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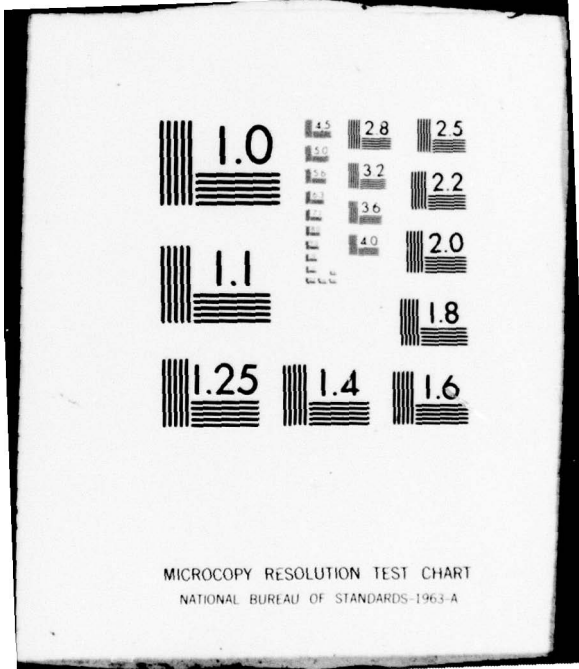
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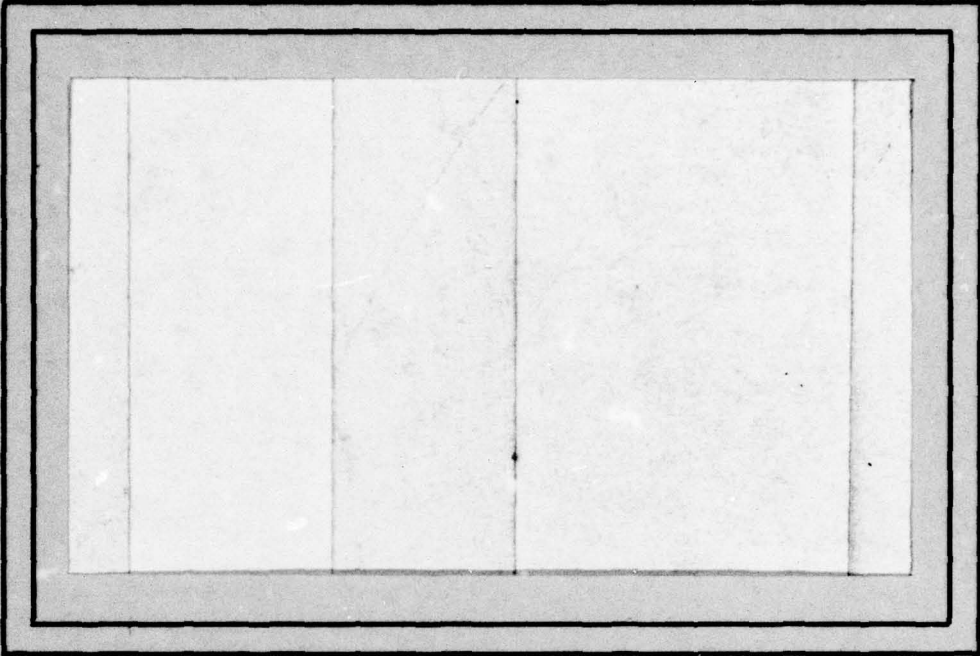
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SECOND-ORDER STATISTICS  
OF TEXTURE PRIMITIVES

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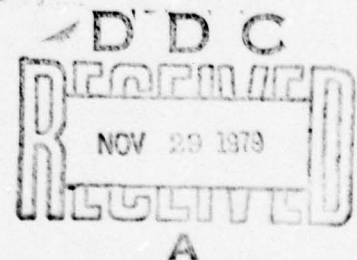
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ABSTRACT

Primitive "elements" were extracted from a set of textures; a set of attributes was measured for each primitive; and second-order statistics of these attributes were computed for pairs of neighboring primitives, using several definitions of "neighboring". In some cases, textures not discriminable using first-order statistics can be discriminated using statistics of the second order.

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1. Introduction

Textures can be considered as composed of primitives placed in a certain spatial arrangement, where each primitive is a connected region satisfying some specified properties. To describe a texture, one needs to describe both the primitives and the placement rules. In [1], attributes of the extracted primitives, such as area, compactness, etc., were measured to discriminate the textures. However, the spatial relationships between the primitives were not studied. This paper studies the spatial relationships between primitives using second-order statistics.

In [2], Haralick et al. computed texture features based on gray level cooccurrence matrices whose elements are relative frequencies  $f(i,j)$  which count the number of times two pixels, one with gray level  $i$  and the other with gray level  $j$ , are separated by a certain distance in a specified direction. Instead of gray level cooccurrence matrices, here we use primitive attribute cooccurrence matrices. An element  $f(i,j)$  of a primitive attribute cooccurrence matrix for a given attribute is the relative frequency with which two neighboring primitives (as defined by some specified relation), one having (quantized) attribute value  $i$  and the other  $j$ , occur in the image. From these matrices, second-order statistics including angular second moment (ASM), inverse difference moment (IDM), entropy

and contrast are computed. The attributes used are area, perimeter, compactness, eccentricity and direction. The sample textures used for the experiment are the same as those used in [1] and [3] so that results can be compared. They include four Brodatz textures [4]: wool, raffia, sand, and grass; and three LANDSAT geological terrain textures: Mississippian limestone and shale, Pennsylvanian sandstone and shale, and Lower Pennsylvanian shale.

