     | FXM1   | REAL    |            | 3744  | FVM2   | REAL    |
| 62     | FXM3   | REAL    |            | 15337 | F1     | REAL    |
| 64     | F2     | REAL    | ARRAY      | 15411 | F7     | REAL    |
| 74     | G      | REAL    |            | 3765  | GAMMA  | REAL    |
| 05     | HM     | REAL    |            | 3714  | HCP    | REAL    |
| 10     | HPX    | REAL    |            | 3775  | H1     | REAL    |
| 142    | H2     | REAL    | ARRAY      | 4377  | H1     | REAL    |
| 717    | I      | INTEGER |            | 3712  | ICOUNT | INTEGER |
| 103    | IJOB   | INTEGER |            | 3676  | IUEL   | INTEGER |
| 135    | I2     | INTEGER |            | 3712  | J      | INTEGER |
| 363    | K1     | REAL    |            | 3565  | K2     | REAL    |
| 367    | K3     | REAL    |            | 3671  | L1     | REAL    |
| 372    | L2     | REAL    |            | 3673  | L3     | REAL    |
| 370    | M      | REAL    |            | 3664  | M1     | REAL    |
| 366    | N2     | REAL    |            | 3723  | OLON1  | REAL    |
| 746    | OLON2  | REAL    |            | 3675  | PI     | REAL    |
| 707    | SIGMA0 | REAL    |            | 3724  | S1     | REAL    |
| 759    | S2     | REAL    |            | 3764  | S3     | REAL    |
| 714    | TEST   | REAL    |            | 3722  | TL     | REAL    |
| 731    | T1     | REAL    |            | 3755  | T2     | REAL    |
| 772    | T3     | REAL    |            | 4454  | U      | REAL    |
| 675    | V      | REAL    | ARRAY      | 15116 | V      | REAL    |
| 713    | X      | REAL    |            | 3715  | XL     | REAL    |
| 741    | XL1    | REAL    |            | 3765  | XL2    | REAL    |
| 774    | XL3    | REAL    |            | 3716  | XM     | REAL    |
| 710    | XM1    | REAL    |            | 3743  | XM2    | REAL    |

## APPENDIX B: TWO DIMENSIONAL (SHALLOW WATER) EDOE WAVES

Consider the conditions under which a long wavelength free surface gravity wave may be trapped against a straight coastline by a topography which varies in a direction normal to the coastline. Taking  $x$  in this normal direction and  $y$  along the coastline, the linearized shallow water-long wavelength equations for a homogeneous fluid on a rotating earth are [e.g. LeBlond and Mysak (1978)]:

$$\begin{aligned} \partial u / \partial t + g \partial \eta / \partial x &= f v \\ (1) \quad \partial v / \partial t + g \partial \eta / \partial y &= -f u \\ \partial \eta / \partial t + H(\partial u / \partial x + \partial v / \partial y) + u \partial H / \partial x &= 0 \end{aligned}$$

where  $(u, v)$  are velocities,  $\eta$  is the free surface elevation,  $H$  is the water depth and  $f$  is the Coriolis parameter.

Assume solutions which propagate along the coastline, e.g.

$$\begin{aligned} u(x, y, t) &= U(x) \cos (ky - \sigma t) \\ v(x, y, t) &= V(x) \sin (ky - \sigma t) \\ \eta(x, y, t) &= E(x) \sin (ky - \sigma t) \end{aligned}$$

The original equations then reduce to

$$\begin{aligned} gE' + \sigma U - fV &= 0 \\ (2) \quad -\sigma V + gkE + fU &= 0 \\ -\sigma E + H(U' + kV) + uH' &= 0 \end{aligned}$$

Clearly  $U$  and  $V$  may be eliminated, leaving a single equation on  $E$ , e.g. LeBlond and Mysak (1978 - p. 220)

$$(3) \quad (HE')' + \left\{ \frac{\sigma^2 - f^2}{g} - k^2 H - \frac{fk}{\sigma} H' \right\} E = 0$$