## 2. Experiment

The steps of the experiment were as follows:

- (1) Extract primitives and compute their attributes.
- (2) For each primitive, find its neighbors.
- (3) Construct primitive attribute cooccurrence matrices for each attribute.
- (4) Compute the second-order statistics.

### 2.1 Primitive extraction and attribute computation

Three different methods were used to extract primitives. Two of the methods, namely, 25th percentile thresholding and Superslice, were used in [1]. These methods are thresholding schemes where primitives are connected sets of pixels having gray level values above some threshold. The third method is an edge-based technique [3] where primitives are areas enclosed by edges. For each of these methods, the output is a binary picture. As in [1], the very small regions (having less than ten pixels), the very large regions, and the regions touching the window boundaries are ignored.

For each primitive, the centroid is found. In addition, five attributes are calculated. They are area, perimeter, compactness, eccentricity, and direction. Their definitions and computational formulas can be found in [1].

## 2.2 Neighbor selection

Given two primitives with centroids  $(x_i, y_i)$ ,  $(x_j, y_j)$  and areas  $A_i, A_j$ , their separation distance is approximated by

$$d = \max\left\{\left((x_i - x_j)^2 + (y_i - y_j)^2\right)^{1/2} - \left(\frac{A_i}{\pi}\right)^{1/2} - \left(\frac{A_j}{\pi}\right)^{1/2}, 0\right\}.$$

Based on this distance function, we used four schemes to select neighbors for the purpose of comparison.

**Method 1 (4-neighbor approach):** The four primitives nearest a primitive are its neighbors.

In the 4-neighbor approach to locating neighbors, only separation distance is considered. In the other three methods, the direction of the primitive, defined as the direction of the major axis of inertia, is also taken into consideration. For each primitive, if we rotate its major axis of inertia 45 degrees clockwise and counterclockwise about its centroid, we get two coordinate axes which divide the plane into four quadrants. As shown in Figure 1, the major axis lies in two of these quadrants and the minor axis lies in the other two.

**Method 2 (Major axis approach):** For each primitive, the nearest primitive in each of the quadrants containing the major axis is a neighbor.

**Method 3 (Minor axis approach):** For each primitive, the nearest primitive in each of the quadrants containing the minor axis is a neighbor.

**Method 4 (All-direction approach):** For each primitive, the nearest primitive in each of the four quadrants is a neighbor.

If the nearest primitive in a quadrant is very far away from the center primitive, then it should not be considered as a neighbor. Hence we use the following procedure to find the neighbors: First we find the six nearest primitives without considering directions, then pick the nearest ones in the appropriate quadrants from among these six. Note that some of the quadrants may not contain any of the six nearest primitives. However, this indicates that the blank quadrant is either facing the picture border or the primitives in that direction are too far away to be considered as neighbors.

### 2.3 Second-order features

For each of the attributes, the values are divided into  $N = 8$  disjoint intervals. Each attribute has its own value partition. The same partitions were used for all the textures. The size of the cooccurrence matrices is  $N \times N$ . Initially all entries of the matrix are zero. In constructing the matrix  $P$  for an attribute, say area, we look at each primitive and its neighbors. If the area of the primitive is an interval  $i$  and the neighbor's area is an interval  $j$ , then we add 1 to  $P(i, j)$ , the  $(i, j)$ th entry of the area cooccurrence matrix. Finally, the matrix is normalized by dividing each entry of  $P$  by  $\sum_{i=1}^N \sum_{j=1}^N P(i, j)$ .

The second-order textural statistics are computed using the following formulas:

- (1) Angular second moment (ASM):

$$\sum_{i=1}^N \sum_{j=1}^N \{P(i, j)\}^2$$

- (2) Entropy:

$$-\sum_{i=1}^N \sum_{j=1}^N P(i, j) \log (P(i, j))$$

- (3) Inverse difference moment (IDM):

$$\sum_{i=1}^N \sum_{j=1}^N \frac{P(i, j)}{1+(i-j)^2}$$

- (4) Contrast:

$$\sum_{k=1}^{N-1} k^2 \left\{ \sum_{\substack{i=1 \\ |i-j|=k}}^N \sum_{j=1}^N P(i, j) \right\}$$

### 3. Second-order textural statistics

ASM shows how consistent and homogeneous the primitive attribute is, while entropy shows how inhomogeneous it is. As an example, consider the ASM and entropy computed from the direction cooccurrence matrix. If all the primitives in an image H have the same quantized direction, ASM has its highest value, while entropy has its lowest. Conversely, ASM has a low value and entropy a high value when the directions of neighboring primitives of an image R are scattered. Therefore ASM can discriminate the textures of H and R. However, it cannot separate the textures shown in Figure 2, while the standard deviation of the directions, a first-order statistic, will separate them.

In Figure 3(a), values of ASM and entropy for four windows of each of the four Brodatz textures are displayed. These values were computed using edge-based primitives and the "major axis" scheme for defining neighbors. Figure 3(b) and (c) show the unnormalized direction cooccurrence matrices of the two windows (one of raffia, the other of grass) that had the minimum and maximum ASM values. Since the primitives of raffia are oriented in two directions only, horizontal and vertical, there exist high value entries in the cooccurrence matrix of Figure 3(b), which contribute to the higher value of ASM. On the other hand, the direction distribution is random in grass

and the matrix of Figure 3(c) looks busier.

In the computation of IDM, entries closer to the main diagonal are given more weight. The opposite is true for contrast. If IDM and contrast are computed for the area cooccurrence matrix, clearly the image of Figure 4(a) will have higher IDM and lower contrast values than that of Figure 4(b).

In Figure 5(a), the values of IDM and contrast for the Brodatz textures are displayed. These values were computed from the compactness cooccurrence matrix, using edge-based primitives and all-direction neighbors. The IDM values decrease from wool to raffia to sand to grass. The unnormalized compactness cooccurrence matrices of selected windows of each texture are shown in Figure 5(b).

Maleson [5] defined two measures of the orientation of primitives. Two regions are said to be collinear if their major axes are similar, and parallel if they are lined up along their minor axes. These measures were used in the discrimination between water, which has more parallel regions, and straw, which has more collinear regions. The IDM values computed from the direction cooccurrence matrix using the "major axis" and "minor axis" approaches for neighbor selection also measure the collinear and parallel relationships. Water will have high IDM value for the minor axis matrix and low

value for the major axis matrix. On the other hand, straw will have low IDM for the minor axis matrix and high IDM for the major axis matrix. For the four Brodatz textures we used, the discrimination ability of IDM in direction cooccurrence is very poor. This indicates that these four textures are not very directional in that the primitives are neither collinear nor parallel.

#### 4. Discrimination

##### 4.1 Brodatz textures

The results of experiments on Brodatz textures are summarized in Table 1. The details about which pairs of textures can be separated are shown in Tables 2, 3, and 4.

##### 4.1.1 Comparison of primitive extraction methods

The edge-based primitive extraction approach is obviously superior to the threshold-based approach in the second-order statistics, even though for compactness, eccentricity and direction, the three schemes give comparable results in the first-order statistics. Figure 6 gives us a good indication as to the explanation of this. Figure 6 shows the original image of grass and the primitives extracted using the three approaches. Clearly, the density of the edge-based primitives is much higher. This shows that many of the neighbors of the primitives are discarded in the threshold approach. Since neighbor information is more important in the computation of second-order textural statistics than in that of first-order statistics, the effect is more noticeable.

Note that the area and perimeter results using the threshold approaches are very poor. This is due to the fact that we get very different area or perimeter cooccurrence matrices from different windows of the same texture. Figure 7 shows the area cooccurrence matrices for two windows of wool using the 25th percentile approach.

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#### 4.1.2 Comparison of neighbor selection methods

Table 1 indicates that there is no significant difference in the results using the four different neighbor selection schemes.

#### 4.1.3 Comparison of first- and second-order statistics

For area and perimeter, the performance of first- and second-order statistics is about the same. For compactness, eccentricity and direction, the performance of the second-order statistics is better. In almost all cases, the texture pairs separable by first-order statistics are also separable by second-order statistics. However, there are a number of pairs not separable by first-order statistics but which can be separated by second-order statistics. For example, the first-order statistics of compactness separate only wool from the others. Almost all pairs can be separated by IDM and entropy (see Figure 5 and Table 1).

#### 4.1.4 Comparison of degrees of quantization

For the edge-based, 4-neighbor approach, we also experimented with using cooccurrence matrices of different sizes (4x4, 8x8, and 16x16). Table 5 indicates that the effect of these three matrix sizes is minimal.

IDM and contrast were also computed using unquantized attribute values. This is the same as if the cooccurrence matrix is  $\infty \times \infty$ . The results are also displayed in Table 5.

ASM and entropy were not computed using unquantized values since entropy will almost always have value zero and ASM will have a value equal to a constant multiple of  $1/n$  where  $n$  is the number of primitives in the texture.

#### 4.2 Terrain textures

The results of the experiments on the terrain textures are summarized in Table 6. The details about which pairs of textures can be separated are shown in Tables 7, 8, and 9.

The edge-based primitive extraction method is still consistently superior to the threshold approach. The second-order statistics for area, compactness, eccentricity and direction are not better than the first-order statistics. However for the perimeter feature, there is a slight improvement of the second-order statistics over the first-order statistics.

## 5. Concluding remarks

Visual discrimination studies [6] show that second-order statistics are important in the description and discrimination of textures. In particular, our study shows that the second-order statistics computed on compactness, eccentricity and direction can distinguish many texture pairs not distinguishable by the first-order statistics (mean and variance) of the same features. However, the overall performance of second-order statistics is not much better than that of first-order statistics. It appears that for the particular textures that were used, the characteristics necessary for good discriminability by second-order statistics, as discussed in Section 3, are not strongly present.