This equation does not force a particular  $x$  dependence on either  $E$  or  $H$ , as was the case for the three dimensional equations, since here  $E$  and  $H$  are coupled while previously they were uncoupled. Instead there are many solutions. If for example,  $E$  is required to be exponentially decaying such as  $E_0 \exp(-lx)$ , equation (3) requires  $H(x)$  to be

$$H(x) = \frac{\sigma^2 - f^2}{g(k^2 - l^2)} + H_0 \exp \left[ - \left( \frac{l^2 - l^2}{l + fk/\sigma} \right) x \right]$$

where  $H_0$  is an arbitrary constant defined in general either by choosing  $H(0) = 0$  or  $U(0) = 0$  to have no net mass flux across the coastline,  $x = 0$ . If  $H(0)$  is chosen to be zero, the topography is given by

$$H(x) = \frac{\sigma^2 - f^2}{g(k^2 - l^2)} \left\{ 1 - \exp \left[ - \left( \frac{k^2 - l^2}{l + fk/\sigma} \right) x \right] \right\}$$

Certain ranges of parameters give negative topographies which must be excluded. Requiring that  $l > 0$ , but recognizing that  $k$  may be either positive or negative yields the following results:

For  $\sigma > f$ ,  $k > l$  gives a positive concave upwards topography, exponentially approaching a uniform depth,  $H_\infty$  equal to  $(\sigma^2 - f^2)/g(k^2 - l^2)$  ;  
 $-l < k < l$  gives a positive, convex upwards, exponentially increasing

depth,  $-\sigma l/f < k < -l$  gives a positive, concave upwards topography again exponentially asymptotic to  $H_\infty$  and finally  $k < -\sigma l/f$  yields a negative topography.

For  $\sigma < f$ ,  $k > l$  and  $-\sigma l/f < k < l$  both yield negative topographies, but  $-l < k < -\sigma l/f$  yields a positive, concave upwards topography again decaying exponentially to the asymptotic depth  $H_\infty$  while  $k < -l$  yields a positive, convex upwards exponentially increasing depth.

The choice of  $U(0)$  equal to zero requires either  $\sigma = -fk/l$ , leading to a constant depth,  $H = f^2/gl^2$ , or the trivial solution,  $E(0)$  equal to zero and thus  $E(x)$  equal to zero everywhere. This choice is then rejected for this particular form for  $E(x)$ .

In general, equation (3) may be considered to define  $E$  for a given  $H$  or alternatively  $H$  for a given  $E$ . While the first view is more realistic physically, the second is simpler mathematically, i.e.

$$\left\{ E' - \frac{fk}{\sigma} E \right\} H' + \left\{ E'' - k^2 E \right\} H = \left\{ \frac{f^2 - \sigma^2}{g} \right\} E$$

may be solved completely for  $H(x)$  for a given  $E$ .

$$\begin{aligned} H(x) = & C_0 \exp \left[ - \int \frac{E''(x) - k^2 E(x)}{E'(x) - \frac{fk}{\sigma} E(x)} dx \right] \\ & + \left\{ \frac{f^2 - \sigma^2}{g} \right\} \exp \left[ - \int \frac{E''(x) - k^2 E(x)}{E'(x) - \frac{fk}{\sigma} E(x)} dx \right] \\ & * \int \frac{E(x)}{E'(x) - \frac{fk}{\sigma} E(x)} \cdot \exp \left[ + \int \frac{E''(x) - k^2 E(x)}{E'(x) - \frac{fk}{\sigma} E(x)} dx \right] dx \end{aligned}$$

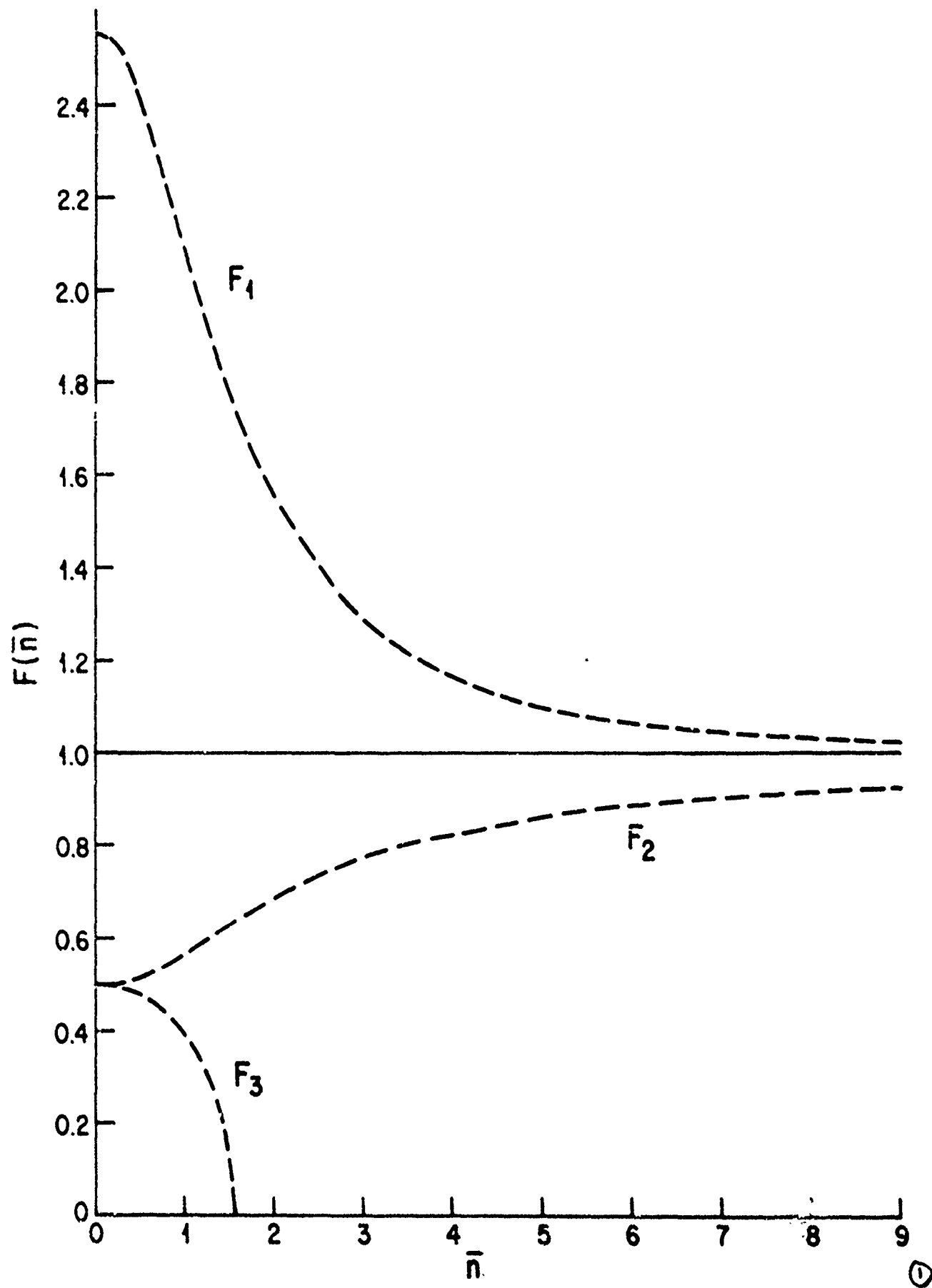
where  $C_0$  is an arbitrary constant to be chosen to have zero net mass flux across  $x = 0$ .