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PRIMITIVES	APPROACH	FEATURE	AREA	PERIMETER	COMPACTNESS	ECCENTRICITY	DIRECTION
25%	FIRST ORDER	- (1)	1 (1)	3 (2)	4 (3)	- (1)	
	4 NEIGHBOR	- - - -	- - - -	2 3 2 2	2 3 - 1	3 3 2 2	
	MAJOR AXIS	- - - -	- - - -	1 3 2 1	3 4 - -	2 2 - 1	
	MINOR AXIS	- - - 2	- - - 2	2 3 2 1	2 3 - 1	- - - 2	
	ALL DIRECTION	- - - -	- - - -	1 3 2 3	2 3 - -	1 3 - 2	
SUPERSLICE	FIRST ORDER	1 (1)	1 (3)	3 (1)	3 (1)	(-)	
	4 NEIGHBOR	- - - -	- - - -	4 3 3 1	3 3 1 -	1 1 1 -	
	MAJOR AXIS	- - - -	- - - -	3 3 2 2	3 3 1 -	- - 1 -	
	MINOR AXIS	- - - -	2 1 - -	4 4 3 1	1 1 2 -	- - 1 -	
	ALL DIRECTION	- - - -	- - - -	4 4 3 1	3 2 1 -	1 1 1 -	
EDGE-BASED	FIRST ORDER	6 (3)	4 (3)	3 (3)	4 (-)	(-)	
	4 NEIGHBOR	5 5 4 3	3 3 3 3	3 5 5 3	4 4 4 -	3 2 1 -	
	MAJOR AXIS	5 5 4 3	3 3 3 3	4 5 5 3	5 4 3 -	4 4 - -	
	MINOR AXIS	5 5 3 3	3 3 3 3	2 4 2 3	2 2 2 1	4 4 - 3	
	ALL DIRECTION	5 5 4 3	3 3 3 3	3 5 5 3	4 4 2 1	3 3 - -	

Table 1. Effectiveness of features in discrimination of pairs of Brodatz textures. The numbers are numbers of separable pairs (at most 6). The format for the first order approach is mean (standard derivation). The format for the second order approach is ASM, entropy, IDM, and contrast.

FEATURE	AREA	PERIMETER	COMPACTNESS	ECCENTRICITY	DIRECTION
TEXTURE PAIRS	W W R R S	W W R R S	W W R R S	W W R R S	W W R R S
APPROACH	/ / / /	/ / / /	/ / / /	/ / / /	/ / / /
	R S G S G G	R S G S G G	R S G S G G	R S G S G G	R S G S G G
MEAN	Y	Y	Y	Y	Y
FIRST ORDER	ST. DEV.	Y	Y	Y	Y
SECOND ORDER	ASM	Y	Y	Y	Y
	ENTROPY	Y	Y	Y	Y
	IDM	Y	Y	Y	Y
	CONTRAST	Y	Y	Y	Y
SECOND ORDER	ASM	Y	Y	Y	Y
	ENTROPY	Y	Y	Y	Y
	IDM	Y	Y	Y	Y
	CONTRAST	Y	Y	Y	Y
SECOND ORDER	ASM	Y	Y	Y	Y
	ENTROPY	Y	Y	Y	Y
	IDM	Y	Y	Y	Y
	CONTRAST	Y	Y	Y	Y
ALL DIRECTION	ASM	Y	Y	Y	Y
	ENTROPY	Y	Y	Y	Y
	IDM	Y	Y	Y	Y
	CONTRAST	Y	Y	Y	Y

Table 2. Effectiveness of the features for the Brodatz textures, when the primitives are extracted by 25th percentile thresholding. "Y" means separable.

FEATURE	AREA	PERIMETER	COMPACTNESS	ECCENTRICITY	DIRECTION
TEXTURE PAIRS	W W R R S / / / / /	W W R R S / / / / /	W W R R S / / / / /	W W R R S / / / / /	W W R R S / / / / /
APPROACH	R S G S G G	R S G S G G	R S G S G G	R S G S G G	R S G S G G
MEAN	Y	Y	Y Y	Y Y	Y
FIRST ORDER	Y	Y Y	Y	Y	Y
ST. DEV.	Y	Y Y	Y	Y	Y
SECOND ORDER					
ASM			Y Y Y Y	Y Y	Y Y
ENTROPY			Y Y Y	Y Y	Y Y
IDM			Y Y Y	Y	Y
CONTRAST			Y		
4 NEIGHBOR					
SECOND ORDER					
ASM			Y Y Y	Y	Y
ENTROPY			Y Y Y	Y	Y
IDM			Y	Y	Y
CONTRAST			Y Y		
MAJOR AXIS					
SECOND ORDER					
ASM		Y Y	Y Y Y	Y	
ENTROPY		Y	Y Y Y	Y	
IDM			Y Y Y	Y	
CONTRAST			Y	Y	Y
MINOR AXIS					
SECOND ORDER					
ASM			Y Y Y Y	Y	Y
ENTROPY			Y Y Y Y	Y	Y
IDM			Y Y Y	Y	Y
CONTRAST			Y Y Y	Y	Y
ALL DIRECTION			Y		
CONTRAST			Y		

Table 3. Analogous to Table 2 for the Superslice primitives.

FEATURE	AREA	PERIMETER	COMPACTNESS	ECCENTRICITY	DIRECTION
TEXTURE PAIRS APPROACH	W W R R S	W W R R S	W W R R S	W W R R S	W W R R S
	/ / / / /	/ / / / /	/ / / / /	/ / / / /	/ / / / /
	R S G S G G	R S G S G G	R S G S G G	R S G S G G	R S G S G G
MEAN	Y Y Y Y Y	Y Y Y Y	Y Y Y	Y Y Y	Y
FIRST ORDER ST. DEV.	Y Y Y	Y Y Y	Y Y Y		
SECOND ORDER	ASM	Y Y Y	Y Y	Y Y Y	Y Y Y
	ENTROPY	Y Y Y	Y Y Y Y	Y Y Y	Y Y
	IDM	Y Y Y	Y Y Y Y Y	Y Y Y	Y
	CONTRAST	Y Y Y	Y Y Y		
4 NEIGHBOR	ASM	Y Y Y	Y Y Y Y	Y Y Y Y	Y Y Y Y
	ENTROPY	Y Y Y	Y Y Y Y	Y Y Y Y	Y Y Y Y
	IDM	Y Y Y	Y Y Y Y Y	Y Y	Y
	CONTRAST	Y Y Y	Y Y Y		
SECOND ORDER	ASM	Y Y Y	Y Y Y Y	Y Y Y Y	Y Y Y Y
	ENTROPY	Y Y Y	Y Y Y Y	Y Y Y Y	Y Y Y Y
	IDM	Y Y Y	Y Y Y Y Y	Y Y	Y
	CONTRAST	Y Y Y	Y Y Y		
MAJOR AXIS	ASM	Y Y Y	Y	Y	Y Y Y Y
	ENTROPY	Y Y Y	Y Y Y	Y Y	Y Y Y Y
	IDM	Y Y Y	Y Y Y Y Y	Y Y	Y
	CONTRAST	Y Y Y	Y Y Y		
SECOND ORDER	ASM	Y Y Y	Y	Y	Y Y Y Y
	ENTROPY	Y Y Y	Y Y Y	Y Y	Y Y Y Y
	IDM	Y Y Y	Y Y	Y	Y
	CONTRAST	Y Y Y	Y Y Y	Y	Y Y Y
MINOR AXIS	ASM	Y Y Y	Y Y Y	Y Y Y	Y Y Y
	ENTROPY	Y Y Y	Y Y Y	Y Y Y	Y Y Y
	IDM	Y Y Y	Y Y Y	Y Y	Y
	CONTRAST	Y Y Y	Y Y Y	Y	Y Y Y
SECOND ORDER	ASM	Y Y Y	Y Y Y	Y Y Y	Y Y Y
	ENTROPY	Y Y Y	Y Y Y	Y Y Y	Y Y Y
	IDM	Y Y Y	Y Y Y Y Y	Y Y	Y
	CONTRAST	Y Y Y	Y Y Y	Y	Y
ALL DIRECTION	ASM	Y Y Y	Y Y Y	Y Y Y	Y Y Y
	ENTROPY	Y Y Y	Y Y Y	Y Y Y	Y Y Y
	IDM	Y Y Y	Y Y Y Y Y	Y Y	Y
	CONTRAST	Y Y Y	Y Y Y	Y	Y

Table 4. Analogous to Table 2 for the edge-based primitives.

FEATURE	AREA	PERIMETER	COMPACTNESS	ECCENTRICITY	DIRECTION
MATRIX SIZE					
4x4	5 5 3 4	4 4 3 3	3 3 3 3	3 4 2 3	1 1 - -
8x8	5 5 3 3	3 3 3 3	3 5 3 3	4 4 - 4	3 2 - -
16x16	5 5 3 3	5 5 3 5	3 3 3 2	4 4 1 3	1 4 - -
unquantized	3 2	3 3	3 2	1 -	- -

Table 5. Comparison of results using different matrix sizes. The samples used are Brodatz textures. Primitives are selected by the edge-based approach. Neighbors are selected by the 4-neighbor approach. The data in each entry represent the number of separable pairs for ASM, entropy, contrast, and IDM respectively. For the unquantized matrices, only contrast and IDM are used.

PRIMITIVES	APPROACH	FEATURE	AREA	PERIMETER	COMPACTNESS	ECCENTRICITY	DIRECTION	
258	FIRST ORDER		- (-)	- (1)	2 (3)	- (-)	(2)	
	SECOND ORDER	4 NEIGHBOR	-	1 2 - 1	- - 1 -	- - - -	- 1 1 2	
		MAJOR AXIS	-	1 2 - 2	- - - 2	- - - -	- - - 1	
		MINOR AXIS	-	1 2 1 1	- - 1 -	1 1 1 1	1 1 - -	
		ALL DIRECTION	-	1 2 - 2	- - 1 -	- - 1 -	- - 2 2	
SUPERSLICE	FIRST ORDER		- (-)	- (-)	- (-)	1 (-)	(1)	
	SECOND ORDER	4 NEIGHBOR	-	2 - - -	2 2 2 -	1 - - -	- - - -	
		MAJOR AXIS	-	- - - -	- - - -	2 2 1 1	- - - -	- - 1
		MINOR AXIS	-	- - - -	2 2 - -	2 2 - -	- - - -	- - - -
		ALL DIRECTION	-	- - - -	- - - -	2 2 1 -	- - - -	- - - -
EDGE-BASED	FIRST ORDER		2 (3)	2 (2)	2 (2)	1 (-)	(2)	
	SECOND ORDER	4 NEIGHBOR	2 2 2 2	2 2 2 2	2 2 2 2	1 1 - -	- - - -	
		MAJOR AXIS	2 2 2 2	2 2 2 2	2 2 2 2	1 - - -	- - - -	
		MINOR AXIS	2 2 2 2	2 2 2 2	2 2 2 2	- - 1 -	- - - 2	
		ALL DIRECTION	2 2 2 2	2 2 2 2	2 2 2 2	1 1 1 -	- - - -	

Table 6. Analogous to Table 1 for the terrain textures. The maximum number of separable pairs is 3.