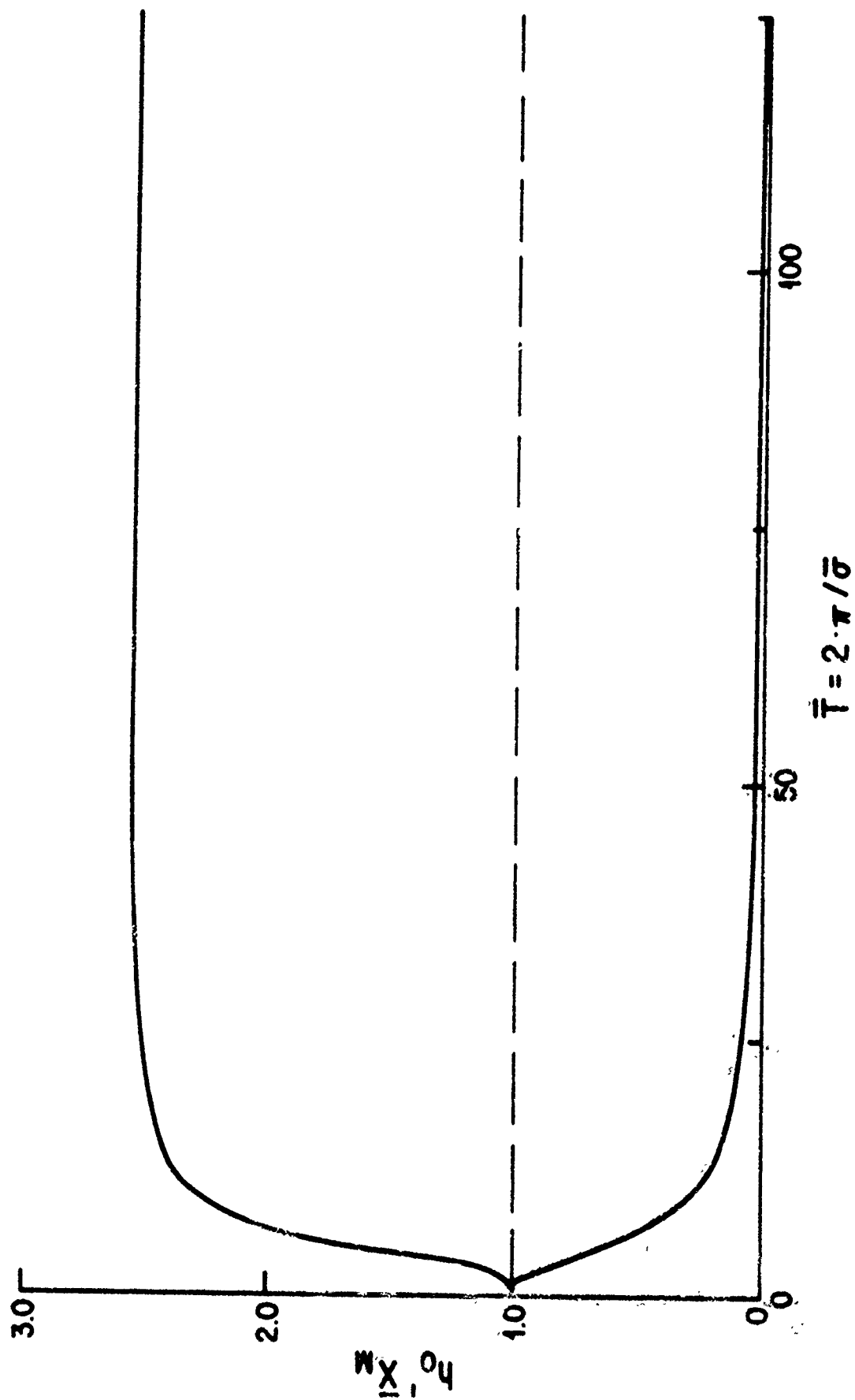
Although this format may seem inappropriate on physical grounds, it represents a "transformation" of  $E(x)$  into  $H(x)$ , i.e. trial functions for  $E$  lead directly to a functional form for  $H$  and in a sense to the two dimensional counterpart to the three dimensional case studied in the main text. The alternative view of specifying  $H$  and solving for  $E$  leads to a second order differential equation on  $E$  with variable coefficients some of which vanish when  $H$  is zero, i.e. represent singular solution points. A number of solutions in this format have been generated, e.g. Ball (1967) obtained a solution in terms of hypergeometric polynomials for an exponential depth variation of the form  $H_0(1-\exp(-ax))$ , Hidaka (1976-b) obtained a solution in terms of modified Mathieu functions for a parabolic depth variation of the form  $H_0(1 + (x^2/a^2))^{1/2}$ , Robinson (1964) used a finite width, linear shelf terminating at a discontinuity to a constant depth ocean with solutions in terms of Laguerre functions, Eckart (1951), Mysak (1968) have obtained solutions in terms of Kummer functions or Laguerre polynomials for a linear topography,  $H(x) = \gamma x$ , and Shaw (1977) has obtained solutions in terms of (both) Kummer functions for the case of a linear topography which did not necessarily break the surface,  $H(x) = H_0 + \gamma x$ . Other solutions could be generated for other topographies using the standard functions of mathematical physics, e.g. Bateman (1954).

In a sense, however, these solutions are all the same type; series solutions, using the method of Frobenius, to the original differential equations with the singular solution (if any) at the origin suppressed, if necessary.

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(2)