FEATURE	AREA		PERIMETER		COMPACTNESS		ECCENTRICITY		DIRECTION	
	M/P	M/L	M/P	M/L	M/P	M/L	M/P	M/L	M/P	M/L
TEXTURE PAIRS										
APPROACH										
MEAN										
FIRST ORDER										
ST. DEV.										
SECOND ORDER										
ASM										
ENTROPY										
IDM										
CONTRAST										
4 NEIGHBOR										
SECOND ORDER										
ASM										
ENTROPY										
IDM										
CONTRAST										
MAJOR AXIS										
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MINOR AXIS										
CONTRAST										
SECOND ORDER										
ASM										
ENTROPY										
IDM										
CONTRAST										
ALL DIRECTION										
CONTRAST										

Table 7. Effectiveness of the features for the terrain textures, when the primitives are extracted by 25th percentile thresholding. "Y" means separable.

FEATURE	AREA M/P / L / M / P / L	PERIMETER M/P / L / M / P / L	COMPACTNESS M/P / L / M / P / L	ECCENTRICITY M/P / L / M / P / L	DIRECTION M/P / L / M / P / L
TEXTURE PAIRS					
APPROACH					
FIRST ORDER					
MEAN				Y	
ST. DEV.					Y
SECOND ORDER					
ASM	Y	Y	Y	Y	
ENTROPY			Y		
IDM			Y		
CONTRAST					
4 NEIGHBOR					
CONTRAST					
SECOND ORDER					
ASM			Y		
ENTROPY			Y		
MAJOR AXIS					
IDM			Y		
CONTRAST			Y		Y
SECOND ORDER					
ASM	Y	Y	Y	Y	
ENTROPY	Y	Y	Y	Y	
MINOR AXIS					
CONTRAST					
SECOND ORDER					
ASM			Y		Y
ENTROPY			Y		Y
IDM					
CONTRAST					
ALL DIRECTION					
CONTRAST					

Table 8. Analogous to Table 6 for the superslice primitives.

FEATURE	AREA	PERIMETER	COMPACTNESS	ECCENTRICITY	DIRECTION		
TEXTURE PAIRS	M/P	M/L	M/P	M/L	M/P	M/L	P/L
APPROACH							
FIRST ORDER	Y	Y	Y	Y	Y	Y	Y
MEAN	Y	Y	Y	Y	Y	Y	Y
ST. DEV.	Y	Y	Y	Y	Y	Y	Y
SECOND ORDER	Y	Y	Y	Y	Y	Y	Y
ASM	Y	Y	Y	Y	Y	Y	Y
ENTROPY	Y	Y	Y	Y	Y	Y	Y
IDM	Y	Y	Y	Y	Y	Y	Y
CONTRAST	Y	Y	Y	Y	Y	Y	Y
4 NEIGHBOR	Y	Y	Y	Y	Y	Y	Y
SECOND ORDER	Y	Y	Y	Y	Y	Y	Y
ASM	Y	Y	Y	Y	Y	Y	Y
ENTROPY	Y	Y	Y	Y	Y	Y	Y
IDM	Y	Y	Y	Y	Y	Y	Y
CONTRAST	Y	Y	Y	Y	Y	Y	Y
MAJOR AXIS	Y	Y	Y	Y	Y	Y	Y
CONTRAST	Y	Y	Y	Y	Y	Y	Y
SECOND ORDER	Y	Y	Y	Y	Y	Y	Y
ASM	Y	Y	Y	Y	Y	Y	Y
ENTROPY	Y	Y	Y	Y	Y	Y	Y
IDM	Y	Y	Y	Y	Y	Y	Y
CONTRAST	Y	Y	Y	Y	Y	Y	Y
MINOR AXIS	Y	Y	Y	Y	Y	Y	Y
CONTRAST	Y	Y	Y	Y	Y	Y	Y
SECOND ORDER	Y	Y	Y	Y	Y	Y	Y
ASM	Y	Y	Y	Y	Y	Y	Y
ENTROPY	Y	Y	Y	Y	Y	Y	Y
IDM	Y	Y	Y	Y	Y	Y	Y
CONTRAST	Y	Y	Y	Y	Y	Y	Y
ALL DIRECTION	Y	Y	Y	Y	Y	Y	Y
CONTRAST	Y	Y	Y	Y	Y	Y	Y

Table 9. Analogous to Table 6 for the edge-based primitives.

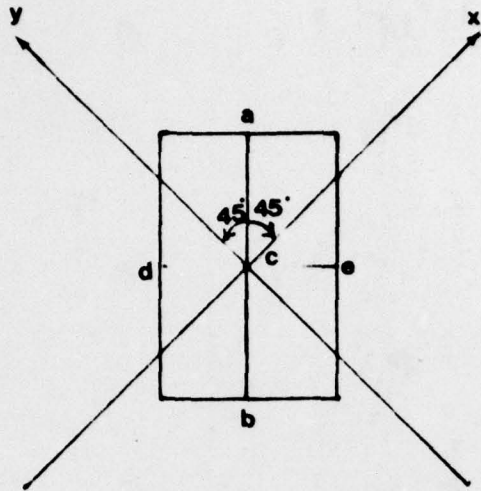


Figure 1. Quadrants of a primitive.  $\overline{ab}$  is the major axis of inertia, and  $c$  is the centroid.

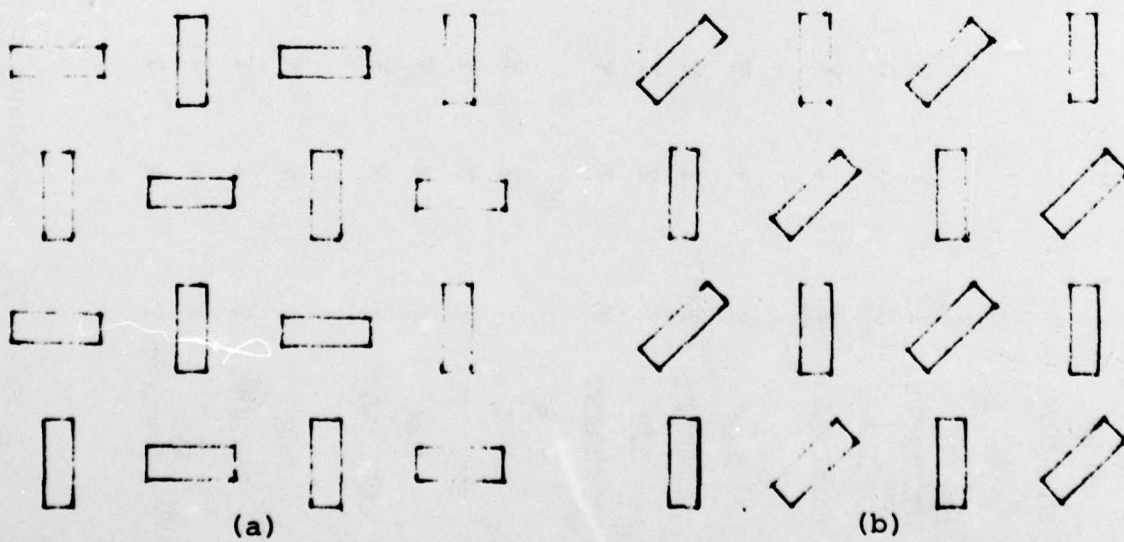
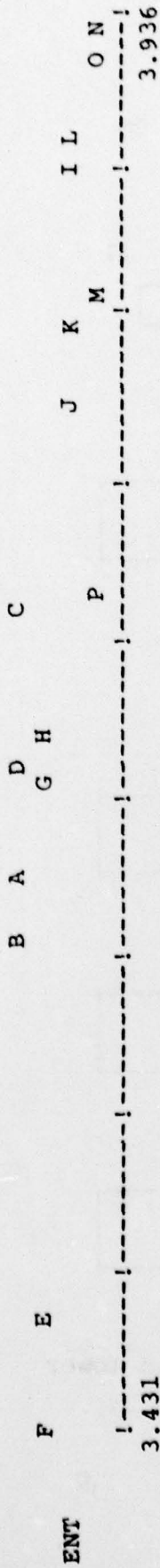
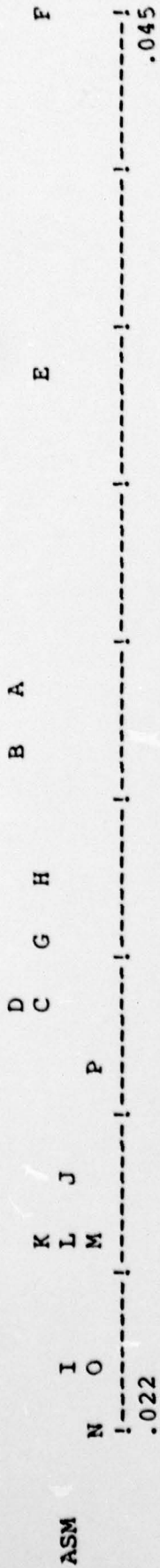


Figure 2. Example of two textures which cannot be separated by ASM or entropy of direction.



(a)

14.	6.	1.	14.	9.	7.	3.	3.
3.	2.	1.	6.	2.	1.	3.	3.
3.	0.	0.	1.	1.	0.	0.	0.
15.	1.	3.	1.	3.	2.	0.	0.
8.	3.	0.	2.	4.	3.	1.	1.
8.	2.	0.	2.	1.	0.	0.	1.
2.	0.	0.	1.	2.	0.	0.	1.
1.	1.	1.	4.	3.	0.	0.	0.

(b)

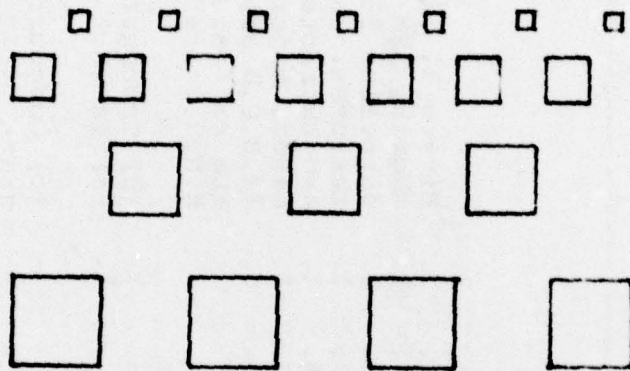
0.	4.	1.	1.	6.	1.	3.	1.
1.	2.	2.	3.	2.	2.	3.	1.
3.	3.	6.	4.	7.	1.	2.	1.
2.	3.	4.	6.	7.	4.	3.	1.
7.	4.	10.	5.	2.	7.	4.	6.
2.	1.	3.	3.	6.	2.	5.	2.
1.	2.	6.	4.	2.	4.	3.	3.
4.	0.	0.	4.	1.	2.	3.	2.

(c)

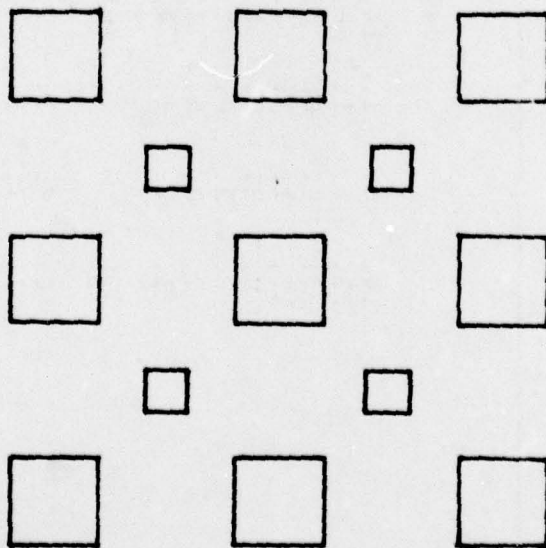
Figure 3. (a) Scatter plots showing ASM and entropy of direction for the Brodatz textures, when the primitives are extracted by the edge-based, major axis approach. (A,B,C,D are wool; E,F,G,H are raffia; I,J,K,L are sand; M,N,O,P are grass.)

(b) Cooccurrence matrix for raffia (F.)

(c) Cooccurrence matrix for grass (N.)



(a)



(b)

Figure 4. Texture (a) has higher IDM and lower contrast than texture (b).

IDM	P	N	M	K	L	I	J	O	E	F	G	H	B	D	C	A
	!	!	!	!	!	!	!	!	!	!	!	!	!	!	!	!
	.463															.708
CONT	A	C	D	B	F	H	J	O	E	KL	G	I	N	P	M	
	!	!	!	!	!	!	!	!	!	!	!	!	!	!	!	
	1.024														7.036	

Figure 5. (a) Scatter plots showing IDM and contrast of compactness for the Brodatz textures, when the primitives are extracted by the edge-based, all-direction approach.

15.	31.	8.	3.	0.	0.	0.	0.	0.	0.	10.	3.	0.	1.	0.	0.	0.
27.	86.	11.	3.	0.	0.	0.	0.	8.	87.	39.	39.	6.	5.	5.	4.	10.
7.	5.	0.	2.	0.	0.	0.	0.	1.	31.	20.	20.	6.	1.	0.	2.	1.
3.	2.	2.	2.	0.	0.	0.	0.	0.	6.	6.	6.	1.	0.	1.	0.	2.
0.	0.	0.	0.	0.	0.	0.	0.	1.	4.	2.	2.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	4.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	5.	1.	1.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	7.	1.	1.	1.	0.	0.	0.	0.

Wool (A)

Raffia (G)

2.	22.	11.	1.	0.	3.	0.	1.	5.	29.	13.	7.	3.	4.	0.	3.
15.	91.	45.	7.	4.	10.	10.	5.	25.	80.	34.	26.	12.	8.	4.	7.
5.	40.	16.	2.	2.	3.	2.	1.	8.	27.	12.	6.	4.	3.	2.	3.
1.	5.	2.	0.	0.	0.	0.	1.	6.	19.	6.	4.	1.	1.	0.	1.
0.	3.	1.	0.	0.	0.	0.	0.	2.	10.	3.	2.	0.	2.	2.	0.
2.	5.	3.	0.	0.	0.	0.	0.	2.	5.	4.	1.	2.	0.	0.	0.
0.	9.	1.	0.	0.	0.	0.	0.	1.	4.	1.	0.	1.	0.	0.	0.
0.	5.	1.	1.	0.	0.	0.	0.	1.	4.	0.	1.	0.	0.	0.	0.

Sand (I)

Grass (N)

Figure 5. (b) Cooccurrence matrices for selected windows.



Figure 6. (a) Original image of grass.  
 (b) Primitives extracted by 25th percentile thresholding.  
 (c) Primitives extracted by Superslice.  
 (d) Primitives extracted by the edge-based approach.

15.	2.	9.	5.	2.	0.	3.	0.
2.	1.	1.	0.	0.	0.	1.	0.
4.	1.	0.	1.	1.	0.	2.	0.
5.	0.	1.	0.	1.	0.	0.	0.
3.	0.	1.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
2.	0.	1.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
31.	19.	4.	5.	0.	0.	0.	0.
17.	1.	3.	0.	0.	0.	0.	0.
6.	2.	0.	0.	0.	0.	0.	0.
4.	1.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

Figure 7. Cooccurrence matrices for two different windows of wool.

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